Exclusionary Pricing and Rebates When Scale Matters*

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Abstract

We consider an incumbent firm and a more efficient entrant, both offering a network good to several asymmetric buyers. The incumbent disposes of an installed base, while the entrant has a network of size zero at the outset, and needs to attract a critical mass of buyers to operate. We analyze different price schemes (uniform pricing, implicit price discrimination - or rebates, explicit price discrimination) and show that the schemes which - for given market structure - induce a higher level of welfare are also those under which the incumbent is more likely to exclude the rival.

JEL classification: L11, L14, L42.

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1 Introduction

This paper deals with exclusionary pricing practices, that is anti-competitive pricing behavior by a firm endowed with a “dominant position” (as called in the EU), or with “monopoly power” (as called in the US). One such practice which has recently received renewed attention is *rebates*, i.e. discounts applicable where a customer exceeds a specified target for sales in a defined period.

There are different types of rebates, or discounts. They can be made contingent on the buyer making most or all of its purchases from the same supplier (”fidelity” or ”loyalty” rebates), on increasing its purchases relative to previous years, or on purchasing certain quantity thresholds specified in absolute terms. It is on this last category of rebates that we focus here.

In the US, rebates have received a very favorable treatment by the courts for many years. Under US case law (see e.g. the Virgin v. British Airways (2001) case), loyalty rebates were said to promote competition on the merits as a rule, and it was for the plaintiff to demonstrate their anticompetitive effect.[1] However, the recent *LePage* (2003) decision - in which the Appeal Court reversed an earlier judgment and found 3M guilty of attempted monopolization for having used (bundled) rebates - may signal the willingness of the judges to use lower standards of proof for the finding of anticompetitive rebates.

In the EU, rebates have long been looked at with suspicion by the European Commission (which is the EU Competition Authority) and the Community Courts, which have systematically imposed large fines on dominant firms applying different forms of rebates.[2] But until the recent *Michelin II* judgment, dominant firms were at least allowed to grant pure quantity discounts, that is standardized rebates given to any buyer whose purchases exceed a predetermined number of units; *Michelin II*, instead, has established that even pure quantity discounts are anticompetitive if used by a dominant firm.[3,4]

One of the objectives of this paper is to take seriously the Community Court’s assessment, and study whether rebates, in the form of pure quantity discounts, can have anticompetitive effects. A key feature of the environment we consider are scale effects, that we choose to model as scale economies on the demand side (but the main insights of the paper would also hold good with production scale economies, as we explain in Section 6).

More precisely, we study an industry exhibiting network effects, and we find that if rebates are allowed, an incumbent firm having a critical customer base

[1] See Kobayashi (2005) for a review of the US case law.

[2] For a review of the EU case law on rebates, see e.g. Gyselen (2003).

[3] Unless they are ’objectively’ justified, that is unless the dominant firm can prove that the discount matches savings from transaction costs.

[4] The (almost) per se illegal status of exclusive contracts, rebates and discriminatory prices by dominant firms in the EU, as well as the difference relative to their treatment in the US (at least until recently), has led to a hot debate on the EU policy towards abuse of dominance. See Gual et al. (2006) for a contribution to the debate.

is more likely to exclude a more efficient entrant that can use the same rebate schemes but does not have a customer base yet. Rebates are a form of implicit discrimination, and the incumbent can use them to make more attractive offers to some crucial group of consumers, thereby depriving the entrant of the critical mass of consumers it needs (in our model, network externalities imply that consumers will want to consume a network product only if demand has reached a critical threshold).

Now, discrimination (implicit and even more so explicit discrimination) will allow the incumbent to play off the different groups of consumers against each other. This strategic use of price discrimination will exacerbate the coordination problems that buyers face, which in turn makes entry even more difficult for the new rival. Only very efficient entrants will be able to overcome the entry barriers that incumbents can raise in this manner.

To give an example of the type of industry that we have in mind, let us briefly review the *Microsoft Licensing Case* of 1994-95 (Civil Action No. 94-1564). Microsoft markets its PC operating systems (Windows and MS-DOS) primarily through original equipment manufacturers ("OEMs"), which manufacture PCs, and has agreements with virtually all of the major microcomputer OEMs. When discussing the substantial barriers to entry for potential rivals of Microsoft, the Complaint explicitly mentions "the difficulty in convincing OEMs to offer and promote a non-Microsoft PC operating system, particularly one with a small installed base". Moreover, "it would be virtually impossible for a new entrant to achieve commercial success solely through license agreements with small OEMs that are not covered by Microsoft's (...) agreements."

The US Department of Justice alleges that Microsoft designed its pricing policy "to deter OEMs from entering into licensing agreements with competing operating system providers", thereby reinforcing the entry barriers raised by the network effects that are inherent in this industry. In particular, the use of two-part tariffs, with high fixed fees and zero per-copy price, is considered strongly anti-competitive. Interestingly, though, the Final Judgment explicitly allows Microsoft to continue granting "volume discounts" (i.e. rebates), as long as Microsoft would use linear prices rather than two-part tariffs. It is not clear why two-part pricing and quantity discounts are treated differently, but our paper does suggest that such rebates, being a form of (implicit) price discrimination as well, can be exclusionary.

Although rebates may have exclusionary effects, it is far from clear that they should be presumed to be welfare-detrimental, even if used by a dominant firm. As John Vickers, then Chairman of the UK Office of Fair Trading, put it:

> "These cases about discounts and rebates, on both sides of the Atlantic, illustrate sharply a fundamental dilemma for the competition law treatment of abuse of market power. A firm with market power that offers discount or rebate schemes to dealers is likely to sell more, and its rivals less, than in the absence of the incentives. But that is equally true of low pricing generally." (Vickers, 2005: F252)

Discriminatory pricing has similar contrasting effects. Consider for instance an oligopolistic industry. On the procompetitive side, it allows firms to decrease prices to particular customers, thereby intensifying competition: each firm can be more aggressive in the rival's customer segments while maintaining higher prices with the own customer base, but since each firm will do the same, discriminatory pricing will result in fiercer competition than uniform pricing, and consumers will benefit from it.[5] On the anticompetitive side, though, in asymmetric situations discriminatory pricing may allow a dominant firm to achieve cheaper exclusion of a weaker rival: prices do not need to be decreased for all customers but only for the marginal customers.[6]

This fundamental dilemma between, on the one hand, the efficiency effects (consumers would buy more and pay less) created by rebates and discriminatory pricing and, on the other hand, their potential exclusionary effects (rival firms would be hurt by such practices, and may be driven out of the market), is possibly the main theme of the paper. Indeed, we shall study here different pricing schemes that both an incumbent and a rival firm can adopt, and show that the schemes which - for given market structure - induce a higher level of welfare are also those under which the incumbent is more likely to exclude the rival. More specifically, we show that explicit price discrimination is the pricing scheme with the highest exclusionary potential (and hence the worst welfare outcomes if exclusion does occur), followed by implicit price discrimination (i.e., rebates, or pure quantity discounts) and then uniform pricing. However, for given market structure (i.e., when we look at equilibria where entry does occur), the welfare ranking is exactly reversed: the more aggressive the pricing scheme the lower the prices (and thus the higher the surplus) at equilibrium. This trade-off between maximizing the entrant's chances to enter and minimizing welfare losses for given market structure, illustrates the difficulties that antitrust agencies and courts find in practice: a tough stance against discounts and other aggressive pricing strategies may well increase the likelihood that monopolies or dominant positions are successfully contested, but may also deprive consumers of the possibility to enjoy lower prices, if entry did occur.

Although it deals with pricing schemes rather than contracts, our paper is closely related to the literature on anticompetitive exclusive dealing. Since Segal and Whinston (2000) is probably the closest work to ours,[7] let us be more specific on the differences with their work. Building on Rasmusen et al. (1991), they show the exclusionary potential of exclusive contracts when the incumbent can discriminate on the compensatory offers it makes to buyers. Our study

[5] See Thisse and Vives (1998). For a recent survey on discriminatory pricing, see e.g. Stole (2005).

[6] See e.g. Armstrong and Vickers (1993).

[7] Bernheim and Whinston (1998) analyze the possible exclusionary effects of exclusive dealing when firms make simultaneous offers (as in our paper), but in *non-coincident markets*: first, exclusivity is offered to a buyer in a first market; afterwards, offers are made to a buyer in a second market. In their terminology, our paper is looking at *coincident market* effects, which makes our analysis closer to Aghion and Bolton (1985), Rasmusen et al. (1991), Segal and Whinston (2000) and Fumagalli and Motta (2006). All these papers, however, study only exclusive dealing arrangements and assume that the entrant can enter the market (if at all) only *after* the incumbent and the buyers have negotiated an exclusive contract.

differs from theirs in several respects: (i) in their game the incumbent has a (first-mover) strategic advantage in that it is allowed to contract with buyers before entry occurs; (ii) if buyers accept the exclusivity offer of the incumbent, they commit to it and cannot renegotiate it even if entry occurs; (iii) buyers are symmetric and only linear pricing is considered. In our paper, instead, (i) the incumbent and the entrant choose price schedules simultaneously, (ii) buyers simply observe prices and decide which firm to buy from (therefore avoiding any problems related to assumptions on commitment and renegotiation); (iii) we explore the role of rebates and quantity discounts in a world where buyers differ in size. Yet, the mechanisms which lead to exclusion in the two papers are very similar: both papers present issues of buyers' miscoordination, and scale economies which are created by fixed costs in their model are created instead by network effects in ours.

Our paper is also related to Innes and Sexton (1993, 1994), who also analyze the anticompetitive potential of discriminatory pricing. In their papers, however, they consider a very different contracting environment, strategic variables, and timing of the game. In particular, after the incumbent made its offers, they allow the buyers to contract with the entrant (or to enter themselves), so as to create countervailing power to the incumbent's. Despite all these differences, Innes and Sexton's insight that discrimination helps the incumbent to 'divide and conquer' consumers reappears in our paper, even if we also allow for the entrant to use the same discriminatory tools available to the incumbent, and even if contrary to Innes and Sexton's (1994) finding, in our case a ban on discrimination cannot prevent inefficient outcomes: in our setting, exclusion can arise also under uniform linear pricing.

Finally, our paper is related to the literature on incompatible entry in network industries. The very nature of network effects provides a strong incumbency advantage, shielding dominant firms against competitors even in the absence of any anticompetitive conduct (Farrell and Klemperer (2006)). Crémer et al. (2000) show that compatibility is a key variable in determining whether or not an entrant can successfully challenge an incumbent. Under incompatibility, entry equilibria may not even exist, and when they exist, the incumbent is likely to maintain a higher market share than under compatibility. Thus, if compatibility is a choice variable, the incumbent can use it strategically to deter entry. Where incompatibility could be overcome through multi-homing, Shapiro (1999) argues that incumbents can use exclusive dealing contracts to block multi-homing, thus excluding a technologically superior firm. Our paper adds to this literature in showing that even simple price discrimination can be sufficient for an incumbent to deter a more efficient firm in such network industries.

The paper continues in the following way. Section 2 describes the model, Section 3 solves the model under the assumption that prices have to be non-negative. Three cases are analyzed: uniform pricing, explicit (or 3rd degree) price discrimination and implicit (or 2nd degree, or rebates) price discrimination. Section 4 studies the effects of the different pricing schemes on consumer surplus. Section 5 discusses some extensions of the model. First, we consider the possibility that firms subsidize customers' usage, i.e., can charge negative linear prices; then, we turn to the case of elastic (linear) demands, allowing for both linear and two-part tariffs; finally, we discuss the case of full (or buyer-specific) discrimination (in the base model we do not allow firms to discriminate across identical buyers). Section 6 concludes the paper.

2 The setup

Consider an industry composed of two firms, the incumbent I, and an entrant E. The incumbent supplies a network good, and has an installed consumer base of size $\beta_I > 0$. (The network good is durable: "old" buyers will continue to consume it but no longer need to buy it.) I incurs constant marginal cost $c_I \in (0, 1)$ for each unit it produces of the network good.

The entrant can supply a competing network good at marginal cost $c_E < c_I$, i.e. it is more efficient than the incumbent. E has not been active in the market so far, that is it has installed base $\beta_E = 0$, but it can start supplying the good any time; in particular, when the game starts it does not have to sink any fixed costs of entry.

The good can be sold to $m + 1$ different "new" buyers, indexed by $j = 1, \ldots, m + 1$. There are $m \geq 1$ identical small buyers, and 1 large buyer.[8] Goods acquired by one buyer cannot be resold to another buyer, but they can be disposed of at no cost by the buyer who bought them (in case the latter cannot consume them). Side payments of any kind between buyers are ruled out. Define firm i's network size s_i (where $i = I, E$) as

$$s_i = \beta_i + q_i^1 + \ldots + q_i^{m+1} \tag{1}$$

i.e. the firm's installed base plus its total sales to all "new" buyers.

To simplify the analysis, we assume that demands are inelastic. (Section 5.2 presents the results for linear demand functions.) A buyer will either buy from the incumbent, or from the entrant (but not from both). The large buyer can consume at most $Q^l = 1 - k$ units, while any small buyer can consume at most $Q^s = \frac{k}{m}$ units. Buyers exert positive consumption externalities on each other: If firm i's network size s_i is below the threshold level $\bar{s}$, consumption of i's good gives zero surplus to its buyer.[9] The goods produced by the two firms

[8] We assume $m \geq 1$ so as to allow for the large buyer to be smaller than the set of all small buyers (which in turn allows for the large buyer to receive better price offers) and to show that prices under rebates depend on the degree of fragmentation of small buyers (and converge to prices under explicit discrimination as $m \to \infty$).

[9] The assumption that a buyer's utility from consuming is positive only if the network in question reaches the threshold size $\bar{s}$ is designed to capture in an admittedly simple way the presence of network effects. Rather than assuming that the utility of a consumer increases continuously with network size, we assume a discontinuous formulation; this also has the advantage that the old generation of buyers can be safely ignored when studying welfare effects: since we shall assume that they have already attained the highest level of utility, new buyers' decisions will never affect old buyers' utility.

are incompatible, so that buyers of firm i do not exert network externalities on buyers of firm j. For a network good of sufficient size, large and small buyers have the same maximum willingness to pay of $\bar{p} = 1$.

We assume that

$$\beta_I \geq \bar{s} \tag{2}$$

i.e. the incumbent has already reached the minimum size, while the entrant's installed base is $\beta_E = 0$. In order to operate successfully, the entrant will have to attract enough buyers to reach $\bar{s}$.[10]

Let the unit prices offered by the two firms to a buyer of type $j = l, s$ be $p_I^j \leq 1$ and $p_E^j \leq 1$. Then, buyer j's demand functions for the incumbent's good, q_I^j, and for the entrant's good, q_E^j, are given by:[11]

$$q_I^j\left(p_I^j, p_E^j, s_I, s_E\right) = \begin{cases} Q^j & \begin{array}{c}\text{if } s_E \geq \bar{s} \text{ and } p_I^j \leq p_E^j \\ \text{or } s_E < \bar{s} \text{ and } -p_E^j < 1 - p_I^j\end{array} \\ 0 & \text{otherwise}\end{cases} \tag{3}$$

$$q_E^j\left(p_I^j, p_E^j, s_I, s_E\right) = \begin{cases} Q^j & \begin{array}{c}\text{if } s_E \geq \bar{s} \text{ and } p_E^j \leq p_I^j \\ \text{or } s_E < \bar{s} \text{ and } -p_E^j \geq 1 - p_I^j\end{array} \\ 0 & \text{otherwise}\end{cases} \tag{4}$$

where the large buyer's demand is $Q^l = 1 - k$, while the typical small buyer's demand is $Q^s = \frac{k}{m}$. If $s_E \geq \bar{s}$ and there is a tie in prices, $p_E^j = p_I^j$, the buyer may either buy from I or from E (we allow for both possibilities).

The parameter $k \in (0, 1)$ is an indicator of the relative weight of the small buyers in total market size: $1 - k$ measures the large buyer's market share, while k measures the market share of the group of small buyers. Assume that $1 - k > k/m$, so that the large buyer's demand is always larger than a small buyer's demand (provided they both demand strictly positive quantities). Note that the assumption $1 - k > k/m$ implies an upper bound on k, namely

$$k < \frac{m}{m+1} \in \left[\frac{1}{2}, 1\right). \tag{5}$$

Total market size is normalized to 1: $m(k/m) + (1 - k) = 1$.

Define a buyer's net consumer surplus as gross consumer surplus minus total

[10] Note that if the entrant manages to reach the minimum size $\bar{s}$, then consumers will consider I's and E's networks as being of homogenous quality, even if $s_I \neq s_E$.

[11] These demand functions apply for general (positive or negative) prices. In the base model we restrict prices to be non-negative. Section 5 considers the case where prices can be negative.

expenditure:

$$CS^l\left(p_i^l, q_i^l, s_i\right) = \begin{cases} q_i^l(1-p_i^l) & \text{if } s_i \geq \bar{s} \text{ and } l \text{ buys } q_i^l \leq 1-k \\ (1-k) - q_i^l p_i^l & \text{if } s_i \geq \bar{s} \text{ and } l \text{ buys } q_i^l > 1-k \\ -q_i^l p_i^l & \text{if } s_i < \bar{s} \text{ and } l \text{ buys } q_i^l \leq 1-k \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$CS^s\left(p_i^s, q_i^s, s_i\right) = \begin{cases} q_i^s(1-p_i^s) & \text{if } s_i \geq \bar{s} \text{ and } s \text{ buys } q_i^s \leq \frac{k}{m} \\ \frac{k}{m} - q_i^s p_i^s & \text{if } s_i \geq \bar{s} \text{ and } s \text{ buys } q_i^s > \frac{k}{m} \\ -q_i^s p_i^s & \text{if } s_i < \bar{s} \text{ and } s \text{ buys } q_i^s \leq \frac{k}{m} \\ 0 & \text{otherwise} \end{cases}$$

The demand functions defined above can be derived from these expressions of net consumer surplus.

Since both types of buyers have the same prohibitive price $\bar{p} = 1$, a monopolist who could charge discriminatory linear prices would set a uniform unit price $p_i^m = 1$. Thus, discriminatory pricing can arise only as a result of the strategic interaction between the incumbent and the entrant.

We assume that neither demand of the large buyer alone, nor demand of all small buyers taken together, is sufficient for the entrant to reach the minimum size:

$$\bar{s} > \max\ \{1-k, k\}\,. \quad (7)$$

In other words, in order to reach the minimum size, the entrant has to serve the large buyer plus at least one (and possibly more than one) small buyer.[12,13]

Note that only units which are actually consumed by a buyer count towards firm i's network size.

We also assume that the threshold level $\bar{s}$ is such that if the entrant sells to all $m+1$ new buyers, then it will reach the minimum size: $\bar{s} \leq 1$.

This, together with the assumption $c_E < c_I$, implies that the social planner would want the entrant (and not the incumbent) to serve all buyers.

The game. Play occurs in the following sequence: At time $t = 0$, the incumbent and the entrant simultaneously announce their prices, which will be binding in $t = 1$. At time $t = 1$, each of the $m+1$ buyers decides whether to patronize the incumbent or the entrant. We also assume that offers are observable to everyone, e.g. because they have to be posted publicly. Then, when the buyers have to decide which firm to buy from, the firms' offers will be common knowledge.

As for the prices that firms can offer in $t = 0$, in the base model (Section 3) we will restrict attention to linear pricing schemes, but we consider three different possibilities: (1) uniform prices (Section 3.1); (2) explicit (or third-degree) price

[12] If either $\bar{s} < 1-k$, or $\bar{s} < k$, then the miscoordination issues, which are at the heart of this paper, would not arise.

[13] We could assume in addition that $\bar{s} \leq 1-k+k/m$, so that the large buyer plus *exactly one* small buyer is sufficient for E to reach the minimum size. This assumption only changes the analysis of miscoordination equilibria when firms can charge negative prices.

discrimination (Section 3.2); and (3) the case of central interest, that is implicit (or second-degree) price discrimination, i.e. the case of standardized quantity discounts or "rebates" (Section 3.3).

Uniform pricing means that a firm must charge the same price to all buyers. Under explicit price discrimination, each firm can set one price for the large buyer, and a different price for the small buyers (all buyers of the same type will be charged the same price). Under implicit price discrimination, all buyers are offered the same price menu, where different prices apply depending on whether the buyer reaches a certain quantity threshold or not: if this menu is designed appropriately, buyers will self-select into different tariffs: small buyers will buy below the threshold, while the large buyer will buy above the threshold, and so the large buyer will end up paying a different price than the small buyers.

Explicit discrimination may not always be feasible, for instance because of informational constraints (firms cannot observe buyer types), or because of policy constraints (explicit discrimination is outlawed, as in the European Union). However, when buyers are asymmetric, pure standardized quantity discounts can induce de facto discrimination, and so allow firms to (imperfectly) replicate outcomes under explicit discrimination.

Section 5 will show that the main results are robust to changes in the assumptions we make in the base model on prices. There, we shall analyze the cases where prices can be negative, where demand is elastic, and where full price discrimination is allowed, that is firms can make buyer-specific offers (in the base model, we do not allow firms to discriminate among buyers of the same type).

3 Equilibrium solutions, under different price regimes

In this Section, we assume that firms set linear (and non-negative) prices, and we find the equilibria under the three different price regimes.

3.1 Uniform pricing

Assume that firms can only use uniform linear prices, p_i with $i = I, E$. Recall that any buyer's demand for E's good, $q_E^j(\ldots, s_E)$, depends on the size of E's network, s_E, which in turn depends on E's sales to the buyers, $\{q_E^1, \ldots, q_E^{m+1}\}$. Thus, in line with Segal and Whinston (2000), we find that our game has two types of pure-strategy Nash equilibria: where all buyers buy from the incumbent, and the other where all buyers (or sufficiently many) buy from the entrant. The following proposition shows the highest prices that can be sustained in each of these two types of equilibria.

Proposition 1 *(equilibria under uniform linear prices) If firms can only use uniform flat prices, the following two pure-strategy Nash equilibria exist under the continuation equilibria as specified:*

(i) ***Miscoordination equilibrium:*** *I sets* $p_I = p_I^m = 1$ *, E sets* $p_E = p_E^m = 1$*, and in all continuation equilibria where* $p_E \leq p_I$*, all buyers buy from* I*.*

(ii) ***Entry equilibrium:*** *E sets* $p_E = c_I$*, I sets* $p_I = c_I$*, and in all continuation equilibria where* $p_E \leq p_I$*, all buyers buy from E.*

The prices in (i) and (ii) are the highest that can be sustained in each type of equilibrium.

Proof: see Appendix A

To understand Proposition 1, note that there are two types of buyers' equilibria when $p_E \leq p_I$. First, there is a miscoordination equilibrium where all buyers buy from the incumbent: despite the higher price p_I, no buyer has an incentive to deviate, since the entrant's network would be below the critical size, and buying from the entrant would then give zero (gross) utility. Second, there is an equilibrium where all buyers buy from the entrant: no buyer has an incentive to deviate given that all others buy from the entrant, since he would pay a (weakly) higher price p_I for a product which is as good as the entrant's (if all buy, the entrant reaches critical size).[14]

Continuation equilibria play a role for the equilibrium at the firms' decision stage. Consider the candidate miscoordination equilibrium where $p_E = p_I = 1$ and all buyers buy from the incumbent. This equilibrium is sustained by having that when $p_E \leq p_I$ the chosen continuation equilibria are those where all buyers will buy from the incumbent.[15] Otherwise, a deviation by the entrant could attract all the buyers, undermining the candidate equilibrium. Likewise, consider the candidate entry equilibrium where $p_E = c_I = p_I$ and all buyers buy from the entrant. Because of the multiplicity of equilibria, when the incumbent deviates by increasing its price, there might also be a continuation equilibrium where $p_E < p_I$ and all buyers buy from the incumbent. To eliminate such counter-intuitive deviations, it is required that in all continuation equilibria where $p_E \leq p_I$ all buyers buy from the entrant.

The equilibria characterized in Proposition 1 represent extreme cases, in the sense that the underlying continuation equilibria are the most favorable ones for the firm that serves the buyers in equilibrium. *These equilibria are by no means the only equilibria that can arise in our game.*

For instance, there are other equilibria where all buyers do miscoordinate on the incumbent, but the latter can at most charge some price $\tilde{p}_I < p_I^m = 1$. Such an equilibrium can be sustained by continuation equilibria where buyers buy from I as long as $p_E \leq p_I \leq \tilde{p}_I$, but would switch to E if p_I exceeded $\tilde{p}_I$. Likewise, there are entry equilibria where the entrant must charge a strictly lower price than c_I to induce buyers to coordinate on E. *For the rest of the paper,*

[14] To be precise, if $p_I = p_E$, there is also a buyers' equilibrium where some buyers (in a sufficient number for the entrant to reach the critical size) buy from the entrant and the remaining buyers buy from the incumbent.

[15] In this situation, the entrant is indifferent among all prices $p_E \geq 0$ it could charge, and might as well offer the monopoly price, which weakly dominates all other possible equilibrium prices.

we will focus on those continuation equilibria which are the most profitable ones for the firm that eventually serves the buyers. The motivation for this choice is two-fold: First, these equilibria are the Pareto-dominant ones from the point of view of the firms. Second, from a policy point of view, the equilibria with the highest profits are those which cause most concern.

3.2 Explicit (3rd degree) discrimination

In this section, we first analyze miscoordination equilibria and then entry equilibria.

3.2.1 Miscoordination equilibria

Proposition 1 gives us the equilibrium for the case of uniform linear pricing. Assume now that the two firms can do 3rd degree (or explicit) discrimination, i.e. each firm chooses a pair of prices $\left(p_I^s, p_I^l\right)$, one price for the large buyer, and another for the small buyers (this is partial discrimination: firms cannot offer different prices to buyers of the same size).

With respect to the uniform pricing case, *nothing changes in the miscoordination equilibria*, the most profitable of which is for the Incumbent still the one where $p_I^s = p_I^l = p_I^m = 1$,[16] while the entrant sets $p_E^m = 1$ and all buyers buy from I. Clearly, the incumbent would have no incentive to deviate from this solution. No buyer would deviate either: if any of them decided to accept a lower price offered by the entrant given that all others buy from the incumbent, he would have zero surplus and would reduce his utility.

Proposition 2 *(miscoordination equilibria under explicit discrimination) Let each firm choose a pair of prices (p_I^s, p_I^l), one for each type of buyer. Under the appropriate continuation equilibria, the miscoordination equilibrium where all buyers buy from I exists for all parameter values. The highest sustainable prices are $p_I^s = p_I^l = p_E^s = p_E^l = p_I^m = 1$.*

Proof: Consider the following continuation equilibria: Following offers where either $p_E^s \leq p_I^s$ or $p_E^l \leq p_I^l$, or both, all buyers buy from I. Then, even if the entrant can charge different prices to both groups (where both prices may be strictly lower than I's prices), no single buyer will have an incentive to switch to the entrant as long as he expects all other buyers to buy from I: E's network cannot reach the minimum size with only one buyer, so its good gives zero utility, and as long as E charges a non-negative price for it, I's offer will (weakly) dominate E's offer. The rest of the proof is analogous to the proof of Proposition 1.□

Thus, the possibility to price discriminate does not allow the entrant to solve the miscoordination problem. Hence, miscoordination equilibria will continue to exist even if we allow for explicit price discrimination.

[16] Note that in our model the monopoly price charged by a firm under explicit discrimination will be the same for all buyers. This is clearly a special feature of the model, which simplifies the analysis without losing much insight.

3.2.2 Entry equilibria

For entry equilibria, things change relative to the uniform pricing case. To fix ideas, start with the candidate entry equilibrium where both firms charge c_I and all buyers buy from the entrant (we have seen that this is an entry equilibrium in the uniform linear pricing case). This equilibrium can be disrupted by the incumbent setting a price $c_I - \epsilon$ to one category of buyers and the monopoly price to the other category: the loss made on the former would be outweighed by the profits made on the latter. Indeed, under this deviation the former category strictly prefers to buy from I, thus preventing the entrant from reaching the minimum size, and the latter category would then prefer to buy from I rather than from the entrant, since they would derive zero utility from buying from E.

Therefore, an entry equilibrium can exist only if it is immune to the deviations outlined above, i.e. if the entrant's prices to both large and small buyers are so low that the incumbent cannot profitably undercut either of the two prices while charging the monopoly price to the other group. This implies that the highest prices that the entrant can charge in any entry equilibrium will be strictly below c_I. Thus, for an entry equilibrium to exist, the efficiency gap between entrant and incumbent must be large enough.

Proposition 3 *(entry equilibria under explicit discrimination) Under explicit price discrimination, entry equilibria only exist if*

$$c_I \geq \min\left\{\frac{1+c_E}{2}, k+c_E, 1-k+c_E\right\}.$$

The highest prices that the entrant can charge in any such entry equilibrium are

$$p_E^s = \begin{cases} \frac{c_I-(1-k)}{k} & \text{if } c_I \geq 1-k \\ 0 & \text{if } c_I < 1-k \end{cases} \quad \textit{and} \quad p_E^l = \begin{cases} \frac{c_I-k}{1-k} & \text{if } c_I \geq k \\ 0 & \text{if } c_I < k \end{cases}.$$

Proof: see Appendix A

Figure 1 illustrates the results of Proposition 3 (recall that miscoordination equilibria exist for all parameter values). The figure shows that, for given k, the larger c_I with respect to c_E, the more likely for entry to be an equilibrium of the game. The intuition is straightforward: if the incumbent is less efficient, it will find it more difficult to profitably make low (discriminatory) price offers to the buyers, which in turn makes it possible for the entrant to sustain higher (more profitable) prices which are immune to incumbent's deviations. The effect of k on equilibrium outcomes is slightly more complex. Entry is more likely at very low levels and very high levels of k. To understand why, consider for instance a candidate entry equilibrium (p_E^s, p_E^l) when k is very small. In order to disrupt this equilibrium, the incumbent could discriminate across buyers, by offering small buyers a very low price and recovering losses on these buyers by setting a

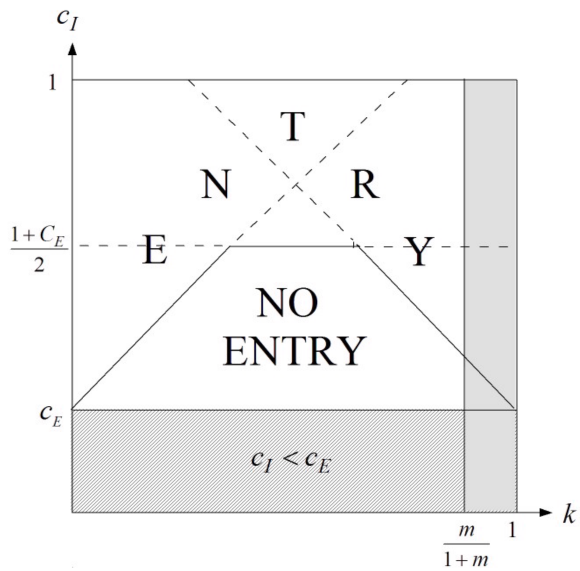

Figure 1: Regions where entry equilibria exist and do not exist under explicit price discrimination (the grey areas are outside of the parameter space)

high price to the large buyer, and vice versa. However, since k is very small, the incumbent cannot offer the large buyer a price (much) below c_I, since the profits it could make on the small buyers are very small (they account for a tiny part of the total market). In contrast, it could use the profits it makes on the (very) large buyer to decrease considerably the price offered to the small buyers. But since prices are restricted to be non-negative here, the incumbent's best offer to the small buyers will be $p_I^s = 0$. In order to avoid deviations, the entrant will therefore have to set $p_E^s = 0$ and p_E^l slightly lower than c_I. As small buyers account for a small proportion of demand (k is very small), the entrant will make positive profits at these prices, and the entry equilibrium will exist. The same argument can be used symmetrically to explain why entry equilibria are more likely to exist if k is sufficiently large. Of course, one important component of this result is that prices cannot go below zero. We shall see below that when prices may be negative, k will affect results monotonically.

To sum up:

- Exclusionary equilibria always exist, and the highest sustainable prices are exactly the same as under uniform linear pricing.
- Entry equilibria only exist if c_I is high enough relative to c_E. When they exist, note that the highest sustainable equilibrium prices are always

strictly below c_I (which is the highest sustainable equilibrium price under uniform linear pricing).

- With respect to uniform pricing, thus, price discrimination (i.e. a more aggressive pricing strategy): (a) on the one hand, makes exclusion more likely; (b) on the other hand, for given market structure, results in (weakly) lower prices.[17]

3.3 Implicit (2nd degree) discrimination (or rebates)

Let us now consider the case where firms cannot condition their offers directly on the type of buyer (large or small), but have to make uniform offers to both types which may only depend on the quantity bought by buyer $j = 1, \ldots, m+1$:

$$T_i(q_i^j) = \begin{cases} p_{i,1}q_i^j & \text{if } q_i^j \leq \bar{q}_i \\ p_{i,2}q_i^j & \text{if } q_i^j \geq \bar{q}_i \end{cases} \tag{8}$$

(If the buyer buys exactly the threshold quantity, $q_i^j = \bar{q}_i$, the firm may either charge $p_{i,1}$ or $p_{i,2}$.) Each buyer can now choose his tariff from this price menu by buying either below the sales target $\bar{q}_i$ or above it.

It is well-known that such quantity discounts or rebates, when applied to buyers who differ in size, will be a tool of (de facto) discrimination, even if the schemes as such are uniform. But to achieve discrimination, the tariffs have to be set in a way that induces buyers to self-select into the right bracket, with small buyers voluntarily buying below target, and the large buyer choosing to buy above it.

Consider the case where $\bar{q}_i < 1-k$. Then, the large buyer can either buy $1-k$ units at price $p_{i,2}$, which yields total surplus $CS^l\left(p_{i,2}, 1-k\right) = (1-k)(1-p_{i,2})$, or he can buy the threshold quantity $\bar{q}_i$ at price $p_{i,1}$ (i.e. a quantity which falls short of his actual demand at this price), in which case his net consumer surplus is $CS^l\left(p_{i,1}, \bar{q}_i\right) = (1-p_{i,1})\bar{q}_i$. If $p_{i,1}$ is sufficiently lower than $p_{i,2}$, it may be worthwhile for the large buyer to buy fewer units than he wants in return for a lower per unit price.[18]

Next, consider the case where $\bar{q}_i > k/m$. A typical small buyer $j = s$ will then have to choose between buying k/m units at price $p_{i,1}$, which yields total surplus $CS^s\left(p_{i,1}, k/m\right) = (1-p_{i,1})(k/m)$, or buying the sales target $\bar{q}_i$ at price $p_{i,2}$ (i.e. a quantity which exceeds his actual demand at this price), giving total surplus of $CS^s\left(p_{i,2}, \bar{q}_i\right) = \frac{k}{m} - p_{i,2}\bar{q}_i$. In this case, if $p_{i,2}$ is sufficiently lower

[17] In the entry equilibria, prices are strictly lower; in the exclusionary equilibrium, prices to both groups of buyers are the same as under uniform pricing.

[18] Assume that each buyer is only allowed one transaction. This rules out the possibility that a large buyer makes "multiple small purchases" so as to buy a large amount of units at the lower price. Presumably, important transaction costs may be invoked to justify this assumption, which in a way is nothing else than the counterpart of the assumption that a small buyer cannot buy a large quantity and then resell it to others. In both cases, it is arbitrage which is prevented.

than $p_{i,1}$, the small buyer will want to purchase more units than he can actually consume in order to qualify for a lower unit price.[19]

We say that firm i's offer satisfies the *"self-selection conditions"* if the large buyer prefers to buy above the threshold, and the small buyers prefer to buy below the threshold, i.e. if

$$\begin{aligned} CS^l\left(p_{i,2}, 1-k\right) &\geq CS^l\left(p_{i,1}, \bar{q}_i\right) \\ \text{and } CS^s\left(p_{i,1}, k/m\right) &\geq CS^s\left(p_{i,2}, \bar{q}_i\right) \end{aligned} \tag{9}$$

For any offer that satisfies the self-selection condition, denote $(p_{i,1})$ by (p_i^s), and $(p_{i,2})$ by $\left(p_i^l\right)$, for $i = I, E$.

We now look for the equilibria that arise in this game when both firms can use quantity discounts.

3.3.1 Miscoordination equilibria

Proposition 4 *(miscoordination equilibria under rebates) Let firms use rebates as defined in (8). Under the appropriate continuation equilibria, the miscoordination equilibrium exists for all parameter values, and the highest (monopoly) prices can be sustained at equilibrium.*

Proof: analogous proof as for the above cases.

Miscoordination arises under rebates for the same reason as under uniform pricing and explicit discrimination, which were discussed at length above. In the most profitable equilibrium, the incumbent does not actually offer a discount to either of the two groups, but charges the same (monopoly) price $p_I^s = p_I^l = 1$ to both large and small buyers. This offer trivially satisfies the self-selection conditions defined in (9).

3.3.2 Entry equilibria

The implicitly discriminatory effect of rebates gives rise to an exclusionary mechanism similar the one under explicit discrimination. Since buyers are asymmetric, they can be induced to self-select either into the high-quantity or the low-quantity bracket of the price menu, thus allowing the incumbent to de facto price-discriminate between them. This in turn enables the incumbent to offer a below-cost price to one group, thus winning their orders, while making up for the resulting losses by charging a high price (possibly the monopoly price) to the other group.

The major difference between explicit and implicit discrimination lies in the self-sorting conditions, which reduce the range of prices that the incumbent can offer. Consider for instance the case where, under explicit discrimination, the

[19] Recall that we exclude reselling of units between buyers (while allowing for free disposal), so the only thing a small buyer can do with units he cannot consume is to throw them away.

incumbent charges the monopoly price $p_I^s = 1$ to the small buyers, and $p_I^l = 0$ to the large buyer. Clearly, this offer does not satisfy the small buyers' self-sorting condition: At a zero price, the small buyers would always prefer to "buy" above the quantity threshold (i.e. receive a large quantity for free, and dispose of the units they cannot consume) rather than paying $p_I^s = 1$ (or any other positive price) for a small quantity.

Likewise, an offer where $p_I^s < c_I$ and $p_I^l = 1$ cannot be replicated through a rebate tariff: in this case, it is the large buyer who would prefer to buy below the threshold and enjoy a positive surplus on the (few) units he consumes, rather than buying above the threshold and being left with zero surplus.[20]

Thus, while rebates still have exclusionary potential, the incumbent's deviation offers will be less aggressive under rebates than under explicit discrimination, allowing for entry equilibria to be sustained where they do not exist if firms can explicitly price discriminate.

Proposition 5 *(entry equilibria under rebates) Under rebates as defined in (8), entry equilibria only exist if*

(i) $c_E < \frac{1}{2(m+1)}$ *and* $c_I \geq \min\left\{c_E(1+m), k+c_E, \frac{m}{1+m}+c_E-k\right\}$

(ii) or if $c_E \geq \frac{1}{2(m+1)}$ *and* $c_I \geq \min\left\{\frac{m+(1+m)c_E}{1+2m}, k+c_E, \frac{m}{1+m}+c_E-k\right\}$

The highest prices that the entrant can charge in any such entry equilibrium are

$$p_E^s = \begin{cases} 1-\frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1-k-k/m \\ 0 & \text{if } c_I < 1-k-k/m \end{cases}$$

$$p_E^l = \begin{cases} \frac{c_I-k}{1-k} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I}{(1-k)(m+1)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

Proof: see Appendix A

Corollary 6 *The parameter space for which entry equilibria exist under explicit discrimination is a proper subset of the parameter space for which entry equilibria exist under rebates.*

Proof: see Appendix A

Figure 2 illustrates the results of the analysis of entry equilibria under rebates and non-negative prices for the case where $c_E \geq \frac{1}{2(m+1)}$ (recall that miscoordination equilibria exist for all parameter values). We see that the region where entry equilibria do not exist is smaller under rebates than under explicit discrimination. While nothing changes for low values of k (rebates exactly replicate the outcome under explicit discrimination), exclusion becomes more difficult for

[20] Such a rebate scheme may appear as somewhat unorthodox, since buyers are "rewarded" for buying little and "penalized" for buying a lot. However, this is a deviation offer which will never be made in equilibrium.

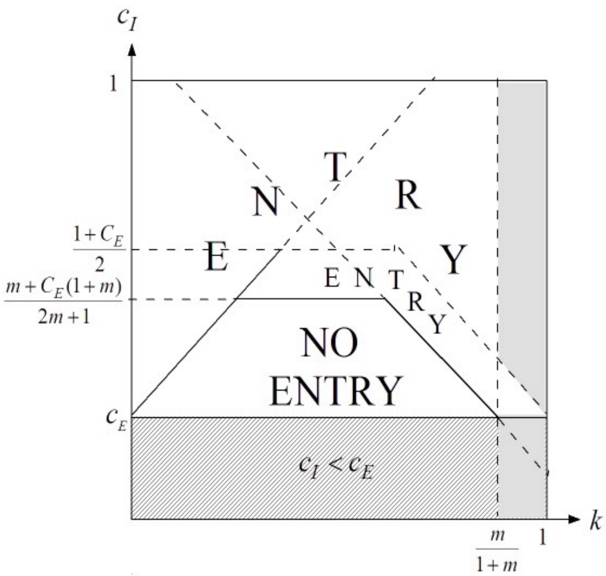

Figure 2: Regions where entry equilibria exist and do not exist under rebates (i.e. implicit price discrimination), compared to explicit discrimination

intermediate and high values of k. Intuitively, given m, the large buyer becomes smaller and smaller the higher k is, and so he becomes more and more similar to the small buyers, making it difficult to discriminate between them through rebates without violating any of the self-sorting conditions.

Note that as m grows, so that a single small buyer becomes smaller and smaller, both the efficiency thresholds and prices under rebates converge to the values under explicit discrimination. In the limit case where $m \to \infty$, the self-selection constraints play no role: the large buyer will never want to behave like a small buyer whose demand is infinitely small, and vice versa for the small buyer, and so the implicit and explicit discrimination cases coincide.

Let us take stock of the results obtained in this section. One of the motivations for this paper was to investigate whether rebates, in particular form of quantity discounts, can be exclusionary. This is especially important in situations where explicit discrimination is not allowed, and competition authorities need to understand how to deal with rebates, that is schemes that implicitly discriminate. Our analysis shows that indeed an incumbent firm could use rebates to exclude a more efficient rival (even if the latter can also make use of rebates). The main intuition is that by relying on quantity discounts the incumbent can (implicitly) discriminate across buyers by making attractive offers to some of them, thus subtracting to the rival firm buyers that it critically needs in order to reach the minimum viable size. Therefore, rebates reduce the likelihood that successful entry takes place.

Nevertheless, precisely because they imply competing aggressively for each group of buyers, rebates also have a procompetitive function: for given market structures (that is, if one compares regions where entry occurs, and we know that there always are parameter configurations for which all new buyers are served by the entrant), prices are lower when rebates are allowed than when prices have to be uniform. It is to explore more formally this basic trade-off between exclusion and lower prices that we now turn to an analysis of consumer welfare under the different price schemes.

4 Consumer Welfare

Recall from Section 2 that, in our model, entry is always socially efficient, because the entrant produces at a lower marginal cost than the incumbent. Thus, all miscoordination equilibria are inefficient. The higher production costs associated with having a less efficient firm serve the buyers are the only source of inefficiencies in our model: buyers have inelastic demand functions, so they will always consume the efficient quantities, no matter how high the prices are.

Yet, prices do matter, as they determine consumer surplus, which is often considered the objective function of antitrust agencies. Now, comparing equilibrium prices across different price regimes is not straightforward because each price regime gives rise to multiple equilibria, both entry and miscoordination equilibria, and each of these can be sustained by a broad range of prices. The approach we take here is to compare the "worst case scenarios" *given* market structure, i.e. the highest sustainable prices under each price regime given that either the incumbent or the entrant serves the buyers.

Proposition 7 *(consumer surplus)*
(i) Miscoordination equilibria: Under all three price regimes (uniform pricing, explicit discrimination, and rebates), the highest equilibrium price is the monopoly price, and so consumer surplus is the same:

$$CS^j_{\text{explicit}} = CS^j_{\text{implicit}} = CS^j_{\text{uniform}} = 0 \text{ for } j = s, l.$$

(ii) Entry equilibria: At the highest sustainable prices under each regime, consumer surplus is maximal under explicit discrimination, intermediate under rebates, and minimal under uniform pricing:

$$\begin{aligned} CS^l_{\text{explicit}} &\geq CS^l_{\text{implicit}} > CS^l_{\text{uniform}} > 0 \text{ with strict inequality if } c_I < \frac{k(1+m)}{m} \\ CS^s_{\text{explicit}} &\geq CS^s_{\text{implicit}} > CS^s_{\text{uniform}} > 0 \text{ with strict inequality if } c_I \geq 1 - k - k/m \end{aligned}$$

Proof: Under all three price regimes, buyers consume the same quantities. Thus, their consumer surplus is solely determined by the price they pay: the higher the price, the lower is consumer surplus.

(i) follows immediately from Propositions 1,2, and 4.

(ii) The following table shows the prices buyers pay under each of the three price regimes. The inequalities follow from simple algebra.

Table 1: Highest Sustainable Prices in Entry Equilibria

	Uniform		Implicit		Explicit
Large Buyer:					
$c_I < k$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I}{(1-k)(m+1)}$	$>$	$p_E^l = 0$
$c_I \in \left[k, \frac{k(1+m)}{m}\right)$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I}{(1-k)(m+1)}$	$>$	$p_E^l = \frac{c_I - k}{1-k}$
$c_I \geq \frac{k(1+m)}{m}$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I - k}{1-k}$	$=$	$p_E^l = \frac{c_I - k}{1-k}$
Small Buyers:					
$c_I < 1 - k - k/m$	$p_E^s = c_I$	$>$	$p_E^s = 0$	$=$	$p_E^s = 0$
$c_I \in [1 - k - k/m, 1 - k)$	$p_E^s = c_I$	$>$	$p_E^s = 1 - \frac{m(1-c_I)}{k(m+1)}$	$>$	$p_E^s = 0$
$c_I \geq 1 - k$	$p_E^s = c_I$	$>$	$p_E^s = 1 - \frac{m(1-c_I)}{k(m+1)}$	$>$	$p_E^s = \frac{c_I - (1-k)}{k}$

The ranking of consumer surplus is the reverse of the ranking of prices.$\square$

The main results can be summarized as follows:

- Under uniform pricing, entry equilibria exist for all parameter values, but the price charged by the entrant is higher than any of the prices under rebates or explicit discrimination.
- Under rebates, entry equilibria exist for a larger region of the parameter space than under explicit discrimination. Indeed, if an entry equilibrium exists under explicit discrimination, it will also exist under rebates; but explicit discrimination may allow the incumbent to break some entry equilibria that would exist under rebates.
- When entry equilibria exist, the prices charged by the entrant to both groups of buyers are (weakly) higher under rebates than under explicit discrimination.

5 Extensions

In this Section, we shall deal with a number of extensions to the basic model. First, we shall analyze in Section 5.1 how results change when we consider the possibility that firms subsidize consumption, i.e. can charge negative prices. This makes the pricing behavior of both firms more aggressive. Not surprisingly, the Incumbent will be able to exclude entry for a wider region of parameter values, but the basic trade-off between exclusion and lower prices acquires now an important dimension. Indeed, the possibility of setting negative prices, i.e. of subsidizing buyers for *using* the product, gives an important tool to the entrant to disrupt miscoordination equilibria. Contrary to the base model (where prices were constrained to be non-negative), if negative-price *discriminatory* offers can be made, miscoordination equilibria do not always exist. In particular, unless the gap between incumbent's and entrant's costs is sufficiently small, miscoordination equilibria do not exist, and if they exist they can be sustained only by lower than monopoly prices.

Next, Section 5.2 will deal with the case of elastic demands, allowing for both linear prices and two-part tariffs.[21] So far, we have assumed that demands are inelastic for simplicity. One possible problem with these demands is that unless a productive inefficiency occurs, total welfare is the same at high or low prices. It is true that lower equilibrium prices will lead to a better social outcome unless consumer surplus and producer surplus have exactly the same weight in the objective function (and most antitrust authorities tend to maximize consumer welfare, not total welfare), but it is still important to look at how our results extend to a setting where demands are elastic.

Finally, Section 5.3 discusses the case where firms are allowed to discriminate even among buyers of the same type, i.e. to set different prices for different small buyers. If uniform pricing is at one end of the extreme, this fully discriminatory price regime is at the other end.

5.1 Allowing for usage subsidies

In this section, we keep inelastic demands, but relax the assumption that prices must be non-negative.

5.1.1 Uniform prices

Under uniform price offers, the results are the same as in the base model. The *miscoordination equilibrium* cannot be disrupted by negative price offers, because the entrant cannot profitably offer negative prices to all buyers. For the same reason, the *entry equilibrium* will also exist for all parameter values. Therefore, *Proposition 1 still holds good.*

5.1.2 Explicit price discrimination

We consider first miscoordination equilibria and then entry equilibria.

Miscoordination equilibria The possibility to offer negative prices changes dramatically the analysis of miscoordination equilibria. Consider for instance a natural candidate equilibrium, that is the miscoordination equilibrium prevailing under uniform (non-negative) prices: ($p_I^s = 1, p_I^l = 1$) and all buyers buy from the incumbent. Under positive prices, this miscoordination equilibrium is sustained by any continuation equilibrium where firm I sets $p_I^s = p_I^l = p_I^m = 1$, firm E sets, for instance, $p_E^l = p_E^m = 1$, $p_E^s = 0$, and all buyers buy from I.

[21] Because of space limitations, we only summarize our findings, without presenting the full treatment. The analytics for the elastic demand case and the case of full discrimination are available from the authors upon request.

This is an equilibrium because if a small buyer, who is offered a zero price by the entrant, decided to switch to the entrant given that all others buy from the incumbent, he would get zero surplus, because the entrant does not reach critical mass and hence the utility derived from consuming the product would be zero. Therefore, the entrant would have no incentive to deviate either.

But this reasoning does not hold any longer when negative prices are admitted. Suppose that firm I sets $p_I^s = p_I^l = 1$. If firm E sets $p_E^l = p_I^l - \varepsilon = 1 - \varepsilon$ and $p_E^s < 0$, then all buyers will buy from the entrant. Indeed, by buying from the entrant each small buyer would receive a strictly positive surplus $(k/m)(-p_E^s) > 0$ even if nobody else consumed the product. Therefore, they will want to consume in order to receive the payment. But since it is a dominant strategy for the small buyers to consume the product, the large buyer will now prefer to buy from the entrant as well, since the critical network size will be met, and since $CS^l(p_E^l) = (1-k)(1-p_E^l) > CS^l(p_I^l) = 0$.

More generally, a miscoordination equilibrium with prices (p_I^s, p_I^l) will not exist if the entrant can offer a negative price $p_E^s < 0$ to the small buyers such that $CS^s(p_E^s, s_E < \bar{s}) > CS^s(p_I^s, s_I \geq \bar{s})$ while slightly undercutting the incumbent's offer to the large buyer, $p_E^l = p_I^l - \varepsilon$.[22]

Proposition 7 *(miscoordination equilibria under negative prices) Let* $\bar{s} > (1-k) + \frac{k}{m}$*. Let buyers buy from the incumbent whenever it is not a dominant strategy to buy from the entrant. Then, if both firms charge negative prices, a miscoordination equilibrium will only exist if* $c_I \leq k + c_E$*.*

(i) If $c_E \leq 1-k$*, the equilibrium is characterized by*

$$\begin{aligned} p_I^l &= 1, p_I^s = 1 - \frac{1}{k}\left[1-k-c_E\right] \\ p_E^l &\in [0,1], p_E^s = -\frac{1-k-c_E}{k} \end{aligned}$$

(ii) If instead $c_E > 1-k$*, the equilibrium is characterized by* $p_I^l = p_I^s = 1$*, and* $p_E^l = p_E^s = 1$*.*

Proof: see Appendix

Figure 3 illustrates in the space (k, c_I) the region where the miscoordination equilibrium arises, for the case $c_E < 1/2$. It shows that this equilibrium exists only if c_I is sufficiently close to c_E.

The main conclusions from the analysis are that:

[22] In the case where $\bar{s} \leq (1-k) + \frac{k}{m}$, the entrant might as well charge a negative price to the *large* buyer, while matching I's offer to the small buyers. In this case, as soon as E attracted the large buyer, E needs just one more buyer to reach the minimum size. Thus, any small buyer will find it optimal to buy from E as well, and the miscoordination equilibrium is broken. This is not the case if $\bar{s} > (1-k) + \frac{k}{m}$, where the entrant needs more than one small buyer to reach the minimum size, so that attracting the large buyer is not sufficient to solve the coordination problem among the small buyers. For simplicity, we will focus on this "asymmetric" case here.

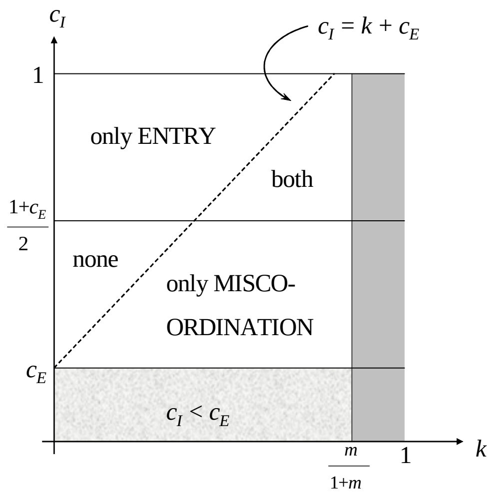

Figure 3: Regions where miscoordination equilibria and/or entry equilibria (or none) exist under negative prices, for $c_E < 1/2$

(1) when negative prices are possible, then allowing for explicit discrimination disrupts miscoordination equilibria when c_I is sufficiently high.

(2) When a miscoordination equilibrium exists under explicit discrimination (with linear prices which can be negative), the incumbent will not be able to enjoy the monopoly outcome ($p_I^s = 1, p_I^l = 1$), unless $c_E > 1 - k$; the incumbent needs to lower its prices to prevent the entrant from stealing its buyers.

Compared to uniform pricing regimes, where a miscoordination equilibrium which reproduces the monopolistic outcome is always possible, allowing for negative prices has the effect of both rendering miscoordination equilibria less likely, and, where such equilibria survive, of reducing the equilibrium prices at those equilibria. Note that in this case, p_I^s may even be below-cost, i.e. $p_I^s < c_I$!

Entry equilibria The analysis of entry equilibria when we allow for negative prices requires just a small modification of the problem already analyzed in Section 3.2 above, i.e. allowing for p_I^s and p_I^l to take negative values, which was not possible before.

Proposition 8 *(entry equilibria under negative prices) If both firms can use explicit price discrimination and charge negative prices, entry equilibria only exist if*

$$c_I \geq \frac{1 + c_E}{2}.$$

The highest prices that the entrant can charge in any such entry equilibrium are

$$p_E^s = \frac{c_I - (1-k)}{k} \text{ and } p_E^l = \frac{c_I - k}{1-k}.$$

Proof: see Appendix A

Figure 3 illustrates entry equilibria. Note that under negative pricing, the incumbent can prevent entry for a larger region of parameter values than under non-negative prices: in the latter case, entry can also occur for values $c_I < \frac{1+c_E}{2}$, whereas under negative prices, the efficiency threshold shifts to $c_I = \frac{1+c_E}{2}$ everywhere.

The figure also shows that under explicit discrimination, there might be a situation where, for given c_E and k, for c_I sufficiently close to c_E a miscoordination equilibrium exists, for intermediate values of c_I no equilibrium in pure strategies exists, and for high values of c_I only the entry equilibrium will exist. (To be precise, such a situation exists if $c_E < 1/3$). For high values of k, there exists an area of parameter values where both miscoordination and entry equilibria will coexist.

To compare results, recall that under uniform pricing both entry and miscoordination equilibria exist under all parameter values. This multiplicity of equilibria in the base case makes it difficult to identify precise policy implications. However incomplete (depending on the values of c_E, there may also exist other regions where no equilibria exist under explicit discrimination, or where multiple equilibria exist also under explicit discrimination), the following Table allows to fix ideas. It shows that for relatively high efficiency gaps between incumbent and entrant, if explicit discrimination schemes are allowed consumer welfare will always be (weakly) higher than under uniform pricing (miscoordination equilibria never exist, and entry equilibria are characterized by (weakly) lower prices). For relatively low efficiency gaps between incumbent and entrant, though, the impact on consumer welfare is not unambiguous: at equilibrium, the incumbent will always serve, and the desirability of explicit discrimination schemes depends on which equilibrium would prevail under uniform pricing: if under uniform pricing a miscoordination equilibrium is played, then explicit discrimination will increase consumer welfare, but if under uniform pricing an entry equilibrium is played, then explicit discrimination leads to exclusion and higher prices. We would then find again the same tension between exclusion and low prices that we have stressed in the main Section above, although it is to be noticed that - apart from very specific cases ($c_E > 1-k$) - exclusion can be achieved by the incumbent only by decreasing equilibrium prices.

	Uniform pricing	Explicit discrim. (neg. prices)
$c_I > \max\left\{\frac{1+c_E}{2}, k + c_E\right\}$	$\begin{cases} I \text{ serves: } p_I^l = p_I^s = 1 \\ \Longrightarrow CS = 0 \\ E \text{ serves: } p_E^l = p_E^s = c_I \\ \Longrightarrow CS = 1 - c_I \end{cases}$	E serves: $\widehat{p}_E^l \leq c_I; \widehat{p}_E^s \leq c_I$ $\Longrightarrow CS \geq 1 - c_I$
$c_I < \min\left\{\frac{1+c_E}{2}, k + c_E\right\}$	$\begin{cases} I \text{ serves: } p_I^l = p_I^s = 1 \\ \Longrightarrow CS = 0 \\ E \text{ serves: } p_E^l = p_E^s = c_I \\ \Longrightarrow CS = 1 - c_I \end{cases}$	I serves: $\widehat{p}_I^l = 1; \widehat{p}_I^s \lesseqgtr c_I$ $\Longrightarrow CS \leq 1 - c_I$

5.1.3 Implicit price discrimination (rebates)

It would be tedious to characterize all the equilibrium solutions for the case of rebates as well. Like for the case of explicit discrimination, the possibility to set negative prices allows the incumbent to make more aggressive offers, eliminating entry equilibria which would have existed under uniform prices; also, and again like for explicit discrimination, it allows the entrant to subsidize a group of buyers and induce them to use the product independently of what other buyers do, thus leading to the disruption of miscoordination equilibria. The fact that the self-selection constraint needs to be satisfied does not therefore eliminate the possibility to disrupt some of the equilibria;[23] however, it does imply that competition is softer under rebates than under explicit discrimination. Even in this case, therefore, we find the result that rebates are less exclusionary than explicit discrimination, but lead to higher prices when similar equilibrium market structures are compared.

5.2 Elastic demands

Here we relax the assumption that demands are inelastic, by assuming a simple linear demand function for the buyers. We briefly deal with two cases:

5.2.1 Linear prices

It turns out that working with elastic demands allows us to uncover an interesting feature of rebates when linear prices are considered. By incorporating a quantity threshold (a certain price is offered for demand up to a certain number of units), a rebate scheme contains a de facto rationing scheme which limits the number of units that a firm has to sell at a given price. Therefore, when offering below-cost prices, a rebate allows a firm to limit losses or, which is the same, for a given amount of losses that it can sustain, it can afford offering lower prices than under an explicit discrimination scheme. This points to an interesting

[23] At first sight, one may wonder why a buyer may want to buy at positive prices when it could mimic a buyer who is offered a negative price. But recall that a large buyer may get more surplus from buying $1-k$ units at a positive price than a smaller number of units k/m at a negative price. However, we have seen in Section 3.3 that small buyers will never be willing to buy at positive price if they have the chance to buy more units than they need at zero price. A fortiori, this is true when the price offered for a large number of units is negative.

comparison between the relative aggressiveness of rebates v. explicit discrimination: on the one hand, the necessity to satisfy the self-selection constraints limits the aggressiveness of rebates, but on the other hand, the presence of an inherent rationing device (the quantity thresholds) allows a rebate scheme to make more aggressive offers. Therefore, the result obtained in Corollary 6, that the parameter space where entry equilibria exist under explicit discrimination is fully included in the corresponding space under rebates, does no longer hold: it is possible to find parameter values for which prices are lower under rebates than under explicit price discrimination, and other values for which the opposite holds.

5.2.2 Two-part tariffs

While the analysis becomes more tedious when working with linear demands and two-part tariffs, the results are very similar in spirit to the ones of our benchmark case, i.e. of inelastic demands and linear prices. Given linear demands, if the firms can use explicitly discriminatory two-part tariffs, the firms will set the variable component of the price at marginal cost (thus maximizing total surplus), and use the fixed fee to transfer rents between buyers (e.g. the incumbent could break an entry equilibrium by extracting all consumer surplus from the large buyer, and sharing it among the small buyers through a negative fixed fee).

Again, the only difference between rebates and explicit discrimination is given by the presence of the self-selection constraints under the former scheme. Under two-part tariffs, these self-selection constraints lead to the well-known usage price distortions: at the miscoordination equilibrium, for instance, the incumbent will charge an above-cost unit price to the small buyers, thus reducing the quantity threshold, and making it less attractive for the large buyer to behave as a small buyer. This allows the incumbent to extract more rent from the large buyer without violating his self-selection constraint.

Unlike the linear-price case, under two-part tariffs, firms will never want to ration buyers when they can explicitly discriminate among them, and so there is no sense in which rebates are superior relative to explicit discrimination. Instead, whenever they lead to usage price distortions, rebates reduce total surplus, and hence the rent that can be appropriated by firms. Thus, under two-part tariffs, rebates are always less aggressive (and therefore less exclusionary) than explicit discrimination.

5.3 Full Discrimination

In this section, we consider the case where firms can discriminate among buyers of the same type. To see why this reinforces the exclusionary potential of discriminatory pricing, consider the following example: there are $m = 5$ small buyers, and to reach the minimum size, the entrant must serve at least $m = 4$ of these small buyers, plus of course the large buyer. Then, to break an entry equilibrium, it is sufficient for the incumbent to "steal" two of the small buyers: this leaves the entrant with only 3 of them, and so it will fall short of the minimum size. This in turn would allow the incumbent to charge the monopoly price not only to the large buyer, but also to the 3 small buyers who are forced to switch to the incumbent once the first two left the entrant. Now, this means that the price offers the incumbent can make to the first two will be a lot more generous than if it had to simultaneously steal all five small buyers, with only the revenue from the large buyer left to compensate for the losses made on the small buyers. Thus, under full discrimination, the incumbent's price offers to the small buyers are more aggressive than under "explicit discrimination" (as discussed in this paper), i.e. when buyers of the same size must be offered the same price. Note that the entrant will have to offer the incumbent's lowest price to all 5 small buyers, and therefore the efficiency threshold for entry equilibria to exist will be even higher than under explicit discrimination. This confirms again the trade-off between exclusionary potential and lower prices given market structure which was emphasized in this paper.

6 Concluding remarks

The purpose of this paper was to demonstrate the exclusionary potential of rebate tariffs in the presence of network externalities. Our findings are particularly interesting insofar as, in our model, the entrant is in a fairly good initial position compared to other papers on exclusionary practices: it does not have to pay any fixed cost to start operating in the industry, entrant and incumbent can approach all buyers simultaneously (i.e. the incumbent has no first-mover advantage in offering contracts to the buyers before the entrant can do so), and the entrant has the same pricing instruments at its disposal. In other words, in our paper the incumbent has an incumbency advantage (when the game starts, it has already reached the minimum threshold number of buyers for its network to be viable, whereas the - more efficient - entrant has not) but no other strategic advantage.

In the base model, we assume that firms can only charge non-negative linear prices. First of all, we find that exclusionary equilibria exist for all parameter values, and that even monopoly prices can be sustained in these exclusionary equilibria under each price regime (uniform pricing, explicit discrimination, and rebates).

As for entry equilibria, we find that the more aggressive the price regime the smaller the region of the parameter space where they exist: under uniform pricing, entry equilibria always exist, whereas under rebates and explicit price discrimination they exist only if the entrant is sufficiently more efficient than the incumbent (and the condition is the tightest under explicit price discrimination). On the other hand, if we look at regions where entry equilibria exist under all three regimes, we find that consumers would be better off under explicit discrimination, followed by rebates (or implicit discrimination) and finally uniform pricing. This trade-off between exclusionary potential and (for given market structure) lower equilibrium prices is one of the main themes of this paper.

Allowing for subsidies (i.e., negative prices) does not change the main insight of our analysis: more aggressive pricing allows the incumbent to exclude the entrant for an even wider region of parameter values, while reducing even further the highest prices that can be sustained in any entry equilibrium. In addition, the possibility of subsidizing buyers for *using* the product, gives an important tool to the entrant to disrupt miscoordination equilibria. If the gap between incumbent's and entrant's costs is sufficiently large, miscoordination equilibria do not exist, and if they exist they can be sustained only by lower than monopoly prices. Overall, usage subsidies (i) make exclusion most likely, but (ii) given market structure, results in the lowest prices.

Finally, the same trade-off appears again if one allows for full price discrimination (under which the same type of buyer can be offered different prices).

Interesting extensions of our model could be to allow for buyers to compete against each other downstream, to see whether fierce downstream competition may eliminate miscoordination problems (as showed by Fumagalli and Motta (2006) in the context of exclusive dealing). Another issue of interest could be to allow for partial compatibility between I's and E's network, and to introduce compatibility as a strategic choice variable.[24]

In this paper, we have chosen to model scale effects as a demand-side variable, by using network effects and by considering a network's installed base as the incumbency advantage. However, our results would be identical if we assumed there are scale economies, and that there is a firm which has already paid its sunk costs, as the incumbency advantage.

Consider the following game. At time 1, firms I and E simultaneously set prices (according to the different price regimes, prices can be uniform or differentiated); at time 2, all buyers decide which firm they want to buy from and make firm orders; at time 3, firm E decides on entry (if it does enter, it has to pay sunk cost $f > 0$); at time 4, payoffs are realized. Like in Section 2, continue to assume that there are m small buyers and 1 large buyer, and let the sunk cost f be large enough so that entry is profitable only if firm E serves the large buyer plus at least one small buyer.[25] With these modifications, results will be of the same nature as those obtained in this paper, and even the calculations will be to a large extent the same. Fumagalli and Motta (2001) set up a model with similar features (economies of scale in production, timing of the game) as those just described and show that miscoordination can indeed prevent an efficient firm from entering the market. However, they have symmetric buyers and do not consider the impact of price discrimination and rebates.

Finally, one may wonder how the existence of switching costs (which play an important role in shaping entry in the real world) would change our model. First

[24] If networks were fully compatible, no issue of miscoordination would arise, but assuming costly interoperability and a demand which rises continuously in the size of the network might render the study of interoperability decisions in our model interesting (see Crémer et al, 2000).

[25] Similarly, in our base model, we have chosen the threshold network size $\overline{s}$ so that the minimum scale was reached only if the large and at least one small buyer were consuming the entrant's network product.

of all, consider our basic model with network effects. One simple way to take switching costs into account would be to assume that (equivalently to the 'old' buyers in the basic model) there are buyers who have arbitrarily large switching costs and therefore would never buy from the entrant, and (equivalently to the 'new' buyers in the basic model) buyers who have switching costs σ which are small enough, so that the entrant's effective marginal cost, $c_E + \sigma \equiv \widetilde{c}_E$, is still lower than the incumbent's: $c_E + \sigma < c_I$. Provided that there are both large and small buyers among the latter category of buyers, and after replacing c_E with the effective marginal cost $\widetilde{c}_E$, the analysis would be the same as in our model, and the comparative statics on the switching costs would be straightforward. An increase in switching costs σ would be equivalent to an increase in the marginal cost of the entrant, $\widetilde{c}_E$, and would thus lead to more likely exclusionary equilibria.

Similarly, one could easily incorporate switching costs into a model where scale effects are due to a minimum efficient scale of production: it would be enough to assume again that all buyers have a small (as defined above) switching cost.

Of course, one could find more sophisticated and interesting ways to incorporate switching costs in the analysis, but it is clear that the basic mechanisms illustrated in this paper would still take place and would be exacerbated by the existence of switching costs. Both under consumption externalities and under economies of scale, switching costs would add to the incumbency advantage provided by the installed base and the sunk cost, respectively. Note, however, that in our framework, switching costs alone (i.e. without installed base or sunk cost) would not be sufficient to obtain the results.

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7 Appendix A - Proofs

Proof of Proposition 1:

(i) Suppose that all buyers buy from I. Then, recall that $\bar{s} > \max\ \{1-k, k\}$, implying that none of the individual buyers alone is sufficient for E to reach the minimum size. Thus, E's product has zero value for any single buyer, and so no buyer will want to deviate and buy from E, even if p_E were strictly lower than p_I. I sets $p_I = p_I^m$, which is the highest among all prices under which buyers will miscoordinate on the incumbent (at a price strictly above the prohibitive price, buyers would stop buying altogether). Thus, I has no incentive to increase or decrease its price. Since in all continuation equilibria buyers will not switch to E even if the price difference between the two firms is maximal, i.e. even if E charges $p_E = c_E$, E has no incentive to decrease its price either.

(ii) With all buyers buying from E at $p_E = c_I$, total demand is $mq_E^s(p_E) + q_E^l(p_E) = 1 \geq \bar{s}$, and so E will reach the minimum size. Thus, E's product has the same value to the buyers as I's, and it sells at the same price. Given that buyers coordinate on the entrant whenever E's offer is at least as good as I's, no buyer has an incentive to deviate and buy from I instead. I will not want to deviate either: To attract the buyers, I would have to set a price $p_I < c_I$, i.e. sell at a loss; and increasing p_I above c_I will not attract any buyers in any continuation equilibria. E has no incentive to change its price either: increasing p_E would imply losing the buyers to I, and decreasing p_E will just reduce profits.

There can be no equilibrium where E serves all buyers at a price $p_E > c_I$: In this case, I could profitably undercut E, and all buyers would switch to I.$\square$

Proof of Proposition 3:

First, the best offer the incumbent can make to the small buyers is given by the solution of the following program:

$$\begin{array}{l} \max_{p_I^s, p_I^l} CS^s(p_I^s) = \frac{k(1-p_I^s)}{m}, \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1], p_I^l \in [0,1], \end{array} \tag{10}$$

where (i) is the profitability constraint of the incumbent.

Next, note that the best offer the incumbent can make to the large buyer is given by the solution of:

$$\begin{array}{l} \max_{p_I^s, p_I^l} CS^l(p_I^l) = (1-k)(1-p_I^l), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1], p_I^l \in [0,1] \end{array} \tag{11}$$

We see that the best offer the incumbent can make to the small buyers is to set $p_I^l = p_I^M = 1$ and lower p_I^s as much as possible while still satisfying the profitability constraint (i); likewise, the best offer to the large buyer is obtained by setting $p_I^s = p_I^M$ and lowering p_I^l as much as allowed by (i).

The offer (p_I^s, p_I^M) to the small buyers is feasible as long as the incumbent breaks even (i.e., constraint (i) must be satisfied):

$$m\frac{k}{m}(-c_I + p_I^s) + (1-k)(1-c_I) \geq 0, \tag{12}$$

The offer (p_I^M, p_I^l) to the large buyer is feasible as long as:

$$(1-k)(-c_I + p_I^l) + m\frac{k}{m}(1-c_I) \geq 0. \tag{13}$$

Call $\widehat{p}_I^s$ and $\widehat{p}_I^l$ respectively the prices that solve the equations associated with inequalities (12) and (13) above. The lowest possible deviation prices of the incumbent are identified by respectively:

$$p_I^s = \max(\widehat{p}_I^s, 0) \quad \text{ and } \quad p_I^l = \max(\widehat{p}_I^l, 0),$$

since we limit attention to non-negative prices (see below for the case where prices can be negative).

The entrant can match the incumbent's deviations if it is able to offer (weakly) more surplus to the buyers, while still making profits. In other words, the entrant will be able to profitably enter at equilibrium if it can set prices (p_E^s, p_E^l) such that:

$$CS^s(p_E^s) = \frac{k}{m}(1 - p_E^s) \geq CS^s(p_I^s) = \frac{k}{m}(1 - p_I^s) \tag{14}$$

$$CS^l(p_E^l) = (1-k)(1-p_E^l) \geq CS^l(p_I^l) = (1-k)(1-p_I^l) \tag{15}$$

$$\pi_E(p_E^s, p_E^l) \geq 0. \tag{16}$$

Optimality requires the entrant offering the highest among all prices that satisfy these conditions, so at equilibrium they will be binding:

$$p_E^s = p_I^s; \; p_E^l = p_I^l,$$

from which condition (16) becomes:

$$k(p_E^s - c_E) + (1-k)(p_E^l - c_E) \geq 0,$$

or:

$$\pi_E(p_I^s, p_I^l) : k(p_I^s - c_E) + (1-k)(p_I^l - c_E) \geq 0. \tag{17}$$

We therefore have to find $\left(p_I^s, p_I^l\right)$. By solving the equalities associated with (12) and (13) above, we obtain:

$$\widehat{p}_I^s = \frac{c_I - (1-k)}{k}; \quad \widehat{p}_I^l = \frac{c_I - k}{1-k}.$$

Note that $\widehat{p}_I^s < c_I$ and $\widehat{p}_I^l < c_I$; also:

$$\widehat{p}_I^s \geq 0 \quad \text{if} \quad c_I \geq 1-k; \text{ and } \widehat{p}_I^l \geq 0 \quad \text{if} \quad c_I \geq k.$$

Therefore, the incumbent's optimal offer will be:

$$p_I^s = \begin{cases} \widehat{p}_I^s & \text{if } c_I \geq 1-k \\ 0 & \text{if } c_I < 1-k \end{cases} \quad p_I^l = \begin{cases} \widehat{p}_I^l & \text{if } c_I \geq k \\ 0 & \text{if } c_I < k \end{cases}$$

This identifies four regions, and for each of them we have to verify whether (16) holds or not:

$$\begin{array}{rcl} \text{if } c_I & \in & [1-k, k] \text{ and } k \geq 1/2\text{:} \quad \pi_E(\widehat{p}_I^s, 0) \geq 0 \\ \text{if } c_I & \in & [k, 1-k] \text{ and } k < 1/2\text{:} \quad \pi_E(0, \widehat{p}_I^l) \geq 0 \\ \text{if } c_I < \min & & \{k, 1-k\}: \pi_E(0,0) \geq 0 \\ \mathit{else} & : & \pi_E(\widehat{p}_I^s, \widehat{p}_I^l) \geq 0 \end{array}$$

After replacing, we can then find that:
(1) $\pi_E(\widehat{p}_I^s, \widehat{p}_I^l) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0$,
which is satisfied for:

$$c_I \geq \frac{1+c_E}{2} \equiv \overline{c}_{I1}$$

(2) $\pi_E(0, \widehat{p}_I^l) = -c_E k + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0$
which holds for:

$$c_I \geq k + c_E \equiv \overline{c}_{I2}$$

(3) $\pi_E(\widehat{p}_I^s, 0) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) - c_E(1-k) \geq 0$
which holds for:

$$c_I \geq 1 + c_E - k \equiv \overline{c}_{I3}.$$

(4) $\pi_E(0,0) = -c_E \geq 0$,

which never holds, apart from the knife-edge case where $c_E = 0$. (Since prices cannot go below zero in this basic model, the best that the incumbent can offer to buyers is to give them the good for free; but when $c_E = 0$, the entrant could match that offer without making losses, and entry equilibria would always exist. Clearly, though, this is a very special case.)

Finally, straightforward algebra shows that if $c_I \geq \max\{k, 1-k\}$, so that threshold $\overline{c}_{I1} = \frac{1+c_E}{2}$ applies, we have that $\overline{c}_{I1} = \min\{\overline{c}_{I1}, \overline{c}_{I2}, \overline{c}_{I3}\}$, and the analogous relation holds for the other two threshold values of c_I: in the parameter region where $\overline{c}_{Ii}$ applies, $\overline{c}_{Ii} = \min\{\overline{c}_{I1}, \overline{c}_{I2}, \overline{c}_{I3}\}$.□

Proof of Proposition 5:

In order to find the conditions under which entry equilibria exist, we proceed in three steps.

First, we look for the best possible offer p_I^s that the incumbent can make to the small buyers; second, we look for the best possible offer p_I^l that the incumbent can make to the large buyer; third, we see whether the entrant is able to make a profitable offer (p_E^s,p_E^l, $\bar{q}_E$) to the small and the large buyer such that they are at least as well off as if they bought from the incumbent.

The incumbent's best offer to the small buyer, $\left(\widetilde{p}_I^s, \overline{p}_I^l, \bar{q}_I^s\right)$, solves Program (18):

$$\begin{array}{l} \max_{p_I^s, p_I^l, \bar{q}_I^s} CS^s(p_I^s, \bar{q}_I^s) = (1-p_I^s)\frac{k}{m}, \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1], p_I^l \in [0,1], \bar{q}_I^s \leq \frac{k}{m} \\ \text{(iii) } CS^l\left(p_I^l, 1-k\right) = (1-p_I^l)(1-k) \geq \bar{q}_I^s\left(1-p_I^s\right), \\ \text{(iv) } CS^s\left(p_I^s, \frac{k}{m}\right) = (1-p_I^s)\frac{k}{m} \geq \left(\bar{q}_I^s + \varepsilon\right)\left(1-p_I^l\right) \end{array} \tag{18}$$

where $k \in \left[0, \frac{m}{m+1}\right]$, and p_I^s applies to all purchases $q^j \leq \bar{q}_I^s$, while p_I^l applies whenever $q^j > \bar{q}_I^s$.

Constraints (i) to (iv) fully determine the solution. Wlog, we can set $\bar{q}_I^s = \frac{k}{m}$, and search for the prices $\left(p_I^s, p_I^l\right)$ that satisfy the remaining constraints. Note that the incumbent would like to set p_I^s as low as possible, while charging the highest possible price to the large buyer. However, I can no longer set $p_I^l = 1$ (as under explicit discrimination), because at this price the large buyer is left with zero surplus, and so his self-sorting condition can never be satisfied (he would prefer to buy even a very small quantity, k/m, at a price $p_I^s < 1$, than a large quantity $1-k$ at the prohibitive price). Hence, the large buyer's self-selection constraint will always be binding under any solution of program (18):

$$p_I^l = 1 - \frac{k(1-p_I^s)}{m(1-k)}$$

In order to satisfy the profitability constraint, the following must hold as well:

$$p_I^l \geq \frac{c_I - kp_I^s}{1-k}$$

At any solution to the program, we must have $p_I^s < p_I^l$, so the small buyers' self-sorting condition, constraint (iv), is never binding. Then, either the break-even constraint (i), or the non-negativity constraint on p_I^s (ii), are binding along with constraint (iii). This gives us the following solutions of the program:

$$\begin{array}{rcl} \widetilde{p}_I^s & = & 1 - \dfrac{m(1-c_I)}{k(m+1)}; \quad \overline{p}_I^l = 1 - \dfrac{(1-c_I)}{(1-k)(m+1)}, \quad \text{if } c_I \geq 1-k-k/m \\ \widetilde{p}_I^s & = & 0; \quad \overline{p}_I^l = 1 - \dfrac{k}{m(1-k)}, \quad \text{if } c_I < 1-k-k/m. \end{array}$$

The incumbent's best offer to the large buyer $(\overline{p}_I^s, \widetilde{p}_I^l, \bar{q}_I^l)$ solves Program (19):

$$\begin{array}{l}\max_{p_I^s, p_I^l, \bar{q}_I^l} CS^l(p_I^l) = (1-k)(1-p_I^l), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \in [0,1], p_I^l \in [0,1], \bar{q}_I^l \leq (1-k) \\ \text{(iii) } CS^l\left(p_I^l, 1-k\right) \geq CS^l\left(p_I^s, \bar{q}_I^l - \varepsilon\right), \\ \text{(iv) } CS_I^s\left(p_I^s\right) = \frac{k}{m}\left(1-p_I^s\right) \geq \frac{k}{m} - p_I^l \bar{q}_I^l \end{array} \tag{19}$$

where $k \in \left[0, \frac{m}{m+1}\right]$, and p_I^s applies to all $q_I^j < \bar{q}_I^l$, while p_I^l applies to all $q_I^j \geq \bar{q}_I^l$. Note that the two quantity thresholds $\bar{q}_I^s$ and $\bar{q}_I^l$ are indexed by s and l to make it clear to which of the two programs they belong.

Now, we can set $\bar{q}_I^l = (1-k)$ wlog. The incumbent would like to set p_I^l as low as possible. (But recall that $p_I^l = 0$ can never satisfy the self-selection constraint of the small buyers, who would always prefer to buy a quantity $(1-k)$ at zero price - and throw away $1-k-k/m$ units - than a smaller quantity k/m at positive price.) In order to satisfy the profitability constraint and the small buyers' self-selection constraint respectively, the following must hold:

$$p_I^s \geq \frac{c_I - (1-k)p_I^l}{k},$$

and respectively:

$$p_I^s \leq \frac{m(1-k)p_I^l}{k}.$$

Under any solution to Program (19), we have that $p_I^s > p_I^l$ (the reason being analogous to Program (18). It follows immediately that constraint (iii) will never be binding at any solution to Program (19). Then, either self-sorting condition (iv), or the $p_I^s \leq 1$ constraint (ii), are binding along with the break-even constraint (i). This gives us the following solutions of the program:

$$\begin{aligned} \widetilde{p}_I^l &= \frac{c_I}{(1-k)(m+1)}; \quad \overline{p}_I^s = \frac{mc_I}{k(m+1)}, \quad \text{if } c_I < \frac{k(1+m)}{m} \\ \widetilde{p}_I^l &= \frac{c_I - k}{1-k}; \quad \overline{p}_I^s = 1, \quad \text{if } c_I \geq \frac{k(1+m)}{m}. \end{aligned}$$

We can now summarize the incumbent's optimal offers as follows:

$$\widetilde{p}_I^s = \begin{cases} 1 - \frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1-k-k/m \\ 0 & \text{if } c_I < 1-k-k/m \end{cases} \quad \widetilde{p}_I^l = \begin{cases} \frac{c_I - k}{1-k} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I}{(1-k)(m+1)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

Again, these are the highest prices that the entrant can charge in any entry equilibrium. For entry to be feasible, $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ must be high enough to allow the entrant to break even. The functions $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ identifies four regions, and for each of them we have to verify whether (16) holds or not:

(i) if $c_I \in \left[1-k-k/m, \frac{k(1+m)}{m}\right]$ and $k \geq \frac{m}{2(1+m)}$: $\quad \pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$
(ii) if $c_I \in \left[\frac{k(1+m)}{m}, 1-k-k/m\right]$ and $k < \frac{m}{2(1+m)}$: $\quad \pi_E(0, \frac{c_I-k}{1-k}) \geq 0$
(iii) if $c_I < \min\left\{\frac{k(1+m)}{m}, 1-k-k/m\right\}$: $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$
(iv) else: $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I-k}{1-k}) \geq 0$

After replacing, we can then find that:
(i) $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for:

$$c_I \geq \frac{m}{1+m} + c_E - k$$

(ii) $\pi_E(0, \frac{c_I-k}{1-k}) \geq 0$ holds for:

$$c_I \geq k + c_E$$

(iii) $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for:

$$c_I \geq c_E(1+m)$$

(iv) $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I-k}{1-k}) \geq 0$ is satisfied for:

$$c_I \geq \frac{m+(1+m)c_E}{1+2m}$$

If $c_E < \frac{1}{2(m+1)}$, then we have that $c_E(1+m) < \frac{m+(1+m)c_E}{1+2m} < \frac{1}{2}$. Tedious algebra shows that in this case, $c_I \geq \frac{m+(1+m)c_E}{1+2m}$ is redundant, and that each of the remaining thresholds is the minimum of all thresholds in the parameter region where it applies. Conversely, if $c_E \geq \frac{1}{2(m+1)}$, then we have that $\frac{m+(1+m)c_E}{1+2m} < c_E(1+m)$ and $c_E(1+m) \geq \frac{1}{2}$. In this case, $c_I \geq c_E(1+m)$ is redundant, and each of the remaining thresholds is the minimum of all thresholds in the parameter region where it applies.

Finally, note that - unlike the case of explicit discrimination - *in principle* it may not be enough if the entrant simply matches the incumbent's offer, because at prices $p_E^s = \widetilde{p}_I^s$ and $p_E^l = \widetilde{p}_I^l$, it may be that one of the self-selection constraints is violated. In other words, the self-selection conditions on the one hand affect the incumbent by obliging it to set (weakly) higher prices but on the other hand also affect the entrant by obliging it to set (weakly) higher prices as well. Thus, we have to verify if each of the possible price pairs identified by $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ will also satisfy the large and small buyers' self-sorting conditions, so that they can actually sustain an entry equilibrium:

(i) Suppose that $\widetilde{p}_I^s < \widetilde{p}_I^l$, and that $\bar{q}_E = \frac{k}{m}$. Then, only the large buyer's self-selection constraint could be violated (but not the small buyers'). But recall that $\widetilde{p}_I^s$ derives from Program (18), i.e. the incumbent's best offer to the *small* buyers, and that this offer $\left(\widetilde{p}_I^s, \overline{p}_I^l, \overline{q}_I^s\right)$ satisfies the large buyer's self-selection constraint by construction. Now, we also have that

$$\overline{p}_I^l > \widetilde{p}_I^l = p_E^l$$

i.e. the price that the large buyer is charged under solution $\left(\widetilde{p}_I^s, \overline{p}_I^l, \overline{q}_I^s\right)$ is always higher than the price under I's best offer to the *large* buyer, $\widetilde{p}_I^l$, which is also the highest price that the entrant can charge the large buyer. Otherwise, $(\overline{p}_I^s, \widetilde{p}_I^l, \overline{q}_I^l)$ cannot be a solution to Program (19). But that implies that $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ must also satisfy the large buyer's self-selection condition.

(ii) For the complementary case $\widetilde{p}_I^s > \widetilde{p}_I^l$, and $\overline{q}_E = 1 - k$, only the small buyers' self-selection constraint could be violated. But now, we have that $(\overline{p}_I^s, \widetilde{p}_I^l, \overline{q}_I^l)$, which solves Program (19), satisfies the small buyers' self-selection constraint by construction, and that $\overline{p}_I^s > \widetilde{p}_I^s$, so that $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ must satisfy the small buyers' self-selection constraint as well.□

Proof of Corollary 6:

Under explicit discrimination, the lower bound on c_I for entry equilibria to exist is $\min\left\{\frac{1+c_E}{2}, k + c_E, 1 - k + c_E\right\}$. Now, if $c_E < \frac{1}{2(m+1)}$, the corresponding condition under rebates reads $c_I \geq \min\left\{c_E(1+m), k + c_E, \frac{m}{1+m} + c_E - k\right\}$. Comparing the components of the two sets, we see that the second component is the same, $k+c_E = k+c_E$. The third component is lower under rebates, $\frac{m}{1+m} + c_E - k < 1 - k + c_E$. Finally, $c_E < \frac{1}{2(m+1)}$ implies that $c_E(1+m) < \frac{1+c_E}{2}$, i.e. the first component is lower under rebates as well. If instead $c_E \geq \frac{1}{2(m+1)}$, the first component under rebates is $\frac{m+(1+m)c_E}{1+2m}$, which is always smaller than $\frac{1+c_E}{2}$. Thus, we can conclude that the parameter space for which entry equilibria exist under rebates fully includes the corresponding parameter space under explicit discrimination.□

Proof of Proposition 7:

To make it a dominant strategy for the small buyers to buy from E, E must offer a price p_E^s that yields a (weakly) higher net surplus as I's offer to the small buyers:

$$-p_E^s \frac{k}{m} \geq \frac{k}{m}(1 - p_I^s)$$

We see immediately that $p_E^s \leq -(1 - p_I^s) < 0$ (E subsidizes small buyers' consumption of its product). If the small buyers consume E's product for sure, then the large buyer will switch to E whenever $p_E^l \leq p_I^l$. Will E be able to break-even under this optimal deviation? Inserting $p_E^s = -(1-p_I^s)$ and $p_E^l = p_I^l$ into the profit function we have that

$$-k(1 - p_I^s) - c_E + p_I^l\,(1 - k) \geq 0$$

Rearranging this break-even constraint, we obtain

$$p_I^s \geq 1 - \frac{1}{k}\left[p_I^l\,(1 - k) - c_E\right]$$

Looking at it from the point of view of the incumbent, this means that given p_I^l, p_I^s must *not* exceed $1 - \frac{1}{k}\left[p_I^l (1-k) - c_E\right]$, or else I becomes vulnerable to the deviation described above. Hence, I's problem reads

$\max_{p_I^s, p_I^l} \pi_I = (p_I^s - c_I)\, k + \left(p_I^l - c_I\right)(1-k)$
s.t. (i) $p_I^l \leq 1$
(ii) $p_I^s \leq \min\left\{1 - \frac{1}{k}\left[p_I^l (1-k) - c_E\right], 1\right\}$

If $(1-k) - c_E < 0$, the problem is trivially solved by

$$p_I^s = p_I^l = 1$$

If instead $(1-k) - c_E \geq 0$, we can insert $p_I^s = 1 - \frac{1}{k}\left[p_I^l (1-k) - c_E\right]$ into the objective function to see that the choice variables drop out, so that the objective function reduces to:

$$\pi_I = k + c_E - c_I$$

Thus, I will be able to break even iff

$$c_I \leq k + c_E$$

(i) Let $c_E \leq 1-k$. If the incumbent raises p_I^l above 1 (the prohibitive price), the large buyer will not buy anything. Reducing p_I^l below 1 would only reduce profits. Note that $c_E \leq 1-k$ implies that $p_I^s \leq 1$. If the incumbent raises p_I^s above $1 - \frac{1}{k}[1-k-c_E]$, the small buyers will find it individually rational to buy from E:

$$-p_E^s \frac{k}{m} = \frac{1}{m}(1-k-c_E) > \frac{k}{m}(1-\tilde{p}_I^s)$$

Reducing p_I^s below $1 - \frac{1}{k}[1-k-c_E]$ would only reduce profits.

Under this equilibrium, all buyers buy from the incumbent, so that the entrant's profits are zero. We argued before that the entrant's optimal deviation is to set $p_E^l = p_I^l = 1$, and to reduce p_E^s below $-\frac{1-k-c_E}{k}$ to attract the small buyers. But such an offer would violate the entrant's break-even condition:

$$\tilde{p}_E^s k - c_E + p_I^l (1-k) < -k(1-p_I^s) - c_E + p_I^l (1-k) = 0$$

The entrant has no incentive either to increase p_E^s above $-\frac{1-k-c_E}{k}$, as it does not make any sales in equilibrium.

Finally, no individual buyer has any incentive to deviate and buy from the entrant instead: each of the small buyers is indifferent between I's and E's offer, and the large buyer strictly prefers to buy from I than being the only buyer to buy from E.

Can there be any other miscoordination equilibrium, where I charges a lower p_I^l, namely $p_I^l < 1$, and an accordingly higher $p_I^s = 1 - \frac{1}{k}\left[p_I^l (1-k) - c_E\right]$? No, because no matter which prices E sets, I would want to increase p_I^l to 1 without changing p_I^s, thereby increasing profits without losing the large buyer to E. Therefore, such a price pair cannot sustain an equilibrium.

(ii) Let $c_E > 1-k$, and let $p_I^s = p_I^l = 1$. Clearly, the incumbent has no incentive to change its prices. Recall that under E's optimal deviation, E's break-even condition reads

$$-k(1-p_I^s) - c_E + p_I^l\,(1-k) \geq 0$$

Inserting $p_I^s = p_I^l = 1$, we get

$$-c_E + (1-k) \geq 0$$

This condition is always violated if $c_E > 1-k$. In other words, business-stealing by the entrant is impossible even if the incumbent charges monopoly prices to both groups of buyers. Therefore, the entrant is indifferent among all the prices it can set such that I serves the buyers: $p_E^s = p_E^l = 1$ dominates all others. The rest of the proof is analogous to the reasoning above.□

Proof of Proposition 8:

The best offer the incumbent can make to the small buyers is given by the solution of the following program:

$$\begin{array}{l} \max_{p_I^s,p_I^l} CS^s(p_I^s) = \frac{k}{m}(1-p_I^s), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \leq 1, p_I^l \leq 1. \end{array} \tag{20}$$

The best offer the incumbent can make to the large buyer is given by the solution of:

$$\begin{array}{l} \max_{p_I^s,p_I^l} CS^l(p_I^l) = (1-k)\,(1-p_I^l), \quad \text{s.to:} \\ \text{(i) } (p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \\ \text{(ii) } p_I^s \leq 1, p_I^l \leq 1. \end{array} \tag{21}$$

By following the same steps as in Section 3.2 one can check that the incumbent's best offers are

$$\widehat{p}_I^s = \frac{c_I - (1-k)}{k}; \quad \widehat{p}_I^l = \frac{c_I - k}{1-k}.$$

An entry equilibrium will exist only if the entrant is able to profitably match simultaneously both best offers, i.e. $p_E^s = \widehat{p}_I^s$, and $p_E^l = \widehat{p}_I^l$. Therefore, such an equilibrium exists if and only if:

$$\pi_E(\widehat{p}_I^s, \widehat{p}_I^l) = k\left(\frac{c_I - (1-k)}{k} - c_E\right) + (1-k)\left(\frac{c_I - k}{1-k} - c_E\right) \geq 0,$$

which is satisfied for:

$$c_I \geq \frac{1+c_E}{2}.$$

□