On Determinacy and Learnability in a New Keynesian Model with Unemployment

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Abstract

We analyze determinacy and stability under learning (E-stability) of rational expectations equilibria in the Blanchard and Gali (2006, 2008) New-Keynesian model of inflation and unemployment, where labor market frictions due to costs of hiring workers play an important role. We derive results for alternative specifications of monetary policy rules and alternative values of hiring costs as a percentage of GDP. Under low hiring costs—a typical part of the U.S. calibration—for policy rules based on current period inflation and unemployment our results are similar to those of Bullard and Mitra (2002). However, we find that the region of indeterminacy and E-instability in the policy space increases with the hiring costs. So, higher hiring costs—consistent with the European 'sclerotic' labor market institutions—seem to play an important part in explaining unemployment instability. Under lagged data based rules the area where monetary policy delivers both determinacy and E-stability shrinks. These rules perform worse according to these two dimensions when hiring costs go up. Finally, under expectations-based rules—unlike Bullard and Mitra (2002)—an additional explosive region is introduced. Here also the scope for determinacy and E-stability oriented monetary policy decreases. Interestingly—under the same rule and European 'sclerotic' labor market institutions - we find that responding too much to expected inflation and too little to expected unemployment may very well be self-defeating. When hiring costs are large, a central bank that follows such a policy rule could very easily end up in the worst-case scenario of both indeterminacy and E-instability.

Keywords: Monetary Policy Rules; Determinacy; Learning; E-Stability.

JEL Codes: E52, E31, D84

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1 Introduction

Business cycle models with forward-looking expectations may be prone to two types of problems. The first is real indeterminacy—the possibility that a unique, stationary rational expectations equilibrium does not exist. The second problem—which has received more attention recently—is expectational instability (in short E-instability a la Evans and Honkapohja (2001)) under private sector learning. In models where monetary policy plays a role in determining inflation and output, such as the New-Keynesian model, one may wonder what sorts of policy rules may lead the economy into indeterminacy and/or expectational instability, so that policymakers can avoid using such undesirable policy rules. Bullard and Mitra (2002) analyze determinacy and learnability of rational expectations equilibria in a purely forward-looking New-Keynesian model of inflation and the output gap. They evaluated different forms of Taylor-type rules for setting the nominal interest rate. While Bullard and Mitra (2002) make an important contribution, their analysis also reveals that the conditions for determinacy and learnability of rational expectations equilibria are affected not only by the form of the monetary policy rule but also by the structure of the economy. The version of the New-Keynesian model that they analyze is simple, in the sense that it does not feature endogenous persistence in inflation and the output gap, and is thus at odds with empirical evidence. Moreover, as argued by, among others, Blanchard and Gali (2006, 2008), the standard New-Keynesian model does not generate movements in unemployment, as it assumes variations in hours worked only. In order to account for unemployment dynamics, the authors extend the standard New-Keynesian model by incorporating search and matching in labor markets.[1] The resulting model of inflation and unemployment has richer dynamics, owing to the presence of lags and leads of unemployment in the Phillips curve and of lags of unemployment in the IS curve.

We use the model of Blanchard and Gali (2008) to analyze determinacy and learnability of rational expectations equilibria under alternative forms of monetary policy rules.[2] Our paper is setup similar to Bullard and Mitra (2002) although we distinguish between a general form (analyzed in Bullard and Mitra (2002)) and a special form of model specification under private sector learning. The general form assumes that even if the structural model does not include a constant term (as in our case) private agents include a constant term in their econometric model

[1] See also Ravenna and Walsh (2008).

[2] Lubik and Krause (2004) analyze the issue of determinacy in a search and matching model of the labor market. Kurozumi and Van Zandweghe (2008) study determinacy and E-stability for search and matching models along the lines of Krause and Lubik (2007) and Christoffel and Kuester (2008).

used for forecasting. Thus bounded rationality is more severe when learning is based on the general form. In this case, indeterminate equilibria that do not fulfill the Taylor principle (where in the long run the nominal interest rate moves more than one-to-one with inflation) are also E-unstable. By contrast, when bounded rationality is not severe, some indeterminate equilibria are E-stable.

Section 2 sets out the linear model of inflation and unemployment. Then section 3 incorporates three policy rules, in each case reporting results on determinacy and learnability of rational expectations equilibria. In section 4 we report additional results, pertaining to the effects of changes in the degree of persistence in productivity shocks, and changes in the intertemporal elasticity of substitution. Finally, section 5 concludes.

2 A model of inflation and unemployment

We work with three key log-linear equations related to the Phillips curve, the IS curve and a monetary policy rule (see in particular Blanchard and Gali (2008)),

$$\pi_t = \beta \pi^e_{t+1} + k_{\pi 0} u_t + k_{\pi f} u^e_{t+1} + k_{\pi l} u_{t-1} + k_{\pi a} a_t \tag{1}$$

where π_t is the rate of inflation, u_t is unemployment rate, and a_t is a labor productivity shock, $0 < \beta < 1$ is the discount factor and $k_{\pi 0} < 0$, $k_{\pi f} < 0$, $k_{\pi l} > 0$.[3] The terms π^e_{t+1} and u^e_{t+1} stand for private sector expectations of inflation and unemployment in period $t+1$ conditional on information available as of period t.

the IS equation is

$$u_t = k_{uf} u^e_{t+1} + k_{ul} u_{t-1} + \tau(i_t - \pi^e_{t+1} - \rho) + k_{ua} a_t \tag{2}$$

where i_t is the nominal interest rate and the parameters $k_{uf} > 0$, $k_{ul} < 0$, $\tau > 0$ are functions of the intertemporal elasticity of substitution in consumption and labor market parameters (such as the hiring cost, equilibrium unemployment, job finding rate and job separation rate).

The productivity shock a_t follows an exogenous first-order autoregressive process $(0 < \rho_a < 1)$

$$a_t = \rho_a a_{t-1} + \eta_t \tag{3}$$

[3] By contrast the Phillips curve considered by Bullard and Mitra (2002) takes a simple form $\pi_t = \beta \pi^e_{t+1} + k_{\pi 0} x_t$ where x_t is the output gap

where η_t is a white noise innovation.

To close the model, we need to specify a particular form of the monetary policy rule. Following Bullard and Mitra (2002), we consider various forms simple policy rules.[4] For each policy rule candidate, we examine which regions in the policy space are associated with (in)determinate and/or E-(un)stable rational expectations equilibria. We also examine whether or not indeterminacy is associated with E-instability.

3 Alternative policy rules

3.1 Policy rule based on contemporaneous data

Following Bullard and Mitra (2002), we first consider the policy rule that responds to current period inflation and unemployment. The exact form shown below is taken from Blanchard and Gali (2008).

$$i_t = \rho + \varphi_\pi \pi_t + \varphi_u u_t \tag{4}$$

where $\varphi_\pi > 0$ and $\varphi_u < 0$. For future reference, we note that the long-run response of the nominal interest rate to changes in inflation is given by $\varphi_\pi + (1-\beta)\varphi_u/k$, where $k = k_{\pi 0} + k_{\pi f} + k_{\pi l}$. If the response is more than one-to-one, then the policy rule satisfies the so-called Taylor principle (see e.g., Woodford (2003)).

Substituting the policy rule (4) in (2) and rearranging we get a system of equations in u_t and π_t. Defining $z_t = (u_t, \pi_t)'$, the system of equations in matrix form is

$$Az_t = A_1 z^e_{t+1} + A_2 u_{t-1} + A_3 a_t \tag{5}$$

where

$$A = \begin{pmatrix} (1-\tau\varphi_u) & -\tau\varphi_\pi \\ -k_{\pi 0} & 1 \end{pmatrix}; A_1 = \begin{pmatrix} k_{uf} & -\tau \\ k_{\pi f} & \beta \end{pmatrix}; A_2 = \begin{pmatrix} k_{ul} \\ k_{\pi l} \end{pmatrix}; A_3 = \begin{pmatrix} k_{ua} \\ k_{\pi a} \end{pmatrix}$$

Multiplying by A^{-1},

$$z_t = Bz^e_{t+1} + Du_{t-1} + \zeta a_t \tag{6}$$

where $B = A^{-1}A_1$, $D = A^{-1}A_2$, and $\zeta = A^{-1}A_3$.

[4]In future work, we intend to consider policy rules derived from optimal monetary policy.

3.1.1 Determinacy

Following the steps of Evans and Honkapohja (2006), introduce the auxiliary variable $u_t^L \equiv u_{t-1}$. Then rewrite (5) as $H_1 z_t' = H_2 z_{t+1}' + other$; where $z_t' = (z_t, u_t^L)$,

$$H_1 = \begin{pmatrix} 1 & 0 & -D_{11} \\ 0 & 1 & -D_{21} \\ 1 & 0 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{11} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the term *other* represents exogenous variables, which not relevant for determinacy. Let $J = H_1^{-1} H_2$. In this model, z_t is a vector of jump variables, while x_t^L is predetermined. Thus, the condition for determinacy is that two eigenvalues of J are inside the unit circle and one eigenvalue is outside the unit circle (see Blanchard and Kahn (1980) for the technical details).

3.1.2 Learning: Regressor set with non-zero constant

Following much of the literature on learning we focus on learning the minimum state variables (MSV) solutions of (6).[5] The state variables are u_{t-1} and a_t, and the reduced form (6) does not have a constant term and π_{t-1}. If agents do not know this fact, they could conjecture that the law of motion of z_t is

$$z_t = c + b z_{t-1} + f a_t \tag{7}$$

where c is a constant and $b = \begin{pmatrix} b_{uu} & b_{up} \\ b_{\pi u} & b_{pp} \end{pmatrix}$ because of the inclusion of π_{t-1}. The learning model is thus overparameterized. As will be seen below, learning about the steady state turns out to be crucial for E-stability of indeterminate rational expectations equilibria.

It is important to note that any MSV solution will necessarily have $c = 0$ and $b = \begin{pmatrix} b_{uu} & 0 \\ b_{\pi u} & 0 \end{pmatrix}$. By contrast, model specification may matter for E-stability. The reason is that under learning the additional coefficients (c, b_{up} and b_{pp}) may be nonzero and the updating of the components of b are interdependent.

Suppose now equation (7) is the perceived law of motion (PLM). Forwarding it one period and taking expectations

$$z_{t+1}^e = c + b(c + b z_{t-1} + f a_t) + f \rho_a a_t = (I + b)c + b^2 z_{t-1} + (bf + f\rho_a) a_t \tag{8}$$

[5]See McCallum (1983) for a justification of focusing on the MSV solution.

where I is a conformable identity matrix. Substituting (13) in (5), we get the actual law of motion (ALM)

$$z_t = B(I+b)c + (Bb^2 + D)z_{t-1} + (Bbf + Bf\rho_a + \zeta)a_t \tag{9}$$

where $D = A^{-1}A_2$ and $A_2 = \begin{pmatrix} k_{ul} & 0 \\ k_{\pi l} & 0 \end{pmatrix}$.

The rational expectations solution is given by $\bar{c}, \bar{b}, \bar{f}$ that satisfy the following system of matrix equations

$$\begin{aligned} B(I+b)c &= c \\ Bb^2 + D &= b \\ Bbf + Bf\rho_a + \zeta &= f \end{aligned} \tag{10}$$

The system is recursive—we first solve for $b_{uu}, b_{\pi u}$, then use that solution in the second equation to get the solution for f_u, f_π. However, the system of equations in $b_{uu}, b_{\pi u}$ is quadratic and in general, it has multiple solutions, which is crucial for stability under learning.[6]. Thus, it may turn out that E-stability is a stricter requirement than determinacy although there could be cases where neither condition implies the other. We point out that in the indeterminate case there maybe non-MSV solutions (that is, sunspot solutions), but following Bullard and Mitra (2002) we focus on the E-stability of MSV solutions.

Under learning, the PLM need not be identical to the ALM. In real-time, the private sector updates its parameter estimates recursively using newly available data. Evans and Honkapohja (2001) have shown that under fairly general conditions the local convergence of real-time learning using recursive least-squares and closely related algorithms is governed by what they call the *E-stability principle* related to the local convergence of a differential equation system in notional time (see p.232 of Evans and Honkapohja (2001)).[7] Since the analysis of local convergence using the E-stability principle is much easier than the real-time counterpart, one usually focuses on E-stability. This is represented by the T-mapping $T(c, b, f) = (B(I+b)c, Bb^2 + D, Bbf + Bf\rho_a + \zeta)$. The private sector's updating

[6] Note however that all MSV solutions are characterized by $\bar{c} = \bar{b}_{up} = \bar{b}_{pp} = 0$

[7] Under real-time learning $b_{t-1} = \begin{pmatrix} b_{uu,t-1} \\ b_{\pi u,t-1} \end{pmatrix}$ and $f_{t-1} = \begin{pmatrix} f_{u,t-1} \\ f_{\pi,t-1} \end{pmatrix}$, where b_{t-1} and f_{t-1} are updated using recursive least-squares.

of its PLM is given by the system of differential equations

$$\frac{d}{dt'}(c, b, f) = T(c, b, f) - (c, b, f) \tag{11}$$

where t' represents notional time. Note that since $T(c, b, f)$ is non-linear, for local stability, we need to linearize the system around a given MSV solution $(\bar{c}, \bar{b}, \bar{f})$. If the above system is locally asymptotically stable at $(\bar{c}, \bar{b}, \bar{f})$, then $(\bar{c}, \bar{b}, \bar{f})$ is E-stable. The necessary and sufficient conditions for E-stability of the MSV solution $(\bar{b}, \bar{f})$ are that the eigenvalues of

$B + B\bar{b}$,

$\bar{b}' \otimes B + I \otimes B\bar{b}$, and

$\rho_a B + B\bar{b}$

all have real parts less than 1.

Because the model features endogenous persistence owing to lags in unemployment, we could not characterize characterize our results analytically.[8] Thus, we show our results numerically based on the calibration used in Blanchard and Gali (2008) for the US economy, with two exceptions. First, in order to compare our results with those of Bullard and Mitra (2002), we assume a CES utility function. Second, noting that Blanchard and Gali (2008) put a hiring cost of 0.1 percent of GDP as an upper bound, we chose a hiring cost of 0.05 percent of GDP, which might seem small but is large enough to give a richer inflation and unemployment dynamics. For this baseline calibration, the IS curve parameters are k_{uf}= 1.07, k_{ul}= -0.07, and τ= 7.16 while the Phillips curve parameters are $k_{\pi 0}$= -0.06, $k_{\pi f}$= -0.05, and $k_{\pi l}$= 0.02. We compare the results of the baseline calibration with two other calibrations. The first serves as a limiting case where hiring costs are insignificant (0.01 percent of GDP), in which case $k_{uf} = 1.01$, $k_{ul} = -0.01$, and $\tau = 6.24, k_{\pi 0} = -0.01, k_{\pi f} = -0.01$, and $k_{\pi l} = 0.003$. The second alternative calibration for the hiring cost is based on the upper bound 0.1 used in Blanchard and Gali (2008). In this case, $k_{uf} = 1.18$, $k_{ul} = -0.18$, and $\tau = 8.78, k_{\pi 0} = -0.13, k_{\pi f} = -0.09$, and $k_{\pi l} = 0.03$. As a benchmark, we set $\rho_a = 0.9$ as in Blanchard and Gali (2008) but we also show results for $\rho_a = 0.8$ and $\rho_a = 0.95$.

Figure 1 shows the E-stability property of the model under the general form of model specification. The result shown in the left panel of the figure based on a

[8] Even for the purely forward-looking New-Keynesian model, Bullard and Mitra (2002) had to resort to a calibrated model once they allow a policy rule that responds to lagged inflation and lagged output gap.

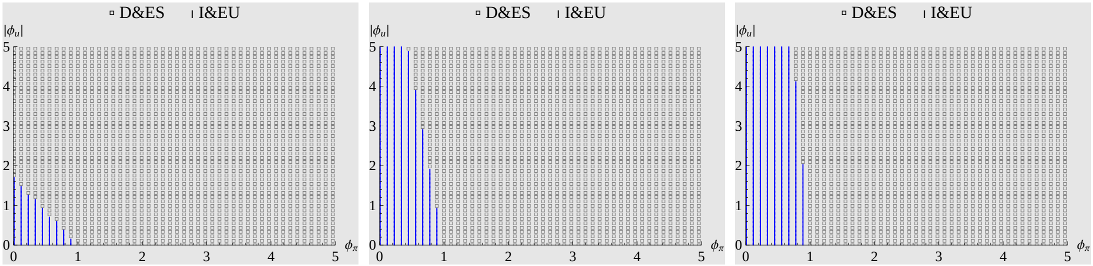

Figure 1: Policy rule based on contemporaneous data. Forecasting model with a constant. Hiring cost (percentage of GDP): 0.1 percent(left panel), 0.5 percent (center panel), 1 percent (right panel).

hiring cost which is very small as a percentage of GDP (0.01), so that this result resembles that of a purely forward-looking model used by Bullard and Mitra (2002). For the figures in the center and right panels, this number is set at 0.05 and 0.1, respectively (Blanchard and Gali (2008) remark that 0.01 is a reasonable upper bound). In all calibrations, we see that even with the general form model the determinate region (which satisfies the Taylor principle) is also E-stable, while the indeterminate region (which violates the Taylor principle) is E-unstable. The difference is that the indeterminate and E-unstable region expands as the hiring cost increases.

3.1.3 Learning with exact MSV specification: zero constant

When the private sector knows the exact MSV form (without zero constant), the relevant state variables are u_{t-1} and a_t, and the MSV solution of the model should take the following form

$$\begin{pmatrix} u_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} b_{uu} \\ b_{\pi u} \end{pmatrix} u_{t-1} + \begin{pmatrix} f_u \\ f_\pi \end{pmatrix} a_t \tag{12}$$

Suppose equation (12) is the perceived law of motion (PLM). Forwarding it one period and taking expectations

$$\begin{pmatrix} u^e_{t+1} \\ \pi^e_{t+1} \end{pmatrix} = \begin{pmatrix} b^2_{uu} \\ b_{uu} b_{\pi u} \end{pmatrix} u_{t-1} + \begin{pmatrix} (b_{uu} + \rho_a) f_u \\ (b_{pu} + \rho_a) f_\pi \end{pmatrix} a_t \tag{13}$$

Substituting (13) in (5), we get the actual law of motion (ALM)

$$\begin{pmatrix} u_t \\ \pi_t \end{pmatrix} = T_b u_{t-1} + T_f a_t \tag{14}$$

where $T_b = B\begin{pmatrix} b_{uu}^2 \\ b_{uu}b_{\pi u}\end{pmatrix} + D$; $T_f = B\begin{pmatrix}(b_{uu}+\rho_a)f_u \\ (b_{\pi u}+\rho_a)f_\pi\end{pmatrix} + A_3$

Under the hypothesis of rational expectations, the ALM is identical to the PLM, and the rational expectations solution is given by $b_{uu}, b_{\pi u}, f_u, f_\pi$ that satisfy the following system of equations

$\begin{pmatrix} b_{uu} \\ b_{\pi u}\end{pmatrix} = T_b$; $\begin{pmatrix} f_u \\ f_\pi\end{pmatrix} = T_f$

Under learning, the T-mapping is $b \to T_b$ and $f \to T_f$, where $b = (b_{uu}, b_{\pi u})'$ and $f = (f_u, f_\pi)'$ and the private sector's updating of its beliefs is given by the system of differential equations

$$\frac{d}{dt'}(b, f) = (T_b, T_f) - (b, f) \tag{15}$$

As before, $T(b, d)$ is non-linear, thus for local stability, we need to linearize the system around a given MSV solution $(\bar{b}, \bar{f})$. If the above system is locally asymptotically stable at $(\bar{b}, \bar{f})$, then we say that $(\bar{b}, \bar{f})$ is expectationally stable (E-stable, in short). The necessary and sufficient conditions for E-stability of the MSV solution are that the eigenvalues of

$\bar{b}' \otimes B + I \otimes B\bar{b}$, and

$\rho_a B + B\bar{b}$

all have real parts less than 1.

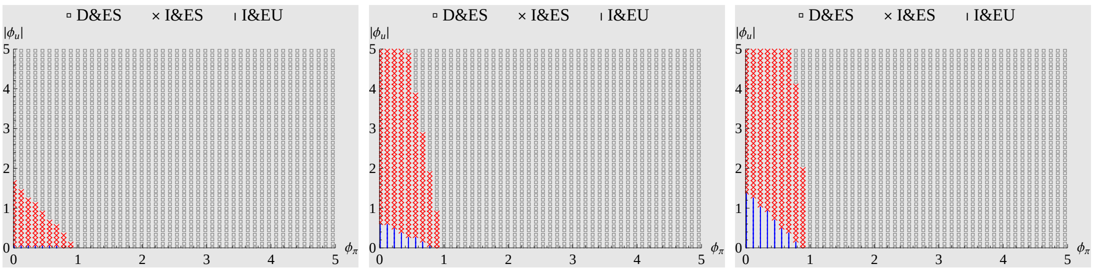

Figure 2: Policy rule based on contemporaneous data. Forecasting model without a constant. Hiring cost (percentage of GDP): 0.1 percent(left panel), 0.5 percent (center panel), 1 percent (right panel).

Figure 2 shows that part of the indeterminate region is E-stable, even though it violates the Taylor principle. Thus the indeterminate and E-stable region expands somewhat compared to that under the general form of the learning model previously considered. The region that is both indeterminate and E-unstable arise when the policy rule assigns small values (in absolute terms) to φ_π and φ_u. Finally, the determinate region (which satisfies the Taylor principle) continues to be E-stable.

3.2 Policy rule based on expectations data

We now analyze the policy rule based on expectations of future inflation and unemployment. It takes the form

$$i_t = \rho + \varphi_\pi \pi^e_{t+1} + \varphi_u u^e_{t+1} \tag{16}$$

The analysis of determinacy and learnability follows analogous to the previous case of a policy rule that responds to current period inflation and unemployment. Substituting the policy rule (16) in (2) and rearranging the system of equations takes the same form as equation (5) where now $A = \begin{pmatrix} 1 & 0 \\ -k_{\pi 0} & 1 \end{pmatrix}$; $A_1 = \begin{pmatrix} k_{uf} + \tau\varphi_u & \tau(\varphi_\pi - 1) \\ k_{\pi f} & \beta \end{pmatrix}$; $A_2 = \begin{pmatrix} k_{ul} & 0 \\ k_{\pi l} & 0 \end{pmatrix}$; $A_3 = \begin{pmatrix} k_{ua} \\ k_{\pi a} \end{pmatrix}$ and therefore, what is new is that the matrices B and D have different elements.

3.2.1 Determinacy

The conditions for determinacy are the same as those for the case of the policy rule based on contemporaneous data, namely that two eigenvalues of J are inside the unit circle and one eigenvalue is outside the unit circle. The results are shown in in Figure 4 and Figure 3. The first thing to note is that, compared to the policy rule based on current period inflation and unemployment, the region of determinacy is smaller. More important is, however, the result that there is indeterminacy for sufficiently small values of ϕ_u and sufficiently large values of ϕ_π (see right panel of Figure 3). This result contrasts with Bullard and Mitra (2002) who find that this region is determinate, which here is roughly captured by the limiting case shown in the left panel of Figure (3).

3.2.2 Learning: Regressor set with non-zero constant

The effect of including a constant in the learning model is similar to the case of contemporaneous data based rule. In particular this specification implies that indeterminate equilibria that fail to satisfy the Taylor principle are also E-unstable, while determinate and indeterminate equilibria that satisfy the Taylor principle continue to be E-stable. The fact that the unmarked region has explosive solutions is also not a feature of Bullard and Mitra (2002).[9]

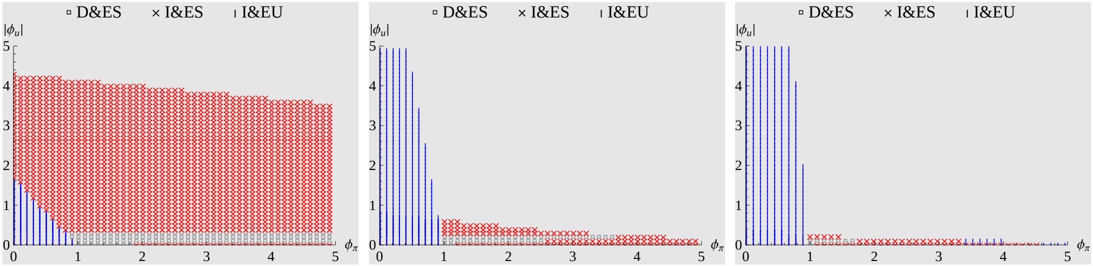

Figure 3: Policy rule based on expectations data. Forecasting model with a constant. Hiring cost (percentage of GDP): 0.1 percent(left panel), 0.5 percent (center panel), 1 percent (right panel). The unmarked region has explosive solutions.

3.2.3 Learning with exact MSV specification: zero constant

Under exact MSV specification of the learning model, some part of the indeterminate region, where the Taylor principle does not hold, is E-stable. This region shrinks as the hiring cost increases.

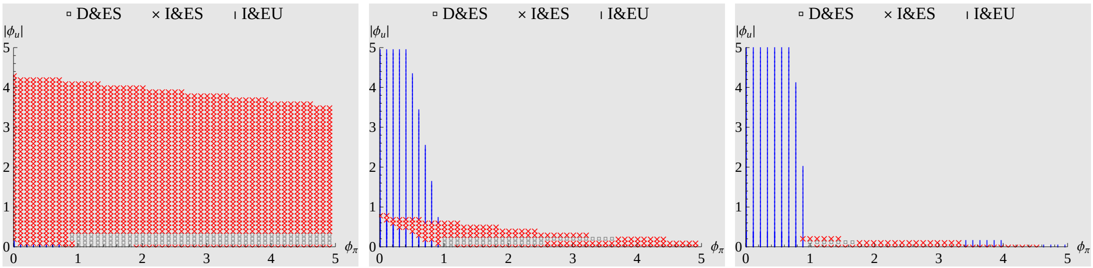

Figure 4: Policy rule based on expectations data. Forecasting model without a constant. Hiring cost (percentage of GDP): 0.1 percent(left panel), 0.5 percent (center panel), 1 percent (right panel). The unmarked region has explosive solutions.

3.3 Policy rule based on lagged data

The policy rule based on lagged data takes the form

$$i_t = \rho + \varphi_\pi \pi_{t-1} + \varphi_u u_{t-1} \tag{17}$$

[9]In the indeterminate region where the Taylor pratincole is violated, we find three solutions, of which two are stationary.

substituting the policy rule (17) in (2) and rearranging the system of equations in matrix form will be (5) where $A = \begin{pmatrix} 1 & 0 \\ -k_{\pi 0} & 1 \end{pmatrix}$; $A_1 = \begin{pmatrix} 1 & -\tau \\ k_{\pi f} & \beta \end{pmatrix}$; $A_2 = \begin{pmatrix} \tau\varphi_u & \tau\varphi_\pi \\ k_{\pi l} & 0 \end{pmatrix}$; $A_3 = \begin{pmatrix} k_{ua} \\ k_{\pi a} \end{pmatrix}$

3.3.1 Determinacy

Analogous to section 3.1.1 introduce two auxiliary variables $u_t^L \equiv u_{t-1}$ and $\pi_t^L \equiv \pi_{t-1}$. Then rewrite (5) as $H_1 z_t' = H_2 z_{t+1}' + other$; where $z_t' = (z_t, u_t^L, \pi_t^L)$,

$$H_1 = \begin{pmatrix} 1 & 0 & -D_{11} & -D_{12} \\ 0 & 1 & -D_{21} & -D_{22} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \; H_2 = \begin{pmatrix} B_{11} & B_{12} & 0 & 0 \\ B_{21} & B_{11} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The system $z_t' = Jz_{t+1}' + other$, with $J = H_1^{-1} H_2$, has two predetermined variables and two jump variables. Thus determinacy requires that two eigenvalues of J be inside the unit circle and two outside the unit circle.

3.3.2 Learning: Regressor set with non-zero constant

Again we repeat the steps in the case of a rule with contemporaneous data. Figure 5 looks similar to that of Bullard and Mitra (2002).[10] This is not that surprising, because with a policy rule that responds to lags of inflation and the output gap their reduced model has endogenous lags, just like ours. In this case, determinacy is necessary but not sufficient for E-stability.

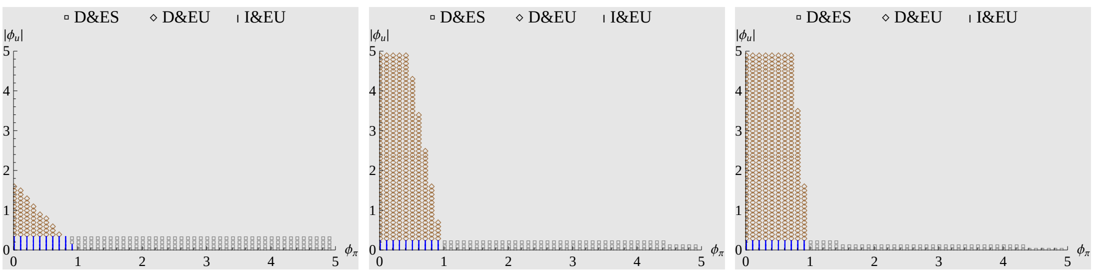

Figure 5: Policy rule based on lagged data. Forecasting model with a constant. Hiring cost (percentage of GDP): 0.1 percent(left panel), 0.5 percent (center panel), 1 percent (right panel). The unmarked region has explosive solutions.

[10]The unmarked region has explosive solutions, as in Bullard and Mitra (2002), and in this case the transversality conditions are violated.

The region of indeterminacy shrinks somewhat with the hiring costs but the policy rule leads to a large region with explosive solutions. Also, while the determinate and E-unstable region expands the determinate and E-stable region shrinks as hiring costs increase.

3.3.3 Learning with exact MSV specification: zero constant

Here we see that with exact MSV specification of the learning model, part of the indeterminate region is E-stable while the determinate region is E-unstable. In this case, determinacy is neither necessary nor sufficient for E-stability. On the other hand, fulfillment of the Taylor principle is necessary for determinacy and sufficient for E-stability.

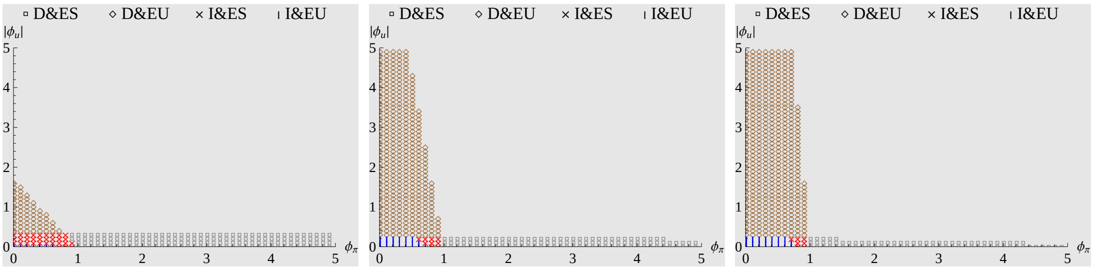

Figure 6: Policy rule based on lagged data. Forecasting model without a constant. Hiring cost (percentage of GDP): 0.1 percent(left panel), 0.5 percent (center panel), 1 percent (right panel). The unmarked region has explosive solutions.

We remark that the indeterminate region has six solutions, of which three are stationary. Of these some are E-stable, while others are not. Again, the indeterminate and E-unstable region is associated with a policy rule which responds very weakly to lags of inflation and unemployment.

4 Model variations

4.1 Persistence of productivity shocks

In our baseline case, we calibrate the degree of persistence in the productivity shock at $\rho_a = 0.9$. How does the degree of persistence in the productivity shock affect E-stability? In order to answer this, we use alternative calibrations, namely, $\rho_a = 0.8$ and $\rho_a = 0.95$. What we show below are results for the policy rule based on forward expectations data.

Note that the degree of persistence in the shock does not affect the conditions for determinacy of equilibria, as what matters for determinacy is the configuration of the parameter matrices B and D. Thus in what follows we concentrate on E-stability, and in order to isolate the effect of the shock, we assume exact MSV specification under learning.

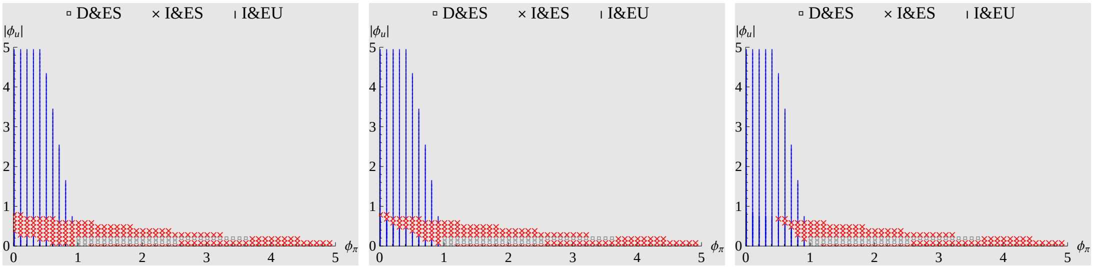

Figure 7: Policy rule based on expectations data. Forecasting model without a constant. $\rho_a = 0.8$ (left), $\rho_a = 0.9$ (center), $\rho_a = 0.95$ (right). The unmarked region has explosive solutions.

As the figure shows, the scope for E-stability decreases with the degree of persistence in the productivity shock.

4.2 Model with log utility

In the baseline specification, we assumed a CES utility function so that we could easily compare our results with those of Bullard and Mitra (2002) derived for a purely forward-looking model. Here our interest is to see how determinacy and learnability of equilibria are affected by the specification of the utility function in households. The alternative specification of utility we consider is of the log form, which is commonly assumed in real business cycle models (actually Blanchard and Gali (2008) assume a log utility). We illustrate our results for the case of expectations based policy rule.

The most notable result is that now the explosive region as well as the indeterminate and E-unstable region shrink. By contrast, the determinate and E-stable region expands, as does the indeterminate and E-stable. Therefore, in an environment with log utility there is more scope for a policy rule to generate determinacy and E-stability.

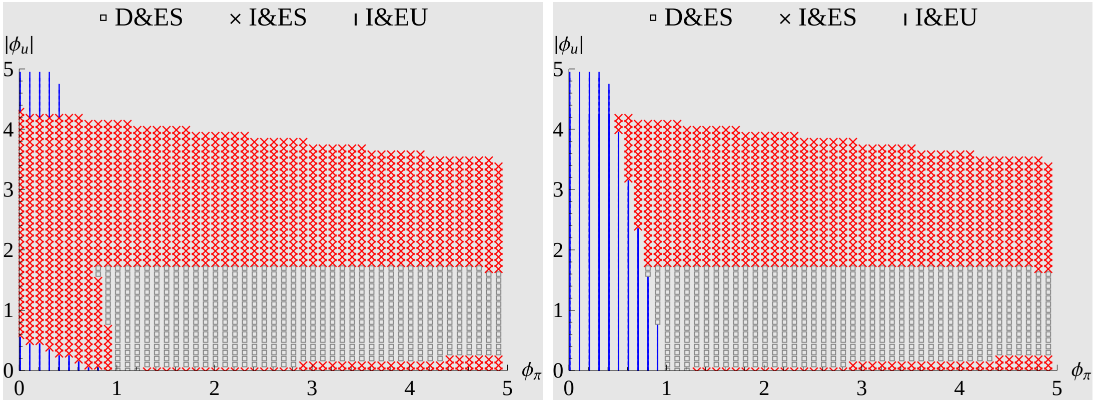

Figure 8: Policy rule with expectations data (log utility). Forecasting model without a constant (left panel), Forecasting model without a constant (right panel). The unmarked region has explosive solutions.

5 Concluding remarks

In this paper we evaluate alternative monetary policy rules with respect to determinacy as well as E-stability (a la Evans and Honkapohja (2001)) of rational expectations equilibria, using recent versions of the New-Keynesian model that generate movements in unemployment due to labor market frictions associated with hiring workers. We derive results for alternative specifications of monetary policy rules as well as alternative values of hiring costs as a percentage of GDP.

We find that for policy rules based on current period inflation and unemployment, the region of indeterminacy and E-instability in the policy space increases with the hiring costs. For policy rules based on expectations of (or lags of) inflation and unemployment, the region of indeterminacy shrinks somewhat with the hiring costs but such policies leads to a large region with explosive solutions. Moreover, expectations based rules can lead to indeterminacy and/or E-instability if they respond very strongly to inflation expectations and very little to unemployment expectations. For lagged data based rules, the determinate and E-unstable region expands while the determinate and E-stable region shrinks as the hiring costs increase. When we allow learning about the steady state of the model, all indeterminate equilibria are also E-unstable. By contrast, when the steady state is assumed to be known, some indeterminate equilibria turn out to be E-stable, and only policy rules which respond very little to inflation and unemployment lead to indeterminacy and E-instability.

The model was calibrated using US data. It would be interesting to see how the model behaves for alternative calibration, for instance using euro area data, because there are important differences between the US and Europe labor market characteristics, namely, (1) the job finding rate, (2) the steady state (average) unemployment rate (3) the job separation rate (determined by (1) and (2)) and (4) the ratio of hiring cost to GDP. These differences lead to different dynamics of inflation and unemployment, with implications for determinacy and learnability of equilibria. Moreover, the analysis in the paper could be extended to allow for other forms of simple policy rules (e.g. with inertia) and policy rules derived from central bank optimization under discretion or commitment.

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