

Portfolio Performance Gauging in Discrete Time Using a Luenberger Productivity Indicator*

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Abstract

This paper proposes a pragmatic, discrete time indicator to gauge the performance of portfolios over time. Integrating the shortage function (Luenberger, 1995) into a Luenberger portfolio productivity indicator (Chambers, 2002), this study estimates the changes in the relative positions of portfolios with respect to the traditional Markowitz mean-variance efficient frontier, as well as the eventual shifts of this frontier over time. Based on the analysis of local changes relative to these mean-variance and higher moment (in casu, mean-variance-skewness) frontiers, this methodology allows to neatly separate between on the one hand performance changes due to portfolio strategies and on the other hand performance changes due to the market evolution. This methodology is empirically illustrated using a mimicking portfolio approach (Fama and French 1996; 1997) using US monthly data from January 1931 to August 2007.

Keywords: shortage function, mean-variance, mean-variance-skewness, efficient portfolios, Luenberger portfolio productivity indicator

1 Introduction

Since Markowitz (1952) foundational work, every investor knows that to gauge the performance of portfolio management risk must be considered in addition to return levels. This dual objective of maximizing returns and minimizing risks turns performance evaluation into a complicated and controversial task. Indeed, no method that is currently available in the literature seems to be universally approved. There is an ever growing literature on this topic in traditional investment contexts (for surveys, see Cuthbertson, Nitzsche, and O’Sullivan (2008) or Le Sourd (2007)), as well as in the specific context of hedge fund management (for instance, Eling and Schuhmacher (2007)), and even a meta-literature criticizing these methods as well (see, for example, Amenc and Le Sourd (2005)).

Performance appraisal is linked to the theory of optimal investment choices, i.e., to the ability of investors to manage assets so as to maximize a utility function (i.e., a function based on a set of various moments characterizing the portfolios’ return distributions). In other words, performance evaluation analyzes the efficiency of an investment at least in terms of a traditional return-risk relationship. It is often assumed in this context that all investors have similar

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behaviors towards these dimensions (representative agent paradigm). The risk characteristics in the utility function depend upon various parameters like investor's objectives, preferences, time horizon,... These simplifications are acceptable in cases where aggregate results suffice, but these are simply problematic in other cases. The methodology proposed in this paper allows for heterogeneity among investors and therefore answers quite a few of these issues.

We explicitly restrict this contribution to the traditional mean-variance (MV) model and a more recently introduced mean-variance-skewness (MVS) framework (see Briec, Kerstens, and Jokung (2007)) for performance evaluation, ignoring any further higher moments. On the one hand, while the MV approach is still a popular reference for practitioners and academics alike, its restrictive nature may lead to erroneous weights in portfolio selection. While some proposals are around allowing investors to maximize a utility function including higher moments (see, for example, Chunchinda, Dandapani, Hamid, and Prakash (1997) or Jondeau and Rockinger (2003)), the empirical evidence provides mixed support at best. Nevertheless, enlarging the classical framework with a MVS model is a potentially interesting improvement for fund managers. On the other hand, the method developed in this research can be easily extended to consider higher moments.

Recently, a new approach has been proposed in the investment literature by Cantaluppi and Hug (2000) that directly measures the performance of a portfolio by reference to its maximum potential on the (ex-ante or ex-post) portfolio frontier. Their proposal is in fact intimately related to some explicit efficiency measures transposed from production theory into the context of portfolio benchmarking by Morey and Morey (1999) in the operations research literature. Informally speaking, their first measure computes the maximum mean return expansion while the risk is fixed at its current level, while an alternative risk contraction function measures the maximum proportionate reduction of risk while fixing the mean return level.¹

These explicit efficiency approaches are generalized by Briec, Kerstens, and Lesourd (2004) who integrate the shortage function (Luenberger (1995)) as an efficiency measure into the MV model and also develop a dual framework to assess the degree of satisfaction of investors preferences. Similar to developments in other fields, this leads to a decomposition of portfolio performance into allocative and portfolio efficiency. The advantage is that this shortage function is compatible with general investor preferences and that it can be extended to higher dimensional spaces (e.g., MVS space: see Briec, Kerstens, and Jokung (2007)).

This paper tackles the problem of tracing the performance of portfolios in discrete time with respect to the ever changing portfolio frontiers by borrowing from recent developments in the theory of productivity indices (see Diewert (2005) for a review). Employing the shortage function, a Luenberger portfolio productivity indicator (Chambers (2002)) is introduced that allows for the estimation of the relative positions of portfolios with respect to changes in the efficient frontier, and that offers an accurate local measure of the eventual shifts of this frontier over time. The proposed methodology for fund performance appraisal in discrete time is therefore founded in a well-established theoretical framework. We show later on that the Luenberger portfolio productivity indicator and especially its decomposition provide an excellent measurement tool to reconsider the traditional performance attribution question: what is the individual contribution of fund managers to portfolio performance and what is due to changes in the financial market. To the best of our knowledge, this contribution is the first to integrate recent developments in index theory into the portfolio performance evaluation framework.

By positioning ourselves into an extended Markowitz-like approach, we do not impose the much stronger assumptions usually maintained in the CAPM context. While the advantage of using a frontier as a benchmarking tool may be obvious, one should be aware of the fundamental relative nature of this frontier with respect to the selected asset universe.² Thus, we do not

¹Cantaluppi and Hug (2000) talk similarly about return loss and surplus risk.

²Obviously, all empirical work within a CAPM framework refers as a matter of fact to geographically limited parts of a potentially universal financial market.

claim our method is a new test of the efficiency of a given portfolio relative to an equilibrium theory of financial markets as proposed in the more traditional literature (e.g., Gibbons, Ross, and Shanken (1989)). We rather propose a method to identify ex-ante or ex-post improvements that can be attributed to funds managers when they optimize their positions relative to a limited asset universe. Indeed, one should notice that the qualification of efficiency is conditioned on its timing. Ex-post efficiency refers to an appraisal of performance once returns (consequently, all moments) are known, while ex-ante efficiency refers to a similar task based on expected returns. Obviously, prospective benchmarking is surrounded with a multitude of problems related to the fundamental uncertainties in the data requiring special attention in terms of statistical inference on the eventual efficiency status of ex-ante decisions regarding the ex-post results (see Markowitz (1952)).

The next section is devoted to a brief presentation of the relevant literature concerning portfolio performance evaluation and the more recently introduced efficiency measures operating relative to the portfolio frontier. Section 3 introduces the basic theoretical building blocks for the analysis. In particular, it introduces the shortage function as proposed by Luenberger (1992) and studies its axiomatic properties. Thereafter, the Luenberger portfolio productivity indicator and its decomposition are presented. Section 4 presents some technical and strategic aspects of the empirical procedures and discusses the choice of data set. Empirical results are provided in Section 5. Conclusions and issues for future work are summarized in the final section.

2 Performance Measurement in Investment: A Brief Review

2.1 Traditional Performance Measures

An enormous literature on portfolio performance evaluation derives more or less directly from the initial work of Markowitz (1952) and the founders of Modern Portfolio Theory with the development of asset pricing theories (e.g., the CAPM). During these early years, performance appraisal evolved from total-risk foundations (e.g., the standard deviation or variance of returns) to performance indexes where the returns in excess of the risk-free rate are matched with some risk measure. Among these early contributions, two classics are on the one hand the Sharpe ratio and on the other hand the Treynor ratio, which gauge performance without any benchmark. More recently, these indicators have taken benefit from the development of value at risk (VaR) techniques (especially in the hedge fund industry context: see Gregoriou and Gueyie (2003)). Another popular performance indicator is Jensen's α , whereby performance is measured by the excess return over the equilibrium reward calculated with the CAPM. Since it does use a benchmark, it is more relative in nature.

This early tradition has received a wide variety of criticisms because of the supposed weaknesses of the underlying equilibrium models on which performance indicators were build and the implicit assumption that financial asset returns are normally, independently and identically distributed, among others.³

The first series of objections touches upon several issues. One is the irrelevance of unconditional performance evaluation: investors are supposed to form expectations about returns irrespective of their expectations over the states of the economy, which may lead to various distortions in performance levels or stability. It has meanwhile been acknowledged that agents use information to condition their expectations (see Fama and French (1989)), making unconditional evaluation techniques rather irrelevant. More generally, the question of the benchmark choice is also clearly central in portfolio performance gauging, especially when funds have different management styles. When the reference point is inappropriate, then the measure is biased (see Grinblatt and Titman (1994)). For instance, the evaluated portfolio might be over-rated

³See, for instance, the debate around CAPM in Fama and French (2004).

if the benchmark is inefficient (see Roll (1978)). Potential solutions consist in improving the model by which expected returns are calculated (using APT or multi-factor models, such as Fama and French (1992; 1993)), or to obtain performance evaluation independent from the market model (for instance, Cornell (1979) or Grinblatt and Titman (1993)). For example, the Fama and French (1993) three-factor model has been developed because the CAPM proved to perform poorly in explaining realized returns.⁴

Another series of problems with the underlying equilibrium models (recognized ever since Jensen (1972)) come from the non-stability of risk-free rates or the volatility of betas. In these cases, performance evaluation is clearly biased because equilibrium returns are misevaluated or simply because a constant beta is irrelevant. One answer is to allow for time-varying betas in equilibrium models (e.g., Shanken (1990) or Ferson and Schadt (1996)). Another answer to this issue and the corresponding misvaluation of Jensen's α has been proposed by Grinblatt and Titman (1989), among others.

Another source of problems is the non-Gaussian nature of stock returns due to dynamic trading strategies (for instance, hedge funds are especially concerned by this issue). Problematic here is the underestimation of risk in performance appraisal. With asymmetric distributions or fat tails, performance gauging must take into account higher order moments (skewness, kurtosis or even beyond: see Ang and Chua (1979)) or lower partial moments (e.g., the Sortino ratio is based on a target return and semi-variance). More recently, various other proposals have been formulated: some of these derive from VaR (Gregoriou and Gueyie (2003)), some are extensions of the Sharpe ratio (Madan and McPhail (2000)) or the Sortino ratio (Kaplan and Knowles (2004)). Others propose generalized methods such as the Omega measure (see Kazemi, Schneeweis, and Gupta (2004)). Finally, in relation to model specification issues in a non-normal world, Harvey and Siddique (2000) have proposed to incorporate a conditional skewness measure to take into account the necessary reward for systematic skewness in funds returns.

Many of these traditional performance measures are frequently associated with a prominent question in the investment industry, namely performance attribution. While it is in blatant contradiction with CAPM theory, performance appraisal is linked to *stock picking* and *market timing*. In other terms, the investment industry is always looking for tools to trace good fund managers that can regularly exploit market anomalies and that could pick stocks in the market to obtain an alpha that is significantly different from zero and manage their portfolios' betas dynamically.

Summing up, the standard approaches to investment performance appraisal may appear unsatisfactory with respect to at least three generic shortcomings: (i) these may yield under- or over-estimations because of the selection of an inappropriate benchmark or equilibrium models for expected returns, (ii) these may be biased due to the non-normal nature of return distributions or unknown utility functions for investors when higher moments have to be considered, and (iii) these may be unstable because of the dependency of the measure with the time-frame in which it is computed. One could also add that these measures usually rely upon other strong assumptions, such as the uniqueness of investor's preferences. We now turn to the rather recent frontier-based measures that may be a solution for some of these shortcomings.

2.2 Frontier-Based Efficiency Measures

Frontier-based measures of fund performance have gained some limited popularity since the late nineties. One of the seminal articles in the finance literature is the work of Cantaluppi

⁴In this model, R_i , the return of fund i , is explained by a combination of market risk factor in excess of the risk free rate and two additional factors, respectively *size risk* (measured as the difference of the returns between a portfolio composed of small firms and one composed of big firms) and *value risk* (measured as the difference between the returns of two portfolios, one composed of firms with high book-to-market ratios and one with low book-to-market ratios).

and Hug (2000) who propose an efficiency ratio in relation to the MV efficient frontier.⁵ In fact, their contribution is similar to the one of Morey and Morey (1999) in the operations research literature. In their search for a more universal approach to portfolio performance measurement, Cantaluppi and Hug (2000) contest the relative nature of most current proposals that define performance with respect to some other, supposedly relevant, portfolio or index. Instead, they suggest looking for the maximum performance that could have been achieved by a given portfolio relative to a relevant portfolio frontier, i.e., a frontier resulting from a particular choice of investment universe and satisfying any additional constraints imposed on the investor. Basically, it is a matter of utilizing the traditional ex-ante computation of optimal portfolios in an ex-post fashion. Ex-ante, one first selects the investment universe; then one determines the investment horizon with corresponding estimates for future returns, risks, and correlations for the asset universe; and finally one computes an efficient frontier based on these estimates and the investment restrictions. This same process can be executed ex-post to benchmark portfolios: computations are then simply performed with historical rather than expected values. Since a portfolio manager that ex-ante would have had perfect foresight could have invested in a frontier optimal portfolio, the ex-post efficient frontier provides a natural benchmark for performance gauging and Cantaluppi and Hug (2000) informally present both a return loss and a surplus risk efficiency measure.

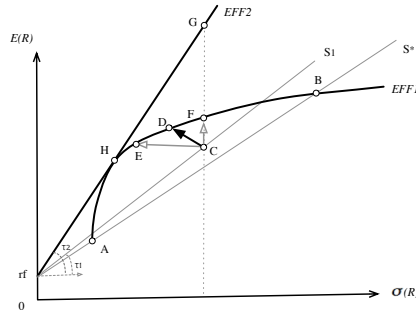


Figure 1: Sharpe Ratio vs. Efficiency Measures

We illustrate this basic point with Figure 1 (in the spirit of Cantaluppi and Hug (2000)) which compares the Sharpe ratio and the efficiency ratio. This figure is drawn in the mean-standard deviation space and depicts three portfolios A, B, and C with respect to a common portfolio frontier. Starting with the Sharpe ratio, it is clear that portfolio C enjoys a higher Sharpe ratio compared to portfolios A and B (i.e., the slope of the line S_1 being greater than the slope of S^*), despite the fact that the latter portfolios are part of the Markowitz frontier ($EFF1$) while portfolio C is not. To remedy this problem, the efficiency ratio approach suggests measuring the inefficiency of portfolio C using either a return loss efficiency measure (vertical projection line, towards point F), or a surplus risk efficiency measure (horizontal projection line, towards point E).

To contrast existing viewpoints, we explicitly position our contribution relative to a seminal article by Gibbons, Ross, and Shanken (1989) proposing a test of the efficiency of a given portfolio within a CAPM framework. Reconsidering Figure 1, when a risk-free asset is available, then the portfolio frontier is a straight line ($EFF2$) with a slope τ_2 , which is tangent to the so-called market portfolio at point E. Thus, τ_2 is the ex-post price of risk as measured within this sample. To evaluate the ex-ante efficiency of portfolio C, considering that it earns a risk price τ_1 (i.e., the slope of S_1), Gibbons, Ross, and Shanken (1989) propose a test statistic based on $\phi = (\sqrt{1 + \tau_2^2})/(\sqrt{1 + \tau_1^2})$ to measure portfolio performance. The bottom line is that the

⁵As stated by these authors, this is not strictly speaking a new method since it has been employed by, e.g., Kandel and Stambaugh (1995) as well.

slopes τ_1 and τ_2 have to be statistically different if one wants to reject the hypothesis of ex-ante efficiency for portfolio C , even if it is clearly situated both under $EFF1$ and $EFF2$. In other terms, ex-post efficiency can be used as an ex-ante efficiency proxy, but this raises serious statistical problems.

Notice that mathematical formulations in this contribution are expressed in terms of expected (forward looking) returns, while the empirical part uses historical returns for illustrative purposes. This raises the traditional ex-ante/ex-post performance appraisal issue. Two reasons justify this choice. First, in view of the efficient market hypothesis, one can view historical returns as a simplified (although weak, see for example, Elton (1999)) mechanism to generate expected return information. Another solution consists in obtaining such expected returns information from scenario analysis or from specialized firms (e.g., the I/B/E/S databases of Thomson Financial that reflect consensus estimates). In a similar vein, we maintain the hypothesis of historical volatility stability instead of using stochastic volatility models or implied volatility derived from option pricing models. This same logic also applies to the higher moment information employed in this research. Second, this ex-ante/ex-post problem is taken into account by mixing several shortage functions based on forward and backward returns (see page 9).

Morey and Morey (1999) are the first to give a precise formal definition of the return loss and surplus risk efficiency measures also proposed by Cantaluppi and Hug (2000).⁶ In the same vein, Briec, Kerstens, and Lesourd (2004) are the first to develop a link between portfolio efficiency measures and MV utility, which leads them to propose an efficiency measure that simultaneously seeks to improve the return and to reduce the variance of a given portfolio.⁷ In Figure 1, this leads -intuitively speaking- to the projection of portfolio C into a diagonal direction towards the Markowitz frontier. Theoretically, these contributions bring portfolio theory in line with developments in production theory and elsewhere in micro-economics, where distance functions as functional representations of choice sets are proven concepts related to efficiency measures that allow to develop dual relations with economic (e.g., MV utility) support functions.

More or less independently, a variety of authors have been transposing efficiency measures, that are related to distance functions from production theory into finance. This literature employs mathematical programming techniques to estimate non-parametric frontiers of choice sets and positions any observation with respect to the boundary of these choice sets.⁸ This has sometimes been accompanied with the utilization of frontiers to rate, for instance, the performance of mutual funds along a multitude of dimensions (rather than mean and variance solely). The -to the best of our knowledge- seminal article of Murthi, Choi, and Desai (1997) employs return as a desirable output to be increased and risk and a series of transaction costs as an input to be reduced, and measure the performance of each mutual fund with respect to a piecewise linear frontier (rather than a traditional non-linear portfolio frontier). More recently, Choi and Murthi (2001) employ a similar framework and compare the resulting efficiency measures to the traditional Sharpe ratio. The same idea has been employed in the context of asset selection, whereby changes in stock performance are related to changes in productive efficiency (see the seminal article of Chu and Lim (1998)). Preliminary results suggest that changes in productive efficiency are at least partially translated into changes in stock prices (see also Edirisinghe and Zhang (2007) for a recent development).

Therefore, it is possible to state that frontier-based portfolio benchmarking methods at least partially remedy some of the generic shortcomings of traditional performance measures mentioned earlier: (i) these select an appropriate benchmark in terms of the ex-post portfolio frontier, and (ii) these can be perfectly generalized to higher moments in case of non-normal

⁶Even though Morey and Morey (1999) and Cantaluppi and Hug (2000) seem to be unaware of one another. Sengupta (1989) is to our knowledge the first author to transpose the idea of a return loss efficiency measure into a MV frontier context.

⁷A generalization of the same approach into MVS space is developed in Briec, Kerstens, and Jokung (2007).

⁸This approach is often referred to with the moniker Data Envelopment Analysis (DEA).

return distributions. It remains to be seen how these behave under extensive stress testing. This contribution aims to remedy to some extent the third defect mentioned in the previous subsection, i.e., the instability of performance measures because of the dependency of these measures with respect to the time-frame in which these are computed. We resolve this at least partially by defining a portfolio productivity indicator based upon general efficiency measures that allows tracking the evolution in financial markets in discrete time. This is -to the best of our knowledge- the first contribution drawing upon index theory to resolve practical portfolio benchmarking issues.

3 Static Portfolio Frontiers and Their Evolution in Discrete Time

3.1 Static Portfolio Frontiers: The Shortage Function as Efficiency Measure

To introduce some basic notation and definitions, consider the problem of selecting a portfolio from n financial assets at time period t . Let $R_{1,t}, \dots, R_{n,t}$ be random returns of assets $1, \dots, n$ in period t . For each time period t , each of these assets is defined through some expected return $E[R_{i,t}]$ for $1, \dots, n$. Furthermore, returns of assets i and j are correlated, so that the variance-covariance matrix Ω_t for time period t is defined as $\Omega_{i,j,t} = Cov[R_{i,t}, R_{j,t}]$ for $i, j \in \{1, \dots, n\}$.

Notice that by adding the skewness-coskewness tensor, the extension to the MVS frontier is rather straightforward. Indeed, the shortage function is compatible with general investor preferences (favoring uneven moments and disliking even moments). Thus, in the MVS space a shortage function is capable to look simultaneously for reductions in risk and augmentations in return and skewness. In view of the familiarity of the traditional MV frontier notion and for reasons of space, the formal analysis is limited to the MV case, while the interested reader is referred to Briec, Kerstens, and Jokung (2007) for details on the use of the shortage function relative to the MVS frontier.

A portfolio $x_t = (x_{1,t}, \dots, x_{n,t})$ at time period t is simply a vector of weights specified over these n financial assets that sums to unity $\left(\sum_{i=1, \dots, n} x_{i,t} = 1 \right)$. If shorting is impossible, then these weights must satisfy the non-negativity conditions ($x_{i,t} \geq 0$). The return of portfolio x_t at time period t is given by $R_t(x_t) = \sum_{i=1, \dots, n} x_{i,t} R_{i,t}$. Therefore, the expected return of portfolio x_t is $E[R_t(x_t)] = \sum_{i=1, \dots, n} x_{i,t} E[R_{i,t}]$, and its variance is $V[R_t(x_t)] = \sum_{i,j} x_{i,t} x_{j,t} Cov[R_{i,t}, R_{j,t}]$.

The set of admissible portfolios \mathfrak{S} can be written in general as:⁹

$$\mathfrak{S} = \left\{ x \in \mathbb{R}^n : \sum_{i=1, \dots, n} x_i = 1, x \geq 0 \right\}. \quad (3.1)$$

Following the seminal approach by Markowitz (1952), one can define at time period t the MV representation of the set \mathfrak{N}_t of portfolios as:

$$\mathfrak{N}_t = \left\{ (V[R_t(x_t)], E[R_t(x_t)]) : x_t \in \mathfrak{S} \right\}. \quad (3.2)$$

Since such a representation cannot be used for quadratic programming because the subset \mathfrak{N}_t is non-convex (see Briec, Kerstens, and Lesourd (2004)), the above set is extended by defining

⁹In this contribution, \mathfrak{S} is time independent. However, this set of admissible portfolios can be modified to include additional constraints (e.g., transaction costs) that can be written as linear functions of asset weights: see Briec, Kerstens, and Lesourd (2004). These additional constraints could eventually be time dependent, thereby imposing time dependency on \mathfrak{S} .

a MV (portfolio) representation set through

$$\mathfrak{R}_t = \mathfrak{N}_t + \mathbb{R}_+ \times (-\mathbb{R}_+). \quad (3.3)$$

Briec, Kerstens, and Lesourd (2004) show that it is useful to rewrite the above subset as follows:

$$\mathfrak{R}_t = \left\{ (V', E') \in \mathbb{R}_+ \times \mathbb{R} : \exists x_t \in \mathfrak{S}, (-V', E') \leq (-V[R_t(x_t)], E[R_t(x_t)]) \right\}. \quad (3.4)$$

The addition of the cone is necessary for the definition of a sort of “free-disposal hull” of the MV representation of feasible portfolios and is compatible with the definition in (Markowitz (1952)). It is of interest to focus on the basic properties of the subset \mathfrak{R}_t on which we define the shortage function below. Briec, Kerstens, and Lesourd (2004) have shown that \mathfrak{R}_t is convex, closed and satisfies a free disposal assumption. These properties of the representation set allow defining an efficiency measure in the context of the Markowitz portfolio theory.

Before generalizing the well-known Markowitz approach, we introduce the shortage function at time period t , a concept introduced by Luenberger 1992; 1995 in a production theory context where it measures the distance between some point of the production possibility set and the Pareto frontier.

Definition 3.1 *The function $S_t : \mathfrak{S} \times \mathbb{R}_+^2 \rightarrow \mathbb{R}_+ \cup \{+\infty\}$ defined by*

$$S_t(x_t; g_t) = \sup \left\{ \delta : (V[R_t(x_t)] - \delta g_{V,t}, E[R_t(x_t)] + \delta g_{E,t}) \in \mathfrak{R}_t \right\},$$

is called the shortage function at time period t for portfolio x_t in the direction of vector $g_t = (g_{V,t}, g_{E,t})$.

The shortage function looks for improvements in the direction of both an increased mean return and a reduced risk. Notice that the efficiency improving direction vector g_t depends on time. The purpose of this time-dependency is to cater for the potentially changing preferences of the investor over time. The pertinence of the shortage function as a portfolio management efficiency indicator results from its properties. In particular, this indicator characterizes the Markowitz frontier, is weakly monotonic and continuous on \mathfrak{S} , and generalizes the Morey and Morey (1999) approaches who look either for return expansions or risk reductions only (see Briec, Kerstens, and Lesourd (2004) for details). Notice that if $g_t = 0$, then $S_t(x_t; g_t) = +\infty$. In general, we assume that $g_t \neq 0$.

Markowitz (1952) also proposed an optimization program in a dual, MV utility based framework to determine the portfolio corresponding to a given degree of risk aversion. To provide a dual interpretation of the shortage function, Briec, Kerstens, and Lesourd (2004) also define a MV indirect utility function as the support function of the Markowitz frontier. From the duality result by Luenberger (1995), who connected expenditure and shortage functions, these same authors derive the shortage function from the indirect MV utility function and conversely through a dual pair of relationships. Following this dual relation, it is also possible to disentangle between various efficiency notions when evaluating potential improvements in portfolios. By analogy with other domains in economics, Briec, Kerstens, and Lesourd (2004) distinguish formally between (i) Portfolio efficiency, (ii) Allocative efficiency, and (iii) Overall efficiency. For reasons of space and since the empirical application ignores the utility approach, we provide the intuition behind this duality relationship and the ensuing efficiency taxonomy in Appendix 1 and refer the reader to Briec, Kerstens, and Lesourd (2004) for details.

3.2 Portfolio Performance Change in Discrete Time: A Luenberger Portfolio Productivity Indicator

This subsection is concerned with the dynamic study of portfolio performance in discrete time. Using a recent Luenberger productivity indicator based on some combinations of shortage functions (see Chambers (2002)), our new proposal applies this Luenberger indicator to measuring dynamic portfolio performance.

However, this requires an adaptation of Definition 3.1 of the shortage function to a dynamic context.

Definition 3.2 *Given two time periods a and b , the function $S_b : \mathfrak{S} \times \mathbb{R}_+^2 \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$ defined by*

$$S_b(x_a; g_a) = \sup_{\delta} \left\{ \delta : (V[R_a(x_a)] - \delta g_{V,a}, E[R_a(x_a)] + \delta g_{E,a}) \in \mathfrak{R}_b \right\}, \quad (3.5)$$

is called the shortage function at time period b in the direction of vector $g_a = (g_{V,a}, g_{E,a})$ for portfolio x_a calculated at time period a .

Remark that $E[R_a(x_a)]$ stands for the expected return of portfolio x_a calculated at time period a , and an analogous interpretation applies to the variance. Notice also that if $t = a = b$, then Definition 3.2 corresponds to Definition 3.1. In this case, the value of δ is always positive. However, for different time periods, this need not be the case. As in Definition 3.1, the direction vector g_a is assumed to be distinct from zero in the general case, although the value of $+\infty$ can be assigned to $S_b(x_a; 0)$. Furthermore, $S_b(x_a, g_a) = -\infty$ if there is no scalar δ such that $(V[R_a(x_a)] - \delta g_{V,a}, E[R_a(x_a)] + \delta g_{E,a}) \in \mathfrak{R}_b$. In the following, we are especially interested in the evolution of the shortage function for two consecutive periods, that is: $(a, b) \in \{t, t+1\} \times \{t, t+1\}$.

The difference derived from expression (3.5) between two periods at $a = t$ and $a = t+1$, given a representation set at $b = t$ yields:

$$\Delta_t(x_t, x_{t+1}; g_t, g_{t+1}) = S_t(x_t; g_t) - S_t(x_{t+1}; g_{t+1}). \quad (3.6)$$

This period t productivity indicator simply computes a difference in the distances between the MV portfolio representations in periods t and $t+1$ relative to the portfolio frontier in period t . Considering the representation set at $b = t+1$, we can compute a similar indicator:

$$\Delta_{t+1}(x_t, x_{t+1}; g_t, g_{t+1}) = S_{t+1}(x_t; g_t) - S_{t+1}(x_{t+1}; g_{t+1}). \quad (3.7)$$

Relative to the portfolio frontier in period $t+1$, this period $t+1$ productivity indicator calculates the difference in the distances between the MV portfolio representations in periods t and $t+1$.

Notice that both these indicators mix various shortage functions which themselves are based on forward and/or backward looking return and other moment information. For example, one can consider $S_{t+1}(x_t; g_t)$ as the ex-ante error made by a portfolio manager in choosing his portfolio weights at time t with respect to information available at time $t+1$, while $S_{t+1}(x_{t+1}; g_{t+1})$ expresses the counterpart ex-post error observed at time $t+1$.

To avoid an arbitrary choice between time periods, it is natural (see, e.g., Chambers (2002)) to take the arithmetic mean of the two indicators defined above to obtain the discrete time Luenberger portfolio productivity indicator of performance change

$$L(x_t, x_{t+1}; g_t, g_{t+1}) = \frac{1}{2} [\Delta_t(x_t, x_{t+1}; g_t, g_{t+1}) + \Delta_{t+1}(x_t, x_{t+1}; g_t, g_{t+1})], \quad (3.8)$$

which is the portfolio analogue of a Luenberger productivity indicator.¹⁰ This portfolio performance change can be equivalently decomposed as:

$$L(x_t, x_{t+1}; g_t, g_{t+1}) = E(x_t, x_{t+1}; g_t, g_{t+1}) + F(x_t, x_{t+1}; g_t, g_{t+1}), \quad (3.9)$$

¹⁰Notice that the Luenberger productivity indicator does not satisfy circularity in this formulation. There are various ways to make it circular. Furthermore, following Diewert (2005), observe that indexes are based on ratios, while indicators are based on differences. Ratio and difference approaches to index numbers differ in terms of basic properties of practical significance: e.g., (i) ratios are unit invariant, differences are not, (ii) differences are invariant to changes in origin, ratios are not, (iii) ratios have difficulties handling zeros, differences have not, etc. In general, a variety of well-known issues in index theory (see, e.g., Diewert (2005)) can probably shed light on some new problems that may crop up when transposing index numbers into portfolio theory.

with

$$E(x_t, x_{t+1}; g_t, g_{t+1}) = S_t(x_t; g_t) - S_{t+1}(x_{t+1}; g_{t+1}), \quad (3.10)$$

and

$$F(x_t, x_{t+1}; g_t, g_{t+1}) = \frac{1}{2} \left[(S_{t+1}(x_{t+1}; g_{t+1}) - S_t(x_{t+1}; g_{t+1})) + (S_{t+1}(x_t; g_t) - S_t(x_t; g_t)) \right]. \quad (3.11)$$

In this decomposition, $E(x_t, x_{t+1}; g_t, g_{t+1})$ measures the efficiency change of the shortage functions between periods t and $t + 1$, while $F(x_t, x_{t+1}; g_t, g_{t+1})$ captures the average change in portfolio performance between the two periods evaluated at the portfolio composition in $t + 1$ and at the portfolio composition in t . Hence, equation (3.9) decomposes portfolio performance change into two components: one representing efficiency change relative to a moving portfolio frontier ($E(x_t, x_{t+1}; g_t, g_{t+1})$), another indicating the average change in the portfolio frontier itself ($F(x_t, x_{t+1}; g_t, g_{t+1})$). This decomposition offers a measurement framework for financial market performance gauging because: on the one hand, $E(x_t, x_{t+1}; g_t, g_{t+1})$ captures the performance of the fund managers over time relative to a shifting portfolio frontier, and on the other hand, $F(x_t, x_{t+1}; g_t, g_{t+1})$ indicates how the financial market itself has locally changed over time and enlarges or reduces the opportunities available to investors. When the Luenberger indicator of portfolio performance change $L(x_t, x_{t+1}; x_t, x_{t+1})$ or any of its components ($E(x_t, x_{t+1}; g_t, g_{t+1})$ or $F(x_t, x_{t+1}; g_t, g_{t+1})$) is positive (negative), then portfolio performance increases (decreases) between the two time periods considered.

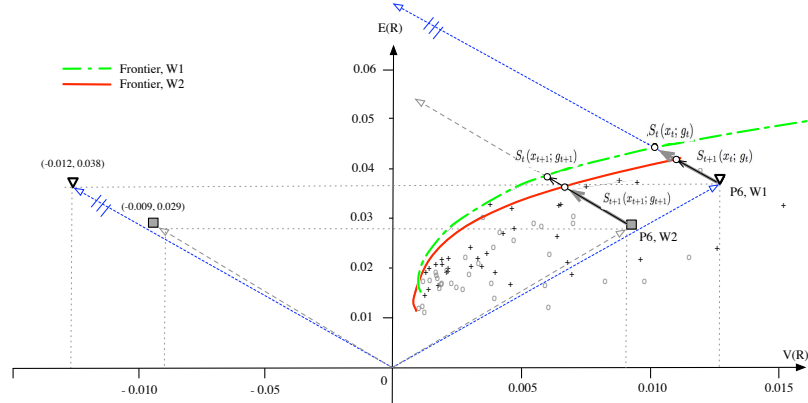


Figure 2: Luenberger Portfolio Productivity Indicator & Its Decomposition: Portfolio Nr. 6

Figure 2 illustrates the above performance indicator with $g_s = (V[R_s(x_s)], E[R_s(x_s)])$ for $s = t, t + 1$. More precisely, we illustrate the Luenberger indicator and its decomposition with the help of a certain portfolio 6 over two overlapping time windows $W1$ and $W2$.¹¹ Figure 2 plots two MV frontiers computed with the returns in the sample over $W1$ and $W2$. Portfolios are plotted using crosses in $W1$ and dots in $W2$, except $P6$ that is once plotted with a black triangle in $W1$ and once with a gray square in $W2$. Arrows indicate the respective distances towards the frontiers in both periods ($S_t(x_t; g_t)$, $S_{t+1}(x_t; g_t)$, $S_t(x_{t+1}; g_{t+1})$, and $S_{t+1}(x_{t+1}; g_{t+1})$ as defined before). The Luenberger indicator must be constructed from its components: $S_t(x_t; g_t) = 0.3795$, $S_{t+1}(x_{t+1}; g_{t+1}) = 0.3475$, $S_{t+1}(x_t; g_t) = 0.3053$, and $S_t(x_{t+1}; g_{t+1}) = 0.4191$. To obtain $E(x_t, x_{t+1}; g_t, g_{t+1})$ (see (3.10)), it suffices to compute:

¹¹This example is drawn from the empirical analysis in sections 4 and 5. The two time windows range respectively from 1934/01 till 1937/01 ($W1$) and 1934/02 till 1937/02 ($W2$).

$0.3795 - 0.3475 = 0.0320$. Clearly, this portfolio has moved closer to the portfolio frontier over time yielding a positive $E(x_t, x_{t+1}; g_t, g_{t+1})$. Computing the $F(x_t, x_{t+1}; g_t, g_{t+1})$ (see (3.11)) requires the following calculations: $0.5((0.3475 - 0.4191) + (0.3053 - 0.3795)) = -0.0729$. This negative number simply reflects the productivity decrease due to the inward shift of the portfolio frontier around portfolio 6. Notice that this inward shift of the portfolio frontier is not a global phenomenon: it does not affect the lower risk-return combinations. The Luenberger indicator is simply the sum of these two components: $0.0320 + (-0.0729) = -0.0409$. In this case, the improvement of the $E(x_t, x_{t+1}; g_t, g_{t+1})$ is overruled by the local deterioration of the $F(x_t, x_{t+1}; g_t, g_{t+1})$ and we end up with a negative portfolio frontier productivity change.

Turning to computational matters, the representation set \mathfrak{R}_t (introduced in 3.3) is used to directly compute the various shortage functions and thus the Luenberger indicators by recourse to standard quadratic programming (QP). Assume a sample of m portfolios $x_t^1, x_t^2, \dots, x_t^m$ are observed over a given finite time horizon $t = 1, \dots, T$. Now, consider a specific portfolio x_t^k for $k \in \{1, \dots, m\}$ at time period t whose performance needs gauging. To calculate the Luenberger indicator, the four different shortage functions composing it must be computed by solving a QP for each. To solve for $S_t(x_t; g_t)$, the following basic QP must be computed:

$$\begin{aligned} \max \delta & & (3.12) \\ \text{s.t. } E[R_t(x_t)] + \delta g_{E,t} &\leq \sum_{i=1, \dots, n} y_{i,t} E[R_{i,t}] \\ V[R_t(x_t)] - \delta g_{V,t} &\geq \sum_{i,j} \Omega_{i,j,t} y_{i,t} y_{j,t} \\ \sum_{i=1, \dots, n} y_{i,t} &= 1, y_{i,t} \geq 0, \delta \geq 0, i = 1, \dots, n, \end{aligned}$$

where δ and $y_{i,t}$, ($i = 1, \dots, n$) are decision variables. This QP is then solved for each portfolio with respect to the portfolio set at periods t and $t + 1$. For the latter computation, one simply replaces the left-hand side of the first two constraints by the return and risk of the evaluated portfolio in period $t + 1$ and also the corresponding direction vector g_{t+1} to end up with $S_t(x_{t+1}; g_{t+1})$. To compute the remaining two shortage functions, one proceeds as follows. To obtain $S_{t+1}(x_{t+1}; g_{t+1})$, all that is needed is to replace the subscript t by $t + 1$ everywhere in (3.12). $S_{t+1}(x_t; g_t)$ is found by replacing the returns, variances and covariances at time period t , occurring on the right-hand side of the first two constraints of model (3.12), by those computed at time period $t + 1$.

We add two remarks on computational issues. First, while in principle several options are available for the choice of direction vector (see Bricc, Kerstens, and Lesourd (2004) for details), we opt here to employ the observation under evaluation itself, that is, $g_t = (g_{V,t}, g_{E,t}) = (V[R_t(x_t)], |E[R_t(x_t)]|)$.¹² In this case, the shortage function measures the maximum percentage of simultaneous risk reduction and expected return augmentation. Second, it is well known that in certain cases the shortage function is not well-defined and achieves a value of infinity (e.g., Luenberger (1995)). Focusing on the choice of direction vector, Bricc and Kerstens (2009) show that the shortage function, one of the most general distance functions available in the literature so far, may not achieve its distance in the general case where a point need not be part of technology and where the direction vector can take any value. As a consequence, the feasibility of the Luenberger productivity indicator can in general not be guaranteed.¹³ Apart from reporting any eventual infeasibilities, these authors show that there is no easy solution in general. Notice that the efficiency measures proposed by Morey and Morey (1999), as special cases of the shortage function approach, are even more vulnerable to the infeasibility issue. Its incidence in a portfolio context has never been reported.

¹²Absolute values for return allow for both positive and negative initial data.

¹³This is related to the property of determinateness in index theory which can be loosely stated as requiring that an index remains well-defined when any of its arguments is not.

Finally, though the Luenberger indicator is not based on a utility approach, it is important to realize that the performance changes traced over time do reflect gains and losses in utility. This interpretation is developed in Briec, Kerstens, and Lesourd (2004).

4 Research Methodology: Implementation Strategy and Data

For the purpose of illustrating how the Luenberger indicator and its components can serve to track individual fund managers' performance, we opt for using a mimicking portfolio approach (Fama and French (1996)). This mimicking portfolio approach employs portfolios categorized on some variable or combination of variables of interest (e.g., Fama and French (1996) form portfolios on firm size and book-to-market equity, while Fama and French (1997) do the same on industry). In our case, we employ portfolios formed on specific factors or styles. To compose these portfolios and compute the corresponding value-weighted monthly returns, the underlying universe of financial assets is restricted to all stocks listed on the main North American stock markets (in particular, NYSE, AMEX and NASDAQ). In particular, we use a data set made available by K. French consisting in series of monthly returns from January 1931 to August 2007 for 36 value-weighted (hence, potentially non-optimal) portfolios denoted P_1, P_2, \dots, P_{36} and formed on specific factors or styles.¹⁴ More details are provided in Appendix 2.

This data set has four important characteristics: (i) the asset universe is common to all portfolios and available over a long time period, (ii) portfolios are not handled by real fund managers over a certain relatively short time span, but represent a variety of management styles that could have been implemented over a long run by some idealized manager, (iii) the value-weighted and non-optimized nature of the portfolios potentially allows for a wide scope of inefficiencies, and (iv) the portfolios have a known time frame (i.e., a month), since they are recomposed each month or each several months depending on factors or styles. By contrast, real world funds have the disadvantage of having no such natural time unit (e.g., the frequency of rescheduling is (i) hard to infer precisely from mission statements, (ii) can vary slightly over time, and (iii) need not coincide across funds).

To test the capabilities of our new methodology for tracking these inefficiencies, we compute the performance of these idealized funds over a series of sliding time windows with respect to a common fund frontier composed of all selected mimicking portfolios. Since the reallocation of assets within the sample of portfolios is at least partly asynchronous, the resulting heterogeneity in portfolio performance under idealized circumstances forms a perfect level playing field to assess the long run success of certain portfolio management strategies conditioned on styles or factors. In particular, this framework allows to highlight two interesting perspectives in the empirical part of this research that are specific to our methodological choices.

First, we can compare these portfolios in terms of the Luenberger indicator and its decomposition over a very long time period and under identical circumstances and contrast it to more traditional performance appraisal tools. Borrowing from the existing literature, we use the Sharpe (*Sharpe*) and Sortino ratios (*Sort*) to evaluate the MV respectively the MVS models employed. We now define their respective variations in discrete time to have a traditional analogue to the difference-based Luenberger portfolio productivity indicator.¹⁵ To be explicit, the

¹⁴The following list provides succinct information on how these 36 portfolios have been composed: (1) Fama-French Benchmark (P1–P6): below and above medium size market equity (ME) portfolios based on Growth, Neutral and Value (according to book-to-market (BTM)) portfolios ; (2) Size (P7–P11): five portfolios (one per quintile) based on size (ME); (3) Growth (P12–P16): five portfolios (one per quintile) based on BTM; (4) Dividend Yield (P17–P21): five portfolios (one per quintile) based on dividend yield; (5) Momentum (P22): picking well-performing stocks from the past; (6) Short Term Reversal (P23): picking poor-performing stocks from the near past; (7) Long Term Reversal (P24): picking poor-performing stocks from the more distant past; and (8) Industry Portfolios (P25–P36): portfolios mimicking returns in 12 different industries. More information is available on the web pages of K. French: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

¹⁵We are unaware of any definitions of these variations on the Sharpe and Sortino ratios in the literature.

change in Sharpe ratio and Sortino ratio are respectively defined as follows:

$$\Delta_t \text{Sharpe} = \text{Sharpe}_{t+1} - \text{Sharpe}_t, \quad \Delta_t \text{Sort} = \text{Sort}_{t+1} - \text{Sort}_t. \quad (4.1)$$

Second, the decomposition of the Luenberger indicator provides a unique tool for the long run assessment of the relative success of implementing different portfolio strategies (e.g., based on various styles, factors, etc.). In particular, the efficiency change component ($E(x_t, x_{t+1}; g_t, g_{t+1})$) provides an alternative, but particularly suitable measurement tool to detect the eventual ability of fund managers for stock picking and market timing, since the measurement is not contaminated by the change in the financial market (i.e., it is separated from the frontier change ($F(x_t, x_{t+1}; g_t, g_{t+1})$)).

With a given set of N portfolios, the minimal size for the time window is $N + 1$ if one wants to avoid the most dramatic estimation error in the variance-covariance matrix (see Disatnik and Benninga (2007)): hence, all computations have been performed with the same time window of 37 months. The sliding tick for this window is one month. Therefore, since we dispose of 920 months in the data set, we end up with 883 time windows.¹⁶ We also use a 3-month T-Bill as reference for the risk-free rate. These data have been obtained from the Federal Reserve Board and are only available since January 1934. Consequently, changes in the traditional ratios (4.1) can only be computed from January 1937 onwards. This difference in availability only affects the comparisons between these traditional measures and the Luenberger portfolio productivity indicator. Furthermore, these risk-free rates of returns were annualized and have been converted to a monthly basis.

Thus, given that all 36 portfolios must be evaluated with 4 different shortage functions over 883 time windows, we end up with 127,152 optimizations in total for the MV model and an equal amount for the MVS model. Recall that in the case of the MV (MVS) model, each portfolio is projected using a shortage function simultaneously looking for return (and skewness) augmentation and risk reduction. Notice the computational advantage of using efficiency measures, since it would be more difficult to compare 883 complete MV frontiers with one another (while ignoring the impossibility to do anything similar in the MVS case). The proposed approach only needs the projections of these 36 portfolios in each of the 883 time windows (the remainder of the MV or MVS frontiers can be safely neglected).

Notice furthermore that the incidence of the infeasibility problem mentioned before, turns out to be rather minor: we observe infeasibilities for only 165 (i.e., 0.519% = 165/(883 × 36)) and 201 (i.e., 0.632% = 201/(883 × 36)) portfolios in the MV respectively the MVS model. Thus, the problem seems to be rather small in this data base.

5 Empirical Results

This section scrutinises these portfolios in terms of their MV and MVS Luenberger portfolio productivity indicators, and also compares these to the $\Delta_t \text{Sharpe}$, respectively $\Delta_t \text{Sort}$ indicators.

A first part of the analysis consists in searching for a common ground in the information provided by this Luenberger productivity indicator and its counterpart traditional performance measures. The idea is to identify whether or not these two categories of performance gauges provide similar results. Rank correlations are computed over the period 02/1937 to 08/2007 (for data availability reasons) between: on the one hand, in MV space $L(x_t, x_{t+1}; g_t, g_{t+1})$ and the $\Delta_t \text{Sharpe}$ indicator; and on the other hand, in MVS space between $L(x_t, x_{t+1}; g_t, g_{t+1})$ and the $\Delta_t \text{Sort}$ indicator. To impose minimal assumptions, these correlations are evaluated by a Spearman rho test. Results are presented in Table 1. Notice that we only report significant results throughout this section.

¹⁶The first time window ranges over the interval [01/1931, 01/1934] and the last one over the interval [08/2004, 08/2007].

Table 1: Portfolios with Significant Correlations between $L(x_t, x_{t+1}; g_t, g_{t+1})$ and $\Delta_t Sharpe$ (MV) resp. $\Delta_t Sort$ (MVS) indicators

Portfolio Groups	Port. Nr	MV		MVS		Portfolio Groups	Port. Nr	MV		MVS	
		ρ	p-value	ρ	p-value			ρ	p-value	ρ	p-value
Fama & French Bench.	1	0.0573	0.0958*	–	–	Momentum	22	–	–	–	–
	4	0.1579	0.0000***	0.1495	0.0000***	S.T. Rever.	23	–	–	0.0730	0.0340**
	5	0.1130	0.0011***	0.1457	0.0000***	L.T. Rever.	24	–	–	0.0760	0.0275**
	6	0.0988	0.0137**	0.0911	0.0088***						
Size	7	0.1765	0.0000***	0.1327	0.0001***		25	0.0661	0.0547*	–	–
	8	0.1222	0.0004***	0.1303	0.0001***		26	0.0695	0.0456**	–	–
	9	0.0967	0.0049***	0.1115	0.0012***	Industry	27	0.0655	0.0568*	0.0765	0.0264**
	10	0.0686	0.0460**	–	–		32	–	–	0.0590	0.0888*
					33		0.0765	0.0271**	0.0595	0.0852*	
					35		0.0618	0.0733*	–	–	
Growth	12	0.0677	0.0490**	0.0782	0.0229**		36	0.0874	0.0112**	–	–
	16	0.0661	0.0557*	0.0695	0.0436**						
Dividend Yield	17	0.0643	0.0619*	–	–						
	18	0.0814	0.0179**	0.0724	0.0359**						

Note: Spearman Correlation coefficient with $H_0: \rho = 0$.

*, ** and *** signs represent 10%, 5% respectively 1% thresholds.

In Table 1, one observes that, for about 50% of portfolios, the Luenberger productivity indicator is positively correlated with the $\Delta_t Sharpe$ indicator in the MV model. However, this result is not uniformly observed across the 8 portfolio families (i.e. *groups* in Table 1). For instance, the rank correlations are the strongest for the families 1 and 2 followed by 8 (i.e., Fama French Benchmark, Size, and Industry portfolios). By contrast, Short Term and Long Term Reversal as well as Momentum portfolios do not appear at all in this table. In the MVS world, less portfolios are significantly correlated. This last result is probably linked to two reasons: (i) the portfolio mimicking approach is fundamentally a non-optimized diversification strategy geared towards a MV framework, and (ii) the $\Delta_t Sort$ indicator does not offer an equally theoretically founded performance measure compared to the Luenberger indicator, which builds upon the shortage function that is suitable to characterize MVS portfolio sets.

Keeping in mind that traditional measures are unable to distinguish the contribution of portfolio managers to the performance evolution, while the Luenberger portfolio productivity indicator and its decomposition allow for such a distinction, we now try to test the relevance of this decomposition. Two questions are considered at this point: (i) is the evolution of $L(x_t, x_{t+1}; g_t, g_{t+1})$, $E(x_t, x_{t+1}; g_t, g_{t+1})$ and $F(x_t, x_{t+1}; g_t, g_{t+1})$ due to mere chance, and (ii) do the series of $L(x_t, x_{t+1}; g_t, g_{t+1})$, $E(x_t, x_{t+1}; g_t, g_{t+1})$ and $F(x_t, x_{t+1}; g_t, g_{t+1})$ have a mean that is different from zero? While the first question is concerned with the detection of any significant influence of portfolio managers on the Luenberger and its components, the second question focuses on the size of any eventual effect.

One basic idea here is simply to identify, if possible, some styles that perform well in terms of efficiency over time (in line with a research stream pioneered by McDonald (1974)). Moreover, since all of these mimicking portfolios belong to a more general active management style (these portfolios being rebalanced on some regular basis), our results could shed some light on the controversy regarding the utility/vacuity of active management. While it is frequently reported that actively managed portfolios fail to outperform passive counterpart strategies (see, for example, Gruber (1996)), some researchers do find some value added for active mutual fund management (e.g., Wermers (2000)). Thus, while we do not expect reporting portfolios with significant non-zero $L(x_t, x_{t+1}; g_t, g_{t+1})$ (given efficient markets), we wonder whether some styles could exhibit some non-zero $E(x_t, x_{t+1}; g_t, g_{t+1})$. Obviously, positive improvements in $E(x_t, x_{t+1}; g_t, g_{t+1})$ could indicate expertise among some portfolio managers (at least over short periods of time) to push portfolios towards the moving portfolio frontier target, while a negative result could point to their inability to do so. This remark applies specifically to this mimicking

portfolio approach, since in this case, virtual portfolio managers simply stick mechanically to some predetermined investment style.

To answer the first question, we utilize a Wald-Wolfowitz run test. Results are proposed in Table 4. Looking at the decomposition, a first major result is that most portfolios exhibit non-random $E(x_t, x_{t+1}; g_t, g_{t+1})$ series in both MV and MVS models. By contrast, $F(x_t, x_{t+1}; g_t, g_{t+1})$ appears to be almost completely random, as could be expected from efficient market theory. Second, the Luenberger indicator $L(x_t, x_{t+1}; g_t, g_{t+1})$, as the sum of both above components, is mainly non-random for the two first portfolio families. These are in particular, the Fama French Benchmark portfolios 3 (not in MV), 4, 5 and 6 (i.e., mainly those that are above the median size, whatever their position in terms of BTM) and portfolios 7, 8, 9 and 10 (not in MVS) (i.e., P7 to P9 are portfolios composed within the subset of the 60% smallest firms).

The second question is answered using a Wilcoxon test for differences. Over the whole time period, we cannot report any portfolio that has non-zero performance indicators except P31 (a significant $L(x_t, x_{t+1}; g_t, g_{t+1})$ in MV) and P23 (a significant $E(x_t, x_{t+1}; g_t, g_{t+1})$ in MVS). Of course, this is in line with the efficient market hypothesis as well, since it is hard to imagine that the portfolio mimicking approach could generate and sustain superior results over such a long run. However, in a sufficiently short time horizon (1 to 3 years: see, for example, Brown and Goetzmann (1995)) and sometimes over longer periods (5 to 10 years: e.g., Elton, Gruber, S.Das, and Blake (1996)), one can imagine that some portfolios (e.g., styles, etc.) may have performed well because, for a variety of reasons, their profile fits into some market niche favored by the economy. Therefore, we look at the short run by fixing a period consisting of the last ten years. The Wilcoxon test is now recomputed and results are reported in Table 2.

Table 2: Wilcoxon Tests for the Luenberger Indicator and its Components (last ten years)

Portfolio Group	Portfolio Port. Nr	$E(x_t, x_{t+1}; g_t, g_{t+1})$		$F(x_t, x_{t+1}; g_t, g_{t+1})$		$L(x_t, x_{t+1}; g_t, g_{t+1})$	
		W	p-value	W	p-value	W	p-value
Mean-Variance							
FF. Benchmark	1	–	–	–	–	0.0028	0.0643*
	2	–	–	0.0058	0.0824*	–	–
Size	11	–	–	0.0047	0.0967*	0.0059	0.0422**
Growth	12	–	–	–	–	0.0006	0.0994*
Dividend Yield	19	–	–	0.0075	0.0311**	0.0133	0.0367**
	20	–	–	0.0091	0.0148**	–	–
Industry	25	–	–	0.0077	0.0203**	0.0123	0.0607*
	29	–	–	0.0108	0.0054***	0.0145	0.0178**
	31	–	–	–	–	–0.0002	0.0521*
	36	–	–	0.0065	0.051*	–	–
Mean-Variance-Skewness							
FF. Benchmark	1	–	–	0.0108	0.0967*	0.0119	0.065*
Dividend Yield	19	–	–	0.0101	0.0698*	0.0164	0.0685*
Industry	25	–	–	0.0277	0.0517*	–	–
	28	–	–	–0.0192	0.0580*	–0.0186	0.0889*
	29	–	–	0.0534	0.0841*	0.0571	0.0961*
	34	–	–	–	–	0.0161	0.0967*
	35	0.0023	0.0736*	–	–	–	–

Note: Wilcoxon test with H_0 : Value is not different of 0.

*, ** and *** signs represent 10%, 5% respectively 1% thresholds.

While no portfolio gets a significant $E(x_t, x_{t+1}; g_t, g_{t+1})$ in MV, and only one (P35) in MVS, quite a few obtain non-zero $L(x_t, x_{t+1}; g_t, g_{t+1})$ and $F(x_t, x_{t+1}; g_t, g_{t+1})$. Notice that not a single portfolio obtains a non-zero $\Delta_t \text{Sharpe}$ or $\Delta_t \text{Sort}$ indicator over the same time span. These portfolios obtain a significant Luenberger indicator value, not because of any capability from the idealized manager, but simply due to changes in the market that temporarily and locally favor certain niches in the portfolio set. Combining this information with the result

regarding the first question, one can conjecture that the non-random $E(x_t, x_{t+1}; g_t, g_{t+1})$ found there must be caused by some coincidentally favorable circumstances situated in some sub-period(s) different from the last ten years. Among these results, one also notices that there is no evidence supporting the relative interest to invest in high book-to-market portfolios (i.e. value portfolios, P15 and P16). This result contrasts with Lakonishok, Shleifer, and Vishny (1994) who provide contradictory illustrations. One explanation for this difference could be the mechanical behavior of our virtual managers who consistently follow certain management styles. This style consistency is known to be insufficient to achieve good performance levels (see Asness, Friedman, Krail, and Liew (2000)): for instance, some appropriately timed rotation between growth and value styles seems necessary to obtain such good results.

Finally, knowing that non-zero performance is at best only observable in the short-term, we wonder whether there is any time-dependency within these indicator-based performance results within the same ten year time span. This question relates to the more general issue of performance persistence in portfolio management. An enormous literature has been devoted to this subject ever since Jensen (1968) illustrated the virtual impossibility to outperform the market over long periods and on a regular basis. Nevertheless, much of the more recent studies illustrate possible persistence in performance for short periods of time, but these results frequently characterize the persistence of poor performances (for example, Hendricks, Patel, and Zeckhauser (1993) and Brown and Goetzmann (1995)). A series of articles has taken a closer look at the linkage between style and performance in terms of persistence (see, e.g., Teo and Woo (2004)). For instance, persistence and momentum strategies have provoked a great deal of interest (and controversy) over the last fifteen years (Jegadeesh and Titman (1993), Carhart (1997), or Barberis and Shleifer (2003)). In Table 3, we report a first-order autocorrelation regression for efficiency change, frontier change, respectively the Luenberger indicator for both MV and MVS models. Since few portfolios reveal non-zero short-term performance, we anticipate finding few, if any, significant AR(1) processes. For the efficiency change component, Table 3 shows that there is a negative persistence for most portfolios. This result partially contradicts Teo and Woo (2004) who illustrate clear connections between styles (essentially value and momentum) and persistence, since value portfolios (P15 and P16) only appear to be performance-persistent for efficiency change in MV (P15) and frontier change in both MV and MVS (P16). This persistence, as well as a possible persistence for a momentum effect (P22 being not significant except in MV for frontier change), is not significant when considering the Luenberger indicator. Thus, any non-zero performance in these non-optimized mimicking portfolios cannot be sustained over time. For the frontier change component and the Luenberger indicator, Table 3 contains more or less the same portfolios and indicate that most of these portfolios enjoy rather a positive persistence. The latter results probably simply reflect the fact that market cycles cover a time span substantially larger than the monthly tick size for the sliding windows in our analysis.

6 Conclusions

The main objective of this contribution is to introduce a general method for measuring the evolution of portfolio efficiency over time inspired by developments in index theory. Benchmarking portfolios by simultaneously looking for risk contraction and mean return (and skewness) augmentation in the MV (MVS) model using the shortage function framework, we have defined a new Luenberger discrete time portfolio productivity indicator. The cardinal virtues of this approach are: (i) it does not require the complete estimation of the efficient frontier and tracing its evolution over time, but simply projects the portfolios on the relevant part of the frontier with the shortage function using non-parametric envelopment methods to obtain an easily interpretable efficiency measure and an ensuing productivity indicator; (ii) the decomposition of the Luenberger portfolio productivity indicator distinguishes between efficiency change and portfolio frontier change. While the latter component measures the local changes in the frontier

movements induced by market volatility, the former can in principle capture efficiency changes attributable to the investor or portfolio manager. This efficiency change component allows testing in an alternative, but conceptually promising way the eventual ability of fund managers to generate superior performances, since this measurement is not contaminated by any changes in the financial market itself.

A simple empirical application on a limited sample of investment portfolios has illustrated the computational feasibility of this general framework in both the MV and MVS frameworks. Given the mimicking portfolio approach adopted, and the long time period available, we have been able to shed some light on the question of the relative performance of implementing different portfolio strategies (e.g., based on various styles, factors, etc.). Summarizing some key empirical results, the Luenberger portfolio productivity indicator is correlated with its counterpart traditional performance measures in both MV and MVS frameworks. Furthermore, most portfolios exhibit non-random efficiency change series in both MV and MVS models, while frontier change series are almost completely random. Additionally, the efficiency change series does almost never yield a non-zero performance. By contrast, the frontier change component of some portfolios can be significantly different from zero in the short run, because the market coincidentally seems to create favorable circumstances. Overall, these results are perfectly concordant with efficient market theory and are probably driven by the mimicking portfolio approach which relies in the selected data base on non-optimized rules. Nevertheless, this new framework opens up possibilities to systematically attribute performance and quantify any eventual individual fund manager performance.

Obviously, the current work has some limitations. One restriction is that it does not account for transaction costs, but assumes that portfolios can be reshuffled in every time period to remain in track with the evolving portfolio frontiers. This can in principle be overcome at the cost of complexifying the analysis slightly. However, we do not anticipate any fundamental problem in extending the proposed Luenberger indicator, since all extensions of basic portfolio models could in principle be fitted into the basic shortage function models. On the positive side, as already pointed out in the text, extensions to higher moments are straightforward (following Briec, Kerstens, and Jokung (2007)).

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Table 3: AR(1) Model for $E(x_t, x_{t+1}; g_t, g_{t+1})$, $F(x_t, x_{t+1}; g_t, g_{t+1})$ and $L(x_t, x_{t+1}; g_t, g_{t+1})$

Portfolio Group	Port. Nr	$E(x_t, x_{t+1}; g_t, g_{t+1})$		$F(x_t, x_{t+1}; g_t, g_{t+1})$		$L(x_t, x_{t+1}; g_t, g_{t+1})$	
		ρ	p-value	ρ	p-value	ρ	p-value
Mean-Variance							
F.F. Benchmarks	1	–	–	0.2176	0.0487**	0.4218	0.0001***
	3	–	–	0.2462	0.0233**	0.2461	0.0233**
	4	–	–	0.1933	0.0764*	–	–
Size	7	–	–	–	–	0.2533	0.0201**
	10	–	–	–	–	0.2276	0.0376**
	11	–	–	–	–	0.3391	0.0018***
Growth	12	–	–	0.2448	0.0259**	0.3222	0.0030***
	15	0.1952	0.0728*	–	–	–	–
	16	–	–	0.2059	0.0582*	–	–
Dividend Yield	17	–	–	0.2502	0.0220**	0.2546	0.0193**
	19	0.2352	0.0330**	–	–	–	–
Momentum	22	–	–	0.2223	0.0409**	–	–
S.T. Rever.	23	–0.1892	0.0826*	–	–	–	–
Industry	25	–	–	–	–	–0.3278	0.0041***
	29	0.2900	0.0090***	–	–	–	–
	30	–	–	0.3165	0.0032**	–	–
	31	–	–	0.2923	0.0069***	0.2555	0.0181**
	32	–	–	0.2075	0.0565*	–	–
	33	–	–	0.2039	0.0633*	–	–
	35	–0.1834	0.0930*	–	–	–	–
	36	–	–	0.2312	0.0341**	–	–
Mean-Variance-Skewness							
F.F. Benchmarks	1	–0.3416	0.0016***	–	–	–	–
	3	–	–	0.2733	0.0140**	–	–
Size	7	–	–	0.2221	0.0412**	0.2385	0.0322**
Growth	16	–	–	0.2181	0.0447**	–	–
Dividend Yield	17	–	–	0.2456	0.0257**	0.1827	0.0965*
	18	–0.2162	0.0466**	–	–	–	–
	19	–0.1946	0.0779*	–	–	–	–
S.T. Reversal	23	–0.3515	0.0010***	–	–	–	–
L.T. Reversal	24	–0.3593	0.0008***	–	–	–	–
Industry	25	–0.4731	0.0000***	–	–	–0.3686	0.0007***
	27	–0.2778	0.0123**	–	–	–	–
	28	–0.3025	0.0059***	–	–	–	–
	32	–	–	0.2073	0.0570**	–	–
	34	–	–	–0.2888	0.0418**	–	–

Note: *, ** and *** signs represent 10%, 5% respectively 1% thresholds.

Table 4: Run Tests for the Luenberger Indicator and Its Components (whole sample)

Portfolio Group	Port. Nr	Mean-Variance			Mean-Variance-Skewness						
		$E(x_t, x_{t+1}; g_t, g_{t+1})$ Z	$F(x_t, x_{t+1}; g_t, g_{t+1})$ Z	$L(x_t, x_{t+1}; g_t, g_{t+1})$ Z	$E(x_t, x_{t+1}; g_t, g_{t+1})$ Z	$F(x_t, x_{t+1}; g_t, g_{t+1})$ Z	$L(x_t, x_{t+1}; g_t, g_{t+1})$ Z				
F.F. Benchmarks	3	-	-	-	-1.115	0.2648	-1.170	0.2419	-1.716	0.0862*	
	4	-3.000	0.0027***	-1.448	0.1477	-3.337	0.0008***	-2.180	0.0292**	-3.468	0.0005***
	5	-2.789	0.0053***	-1.116	0.2645	-4.634	0.0000***	-1.649	0.0991*	-5.031	0.0000***
	6	-3.764	0.0002***	-1.689	0.0912*	-3.045	0.0023***	-4.476	0.0000***	-1.680	0.0929***
	7	-6.503	0.0000***	-2.063	0.0391**	-4.354	0.0000***	-8.303	0.0000***	-2.294	0.0218***
	8	-2.668	0.0076***	-1.284	0.199	-3.169	0.0015***	-1.811	0.0701*	-1.274	0.2029
Size	9	-1.517	0.1293	-1.034	0.301	-2.655	0.0079***	-0.772	0.4399	-1.314	0.1887
	10	-1.716	0.0861*	-0.027	0.9783	-2.727	0.0064**	-	-	-	-
Growth	14	-	-	-	-	-	-	-1.986	0.0470**	-0.096	0.9232
	15	-2.140	0.0323**	0.372	0.7097	1.314	0.1891	-2.188	0.0287**	1.352	0.1765
	16	-2.059	0.0395**	0.204	0.8388	-0.966	0.3341	-4.860	0.0000***	-0.776	0.4377
Dividend Yield	17	-1.137	0.2554	0.086	0.9314	-2.221	0.0263**	-1.964	0.0495**	-0.120	0.9048
	20	-2.256	0.0241**	-0.676	0.4992	-1.515	0.1298	-2.724	0.0064***	1.225	0.2207
Momentum	22	-4.495	0.0000***	-0.961	0.3366	-0.919	0.3582	-6.263	0.0000***	-0.805	0.4206
	23	-2.473	0.0134**	-1.700	0.089*	-0.769	0.4421	-6.686	0.0000***	-1.204	0.2288
S.T. Reversal	24	-2.230	0.0258**	-0.683	0.4949	-1.747	0.0806**	-7.022	0.0000***	1.223	0.2212
	25	-2.765	0.0057***	-1.421	0.1554	-0.745	0.4562	-2.237	0.0253**	0.738	0.4603
Industry	26	-2.199	0.0278**	-0.023	0.982	0.104	0.9174	-6.850	0.0000***	-1.655	0.0979*
	28	-3.410	0.0006***	-0.467	0.6402	0.273	0.7849	-8.426	0.0000***	-0.977	0.3287
	29	-1.683	0.0924*	0.371	0.7108	1.449	0.1474	-1.869	0.0617**	0.453	0.6503
	30	-2.800	0.0051***	-0.334	0.7388	-0.595	0.5519	-5.298	0.0000***	0.409	0.6824
	31	-3.740	0.0002***	-0.354	0.7235	-0.564	0.5727	-7.750	0.0000***	-0.554	0.5793
	32	-	-	-	-	-	-	-5.008	0.0000***	-0.235	0.8139
33	-2.978	0.0029***	-1.416	0.1568	-0.576	0.5649	-3.486	0.0005***	-0.337	0.7364	
34	-3.444	0.0006***	0.081	0.9354	-1.307	0.1912	-6.388	0.0000***	0.728	0.4665	
35	-	-	-	-	-	-	-1.716	0.0861*	1.498	0.1342	
36	-1.383	0.1667	-1.651	0.0988*	-1.773	0.0763*	-	-	-	-	

Note: Wald-Wolfowitz run test with H_0 : Series X follows a random process.
*, **, and *** signs represent 10%, 5% respectively 1% thresholds.

Appendices

1. The Shortage Function and Duality in a Mean-Variance Utility Based Framework: A Decomposition

Markowitz (1952) also proposed a dual, MV utility based framework to determine the portfolio corresponding to a given degree of risk aversion.

Definition: *The direct MV utility function at time period t is defined by*

$$U_{\rho,\mu,t}(x_t) = \mu E[R_t(x_t)] - \rho V[R_t(x_t)], \quad (6.1)$$

with $\mu \geq 0$ and $\rho \geq 0$. The function $U_t^*(\rho, \mu) = \sup\{U_{\rho,\mu,t}(x_t) : x_t \in \mathfrak{S}\}$ is called the indirect MV utility function at time period t .

Thus, the support function of the Markowitz frontier is given by the indirect utility function $U_t^*(\rho, \mu)$. The quadratic optimization program needed to obtain the indirect MV utility function can simply be written as:

$$\begin{aligned} \sup U_{\rho,\mu,t}(x_t) &= \mu E[R_t(x_t)] - \rho V[R_t(x_t)] \\ \text{s.t. } \sum_{i=1,\dots,n} x_{i,t} &= 1, x_t \geq 0. \end{aligned} \quad (6.2)$$

The ratio $\varphi = \rho/\mu \in [0, +\infty]$ is known as the risk aversion.

Following the dual relation between shortage function and MV utility function, it is also possible to disentangle between various efficiency notions when evaluating potential improvements in portfolios: (i) Portfolio efficiency (*PEFF*), (ii) Allocative efficiency (*AEFF*), and (iii) Overall efficiency (*OEFF*) (see Briec, Kerstens, and Lesourd (2004) for technical details).

Starting from a portfolio under evaluation, portfolio efficiency guarantees only reaching a point on the Markowitz frontier using the shortage function. However, this point need not necessarily coincide with a portfolio maximizing the investor's indirect MV utility function. Starting again from a portfolio under evaluation, it is possible to define another efficiency measure that guarantees reaching the point on the Markowitz frontier maximizing the MV utility function. For this purpose, the overall efficiency is the ratio between, (i) the difference between indirect MV utility and the value of the MV utility function for the evaluated portfolio, and (ii) a normalization based on the direction vector. Finally, since the overall efficiency notion is clearly more demanding than the portfolio efficiency concept, one can define a residual notion of allocative efficiency which is simply the difference between overall efficiency and portfolio efficiency. Thus, allocative efficiency measures the needed portfolio reallocation along the portfolio frontier to achieve the maximum of the indirect MV utility function.

For a given time period, this approach is illustrated in Figure 3 in MV space. The shortage function looks for improvements in the direction of both an increased mean return and a reduced risk. For instance, the inefficient portfolio A is projected onto the MV frontier at point B . Furthermore, given the knowledge about the investor's risk-aversion, one can establish the ideal point on the portfolio frontier conforming to his/her preferences (i.e., the tangency point of the MV utility function and the Markowitz frontier). In Figure 3, point D maximizes the direct utility function. To illustrate the above decomposition starting from the portfolio denoted by point A , it can be shown that

$$OEFF = \frac{|CA|}{|OA|}, PEFF = \frac{|BA|}{|OA|}, \text{ and } AEFF = \frac{|CB|}{|OA|}.$$

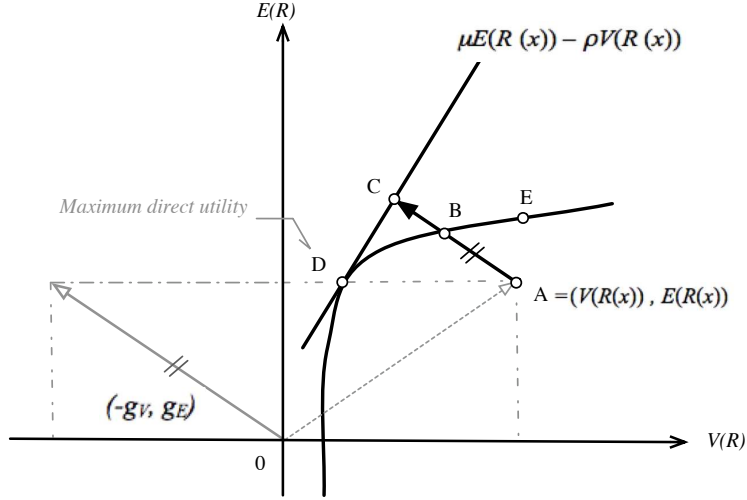


Figure 3: Shortage Function & OE Decomposition

2. Description of K. French Database

The following list provides essential information on how these 36 portfolios have been composed by K. French:

- (1) **Fama-French Benchmark (P1–P6):** These portfolios combine stocks with respect to two main characteristics. The first one is their book-to-market ratio (BTM). On this basis, 3 categories are established (Growth, Neutral and Value portfolios). The second characteristic is the size of the firm proxied by its market equity (ME). Mixing these categories results in 6 profiles (see Table 5).¹⁷

Table 5: Portfolio Profile

	30% Smallest BTM	In-Between BTM	30% Biggest BTM
Below median size	Buy + Growth Firms (P1)	Buy + Neutral Firms (P2)	Buy + Value Firms (P3)
Above median size	Sell + Growth Firms (P4)	Sell + Neutral Firms (P5)	Sell + Value Firms (P6)

Note: Breakpoints for each category are computed over the NYSE data, although each portfolio combines stocks from NYSE, AMEX, and NASDAQ.

- (2) **Size (P7–P11):** Five portfolios (one per quintile) based on firms' size composing each portfolio. Size is proxied by market equity. For instance, P7 is based on the 20% smallest firms listed on the NYSE, the AMEX, and the NASDAQ while P11 draws on the 20% biggest firms.
- (3) **Growth (P12–P16):** Same logic as for size-based portfolios, but book-to-market (BTM) serves as a proxy for growth opportunity. In other words, P12 is a portfolio composed of the smallest firms while P16 combines the biggest ones.

¹⁷It could be interesting to compare these with the Morningstar classification system which is based on the same criteria. ME is divided in 3 categories, therefore each fund receives a pictogram indicating its synthetic position in a 3×3 matrix. In this research we only have a 3×2 matrix.

- (4) **Dividend Yield (P17–P21):** Ibidem, with dividend yields (DY) replacing ME or BTM.
- (5) **Momentum (P22):** We focus here on the more typical momentum portfolio. For each month t , stocks are included in this portfolio provided (i) these are ranked in the 10% most performing stocks in terms of return at the end of the previous month ($t - 1$), and (ii) these were already listed one year and a month before ($t - 13$).¹⁸ Some investors believe that well performing stocks in the past will deliver the well performing stocks of the future, whence they play a momentum strategy.
- (6) **Short Term Reversal (P23):** Similarly to the Momentum portfolio (P22). P23 is a typical short term reversal portfolio. For each month t , stocks are included in this portfolio provided (i) these have been ranked in the 10% least performing stocks in terms of return at the end of the previous month ($t - 1$), and (ii) these were already listed one month before ($t - 2$).¹⁹ Contrary to the beliefs of momentum traders, short term reversal investors think that returns inevitably tend to revert to the mean over time. Therefore, it is worth buying poorly performing stocks to benefit from their possible appreciation in the short term.
- (7) **Long Term Reversal (P24):** Same logic as for P23, but stocks are now picked (i) on the basis of their poor performance observed in $t - 13$, and (ii) provided these were listed five years before ($t - 61$).²⁰
- (8) **Industry Portfolios (P25–P36):** Portfolios are based on all stocks listed on NYSE, AMEX and NASDAQ with respect to their four-digit SIC. These portfolios simply aim at mimicking industry returns. These are coded by a number ranging from 25 to 36 corresponding to (i) Non Durable Goods, (ii) Durable Goods, (iii) Manufactured Goods, (iv) Energy, (v) Chemicals, (vi) Business Equipment, (vii) Telecommunication, (viii) Utils, (ix) Shops, (x) Health, (xi) Money, and (xii) Others.

¹⁸P22 corresponds to the last quintile portfolio in the data file **Momentum Portfolio**.

¹⁹P23 corresponds to the first quintile portfolio in the data file **Short-Term Reversal Portfolio**.

²⁰P24 corresponds to the first quintile portfolio in the data file **Long-Term Reversal Portfolio**.