Unemployment dynamics and the cost of business cycles

Jean-Olivier Hairault *

Paris School of Economics (PSE) & Paris 1 University & IZA

joh@univ-paris1.fr

François Langot

GAINS-TEPP (Le Mans University) & ERMES (Paris 2 University) & IZA

flangot@univ-lemans.fr

Sophie Osotimehin †

CREST-INSEE & Paris School of Economics (PSE) & Paris 1 University

sophie.osotimehin@ensae.fr

June 2009

Abstract

We investigate the welfare cost of business cycles implied by matching frictions. First, using the reduced-form of the matching model, we show that job finding rate fluctuations generate intrinsically a non-linear effect on unemployment: positive shocks reduce unemployment less than negative shocks increase it. For the observed process of the job finding rate in the US economy, this intrinsic asymmetry increases average unemployment, which leads to substantial business cycles costs. Moreover, the structural matching model embeds other non-linearities, which alter the average job finding rate and consequently the welfare cost of business cycles. Our theory suggests to subsidize employment in order to dampen the impact of the job finding rate fluctuations on welfare.

Keywords: Business cycle costs, unemployment dynamics, matching

JEL Codes: E32, J64

*Address: MSE, 106 Boulevard de l'Hopital, 75013 Paris, France.

†We thank Guy Laroque, Etienne Lehmann, Franck Portier and Christian Zimmermann and all participants at the following seminars and conferences for helpful comments: Crest (2008), PSE (2009), Boston College (2009), British Columbia (2009), Boston University (2009), UQAM (2009), Malaga (2009) and Mannheim (2009). This paper is a revised version of the IZA Working Paper 3840. Any errors and omissions are ours.

1 Introduction

In a very famous and controversial article, Lucas (1987) argues that the costs of business cycles are negligible: using empirically plausible values for risk aversion, he shows that individuals would only sacrifice a mere 0.008% of their consumption to get rid of all aggregate variability in consumption. This leads him to argue that further improvements in stabilization policies are not the priority. This claim is completely at odds with the neoclassical synthesis and the Keynesian legacies, but also with conventional wisdom. In subjective data studies, aggregate unemployment volatility undermines the household's perceived well-being (Wolfers (2003)). Survey data also suggest substantial benefits from improved stabilization policies (Shiller (1997)).

Our objective in this paper is to show that the matching unemployment theory leads to sizable welfare costs of business cycles. Due to strong non-linearities, average employment and therefore average consumption are lowered by the mere process of alternate expansions and contractions: the losses during recessions outweigh the gains during economic booms. First, the volatility of the job finding rate may affect average unemployment as the unemployment dynamics depend non-linearly on the job finding rate. Secondly, the canonical matching approach also predicts that the job finding rate depends itself in a non-linear way on the shocks hitting the fundamentals of the economic structure like productivity shocks. The welfare cost of unemployment fluctuations stemming from these non-linearities has been ignored despite the high interest in the recent literature with matching frictions for the cyclical volatility of unemployment (Shimer (2005) and Hall (2005*b*)) and for the design of the optimal monetary policy (Blanchard and Gali (2008), Faia (2009) and Sala et al. (2008)). By filling this gap, our paper adds a new dimension to the analysis of the matching model with aggregate uncertainty.

Following Cole and Rogerson (1999), our analysis is firstly based on the reduced form of the matching model where the job finding and separation rates are considered as exogenous stochastic processes. The use of a reduced form model has the advantage of making our results robust to several changes to the standard matching model and of easily unveiling the asymmetric impact of job finding and separation rate fluctuations on unemployment. During booms, the increase in the job finding rate is partly offset by the decrease in unemployment. In recessions, the decrease in the job finding rate is amplified by the increase in unemployment. Because of this asymmetry, average unemployment is increased by fluctuations in the job finding rate. Conversely, fluctuations in the job separation rate lower the average unemployment rate[1]. We show that these asymmetries explain why the volatility and the persistence of the job finding and separation rate processes are potentially key variables in the analysis of business cycle costs. More surprisingly, we find that the costs of fluctuations also depend on the structural level of unemployment. Economies in which the steady state unemployment rate is high are economies

[1]During booms, the decrease in the job separation rate is amplified by the increase in employment. In recessions, the increase in the job separation rate is compensated by the decrease in employment.

also suffering from high costs of fluctuations. For the observed process of the job finding rate in the US economy, we show that the asymmetry quantitatively matters and generates sizable business cycle costs. Conversely, the observed volatility of the separation rate is too small for this non linearity to manifest: the volatility of the job separation rate has virtually no impact on average unemployment.

However, this reduced form approach suffers from several shortcomings. First, beyond the employment loss, one needs a better founded criterion in order to evaluate the business cycle cost. Only a structural matching model can provide a well-defined welfare cost of business cycles. Second, the reduced form analysis supposes that business cycles do not have any effect on the average job finding rate: the counterfactual stabilized job finding rate is assumed to be equal to the average job finding rate. But what if fluctuations also modify the average job finding rate? Finally, in the reduced form approach, stabilization policy cannot be explicitly addressed, as there is no operative way of getting rid of the cycles. A structural matching model is then necessary to take into account how the shocks hitting the fundamentals of the economy affect the job finding rate[2]. However, since Shimer (2005), it is well known that the standard matching model fails to generate realistic fluctuations in the job finding rate. Productivity shocks cause strong movements in wages that offset the firm's incentive to change hirings, thus dampening the variations in the job finding rate.

A matching model with rigid wages in the lines of Hall (2005*b*) allows us to address these issues altogether. It generates enough job finding rate volatility[3]. Moreover, it enables us, by eliminating the wage retroaction, to focus on the basic mechanisms embedded in the matching function which cause the business cycles to influence the average job finding rate. The matching model naturally predicts that the job finding rate is a concave function of the vacancy-unemployment ratio. This could imply that fluctuations decrease the average job finding rate. But as the vacancy-unemployment ratio is a convex function of productivity, the impact of productivity fluctuations on the average job finding rate is a priori ambiguous. These non-linearities come from the decreasing marginal returns in the matching function. The matching elasticity to vacancy plays a crucial role in the size of the business cycle costs: the lower the elasticity, the lower the average job finding rate and the higher the business cycle costs. The internal mechanisms of

[2] The Lucas' approach can be considered as sharper than ours because it is model-free. But it cannot address the stabilization issue.

[3] This hypothesis increases the sensitivity of the model to productivity shocks, but at the expense of the observed flexibility in wages (Pissarides (2007)). Other routes have been followed to elucidate the Shimer puzzle. Hagedorn and Manovskii (2008) show that a higher parametrization for the utility of being unemployed and a lower bargaining power for workers enable the standard matching model to yield realistic fluctuations in the unemployment rate. Whereas Hornstein et al. (2007) introduce investment-specific technological shocks, Kennan (2006) emphasizes the role of procyclical informational rents: the gain that firms obtain by being more informed than workers increases in booms. The inclusion of turnover costs (Pissarides (2007), Mortensen and Nagypal (2003) and Silva and Toledo (2008)) and match-specific technological change (Costain and Reiter (2008)) have also been investigated.

the matching model then lead to quite strong additional business cycle costs when the vacancy elasticity of the matching function is sufficiently low.

All in all, our results challenge Lucas's controversial view on business cycle costs by exploring an original mechanism in a canonical framework. These costs reach 0.55% of permanent consumption in the case considered as the most empirically relevant. We believe that the underlying mechanisms are quite general since the unemployment dynamics and the non-linearity embedded in the matching function are key features of any matching model. In order to show the robustness of our results, we also consider the flexible wage approach proposed by Hagedorn and Manovskii (2008).

Finally, we explicitly address the policy issue. Besides stabilizing the job finding rate, our theory suggests to dampen the impact of the job finding rate fluctuations on welfare, by subsidizing employment in order to increase the average job finding rate. Note that such subsidies in our model, characterized by a suboptimal level of employment, permit to reach both the stabilization and the resource allocation objectives.

The literature following Lucas (1987) has mostly focused on the consequences of business cycles on the volatility of individual consumption. More precisely, because business cycles amplify individual income risks, they could generate higher welfare losses than Lucas's predictions when financial markets are incomplete. However, in as far as individual income fluctuations are transitory, the costs of business cycle are still low, even negligible (Krusell and Smith (1999)), mainly because consumption can be smoothed through capital accumulation. On the other hand, when individual income variations are more persistent, the cost of business cycles becomes substantial (Beaudry and Pages (1999), Krebs (2007), Storesletten et al. (2001), De Santis (2007)). Reis (2009) shows that the persistence also plays a key role when only aggregate shocks are considered. In the literature, very few papers focus on the consequences of business cycles on average consumption. This idea has been sketched out by De Long and Summers (1988); they argue that rather than steadying economic activity at its average level, stabilization would prevent output from deviating from its potential level. Ramey and Ramey (1993) explore this mechanism in a model where firms have to pre-commit to a specific technology before starting production. In this context, stabilization enhances welfare by increasing the efficiency of production. Gali et al. (2007) emphasizes that fluctuations in markups potentially generate efficiency costs on average, but they fail to show that these costs are large when averaged across booms and recessions. Stabilization may also increase welfare through its effect on capital accumulation (Matheron and Maury (2000) and Epaulard and Pommeret (2003)). Barlevy (2004) shows that the business cycle costs become sizable when returns to investment are decreasing. Eliminating fluctuations reallocates investment from periods where the marginal return to investment is low to periods where this return is high, and therefore leads to a higher growth rate of consumption. Finally, a very recent paper by Jung and Kuester (2008) emphasizes the non-linear relation between unemployment and the job finding rate. They however propose a quite complex matching model with capital accumulation, liquidity constraints and human capital, which prevents them to clearly evaluate the basic non-linearities embedded in the matching approach. Moreover, all these additional features bring very small business cycle costs. This makes a clear difference with our paper: we aim at unveiling the mechanisms in the basic matching model, which lead to sizable business cycle costs, namely the stock-flow unemployment dynamics and the congestion effects due to decreasing marginal returns in the matching function.

The paper is organized as follows. In the second section, we use a reduced form of the matching model to investigate the consequences of the non-linearity in the unemployment dynamics. Given the observed processes of the job separation and finding rates, we then derive its implication for the costs of business cycles. The third section takes into account the non-linearity in the job finding rate dynamics embedded into the matching model. The last section concludes.

2 Asymmetry in the unemployment dynamics: a reduced form approach

We believe that labor market frictions naturally generate asymmetries in the unemployment dynamics. Because of these asymmetries, aggregate fluctuations may have an impact on average unemployment. We first present our theoretical framework, and then analyze the unemployment dynamics. Finally, a quantitative evaluation of the business cycle costs is proposed.

2.1 Framework

Following Cole and Rogerson (1999), our theoretical framework is based on the reduced form of the matching model. We consider unemployment dynamics as the result of exogenous job separation and job finding fluctuations. By shutting down any non linearities that may affect the job finding and separation rates, the reduced form model allows us to focus on the non linearity embedded in the unemployment dynamics.

Unemployment. The unemployment dynamics arise from the entries to and exits from employment. The former are determined by the job finding rate p, the latter by the separation rate s.

$$u_{t+1} = s_t(1 - u_t) + (1 - p_t)u_t \tag{1}$$

Shocks. The economy is hit only by aggregate shocks which generate some fluctuations in the job finding rate p_t and in the separation rate s_t. The job finding rate and separation rate are exogenous with respect to u. This key assumption derives from the matching theory. The aggregate shocks affect linearly[4] both the job finding rate and the separation rate which are assumed to follow an AR(1) process:

$$p_t = (1-\rho_p)\bar{p} + \rho_p p_{t-1} + \varepsilon_t^p \tag{2}$$

$$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \varepsilon_t^s \tag{3}$$

The shocks ε^p and ε^s have a zero-mean and a standard deviation equal to σ_{ε^p} and σ_{ε^s} respectively. $\bar{p}$ and $\bar{s}$ denote the average job finding rate and the average separation rate respectively.

Business cycle costs. Aggregate shocks may cause average unemployment to differ from the level it would have reached in an economy without any shocks. Following the convention in the literature, we refer to the latter as the stabilized economy. How the stabilized economy is reached is not explicitly presented, in particular nothing is said in this section on the design and the efficiency of stabilization policies.

The cost of fluctuations is the cost of being in an economy hit by aggregate shocks, rather than being in an economy without aggregate shocks. In the former economy, the job finding rate and the separation rate fluctuate around their means, whereas in the latter they are set forever at their average value $\bar{p}$ and $\bar{s}$. The stabilized unemployment (or the structural unemployment) is equal to:

$$\bar{u} = \frac{\bar{s}}{\bar{s}+\bar{p}}$$

The percentage of aggregate employment lost in the business cycle is then given by:

$$\lambda_u = \frac{1-\bar{u}}{1-\mathbb{E}(u)} - 1 = \frac{\mathbb{E}(u)-\bar{u}}{1-\mathbb{E}(u)}$$

with $\mathbb{E}(u)$ the unconditional expectation of unemployment. Traditionally, since Lucas (1987), the business cycle cost is defined as the percentage of the consumption flow that agents would accept sacrificing in order to get rid of aggregate fluctuations. To what extent employment losses are transformed into welfare costs depends on the links between employment, income and consumption. This issue requires a more structural approach and we will show in Section 3 that the employment loss can be considered as a good approximation of the welfare costs in a matching economy. But it is already straightforward that the employment loss λ_u is of the same order of magnitude as the welfare cost in an economy populated by risk-neutral agents without savings.

2.2 The analysis of the non-linearities in the unemployment dynamics

By considering equation (1), it is fairly intuitive that shocks on the job finding rate and on the separation rate have non linear effects on unemployment. The impact of these shocks depends

[4] We assume symmetrical shocks in order to identify the endogenous asymmetries generated by equation (1). We show in Appendix D that our results are not sensitive to this hypothesis.

on the level of unemployment. During booms, the decline in unemployment offsets the increase in the job finding rate. The small search pool in booms implies that a higher job finding rate increases job creations less than it otherwise would. On the contrary, as unemployment is higher during a recession, the job finding rate shocks have a greater impact in recession; the decline in the job finding rate is magnified by the increase in unemployment. As a result, fluctuations in the job finding rate tend to increase average unemployment.

Conversely, as the impact of the job separation rate shocks depends on the level of employment, the fluctuations in the separation rate lead unemployment to decrease more in booms than to increase in recession: fluctuations in the job separation rate tend to reduce average unemployment.

In this section, we assess precisely these different effects. First, as it is traditionally done in the matching approach with aggregate shocks (see for instance Hall (2005)), a steady state analysis of equation (1) is conducted and so fluctuations in the conditional steady states are considered. This analysis delivers very easily the basics of the non-linearity embedded in the unemployment dynamics. This simple framework highlights the importance of the volatility of the job finding and separation rates for the size of business cycle costs, but also the less expected role played by the structural unemployment rate. Secondly, taking into account the inertia embedded in equation (1), we derive the full non-linear properties of the unemployment dynamics and show that the persistence of the aggregate shocks also matters.

2.2.1 Steady state analysis

Let us assume for now that the speed of convergence of unemployment is infinite: fluctuations cause unemployment to jump directly from one conditional steady state to another. A conditional steady state unemployment corresponds to the level $\widetilde{u}_i$ toward which the unemployment rate would converge if the separation and job finding rates forever keep the same value p_i and s_i, i.e. if the economy remains in the same state i. Let us define π_i the unconditional probability of being in state i. The value taken by p and s in each state i and the probability associated π_i define the Markov chains associated with p and s, consistently with equations (2) and (3). The average job finding rate $\bar{p}$ is therefore equal to $\sum_i \pi_i p_i$ and the average separation rate $\bar{s}$ to $\sum_i \pi_i s_i$. Jointly, they determine the structural unemployment rate $\bar{u}$. On the other hand, as unemployment is assumed to jump directly from one conditional steady state to another, the average unemployment in this economy is then equal to the average of the conditional steady states:

$$\tilde{u} = \sum_i \pi_i \widetilde{u}_i$$

The non-linearity embodied in equation (1) implies that average unemployment $\tilde{u}$ has no reason to coincide with structural (stabilized) unemployment $\bar{u}$.

Job finding rate shocks

To understand the specific role of the non linearity in job findings, let us assume first that the separation rate is constant and equal to its mean $\bar{s}$. This non-linearity implies that $\widetilde{u}_i$ is a convex function of the state-dependent job finding rate p_i:

$$\widetilde{u}_i = \frac{\bar{s}}{\bar{s} + p_i} \tag{4}$$

Because unemployment is a convex function of the job finding rate, the average unemployment is higher than the structural (stabilized) unemployment $\bar{u}$, i.e. the unemployment level in the counterfactual economy where the job finding rate is forever set at its mean $\bar{p}$:

$$\bar{u} = \frac{\bar{s}}{\bar{s} + \bar{p}} < \widetilde{u} = \sum_i \pi_i \widetilde{u}_i$$

It is fairly intuitive that the gap between average unemployment $\widetilde{u}$ and stabilized unemployment $\bar{u}$ depends on the unconditional variance σ_p^2 of the job finding rate. For uniformly small deviations, using a second order Taylor expansion of equation (4), this gap can be written as [5]:

$$\tilde{u} - \bar{u} \approx \frac{\bar{s}}{(\bar{s} + \bar{p})^3} \sigma_p^2 \tag{5}$$

A mean-preserving increase in the volatility widens the gap between the stabilized and fluctuating unemployment rates. The more volatile the economy, the greater the business cycle cost.

Equation (5) indicates that the unemployment gap also depends on the mean of the job finding rate and of the separation rate. More particularly, a lower value of $\bar{p}$ imply a more convex economy and leads to higher business cycle costs. This result is important as it generates strong interactions between structural and cyclical unemployment. Furthermore, this suggests that labor market institutions affect the costs of fluctuations as they have an impact on the average job finding and separation rates.

Job separation rate shocks

So far, the separation rate was assumed to be constant. However, it appears clearly from equation (1) that the asymmetry in the unemployment dynamics could also come from fluctuations in the separation rate. The resulting unemployment gap would then read:

$$\widetilde{u} - \bar{u} \approx -\frac{\bar{p}}{(\bar{s} + \bar{p})^3} \sigma_s^2 \tag{6}$$

with σ_s^2 the unconditional variance of the separation rate. Contrary to the job finding rate case, fluctuations in the separation rate tend to reduce average unemployment. Job separations

[5] See Appendix A for the derivation.

decrease more in expansion than they increase in recessions. As shown by equation (6), the unemployment gap depends again on the average job finding and separation rates. Further, the asymmetry embodied in equation (1) translates into average unemployment only if the separation rate is volatile enough. As the separation rate is one order of magnitude lower than the job finding rate, its variance must be two orders of magnitude lower if the two standard deviations are similar in relative terms. The employment gains are then potentially smaller than the employment losses[6].

2.2.2 Considering unemployment inertia

All previous calculations have been made with the assumption that unemployment jumps directly to the conditional steady states. Under this assumption, average unemployment in the business cycle economy is equal to the average of the steady states. Because of unemployment inertia, the asymmetry embodied in the conditional steady state does not necessarily manifest in average unemployment. This asymmetry affects the average unemployment rate only if the aggregate shocks are persistent enough.

To see in a synthetic formula the role of the mean, the volatility and the persistence of aggregate shocks, let us solve equation (1). We first compute average unemployment in the case where only the job finding rate is fluctuating. Rewrite simply the unemployment dynamics in the following way:

$$u_{t+1} = \bar{s} + \phi_t u_t$$

where $\phi_t \equiv 1 - \bar{s} - p_t$ and $\mathbb{E}[\phi] \equiv \bar{\phi} = 1 - \bar{s} - \bar{p}$. The autoregressive process defined by equation (2) implies that ϕ_t follows an autoregressive stationary process:

$$\phi_t = \rho_p \phi_{t-1} + (1 - \rho_p)\bar{\phi} - \varepsilon_t^p$$

A backward substitution gives:

$$\mathbb{E}[u] \quad = \quad \bar{s}\left(1 + \sum_{k=0}^{+\infty} \mathbb{E}[\prod_{j=0}^{k} \phi_{t-j}]\right)$$

As shown in Appendix B, the additional unemployment created by business cycles can then be approximated by [7]:

$$\mathbb{E}[u] - \bar{u} \approx \frac{\bar{s}}{(\bar{s}+\bar{p})^2} \frac{\rho_p}{1 - \rho_p(1 - \bar{s} - \bar{p})} \sigma_p^2 \tag{7}$$

[6] We will investigate this point further in the quantitative section.

[7] The approximation consists of neglecting moments of order above 2. This is line with our approach : as we want to understand how symmetrical shocks can yield non symmetrical effects on unemployment, we disregard in particular the consequences of non-zero skewness

The difference between average and stabilized unemployment depends on the exogenous volatility of p and on the propagation of these shocks which in turn results from the exogenous persistence ρ_p and the unemployment inertia. If p_t and hence ϕ_t were not serially correlated ($\rho_p = 0$), fluctuations in the job finding rate would not affect average unemployment: $\mathbb{E}[u] = \bar{u}$. When aggregate shocks are persistent, average unemployment in the business cycle economy is no longer equal to stabilized unemployment. Volatility then matters: the greater the variance of job finding rate shocks, the higher the unemployment rate. In line with our intuition, equation (7) also shows some interactions between the volatility, the persistence and the mean of the job finding rate. An increase in the variance of the shocks raises average unemployment all the more so when the average job finding rate is low and the persistence of the shocks is high. The magnitude of business cycle costs will then depend on the observed characteristics of the job finding rate shocks. Business cycle costs also depend on the average separation rate : like the persistence and the mean of the job finding rate, the level of the separation rate can amplify the costs generated by job finding rate fluctuations.

Symmetrically, in the case where only the separation rate is fluctuating, the gain of fluctuations positively depends on the volatility and on the persistence of the separation rate:

$$\mathbb{E}[u] - \bar{u} \approx -\frac{\bar{p}}{(\bar{s}+\bar{p})^2}\frac{\rho_s}{1-\rho_s(1-\bar{s}-\bar{p})}\sigma_s^2 \tag{8}$$

Again, the average job separation and finding rates interact with the volatility and the persistence of the shocks.

2.3 Quantifying the employment loss

To investigate whether observed fluctuations in the job finding and separation rates affect average unemployment and hence the costs of business cycles, it is necessary to estimate the AR(1) processes[8] described by equations (2) and (3). As equations (7) and (8) give only an approximation of average unemployment, we resort to simulations to obtain a more accurate estimate of the costs of business cycles. Consistently with the AR(1) estimations, we simulate job finding and separation shocks in order to obtain artificial series for the job finding and separation rates. We then use them to simulate equation (1), which allows us to compute the average unemployment rate in the business cycle economy and the business cycle costs.

2.3.1 Data

The behavior of job finding and job separation rates over the business cycle is still a debated subject. It especially depends on the underlying conception of "unemployment". Contrary to

[8] It must be emphasized that considering a log-normal distribution for p and s would have led to very similar results. See Appendix D for more details.

Shimer (2005), Hall (2005a) uses a measure of unemployment expanded to include "discouraged workers" and "marginally attached workers". Although those workers are classified as being out of the labor force, their behavior is close to that of workers classified as unemployed. This measure seems particularly relevant in an analysis of the business cycle costs. Although our benchmark result is based on Hall's data, we also provide the results obtained when Shimer's approach is used.

The job finding rate

The Hall (2005a) and Shimer (2005) measures of the job finding rate both exhibit pro-cyclicality. The job finding rate plunges at each recession and recovers at each expansion (Figures 2 and 3, Appendix C). Both measures show a downward trend in the 1970s and in the early 1980s. As some of these movements could be due to factors unrelated to business cycles, we primarily focus hereafter on series detrended by a low frequency filter[9]. However, note that two elements call for not detrending the data. First, the downward trend does not necessarily result from non cyclical factors ; it could also be explained by the increase in the frequency of recessions observed during the 1970s. Secondly, as the mechanism studied in this paper relies on non linearities, the method used to isolate the cyclical component of the job finding rate could affect the cost of fluctuations. This is why non detrended (raw) data are also considered hereafter.

Table 1: Job finding rate statistics

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Mean $\bar{p}$	0.285		0.450	
Standard deviation σ_p	0.084	0.069	0.068	0.053
Autocorrelation	0.942	0.913	0.939	0.915

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005a) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both sets of data are detrended with a HP smoothing parameter of 10^5.

We then estimate the parameters characterizing the process of the job finding rate as described by equation (2). As expected, the "expanded job finding rate" has a lower mean than Shimer (2005)'s measure (Table 1). As shown in the previous section, this may exacerbate the asymmetry in the unemployment dynamics and induce larger business cycles costs. More importantly, including low intensive job seekers also implies a higher variability[10]. Hall (2005a)'s measure should therefore lead to a higher cost of business cycles. On the other hand, these figures are quite sensitive to the use or not of the HP filter.

The job separation rate

[9]As in Shimer (2005), we used a Hodrick-Prescott filter with smoothing parameter 10^5.

[10]Table 1 shows the standard deviation of the job finding rate, and not of its innovation.

The discrepancy between Shimer's and Hall's measures of the separation rate is also noteworthy. Hall (2005*a*) computes a series for the overall separation rate (which includes layoffs, quits, end of short-term contracts) from gross flows on separations. By contrast, Shimer (2005) focuses only on transitions from employment to unemployment. He infers the job separation rate from short term unemployment. Although both series capture the NBER dated recessions quite well, their trends are completely opposed (Figures 4 and 5, Appendix C). Furthermore, Hall's job separation rate is 10 times less volatile than Shimer's for a similar persistence (Table 2).

Table 2: Job separation rate statistics

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Mean $\bar{s}$	0.031		0.034	
Standard deviation σ_s $\times 10^2$	0.058	0.029	0.540	0.260
Autocorrelation	0.970	0.872	0.946	0.756

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005*a*) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both sets of data are detrended with a HP smoothing parameter of 10^5.

2.3.2 Business cycle costs with only job finding shocks

Table 3: Average unemployment and job finding rate fluctuations

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Unemployment	10.50%	10.22%	7.14%	7.08%
Stabilized unemployment	9.83%	9.83%	7.00%	7.00%
Cost of fluctuations λ_u	0.74%	0.44%	0.13%	0.08%

Table 3 presents the employment loss due to business cycles in the US economy for the four measures of the job finding rate. Non-linearities in the unemployment dynamics are enough to generate sizable costs of business cycles. In particular, these costs are between one and two orders of magnitude greater than the costs found by Lucas (1987)[11]. The method chosen to measure the job finding rate has strong consequences on the costs of fluctuations. When some non-employed job seekers (Hall's method) are taken into account, fluctuations in the job finding rate induce at least a 0.44% employment loss. This cost increases to 0.74% if all job finding rate fluctuations are related to business cycles factors. As expected, Shimer's measure leads to lower business cycle costs: if the job finding rate is computed using only transitions from

[11] At this stage, we agree that the two approaches are not directly comparable. We will see in the next section that the employment loss is not so far from a cost expressed in terms of consumption.

unemployment, the cost of business cycles reduces to 0.08%. Such a result was expected as Shimer's job finding rate series display both a lower volatility and a higher mean (Table 1), two characteristics that we identified as cost-reducing.

As Table 4 shows, consistently with equation (7), volatility plays a crucial role in this result. The gap between the results inferred from the Shimer and Hall data comes mainly from differences in volatility. Considering a lower average job finding rate[12] for a relatively low volatility modifies only marginally the magnitude of the business cycle costs (0.10% versus 0.08%). The same statement can be made when a different value for s is considered for a given volatility (0.07% versus 0.08%). On the other hand, if Shimer's job finding rate was characterized by a higher level of volatility (equal to Hall's data)[13], the corresponding cost of fluctuations would amount to 0.35% which is significantly different from 0.08%. However, it is also significantly different from the 0.44% cost induced from Hall's data. The remaining difference is due to the influence of the average value of the job finding rate and of the separation rate. The influence of the structural unemployment then depends on the level of the volatility: the higher the latter, the higher the influence of $\bar{p}$ and $\bar{s}$. This result reveals significant interactions between structural and cyclical unemployment[14]. A higher structural unemployment rate amplifies the cost of business cycles when the volatility in unemployment is high. This result suggests that business cycles could reduce average employment by more in continental European countries[15] which would then suffer from both higher structural unemployment and more costly unemployment fluctuations.

When we consider an arbitrarily higher persistence[16] (ρ equal to 0.98), the cost of business cycles is higher for both Hall's and Shimer's measure, but the increase is greater when volatility and structural unemployment are higher. An increase in the persistence of shocks reinforces the asymmetry in the unemployment process, especially when volatility is high.

These results bring new insights to the analysis of the costs of fluctuations. Business cycle costs mainly depend on the variability of aggregate shocks, but the three characteristics of the job finding rate process deeply interact. The marginal effect of volatility crucially depends on the mean and on the persistence of the job finding rate, but also on the value of the separation rate. Furthermore, this suggests that labor market institutions have an impact on the costs of

[12] A lower average job finding rate implicitly increases the relative volatility of the job finding rate. To understand the specific role of the mean of the job finding rate, we ensure that its relative volatility is held constant. We modify the standard deviation of the shocks to maintain the coefficient of variation constant.

[13] We focus here on the relative volatility of the job finding rate, measured by the coefficient of variation.

[14] That higher unemployment is caused by a higher separation rate or a lower job finding rate influences the cost of fluctuations only marginally.

[15] To the best of our knowledge, job finding rate data for European countries do not go back enough to infer their cyclical properties. Petrongolo and Pissarides (2001) have constructed series for France and Spain, but these series start respectively in 1991 and in 1987. However, all empirical evidence points to a lower average job finding rate in Europe.

[16] The variance of the shock has been to modified to maintain the overall variance of the job finding rate constant.

Table 4: Understanding the "Hall-Shimer wedge", detrended data

From Shimer data	$\bar{p}$,σ_p,$\bar{s}$,ρ_p		$\bar{p}$,$\bar{s}$,ρ_p	$\frac{\sigma_p}{\bar{p}}$,$\bar{s}$,ρ_p	$\bar{p}$,σ_p,ρ_p		$\bar{p}$,σ_p,$\bar{s}$, 0.98
From Hall data		$\bar{p}$,σ_p,$\bar{s}$,ρ_p	$\frac{\sigma_p}{\bar{p}}$	$\bar{p}$	$\bar{s}$	$\bar{p}$,σ_p,$\bar{s}$, 0.98	
Unemployment	7.08 %	10.22%	7.33%	10.71%	6.53%	10.33%	7.08%
Stabilized unemployment	7.00%	9.83%	7.00%	10.62%	6.46%	9.83%	7.00%
Cost of fluctuations λ_u	0.08%	0.44%	0.35%	0.10%	0.07%	0.56%	0.09%

Notes: This table shows the welfare cost of fluctuations for job finding rate processes that share characteristics from both Hall (2005*a*) and Shimer (2005) data. For instance, column 3 presents the same average job finding and separation rates and the same autocorrelation as Shimer (2005) but the same dispersion (measured by the coefficient of variation) as Hall (2005*a*). In column 4, we modify the variance of the shocks to ensure that the relative volatility (again measured by the coefficient of variation) of the job finding rate is held constant. Regarding the persistence experiment (columns 7 and 8), the variance of the shocks has been to modified to maintain the unconditional variance of the job finding rate constant.

business cycles, not only because they affect the amount of volatility faced by individuals (with unemployment benefits for example), but also because they affect structural unemployment.

2.3.3 Business cycle costs with job separation shocks

If shocks on the job finding rate lead to business cycle costs, those on the separation rate could imply business cycle gains. Let us simulate equation (1) with job separation shocks, first without job finding rate shocks, secondly with those shocks and taking into account the covariance between the two. We again consider both Shimer's and Hall's data. Whatever the data considered, the employment gains brought by these fluctuations are negligible. They are even nil considering Hall's data which display little volatility in the separation rate. We show in Table 5 the results relative to Shimer's data. Even in this case, the volatility is not enough to generate strong asymmetries in the unemployment dynamics: the business cycle gains are of the same magnitude as the business cycle costs shown by Lucas (1987). In our investigation of business cycle costs, the absolute level of volatility matters. This is an intrinsic limitation to the impact of separation shocks on business cycle costs. It explains why the volatility of the job separation rate has no real impact on average unemployment[17].

However, if their specific role is limited, job separation shocks can interact with job finding ones. Taking into account its covariance with the job finding rate may enhance the consequences of job finding rate fluctuations. If the separation rate is negatively correlated to the job finding rate, movements in the separation rate cause the unemployment rate to increase further during recessions and decrease further during booms. This amplifies the asymmetry in job creation

[17]Note that this does not necessarily imply that the volatility of the separation rate does not significantly contribute to the volatility of the unemployment rate when a first-order approximation of the unemployment dynamics is considered. The variance of the separation rate is then measured in relative terms. Considering Shimer's data, Fujita and Ramey (2008) estimate that fluctuations in the separation rate account for at least 28% of the unemployment volatility.

Table 5: The welfare cost of fluctuations in the separation rate

	no shock on p		two correlated shocks	
	Raw	Detrended	Raw	Detrended
Unemployment	6.99%	7.00%	7.16%	7.10%
Stabilized unemployment	7.00%	7.00%	7.00%	7.00%
Cost of fluctuations λ_u	-0.009%	0.00%	0.18%	0.10%

Note: The two first columns show the welfare cost of fluctuations in the case where only the separation rate fluctuates. Columns 3 and 4 give the consequences of the co-variance between the job finding rate and the separation rate. These results are computed using Shimer (2005)'s data for which the covariance is $-1.6.10^{-4}$ ($-0.8.10^{-4}$) for raw (detrended) data.

between periods of boom and recession. During booms, the increase in the job finding rate is all the more offset as the separation rate amplifies the decline in unemployment. During recessions, the increase in the separation rate enhances the magnifying effect of the rise in unemployment. A negative correlation between the job finding rate and the job separation rate in the cycle could lead to exacerbate the costs induced by the job finding volatility.

We then simulate equation (1) taking into account both shocks and the observed negative correlation (equal to -0.48 and -0.59 in the raw and detrended cases respectively) between them (only for Shimer (2005)'s data). The last two columns of Table 5, compared to those of Table 3, show how this negative correlation amplifies the welfare consequences of the job finding rate volatility. The welfare cost of fluctuations increases from 0.08% to 0.10% when considering HP filtered data, and from 0.13% to 0.18% when the data are not filtered. However, as this effect modifies only marginally the costs induced directly by the volatility in the job finding rate, we choose to disregard the fluctuations in the separation rate in the rest of the paper for the sake of simplicity.

3 Endogenizing the job finding rate: a structural approach

The previous section showed that the observed volatility and persistence in the job finding rate lead to sizable costs of business cycles. The structural matching model is a candidate for generating such costs. However, Shimer (2005) shows that the canonical matching model fails to generate realistic fluctuations in the job finding rate. The standard deviation of the job finding rate is much greater in the data than in the model (Shimer (2005)). An increase in labor productivity increases the expected profit from a filled job, and thus firms tend to open more vacancies. But there are internal forces in this framework which partially offset the initial increase in expected profits and then dampen the incentives for vacancy creations.

There already exist in the literature different approaches which solve the Shimer puzzle[18]. Do we care about identifying the mechanism at the origin of the high fluctuations in the job finding rate? From the analysis conducted in the first part, it could be tempting to say that it is enough to know that at least one theory is able to replicate the job finding rate dynamics. Actually, we do care. Indeed, the results obtained in the first part are derived from a model in which the job finding rate is exogenous, and in which fluctuations are neutral regarding the average job finding rate. To assess the costs of fluctuations, one must take into account the consequences of stabilization on the average job finding rate. If productivity shocks and the job finding rate are linearly related, the results found in the previous section should *a priori* be close to the endogenous job finding rate case. But if not, the average job finding rate can then be affected by business cycles. Why do we suspect the presence of a non-linear effect of business cycles on the job finding rate? The job finding rate is a non-linear function of the labor market tightness which also depends non-linearly on productivity changes. To show and quantify these different effects, a structural model is then required and the costs of business cycles could differ according to the model specification. This last statement is all the more true as the cost of business cycles will rely on a welfare criterion consistent with the structural model. In particular, more attention must be paid to the cyclical behavior of vacancies.

The choice of the theoretical model is then potentially crucial. The studies aiming at elucidating the Shimer puzzle emphasize different mechanisms and none seems to close the debate. We choose to study the business cycle cost implied by the wage rigidity approach as suggested initially by Hall (2005*b*) in a first response to the puzzle. This framework fits perfectly well with our objectives: it allows us to replicate all the volatility in the job finding rate[19], but also to reveal, in a very transparent way, the basic non-linearity embodied in the matching model. As the wage retroaction is neutralized in the job creation condition[20], it allows us to focus only on the implications of the basic non-linearity introduced by the matching function, independently of other assumptions (hiring and/or separation costs, insider/outsider wages). We then present different calibrations of the matching function elasticity in order to unveil these implications. Each replicates the job finding rate process (standard deviation and mean)[21]. The implied business cycle costs are not necessarily identical and equal to that obtained in the reduced-form part. In a robustness analysis, considering a flexible wage framework (Hagedorn and Manovskii (2008)), we show that these mechanisms are intrinsic to the matching model, and then not specific to the rigid wage approach.

[18]See for instance Hall (2005*b*), Hall and Milgrom (2008), Pissarides (2007), Hagedorn and Manovskii (2008), Hornstein et al. (2007), Kennan (2006), Mortensen and Nagypal (2003), Silva and Toledo (2008) and Costain and Reiter (2008).

[19]Note that we choose to replicate all the volatility in the job finding rate with only productivity shocks, even if it is now well-established that the volatility generated by these shocks only is lower (Pissarides (2007) and Mortensen and Nagypal (2003)).

[20]Note that considering Hall and Milgrom (2008) would have led to the same simplification.

[21]The persistence would be naturally matched by that of productivity shocks.

3.1 A canonical matching model

The model considered hereafter is a version of the matching model *à la* Pissarides with aggregate uncertainty and exogenous separation.

3.1.1 Matching technology

Output per unit of labor is denoted by y_t and is assumed to follow a first-order Markov process according to some distribution $G(y, y') = Pr(y_{t+1} \leq y'|y_t = y)$. To hire workers, firms must open vacancies at unit cost κ. Jobs and workers meet pairwise at a Poisson rate $M(u, v)$, where $M(u, v)$ stands for the flows of matches and v the number of vacancies. This function is assumed to be strictly increasing and concave, exhibiting constant returns to scale, and satisfying $M(0, v) = M(u, 0) = 0$. Under these assumptions, unemployed workers find a job with a probability $p(\theta) = M(u, v)/u$ that depends on the ratio of vacancies to unemployment ($\theta = v/u$). The probability of filling a vacancy is given by $q(\theta) = M(u, v)/v$. Hereafter, we impose that the matching function is Cobb-Douglas: $M(u, v) = \varphi u^{1-\alpha} v^{\alpha}$ with $0 < \alpha < 1$.

The unemployment dynamics in the economy (equation (9)) are similar to equation (1), except that the job finding rate is now endogenous. Equations (10) and (11) define the job finding rate and the job filling rate respectively which depends[22] on the current productivity state y:

$$u' = s(1-u) + (1 - p(\theta_y))u \tag{9}$$

$$p(\theta_y) = \varphi \theta_y^{\alpha} \tag{10}$$

$$q(\theta_y) = \varphi \theta_y^{\alpha-1} \tag{11}$$

3.1.2 Workers

Workers are risk neutral. They have no access to financial markets. This simplifying assumption is not restrictive as our results do not rely on the impossibility of individuals to smooth their income. As agents are risk-neutral, the excessive volatility of consumption implied by this assumption is not captured in the welfare calculations. The focus is here on the impact of business cycles on average consumption, which is not affected by smoothing behaviors.

Workers can either be employed or unemployed. Employed workers receive wage w until their job is destroyed (at rate s); we do not take into account on-the-job search and voluntary quits. An unemployed worker gets an unemployment benefit[23] z which is equally financed by workers through lump-sum taxes. We choose to consider that the disutilities of working and of not

[22] Throughout the paper the notation x_y indicates that a variable x is a function of the aggregate productivity level y and $\mathbb{E}_y$ is the expected value conditional on the current state y.

[23] At this stage, the non market value does not include home production. See the robustness analysis for a discussion of this assumption.

working are both equal to the same value χ. It is then straightforward to derive the representative agent intertemporal preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (c_t - \chi)$$

where $\mathbb{E}_0$ denotes the expectation operator conditional on information at time 0 and β the discount factor.

3.1.3 Aggregate consumption and the welfare cost of business cycles

In our economy, aggregate consumption is equal to the aggregate production net of the vacancy costs.

$$c_t = y_t(1 - u_t) - \kappa\ v_t \tag{12}$$

In this economy, the only source of fluctuations is the labor productivity shocks. The welfare cost of fluctuations is therefore defined relatively to a counterfactual economy, in which labor productivity remains at its average value. The welfare cost of business cycles λ is defined as the percentage of the consumption flow that the agent would accept sacrificing in order to get rid of aggregate fluctuations:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\lambda)c_t\right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \bar{c}_t$$

where $\bar{c}_t$ is the level of consumption in the economy without aggregate shocks. It needs to be derived from a counterfactual experiment based on an artificial economy without any shocks. The computation of welfare costs takes explicit account of the transition path to the stabilized economy. However, in order to highlight the welfare cost of business cycles in the matching economy, let us consider the following expression, which leaves aside the transition path[24]:

$$\lambda \approx \frac{\bar{c} - \mathbb{E}(c)}{\mathbb{E}(c)} = \frac{[(1-\bar{u})\bar{y} - \kappa\bar{v}] - \mathbb{E}[(1-u)y - \kappa v]}{\mathbb{E}(c)} \tag{13}$$

Business cycles are costly when they make the production net of the vacancy cost lower than its stabilized level. In order to make a link with the reduced form analysis, the welfare cost of business cycles can be written as follows:

$$\lambda \approx \frac{(\mathbb{E}(u) - \bar{u}) - cov(y, 1-u) + \kappa(\mathbb{E}(v) - \bar{v})}{\mathbb{E}(c)} \tag{14}$$

The first part of the welfare cost of business cycles is the employment loss $(E(u) - \bar{u})$ present in the reduced-form analysis. Let us emphasize that the size of the employment loss is not necessarily of the same magnitude in the structural model, as the job finding rate is now endogenous. The lower the average job finding rate, the higher the employment loss and the higher the welfare cost.

[24]The transition path does not significantly matter in our economy: it decreases only very slightly the business cycle cost.

The second part $cov(y, 1-u)$ comes from the interaction between productivity and employment. Just as making the job finding rate negatively correlated with the stock of job searchers lowers mean employment, making productivity positively correlated with the stock of workers raises mean output. This explains why a positive covariance leads to decrease the business cycle costs. Finally, the third part shows that more vacancies in the fluctuating economy than in the stabilized one generate higher costs.

3.1.4 The value functions

The worker's utility

Define U_y and W_y to be the state contingent present value of an unemployed worker and an employed worker[25]:

$$U_y = z + \beta\{(1 - p(\theta_y))\mathbb{E}_y[U_{y'}] + p(\theta_y)\mathbb{E}_y[W_{y'}]\} \tag{15}$$

$$W_y = w_y + \beta\{(1 - s)\mathbb{E}_y[W_{y'}] + s\mathbb{E}_y[U_{y'}]\} \tag{16}$$

The firm's surplus

The firm's value of an unfilled vacancy V_y is given by:

$$V_y = -\kappa + \beta\{q(\theta_y)\mathbb{E}_y[J_{y'}] + (1 - q(\theta_y))\mathbb{E}_y[V_{y'}]\} \tag{17}$$

with J_y the state contingent present value of a filled job and $q(\theta_y)$ the probability of filling a vacancy conditionally on the productivity state y. When the job is filled, the firms operate with a constant return to scale technology with labor as only input. The firm's value of a job is given by:

$$J_y = y - w_y + \beta\{(1 - s)\mathbb{E}_y[J_{y'}] + s\mathbb{E}_y[V_{y'}]\} \tag{18}$$

Free entry implies $V_y = 0$ for all y. Therefore, the job creation condition is:

$$\kappa = \beta q(\theta_y)\mathbb{E}_y[J_{y'}] \tag{19}$$

3.1.5 Equilibrium

The labor market equilibrium depends on the way the wage is determined in the economy. Though our benchmark is the rigid wage model, we first present the traditional equilibrium with a Nash-bargained flexible wage. It allows us to compare the non-linearities embedded in these two equilibria.

[25] For the sake of simplicity, we omit from these equations the disutility of working and not working, the lump-sum tax financing the unemployment benefits and the dividend paid by firms to workers, as these variables are assumed to be identical across individuals.

Equilibrium with flexible wages.

When a worker and an employer meet, the expected surplus from trade is shared according to the Nash bargaining solution. The joint surplus S_y is defined by $S_y = W_y + J_y - U_y$. The worker gets a fraction γ of the surplus, with γ her bargaining power. The equilibrium with flexible wages is defined by the job creation condition and the wage rule (equations (20) and (21)), plus equations (9) to (11):

$$\frac{\kappa}{q(\theta_y)} = \beta \mathbb{E}_y \left[y' - w(\theta_{y'}) + (1-s)\frac{\kappa}{q(\theta_{y'})} \right] \tag{20}$$

$$w(\theta_y) = \gamma(y + \kappa\theta_y) + (1-\gamma)z \tag{21}$$

As Shimer (2005) points out, the adjustment of wages is responsible for the insensitivity of the labor market tightness to the productivity shocks. It also makes the interplay of the non-linearities in the model more complex relative to the rigid wage equilibrium, due to the retroaction of wages in the job creation condition (equation (20)).

Equilibrium with rigid wages

Incorporating wage rigidity in the matching model is a natural way to generate enough volatility. Moreover, this allows us to focus on the basic non-linearities introduced by the matching function, present in any matching model.

Following Hall (2005*b*), we consider a constant wage $w_y = w$, $\forall y$. This constant wage is an equilibrium solution if $z \leq w \leq \min \pi_y$, where π_y denotes the annuity value of the expected profit[26]. The wage is set at the Nash bargaining solution relative to the average state of productivity $\bar{y}$. This wage is an equilibrium wage provided it lies in the bargaining set defined by the participation constraints of the firms and the workers.

The rigid wage equilibrium is then defined by substituting equations (20) and (21) by equations (22) and (23), again in addition to the conditions (9) to (11):

$$\frac{\kappa}{q(\theta_y)} = \beta \mathbb{E}_y \left[y' - \bar{w} + (1-s)\frac{\kappa}{q(\theta_{y'})} \right] \tag{22}$$

$$\bar{w} = \gamma(\bar{y} + \kappa\theta_{\bar{y}}) + (1-\gamma)z \tag{23}$$

Figure 1 summarizes all the non-linearities embedded in the structural matching model, ie. the impact of the variance $V(.)$ on the mean $E(.)$. The non-linearity arising from the unemployment dynamics (u-convexity effect), equation (9), has been intensively investigated in the previous section. Let us concentrate here on the additional non-linearity that appears once p is endogenous.

[26]This annuity value is simply computed using the value an employer attaches to a new hired worker who never receives any wage:

$$\widetilde{J}_y = y + \beta(1-s)\mathbb{E}_y[\widetilde{J}_{y'}]$$

The annuity value is then given by $\pi_y = [1 - \beta(1-s)]\widetilde{J}_y$.

Secondly, equation (22), combined with the job filling rate condition (equation (11)), shows that the average labor market tightness could also be affected by productivity fluctuations. If expansions and recessions have the same marginal impact on the firm's profits, the free entry condition is satisfied for greater variations in job creation in booms than in recessions. This effect tends to increase the average job finding rate in the fluctuating economy (θ-convexity effect). From equation (11), it can be seen that this convexity is amplified by a high elasticity α of the matching function to vacancies. *Ceteris paribus*, it leads to lower business cycle costs only in the case where the deterministic equilibrium level of employment is below its optimal value. This is the case in our economy, whatever the parameters we consider, as we impose relatively

[27] In this latter case, a lower labor market tightness could be welfare-improving, if the deterministic equilibrium level of employment was above its optimal value (too many vacancy costs)

high unemployment benefits.[28]

These basic non-linearities are not specific to the rigid wage model. It is obvious that the flexible wage equilibrium shares the same fundamental non-linearities, as they stem from the intrinsic characteristics of the unemployment dynamics, of the job finding rate and of the job filling rate which are exactly the same in the two equilibria. However, in the flexible wage case, the θ-convexity effect is modified by the retroaction of the wage in the job creation condition (equations (20) and (21)). Hence, the non-linearity between y and θ_y also depends on the assumption about the wage bargaining process, and more generally on the particular assumptions considered, such as the existence or not of fixed hiring and separation costs. In this sense, the flexible wage equilibrium may introduce non-linearities which are not intrinsic to the matching process. The use of the rigid wage framework allows us to focus on the basic non-linearities which are common to a large class of matching models.

3.1.6 Welfare cost and employment loss: the rigid wage case

In addition to the u-convexity effect, the p-concavity and the θ-convexity interplay to determine the cost of the business cycles in our rigid wage economy. Actually, when wages are rigid, only the effect of these non-linearities on the job finding rate and the unemployment rate really matters: the employment loss can be considered as a good approximation of the welfare cost. Indeed, as the production net of vacancy costs is approximately equal to labor earnings (for a discount factor β sufficiently close to 1)[29], it is straightforward to show that:

$$\lambda \approx \frac{\bar{w}(1-\bar{u}) - \mathbb{E}(\bar{w}(1-u))}{\mathbb{E}(\bar{w}(1-u))} = \frac{\mathbb{E}(u) - \bar{u}}{1-\mathbb{E}(u)} \quad (\beta \to 1) \tag{24}$$

Comparing with equation (14), the cost-decreasing effect of the covariance between productivity and employment and the cost-increasing effect of higher vacancies compensate each other for a discount factor β sufficiently close to 1. More generally, the wage rigidity in the business cycles makes the welfare cost very close to the employment loss.

The total impact of productivity fluctuations on the unemployment rate is then key to understand the business cycle costs. It is the result of the u-convexity as in the reduced-form section, but also of the p-concavity and of the θ-convexity, as the average job finding rate is now affected by the productivity volatility. To make this point explicit, let us approximate the average unemployment using the comparative statics of the model without aggregate shocks[30]. It can be

[28] Note that this is a conservative choice with regard the size of the business cycle costs.

[29] In the general case, it is necessary to also take into account the dividends paid by the firms to the workers.

[30] The comparative static of the model without aggregate shocks can be used to approximate the dynamic stochastic model if the shocks are persistent enough. See Mortensen and Nagypal (2003) for more details.

shown that:

$$\begin{aligned}\mathbb{E}(u)-\bar{u} &\approx \frac{s}{[s+\bar{p}]^3}\sigma_p^2-\frac{s}{[s+\bar{p}]^2}\left(\mathbb{E}(p)-\bar{p}\right)\\ &\approx \underbrace{\frac{s}{[s+\bar{p}]^3}\sigma_p^2}_{u\text{-convexity}}-\frac{s}{[s+\bar{p}]^2}\frac{1}{2}\left(\underbrace{p''(\bar{\theta})(\theta'(\bar{y}))^2}_{p\text{-concavity}}+\underbrace{p'(\bar{\theta})\theta''(\bar{y})}_{\theta\text{-convexity}}\right)\sigma_y^2 \qquad (25)\end{aligned}$$

The first part of the welfare cost of business cycle captures the impact of the job finding rate volatility on average unemployment (u-convexity). The second part comes from the impact of the productivity volatility on the average job finding rate. It combines both the p-concavity and the θ-convexity and thus depends on the elasticity α of the matching function to vacancies. This can be shown more formally as follows[31]:

$$p''(\theta)(\theta'(y))^2+p'(\theta)\theta''(y)=\Gamma(\theta)(2\alpha-1) \qquad \Gamma(\theta)>0$$

The stabilization of labor productivity either decreases or increases the average job finding rate, depending on the value of the elasticity of the matching function α. The job finding rate is a concave (convex) function of labor productivity if α is below (above) 1/2 and the stabilized job finding rate is then higher (lower) relatively to that of the volatile economy. In the case $\alpha = 1/2$, the θ-convexity and the p-concavity effects exactly compensate each other. In this case, the productivity fluctuations will lead to the same increase in average unemployment as in the reduced form analysis.

3.2 Quantifying the business cycles costs

We first calibrate the rigid wage economy. As equations (24) and (25) give only an approximation of the welfare cost of business cycles, we resort to simulations to obtain an accurate estimate.

3.2.1 Calibration

The model is calibrated to match US data. We calibrate the productivity process to match the US labor productivity standard deviation and persistence[32]. The monthly discount rate is set to 0.42%. The job separation rate is set at Hall's estimate for the US economy, 0.031. We choose the elasticity of the matching function α to be 0.5, in Petrongolo and Pissarides (2001) range, in order to start with a structural model as close as possible to the reduced form analysis. Following Mortensen and Pissarides (1999), γ is set at 0.5. The scale of the matching function

[31] $\Gamma(\theta)=\frac{\varphi\alpha}{(1-\alpha)^2}\left(\frac{\beta}{\kappa(1-\beta(1-s))}\right)^2\theta^{3\alpha-2}$

[32] We use the same data as Shimer (2005), the real output per worker in the non farm business sector, detrended with a HP smoothing parameter of 10^5.

φ is chosen to pin down the US average vacancy-unemployment ratio. Unemployment benefits and vacancy costs are then calibrated to reproduce the volatility and the mean of the job finding rate over the cycle. These two targets are computed using Hall (2005a)'s measure of the job finding rate (Table 1).

Table 6: Benchmark calibration of the matching model

Parameter	Calibration	Target
Labour productivity		
Average	1	Normalization
Persistence ρ_y	0.90	US data (1951-2003)
Standard deviation σ_y	0.9%	US data (1951-2003)
Discount rate r	0.0042	Corresponds to 5% annually
Job destruction rate	0.031	Hall (2005)
Elasticity of the matching function α	0.5	Petrongolo-Pissarides (2001)
Workers' bargaining power γ	0.5	Mortensen and Pissarides (1999)
Scale of the matching function φ	0.346	Matches US average v-u ratio of 0.72 (Pissarides, 2007)
Cost of vacancy κ	0.239	Matches US average job finding rate of 0.285 and job
Unemployment benefits z	0.796	finding rate volatility of 0.068

3.3 Simulation

Table 7: The welfare cost of fluctuations in a matching model with rigid wages

	Business cycles economy				Stabilized economy			Cost of
	$E(u_t)$	$E(p_t)$	σ_p	$E(\theta)$	$\bar{u}$	$\bar{p}$	$\bar{\theta}$	fluctuations
$\alpha = 0.5$	10.22%	0.285	0.069	0.72	9.82%	0.285	0.681	0.44%
$\alpha = 0.4$	10.22%	0.285	0.069	0.72	9.75%	0.288	0.678	0.55%
$\alpha = 0.6$	10.22%	0.285	0.069	0.72	9.89%	0.283	0.685	0.32%

Table 7, Line 1, presents the results for the benchmark calibration of the rigid wage model. These results show that the average unemployment rate is higher in the fluctuating economy. This is also the case for the average labor market tightness, and so for the average vacancies as well. As expected, there is no influence of productivity fluctuations on the average job finding rate as $\alpha = 0.5$, and the welfare cost of business cycles is of the same magnitude as in the reduced form analysis[33]. The size of the business cycle cost is only determined by the u-convexity effect.

We then simulate two other cases: $\alpha = 0.4$ and $\alpha = 0.6$ (last two lines of Table 7). The values

[33]Note that the cost-decreasing effect of the covariance between productivity and employment, offset by the cost-increasing effect of higher vacancies, generates a small change in the business cycle costs relative to the employment loss (less than 0.06 percentage point compared to 0.44).

of the parameters κ and z have been changed accordingly in order to still match the job finding rate characteristics[34]. Depending on α, the US welfare cost of fluctuations could reach 0.55% or reduce to 0.32%. Petrongolo and Pissarides (2001) estimate this elasticity to be between 0.3 and 0.5. This suggests that with $\alpha = 0.5$, our benchmark calibration gives a lower bound of the welfare costs of fluctuations. In the more realistic case ($\alpha = 0.4$), the average job finding rate in the fluctuating economy is lower than the value which would be reached in the stabilized economy. A lower elasticity strengthens the p-concavity effect and dampens the θ-convexity effect. In this case, the internal mechanism of the matching model leads to quite high business cycle costs, since these costs are increased by one fourth. Note that it occurs only when the labor market tightness θ (and so the job finding rate $p(\theta)$) is volatile enough to make the non-linearity operating. Replicating the volatility of both the labor market tightness and the job finding rate leads to sizable business cycle costs through different channels which are all at work in this structural model[35].

We believe that these results are quite general and do not depend on the rigid wage framework we consider here[36]. Firstly, the business cycle costs are mainly generated by the u-convexity, which is a key feature of any matching model. Secondly, the non-linearity of the job finding rate due to the matching function is not specific to the rigid wage economy. The flexible wage case actually adds other sources of non-linearity which depend on the specification of the matching model considered. The next section explicitly addresses this robustness issue.

3.4 Robustness

Hagedorn and Manovskii (2008) have recently proposed an alternative response to the Shimer puzzle by suggesting that the problem is more in the way the model is calibrated than in the model itself. The matching framework with flexible wages can generate a realistic volatility in the job finding rate when the value of unemployment and of the workers' bargaining power are judiciously calibrated. Considering this alternative view enables us to qualify our results.

In this flexible wage framework, the welfare cost of business cycles may differ from the results derived under rigid wages for several reasons. Firstly, the non linearity in the job finding rate is modified due to the retroaction of wages in the job creation condition. Appendix E indeed

[34]Note that, in the $\alpha = 0.4$ case, the rigid wage defined at the median productivity is no longer in the bargaining set. The wage is then fixed at its highest value ensuring that the firm's value is still positive ($w = \text{argmin}_y \pi_y$).

[35]Note that the higher volatility implied by the unfiltered job finding rate process would lead to an even more substantial business cycle cost in the ($\alpha = 0.4$) case.

[36]Our result suggests another interpretation to the cost of the wage rigidity given by Shimer (2004). This cost is of the same order of magnitude as our measure. He interprets this cost as the result of the non-optimality of the labor market tightness volatility. Our result rather suggests that it comes from the employment loss generated by the volatility of the job finding rate. Note that Shimer (2004) indeed emphasizes that the average output in the rigid wage economy is 0.12 percent below the level in the flexible wage one.

shows that the condition which ensures that the job finding rate is lowered by business cycles is less stringent in this flexible wage environment. For the same value of α, the flexible wage framework potentially leads to higher business cycle costs. Secondly, the employment loss is no longer necessarily a good approximation of the welfare cost of business cycles. The covariance between productivity and employment and the behavior of vacancies over the cycle matter for the size of the market production loss (equation (14)). Further, the calibration of Hagedorn and Manovskii (2008) adds another difference: they show that the volatility of the job finding rate is high enough only if the value of the non-market activity z is calibrated sufficiently close to the average productivity. This implies calibrating z at a much higher value than its strict interpretation as an unemployment benefit would imply. Following Hagedorn and Manovskii (2008), we consider that z now includes home production l. It leads to a redefinition of aggregate consumption as follows: $c = y(1-u) + ul - \kappa v$. It implies that the business cycle welfare cost is different from the market production loss defined in equation (14) and is then sensitive to the value of l:

$$\lambda_l \approx \frac{(\mathbb{E}(u) - \bar{u})(1-l) - cov(y, 1-u) + \kappa(\mathbb{E}(v) - \bar{v})}{\mathbb{E}(c)} \tag{26}$$

Let us emphasize that, for a given level of z, the value of l matters only for the magnitude of the welfare costs and has no effect on average employment and vacancy rates.

In order to check the robustness of our results, we now calibrate the flexible wage equilibrium, defined by the equations (9), (11) (20) and (21), along the lines of Hagedorn and Manovskii (2008). We consider the same values as in our benchmark economy for all parameters, except for the workers' bargaining power γ, the non-market value z and the vacancy cost κ which are calibrated in order to match the same targets as in the rigid wage case for internal consistency[37].

Table 8: Employment and market production loss with flexible wages

Business cycles economy				Stabilized economy			Employment	Market production
$E(u_t)$	$E(p_t)$	σ_p	$E(\theta)$	$\bar{u}$	$\bar{p}$	$\bar{\theta}$	loss	loss
10.22%	0.285	0.069	0.72	9.74%	0.288	0.697	0.52%	0.49%

Table 8 shows that the effects of the non-linearities present in the matching model appear to be quite robust to variations in the degree of wage rigidity. The employment loss is even larger than in the rigid wage approach (0.52% against 0.44% in Table 7), as the stabilized job finding rate is now higher than its cyclical counterpart for α equal to 0.5. For the calibration considered in this flexible wage framework, the threshold for α under which the job finding rate is higher in the stabilized economy is 0.63 (vs. 0.5 in the rigid wage economy)[38]. However, as wages

[37]Note that the targets are different in Hagedorn and Manovskii (2008). The values of κ, γ and z are respectively set at 0.21, 0.033 and 0.97.

[38]See Appendix E.

are now flexible, this employment loss does not necessarily coincide with the market production loss. Actually, the last column in Table 8 shows that these two losses are very close. The interaction between productivity and employment is nearly compensated by the the vacancy gain. This result is not surprising: as the calibration is such that the bargained wage is close to the constant outside opportunity of the employed workers, this flexible wage model "is a close cousin of others that rationalize wage rigidity by dropping Nash wage bargaining" (Hall (2006), p.16).

However, the extent to which the employment (or the market production) loss can be interpreted as a welfare cost crucially depends on the home production value l. The standard calibration of the unemployment benefits for the US economy is between 0.3 and 0.6 (Kitao et al. (2008), Nickell et al. (2005) and Shimer (2005)). For the calibrated value of 0.97 for z, this implies that l is between 0.37 and 0.67, leading to welfare costs equal to 0.30% and 0.14% respectively. Obviously, a high value of l automatically lowers the costs of business cycles. This question of the value of non-market activities is key for the welfare costs of business cycles. Without any doubt, our choice in the benchmark economy ($l = 0$) increases the magnitude of these costs.

However, it must be emphasized that we have neglected other dimensions that could have increased our quantitative measure of the business cycle costs. Once the non-market production is considered to be as efficient as the market production, the disutility of "not working" may dominate the disutility of working when job search costs, as well as indirect costs such as psychological damage and skill obsolescence, are taken into account. There are no clear empirical answers on this crucial point. On the other hand, unemployment benefits do not lead to distortive taxation in our theoretical framework. Business cycles, by increasing average unemployment, could imply higher taxes, which would, in turn, weigh employment down. This could have been counted as a cost of business cycles. Another dimension which could magnify these costs is the loss of human capital generated by unemployment spells and reflected in the permanent decrease in wages observed in data (Krebs (2007), Jung and Kuester (2008)). Is this compensated for by more intense human capital accumulation during expansions? All these points would deserve to be addressed to obtain a more general assessment of the welfare cost of unemployment fluctuations.

4 Structural policy as a stabilizer

The matching model displays sizable costs of business cycles, provided that home production is less efficient than market work. We agree that these costs cannot be interpreted as the gain of a stabilization policy since the productivity shocks have been exogenously shut off. However, it can be argued that the business cycle costs give an upper bound of the benefits of stabilization policies.

The next step could have been to address more explicitly the question of stabilization, especially that of the job finding rate p through fiscal or monetary policy, as the fluctuations in p are welfare-degrading. But studying the design of these macroeconomic policies is clearly beyond the scope of this paper. We rather exploit the fact that the structural and the cyclical dimensions are strongly interrelated in our framework. Section 2 has revealed that the costs of business cycles are sensitive to the mean of the job finding rate. The higher the latter, the lower the cost of fluctuations. Beyond the stabilization of the job finding rate volatility, our theory suggests dampening the impact of its fluctuations by increasing the mean of the job finding rate. However, any public intervention in this direction must take care not to introduce any additional distortions in the economy. Due to the existence of unemployment benefits, the rigid wage economy is characterized by a suboptimal level of employment (as the Hosios condition is satisfied). We therefore consider a constant subsidy to firms, equal to the unemployment benefits, and financed by means of lump-sum taxes equally paid by employed and unemployed workers[39]. It is straightforward to show that providing such a subsidy allows the economy to reach the first best allocation in an environment without shocks. This policy implies no trade-off between the stabilization and the resource allocation objectives. Let us note that this subsidy policy has the advantage of not being state-dependent and therefore does not suffer from a potential lack of information[40].

Leaving aside the welfare gain due to the reduction of the structural distortions, we focus on the additional gain implied by the reduction of the cost of fluctuations. We then consider the subsidized rigid wage economy[41] with and without productivity shocks for our benchmark calibration. Let us note that the mean of the job finding rate at the first best is equal to 0.677 (Table 9), much higher than its value in the economy without subsidies (Table 7). The business cycle costs implied by the productivity shocks are then considerably dampened by the structural policy. Whereas the volatility of the job finding rate is not modified, the mean of the unemployment rate is no longer significantly increased by these fluctuations (Table 9). The structural policy reduces the costs of business cycles by one order of magnitude: these costs drop to 0.03%. This result illustrates the fact that reducing Harberger triangles may lead to

[39] In our framework, this policy is strictly equivalent to removing unemployment benefits though they do not have the same distributional implications.

[40] Alternatively, it is possible to think of counter-cyclical subsidies to firms. The government would provide subsidies τ_y to firms in order to fully compensate for productivity shocks. This would make the firm's value of a job independent of the business cycle. However, the implementation of this policy would require more information than is usually available on aggregate productivity shocks.

[41] In the flexible wage framework, there is the same interaction between the structural and the cyclical dimensions. But the subsidy, which must take into account the existence of both the domestic production and the low bargaining power of the workers, would also have an effect on the volatility of the job finding rate. On the other hand, there would be still no conflict between the structural and cyclical objectives. Even in the case where a high level of home production would imply the economy to be without ambiguity in over-employment, decreasing the job finding rate would not inflict significant business cycle costs, given the high level of home production.

dampening the welfare cost of Okun gaps. This is the natural policy implication of the existence of strong non-linearities in the matching model. Sizable business cycle costs do not necessarily imply the need for stabilization policies.

Table 9: The welfare cost of fluctuations with structural subsidies

Business cycles economy				Stabilized economy			Cost of
$E(u_t)$	$E(p_t)$	σ_p	$E(\theta)$	$\bar{u}$	$\bar{p}$	$\bar{\theta}$	fluctuations
4.43%	0.677	0.069	3.880	4.39%	0.677	3.830	0.03%

5 Conclusion

This paper shows that non-linearities in the unemployment dynamics caused by frictions on the labor market can generate sizable costs of fluctuations. Using a reduced-form model of the labor market, these costs are estimated to be two orders of magnitude greater than those computed by Lucas (1987). We also show in the rigid wage version *à la* Hall (2005) that the internal mechanisms of the matching model matter for the magnitude of business cycle costs as they impact the average job finding rate through different non-linearities. Our results emphasize that the welfare cost of fluctuations does not only depend on the variability of aggregate shocks. The persistence of these shocks, but also the level of structural unemployment have important implications. Furthermore, a high structural unemployment rate magnifies the welfare consequences of the volatility and the persistence of macroeconomic shocks. We then show that an employment subsidy, by increasing the mean job finding rate, acts as a stabilization policy. This also suggests that business cycles may reduce average consumption by more in continental European countries which would then suffer from both higher structural unemployment and more costly unemployment fluctuations. Business cycle costs would not be alike across countries. These results have been shown to be robust to a flexible wage environment (Hagedorn and Manovskii (2008)).

Overall, the welfare implications of aggregate fluctuations in the matching model question the optimism of Lucas (1987) about the weakness of business cycle costs. Our results are obtained in a canonical framework without taking into account the individual risks associated with aggregate unemployment, which has received more attention in the literature since the seminal work of Krusell and Smith (1999). Unemployment fluctuations could then imply welfare costs through both a decrease in aggregate consumption and an increase in individual consumption volatility. There are no reasons to think that these two dimensions are not cumulative, leading to substantial business cycle costs. The verification of this assertion is left to further research.

Finally, our results give strong support to the recent line of research which takes into account labor market frictions in the design of optimal monetary policy (Blanchard and Gali (2008), Faia (2009) and Sala et al. (2008)). However, they also emphasize that leaving aside the non-linearities arising from the matching frictions leads to ignore an important source of business cycle costs and then potentially to derive misleading policy recommendations. From a methodological standpoint, our paper then questions first-order approximations to the equilibrium conditions which are extensively used to approximate welfare up to second order in the optimal policy literature[42].

[42]See Schmitt-Grohé and Uribe (2007) and Faia (2009) for a similar point of view.

References

Barlevy, G. (2004), 'The cost of business cycles under endogenous growth', *American Economic Review* **94(4)**, 964–990.

Beaudry, P and C. Pages (1999), 'The cost of business cycles and the stabilization value of unemployment insurance', *European Economic Review* **45(8)**, 1545–1572.

Blanchard, O. and J Gali (2008), Labor markets and monetary policy: a new-keynesian model with unemployment. NBER WP 13897.

Cole, H. and R. Rogerson (1999), 'Can the mortensen-pissarides matching model match the business-cycle facts', *International Economic Review* **40**(4), 933–1959.

Costain, J. and M. Reiter (2008), 'Business cycles, unemployment insurance, and the calibration of matching models', *Journal of Economic Dynamics and Control* **32**, 1120–1155. forthcoming.

De Long, B. and L. Summers (1988), 'How does macroeconomic policy affect output?', *Brookings Papers on Economic Activity* **2**, 433–480.

De Santis, M. (2007), 'Individual consumption risk and the welfare cost of business cycles', *American Economic Review* **97**, 1488–1506.

Epaulard, A. and A. Pommeret (2003), 'Recursive utility, growth, and cost of volatility', *Review of Economic Dynamics* **6(2)**, 672–684.

Faia, E. (2009), 'Ramsey monetary policy with labour market frictions', *Journal of Monetary Economics* **56**.

Fujita, S. and G. Ramey (2008), 'The cyclicality of separation and job finding rates', *International Economic Review* . forthcoming.

Gali, J., M. Gertler and D. Lpez-Salido (2007), 'Markups, gaps, and the welfare costs of business fluctuations', *The Review of Economics and Statistics* **89**, 44–59.

Hagedorn, M. and I. Manovskii (2008), 'The cyclical behaviour of equilibrium unemployment and vacancies revisited', *American Economic Review* **98(4)**, 1692–1706.

Hall, R. (2005*a*), 'Employment efficiency and sticky wages: Evidence from folows in the labor market', *Review of Economics and Statistics* **87**(3), 397–407.

Hall, R. (2005*b*), 'Employment fluctuations with equilibrium wage stickiness', *American Economic Review* **95**(1), 50–65.

Hall, R. (2006), Work-consumption preferences and employment volatility. Unpublished.

Hall, R. and P. Milgrom (2008), ‘The limited influence of unemployment on the wage bargain’, *American Economic Review* **98(4)**, 1653–1674.

Hornstein, A., P. Krusell and G Violante (2007), Modelling capital in matching models: Implications for unemployment fluctuations. mimeo,.

Jung, P. and K. Kuester (2008), The (un)importance of unemployment fluctuations for welfare. Federal Reserve Bank of Philadelphia Working Paper.

Kennan, J. (2006), Private information, wage bargaining and employment fluctuations. NBER Working Papers.

Kitao, S., L. Ljungqvist and T. Sargent (2008), A life cycle model of trans-atlantic employment experiences. NYU mimeo.

Krebs, T. (2007), ‘Job displacement risk and the cost of business cycles’, *American Economic Review* **97(3)**, 664–686.

Krusell, P. and A. Smith (1999), ‘On the welfare effects of eliminating business cycles’, *Review of Economic Dynamics* **2(1)**, 245–272.

Lucas, Robert (1987), *Models of Business Cycles*, Blackwell Publishing.

Matheron, J. and T. Maury (2000), The welfare cost of fluctuations in ak growth models. mimeo, University of Paris.

Mortensen, D. and C. Pissarides (1999), *Job reallocation, employment fluctuations and unemployment*, Vol. 1, 1 edn, Elsevier, chapter 18, pp. 1171–1228.

Mortensen, D. and E. Nagypal (2003), ‘More on unemployment and vacancy fluctuations’, *Review of Economic Dynamics* **61**, 397–415.

Nickell, S., L. Nunziata and W. Ochel (2005), ‘Unemployment in the oecd since the 1960s, what do we know’, *Economic Journal* **115**, 1–27.

Petrongolo, B. and C. Pissarides (2001), ‘Looking into the black box: a survey of the macthing function’, *Journal of Economic Literature* **39**, 716–741.

Pissarides, C. (2007), The unemployment volatility puzzle: Is wage stickiness the answer? The Walras-Bowley lecture, North American Summer Meetings of the Econometric Society, Duke University.

Ramey, G. and V. Ramey (1993), ‘Technology commitment and the cost of economic fluctuations’, *NBER Working Paper No. W3755* pp. 433–480.

Reis, R. (2009), 'The time-series properties of aggregate consumption: implications for the costs of fluctuations', *Journal of European Economic Association* **7**.

Sala, L., U. Soderstrom and A. Trigari (2008), 'Monetary policy under uncertainty in an estimated model with labour market frictions', *Journal of Monetary Economics* **55**, 983–1006.

Schmitt-Grohé, S. and M. Uribe (2007), 'Optimal, simple, and implementable monetary and fiscal rules', *Journal of Monetary Economics* **54**, 1702–1725.

Shiller, R (1997), *Why Do People Dislike Inflation? in Christina Romer and David Romer (eds.), Reducing Inflation: Motivation and Strategy*, University of Chicago Press.

Shimer, R. (2004), 'The consequences of rigid wages in search models', *Journal of the European Economic Association* **2**(2-3), 469–479.

Shimer, R. (2005), 'The cyclical behaviour of equilibrium unemployment and vacancies', *American Economic Review* **95**(1), 25–49.

Silva, J. and M. Toledo (2008), 'Labour turnover costs and the behavior of vacancies and unemployment', *Macroeconomic Dynamics* . forthcoming.

Storesletten, K., C. Telmer and A. Yaron (2001), 'The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk', *European Economic Review* **45**, 1311–1339.

Wolfers, J. (2003), 'Is business cycle volatility costly? evidence from surveys of subjective wellbeing', *International Finance* **6**(1), 1–26.

A The steady state unemployment gap

The conditional steady state can be written as a function of the job finding rate:

$$\widetilde{u}_i = \widetilde{u}(p_i)$$

Let $\nu_i = p_i - \bar{p}$, the conditional unemployment rate is therefore:

$$\widetilde{u}_i = \widetilde{u}(\bar{p} + \nu_i)$$

Because the unemployment rate is a convex function of the job finding rate, volatility in the job finding rate affects average unemployment. The unemployment gap ψ_p between an economy characterized by a stable job finding rate and an economy with a volatile job finding rate can be computed as follows:

$$\sum_i \pi_i \widetilde{u}(\bar{p} + \nu_i) = \bar{u} + \psi_p$$

A second order approximation of the left hand side yields:

$$\sum_i \pi_i \left[\widetilde{u}(\bar{p}) + \nu_i \widetilde{u}'(\bar{p}) + \frac{\nu_i^2}{2} \widetilde{u}''(\bar{p}) \right] \approx \bar{u} + \psi_p$$

Which gives:

$$\begin{aligned} \psi_p &\approx \frac{\sigma_p^2}{2} \widetilde{u}''(\bar{p}) \\ \psi_p &\approx \sigma_p^2 \frac{\bar{s}}{(\bar{s} + \bar{p})^3} \end{aligned}$$

A similar calculation gives for the job separation rate:

$$\begin{aligned} \psi_s &\approx \frac{\sigma_s^2}{2} \widetilde{u}''(\bar{s}) \\ \psi_s &\approx -\sigma_s^2 \frac{\bar{p}}{(\bar{s} + \bar{p})^3} \end{aligned}$$

B The unemployment gap: the general case

The unemployment dynamics read:

$$u_{t+1} = \bar{s} + (1 - \bar{s} - p_t)u_t \tag{27}$$

Define $\phi_t \equiv 1 - \bar{s} - p_t$ and $\mathbb{E}[\phi] \equiv \bar{\phi} = 1 - \bar{s} - \bar{p}$.

$$\phi_t = \rho_p \phi_{t-1} + (1 - \rho_p)\bar{\phi} - \varepsilon_t^p$$

Where ε_t^p is the innovation of the job finding rate process. It is iid, has mean zero and variance $\sigma^2_{\varepsilon_p}$.

The unemployment dynamics can be written:

$$u_{t+1} = \bar{s} + \phi_t u_t \tag{28}$$

A backward substitution gives:

$$\begin{aligned} u_{t+1} &= \bar{s} + \sum_{k=0}^{+\infty} \prod_{j=0}^{k} \phi_{t-j} \bar{s} \\ \mathbb{E}[u_{t+1}] &= \bar{s} \left(1 + \sum_{k=0}^{+\infty} \mathbb{E}[\prod_{j=0}^{k} \phi_{t-j}] \right) \end{aligned}$$

Write ϕ as an infinite moving average:

$$\phi_t = \bar{\phi} - \Sigma_{j=0}^{+\infty} \rho_p^j \varepsilon_{t-j}^p$$

The mean of unemployment can then be written:

$$\mathbb{E}[u] = \bar{s} \left(1 + \mathbb{E}\Sigma_{k=0}^{+\infty}[(\bar{\phi} - \Sigma_{j=0}^{+\infty}\rho_p^j \varepsilon_{t-j}^p)...(\bar{\phi} - \Sigma_{j=0}^{+\infty}\rho_p^j \varepsilon_{t-j-k}^p)] \right)$$

Neglecting moments of order above 2, the mean of unemployment can be approximated by:

$$\mathbb{E}[u] \approx \bar{s} \left(\frac{1}{\bar{s} + \bar{p}} + \frac{\sigma^2_{\varepsilon^p}}{1 - \rho_p^2} \sum_{k=2}^{+\infty} (1 - \bar{s} - \bar{p})^{k-2} \sum_{i=0}^{k-1} (k - 1 - i)\rho_p^{i+1} \right)$$

which simplifies to:

$$\mathbb{E}[u] \approx \bar{u} + \bar{s} \frac{\sigma^2_{\varepsilon^p}}{1 - \rho_p^2} \sum_{k=1}^{+\infty} \frac{\rho_p k (1 - \bar{s} - \bar{p})^{k-1}}{1 - \rho_p(1 - \bar{s} - \bar{p})}$$

This yields equation (7).

A similar calculation gives the consequences of job separation rate fluctuations on the average unemployment rate:

$$\mathbb{E}[u] - \bar{u} \approx -\frac{\sigma^2_{\varepsilon^s}}{1-\rho_s^2}\left[\sum_{k=1}^{+\infty}(1-\bar{s}-\bar{p})^{k-1}\sum_{i=0}^{k-1}\rho_s^{i+1} - \sum_{k=2}^{+\infty}\bar{s}(1-\bar{s}-\bar{p})^{k-2}\sum_{i=0}^{k-1}(k-1-i)\rho_s^{i+1}\right]$$

which simplifies to:

$$\mathbb{E}[u] - \bar{u} \approx -\frac{\sigma^2_{\varepsilon^s}}{1-\rho_s^2}\left[\frac{\rho_s}{1-\rho_s}\left(\frac{1}{\bar{s}+\bar{p}} - \frac{\rho_s}{1-\rho_s(1-\bar{s}-\bar{p})}\right) - \frac{\rho_s}{1-\rho_s(1-\bar{s}-\bar{p})}\frac{\bar{s}}{(\bar{s}+\bar{p})^2}\right]$$

This yields equation (8).

C Job finding rate and separation rate data

Figure 2: Job finding rate - Shimer data

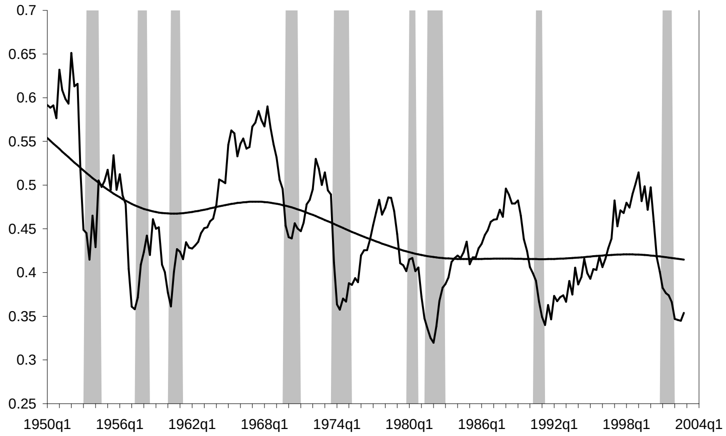


Figure 3: Job finding rate - Hall data

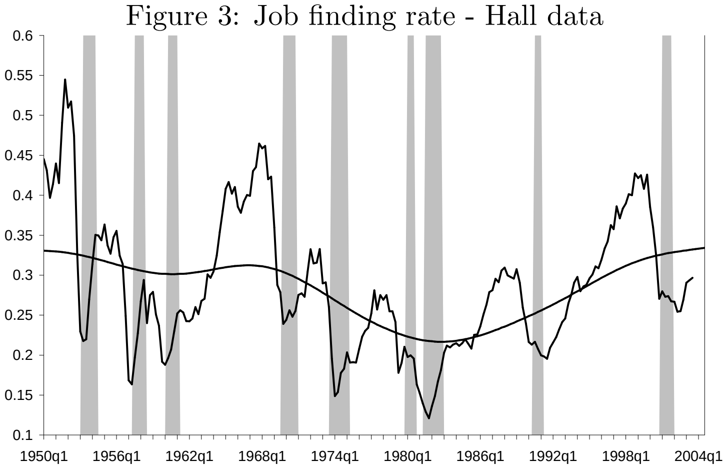


Figure 4: Separation rate - Shimer data

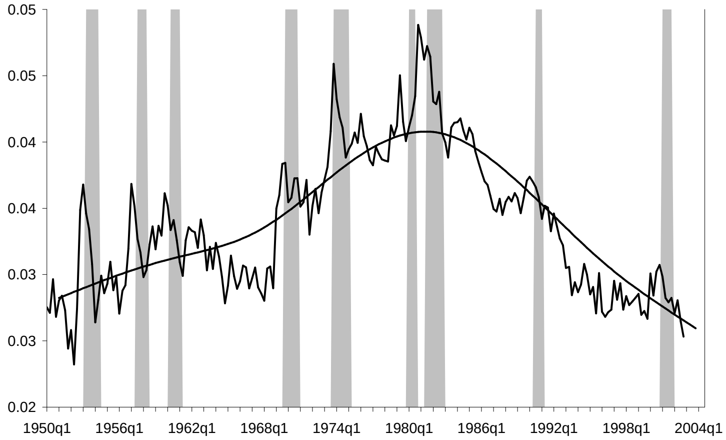
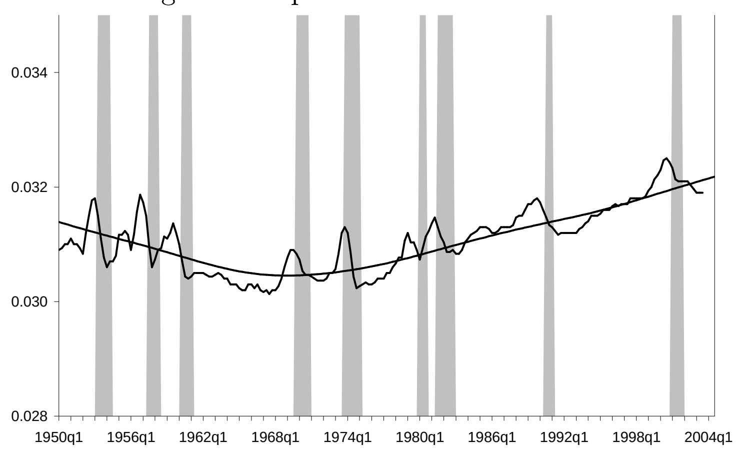


Figure 5: Separation rate - Hall data



D Alternative specification: log-normal shocks

We estimate the following AR(1) processes:

$$ln(p_t) = a_{lp} + \rho_{lp} ln(p_{t-1}) + \varepsilon_t^{lp} \quad (29)$$

$$ln(s_t) = a_s + \rho_{ls} ln(s_{t-1}) + \varepsilon_t^{ls} \quad (30)$$

Tables 10 and 11 show that the implied characteristics for the level of p and s are relatively close to those displayed in Tables 1 and 2. This is particularly the case when considering H-P filtered data. Using the HP filter makes the choice of estimating in log or in level pointless. On the other hand, without filtering, it appears that the log-linearity decreases the cost of business cycles. As the distribution of p is shifted to the right, the non-linear effect of the fluctuations in p on u is weaker: p fluctuates over a less convex region (Figure 6).

Figure 6: The implications of log-normality

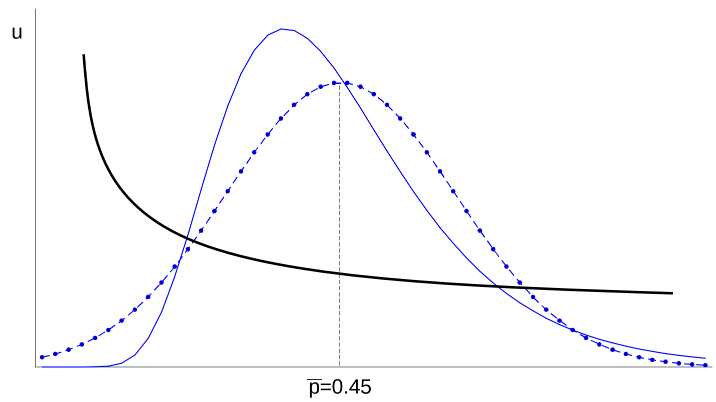


Table 10: Job finding rate fluctuations under log-normality

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Estimated process of $ln(p)$				
Standard deviation of ε^{lp}	0.098	0.097	0.049	0.048
Autocorrelation ρ_{lp}	0.941	0.913	0.941	0.914
Implied dynamics of p				
Mean $\bar{p}$	0.285		0.450	
Standard deviation σ_p	0.084	0.068	0.068	0.053
Autocorrelation	0.942	0.913	0.939	0.915
Business cycle cost	0.63%	0.43%	0.13%	0.08%

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005*a*) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both sets of data are detrended with a HP smoothing parameter of 10^5.

E Non linearities in the endogenous job finding rate: the flexible wage case

In this Appendix, we show that the condition ensuring the concavity of the job finding rate is less stringent when wages are flexible than when wages are rigid. We use the comparative statics of the model without aggregate shocks to approximate the response of the job finding rate.

For a level y of productivity, the equilibrium vacancy-unemployment ratio θ is characterized by the following equation.

$$H(\theta, y) \equiv \frac{\kappa}{q(\theta)} - \Psi\left((1-\gamma)(y-z) - \gamma\kappa\theta\right) = 0$$

where $\Psi \equiv \frac{\beta}{1-\beta(1-s)}$

Table 11: Job separation rate fluctuations under log-normality

	Hall data		Shimer data	
	Raw	Detrended	Raw	Detrended
Estimated process of $ln(s)$				
Standard deviation of ε^{ls}	0.005	0.004	0.055	0.051
Autocorrelation ρ_{ls}	0.970	0.873	0.941	0.734
Implied dynamics of s				
Mean $\bar{s}$	0.031		0.034	
Standard deviationσ_s $\times 10^{-2}$	0.058	0.029	0.540	0.260
Autocorrelation	0.970	0.873	0.946	0.734
Business cycle cost	0%	0%	-0.006%	0%

Note: Quarterly average of monthly data. Sample covers 1948q3-2004q3 for Hall (2005*a*) and 1951q1-2003q4 for Shimer (2005). Following Shimer (2005), both data are detrended with a HP smoothing parameter of 10^5.

Let us define

$$G(\alpha) \equiv \frac{\partial^2 p}{\partial y^2} = p''(\theta)(\theta'(y))^2 + p'(\theta)\theta''(y)$$

Using the function $H(\theta, y)$ to compute the implicit derivatives $\theta'(y)$ and $\theta''(y)$, we then obtain:

$$G(\alpha) = \frac{\alpha(1-\alpha)\Psi^2(1-\gamma)^2}{\Gamma(\theta)^2}\left[-\varphi\theta^{\alpha-2} + \frac{\alpha\kappa\theta^{-2}}{\Gamma(\theta)}\right]$$

where $\Gamma(\theta) = \kappa\varphi^{-1}(1-\alpha)\theta^{-\alpha} + \Psi\gamma\kappa$

We deduce that the job finding rate is a concave function of productivity if:

$$G(\alpha) < 0 \quad \Leftrightarrow \quad 2\alpha - 1 - \Psi\varphi\gamma\theta^{\alpha} < 0$$

If $\alpha = 1/2$, then:

$$G(1/2) = -\Psi\varphi\gamma\theta^{1/2} < 0$$

The job finding rate is a strictly concave function of productivity when $\alpha = 1/2$ (i.e at the rigid wage threshold). Because $2\alpha - 1 < 0$ for $\alpha < 1/2$, this restriction is also satisfied for any $\alpha \in [0, 1/2]$. Then, we deduce that the concavity of the job finding rate is more probable in the case of flexible wages.