A Simultaneous Unobserved Components Analysis of US Output and the Great Moderation[1]

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Abstract

In an unobserved components framework of US output trend and cycle, this paper seeks to determine the causal interaction between permanent and transitory innovations. For the purpose of identification, strategies of augmenting the cyclical dynamics as well as allowing for shifts in volatility are proposed. In the early 1980s, substantial predominance of cycle shocks gives way to strong negative spillovers of trend impulses, consistent with real business cycle theories. The coincident reduction of macroeconomic volatility mainly traces back to pronounced dampening of transitory disturbances. This ascribes an important role to the mitigation of policy interventions in explaining the Great Moderation.

Keywords: Unobserved Components, Trend, Cycle, Identification, Great Moderation
JEL classification: C32, E32

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1 Introduction

What is the nature of business cycles? Are they a phenomenon simply triggered by transitory shocks to the economy, or is it permanent innovations that drive both long-run growth of output as well as its periodicity? Reversely, might the course of the business cycle leave a persistent imprint on a country's development? These issues have initiated important progress in theoretical research, including prominent concepts like real business cycles (RBC) and New Keynesian economics, amongst others.

On the empirical side, unobserved components (UC) models, which specify the latent growth and cycle paths directly, naturally bear the potential to answer the introductory questions. However, the first UC models built to decompose GDP into trend and cycle assumed uncorrelated innovations (e.g. Harvey 1985, Clark 1987), thereby neglecting potential interactions a priori. More recently, Balke and Wohar (2002) as well as Morley et al. (2003) showed that correlation between permanent trend and transitory cycle shocks can be taken into account while maintaining identifiability of the structural model form. Indeed, the latter authors found a large negative correlation in their application to US GDP, just as the former for real dividend growth.

Economically, the prevalent interpretation sees the correlation as a causal effect from trend to cycle in the sense of partial GDP adjustment: When a positive permanent shock shifts up the long-run output path, we will see a negative transitory component, which vanishes over time while realigning real output with the elevated production potential. This view is consistent with Stock and Watson (1988) as well as RBC theories, see Kydland and Prescott (1982). The latter suggest that transitory fluctuations represent dynamic reactions of output to real shocks, delayed by time-to-build effects. A further theoretical interpretation stresses the role of nominal rigidities triggering negative initial impacts of positive supply or technology shocks (e.g. Blanchard and Quah 1989, Galí 1999). Even though these particular explanations for the estimated correlation might appear plausible, in terms of statistics no case can be made to exclude alternative ones, even comprising totally reversed causality. That is to say, spillovers of cycle shocks to the trend can produce an observationally equivalent outcome.

Different prominent approaches bear the potential to rationalise such a reversed mechanism: For instance, Okun (1962) argued that transitory recessions might leave their mark on permanent output, when the average age of the nation's capital stock rises (i.e., the vintage effect). The same effect on GDP is likely to occur in case unemployment does not regress to its starting point after a temporary increase, so-called hysteresis (e.g.

Blanchard and Summers 1986). Clark (1989) gives the example of a surge in investment improving short-run demand along with long-run capacity. However, the previous considerations direct at a positive linkage of trend and cycle disturbances. As Proietti (2006) notes, negative correlation would go in line with adverse effects of temporary shocks on the permanent GDP component. For example, Clark (1987) argues that initially positive demand effects of fiscal policy shocks may be followed by rising tax and interest rates, lowering production potential hand in hand with output. The same may hold true for inflationary, e.g. monetary, shocks, if they provoke increased uncertainty, dampened trade development or inefficient product and labour substitution under price staggering of Calvo or Taylor type. Moreover, labour market policy actions like increases in unemployment compensation (or disability benefits, following Clark 1989) might trigger short-run consumption-based upturns, but discourage productive work in the long run.

This list is surely extendable. Economically, the different arguments call for answering the much discussed question whether the dynamics of output are governed by permanent or transitory shocks; see King et al. (1991) and many others. Empirically, a decision between the two potential directions of causality critically hinges on the ability to identify two simultaneous effects from the data. To this end, the present paper introduces the so-called simultaneous unobserved components (SUC) model. Precisely, I seek to overidentify correlated UC models in order to reveal statistical evidence discriminating between the offered economic interpretations. For that purpose, I first propose to enhance the set of available information by extending the dynamic specification of the cyclical component and the reduced-form model version. Since this strategy turns out to suffer from weak empirical identification, a second innovative model is put forward exploiting the shift in shock variances going along with the so-called Great Moderation: As has been described by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), amongst others, the early 1980s saw a distinct decrease of variability in major macroeconomic indicators.

I show that implementing an additional variance regime allows identification of a random-walk trend, an autoregressive cycle, two according innovations as well as simultaneous cross-impacts between them from the output series alone. Furthermore, changes in the trend-cycle composition of output and even the structural origins of the Great Moderation can be assessed. Empirical evidence is twofold: On the one hand, transitory disturbances clearly dominate the first post-war decades, possibly hinting at important policy influences on economic activity. This is in notable contrast to the above-mentioned mainstream interpretation given to the negative correlation phenomenon. However, permanent shocks survive as the only relevant source of both macroeconomic growth and fluctuations since the early 1980s. I.e., the correlation of reduced-form residuals can be traced back to spillovers of trend innovations to the cycle. This supports approaches ascribing a leading role to real shocks, such as RBC theories. Concerning the discussion on the causes of the Great Moderation, the study makes a case for an important influence of changes in macroeconomic policies, associated to the tremendous reduction in genuine cyclical variability.

The reader can expect the following: Subsequently, the SUC model, including several variants, is discussed along with key considerations on identification. Section 3 then presents the application to US industrial production (IP). The last section summarises and discusses the results and sets out implications for further research. Two appendices cover identification and estimation issues, respectively.

2 Specification and Identification

The classical UC model is built on the idea that (seasonally adjusted) log output y_t can be represented as the sum of a stochastic trend τ_t and transitory deviations c_t, called the cycle. Formally, this is

$$y_t = \tau_t + c_t \tag{1}$$

$$\tau_t = \tau_{t-1} + \mu + \eta_t \quad , \quad \eta_t \sim N(0, \sigma_\eta^2) \tag{2}$$

$$c_t = b_1 c_{t-1} + \ldots + b_p c_{t-p} + \varepsilon_t \quad , \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \ , \tag{3}$$

where in modulus, all roots of the lag polynomial $B(L) = 1 - \sum_{i=1}^{p} b_i L^i$ lie outside the unit circle. Thus, the cycle is described by a stationary autoregressive process of order p (AR(p)). Periodic behaviour would only result for complex roots of $B(L)$, but in any case, I will stick to the name "cycle" for the transitory part of the output fluctuations. The trend component follows a random walk with a drift term μ that captures the steady-state growth rate of the economy. As explained later on in section 3.4, more sophisticated specifications for the drift proved inessential for the underlying analysis.

While the original contributions assumed zero covariance between the permanent and transitory innovations, Balke and Wohar (2002) and Morley et al. (2003) relaxed this constraint. The latter specified $E(\eta_t \varepsilon_t) = r\sigma_\eta \sigma_\varepsilon$, with r being the contemporaneous correlation. Decisively, r becomes identifiable by setting the AR order $p = 2$, so that the structural UC model translates into a reduced-form autoregessive integrated moving average − ARIMA(2,1,2) − by virtue of Granger's lemma (Granger and Newbold 1977). More precisely, substituting (2) and (3) into (1) leads to

$$B(L)\Delta y_t = B(1)\mu + B(L)\eta_t + \Delta\varepsilon_t \ . \tag{4}$$

The ARIMA representation in conventional form and notation is obtained as

$$B(L)\Delta y_t = c + A(L)u_t \quad , \quad u_t \sim N(0, \sigma_u^2) \ , \tag{5}$$

where A(L) is a p-dimensional lag polynomial. Its coefficients are in general determined along with the variance σ_u^2 by matching autocovariances between the MA parts in (4) and (5). Evidently, the b_i, $i = 1, \ldots, k$, from the cycle equation (3) are directly identified by the autoregressive parameters in the reduced-form ARIMA process. Then, the drift term μ can be easily recovered from the constant c. Furthermore, the right-hand-side MA part delivers $p + 1$ non-zero autocovariances $\gamma(0), \ldots, \gamma(p)$, which are theoretically given as $\gamma(j) = E[(B(L)\eta_t + \Delta\varepsilon_t)(B(L)\eta_{t-j} + \Delta\varepsilon_{t-j})]$. For $p = 2$, the MA structure thus provides sufficient information to exactly identify three unknown parameters given as the correlation r in addition to the variances σ_η^2 and σ_ε^2.

As has been set out in the Introduction, the present paper aims at incorporating further structure into the model in order to represent the causal mechanisms underlying the correlation of residuals. Thus, I split up the trend and cycle shocks from (2) and (3) according to the linear combinations

$$\eta_t = k_{11}\tilde{\eta}_t + k_{12}\tilde{\varepsilon}_t \ , \tag{6}$$

$$\varepsilon_t = k_{21}\tilde{\eta}_t + k_{22}\tilde{\varepsilon}_t \ . \tag{7}$$

This simultaneous system is normalised by $E(\tilde{\eta}_t^2) = 1$ and $E(\tilde{\varepsilon}_t^2) = 1$ as well as $k_{11} \geq 0$ and $k_{22} \geq 0$. $\tilde{\eta}_t$ and $\tilde{\varepsilon}_t$ denote structural uncorrelated trend and cycle shocks, respectively. Therefore, k_{12} and k_{21} pick up the mutual spillover effects between both unobserved components.

Note that the equation system (6), (7) replaces the three parameters σ_η^2, σ_ε^2 and r by the four k_{ij}, $i, j = 1, 2$. Naturally, the fully simultaneous SUC specification lacks one piece of information for identification. A straightforward solution to this problem works through raising the AR order p of the cycle. That is, $p = 3$ implies an ARIMA(3,1,3) structure for the reduced form, which delivers one additional AR coefficient and a third non-zero autocorrelation from the MA part. Thereby, one gains the required piece of information, because the number of unknowns in the structural form rises only by one (b_3).

While the application in the next section might even be able to justify $p = 3$ for US IP, the likelihood function around the estimates for the spillover coefficients k_{12} and k_{21} will be extremely flat. This hints at weak identification power of the determining equations arising from the autocovariances of the MA part in (5). More specifically, the expressions for $\gamma(0)$ till $\gamma(3)$ (see Appendix A for details) are to be solved for the k_{ij}, $i, j = 1, 2$. Amongst other cases, it can be shown that this equation system possesses no unique solution, and therefore fails to meet the sufficient condition for identification, if $b_2 = 0$ or $b_3 = 0$. Empirically, inference on the spillover parameters can thus not be expected to effectively discriminate between different hypotheses, if the estimate of one of the aforementioned AR parameters cannot be convincingly distinguished from zero. In short, empirical identification may fail.

Since the aim of the underlying paper requires distinct discriminating power, I offer a more elaborate solution to the fundamental identification problem. In essence, the task is to enlarge the set of information obtainable from the reduced form while extending the set of unknowns by as little as passable. For that purpose, imagine two regimes for the generating processes of the structural SUC shocks $\tilde{\eta}_t$ and $\tilde{\varepsilon}_t$, one of high and one of low volatility. One retains the variance normalisations say for the first regime, that is $\sigma^2_{\eta 1} = E(\tilde{\eta}^2_t | t \in R_1) = 1$ and $\sigma^2_{\varepsilon 1} = E(\tilde{\varepsilon}^2_t | t \in R_1) = 1$, where R_i denotes the set of time points belonging to the ith regime. Accordingly, the variances for the second regime $\sigma^2_{\eta 2} = E(\tilde{\eta}^2_t | t \in R_2)$ and $\sigma^2_{\varepsilon 2} = E(\tilde{\varepsilon}^2_t | t \in R_2)$ are free parameters differing from unity in case breaks indeed occur.

Clearly, this specification introduces two additional unknown variance coefficients ($\sigma^2_{\eta 2}$ and $\sigma^2_{\varepsilon 2}$) into the structural model. However, for the second variance regime, a completely new[2] set of autocovariances from the reduced-form MA part can be calculated, providing $p+1$ additional determining equations. It follows that for $p \geq 2$, the necessary summing-up condition for identifying the four k_{ij} in addition to $\sigma^2_{\eta 2}$ and $\sigma^2_{\varepsilon 2}$ is fulfilled. In detail, the number of unknowns (constant, AR parameters, shock loadings, variances) is given by $1+p+4+2(s-1)$, where s is the number of regimes. The number of pieces of information

[2]"Completely new" holds as long as the variances do not break proportionally to each other. Otherwise, the new set of autocovariances would linearly depend on the existing one, delivering no additional identifying information. Again, see Appendix A for details.

from the reduced form (constant, AR parameters, MA autocovariances) amounts to $1 + p + s(1 + p)$. Comparing these terms, identification requires $s(p - 1) \geq 2$, what is met by AR orders of at least three with a single regime (the case discussed first in this section) and of at least two in the presence of multiple regimes.

The idea of attaining identification by non-constant variances goes back as far as Wright (1928) and is comparable to Sentana and Fiorentini (2001) and Rigobon (2003), who treat factor models and simultaneous systems. However, while these authors rely on the contemporaneous residual covariance matrix as source of identifying information, the present approach involves the whole autocorrelation structure of the data. Furthermore, it identifies simultaneous impacts between *un*observed components (i.e., SUC) determined from a single observed series. In contrast, the existing approaches employ the conventional setup of left-hand-side observed variables depending on latent factors right hand side.

For estimation purposes, the structural model is cast in state-space form, see Appendix B. Maximum Likelihood is applied to estimate the model parameters. Thereby, the likelihood function is constructed using the prediction error decomposition from the Kalman filter, which delivers estimates of the states of the unobserved components.

3 Application to US Output

3.1 Data

Previous unobserved components studies have routinely employed quarterly US GDP. While in principle, applying the newly developed methodology to this series is feasible, I encountered no sufficient significance for effectively discriminating between competing theoretical explanations. In particular, estimation uncertainty around the spillover coefficients k_{12} and k_{21} proved to be too large. Instead, I use IP, what considerably raises the number of observations given that monthly data are available.[3] Two well-known points should be addressed: First, IP development can differ quite substantially from GDP. However, IP is quite common as an output measure in the macroeconomic literature, and in particular, the mere point estimates in my model lead to similar conclusions for GDP and IP. That is, it is solely the precision of estimates that was improved by raising the

[3]Quarterly GDP could be included along with monthly IP in a mixed-frequency state space setup. I leave this issue for future research, focusing the interest of the current paper on the fundamental identification problem in the case of a single observed variable.

frequency. Second, augmenting the number of observations through higher data frequency often does not provide the same quality of additional information as collecting time series of higher overall length. Nonetheless, in the present case, the gains in significance allow clear-cut economic interpretation based on statistical evidence.

The monthly seasonally adjusted IP index of the United States for the sample 1947:1-2008:12 is obtained from the Federal Reserve. Slight changes in the start and end points would be uncritical, see section 3.4. Log IP (multiplied by 100) and its first differences are plotted in Figure 1.

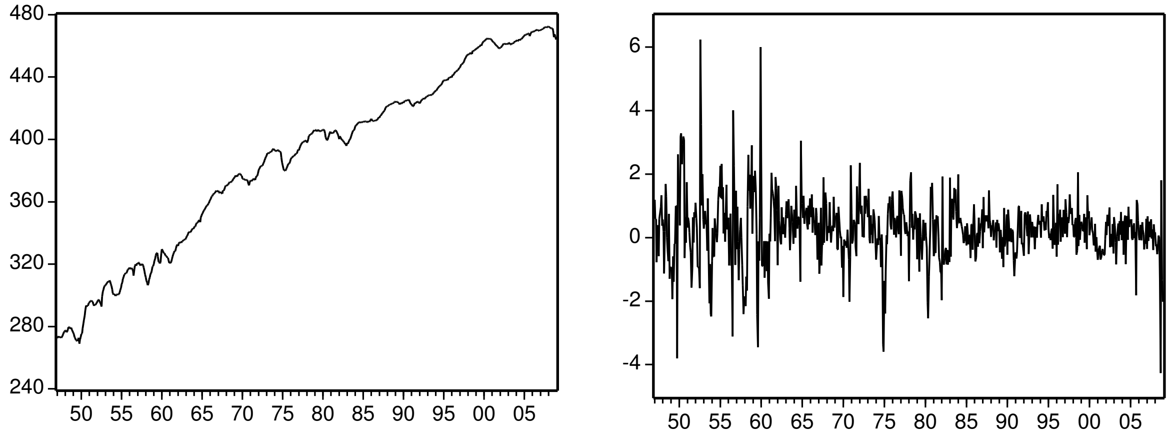

Figure 1: Log real US IP ($\times$100) and first differences

3.2 Preliminary Steps

In a first step, I estimate a correlated UC model similar to Morley et al. (2003). For determining the lag length p of the cycle, ARIMA(p,1,p) models as in (5) are specified for the log IP series. Both the Akaike and the Schwarz criterion consistently prefer a lag length of $p = 3$. All coefficients (apart from the MA(1) parameter) are significant[4], and the residual autocorrelations are rather low. Therefore, it seems justified to let the unobserved cyclical component follow an AR(3) process.

The correlation r is found to be close to -1. Statistical significance can be conveniently assessed by means of confidence sets based on the likelihood ratio (LR) principle: In

[4] Admittedly, these coefficients might be subject to near cancellation, causing bias in t-tests (e.g. Nelson and Startz 2007). Indeed, the largest AR and MA roots of the ARIMA(3,1,3) model almost lie on the unit circle. However, for the moment I continue with the decision of the information criteria, which should be fairly reliable.

detail, LR tests reject the null hypothesis $r = r_0$ for all $r_0 \geq -0.55$ (i.e., closer to zero than -0.55 or positive) on the 5% level. Morley et al. (2003), using quarterly data until 1998:2, nearly failed to reject even a zero correlation. Evidently, the additional ten years in my sample and the monthly frequency provide essential information for precise estimation. This fact shall be of avail for the analysis of the even more demanding SUC model below.

The large negative estimate for the correlation leads Morley et al. (2003) to the interpretation that positive real trend shocks leave a lower transitory component, which gradually adjusts to the permanent output path with a lag. Balke and Wohar (2002) propose a similar explanation with regard to their real dividend growth model. I now head to reassess this assertion by identifying the causal structure underlying the inferred residual correlation. Particularly, this correlation could be either generated by two shocks with strong mutual spillovers or by a single relevant shock affecting trend and cycle alike. Here, one obviously faces a fundamental identification problem.

The first solution presented in the methodological section was based on augmented cyclical dynamics. The selected lag length of $p = 3$ should be appropriate for this strategy, since it fulfils the necessary identification condition. The trend and cycle equations, with standard errors in parentheses, are estimated as

$$\tau_t = \tau_{t-1} + \underset{(0.020)}{0.116} + \underset{(1.051)}{0.370}\tilde{\eta}_t - \underset{(0.806)}{0.488}\tilde{\varepsilon}_t \tag{8}$$

$$c_t = \underset{(0.074)}{1.473}c_{t-1} - \underset{(0.149)}{0.272}c_{t-2} - \underset{(0.093)}{0.241}c_{t-3} - \underset{(0.650)}{0.232}\tilde{\eta}_t + \underset{(0.512)}{0.306}\tilde{\varepsilon}_t \,. \tag{9}$$

Strikingly, all shock loadings are clearly smaller than their standard errors. Indeed, the likelihood differences between the model in its general form and under both hypotheses $k_{12} = 0$ and $k_{21} = 0$ are negligible. That is, identification of the simultaneous structure is extremely weak, as it has been anticipated in section 2. Since we should not draw economic conclusions based on such an empirical result, I turn towards the alternative strategy of identification by variance regimes.

3.3 Identification and the Great Moderation

To begin with, reconsider the IP growth rates in Figure 1. The early 1980s witnessed a striking reduction in the volatility of macroeconomic fluctuations.[5] This phenomenon, mainly for GDP growth, which features an even clearer effect than IP, found its way into the literature as the Great Moderation (see Kim and Nelson 1999, McConnell and Perez-Quiros 2000). Concerning its reasons, the debate goes on "good policies" (e.g. Clarida et al. 2000) versus "good luck", meaning a simple reduction in the size of shocks hitting the economy (e.g. Stock and Watson 2003).[6] I do not claim to be able to decide this discussion based on inference on a single time series, i.e. IP. However, one might still arrive at straightforward conclusions even on a high level of abstraction. So, if the origin of the Great Moderation lies in better policies, and if one is willing to accept that policy shocks exert a transitory impact on the real economy, then the policy argument might be roughly associated with the cycle innovation in the present framework. Accordingly, the trend disturbance is more prone to represent structural growth shocks not under the control of single political institutions. That is, identifying the SUC structure provides the means for discriminating between competing explanations of the Great Moderation by determining the contributions of both types of shocks prior and subsequent to the breakpoint. Of course, this comes in addition to the potential of assessing strength and nature of the trend-cycle interaction and the consequences for output dynamics.

Technically, I introduce shift dummies for the variances of both innovations, letting the data decide about the respective contributions. This shift in variability provides the statistical information required for identifying the simultaneous structure. As for the exact date of the change in regimes, I pick February 1984 based on visual inspection of the growth rates in Figure 1; that is, the last in a row of pronounced spikes might have immediately occurred in the preceding month. Indeed, for GDP, the literature (as cited above) has often identified the first quarter of 1984. However, one or two years more or less would not make a decisive difference, neither to the parameter estimates nor to the likelihood. After the third AR coefficient has been eliminated due to insignificance, the model is estimated as follows:

[5] Given the available sample, it is too early to assess the high fluctuations in the 2008 financial and economic crisis. Notwithstanding, cutting the last few observations does not change the outcome of this paper.

[6] A third issue, changes in the structure of the economy, shall be left open in the present paper.

$$\tau_t = \tau_{t-1} + \underset{(0.025)}{0.088} + \underset{(0.079)}{0.618}\tilde{\eta}_t - \underset{(0.182)}{0.760}\tilde{\varepsilon}_t \tag{10}$$

$$c_t = \underset{(0.154)}{1.074}c_{t-1} - \underset{(0.129)}{0.247}c_{t-2} - \underset{(0.094)}{0.316}\tilde{\eta}_t + \underset{(0.199)}{0.936}\tilde{\varepsilon}_t \tag{11}$$

$$\sigma^2_{\eta 2} = \underset{(0.160)}{0.599} \quad , \quad \sigma^2_{\varepsilon 2} = \underset{(0.090)}{0.028} \, . \tag{12}$$

The new identification strategy shows its merits in a tremendous reduction of the standard errors, leading to highly significant impact coefficients. In the first regime, where both variances are normalised to 1, the SUC system is dominated by the cycle shock $\tilde{\varepsilon}_t$. It hits the cyclical component three times stronger than the trend innovation does, and even prevails in its effect on the trend component itself. It follows that until the Great Moderation, real persistent shocks driving output dynamics are not at the heart of an appropriate economic interpretation.[7] This stands in notable contrast to the currently prevalent interpretations discussed above.

However, the situation changes in the second regime: Here, variability of the cycle disturbances nearly vanishes, while it is reduced by only 40% for $\tilde{\eta}_t$. The LR test statistic of $H_0 : \sigma^2_{\varepsilon 2} = 0$ amounts to 0.049. Usual significance levels are unlikely to hold in this case, since under the null hypothesis, the variance is on the boundary of the admissible parameter space. Nevertheless, statistical (and economic) insignificance is too clear to be revised by any modification of the inference procedure. Consequently, the negative correlation between the reduced-form error terms can be fully traced back to transmission of trend shocks to the cyclical component. The phenomenon of the Great Moderation can thus be explained by (nearly) complete disappearance of genuine cyclical volatility, complemented by a much less important reduction of the size of trend innovations.

Finally, to gain a graphical impression, Figure 2 plots the filtered unobserved components. As it has usually been discovered in Beveridge-Nelson-type decompositions and correlated UC models, the cycle is highly volatile. Since the lag polynomial in (11) has no complex roots, one cannot observe any pronounced periodicity. In this context, recall the important result that volatility of the cycle shock $\tilde{\eta}_t$ declined enormously during the Great Moderation. However, this does not imply that business cycles totally disappear − instead, they are driven by permanent rather than by transitory innovations. Figure 2 reflects exactly this fact, since the amplitude of the cyclical UC drops, but not as overwhelmingly as exclusive inspection of $\sigma^2_{\varepsilon 2}$ would suggest.

[7] Note that since in the first regime both variances are normalised to one, variance decompositions yield identical conclusions.

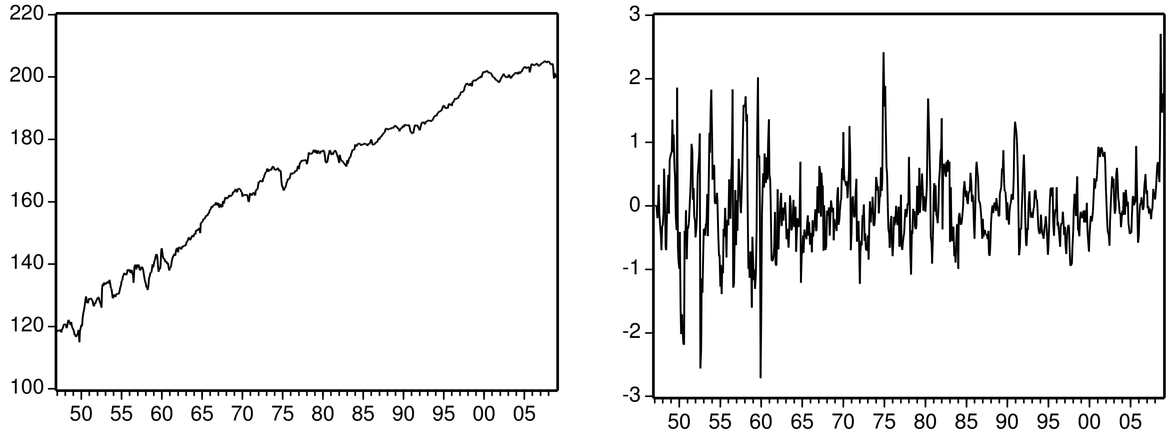

Figure 2: Filtered IP trend and cycle

3.4 Robustness Checks

Robustness of the model estimates to the following issues was checked. The tests support the reported empirical results, notably including the identified causalities.

- The deterministic drift term μ was allowed to follow a random walk. However, I encountered no relevant differences to the current investigation. This is in line with Oh and Zivot (2006), who showed that the "double-drift" specification yields results similar to Morley et al. (2003).
- Likewise, no important contribution of a trend break in 1973, as proposed by Perron and Wada (2005), could be found.
- While in section 3.2, the cycle had been specified as an AR(3) process, the third lag was omitted in section 3.3 due to insignificance. Keeping it in the model did not change the conclusions.
- Quasi Maximum Likelihood was adopted in order to make statistical inference robust against deviations from the normality assumptions. Since the standard errors did not change by much, all test results remained unaltered. In particular, the impact coefficients identified by use of variance regimes were still precisely estimated.
- The sample could be shortened without notable effects either at the beginning to leave out the Korean War, or at the end to cut off the observations affected by the subprime crisis.

4 Summary and Discussion

The underlying paper has presented the novel SUC approach to assess the importance of trend and cycle shocks in a macroeconomy. Building on the conventional UC model and its correlated extension, the focus was on determining causal structure in the interaction of permanent and transitory output components. This task makes high demand on the extraction of identifying information from the data. A first solution to this problem relied on exploiting the autocovariance structure of augmented cyclical dynamics. However, empirical identification proved to be insufficient. A more effective strategy was developed by specifying a shift in the volatility of the structural disturbances. By providing more statistical information than introducing additional unknowns, such a type of heteroscedasticity bears the potential to identify simultaneity among unobserved components.

The application to US IP revealed strikingly different patterns in the periods prior and subsequent to the Great Moderation. The first post-war decades were dominated by cycle shocks driving the transitory component and even leaving a sustained mark on the long-run growth path. As for the first regime, this stands in contrast to the popular view of prevailing real or permanent shocks. Recurring to the Introduction, one might instead locate an important source of both transitory and persistent output variability in the conduct of interventionist and discretionary monetary, fiscal and labour market policy, which might have characterised the decades until the 1980s.

Coinciding with the Great Moderation, the cyclical influence has disappeared, leaving a permanent-transitory composition of IP largely governed by trend impulses. Logically, in the second regime the spillovers underlying the residual correlation found in UC models mainly originate from the trend innovations. The Great Moderation phenomenon seems to be a product of sustained reduction of the size of transitory shocks. In other words, until the early 1980s, the cycle was predominantly triggered by transitory shocks with independent variation. Since then, however, it represents temporary adjustments of actual IP to the development of the production potential. The latter fact is in line with RBC-type theories emphasising the role of real innovations in driving business cycles in addition to long-run economic growth.

Furthermore, the results are compatible with the "good policies" position in the discussion on the origins of the Great Moderation. So, it is plausible that the reduction of excessive policy interventions, most likely in place until the 1980s, at least in part stands behind the decline of the size of transitory shocks. Nevertheless, it is clear that policy impacts cannot be uniquely identified for instance from other demand shocks within the underlying highly parsimonious framework. That is, the statistical evidence should not be unduly stressed, since inference on a single time series cannot be expected to deliver imperatively compelling arguments on a topic as complex as the functioning of a whole macroeconomy. By the same token, the potentially important issue of changes in the structure of the economy was relieved. Including policy variables in an augmented model and explicitly identifying further shocks can be expected to dissolve the remaining ambiguity. Moreover, it should not be overlooked that the general volatility decrease also includes the trend disturbances, to a much lesser extent though. As far as those can be associated to structural growth shocks exogenous to macroeconomic policy, the "good luck" hypothesis additionally helps in explaining the Great Moderation.

There is considerable potential for future research drawing on this paper's accomplishments. Particularly, it seems promising to extend multivariate approaches as in Cochrane (1994), Morley (2007), Basistha (2009) or Sinclair (2009). Clearly, before having confidence in the conclusions of the underlying investigation, further time series containing valuable information on macroeconomic trends and cycles should be employed. Those might put the permanent-transitory identification on an empirically even more firm footing and allow more precise economic interpretation. By the same token, richer causal structures implied by economic theory might be assessed econometrically. Finally, interesting studies may gauge how both the simultaneous setup and heteroscedasticity relate to identifiability of further UC specifications, for example incorporating cyclical growth, hysteresis or ARMA cycles as discussed in Proietti (2006).

5 Appendix

A Sufficient Identification Conditions

Applying the simultaneous specification (6), (7) to the ARIMA model (4) leads to

$$B(L)\Delta y_t = B(1)\mu + B(L)(k_{11}\tilde{\eta}_t + k_{12}\tilde{\varepsilon}_t) + k_{21}\Delta\tilde{\eta}_t + k_{22}\Delta\tilde{\varepsilon}_t \ . \tag{13}$$

The autocovariances $\gamma(j)$ of the MA part are calculated according to

$$\gamma(j) = E[(B(L)(k_{11}\tilde{\eta}_t + k_{12}\tilde{\varepsilon}_t) + k_{21}\Delta\tilde{\eta}_t + k_{22}\Delta\tilde{\varepsilon}_t)\cdot (B(L)(k_{11}\tilde{\eta}_{t-j} + k_{12}\tilde{\varepsilon}_{t-j}) + k_{21}\Delta\tilde{\eta}_{t-j} + k_{22}\Delta\tilde{\varepsilon}_{t-j})] \,. \tag{14}$$

For $j = 0, \ldots, p$, one gets $p + 1$ equations. This nonlinear equation system has a locally unique solution, if the first derivatives matrix $\partial\gamma/\partial k'$ has full rank $p + 1$. Therein, the column vectors γ and k stack all $\gamma(j)$, $j = 0, \ldots, p$, respectively k_{ij}, $i, j = 1, 2$. In presence of nonlinear terms, the rank naturally depends on unknown parameters k_{ij}. Plugging in estimates from the system (8), (9), the rank is numerically not reduced. Nonetheless, since the determinant of $\partial\gamma/\partial k'$ is near zero, empirical identification is likely to be weak. Concerning the reasons for that outcome, it can for instance be shown that the matrix is in general irregular if $b_2 = 0$ or $b_3 = 0$.

Above, $E(\tilde{\eta}_t^2) = 1$ and $E(\tilde{\varepsilon}_t^2) = 1$ applied. In the two-regime case, these normalisations are retained for the first regime, while the variances $\sigma^2_{\eta 2}$ and $\sigma^2_{\varepsilon 2}$ in the second regime are freely estimated. The autocovariances $\gamma_1(0), \ldots, \gamma_1(p)$ and $\gamma_2(0), \ldots, \gamma_2(p)$ can thus be calculated separately for both regimes, providing extra equations available for identifying the simultaneity. In this, a further (sufficient) condition must be taken into account: A proportional break would occur for $\sigma^2_{\eta 2} = \sigma^2_{\varepsilon 2}$ (since the variances are identical in the first regime by definition). Then, the $\gamma_2(j)$, $j = 0, \ldots, p$, would simply result as multiples of their first-regime counterparts, a special case of linear dependence. Formally, let vector γ_R (R for "regime") stack all $\gamma_1(j)$ and $\gamma_2(j)$ and vector k_R stack all k_{ij}, as well as $\sigma^2_{\eta 2}$ and $\sigma^2_{\varepsilon 2}$. Then, even though the dimension of the first derivatives matrix $\partial\gamma_R/\partial k'_R$ would rise compared to the constant variance case, its rank would not be augmented by the introduction of the shift in volatility. Finally, note that the condition of linear independence does *not* require breaks in *both* of the structural variances.

B State-Space Model

Setting up a spate-space model, both trend and cycle are treated as state variables. According to (1), the observation equation is the simple identity

$$y_t = \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tau_t \\ c_t \\ c_{t-1} \\ c_{t-2} \end{pmatrix} . \tag{15}$$

Combining (2) and (3) gives the transition equation

$$\begin{pmatrix} \tau_t \\ c_t \\ c_{t-1} \\ c_{t-2} \end{pmatrix} = \begin{pmatrix} \mu \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b_1 & b_2 & b_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ c_{t-3} \end{pmatrix} + \begin{pmatrix} \eta_t \\ \varepsilon_t \\ 0 \\ 0 \end{pmatrix} . \tag{16}$$

Based on the simultaneous extensions (6) and (7), the covariance matrix of the vector of transition errors can be written as

$$\begin{pmatrix} k_{11}^2 + k_{12}^2 & k_{11}k_{21} + k_{12}k_{22} & 0 & 0 \\ k_{11}k_{21} + k_{12}k_{22} & k_{21}^2 + k_{22}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \tag{17}$$

Note that the covariance matrix of η_t and ε_t (the upper left block) as a quadratic form in the matrix of the k_{ij}s is guaranteed to be positive definite. In the two-regime case, for the observations after the shift a second covariance matrix applies. It differs from (17) only in that each k_{i1} is to be multiplied by $\sigma_{\eta 2}$ and each k_{i2} by $\sigma_{\varepsilon 2}$, $i = 1, 2$.

Initial values for the AR parameters and the constant are obtained from estimating the appropriate ARIMA process. The trend starts at the first observation of the series y_t and is assigned an extremely large variance. The cycle is initialised at zero with the variance of the IP growth rates. Then, the log-likelihood function can be constructed and numerically maximised passing through the standard Kalman filter equations.

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