# Estimating Dynamic Treatment Effects from Project STAR 

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## MOTIVATION

Program Evaluation: Non-Compliance of Human Subjects Multi-Period Nature of the Interventions
Past Studies treat non-compliance as random and ignore multiperiod nature of the experimental design

Economics: Structural Parameters of the Education Production Functions, Flexible Estimation Approaches
Relationship between Economic Estimates and Dynamic Treatment Parameters in Policy Evaluation

Public Policy: Multi-Billion Dollar Class Size Reduction Initiatives Throughout North America Over the Past Decade Incredible Attention Paid to Project STAR in Policy Debates

## POLICY RESPONSE TO PROJECT STAR

## United States Department of Education (1998)

"In sum, due to the magnitude of the Project STAR longitudinal experiment, the design, and the care with which it was executed, the results are clear: This research leaves no doubt that small classes have an advantage over larger classes in student performance in the early primary grades"

## The Manitoba Teachers' Society: Written Submission to Class Size and Composition Commission <br> "Tennessee's Project STAR the most reputable and frequently cited study of class size"; "Nevertheless, the "tipping point" does seem to be between 19 and 20 students"

Frederick Mosteller (1995), the STAR Project is considered "one of the most important educational investigations ever carried out and illustrates the kind and magnitude of research needed in the field of education to strengthen schools."

Highly Politicized Area of Debate

## PAST FINDINGS

Jeremy Finn and C.M. Achilles (Fall 1990) state "This research leaves no doubt that small classes have an advantage over larger classes in reading and math in early primary grades." "The effects of reduced-size classes were found on every achievement measure administered in Project STAR ".

Krueger (1999) finds ("smaller overall effects") "test score ad-vantage of students in small classes expands by $1 \%$ per year in subsequent years"

Hanushek (1999) "one would expect the differences (between class types) to get wider through the grades as they continue to get more resources (treatment)"

Finn and Achilles (1999) recast STAR results using grade equivalent scores as achievement outcomes to show effects increase with each grade.

What's the nature and magnitude of small class effect?
Is the effect persistent?
Is non-compliance a problem with STAR data?

## PROJECT STAR

The Student Teacher Achievement Ratio (STAR) project was a largescale, four-year, experimental study of reduced class size

Over 6,000 students in 79 schools upon entering kindergarten were randomly assigned into one of three interventions: small class, regular class and regular-with-aide class.
Classroom teachers were also randomly assigned to classes.
The interventions were initiated as the students entered school in kindergarten and continued through the third grade.

Random assignment is designed to circumvent selection in treatment.
A variety of implementation problems came up in those four years.

Figure 1: Transitions During Project STAR for Kindergarten Cohort


## STAR IMPLEMENTATION PROBLEMS

Only $45.16 \%$ of the kindergarten sample initially assigned to small classes conformed to their assignments throughout all four years of the STAR experiment.

Only $39.73 \%$ of those initially assigned to regular classes conformed.
Refreshment samples NOT randomly assigned w.r.t. class type.
Subjects exit the sample differently with regard to initial random assignment. Coefficient 0.065 (0.030)

Simple test: Are individuals who subsequently leave the STAR experiment systematically different from those who remain in terms of initial behavioral relationships?

$$
\mathrm{A}_{\mathrm{ijK}}=\alpha_{\mathrm{x}}{ }^{\prime} \mathrm{X}_{\mathrm{ijK}}+\alpha_{1}{ }^{\prime} \mathrm{L}_{\mathrm{ij}} \mathrm{X}_{\mathrm{ijK}}+\mathrm{v}_{\mathrm{j}}+\varepsilon_{\mathrm{ijK}}
$$

Results suggest attrition is non-random. (Table 1)

Table 1: Are Attritors Different from Non-attritors

| Subject Area | Mathematics | Reading | Word Recognition |
| :---: | :---: | :---: | :---: |
| Kindergarten Class Type | $\begin{aligned} & 10.434 \\ & (2.332) \end{aligned}$ | $\begin{aligned} & \hline 6.513 \\ & (1.440) \end{aligned}$ | $\begin{aligned} & \hline 7.370 \\ & (1.628) \end{aligned}$ |
| White or Asian Student | $\begin{aligned} & 20.499 \\ & (2.760) \end{aligned}$ | $\begin{aligned} & 8.608 \\ & (2.005) \end{aligned}$ | $\begin{aligned} & 8.505 \\ & (2.524) \end{aligned}$ |
| Female Student | $\begin{aligned} & 2.587 \\ & (1.363) \end{aligned}$ | $\begin{aligned} & 3.349 \\ & (1.074) \end{aligned}$ | $\begin{aligned} & 2.488 \\ & (1.296) \end{aligned}$ |
| Student on Free lunch | $\begin{aligned} & -13.729 \\ & (1.679) \\ & \hline \end{aligned}$ | $\begin{aligned} & -12.239 \\ & (1.187) \\ & \hline \end{aligned}$ | $\begin{gathered} -13.916 \\ (1.480) \\ \hline \end{gathered}$ |
| Years of Teaching Experience | $\begin{aligned} & 0.323 \\ & (0.220) \end{aligned}$ | $\begin{aligned} & 0.255 \\ & (0.123) \end{aligned}$ | $\begin{aligned} & 0.329 \\ & (0.135) \end{aligned}$ |
| White Teacher | $\begin{aligned} & -.926 \\ & (4.366) \end{aligned}$ | $\begin{aligned} & -1.577 \\ & (3.068) \end{aligned}$ | $\begin{aligned} & -1.578 \\ & (3.506) \end{aligned}$ |
| Teacher has Master Degree | $\begin{aligned} & -1.482 \\ & (2.396) \\ & \hline \end{aligned}$ | $\begin{aligned} & -1.211 \\ & (1.423) \\ & \hline \end{aligned}$ | $\begin{aligned} & -0.491 \\ & (1.729) \\ & \hline \end{aligned}$ |
| Attrition Indicator | $\begin{aligned} & -17.305 \\ & (3.838) \\ & \hline \end{aligned}$ | $\begin{aligned} & -13.674 \\ & (2.537) \\ & \hline \end{aligned}$ | $\begin{aligned} & -13.198 \\ & (3.251) \\ & \hline \end{aligned}$ |
| Attrition Indicator Interacted with Kindergarten Class Type | $\begin{aligned} & -5.383 \\ & (2.616) \\ & \hline \end{aligned}$ | $\begin{aligned} & -2.069 \\ & (1.686) \\ & \hline \end{aligned}$ | $\begin{aligned} & -3.004 \\ & (2.045) \\ & \hline \end{aligned}$ |
| Attrition Indicator Interacted with White or Asian Student | $\begin{aligned} & \hline-3.949 \\ & (2.732) \end{aligned}$ | $\begin{aligned} & \hline-.259 \\ & (1.824) \end{aligned}$ | $\begin{aligned} & -1.177 \\ & (2.368) \end{aligned}$ |
| Attrition Indicator Interacted with Female Student | $\begin{aligned} & 5.597 \\ & (2.078) \\ & \hline \end{aligned}$ | $\begin{aligned} & 2.943 \\ & (1.454) \\ & \hline \end{aligned}$ | $\begin{aligned} & 3.750 \\ & (1.739) \\ & \hline \end{aligned}$ |
| Attrition Indicator Interacted with Student on Free lunch | $\begin{aligned} & \hline-5.186 \\ & (2.384) \end{aligned}$ | $\begin{aligned} & \hline-0.496 \\ & (1.554) \end{aligned}$ | $\begin{aligned} & \hline 0.549 \\ & (1.891) \end{aligned}$ |
| Attrition Indicator Interacted with Years of Teaching Experience | $\begin{aligned} & 0.188 \\ & (0.210) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.075 \\ & (0.131) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.060 \\ (0.164) \\ \hline \end{gathered}$ |
| Attrition Indicator Interacted with White Teacher | $\begin{aligned} & \hline 1.263 \\ & (3.490) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 2.269 \\ & (2.133) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0.642 \\ & (2.678) \end{aligned}$ |
| Attrition Indicator Interacted with Teacher has Master Degree | $\begin{aligned} & -1.370 \\ & (2.490) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.939 \\ & (1.586) \end{aligned}$ | $\begin{aligned} & 1.552 \\ & (1.876) \\ & \hline \end{aligned}$ |
| Number of Observations (R-Squared) | 5810 (0.305) | 5729 (0.295) | 5789 (0.259) |
| Joint Effect of Attrition on Constant and Coefficient Estimates |  | $\begin{aligned} & 32.68 \\ & {[0.000]} \\ & \hline \end{aligned}$ | $\begin{aligned} & 25.76 \\ & {[0.000]} \\ & \hline \end{aligned}$ |
| Joint Effect of Attrition on all Coefficient Estimates but not constant | $\begin{aligned} & 3.14 \\ & {[0.003]} \end{aligned}$ | $\begin{aligned} & 1.23 \\ & {[0.280]} \end{aligned}$ | $\begin{aligned} & 1.45 \\ & {[0.181]} \end{aligned}$ |
| Effect of Attrition on Constant Alone | $\begin{aligned} & 20.33 \\ & {[0.000]} \\ & \hline \end{aligned}$ | $\begin{aligned} & 29.06 \\ & {[0.000]} \\ & \hline \end{aligned}$ | $\begin{aligned} & 16.48 \\ & {[0.000]} \end{aligned}$ |

Note:Regressions include school indicators. Standard errors corrected at
the classroom level are in ( ) parentheses. Probability > F are in [ ] parentheses.

## THE BASIC CASE

Define $\mathrm{M}=1$ Initially assigned to small class
$\mathrm{M}=0$ Initially assigned to regular class
$\mathrm{S}_{\mathrm{t}}=1$ Attending small class in period $t$
$\mathrm{S}_{\mathrm{t}}=0$ Attending regular class in period $t$
$\mathrm{A}_{\mathrm{t}}$ is Achievement at the end of period $t$.

$$
\mathrm{A}_{\mathrm{t}}=\mathrm{S}_{\mathrm{t}} \mathrm{~A}_{1 \mathrm{t}}+\left(1-\mathrm{S}_{\mathrm{t}}\right) \mathrm{A}_{0 \mathrm{t}}
$$

GOAL: Estimate $A T E=E\left(A_{1 t}-A_{0 t}\right)$ : Causal Effect of Treatment.
Sometimes we are interested in $I T T=\bar{A}_{M=1}-\bar{A}_{M=0}$ : Causal Effect of Treatment Assignment (not proper for mandatory policy like class size).

## NON-COMPLIANCE

Dropout Bias $\mathrm{M}=1, \mathrm{~S}=0$
Substitution Bias M=0, $\mathrm{S}=1$
Under dropout and substitution, we can use initial random assignments as instrument to estimate Local Average Treatment Effects (LATE or CACE; Angrist, Imbens and Rubin (1996))

$$
I T T^{I V}=\bar{A}_{M=1}^{C}-\bar{A}_{M=0}^{C}=\frac{\bar{A}_{M=1}-\bar{A}_{M=0}}{\operatorname{Pr}\left(S_{t}=1 \mid M=1\right)-\operatorname{Pr}\left(S_{t}=1 \mid M=0\right)}
$$

Note that in general LATE $\neq$ ATE

## ATTRITION

Define $\quad \mathrm{L}_{\mathrm{t}+1}=1$ Leave the STAR sample at the end of year $t \mathrm{~L}_{\mathrm{t}+1}=0$
Remain in the STAR sample at end of year $t$
Frangakis and Rubin (1999) demonstrate that neither ITT or LATE (CACE) are robust to attrition bias.

Attrition Bias
-Selection Due to Observables
-Selection Due to Unobservables
Balke and Pearl (97) demonstrate that in the face of imperfect compliance traditional estimates are potentially misleading as they may lie entirely outside the bounds for an average causal effect of the intervention.

Construct bounds using 2 approaches Horowitz and Manski (2000) and Lee (2005)

Table 3: Traditional Single Period Causal Estimates of The Impacts of Reduced Class Size

| Method | Mathematics |  |  | Reading |  |  |  | Word Comprehension |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Grade 1 | Grade 2 | Grade 3 | Grade 1 | Grade 2 | Grade 3 | Grade 1 | Grade 2 | Grade 3 |  |
| ITT Analysis | 9.023 | 5.389 | 4.329 | 10.409 | 4.698 | 7.189 | 9.036 | 4.622 | 8.127 |  |
|  | $(1.946)$ | $(2.48)$ | $(2.048)$ | $(2.288)$ | $(2.316)$ | $(1.917)$ | $(2.377)$ | $(2.314)$ | $(2.45)$ |  |
| IV Analysis (LATE) | 10.353 | 6.417 | 5.781 | 11.948 | 5.602 | 9.584 | 10.296 | 5.513 | 10.862 |  |
|  | $(2.231)$ | $(2.953)$ | $(2.035)$ | $(2.625)$ | $(2.536)$ | $(2.633)$ | $(2.696)$ | $(2.748)$ | $(3.351)$ |  |
| Attrition Test for ITT | 61.60 | 79.89 | 86.12 | 62.21 | 89.73 | 78.22 | 36.55 | 73.21 | 63.19 |  |
|  | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ |  |
| Attrition Test for IV | 26.93 | 22.43 | 22.11 | 22.21 | 25.69 | 18.90 | 14.22 | 18.93 | 13.41 |  |
|  | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ |  |
| Horowitz-Manski Bounds | $\{-73.174$, | $\{-111.487$, | $\{-144.049$, | $\{-67.039$, | $\{-114.397$, | $\{-130.950$, | $\{-102.133$, | $\{-97.005$, | $\{-112.028$, |  |
|  | $86.647\}$ | 117.794 | $149.045\}$ | $81.720\}$ | $120.315\}$ | $135.940\}$ | $119.564\}$ | $105.765\}$ | $118.986\}$ |  |
| Horowitz-Manski Bounds | $\{-93.935$, | $\{-87.347$ | $\{-97.246$, | $\{-82.452$, | $\{-89.447$, | $\{-87.716$, | $\{-98.434$, | $\{-78.441$, | $\{-77.924$, |  |
| with school covariates | $94.805\}$ | $87.861\}$ | $98.414\}$ | $83.447\}$ | $90.532\}$ | $88.713\}$ | $98.632\}$ | $78.359\}$ | $79.558\}$ |  |
| Lee Bounds with no | $\{5.844$, | $\{-3.010$ | $\{-3.296$, | $\{5.443$, | $\{-2.575$, | $\{-0.834$, | $\{5.471$, | $\{-1.097$, | $\{-2.523$, |  |
| covariates | 11.126 | $6.502\}$ | $6.005\}$ | $12.152\}$ | $6.387\}$ | $7.627\}$ | $11.407\}$ | $7.110\}$ | $8.547\}$ |  |
| Lee Bounds with school | $\{0.487$, | $\{-6.782$ | $\{-8.055$, | $\{-2.024$, | $\{-8.177$ | $\{-6.286$, | $\{-4.357$, | $\{-9.301$, | $\{-8.796$, |  |
| covariates | $20.992\}$ | $20.011\}$ | $17.619\}$ | $24.360\}$ | $18.779\}$ | $19.227\}$ | $26.532\}$ | $21.683\}$ | $22.948\}$ |  |
| ITT Ignoring Selective | 9.297 | 5.554 | 4.034 | 10.659 | 5.488 | 5.437 | 9.737 | 5.649 | 6.436 |  |
| Attrition | $(1.894)$ | $(2.070)$ | $(1.637)$ | $(2.141)$ | $(1.86)$ | $(1.545)$ | $(2.283)$ | $(2.004)$ | $(1.922)$ |  |
| IV Ignoring Selective | 10.852 | 6.707 | 5.642 | 12.45 | 6.783 | 7.56 | 11.323 | 6.918 | 9.004 |  |
| Attrition | $(1.449)$ | $(1.854)$ | $(2.035)$ | $(1.87)$ | $(1.837)$ | $(1.959)$ | $(1.951)$ | $(2.081)$ | $(2.352)$ |  |

Note: All of the ITT and IV estimates are statistically significant at the $5 \%$ level. The IV and ITT analyses include the full history of teacher inputs, free lunch status, race, gender and school indicators. Standard errors corrected at the classroom level are in parentheses. For the specification tests the Probability that the Null is rejected is contained in [] brackets. For all the bounds on the ATE analysis \{lower bound, upper bound\}.

## OVERVIEW OF ESTIMATION STRATEGY

More generally,

$$
A_{i j 2}=S_{i 1} S_{i 2} A_{i}^{11}+\left(1-S_{i 1}\right) S_{i 2} A_{i}^{01}+S_{i 1}\left(1-S_{i 2}\right) A_{i}^{10}+\left(1-S_{i 1}\right)\left(1-S_{i 2}\right) A_{i}^{00}
$$

Linearized version of Education Production Function

$$
\begin{gathered}
\mathrm{A}_{\mathrm{i} 2}=\beta_{\mathrm{x} 2}{ }^{\prime} \mathrm{X}_{\mathrm{i} 2}+\beta_{\mathrm{x} 1}{ }^{\prime} \mathrm{X}_{\mathrm{i} 1}+\beta_{\mathrm{S} 2}{ }^{\prime} \mathrm{S}_{\mathrm{i} 2}+\beta_{\mathrm{S} 1}{ }^{\prime} \mathrm{S}_{\mathrm{i} 1}+\beta_{\mathrm{S} 12}{ }^{\prime} \mathrm{S}_{\mathrm{i} 2} \mathrm{~S}_{\mathrm{i} 1}+\mathrm{v}_{\mathrm{i}}+\mathrm{t}_{2}+\varepsilon_{\mathrm{i} 2} \\
\mathrm{~A}_{\mathrm{i} 1}=\alpha_{\mathrm{x} 1}{ }^{\prime} \mathrm{X}_{\mathrm{i} 1}+\alpha_{\mathrm{s} 1} \mathrm{~S}_{\mathrm{i} 1}+\mathrm{v}_{\mathrm{i}}+\varepsilon_{\mathrm{i} 1}
\end{gathered}
$$

Remove effect of $v_{i}$ by taking first differences
$\mathrm{A}_{\mathrm{i} 2}-\mathrm{A}_{\mathrm{i} 1}=\beta_{\mathrm{x} 2}{ }^{\prime} \mathrm{X}_{\mathrm{i} 2}+\left(\beta_{\mathrm{x} 1}-\alpha_{\mathrm{x} 1}\right)^{\prime} \mathrm{X}_{\mathrm{i} 1}+\beta_{\mathrm{S} 2} \mathrm{~S}_{\mathrm{i} 2}+\left(\beta_{\mathrm{S} 1}-\alpha_{\mathrm{S} 1}\right)^{\prime} \mathrm{S}_{\mathrm{i} 1}+\beta_{\mathrm{S} 12} \mathrm{~S}_{\mathrm{i} 2} \mathrm{~S}_{\mathrm{i} 1}+\varepsilon_{\mathrm{i} 2}{ }^{*}$

$$
\mathrm{A}_{\mathrm{i} 1}=\alpha_{\mathrm{x} 1}{ }^{\prime} \mathrm{X}_{\mathrm{i} 1}+\alpha_{\mathrm{s} 1} \mathrm{~S}_{\mathrm{i} 1}+\varepsilon_{\mathrm{i} 1}{ }^{*}
$$

Estimate System of Equations.

## INVERSE PROBABILITY WEIGHTING

1) Estimate Attrition Logit

$$
\begin{aligned}
\operatorname{Pr}\left(\mathrm{L}_{\mathrm{i} 2}=0 \mid \mathrm{A}_{\mathrm{i} 1}, \mathrm{X}_{\mathrm{i} 1}\right\} & =1\left\{\alpha^{\prime} \mathrm{Z}_{\mathrm{i} 1}+\omega_{\mathrm{i} 1} \geq 0\right\} \\
\hat{p}_{i 1} & =\mathrm{F}\left({ }^{\alpha^{\prime}} Z_{i 1}\right)
\end{aligned}
$$

2) Use $\hat{P}_{i 1}$ to reweight data


$$
\mathrm{A}_{\mathrm{i} 1}=\alpha_{\mathrm{x} 1}{ }^{\prime} \mathrm{X}_{\mathrm{i} 1}+\alpha_{\mathrm{S} 1}{ }^{\prime} \mathrm{S}_{\mathrm{i} 1}+\varepsilon_{\mathrm{i} 1}{ }^{*}
$$

Use structural parameters to calculate dynamic treatment effects

$$
\begin{gathered}
\mathrm{T}^{(1,1)(0,0)}(1,1)=\beta_{\mathrm{S} 2}+\beta_{\mathrm{S} 1}+\beta_{\mathrm{S} 12} \\
\mathrm{~T}^{(1,1)(1,0)}(1,1)=\beta_{\mathrm{S} 2}+\beta_{\mathrm{S} 12} \\
\mathrm{~T}^{(0,1)(0,0)}(0,1)=\beta_{\mathrm{S} 2}
\end{gathered}
$$

## DETAILS

Miquel (2003) proves that the full sequence of causal effects are estimated under the standard assumptions of

- common trend
- no pretreatment effects
- common support condition

Easy to extend to T periods.
-Assume attrition is an absorbing state
Note we also consider several methods that place bounds on the ATE.

## DATA

Use scaled test scores for norm referenced test as outcome measures Three subjects separately
Only the kindergarten cohort

Table 2: Testing Randomization of Student Characteristics across Class Types

|  | Kindergarten | Grade One | Grade Two | Grade Three |
| :---: | :---: | :---: | :---: | :---: |
| INCOMING STUDENTS |  |  |  |  |
| White or Asian Student | $\begin{aligned} & 2.35^{*} 10 \mathrm{E}-4 \\ & (0.012) \end{aligned}$ | $\begin{aligned} & \hline-0.275 \\ & (0.193) \end{aligned}$ | $\begin{aligned} & -0.061 \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 7.63^{*} 10 \mathrm{E}-4 \\ & (0.063) \end{aligned}$ |
| Female Student | $\begin{aligned} & 0.012 \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.199 \\ & (0.126) \end{aligned}$ | $\begin{aligned} & -0.020 \\ & (0.021) \end{aligned}$ | $\begin{aligned} & -0.017 \\ & (0.028) \end{aligned}$ |
| Student on Free lunch | $\begin{aligned} & -8.74^{*} 10 \mathrm{E}-3 \\ & (0.017) \end{aligned}$ | $\begin{aligned} & -0.262 \\ & (0.167) \end{aligned}$ | $\begin{aligned} & 0.013 \\ & (0.022) \end{aligned}$ | $\begin{aligned} & -0.057 \\ & (0.037) \end{aligned}$ |
| Joint Test of Student Characteristics | $\begin{aligned} & 0.29 \\ & {[0.831]} \end{aligned}$ | $\begin{aligned} & 1.83 \\ & {[0.150]} \end{aligned}$ | $\begin{aligned} & 1.24 \\ & {[0.301]} \end{aligned}$ | $\begin{aligned} & 1.01 \\ & {[0.392]} \end{aligned}$ |
| Number of Observations | 6300 | 2211 | 1511 | 1181 |
| R Squared | 0.318 | 0.360 | 0.248 | 0.411 |
| FULL SAMPLE |  |  |  |  |
| White or Asian Student | $\begin{aligned} & 2.35 * 10 \mathrm{E}-4 \\ & (0.012) \end{aligned}$ | $\begin{aligned} & \hline-0.003 \\ & (0.021) \end{aligned}$ | $\begin{aligned} & -0.008 \\ & (0.025) \end{aligned}$ | $\begin{aligned} & -0.021 \\ & (0.027) \end{aligned}$ |
| Female Student | $\begin{aligned} & \hline 0.012 \\ & (0.019) \end{aligned}$ | $\begin{aligned} & \hline 0.007 \\ & (0.009) \end{aligned}$ | $\begin{aligned} & \hline 0.004 \\ & (0.009) \end{aligned}$ | $\begin{aligned} & \hline 0.008 \\ & (0.009) \end{aligned}$ |
| Student on Free lunch | $\begin{aligned} & -8.74^{*} 10 \mathrm{E}-3 \\ & (0.017) \end{aligned}$ | $\begin{aligned} & -0.038 \\ & (0.016) \end{aligned}$ | $\begin{aligned} & -0.030 \\ & (0.016) \end{aligned}$ | $\begin{aligned} & -0.044 \\ & (0.016) \end{aligned}$ |
| Joint Test of Student Characteristics | $\begin{aligned} & \hline 0.29 \\ & {[0.831]} \end{aligned}$ | $\begin{aligned} & \hline 2.05 \\ & {[0.114]} \end{aligned}$ | $\begin{aligned} & 1.38 \\ & {[0.255]} \end{aligned}$ | $\begin{aligned} & \hline 2.98 \\ & {[0.037]} \end{aligned}$ |
| Number of Observations | 6300 | 6623 | 6415 | 6500 |
| R Squared | 0.318 | 0.305 | 0.328 | 0359 |

Note:Regressions include school indicators. Standard errors corrected at the school level are in ( ) parentheses. Probability >F are in [ ] parentheses.

Table 1: Summary Statistics

|  | Kindergarten | Grade One | Grade Two | Grade 3 |
| :---: | :---: | :---: | :---: | :---: |
| Class Size | $\begin{array}{r} 19.914 \\ (3.827) \\ \hline \end{array}$ | $\begin{array}{r} 20.334 \\ (4.017) \\ \hline \end{array}$ | $\begin{aligned} & 20.217 \\ & (4.118) \\ & \hline \end{aligned}$ | $\begin{aligned} & 20.400 \\ & (4.441) \\ & \hline \end{aligned}$ |
| Receiving Small Class Treatment | $\begin{gathered} 0.314 \\ (0.464) \end{gathered}$ | $\begin{gathered} 0.350 \\ (0.477) \end{gathered}$ | $\begin{gathered} \hline 0.373 \\ (0.484) \\ \hline \end{gathered}$ | $\begin{gathered} 0.396 \\ (0.489) \end{gathered}$ |
| Math Test Score | $\begin{array}{r} 500.038 \\ (44.979) \\ \hline \end{array}$ | $\begin{array}{r} 545.939 \\ (40.405) \\ \hline \end{array}$ | $\begin{aligned} & 594.427 \\ & (43.499) \\ & \hline \end{aligned}$ | $\begin{array}{r} 627.977 \\ (40.181) \\ \hline \end{array}$ |
| Reading Test Score | $\begin{aligned} & 445.673 \\ & (31.438) \end{aligned}$ | $\begin{aligned} & 541.754 \\ & (52.412) \end{aligned}$ | $\begin{aligned} & 599.326 \\ & (43.390) \\ & \hline \end{aligned}$ | $\begin{aligned} & 625.634 \\ & (37.125) \\ & \hline \end{aligned}$ |
| Word Recognition Test Score | $\begin{array}{r} 444.702 \\ (37.295) \\ \hline \end{array}$ | $\begin{array}{r} 532.811 \\ (46.788) \\ \hline \end{array}$ | $\begin{aligned} & 600.021 \\ & (47.118) \\ & \hline \end{aligned}$ | $\begin{array}{r} 622.771 \\ (43.932) \\ \hline \end{array}$ |
| Free Lunch Status | $\begin{gathered} 0.359 \\ (0.480) \\ \hline \end{gathered}$ | $\begin{gathered} 0.371 \\ (0.483) \\ \hline \end{gathered}$ | $\begin{gathered} 0.354 \\ (0.478) \\ \hline \end{gathered}$ | $\begin{gathered} 0.353 \\ (0.478) \\ \hline \end{gathered}$ |
| Student is White of Asian | $\begin{gathered} \hline 0.753 \\ (0.432) \\ \hline \end{gathered}$ | $\begin{gathered} 0.753 \\ (0.432) \\ \hline \end{gathered}$ | $\begin{gathered} 0.753 \\ (0.432) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.753 \\ (0.432) \\ \hline \end{gathered}$ |
| Student is Female | $\begin{gathered} 0.518 \\ (0.500) \\ \hline \end{gathered}$ | $\begin{gathered} 0.518 \\ (0.500) \\ \hline \end{gathered}$ | $\begin{gathered} 0.518 \\ (0.500) \\ \hline \end{gathered}$ | $\begin{gathered} 0.518 \\ (0.500) \\ \hline \end{gathered}$ |
| Teacher Race is NonWhite | $\begin{gathered} \hline 0.129 \\ (0.335) \end{gathered}$ | $\begin{gathered} \hline 0.140 \\ (0.347) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.178 \\ (0.383) \end{gathered}$ | $\begin{gathered} \hline 0.165 \\ (0.372) \\ \hline \end{gathered}$ |
| Teacher has a Masters Degree | $\begin{gathered} 0.377 \\ (0.485) \end{gathered}$ | $\begin{gathered} 0.343 \\ (0.475) \end{gathered}$ | $\begin{gathered} 0.363 \\ (0.481) \end{gathered}$ | $\begin{gathered} 0.443 \\ (0.497) \end{gathered}$ |
| Teacher Years of Experience | $\begin{gathered} \hline 9.447 \\ (5.497) \end{gathered}$ | $\begin{aligned} & 11.713 \\ & (8.625) \end{aligned}$ | $\begin{aligned} & 13.076 \\ & (8.567) \end{aligned}$ | $\begin{array}{r} 13.547 \\ (8.471) \\ \hline \end{array}$ |

Note: Each cell reports the mean and standard deviations are presented in parentheses. There are 2239 students who participated and completed all three exams in each year of the experiment.

## RESULTS I

We find that small class attendance is effective in kindergarten.

## GRADE ONE (Table 4)

- Small class attendance in kindergarten and grade one are both positive.
- No additional non-linear benefit.
- However the effect of attending small classes in both years is not significantly different from attending in either year only.
- Economic significance of grade one benefit is larger.


## RESULTS II

## GRADE TWO AND THREE

- No lasting beneficial small class impacts in mathematics, reading and word recognition.
- Grade Two: Positive in Math Only.
- Grade Three: Negative in all Subject Areas


## What is going on?

We investigate who gains the most in the classroom.
Weakest Students in Mathematics
Environmental Stability
Increased Variation in Small Classes Relative to Regular Classes
Small Classes do not seem to work unconditionally

Table 4: Structural Estimates of the Treatment Parameters in Education Production Functions

| Subject Area | Mathematics | Reading | Word Recognition |
| :---: | :---: | :---: | :---: |
| Kindergarten |  |  |  |
| $S_{i K}$ | $8.595(1.120)^{* * *}$ | $5.950(0.802)^{* * *}$ | $6.342(0.945)^{* * *}$ |
| Grade One |  |  |  |
| $S_{i K}$ | $7.909(4.625)^{* *}$ | $8.785(5.284)^{* *}$ | 11.868 (6.722)** |
| $S_{i 1}$ | 9.512 (3.307)*** | 9.315 (4.350)*** | 15.394 (5.730)*** |
| $S_{i K} S_{i 1}$ | -6.592 (5.648) | -2.229 (6.992) | -11.060 (8.965) |
| Grade Two |  |  |  |
| $S_{i K}$ | -2.078 (7.276) | 11.320 (7.240) | 9.959 (8.438) |
| $S_{i 1}$ | -4.010 (3.855) | -20.036 (19.189) | 4.298 (7.763) |
| $S_{i 2}$ | 15.150 (5.430) ${ }^{* * *}$ | 3.040 (4.428) | 0.526 (5.814) |
| $S_{i K} S_{i 1}$ | 3.851 (11.678) | 1.148 (24.059) | -12.074 (17.673) |
| $S_{i K} S_{i 2}$ | -4.049 (13.112) | -31.513 (17.366)** | $-23.084(13.237)^{* *}$ |
| $S_{i 1} S_{i 2}$ | -4.944 (6.617) | 25.122 (19.480) | 7.868 (8.537) |
| $S_{i K} S_{i 1} S_{i 2}$ | 6.653 (16.067) | 23.634 (28.632) | 30.111 (19.851) |
| Grade Three |  |  |  |
| $S_{i K}$ | -7.298 (10.901) | 1.215 (10.372) | 13.071 (12.202) |
| $S_{i 1}$ | 43.514 (32.898) | 22.083 (30.097) | -6.920 (37.200) |
| $S_{i 2}$ | 25.263 (42.080) | -22.085 (26.069) | -25.024 (22.031) |
| $S_{i 3}$ | $-6.835(3.932)^{* *}$ | -10.590 (4.179)*** | -12.738 (5.952)*** |
| $S_{i K} S_{i 1}$ | -38.612 (30.944) | 7.978 (39.071) | -18.002 (32.872) |
| $S_{i K} S_{i 2}$ | 37.355 (28.625) | -42.740 (25.731)** | -2.932 (22.527) |
| $S_{i K} S_{i 3}$ | -39.819 (19.922) | 17.870 (18.147) | 7.328 (14.855) |
| $S_{i 1} S_{i 2}$ | -61.947 (52.749) | 25.388 (35.964) | -7.586 (36.814) |
| $S_{i 1} S_{i 3}$ | 17.163 (43.057) | -6.613 (32.183) | -7.954 (29.718) |
| $S_{i 2} S_{i 3}$ | -14.366 (42.280) | 35.547 (22.836) | 29.203 (26.267) |
| $S_{i K} S_{i 1} S_{i 3}$ | -4.651 (52.881) | -41.180 (43.335) | -14.706 (35.985) |
| $S_{i K} S_{i 1} S_{i 2} S_{i 3}$ | 48.084 (48.704) | 6.834 (30.521) | 14.377 (33.920) |

Note: Corrected standard errors in parentheses. The
sequences $S_{i K} S_{i 1} S_{i 2}, S_{i K} S_{i 2} S_{i 3}$ and $S_{i 1} S_{i 2} S_{i 3}$ lack
unique support to permit identification in grade 3 .

Table 5: Dynamic Average Treatment Effect for the Treated Estimates

| Subject Area | Mathematics | Reading | Word Recognition |
| :---: | :---: | :---: | :---: |
| Kindergarten |  |  |  |
| $\tau^{(1)(0)}(1)$ | 8.595 (1.120)*** | $5.950(0.802)^{* * *}$ | 6.342 (0.945)*** |
| Grade One |  |  |  |
| $\tau^{(0,1)(0,0)}(0,1)$ | $9.512(3.307)^{* * *}$ | 9.315 (4.350)*** | $15.394(5.730)^{* * *}$ |
| $\tau^{(1,0)(0,0)}(1,0)$ | 7.909 (4.625)** | 8.785 (5.284)** | 11.868 (6.722)** |
| $\tau^{(1,1)(0,0)}(1,1)$ | 10.829 (8.021)* | 15.872 (9.787)* | 16.203 (12.587)* |
| $\tau^{(1,1)(1,0)}(1,1)$ | 2.920 (6.544) | 7.086 (8.235) | 4.334 (10.640) |
| $\tau^{(1,1)(0,1)}(1,1)$ | 1.317 (7.300) | 6.556 (8.764) | 0.808 (11.205) |
| $\tau^{(0,1)(1,0)}(0,1)$ | 1.603 (5.686) | 0.530 (6.844) | 4.066 (8.833) |
| Grade Two |  |  |  |
| $\tau^{(0,0,1)(0,0,0)}(0,0,1)$ | 15.150 (5.430)*** | 3.040 (4.428) | 0.526 (5.814) |
| $\tau^{(1,0,0)(0,0,0)}(1,0,0)$ | -2.078 (7.276) | 11.320 (7.240)* | 9.959 (8.438) |
| $\tau^{(1,1,1)(0,0,0)}(1,1,1)$ | 10.574 (26.606) | 12.714 (50.199) | 17.603 (33.463) |
| $\tau^{(1,1,1)(1,0,0)}(1,1,1)$ | 12.651 (25.589) | 1.394 (49.674) | 7.644 (32.381) |
| $\tau^{(1,1,1)(1,1,0)}(1,1,1)$ | 12.810 (22.436) | 20.282 (38.993) | 15.421 (25.999) |
| $\tau^{(0,1,1)(0,0,0)}(0,1,1)$ | 6.196 (9.400) | 8.125 (27.700) | 12.691 (12.920) |
| $\tau^{(0,0,1)(1,0,0)}(0,0,1)$ | 17.228 (9.084)** | -8.208 (8.490) | -9.433 (10.249) |
| Grade Three |  |  |  |
| $\tau^{(0,0,0,1)(0,0,0,0)}(0,0,0,1)$ | -6.835 (3.932)** | -10.590 (4.179)*** | -12.738 (5.952)*** |
| $\tau^{(1,1,1,1)(0,0,0,0)}(1,1,1,1)$ | -2.148 (129.436) | -17.192 (93.135) | -20.985 (102.228) |
| $\tau^{(1,1,1,1)(1,1,0,0)}(1,1,1,1)$ | 0.247 (120.810) | -22.487 (81.117) | -35.114 (85.973) |
| $\tau^{(1,1,1,1)(1,1,1,0)}(1,1,1,1)$ | -0.424 (96.033) | 10.115 (63.543) | 7.262 (70.360) |
| $\tau^{(1,1,1,1)(0,1,1,1)}(1,1,1,1)$ | -4.940 (86.378) | -20.263 (64.365) | -30.626 (75.468) |
| $\tau^{(0,1,1,1)(0,0,0,0)}(0,1,1,1)$ | 2.792 (96.397) | 3.071 (67.314) | 9.641 (68.958) |
| $\tau^{(0,0,1,1)(0,0,0,0)}(0,0,1,1)$ | 4.062 (59.781) | -3.472 (37.243) | -2.215 (32.284) |
| $\tau^{(0,0,1,1)(1,1,0,0)}(0,0,1,1)$ | 6.458 (75.714) | -8.767 (59.001) | -16.344 (64.043) |
|  |  |  |  |

Note: Standard Errors in parentheses.
${ }^{* * *}, * *$ indicate statistical significance at the $5 \%$, and $10 \%$ level respectively

## RESULTS III

DuMouchel and Duncan Test confirm accounting for attrition due to observables is crucial.

Likelihood ratio tests reject Null of no selection on unobservables.

## ROBUSTNESS CHECK

1) No non-linear effects of treatment
2) Simpler attrition model (less lags, more data)
3) Always versus never

- No major differences in structural parameters
- Larger Sample supports limited lasting effect of kindergarten small class attendance to reading or word recognition later on (not strong).
- Treatment benefits disappear between two prime arms

Table 6: Tests of Weighted versus Unweighted Estimates

| Subject Area | Mathematics | Reading | Word Recognition |
| :--- | :--- | :--- | :--- |
| Grade One | 8.74 | 3.39 | 1.35 |
|  | $[0.000]$ | $[0.000]$ | $[0.169]$ |
| Grade Two | 1.48 | 3.86 | 2.08 |
|  | $[0.071]$ | $[0.000]$ | $[0.002]$ |
| Grade Three | 1.72 | 1.91 | 1.03 |
|  | $[0.008]$ | $[0.002]$ | $[0.424]$ |
| Note: Probability $>$ F are in [] parentheses. |  |  |  |

Table 7: Likelihood Ratio Tests for the Presence of Selection on Unobservables

| Subject Area | Mathematics | Reading | Word Recognition |  |
| :--- | :--- | :--- | :--- | :---: |
| Grade One | 2661.91 <br> $[0.000]$ | 4468.98 <br> $[0.000]$ | 3293.98 <br> $[0.000]$ |  |
|  | 1648.11 | $[0.000]$ | $[478.86$ |  |
|  | $[0.000]$ | $[0.000]$ |  |  |
| Grade Three | 1606.95 | 1421.94 | 839.84 |  |
|  | $[0.000]$ | $[0.000]$ | $[0.000]$ |  |
| Note. Probability $>\chi^{2}$ are in [] parentheses. |  |  |  |  |

Note: Probability $>\chi^{2}$ are in [] parentheses.

Table 8: Structural Estimates of the Treatment Parameters in Education Production Functions using Simpler Attrition Model to Account for Test Completion

| Subject Area | Mathematics | Reading | Word Recognition |
| :---: | :---: | :---: | :---: |
| Kindergarten |  |  |  |
| $S_{i K}$ | $8.595(1.120)^{* * *}$ | $5.950(0.802)^{* * *}$ | $6.342(0.945)^{* * *}$ |
| Grade One |  |  |  |
| $S_{i K}$ | $12.794(4.742)^{* * *}$ | 11.221 (5.088) ${ }^{* * *}$ | 12.580 (5.433)*** |
| $S_{i 1}$ | 10.322 (2.798)*** | 4.032 (2.962) | 9.282 (3.568)*** |
| $S_{i K} S_{i 1}$ | -12.748 (5.461)*** | -3.164 (5.914) | -10.514 (6.603) |
| Grade Two |  |  |  |
| $S_{i K}$ | 8.993 (7.063) | 17.40 (8.054)*** | -1.690 (4.068) |
| $S_{i 1}$ | -15.755 (11.672) | $-37.592(16.710)^{* * *}$ | -23.035 (16.522) |
| $S_{i 2}$ | $9.001(4.839)^{* *}$ | -2.471 (4.4149) | 7.278 (8.297) |
| $S_{i K} S_{i 1}$ | 0.437 (15.122) | -0.044 (22.636) | 0.061 (21.173) |
| $S_{i K} S_{i 2}$ | -0.933 (8.931) | -19.001 (11.704) | -10.165 (21.262) |
| $S_{i 1} S_{i 2}$ | 14.477 (12.686) | 43.044 (17.248)*** | 29.128 (17.002)** |
| $S_{i K} S_{i 1} S_{i 2}$ | -7.712 (16.250) | 8.050 (24.184) | 9.189 (28.858) |
| Grade Three |  |  |  |
| $S_{i K}$ | 2.512 (11.252) | 12.487 (9.726) | 20.241 (11.072)** |
| $S_{i 1}$ | 7.347 (11.921) | 3.743 (19.584) | 3.533 (27.390) |
| $S_{i 2}$ | 32.700 (25.589) | -14.059 (11.435) | -16.140 (8.272)** |
| $S_{i 3}$ | -2.991 (3.932) | -3.547 (3.411) | -5.491 (4.815) |
| $S_{i K} S_{i 1}$ | -2.424 (19.982) | -14.738 (27.662) | -18.626 (33.645) |
| $S_{i K} S_{i 2}$ | 42.515 (28.165) | -19.929 (26.944) | -49.423 (35.623) |
| $S_{i K} S_{i 3}$ | -9.926 (26.641) | 20.363 (23.145) | 29.862 (26.369) |
| $S_{i 1} S_{i 2}$ | -30.957 (29.537) | 6.710 (27.010) | -3.718 (36.282) |
| ${ }_{S i 1} S_{i 3}$ | -34.354 (28.549) | -45.065 (25.648)** | -65.591 (29.914)*** |
| $S_{i 2} S_{i 3}$ | -27.291 (25.802) | 13.957 (11.755) | 25.368 (9.699)*** |
| $S_{i K} S_{i 1} S_{i 2}$ | -43.321 (34.722) | 38.333 (40.920) | 94.618 (53.809)** |
| $S_{i 1} S_{i 2} S_{i 3}$ | 66.369 (39.566) ${ }^{* *}$ | 46.807 (31.803) | 69.728 (38.514)** |
| $S_{i K} S_{i 1} S_{i 2} S_{i 3}$ | 8.646 (28.371) | -34.171 (28.758) | $-72.552(36.493)^{* * *}$ |

Note: Corrected standard errors in parentheses. The
sequences $S_{i K} S_{i 1} S_{i 3}$ and $S_{i K} S_{i 2} S_{i 3}$ lack unique
support to permit identification in grade 3.

Table 3: Traditional Single Period Causal Estimates of The Impacts of Reduced Class Size

| Method | Mathematics |  |  |  | Reading | Word Comprehension |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Grade 1 | Grade 2 | Grade 3 | Grade 1 | Grade 2 | Grade 3 | Grade 1 | Grade 2 | Grade 3 |
| ITT Analysis | 9.023 | 5.389 | 4.329 | 10.409 | 4.698 | 7.189 | 9.036 | 4.622 | 8.127 |
|  | $(1.946)$ | $(2.48)$ | $(2.048)$ | $(2.288)$ | $(2.316)$ | $(1.917)$ | $(2.377)$ | $(2.314)$ | $(2.45)$ |
| IV Analysis (LATE) | 10.353 | 6.417 | 5.781 | 11.948 | 5.602 | 9.584 | 10.296 | 5.513 | 10.862 |
|  | $(2.231)$ | $(2.953)$ | $(2.035)$ | $(2.625)$ | $(2.536)$ | $(2.633)$ | $(2.696)$ | $(2.748)$ | $(3.351)$ |
| Attrition Test for ITT | 61.60 | 79.89 | 86.12 | 62.21 | 89.73 | 78.22 | 36.55 | 73.21 | 63.19 |
|  | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ |
| Attrition Test for IV | 26.93 | 22.43 | 22.11 | 22.21 | 25.69 | 18.90 | 14.22 | 18.93 | 13.41 |
|  | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ | $[0.00]$ |
| Horowitz-Manski Bounds | $\{-73.174$, | $\{-111.487$, | $\{-144.049$, | $\{-67.039$, | $\{-114.397$, | $\{-130.950$, | $\{-102.133$, | $\{-97.005$, | $\{-112.028$, |
|  | $86.647\}$ | 117.794 | $149.045\}$ | $81.720\}$ | $120.315\}$ | $135.940\}$ | $119.564\}$ | $105.765\}$ | $118.986\}$ |
| Horowitz-Manski Bounds | $\{-93.935$, | $\{-87.347$ | $\{-97.246$, | $\{-82.452$, | $\{-89.447$, | $\{-87.716$, | $\{-98.434$, | $\{-78.441$, | $\{-77.924$, |
| with school covariates | $94.805\}$ | $87.861\}$ | $98.414\}$ | $83.447\}$ | $90.532\}$ | $88.713\}$ | $98.632\}$ | $78.359\}$ | $79.558\}$ |
| Lee Bounds with no | $\{5.844$, | $\{-3.010$ | $\{-3.296$, | $\{5.443$, | $\{-2.575$, | $\{-0.834$, | $\{5.471$, | $\{-1.097$, | $\{-2.523$, |
| covariates | 11.126 | $6.502\}$ | $6.005\}$ | $12.152\}$ | $6.387\}$ | $7.627\}$ | $11.407\}$ | $7.110\}$ | $8.547\}$ |
| Lee Bounds with school | $\{0.487$, | $\{-6.782$ | $\{-8.055$, | $\{-2.024$, | $\{-8.177$ | $\{-6.286$, | $\{-4.357$, | $\{-9.301$, | $\{-8.796$, |
| covariates | $20.992\}$ | $20.011\}$ | $17.619\}$ | $24.360\}$ | $18.779\}$ | $19.227\}$ | $26.532\}$ | $21.683\}$ | $22.948\}$ |
| ITT Ignoring Selective | 9.297 | 5.554 | 4.034 | 10.659 | 5.488 | 5.437 | 9.737 | 5.649 | 6.436 |
| Attrition | $(1.894)$ | $(2.070)$ | $(1.637)$ | $(2.141)$ | $(1.86)$ | $(1.545)$ | $(2.283)$ | $(2.004)$ | $(1.922)$ |
| IV Ignoring Selective Attrition | 10.852 | 6.707 | 5.642 | 12.45 | 6.783 | 7.56 | 11.323 | 6.918 | 9.004 |
|  | $(1.449)$ | $(1.854)$ | $(2.035)$ | $(1.87)$ | $(1.837)$ | $(1.959)$ | $(1.951)$ | $(2.081)$ | $(2.352)$ |

Note: The IV and ITT analyses include the full history of teacher inputs, free lunch status, race, gender and school indicators. Standard errors corrected at the classroom level are in parentheses. For the specification tests the Probability that the Null is rejected is contained in [] brackets. For all the bounds on the ATE analysis \{lower bound, upper bound \}.

Table 8:

| Subject Area | Mathematics | Reading | Word Recognition |
| :---: | :---: | :---: | :---: |
| Grade One |  |  |  |
| $\mathrm{S}_{\mathrm{iK}}$ | 4.174 (2.829) | 9.351 (2.805) | 5.434 (3.250) |
| $\mathrm{S}_{\mathrm{i} 1}$ | 6.608 (2.488) | 2.779 (2.582) | 6.415 (3.016) |
| Grade Two |  |  |  |
| $\mathrm{S}_{\mathrm{iK}}$ | 6.191 (4.034) | 10.479 (4.340) | 6.035 (4.659) |
| $\mathrm{S}_{\mathrm{i} 1}$ | -8.916 (5.191) | -6.529 (5.949) | 0.742 (5.784) |
| $\mathrm{S}_{\mathrm{i} 2}$ | 12.805 (4.152) | 5.730 (4.659) | 4.114 (4.138) |
| Grade Three |  |  |  |
| $\mathrm{S}_{\mathrm{iK}}$ | 0.131 (5.286) | 8.885 (5.088) | 12.057 (5.940) |
| $\mathrm{S}_{\mathrm{i} 1}$ | -1.168 (7.588) | -0.057 (7.500) | -5.097 (8.118) |
| $\mathrm{S}_{\mathrm{i} 2}$ | 11.747 (7.162) | 3.152 (6.784) | 11.079 (7.655) |
| $\mathrm{S}_{\mathrm{i} 3}$ | -2.596 (3.717) | -1.370 (3.244) | -6.679 (4.691) |
| DYNAMIC TREATMENT EFFECTS |  |  |  |
| Grade One |  |  |  |
| $\tau^{(1,1)(0,0)}(1,1)$ | 10.782 (3.767) | 11.933 (3.813) | 11.849 (4.434) |
| $\tau^{(1,1)(1,0)}(1,1)$ | 6.608 (2.488) | 2.779 (2.582) | 6.415 (3.016) |
| $\tau^{(1,1)(0,1)}(1,1)$ | 4.174 (2.829) | 9.351 (2.805) | 5.434 (3.250) |
| $\tau^{(0,1)(1,0)}(0,1)$ | 2.434 (3.767) | -6.572 (3.813) | 0.981 (4.434) |
| Grade Two |  |  |  |
| $\tau^{(0,0,1)(0,0,0)}(0,0,1)$ | 12.805 (4.152)* | 5.730 (4.659) | 4.114 (4.138) |
| $\tau^{(1,0,0)(0,0,0)}(1,0,0)$ | 6.191 (4.034) | 10.479 (4.340)* | 6.035 (4.659) |
| $\tau^{(1,0,0)(0,1,0)}(1,0,0)$ | 15.107 (6.574)* | 8.942 (7.364) | 17.154 (7.427)* |
| $\tau^{(1,1,1)(0,0,0)}(1,1,1)$ | 10.080 (7.776) | 9.680 (8.714) | 10.891 (8.502) |
| $\tau^{(11,1)(1,0,0)}(1,1,1)$ | 3.889 (6.647) | -0.799 (7.556) | 3.372 (7.112) |
| $\tau^{(0,0,1)(0,1,0)}(0,0,1)$ | 21.721 (6.647)* | $12.259(7.556)^{* * *}$ | 4.856 (7.112) |
| $\tau^{(0,0,1)(1,0,0)}(0,0,1)$ | 6.614 (5.789) | -4.749 (6.367) | -1.921 (6.231) |
| Grade Three |  |  |  |
| $\tau^{(0,0,0,1)(0,0,0,0}(0,0,0,1)$ | -2.596 (3.717) | -1.370 (3.244) | -6.679 (4.691) |
| $\tau^{(1,0,0,0)(0,0,0,0}(1,0,0,0)$ | 0.131 (5.286) | 8.885 (5.088)*** | 12.057 (5.940)** |
| $\tau^{(1,1,1,1)(0,0,0,0}(1,1,1,1)$ | 8.114 (12.273) | 10.580 (11.776) | 11.360 (13.483) |
| $\tau^{(1,1,1,1)(1,1,0,0)}(1,1,1,1)$ | 9.151 (8.069) | 1.782 (7.520) | 4.400 (8.798) |
| $\tau^{(1,1,1,1)(1,1,1,0)}(1,1,1,1)$ | -2.596 (3.717) | -1.370 (3.244) | -6.679 (4.691) |
| $\tau^{(1,1,1,1)(0,1,1,1)}(1,1,1,1)$ | 0.131 (5.286) | 8.885 (5.088)*** | 12.057 (5.940)** |
| $\tau^{(0,1,1,1)(0,0,0,0)}(0,1,1,1)$ | 7.983 (11.076) | 1.695 (10.621) | -0.697 (12.104) |
| $\tau^{(1,1,1,1)(1,0,0,0)}(1,1,1,1)$ | 7.983 (11.076) | 1.695 (10.621) | -0.697 (12.104) |
| $\tau^{(0,0,1,0)(, 0,0,0,0)}(0,0,1,0)$ | 11.747 (7.162) | 3.152 (6.784) | 11.079 (7.655) |
| $\tau^{(1,0,1,0)(0,0,0,0)}(1,0,1,0)$ | 11.878 (6.426)*** | 12.037 (6.034)** | 23.129 (7.570)* |
| $\tau^{(1,0,0,0)(0,0,1,0)}(1,0,0,0)$ | -11.616 (6.426)*** | 5.733 (6.034) | 0.971 (7.570) |

Note: Standard Errors in parentheses.
***,** indicate statistical significance at the $5 \%$, and $10 \%$ level respectively

Table 9

|  | Reading | Math | Word |
| :--- | :---: | :---: | :---: |
| Weighted Estimates |  |  |  |
| Kindergarten | 5.95 | 8.595 | 6.342 |
|  | $(1.28)$ | $(2.025)$ | $(1.415)$ |
| Grade One | -11.642 | -4.655 | -16.51 |
|  | $(14.768)$ | $(11.692)$ | $(16.263)$ |
| Grade Two | 2.131 | 10.2 | -1.095 |
|  | $(6.533)$ | $(6.844)$ | $(7.552)$ |
| Grade Three | 15.231 | 13.134 | 5.714 |
|  | $(7.322)$ | $(7.882)$ | $(8.913)$ |
|  | Unweighted Estimates |  |  |
| Grade One | -0.044 | -3.576 | -1.770 |
|  | $(2.97)$ | $(2.369)$ | $(3.426)$ |
| Grade Two | 6.032 | 7.498 | 5.969 |
|  | $(2.258)$ | $(2.465)$ | $(2.497)$ |
| Grade Three | 4.626 | 2.438 | 5.868 |
|  | $(2.624)$ | $(2.902)$ | $(3.366)$ |

Note: Each coefficient. Standard errors corrected at the classroom level are in parentheses.

## CONCLUSIONS

Multi-period trials with non-compliance are common in both economics and clinical medicine
-Traditional Estimators may not recover parameters of interest We adopt a simple estimation approach for this setting and apply it to data from Project STAR.
-Small classes does not seem to have Non-Linear Benefit.
-One dose in kindergarten may be all that's needed.
-No significant dynamic benefits in the higher grade.
-Achievement gap closing or reversed in higher grades may be due to a tradeoff between variation in past performance and class size as well as teachers teaching to the bottom, particularly in mathematics.

- More understanding of the trade-off between increased student variability, class size and teaching methods is needed
THE FULL DATA SET SHOULD BE MADE AVAILABLE TO THE GENERAL RESEARCH COMMUNITY (attendance, teaching strategy, group size, teaching time to each group, principal evaluation, teacher evaluation of student classroom behavior, which district 42, teaching attitude etc. observational data to get to the nature of how class size operates)

