

Housing, Taxation, and Retirement Provision

by

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Abstract

A simple life-cycle model is developed, in which both housing and financial assets are available for retirement provision. Individuals face perfect capital markets and also choose between owner-occupied and rented dwellings. It is shown that, while without taxes, optimal lifetime consumption and maximum utility are the same for both tenant and owner-occupier, there is a strong impact of taxation rules on optimal lifetime consumption. With existing taxation rules, owner-occupied housing is generally a favourable means of retirement provision, even if imputed rents are subject to income taxation. On the other hand, tax allowances for private retirement provision typically focus on financial assets, thereby excluding housing. It is argued that deferred income taxation could integrate both retirement provision grants and optimal taxation.

JEL-Classification: D, H, J

1. Introduction

With the challenge of demographic change, the pros and cons of home ownership, as compared to other forms of retirement provision, have become a most relevant issue. In many countries, owner-occupied housing plays an important role in private asset formation. However, public allowances typically promote other forms of retirement provision, in particular occupational pensions and long term financial assets, thereby indirectly discriminating against real estate investments. On the other hand, there are also lots of subsidies and tax privileges in the housing market, which make the balance of advantages vague, at the very least.

There is a broad literature on proper housing taxation (Goode 1960, Aaron 1970, Rosen 1977, Hamilton et. al 1985, Nakagami et. al. 1994, Broadbent et. al.2001.), and also a large international variety of existing taxing schemes (van der Hoek/Radloff 2007). Most authors argue that imputed rent of owner-occupied housing should be taxed to ensure efficient capital allocation (Goode 1960, 526;; Merz 1965, 255; Aaron 1970; 803, Berkovec 1989, 157). It has also been largely discussed to what extent the tax privilege of owner-occupied housing does reduce investment in the productive sector (van Order 1990, Nakagami et.al. 1994). While some authors find substantial welfare costs (Hendershott et.al. 1983, Skinner 1996, Gervais 2002), others query that there is a general negative impact on non-housing investments (Broadbent et.al. 2001).

The present paper targets the housing issue from the particular retirement provision point of view. There is remarkably rare literature on this issue, which is, for example, only mentioned in passing by in the review article by Leung (2004). There is, however, a theoretical literature on housing and the life cycle to be build upon (Artle et.al. 1977, King 1982, Pines et.al. 1985,

Grossman et.al. 1990, Turnowsky 1994, Flavin et.al.2004). In section (2), a simple lifetime-model is developed, following the OLG approach in the tradition of Samuelson (1958), Diamond (1965), Kotlikoff (2006) and Conesa et al. (2007). In contrast to most of the recent articles on lifetime consumption we adopt a discrete three-period model, both because of its simplicity and intuitive applicability to practical taxation schemes. In particular, we discuss for a world without taxes, but with perfect capital markets, if there is any general advantage or disadvantage for owner-occupied compared with rented housing.¹ While the building societies use to promote their loan contracts with the slogan “property ownership is the only retirement provision you can live in”, it is argued that there is no such general advantage with the assumptions stated above, nor is there any impact of individual time preference on housing tenure choice. It is also shown that, for the owner-occupier, generally a successive investment in his property is optimal rather than a fixed amount of housing over time.²

In section (3), the impact of typical income taxation rules on the rent vs. own decision is examined, referring to both positive and normative issues. We show that, even with imputed rent taxation, the tax advantage of owner-occupied housing is not entirely eliminated. This is because the tenant`s interest expenses, unlike the owner-occupiers mortgage charges, are generally non deductible. Hence, there remains a tax-induced bias in favour of owner-occupied housing with respect to optimal lifetime consumption and retirement provision.

This is also true for an alternative approach, where imputed interest on net housing capital is taxed instead of imputed rent. However, being a much simpler equivalent to the imputed rent approach, the imputed interest approach turns out to be a good starting point for the integration of housing taxation and public allowances for private retirement provision. In

¹ In this context, “general advantage” is not meant in terms of risk, fungibility or liquidity, but refers solely to the maximum utility which the individual can achieve by optimizing her lifetime consumption.

² We thereby ignore transaction costs, which could distort the optimal timing of housing investment (Grossman et. al. 1990, Flavin et.al.2004).

section (4), a proposal for the latter is made, being outlined within the theoretical framework developed above. Section (5) summarizes the analyses and comments on its limitations.

2. A Simple Life-Cycle-Model of Housing

We assume all individuals to live for three periods and have the same income and taste. Let $\mathbf{w} = (w_1; w_2; w_3)$ be the vector of wage incomes, $\mathbf{c} = (c_1; c_2; c_3)$ the vector of residential consumption, and $\mathbf{c}^j = (c^j_1; c^j_2; c^j_3)$ the vector of non-residential consumption in three periods j of lifetime, where $j = 3$ stands for the retirement period. Non-residential consumption is only used as a numeraire good with its price being normalized to 1. With all accommodation having the same quality, residential consumption \mathbf{c} , can simply be measured by the living space. Individuals have identical, well behaved temporal utility functions $U(\mathbf{c}; \mathbf{c}^j)$, which they seek to maximize. It is supposed that they are purely self-interested and, hence, that there are no bequests.

In contrast to non-residential goods, houses are assumed to be durable goods. For simplicity, neither depreciation nor maintenance costs are assumed. Housing consumption can be accomplished either by renting or buying. It is also possible to hire out part of ones property. Moreover, individuals can borrow from and lend money to a perfect capital market at an interest rate i , with $q = (1+i)$ denoting the interest factor. Note, however, that this does not apply to the final phase of life, because individuals are no longer alive in the following period and will therefore neither save nor obtain any credit in the period before. Hence, in order to maximize their utility, they will sell their property in Period 2 at the latest, in order to rent an appropriate apartment in Period 3, possibly in an old-age home.

2.1. Optimum Lifetime Consumption with Rented Accommodation

As a benchmark for the analysis, we examine optimal lifetime consumption for a tenant who does not consider living in a house of his own, for whatever reason. Hence, in order to provide for his retirement, he has to save in terms of financial assets.³ Let $\mathbf{s} = (s_1; s_2; 0)$ be the vector of his net monetary savings. For simplicity, we normalize to unity both the price of the numeraire good and the purchase price of an – appropriately defined – unit of living space. Then, with r denoting the rental charge as a percentage of the value of the accommodation, the tenant faces the following set of budgetary constraints:

$$\begin{aligned} (1) \quad rc_1 + \dot{c}_1 &= w_1 - s_1 \\ (2) \quad rc_2 + \dot{c}_2 &= w_2 + s_1q - s_2 \\ (3) \quad rc_3 + \dot{c}_3 &= w_3 + s_2q \end{aligned}$$

We specify the utility function $U(\mathbf{c}; \mathbf{c}')$ Cobb-Douglas-type:

$$(4) \quad U(\mathbf{c}; \mathbf{c}') = \left(c_1^{\alpha_1} c_2^{\alpha_2} c_3^{\alpha_3} \right)^\beta \left(c_1^{\alpha_1} c_2^{\alpha_2} c_3^{\alpha_3} \right)^{1-\beta}$$

where the α_j can be interpreted as consumption weights for the respective periods,⁴ while β and $(1-\beta)$ are the respective weights for non-residential and residential consumption.

Maximizing (4) with respect to (1) to (3), after some manipulation of terms⁵, yields

³ It is generally assumed that individual income is less in period 3 than it is on average of lifetime, and that time preference is positive, so that there is actually the need for private retirement provision. These assumptions are, however, not essential for our analytical results.

⁴ By rewriting utility function (4) as $\ln U = \alpha_1 \ln c_1 + \alpha_2 \ln c_2 + \alpha_3 \ln c_3$ it follows that time preference is positive if $\alpha_j < \alpha_{j-1}$ and vice versa. In the Samuelson case with $\alpha_j = 1 \quad \forall j$, there is no explicit time preference, and hence the optimisation problem results only from diminishing marginal utility of consumption in the respective periods. The sum of the α_j may or may not be unity without any consequences of our general results.

⁵ For more algebraic details see appendix AI.

$$(5) \quad c_1^* = \frac{\alpha_1(1-\beta)}{rq^2} \left(\frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(6) \quad c_2^* = \frac{\alpha_2(1-\beta)}{rq} \left(\frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(7) \quad c_3^* = \frac{\alpha_3(1-\beta)}{r} \left(\frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

The optimum solutions for non-residential consumption are quite similar, with the only difference that $(1-\beta)$ is substituted by βr in equations (5) to (6). Hence we have

$$c_j^* = r_i c_j^* \beta / (1-\beta) \quad \forall j \text{ in each period, which makes sense because } r \text{ is the price of housing,}$$

while the price of non-residential consumption has been normalized to unity. Not surprisingly with the Cobb-Douglas function, we find a constant share of the respective commodity expenses.

In the sequel we concentrate entirely on the housing side of the model.⁶ Since the terms in brackets are the same in each period j , housing consumption rises at the factor $q\alpha_j / \alpha_{j-1}$ with a constant rental charge. For $\alpha_j = 1 \quad \forall j$, i.e. if there were no time preference at all, housing consumption would simply rise at the interest rate i .

2.2. Optimum Lifetime Consumption with Owner-Occupied Accommodation

If the dwelling is occupied by the owner himself, the situation is slightly more complicated.

After having bought his property in period 1, the owner's housing consumption remains the same in period 2 even if he does not further spend any money on it. Therefore, if he buys e_1

⁶ In fact there would not be any change of our general results if β was set to zero and, hence, non-residential consumption was entirely neglected. That is also true for the more sophisticated cases of taxation to be discussed later on, where it might be, however, less obvious. Therefore, we stick here to the more general two-commodity approach.

and e_2 units in Periods 1 and 2 respectively, his housing consumption is $c_1 = e_1$ in Period 1, but $c_2 = e_1 + e_2$ in Period 2. Selling all his property in Period 3, he will therefore have an extra revenue of $e_1 + e_2 (= c_2)$ in that period. His housing consumption in Period 3 – now as a tenant – is rc_3 . Hence, he faces the following set of temporal budgetary constraints:

$$(8) \quad e_1 + c_1 = c_1 + c_1 = w_1 - s_1$$

$$(9) \quad e_2 + c_2 = c_2 - c_1 + c_2 = w_2 + s_1q - s_2$$

$$(10) \quad rc_3 + c_3 = w_3 + s_2q + c_2$$

In market equilibrium - if there are tenancies at all - housing investment must yield exactly the market interest rate i to the landlord, to make the present value of his investment zero. He will then, both in terms of profit rate and utility, be indifferent between financial and housing investment. Hence, if an investment e and its first rental yield re both accrue in period 1,⁷ and if the estate is sold in Period 3, the equilibrium relation of the rental charge and the respective property price is given by

$$(11) \quad 0 = -e + re + \frac{re}{q} + \frac{e}{q^2}$$

$$\Rightarrow r = 1 - \frac{1}{q} = \frac{i}{1+i}$$

With this inserted into (10), maximization of utility function (4) with respect to restrictions (8) to (10) yields exactly the same solution for $\mathbf{c} = (c_1; c_2; c_3)$ as in the tenant's case (see Equations 5 to 7).⁸

Therefore, in contrast to the advertisement quoted above, owner-occupied accommodation does not have a general advantage compared to renting ones home. Starting with the same set of temporal income and taste, both the owner-occupier and the tenant achieve the same maximum level of utility and consume the same quantities of both housing and non-housing

⁷ This assumption saves the symmetry of the landlord case to the owner-occupied accommodation case. It implies that houses can be used instantly after acquisition.

⁸ For more details see appendix AII.

commodities in each period of lifetime. The only difference is that the tenant provides for his retirement by the way of financial investments, while the owner-occupier mainly⁹ invests in his property. Therefore, it would also be false to attribute a generally higher rate of time preference to the tenant as compared to the self-occupying owner. At least in a world with perfect capital markets, but without taxes, their different choice cannot be explained that way.

A numerical example might be helpful. Assume that $\beta = 0.6$, $\alpha_j = (1.1; 1.0; 0.9)$, with the interest factor $q = 1.5$ and hence, according to (11), $r = 0.33$. Then, if the vector of wages is $w = (1; 3; 2)$, both the tenant and the owner-occupier choose the same consumption patterns c^* and c^* , thereby achieving the same utility level U^* (see Table I). Their financial savings s^* , however, are different. While the tenant chooses a positive amount of saving in Period 2, the owner-occupier takes a debt in both periods. Yet their overall consumption-pattern is lastly identical, because of the owner's additional housing investment $e^* = (1.7108; 0.6227; -2.3334)$.¹⁰ Note that, generally, for the owner-occupier both e_1^* and e_2^* are positive and, hence, a fixed amount of housing e that is constant over time would not be optimal (see also Grossman et.al 1990). This corresponds to the practical experience that owners normally continue to invest in their property.

The owner-occupier could also choose to let a part of his property, as it is assumed in the last column of Table I. In fact, with a rental charge according to (11), he will be indifferent concerning housing and financial saving. In our example he borrows $s = (-1.8; -1.8; 0)$, thereby expanding his real estate investment to $e = (2.0611; 1.2639; -3.3250)$. This would earn him

⁹ If he makes financial provisions in addition depends on his lifetime income structure and his rate of time preference.

¹⁰ Note that, in the first period, the tenant needs less income to achieve the same consumption level as the owner, because he must only pay re_1 instead of e_1 for c_1 . Therefore, in spite of the identical c , the sum of s and e is not identical for them in the respective periods.

additional rental revenue of $\mathbf{re} = (0.1167; 0.3306; 0)$, but leaves both his total utility and his consumption unaffected.

Table I: Optimal Housing Consumption without Taxes

	Tenant	Owner-occupier	Owner-occupier, renting part of his property
consumption c_1	1.7111	1.7111	1.7111
consumption c_2	2.3333	2.3333	2.3333
consumption c_3	3.1500	3.1500	3.1500
savings s_1	-0.4259	-1.5665	-1.8000
savings s_2	0.4167	-1.1392	-1.8000
savings s_3	0	0	0
Utility level U	3.3979	3.3979	3.3979

Hence, with the assumptions made above, there is in principal no efficiency bias I the housing market in terms of the mode of providing for one`s old age. Accordingly, different choices on the issue must be explained otherwise.

3. Income Taxation and Housing Tenure Choice

We now introduce income taxation into the model, but neglect wage taxes, by simply assuming that the vector \mathbf{w} already denotes net wage incomes. Interest is assumed to be taxed by means of a proportional and constant income tax t . We assume for now that private interest expenses are fully deductible. While this assumption does usually not apply to private debt other than mortgage, it substantially simplifies the algebra. It will be modified later on.¹¹

¹¹ It is entirely consistent with existing taxing schemes if the optimum solution for the tenant is such that he does not take credit anyway, i.e. if $s_j^* \leq 0 \quad \forall j$.

If we assume that rental receipts are taxed by the same rate as interest income, the market equilibrium relation between the rental charge and the value of the property rises from r to r_t , according to

$$(1\Gamma) 0 = -e + r_t e(1-t) + \frac{r_t e(1-t)}{1+i(1-t)} + \frac{e}{[1+i(1-t)]^2}$$

$$\Rightarrow r_t = \frac{i}{1+i(1-t)}$$

Hence, the landlord will raise the rental charge, which means that the tenant now in fact must bear a double tax-burden: Not only the rent rises, but also his interest receipts are subject to taxation. Accordingly, his set of restrictions changes as follows:

$$(1') r_t c_1 + \dot{c}_1 = w_1 - s_1 - T_1$$

$$(2') r_t c_2 + \dot{c}_2 = w_2 + s_1 q - s_2 - T_2$$

$$(3') r_t c_3 + \dot{c}_3 = w_3 + s_2 q - T_3$$

with

$$(12) T_j = s_{j-1} i t$$

Maximizing utility function (4) subject to restrictions (1') to (3') yields for optimal residential consumption:¹²

$$(5') c_1^* = \frac{\alpha_1(1-\beta)}{r_t(1+i-it)^2} \left(\frac{w_1(1+i-it)^2 + w_2(1+i-it) + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(6') c_2^* = \frac{\alpha_2(1-\beta)}{r_t(1+i-it)} \left(\frac{w_1(1+i-it)^2 + w_2(1+i-it) + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(7') c_3^* = \frac{\alpha_3(1-\beta)}{r_t} \left(\frac{w_1(1+i-it)^2 + w_2(1+i-it) + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

¹² See appendix AIII

The respective optimum quantities of non-residential consumption are now given by

$\hat{c}_j^* = r_t c_j^* \beta / (1 - \beta) \quad \forall j$. As $r_t < r$ holds, the residential/non-residential relation \hat{c}_j^* / c_j^* is higher than without taxation, though it is still equal in each period.¹³

3.1. The Consumer Good Approach

The corresponding consequences for the owner-occupier depend crucially on the particular taxation rules to which he is subject. If living in one's own home is viewed as mere consumption, it appears natural to leave it tax-free. Accordingly, it follows that interest on debt should not be deductible. Hence, the consumer good approach results in the following new budgetary constraints for the owner-occupier:

$$(8^*) \quad e_1 + \hat{c}_1 = c_1 + \hat{c}_1 = w_1 - s_1 - T_1$$

$$(9^*) \quad e_2 + \hat{c}_2 = c_2 - c_1 + \hat{c}_2 = w_2 + s_1 q - s_2 - T_2$$

$$(10^*) \quad r_t c_3 + \hat{c}_3 = w_3 + s_2 q + c_2 - T_3$$

with

$$(12^*) \quad T_j = s_{j-1} i t \quad \text{if } s_{j-1} > 0$$

$$T_j = 0 \quad \text{if } s_{j-1} \leq 0$$

Provided there is no additional financial saving,¹⁴ the owner-occupier would then choose the following optimum quantities of residential consumption:¹⁵

¹³ We do not take into account here that, depending on the capital intensity of producing the non-residential good, the latter's price would also be changed by the taxation of interest income, but still entirely concentrate on residential consumption.

¹⁴ Additional financial savings would not change our general results but would only complicate the algebra.

¹⁵ See appendix AIV.

$$(5'') \quad c_1^* = \frac{\alpha_1(1-\beta)}{rq^2} \left(\frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(6'') \quad c_2^* = \frac{\alpha_2(1-\beta)}{rq} \left(\frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

$$(7'') \quad c_3^* = \frac{\alpha_3(1-\beta)}{r_t} \left(\frac{q^2 w_1 + qw_2 + w_3}{\alpha_1 + \alpha_2 + \alpha_3} \right)$$

Equations (5'') and (6'') are identical to the owner-occupiers optimum conditions without taxation (5) and (6). In contrast, Equation (7'') slightly differs from (7), because r is now replaced by r_t in the denominator. The latter occurs, because the increased rental charge also affects the owner-occupier in the last period of his lifetime, when he changes in a tenant.

Nevertheless, with this taxation rule the owner-occupier is clearly better off than he would be as a tenant. He achieves higher levels of both residential consumption and maximum utility by owning rather than renting.¹⁶ His tax advantage is the higher, the more he would have to save as a tenant in order to optimise his lifetime consumption. By this bias the consumption approach tends to yield a relatively high share of owner-occupied housing and would - other things being equal - in fact erase the rental market (Pines et.al. 1985, 4).¹⁷

3.2. The Investment Approach

According to the so-called investment approach, the imputed rent of an owner-occupied dwelling should be taxed in the same manner as interest payments. The usual proposal is for the taxation of the (hypothetical) rent which the owner could earn from letting his accommodation, minus ownership expenses like mortgage debt and depreciation (Goode

¹⁶ Concerning non-housing consumption, there is a negative substitution effect in favour of housing, but also a positive income effect due to the tax advantage, the result of which is ambiguous.

¹⁷ In reality, the tax-effect could, of course, be superposed by other variables which are not included in our model, such as the need for mobility, individual preferences for or against renting etc.

1960, 505). Still neglecting depreciation and maintenance costs, the taxation formula (12[`]) would then change to (12^{``}):

$$(12^{``})T_j = t \left[\sum_{j=1}^3 (r_t e_j) + i s_{j-1} \right].$$

The first term in the bracket is the (hypothetical) rental charge, with the value of the property being measured by cumulated net investment.¹⁸ The second term $i s_{j-1}$ is the owner's interest receipt from financial assets. Hence, the owner is taxed *as if* he would have let his property, although he actually occupies it himself.

By inserting both the rent/price relation r_t (11[`]) and the tax-formula (12^{``}) into restrictions (8[`]) to (10[`]), both the owner-occupier's optimal lifetime consumption and his achievable utility turn out to be the same as if he were a tenant (see equations 5[`] to 7[`]).¹⁹ Hence, the investment approach in so far seems to heal the tax-bias in favour of owner-occupied housing.

However, the deductibility of interests which was hitherto assumed for the tenant is not really adopted in most countries. In contrast to the owner-occupier, the tenant must rather pay all of his interest expenses from taxed income, even if his debts are entirely used to finance accommodation. Hence, the investment approach does in practice not fully remove the tax advantage of owner-occupied housing, if the tenant's optimal lifetime consumption plan should imply some borrowing (see the example in Table II below). Only from this taxation bias does it follow that individuals with a strong preference for present consumption, and also individuals with a relatively low income in their early periods of life, will tend to buy rather than rent their accommodation.

¹⁸ Remember that in Period 3 the property is sold and, hence, cumulated net investment becomes zero.

¹⁹ See appendix AIV

In our example above, i.e. with $t = 0.4$, $q = 1.5$, $\mathbf{w} = (1;3;2)$, $\beta = 0.6$ and $\mathbf{a} = (1.1;1.0;0.9)$, the owner-occupier achieves both a consumption bundle \mathbf{c}^* and a utility level U^* , that is still above the maximum utility to be achievable for the tenant, if the latter is not eligible to set off interest charges against his tax liability (see Table II).²⁰ Only if the initial vectors \mathbf{w} and $\mathbf{a} \equiv (\alpha_1; \alpha_2; \alpha_3)$ were such that no debt must be taken by the tenant for achieving his optimal consumption bundle \mathbf{c} , the advantage of the owner-occupier would vanish and, hence, the investment approach would be appropriate (see section 4 below).

Table II: Optimal Consumption Pattern with different Taxation Schemes

	Tenant (with interest payments non- deductible)	Owner-occupier, Consumption good approach	Owner-occupier, investment approach (imputed rent)	Owner-occupier, investment approach (imputed interest)
consumption c_1	1.5351	1.7111	1.7126	1.7126
consumption c_2	2.0934	2.3333	2.0240	2.0240
consumption c_3	2.4492	2.7300	2.3681	2.3681
savings s_1	-0.4761	-1.5666	-1.9641	-1.7007
savings s_2	0.2731	-1.1389	-1.3438	-1.0325
savings s_3	0	0	0	0
Utility level U	2.7908	3.2273	2.9526	2.9526

The same results as with imputed rent taxation can be achieved by an alternative version of the investment approach, where the owner-occupier has to pay a tax on the imputed interest receipt from his net capital instead of a tax on imputed rental income (see the last column in Table II). The taxation formula then changes to

$$(12''') \quad T_j = ti(k_{j-1} + e_j + s_j - s_{j-1}) = it(c_{j-1} + s_{j-1}) \quad ,$$

²⁰ As the algebraic derivation of the general optimality conditions for this case would be rather cumbersome, the respective figures in the first column of Table II have been derived by numerical methods.

where k denotes net capital, which is initially zero and rises by e_j plus the respective net financial saving $s_j - s_{j-1}$ in each period j . The insertion of (12'') instead of (12) into restrictions (8) to (10) yields the same quantities of optimal residential consumption (5) to (7) as with the conventional investment approach.²¹ Note that no imputed rent has to be assessed with this version of the investment approach. Rather, the taxable base can simply be fixed by multiplying the market interest rate by the net value of the property.

It is sometimes argued by real estate practitioners that the investment approach is advantageous for the owner-occupier, as compared to the consumer good approach, if interest rates are high and preferences are such that high levels of debt must be incurred to optimise lifetime consumption. However, in the light of our analysis, this statement appears to be wrong. The consumption good approach is definitely the preferred taxation scheme from the owner-occupier perspective. This can readily be seen from the imputed interest version of the investment approach. Because the internal rate of return for the owner-occupied dwelling must equal the market interest rate, from that approach, it follows that the tax burden for the owner-occupier is a monotonically increasing function of the interest rate.²²

4. Integrating Taxation and Allowances for Retirement Provision

The relevant literature almost uniquely suggests abandoning the tax advantage of owner-occupied housing in order to ensure efficiency of capital allocation. As our analysis has shown, an adoption of the investment approach would not even be enough. An equal treatment of both the tenant and the owner-occupier would rather require to make the former's interest charges equally deductible as mortgage interests. Only that would really put the tenant on par with the owner-occupier in case that his optimal lifecycle plan requires borrowing in

²¹ See appendix AV

²² Note that also the imputed rent is positively related to the interest rate because of equation (11).

any period. For example, with $q = 1.5$, $t = 0.4$, $\mathbf{w} = (1;3;2)$, $\beta = 0.6$ and $\boldsymbol{\alpha}_j = (1.1;1.0;0.9)$, we achieve the optimal consumption plans shown in Table III, all of which yield the same levels of both consumption and utility. Note that periodical savings and also periodical tax payments nevertheless differ substantially with the respective taxation rules.

Table III: Optimum Consumption Plans with Fully Deductible Interests

	Tenant, with interest payments fully deductive	Owner-occupier, investment approach with taxing imputed rent	Owner-occupier, investment approach with taxing imputed interest receipts
consumption c_1	1.7126	1.7126	1.7126
consumption c_2	2.0240	2.0240	2.0240
consumption c_3	2.3681	2.3681	2.3681
savings s_1	-0.6467	-1.9641	-1.7007
savings s_2	0.2131	-1.3438	-1.0325
savings s_3	0	0	0
Tax payment T_1	0	0.2635	0
Tax payment T_2	-0.1293	-0.0814	0.0024
Tax payment T_3	0.0426	-0.2688	0.1983
Utility level U	2.9526	2.9526	2.9526

One might object to the full deductibility of interests for the tenant that it could be difficult to separate housing-related debt from his other borrowing in practice. However, it is not at all clear that such a distinction would really be useful. Why not pre-draw consumption by buying vehicles, jewellery or human capital instead of building a home? Moreover, even the purpose for which an owner occupier really uses his credits cannot ultimately be controlled. Hence, both from the theoretical and from the pragmatic point of view, interest allowances should not be attempted to be earmarked by the taxation authorities. The investment approach would therefore best be combined with full deductible interests on any private debt of everyone. This, combined with the imputed interest version of the investment approach, would result in a both elegant and viable taxation rule.

From a more general perspective, however, it is thoroughly debateable whether the investment approach is appropriate at all. This is not only because many people, who are less familiar with optimum taxation theory, feel uncomfortable with paying income taxes on their property although they do not earn any real money from it. In fact, there are both many other durable goods and non-market services which are left untaxed in all but every country, although they could be subject to very similar arguments as for the taxation of imputed rent. Obvious examples are private cars (which could also serve as a taxi), private gardening (instead of working as a professional gardener) or DIY (rather than employing a craftsman). Not least, the services which a housewife provides to her husband and family are left tax-free, whereas the same services, if bought in the market, have to bear a tax burden. All this leads to substantial misallocations from a purely economic point of view, but yet no one even thinks of taxing these goods.

Moreover, the issue of optimal housing taxation should also be viewed from the broader scope of the demographical challenge. Due to the default of public pension systems, public assistance for private retirement has now become a most relevant issue. In particular, there are tax allowances for private financial savings in many states, which could readily outweigh the tax advantage of owner-occupied accommodation. Therefore, an integrated taxation approach on these issues seems appropriate.

A very simple solution to the problem would be to abandon any taxation of interest. However, implying both a possible breakdown of tax revenues and a violation of widely acknowledged principles of justness, this radical approach appears far removed from real political options and is, therefore, not pursued here in more detail.

However, the less far-reaching approach of deferred income taxation is well worthwhile to be examined some closer, in particular because it is already adopted on private retirement provisions in many countries, although up to now only to a limited extent (Boersch-Supan 2004).

Deferred income taxation is closely related to a cash flow tax, making all private retirement outflows deductible and all private pension payments (i.e. the inflows) taxable. For simplicity, by sketching the consequences of this tax rule within our model, we assume that it is generally adopted on all interest income. As is well known from the literature on cash flow taxation (Katz 1999, 5), both the after-tax-rate of return and the interest rate will thereby be lastly unaffected. The rent/house price relation (11) can now be written as

$$(11'') \quad 0 = (-e + r_t e)(1-t) + \frac{r_t e(1-t)}{q} + \frac{e(1-t)}{q^2}$$

$$\Rightarrow r_t = 1 - \frac{1}{q} = \frac{i}{1+i}$$

With the deferred income taxation approach adopted to the tenant, the tax rule changes from Equation (12) to

$$(12''') \quad T_j = t((1+i)s_{j-1} - s_j)$$

which is now also relevant for the owner-occupier. Equation (12''') implies that both the tenant and the owner-occupier pay income taxes on every inflow of interest or withdrawal from previous savings, while each interest payment or new financial saving s is deductible. Note that, other than for the investment approach discussed above, neither housing purchases e nor imputed rent re is any longer relevant.

As can be shown in our example above, the deferred income taxation approach yields the same optimum solution (5) to (7) as if there were no taxes on interest income at all. This is

because it ultimately implies a zero present value of the tax burden, with alternating signs of tax payments within the respective periods of lifetime (see Table IV below).²³

Table IV: Optimum consumption plans with deferred income taxation

	Tenant, with deferred income taxation	Owner-occupier, with deferred income taxation	Owner-occupier, without income taxation at all
consumption c_1	1.7111	1.7111	1.7111
consumption c_2	2.3333	2.3333	2.3333
consumption c_3	3.1500	3.1500	3.1500
savings s_1	-0.7099	-2.6110	-1.5665
savings s_2	0.6945	-1.8978	-1.1392
savings s_3	0	0	0
Tax payment T_1	0.2840	1.0444	0
Tax payment T_2	-0.7038	-0.8075	0
Tax payment T_3	0.4167	-1.1387	0
Utility level U	3.3979	3.3979	3.3979

This approach apparently removes the tax advantage of owner-occupied housing, namely by generally leaving private retirement savings untaxed (in terms of present value at least) instead of imposing imputed taxes on self occupied homes. It thereby should also remove a bulk of assessment problems and bureaucracy, and, not at least, the old bone of contention called owner occupier taxation privilege. This means of course also a loss in tax receipts, as it is implied by any grants to spur private retirement provision. Therefore, the way towards deferred income taxation can be gone – and actually is gone - only by small steps.

5. Concluding remarks

Within the limits of our model, it has been shown that there is no principal advantage or disadvantage of owning vs. renting ones home. However, income taxation can – and usually

²³ The optimum quantities of the non residential good are still given by $c_j^* = r_j c_j^* \beta / (1 - \beta) \quad \forall j$.

does - create a bias in favour of owner-occupied housing. This applies in particular to the consumption good approach, which is definitely more advantageous for the owner-occupier than the investment approach. Concerning the latter, it has been shown that taxing the imputed interest on net capital in each period would be a perfect equivalent to the conventional imputed rent approach. It has also been argued that private interest payments should be made generally deductible, irrespective of the purpose of private borrowing.

Our main point to make here, however, is the integration of both efficient housing taxation and private retirement provision allowances. The deferred income taxation approach is already adopted in many countries for private retirement provision. Its general adoption to private long term savings would remove the housing taxation advantage in a most elegant way, thereby providing a both efficient and just public assistance for private retirement provision at the same time.

Several limits of our analysis must be regarded. In particular, no explicit allowance was made for depreciation, maintenance and repair. We also neglected inflation, and our analysis is confined to perfect capital markets and proportional income taxation. It might, nevertheless, be helpful in designing practical taxation rules for housing and retirement provisions.

Appendix

(AI) We first prove that equations (5) to (7) solve the tenant's optimisation problem with respect to restrictions (1) to (3). This can also be expressed as

$$(A1) \text{Max}_{\mathbf{c}; \mathbf{c}^{\cdot}} U(\mathbf{c}; \mathbf{c}^{\cdot}) = \left(c_1^{\alpha_1} c_2^{\alpha_2} c_3^{\alpha_3} \right)^{\beta} \left(c_1^{\alpha_1} c_2^{\alpha_2} c_3^{\alpha_3} \right)^{1-\beta}$$

s.t.

$$(A2) \sum_{j=1}^3 (q^{j-1} r c_j + q^{j-1} c_j^{\cdot}) = \sum_{j=1}^3 q^{j-1} w_j$$

For the partial derivations of the utility function we find

$$(A3) \frac{\delta U / \delta c_j}{\delta U / \delta c_{j+1}} = \frac{\alpha_j c_{j+1}}{\alpha_{j+1} c_j}$$

$$(A4) \frac{\delta U / \delta c_j^{\cdot}}{\delta U / \delta c_{j+1}^{\cdot}} = \frac{\alpha_j c_{j+1}^{\cdot}}{\alpha_{j+1} c_j^{\cdot}}$$

$$(A5) \frac{\delta U / \delta c_j}{\delta U / \delta c_j^{\cdot}} = \frac{(1-\beta) c_j^{\cdot}}{\beta c_j}$$

Solving the Lagrangian yields

$$(A6) \frac{\delta U}{\delta c_j} = \lambda r q^{3-j}$$

$$(A7) \frac{\delta U}{\delta c_j^{\cdot}} = \lambda q^{3-j}$$

from which it follows that

$$(A8) \frac{\delta U / \delta c_j}{\delta U / \delta c_{j+1}} = \frac{\delta U / \delta c_j^{\cdot}}{\delta U / \delta c_{j+1}^{\cdot}} = q$$

$$(A9) \frac{\delta U / \delta c_j}{\delta U / \delta c_j^{\cdot}} = r$$

Hence we lastly have

$$(A10) c_{j+1} = \frac{\alpha_{j+1}}{\alpha_j} q c_j$$

$$(A11) \dot{c}_{j+1} = \frac{\alpha_{j+1}}{\alpha_j} q \dot{c}_j$$

$$(A12) \dot{c}_j = \frac{\beta}{1-\beta} r c_j$$

Inserting the corresponding relations into the budgetary restriction, after some manipulations of terms, yields solution (5) to (7), q.e.d.

(AII) It is now proved that equations (5) to (7) also solve the owner-occupier's maximization problem. Adding the latter's budgetary restrictions (8) to (9) yields

$$(A13) q^2 w_1 + q w_2 + w_3 + c_1(q - q^2) + c_2(1 - q) - r c_3 - c_1 \dot{q}^2 - c_2 \dot{q} - c_3 \dot{=} 0$$

From equilibrium condition (11) it is known that $(1 - q) = -r q$ resp. $(q - q^2) = -r q^2$, the insertion of which into (A13) yields restriction (A2) and, hence, the same solution as in the tenant's case, q.e.d.

(AIII) Next we show that, with the taxation scheme (12), equations (5') to (7') solve the tenant's maximisation problem. After some manipulations of terms, (12) inserted into (1') to (3') yields

$$(A14) (r_1 c_1 + c_1')(1 + i - it)^2 + (r_1 c_2 + c_2')(1 + i - it) + r_1 c_3 + c_3' = w_1(1 + i - it)^2 + w_2(1 + i - it) + w_3$$

Solving the Lagrangian now yields

$$(A15) \frac{\delta U}{\delta c_j} = \lambda r_i (1 + i - it)^{3-j}$$

$$(A16) \frac{\delta U}{\delta \dot{c}_j} = \lambda (1 + i - it)^{3-j}$$

From that and equations (A4) to (A6) it is easily derived that now

$$(A17) \frac{\delta U / \delta c_j}{\delta U / \delta c_{j+1}} = \frac{\delta U / \delta \hat{c}_j}{\delta U / \delta \hat{c}_{j+1}} = (1+i-it)$$

$$(A18) \frac{\delta U / \delta c_j}{\delta U / \delta \hat{c}_j} = r_t$$

Hence we lastly have

$$(A19) c_{j+1} = \frac{\alpha_{j+1}}{\alpha_j} (1+i-it) c_j$$

$$(A20) \hat{c}_{j+1} = \frac{\alpha_{j+1}}{\alpha_j} (1+i-it) \hat{c}_j$$

$$(A21) \hat{c}_j = \frac{\beta}{1-\beta} r_t c_j$$

Inserting (A19) to (A21) into (A14) finally yields equations (5[^]) to (7[^]), q.e.d.

(AIV) We now show that (5[^]) to (7[^]) solve the owner-occupier's maximisation problem, if the investment approach of taxation is adopted. For the conventional imputed rent version of the investment approach we have (12^{^^}), the insertion of which into restrictions (8[^]) to (10[^]) yields

$$(A22) c_1(1+i-it) + c_2 + \frac{r_t c_3 + \hat{c}_3}{1+i-it} + c_1[(1+tr_t)(1+i-it)-1] + c_2 \left[(1+tr_t) - \frac{1}{1+i-it} \right] \\ = w_1(1+i-it) + w_2 + \frac{w_3}{1+i-it}$$

Substituting r_t by the equilibrium condition (11[^]) in the squared brackets and rearranging terms finally yields the tenant's restriction (A14) again. Hence, solution (5[^]) to (7[^]) is valid also for the owner occupier, q.e.d.

(AV) The same is true if the imputed interest approach is adopted instead of the conventional imputed rent approach. Inserting (12^{^^^}) instead of (12^{^^}) into restrictions (8[^]) to (10[^]) yields

$$(A23) c_1(1+i-it) + c_2 + \frac{r_t c_3 + \hat{c}_3}{1+i-it} + c_1 i + \frac{c_2 i}{1+i-it} = w_1(1+i-it) + w_2 + \frac{w_3}{1+i-it}$$

From equilibrium condition (11') it follows that $i = (1 + i - it)r_t$, the insertion of which into the last two terms on the left hand side of (A23) again yields the tenant's restriction (A14), q.e.d.

References

Aaron, H. (1970), Income Taxes and Housing, *The American Economic Review*, Vol 60, No. 5, pp. 789-806.

Artle, Roland/Varaiya, Pravin (1978), Life Cycle Consumption and Homeownership, *Journal of Economic Theory* 18, 38-58.

Berkovec, James (1989), A General Equilibrium Model of Housing Consumption and Investment, *Journal of Real Estate Finance and Economics* 2, 157-172.

Boersch-Supan, Axel (2004), Mind the Gap: The Effectiveness of Incentives to Boost Retirement Saving in Europe, working paper, Mannheim

Boskin, Michael J. (1975), Efficiency Aspects of the Differential Tax Treatment of Market and Household Economic Activity, *Journal of Public Economics* 4, 1-25.

Broadbent, Ben/Kremer, Michael (2001), Does favourable tax-treatment of housing reduce non-housing investment? *Journal of Public Economics* 81, 369-391.

Conesa, J.C./Kitao, S./Krueger, D. (2007), Taxing Capital? Not a Bad Idea After All! *Mimeo. (Revised and resubmitted to the American Economic Review)*

Diamond, P.A. (1965), National Debt in a Neoclassical Growth Model, *The American Economic Review*, Vol. 55, No. 5, Part 1, pp. 1126-1150

Feldstein, Martin (1978), The Welfare Cost of Capital Income Taxation, *The Journal of Political Economy* 86, 29-51

Flavin, Marjorie/Nakagawa, Shinobu (2004), A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence, *NBER Working Paper* 10458, Cambridge MA.

Gervais, Martin (2002), Housing taxation and capital accumulation, *Journal of Monetary Economics* 49, 1461-1489.

Goode, R. (1960), Imputed Rent of Owner-Occupied Dwellings Under the Income Tax, *The Journal of Finance*, Vol. 15, No. 4, pp 504-530.

Grossman, Sanford J./Laroque, Guy (1990), Asset pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods, *Econometrica* 58, 25-51.

Hamilton, Bob/Whalley, John (1985), Tax Treatment of Housing in a Dynamic Sequenced General Equilibrium Model, *Journal of Public Economics* 27, 157-175.

Hendershot, Patric.H./Hu, Sheng Cheng., The allocation of capital between residential and non-residential uses: Taxes, inflation and market constraints, *Journal of Finance* 38, 795-812

Hendershott, Patric H./Won, Yunhi (1992), Introducing risky housing and endogenous tenure choice into a portfolio-based general equilibrium model, *Journal of Public Economics* 48, 293-316.

Henderson, J.V./Ioannides, Y.M. (1983), A Model of Housing Tenure Choice, *American Economic Review* 73, 98-113.

Van der Hoek, M.P./Radloff, S. (2007), Taxing owner-occupied housing: comparing the Netherlands to other European Union countries, *MPRA Paper* No. 5876, Rotterdam

Hoyt, W.H./Rosenthal, S.S. (1990), Capital Gains and the Demand for Owner-Occupied Housing, *The Review of Economics and Statistics*, Vol. 72, No. 1, pp 45-54.

Katz, D.(1999), Towards a Practical Cash-Flow Tax, *Treasury Working Paper* 99/1, New Zealand.

King, M.A./Dicks-Mireaux, L.-D. L.(1982), Asset Holdings and the Life-Cycle, *The Economic Journal* 92, 247-267.

Kotlikoff, L.A. (2006), Paul Samuelson's Amazing Intergenerational Transfer," in *Samuelsonian Economics and the 21st Century*, Michael Szenberg, editor, Oxford University Press

Leung, Charles (2004), Macroeconomics and housing: a review of the literature

Merz, P.E. (1965), The Income Tax Treatment of Owner-Occuopied Housing, *Land Economics*, Vol 41, No. 3, pp 247-255.

Nakagami, Yasuhiro (1996), Budgetary and Efficiency Effects of Housing Taxation in the United States, *Journal of Urban Economics* 39, 68-86.

Van Order, Robert (1990), Housing, Taxes, and Capital Allocation, *Journal of Public Economics* 42, 387-398.

Pines, David/Sadka, Efraint/Sheshinski, Eytan (1985), The Normative and Positive Aspects of the Taxation of Imputed Rent on Owner-Occupied Housing, *Journal of Public Economics* 27, 1-23.

Rosen, Harvey S.(1979), Housing Decisions and the U.S. Income Tax, *Journal of Public Economics* 11, 1-23

Samuelson, P.A. (1958), An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money, *The Journal of Political Economy*, Vol. 66, No. 6, pp. 467-482

Skinner, Jonathan (1996), The dynamic efficiency cost of not taxing housing, *Journal of Public Economics* 59, 397-417.

Turnovsky, Stephen (1994), Taxes, housing, and capital accumulation in a two-sector growing economy, *Journal of Public Economics* 53, 245-267.