Inflation and Output Dynamics in a Model with Labor Market Search and Capital Accumulation

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Abstract:
In a sticky-price model with labor market search and habit persistence, Walsh (2005) shows that inertia in the interest rate policy helps to reconcile the inflation and output persistence with empirical observations for the US economy. We show that this finding is sensitive with regard to the introduction of capital formation. While we are able to replicate the findings for the inflation inertia in a model with capital adjustment costs and variable capacity utilization, the output response to an interest shock is found to be too large and no longer hump-shaped in this case. In addition we find that the response of output to a technology shock can only be reconciled with empirical findings if either the adjustment of the utilization rate is very costly or there is only a modest amount of nominal rigidity in the economy.

1 Introduction

There is ample evidence from structural vector autoregressions using different identification schemes and data sets that a sudden increase of the short term nominal interest rate produces a persistent and hump-shaped response of output and inflation.[1] In recent studies, labor market imperfections have been introduced into monetary business cycle models in order to replicate these findings. Christiano et al. (2005) model nominal rigidities in the form of both price and wage staggering in order to explain the observed inertia in inflation after a monetary expansion. Walsh (2005), Trigari (2006, 2009), and Christoffel et al. (2009) consider search and matching frictions in the labor market. Walsh (2005) finds that the inertia of the interest rate policy itself is an important contributing factor for the explanation of the inflation and output inertia. Trigari (2006, 2009) considers the effects of the wage bargaining process on the variation of both inflation and real wages following a monetary shock, while Christoffel et al. (2009) study the sensitivity of her results with regard to the introduction of wage rigidity, on-the-job search, and endogenous separation. In addition to Walsh (2005), the latter two studies also allow for the variation of labor at both the intensive and the extensive margin. [2]

In this paper, we consider the sensitivity of these recent studies with respect to the introduction of capital. Our economy is based upon the model of Walsh (2005) in which we introduce capital as a second production factor besides labor. The reasoning why capital may introduce a different dynamic response of inflation and output to a monetary shock is as follows: In the model of Walsh (2005) the marginal costs of price setters equal the relative price of intermediate goods in terms of the final good. Intermediate good firms adjust their nominal price immediately while wholesale firms respond only sluggishly to a demand or supply shock. Thus, marginal costs of price setters decrease in response to a negative demand shock. The size of this shock depends on the response of the household sector to an increase of the nominal interest rate. Without capital and with habit persistence in consumption this effect is small. However, if capital allows for intertemporal substitution, overall demand can decrease significantly. Obviously, the adjustment of capital as a second

[1] See, among others, Sims (1992), Leeper et al. (1996), and Christiano et al. (1999, 2005).

[2] Subsequently, the labor market search model has also been prominently applied to the analysis of the Ramsey policy as, e.g., in Faia (2008), or the study of the business-cycle dynamics of wages as in Rotemberg (2006).

factor of production also affects the dynamics of output.

As one of our main results, our model with capital is able to generate inflation dynamics following an interest rate shock that is in accordance with empirical observations. Therefore, we are able to confirm this finding of Walsh (2005) who considers a model without capital. Similar to Christiano et al. (2005), we also find that the introduction of variable capital utilization is an important factor for the modelling of the inertia in the inflation dynamics. In this case, rather the capacity than the investment demand increases after a fall in the interest rate so that the real interest rate displays a smaller variation. In the model with capital, however, an unexpected rise in the nominal interest rate does not trigger a hump-shaped response of output, quite contrary to the model without capital.

The main reason why the model of Christiano et al. (2005) is able to generate a more persistent and hump-shaped response of output than our model is the introduction of real wage rigidity in their model. In their Fig. 4, Christiano et al. (2005) show that, in the case of flexible rather than sticky wages, the impulse response of output also peaks in the first period following the monetary shock. With sticky wages, however, marginal costs do not surge in the period after the shock and output does not return quickly to its steady state value. In a standard labor market model with search frictions, however, the real wage plays no allocational role but splits the rents associated with a successful match between the firm and the worker. It requires additional assumptions, as in Trigari (2006) and Christoffel et al. (2009) where the firms decide about working hours after the wage bargain, to create a channel from wages to marginal costs. In this setting sticky wages can generate a more inertial response of inflation (Christoffel et al. (2009)).

In addition to the studies, we also analyze the effects of a technology shock on the output-inflation dynamics. Most studies including Walsh (2005), Christiano et al. (2005), or Trigari (2006, 2009) neglect this question. We consider it an interesting problem because a researcher is ultimately aiming for a monetary general equilibrium model that is able to match the empirical responses to various kinds of supply, demand, and policy shocks simultaneously. As one prominent example, consider the analysis of optimal monetary policy and to what extent the monetary authority should respond to a productivity shock. Here, too, we find that while the inflation dynamics is insensitive to the assumption of fixed capital services the output dynamics is not. In line with empirical evidence, we get a protracted hump-shaped decline of the rate of inflation in response to a productivity shock in our model with capital accumulation and a variable utilization rate of capital. However, this model also implies a significant immediate decrease of output that is not observed in estimated impulse response functions. We can reconcile the model with empirical evidence if we either assume that it is very costly to adjust the utilization rate of capital or that the degree of nominal rigidity in our model economy is small.

The remainder of the paper is structured as follows. Sections 2 introduces the model. In Section 3, we describe the calibration and computation of the model. Sections 4-6 present our results. In Section 4 we study the second moments of the model. In Sections 5 and 6, we analyze the impulse responses of the model following an interest rate and a technology shock, respectively, and compare them to empirical estimates. Section 7 concludes. Technical Appendices A-D document details of our computation and estimation procedures.

2 The model economy

In this section, we describe our model that is based upon Walsh (2005). Three different sectors are depicted: firms, households, and the monetary authority.

2.1 Firms

2.1.1 Retail sector

A final goods or retail sector buys differentiated goods Y_{jt} distributed over the unit interval, $j \in [0,1]$, from wholesale firms and assembles the final output Y_t according to

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \tag{1}$$

Profit maximization of retail firms,

$$\max_{\{Y_{jt}\}_{j=0}^1} \quad Y_t - \int_0^1 P_{jt} Y_{jt} dj,$$

implies the demand function

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t, \tag{2}$$

where P_{jt} is the nominal price of good $j \in [0,1]$ and P_t is the price level. The zero profit condition for the retail sector implies that P_t is given by

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta} dj \right)^{1/(1-\theta)}. \tag{3}$$

2.1.2 Wholesale sector

Firms in the wholesale sector[3] purchase intermediate goods y_{jt}, $j \in [0,1]$ from the production sector that is described below. The profit of a wholesaler in terms of the final output is given by

$$\left(\frac{P_{jt}}{P_t} - g_t \right) Y_{jt}, \tag{4}$$

where

$$g_t = \frac{P_t^W}{P_t} \tag{5}$$

is the price of the output of the production sector P_t^W in terms of the final good. From the perspective of the wholesale sector g_t are the real marginal costs faced by any firm in this sector.

Prices are set according to the mechanism spelled out in Calvo (1983). In each period $(1-\omega)$ of the wholesale firms are allowed to set their relative price P_{jt}/P_t optimally. Walsh (2005) follows Christiano et al. (2005) and assumes that prices must be set before the monetary shock is observed. The remaining fraction of the wholesale firms, indexed by N, adjusts their price according to a rule of thumb: They increase their price according to the inflation factor (one plus the rate of inflation) of the previous period π_{t-1}:

$$P_{Nt} = \pi_{t-1} P_{Nt-1}, \quad \pi_t := \frac{P_t}{P_{t-1}}. \tag{6}$$

[3] The model of Walsh (2005) does not need a wholesale sector. In this model the household consumes a basket of differentiated products and since consumption is the single use of output the demand functions derive from the household's optimization problem. In our model, output is used for both consumption and investment. Therefore, we must first aggregate the differentiated products into a single good Y. Otherwise, we could have assumed similar baskets for consumption and investment, which is the less usual way to tackle this problem.

This price setting behavior implies the following log-linear Phillips curve equation:

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_{t-1}\hat{\pi}_{t+1} + \Gamma E_{t-1}\hat{g}_t \tag{7}$$

with $\Gamma = \frac{(1-\omega)(1-\beta\omega)}{(1+\beta)\omega}$, $E_t x_{t+1}$ expectations of x_{t+1} based on information available up to period t, and where a hat over a variable denotes its percentage deviation from its steady-state value. β denotes the discount factor of the household that will be introduced below. We also consider the effect of a monetary policy shock, if the firms which are allowed to set their price optimally do so after they have observed the shock. This implies the following Phillips curve:

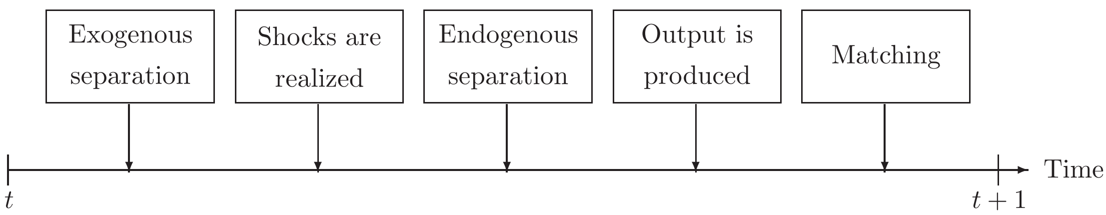
$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \Gamma\hat{g}_t. \tag{8}$$

Note, finally, that in the steady state of the deterministic counterpart of the model, $g_t = P_t^W/P_t$ is constant and equals $g = (\theta-1)/\theta < 1$.

2.1.3 Intermediate goods sector

Employment relationships consist of a worker and a firm. At the beginning of each period there are N_t employed workers and, thus, N_t worker-firm pairs indexed by i. For reasons outside of the model, the fraction of ρ^x of those pairs separate. The remaining pairs observe the current state of the environment and decide whether or not to continue their relationship. Those that do not separate produce output. Finally, at the end of the period vacancies posted by firms are filled with job applicants via a matching technology described below. Figure 1 depicts the timing of events in this model.

Figure 1: Timing of Events within a Period



Output produced by a worker-firm pair i is given by

$$y_{it} = Z_t a_{it} k_{it}^{\alpha}, \quad \alpha \in (0,1). \tag{9}$$

k_{it} are capital services, Z_t is a random productivity disturbance that is common to all firms, and a_{it} is a random productivity disturbance that is specific to relationship $i \in [0, 1]$, respectively. The worker and the firm observe both shocks and choose k_{it} to maximize their joint payoff:

$$\max_{k_{it}} \quad g_t Z_t a_{it} k_{it}^{\alpha} - r_t k_{it} - l,$$

where l denotes the consumption value of the disutility of work. Firm i pays the real rental rate r_t on its capital services k_{it}. Profit maximization implies

$$k(a_{it}) = \left(\frac{\alpha g_t Z_t a_{it}}{r_t} \right)^{1/(1-\alpha)}. \tag{10}$$

Job creation. The decision to severe the relationship depends on the outside options of the worker and the firm and on the present value of continuing the relationship into the next period v_{it}. Note that except for the realization of a_{it} all employment relationships face the same conditions. Thus, if a_{it} is distributed identically and independently over time, v_{it} must be equal for all worker-firm pairs and we can drop the index i from this variable and all others as well. In equilibrium, the present value of the firm's outside opportunities is zero, and the value of the worker's outside opportunities equals the present value of being unemployed w_t^u. The surplus of an employment relationship thus can be written as

$$s_t = g_t Z_t a_t k(a_t)^{\alpha} - r_t k(a_t) - l + v_t - w_t^u. \tag{11}$$

The firm and the worker will terminate their relationship if $a_{it} < \underline{a}_t$, where $\underline{a}_t$ is determined as solution to

$$g_t Z_t \underline{a}_t k(\underline{a}_t)^{\alpha} - r_t k(\underline{a}_t) - l + v_t - w_t^u = 0. \tag{12}$$

Note that due to (10) and (12) the surplus of an employment relationship can also be written as

$$s_t = (1-\alpha) \left(\frac{\alpha}{r_t} \right)^{\alpha/(1-\alpha)} (g_t Z_t)^{1/(1-\alpha)} \left[a_t^{1/(1-\alpha)} - \underline{a}_t^{1/(1-\alpha)} \right]. \tag{13}$$

Given the job destruction margin $\underline{a}_t$ the endogenous job destruction rate ρ_t^n is obtained from

$$\rho_t^n = \int_0^{\underline{a}_t} f(a) da = F(\underline{a}_t), \tag{14}$$

where $f(a)$ and $F(a)$ denote the probability density function and the distribution function of a_{it}, respectively. Since new matches from period t will not produce before period $t+1$ the mass of workers that are unemployed during period t equals

$$\begin{aligned} U_t &= \underbrace{1 - N_t}_{\text{stock of unemployed}} + \underbrace{\rho^x N_t}_{\text{exogenous separations}} + \underbrace{(1-\rho^x)\rho_t^n N_t}_{\text{endogenous separations}}, \\ &= 1 - (1-\rho^x)(1-\rho_t^n)N_t. \end{aligned} \tag{15}$$

Matching technology. At the end of period t unemployed workers are matched to vacancies. Therefore, at the beginning of period $t+1$ the mass of employed workers is determined by

$$N_{t+1} = (1-\rho^x)(1-\rho_t^n)N_t + m(U_t, V_t), \tag{16}$$

where $m(U_t, V_t)$ is the number of aggregate matches. The matching function is assumed to be Cobb-Douglas:

$$m(U_t, V_t) = \psi U_t^{\chi} V_t^{1-\chi}, \quad \chi \in (0,1). \tag{17}$$

The probability that a firm offering a job in period t will find a worker is given by

$$\kappa_t^f = \frac{m(U_t, V_t)}{V_t} = \psi \left(\frac{V_t}{U_t}\right)^{-\chi}. \tag{18}$$

Similarly, the probability that the unemployed worker is finding a job is given by

$$\kappa_t^w = \psi \left(\frac{V_t}{U_t}\right)^{1-\chi}. \tag{19}$$

Job creation. We assume that the firm obtains the share $1-\eta \in (0,1)$ from an employment relationship that produces in period t. The probability that a worker-firm pair which is matched in period t will produce in period $t+1$ is $(1-\rho^x)(1-\rho_{t+1}^n)$. The expected value of this match in period $t+1$ equals

$$\int_{\underline{a}_{t+1}}^{\infty} s_{t+1} \frac{f(a)}{1-\rho_{t+1}^n} da,$$

where $f(a)/(1-\rho_{t+1}^n)$ is the conditional density of the event $a | a \geq \underline{a}_{t+1}$. We assume free entry of firms and a cost of γ for offering a job. Thus, the number of vacancies is determined by the condition

$$\gamma = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\eta)\kappa_t^f (1-\rho^x) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da \right\}, \tag{20}$$

where $\beta(\lambda_{t+1}/\lambda_t)$ is the stochastic discount factor and λ_t the marginal utility of consumption that we will introduce in a moment. Equation (20) establishes that the outside value of a firm equals zero.

The present value of unemployment. In period t an unemployed worker faces the probability κ_t^w in (19) to find a job. The probability that he will not loose this job in the next period is $(1-\rho^x)(1-\rho_{t+1}^n)$. Since the worker always receives the value of his outside option, the present value of being unemployed is determined by

$$w_t^u = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \eta \kappa_t^w (1-\rho^x) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da + w_{t+1}^u \right\}, \tag{21}$$

where b is the worker's valuation of leisure time in units of consumption goods.

The present value of a continuing employment relationship. A worker-firm pair that produces in the next period receives the expected value of its surplus. Since the worker always receives the value of its outside option w_{t+1}^u and since the value of the firm's outside option equals zero, the present discounted value of a match that continues to produce in $t+1$ is given by

$$v_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\rho^x) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da + w_{t+1}^u \right\}. \tag{22}$$

Note that equations (22) and (21) imply

$$q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\rho^x)(1-\eta\kappa_t^w) \int_{\underline{a}_{t+1}}^{\infty} s_{t+1} f(a) da, \right\}, \tag{23}$$

where

$$q_t = v_t - w_t^u + b.$$

Division of (23) by equation (20) yields

$$q_t = \frac{\gamma(1-\eta\kappa_t^w)}{(1-\eta)\kappa_t^f}. \tag{24}$$

Job creation and destruction. Let $\rho_t = \rho^x + (1-\rho^x)\rho_t^n$ denote the share of workers N_t that loose their job for either exogenous or endogenous reasons. Den Haan et al. (2000), p. 490 assume that firms which have been separated from their workers endogenously do not post vacancies in the current period. Accordingly, $\kappa_t^f \rho^x$ is the fraction of jobs lost at the beginning of the period which are successfully advertised and filled at the end of the period. The rate of job destruction (as a percentage of employment N_t), thus, equals

$$jdr_t = \rho_t - \kappa_t^f \rho^x = (1-\rho^x)\rho_t^n + \rho^x(1-\kappa_t^f). \tag{25}$$

The job creation rate jcr_t – as defined by den Haan et al. (2000), p. 490 – is the mass of all firms that have no workers at the beginning of the period but find workers in the matching phase as a percentage of the mass of employed workers N_t:

$$jcr_t = \frac{\kappa_t^f (V_t - \rho^x N_t)}{N_t}. \tag{26}$$

2.2 Households

Employed and unemployed workers pool their income so that we can ignore distributional issues. Employed workers supply one unit of labor inelastically with disutility $c_l l$ while unemployed workers enjoy leisure at utility value $c_b b$, where both l and b are measured in units of the consumption good so that the constants $1/c_l > 0$ and $1/c_b > 0$ transform the utility value of leisure and home work to consumption units. As in Walsh (2005), we introduce habit formation in the utility function. In addition, the household obtains utility from real money M_t/P_t. The households current-period utility function is given by:[4]

$$\begin{aligned} u(C_t, C_{t-1}, \zeta_t, M_t/P_t) &:= \frac{(C_t - hC_{t-1})^{1-\sigma} - 1}{1-\sigma} + (1-\zeta_t)c_b b - \zeta_t c_l l + \phi(M_t/P_t), \\ & h \in [0,1),\ c_b, c_l > 0, \qquad \zeta_t = \begin{cases} 1 & \text{if employed,} \\ 0 & \text{if unemployed.} \end{cases} \end{aligned} \tag{27}$$

[4]Our specification departs from Walsh's utility function given in his equation (1). He uses

$$\tilde{C}_t := C_t + (1-\zeta_t)b - \zeta_t l$$

as argument of u. However, his Matlab program takes C_t as argument of u. We consider the consequences of using $\tilde{C}_t$ instead of C_t in Appendix A.

According to this specification the household's marginal utility of consumption also depends upon his level of consumption in the previous period. In particular, the marginal utility of consumption is higher if C_t is closer to C_{t-1}.

The household sector receives wage and profit income from employment relationships that are not severed at the beginning of period t given by

$$Inc_t = (1-\rho^x)(1-\rho_t^n)N_t \int_{\underline{a}_t}^{\infty} \left[g_t Z_t a_t k(a_t)^\alpha - r_t k(a_t)\right] \frac{f(a_t)}{1-\rho_t^n} da_t. \tag{28}$$

In addition, the household receives profits Ω_t from the wholesale sector and transfers T_t from the monetary authority.[5]

The household holds beginning-of-period nominal money M_t and bonds B_t, as well as the real physical capital stock $\bar{K}_t$. Bonds are issued by other households and pay a nominal rate of interest i_t. The nominal interest rate factor is denoted by $R_t := 1 + i_t$. Following Christiano et al. (2005), capital services K_t are related to the physical stock of capital $\bar{K}_t$ by $K_t = u_t \bar{K}_t$, where u_t denotes the utilization rate of capital.[6] The household's budget constraint is given by:

$$\frac{B_{t+1} + M_{t+1}}{P_t} \leq Inc_t + \Omega_t + r_t u_t \bar{K}_t + T_t + R_t \frac{B_t}{P_t} + \frac{M_t}{P_t} - \gamma V_t - C_t - I_t - \iota(u_t)\bar{K}_t, \tag{29}$$

where I_t and $\iota(u_t)$ denotes investment and the costs of setting the utilization rate to u_t, respectively. In the non-stochastic steady state, $\bar{u} = 1$ and $\iota(\bar{u}) = 0$.

The stock of capital evolves according to

$$\bar{K}_{t+1} = \Phi\left(\frac{I_t}{\bar{K}_t}\right)\bar{K}_t + (1-\delta)\bar{K}_t. \tag{30}$$

We assume that the concave function $\Phi(\cdot)$ does not change the non-stochastic steady state of the model. Thus, $I = \delta K$ implying $\Phi(\delta) = \delta$ and $\Phi'(\delta) = 1$. The absolute value of the elasticity of Φ' with respect to its argument I/K is given by the parameter σ_Φ.

Households maximize

$$E_t \sum_{s=0}^{\infty} \beta^s u\left(C_{t+s}, C_{t+s-1}, \frac{M_{t+s}}{P_{t+s}}\right)$$

[5] As in the case of capital income we should have included $-l$ in the definition of the income flow from active worker-firm pairs and added $(1-U_t)l$ as compensation to workers on the right-hand-side of the budget constraint of the representative household. Since both terms cancel, we decided to suppress l both in the definition of Inc_t in (28) and in the budget constraint (29), given below.

[6] To keep the model simple, we assume that the household rather than the firm chooses u_t.

with regard to M_{t+1}, B_{t+1}, $\bar{K}_{t+1}$, C_t, I_t, and u_t subject to (29) and (30). The first-order conditions of the household are given by:

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t (C_{t+1} - hC_t)^{-\sigma}, \tag{31a}$$

$$\iota'(u_t) = r_t, \tag{31b}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}}, \tag{31c}$$

$$\lambda_t = \beta E_t \left\{ \frac{\phi'(M_{t+1}/P_{t+1}) + \lambda_{t+1}}{\pi_{t+1}} \right\}, \tag{31d}$$

$$\xi_t = \frac{1}{\Phi'(I_t/\bar{K}_t)}, \tag{31e}$$

$$\xi_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Big[r_{t+1} u_{t+1} - \iota(u_{t+1}) - \frac{I_{t+1}}{\bar{K}_{t+1}} + \xi_{t+1} \Big(1 - \delta + \Phi(I_{t+1}/\bar{K}_{t+1}) \Big) \Big]. \tag{31f}$$

Equations (31a) and (31b) are the optimal conditions for the current-period consumption level C_t and utilization rate u_t, respectively. Condition (31c) ensures that bonds have the same expected rate of return as capital. Note, that $B_t \equiv 0$ in equilibrium, since we are aggregating the holdings of bonds over the members of the representative household. Equation (31d) induces a money demand function. Since the central bank will pursue an interest rate policy, we can disregard this equation. In (31e), the variable ξ_t is Tobin's q and gives the number of units of output which must be forgone to increase the stock of capital by one unit (this equals Θ_t/λ_t, where Θ_t is the Lagrange multiplier of the constraint (30) in the household's optimization problem).

2.3 Monetary authority

The central bank targets the nominal interest rate and supplies the amount of money necessary to achieve its target rate. We use the following rule:

$$R_{t+1} = \bar{\pi}^{(1-\rho_R)(1-\phi_\pi)} \beta^{-(1-\rho_R)} R_t^{\rho_R} \pi_t^{\phi_\pi(1-\rho_R)} e^{\phi_t}, \quad \phi_t \sim N(0, \sigma_\phi). \tag{32}$$

It is well-known that the exponent of the inflation factor ϕ_π is crucial for the existence of a determinate transition path to the equilibrium. In our benchmark calibration the critical root passes one from below if ϕ_π exceeds one (see Figure 9 in Appendix B). The value of $\phi_\pi = 1.1$ that we take from Walsh (2005) and use in all our numerical experiments always ensures determinacy. In the non-stochastic stationary equilibrium of the model the Euler equation (31c) implies $\pi = \beta R$ and the Taylor rule delivers $\pi = \bar{\pi}$.[7]

Given the monetary policy, the nominal quantity of money adjusts so that the money market is in equilibrium. Seignorage T_t is transferred to the households:

$$T_t = \frac{M_{t+1} - M_t}{P_t}. \tag{33}$$

2.4 Equilibrium

In equilibrium

$$K_t = u_t \bar{K}_t$$

and the aggregate amount of capital services, K_t, is given by the sum of the individual capital services

$$K_t = (1-\rho^x)(1-\rho_t^n) N_t \int_{\underline{a}_t}^{\infty} k(a_t) \frac{f(a_t)}{1-\rho_t^n}\, da_t,$$

implying

$$K_t = (1-\rho^x) N_t H(\underline{a}_t) \left(\frac{\alpha g_t Z_t}{r_t} \right)^{1/(1-\alpha)}, \quad H(\underline{a}_t) := \int_{\underline{a}_t}^{\infty} a_t^{1/(1-\alpha)} f(a_t) da_t. \tag{34}$$

Aggregating y_{it} in (9) over all productive worker-firm pairs using this definition of capital yields the aggregate production function

$$Y_t = Z_t \Big[(1-\rho^x) N_t H(\underline{a}_t) \Big]^{1-\alpha} K_t^{\alpha}. \tag{35}$$

Firms redistribute all profits to the households, and the monetary authority transfers the seignorage. In equilibrium and using the definition of income from (28), the resource constraint of the economy is given by

$$Y_t = C_t + I_t + \gamma V_t + \iota(u_t) \bar{K}_t. \tag{36}$$

[7] The policy rule of Walsh (2005) in his equation (18) is only consistent with zero inflation, $\bar{\pi} = 1$ so we decided to use the more general equation (32). However, as the log-linear equations in Appendix B show none of the coefficients depends upon the value of $\bar{\pi}$ and the derivation of the Phillips curve goes through with any value of $\bar{\pi}$ as shown in Maußner (2007). Therefore, we do not need to fix the value of $\bar{\pi}$ in our simulations.

3 Calibration and computation

If not mentioned otherwise, the choice of the functional forms and the parameterization follows Walsh (2005). We will refer to this case as our benchmark. In Sections 4 and 5, we also analyze cases where the parameters, for which we do not have robust empirical evidence, are chosen in order to optimize the statistical properties of the model. Furthermore, Appendix A covers – among other issues – minor differences between Walsh's and our numerical implementation.

3.1 Functional form assumptions

We assume that the firm-specific productivity shock a is log-normally distributed with mean zero and standard deviation $\sigma_a = 0.13$:

$$f(a) = \frac{1}{a\sigma_a\sqrt{2\pi}} e^{-0.5(\ln a/\sigma_a)^2}.$$

Thus,

$$z := \frac{\ln a}{\sigma_a}$$

has a standard normal distribution, and we get z from the inverse of the cumulative distribution function of the standard normal distribution at the steady state value of ρ^n. Given $\underline{a} = e^{\sigma_a z}$=0.7892 in steady state, we compute

$$H(\underline{a}) := \int_{\underline{a}}^{\infty} a^{1/(1-\alpha)} f(a) da$$

using Simpson's method.

According to our specification of the functions Φ and ι the dynamics of the model only depends on the elasticities σ_Φ and σ_ι of the functions Φ' and ι' with respect to their arguments, respectively.

3.2 Parameterization

We analyze the sensitivity of our model with respect to the introduction of capital adjustment costs and variable capital utilization. Therefore, our main interest is the sensitivity of the model with regard to the choice of the parameters σ_Φ and σ_ι. In addition, we study the model's behavior depending on the parameter values for the price rigidity ω and the habit parameter h. Periods correspond to quarters.

Preferences. Following Walsh (2005), we set the discount factor $\beta = 0.989$, the intertemporal elasticity of substitution $1/\sigma = 0.5$, and the habit parameter $h = 0.78$. The parameter values are summarized in Table 1.

Table 1:
Benchmark calibration

Preferences	β=0.989	σ=2	h=0.78	
Labor Market	ρ^x=0.068	ρ^n=0.0343	κ^f=0.7	κ^w=0.6
	η=0.5	χ=0.4		
Production	α=0.36	δ=0.025	σ_a=0.13	σ_Φ=0.5
	σ_ι=0.01	θ=11.0	ρ_z=0.95	σ_ϵ=0.01
Price adjustment	ω=0.85			
Monetary policy	ρ_R=0.9	ϕ_π=1.1	σ_π=0.002	

Matching and the labor market. Den Haan et al. (2000) bear on evidence provided by Hall (1995) and Davis et al. (1996) to determine the steady state separation rates ρ^x and ρ^n. They employ a total separation rate $\rho = 1 - (1 - \rho^x)(1 - \rho^n)$ equal to 0.1 and an exogenous separation rate $\rho^x = 0.068$. The endogenous separation rate therefore amounts to $\rho^n = 0.0343$. Walsh (2005) adopts this choice. In the matching function, he sets χ equal to 0.4 in accordance with empirical estimates by Blanchard and Diamond (1989). Furthermore, he chooses the steady state values of the matching probabilities as $\kappa^f = 0.7$ and κ^w=0.6, and assumes (as in den Haan et al. (2000)) that workers and firms split the surplus evenly implying $\eta = 0.5$.

Production and capital adjustment. In addition to Walsh (2005), we introduce capital into production. The capital elasticity of output is set equal to $\alpha = 0.36$. Capital depreciates at the rate $\delta = 0.025$. Following Christiano et al. (2005), we set $\sigma_\iota = 0.01$, but we will also consider the case of a constant utilization rate with $u_t \equiv 1.0$. As empirical estimates of the adjustment-cost elasticity vary considerably, we consider a wide range of values for $\sigma_\Phi \in \{1/15, 0.5, 1.3\}$. In our benchmark case, we choose $\sigma_\Phi = 0.5$. In our sensitivity analysis, we apply $\sigma_\Phi = 1/15$ (in accordance with Baxter and Crucini (1993)) and estimate $\sigma_\Phi = 1.3$ in Section 4.

The log of the aggregate technology shock follows an AR(1) process, $\log Z_t = \rho_z \log Z_{t-1} + \epsilon_t$, with autoregressive parameter $\rho_z = 0.95$ and standard deviation $\sigma_\epsilon = 0.01$ as in Walsh (2005). In the wholesale sector, the demand elasticity is equal to $\theta = 11$ implying an average mark-up equal to 10%.

Price rigidity. We set the probability ω that a firm is not allowed to change its price optimally in a given period equal to 0.85. Walsh (2005) uses the same value that implies the average time between price adjustment of 6.5 quarters. Alternatively, we will also consider a more frequent price adjustment $\omega = 0.5$ in our sensitivity analysis.

Monetary policy. The parameters of the monetary policy rules applied by Walsh (2005) reflect a high degree of inertia in the interest rate, $\rho_R = 0.9$, and a long-run response of the interest rate to the inflation rate by 1.10 implying $\phi_\pi = 1.10$. The monetary policy shock displays a standard deviation $\sigma_\phi = 0.002$.

3.3 Computation

We use a log-linear approximation of the model around the steady state of its deterministic version in order to compute the dynamics. The derivation of the steady state and the log-linearized version of the model are provided in Appendix B. For the numerical solution, we use the techniques proposed by King and Watson (2002). It relies upon the Schur factorization of the matrix that is describing the autoregressive part of the dynamic system.[8]

[8] See Section 2.3 in Heer and Maußner (2009) for a detailed description.

4 Summary statistics

In the next three sections, we present our results on the dynamics of output and inflation in the labor market search model with capital. We begin with the description of the summary statistics from simulations of the model.

Table 2 presents the standard deviations of a few key variables relative to the standard deviation of output shown in the first row below the table head. Column I displays the empirical values taken from Table 2 of Walsh (2005). They are based on HP-filtered US data from 1959.i through 1995.iv.[9]

Table 2:
Standard deviations

Variable	Standard deviations of key variables: σ_x/σ_Y					
	US data	I	II	III	IV	V
Output	σ_Y=1.60	σ_Y=1.25	σ_Y=0.89	σ_Y=1.24	σ_Y=4.21	σ_Y=2.11
Employment	0.62	0.90	1.69	0.92	0.53	0.51
Job creation rate	2.89	7.84	17.13	7.99	4.61	3.63
Job destruction rate	4.26	13.69	34.56	14.06	8.85	3.94
Inflation	0.35	0.47	0.77	0.48	0.11	1.04

Notes:
Standard deviations as averages from 300 simulations. The length of the time series in each simulation is 300. All simulated data are HP-filtered with weight $\lambda = 1,600$. The same set of random numbers was employed to compute the moments in columns I-V.
I: Walsh's model (our solution, see Appendix A).
II: Our model with fixed capital and fixed utilization rate: $\sigma_\Phi = \sigma_\iota = 10,000$.
III: II but with $\alpha = 0.01$.
IV: Benchmark calibration.
V: Our model with parameters from Table 3.

The numbers in columns I-V are averages from 300 simulations. The simulated time series are of length 300 and HP-filtered with weight $\lambda = 1,600$. The results from Walsh's original model are shown in column I.[10] The model underpredicts the standard deviation of US

[9] See Cooley and Quadrini (1999), Table 4, for the source of Walsh's data.
[10] We explain in Appendix A why the numbers differ from those in Walsh's Table 2.

output and it implies rates of job creation and job destruction that are much larger than those found in US labor market data. The match between the actual relative standard deviation of inflation and the one predicted by the model is much closer.

Column II displays the results from our model for the case of a constant rate of capital utilization and a constant stock of capital.[11] We find a smaller variance of output and a higher variance of employment than the original model. This reflects the smaller elasticity of output with respect to employment ($1 - \alpha$ versus 1): Setting α to a very small value provides the results displayed in column III, which are very close to those of column I.

Column IV presents summary statistics from our benchmark calibration, where both the rate of investment and the rate of capacity utilization are flexible. It is immediately obvious from the high standard deviation of output of 4.21 that the introduction of capital leads to a further deterioration of the model's fit.

The last column of Table 2 reports the results of a simulation where we have chosen the parameters of the model for which direct evidence is hard to come by so that the sum of squared differences between the empirical moments and those implied by the model is minimized. These parameters are the habit parameter h, the share of firms that are not allowed to post their optimal price ω, the elasticities σ_Φ and σ_ι which determine the flexibility of capital, the standard deviation of the idiosyncratic productivity shock σ_a, and the share of workers in the surplus of a successful match η. We placed a coarse grid over the six dimensional cube within which the respective tuple of parameters must lie and chose the minimizer from this set. Table 3 presents the outcome from this exercise.

The model that best fits the output, labor market, and inflation statistics is a model without nominal rigidities (prices are not preset and $\omega = 0.01$), without a consumption habit $h = 0.01$, a constant rate of capital utilization ($\sigma_\iota = 3000$), and flexible capital adjustment $\sigma_\Phi = 1.3$. But note, in as much as we get the labor market statistics closer to their empirical counterparts the volatility of employment decreases and the volatility of inflation becomes much larger than observed empirically.

[11] Technically, we implement these assumptions by setting both the elasticity of the function $\Phi(I_t/K_t)$ and the elasticity of the function $\iota(u_t)$ at prohibitively high values.

Table 3:
Estimated parameters from the optimal adjustment of second moments

Preferences	h=0.01	
Labor market	η=0.67	
Production	σ_Φ=1.3	σ_ι=3,000
	σ_a=0.063	
Price adjustment	ω=0.01	

5 Interest rate shock

In this section we consider the impulse responses of our model with respect to an interest rate shock. Before we report and discuss the results of our benchmark calibration, we consider the case of a fixed capital stock. Here we mainly replicate the findings of Walsh (2005) (his Figure 1). Nevertheless we undertake this endeavor, since our solution differs from Walsh's one in several details as explained in Appendix A. In Sections 5.2-5.4, we study how sensitive Walsh's results are to the introduction of capital. In Section 5.5, we consider the case when the model parameters are chosen optimally in order to approximate the impulse responses from our empirical vector autoregressive model.

5.1 Fixed capital

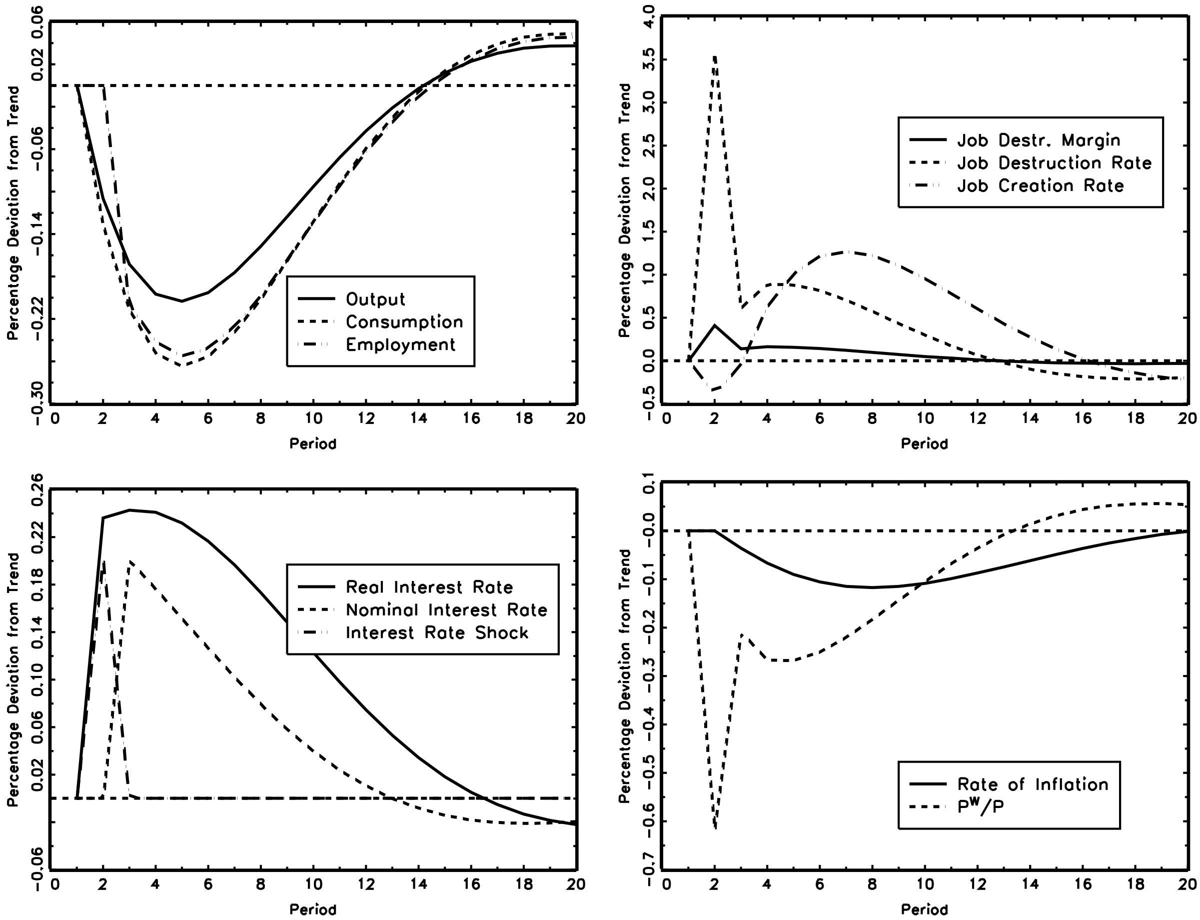
Figure 2 plots the impulse responses of the model's variables following an unexpected rise of the interest rate by one standard deviation (equal to 0.2 percentage points) in quarter $t = 2$. They rest on Walsh's parameter choice, in particular, we employ $\{\omega, \rho_R, h\} = \{0.85, 0.9, 0.78\}$, and assume prohibitively high costs of both capacity utilization ($\sigma_\iota = 10,000$) and capital stock adjustment ($\sigma_\Phi = 10,000$).[12] Therefore, capital input remains constant.

Following an increase of the nominal interest rate by 0.2 percentage points, output falls by 0.2% and displays a hump-shaped response, while inflation inertia is pronounced and

[12]Our impulse response functions are smaller by the factor 5 as we consider a shock of one standard deviation rather than one percentage point.

inflation attains its minimum value at 0.12 percentage points below its steady state value after six quarters.

Figure 2: Effects of a negative interest rate shock, preset prices, constant capital



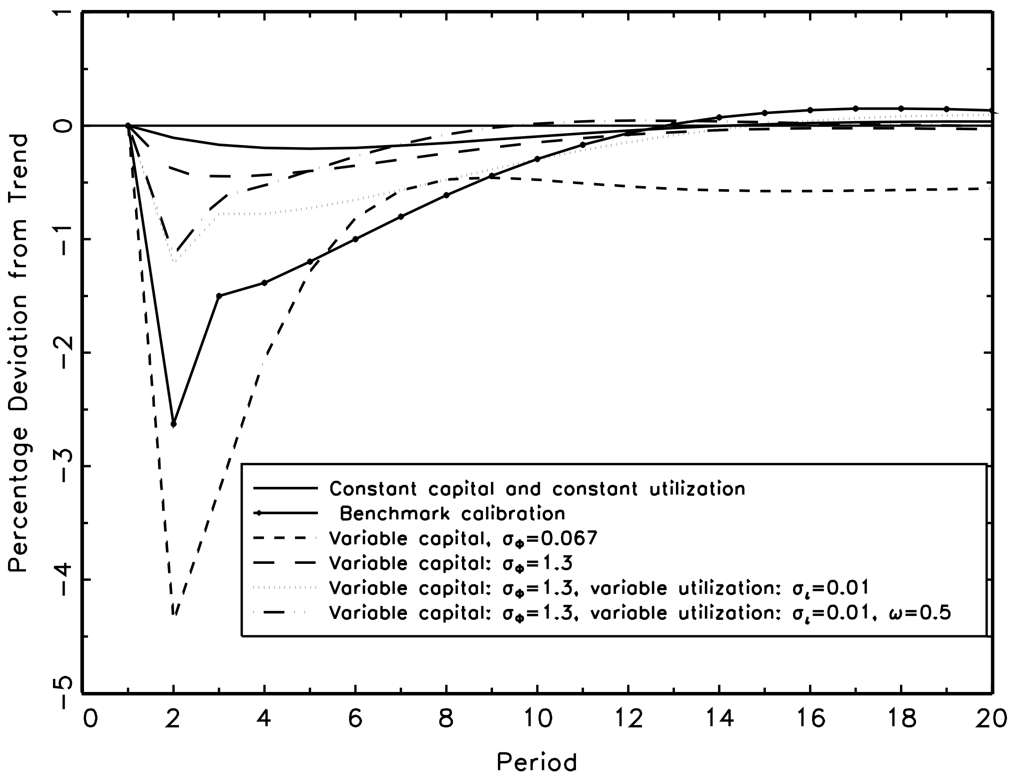
In the presence of sticky prices, a rise in the nominal interest rate R_{t+1} on bonds results in a rise of the real interest rate on bonds, R_{t+1}/π_{t+1}. As a consequence, households want to shift consumption into the future so that demand and, hence, output declines. Since intermediate sector prices are flexible while wholesale prices are sticky, the relative price of the intermediate sector output g_t deteriorates (see the line labeled P^W/P in Figure 2) raising the job destruction margin. The rate of job destruction jumps upward (see the upper right panel of Figure 2) and since firms post less vacancies the rate of job creation temporarily falls. The economy recovers only slowly from this shock: the habit in consumption prevents a fast increase in demand, and the high degree of inertia in the

Taylor rule (32) entails high interest rates for several future quarters. As a consequence, the response of output is hump-shaped and peaks after four quarters.[13]

In the following, we depart from this model and study the sensitivity of these results with regard to the introduction of capital. In addition, we will consider the role of sticky prices, preset prices, and the inertia of the central bank policy.

5.2 Variable investment and capacity utilization

Figure 3: Effects of a negative interest rate shock on output



In Figure 3, we graph the effects of variable capital on the dynamics of output. If not noted otherwise, the graphs rest on the parameter settings presented in Table 1. The solid line displays the response of output to an interest rate shock in period $t = 2$, if capital and its utilization rate remain constant ($\sigma_\Phi = 10,000$ and $\sigma_\iota = 10,000$). The solid line with

[13]If we set ρ_R equal to zero, the maximum absolute response of output already takes place in the first period of the shock.

dots shows the response for our benchmark settings of $\sigma_\Phi = 0.5$ and $\sigma_\iota = 0.01$. Notice that if we introduce very elastic capital adjustment costs ($\sigma_\Phi = 0.067$) together with a fixed utilization rate ($\sigma_\iota = 10,000$), investment demand falls significantly in response to a rise in the interest rate of only 0.2 percentage points. In this case, we observe an output response that is much larger than observed empirically. According to our estimates reported in Appendix D, the maximum response of output due to a one-percent shock in the federal funds rate equation implies a maximum decline of output in the seventh quarter after the shock of about 0.36 percentage points. The model predicts that after a 0.02 percentage point increase of the nominal interest rate output falls with a maximum deviation of 4.3 percentage points occurring in the first quarter (see the line with short dashes in Figure 3).[14] It requires an elasticity of $\sigma_\Phi = 1.3$, which is much larger than the values considered so far in business cycle models,[15] to bring the impulse response of output close to the response of the Walsh (2005) model (compare the dashed line with the solid line in Figure 3). If capacity utilization is also variable ($\sigma_\iota = 0.01$), capital services are reduced by a fall in the utilization rate u_t rather than by a decrease of the capital stock K_t. This again makes output more responsive to an interest rate shock, albeit to a much lesser extent than in the case of a small σ_Φ. Notice, however, that the response of output is not hump-shaped (the line with closely spaced dots in Figure 3). The maximum impact of the interest rate shock on output occurs in the first period. Finally, less rigid prices ($\omega = 0.5$) slightly dampen the negative effect of interest rates shock (see the line with dots and dashes).

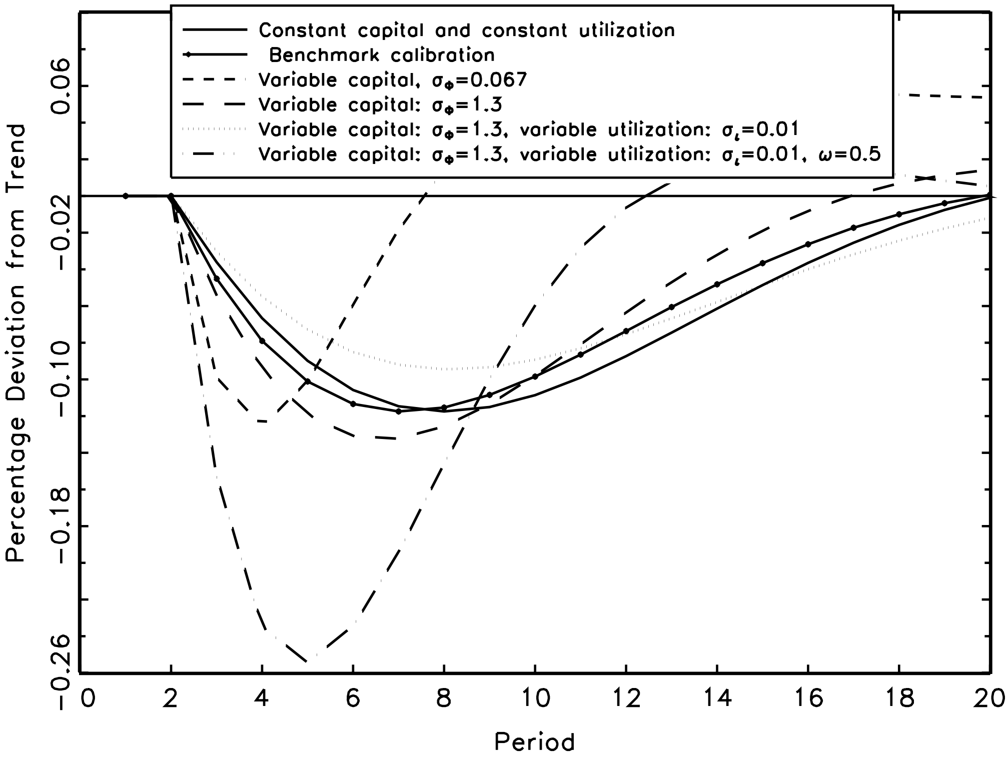
Figure 4 plots the dynamics of inflation in response to a rise of the nominal interest rate by one standard deviation. The picture is similar to the one of output in Figure 3. First note, that in our model with flexible capital and flexible utilization of capital ($\sigma_\Phi = 0.5$ and $\sigma_\iota = 0,01$) the response of output (the solid line with dots) is relatively close to the inflation response of the model with fixed capital and constant utilization rate (the solid line). If capital adjustment is easy and the rate of utilization is fixed ($\sigma_\Phi = 0.067$ and $\sigma_\iota = 10,000$) the rate of inflation drops and its deviation from the steady state attains a maximum after three quarters (see the line with short dashes).[16] With fixed capital and

[14]In Appendix D we compare the impulse response of our model for the benchmark calibration in Table 1 to empirical estimates and show that the model widely overpredicts not only the response of output but also the response of consumption, investment, inflation, and capacity utilization.

[15]For instance, Correia et al. (1995) use $\sigma_\Phi = 1/30$ and Baxter and Cruccni (1993) consider σ_Φ not larger than 1 and use $\sigma_\Phi = 1/15$ as a benchmark value.

[16]If prices were not preset, the biggest impact would be in the very first period following the shock.

Figure 4: Effects of a negative interest rate shock on inflation



fixed utilization it takes four more quarters before inflation starts to revert to its long-run equilibrium value. High costs of capital adjustment ($\sigma_\Phi = 1.3$) shift the impulse response (the dashed line in Figure 4) close to the case with fixed capital and fixed capital utilization (the solid line). In the case of variable capacity utilization, investment demand changes little and all the adjustment takes place by using the existing capital stock less intensively. As a consequence, the change in marginal costs is smaller and smoother (see the dotted line in Figure 4).[17] If firms can adjust their prices on average every second quarter ($\omega = 0.5$), a sharp decline in price inflation (shown by the line with dots and dashes) takes the burden of adjustment.

[17]This has been pointed out by Christiano et al. (2005) who write on p. 3 "..variable capital utilization helps dampen the large rise in the rental rate of capital that would otherwise occur. This in turn dampens the rise in marginal costs and, hence, prices."

5.3 Preset prices and habit persistence

In this paragraph we study the sensitivity of our results by changing only one of the parameters from our benchmark calibration with variable capacity utilization ($\sigma_\iota = 0.01$) and capital adjustment costs ($\sigma_\Phi = 0.5$) as presented in Table 1 at a time. To save space, we do not present the respective graphs.[18] The assumption that prices are set prior to the observation of the interest rate shock is rather innocuous. If firms can adjust their price after the realization of the shock so that the New-Keynesian Phillips curve is presented by equation (8) rather than by (7) the impulse response of output does not differ noteworthy from the benchmark case, whereas the effect on inflation is more immediate and more pronounced in the first six quarters after the impact of the shock.

As can be expected, the effect on output is more pronounced if past consumption plays a minor role in the household's utility function. If we reduce h from our benchmark value of 0.78 to 0.5, output drops on impact about 3.3 percent below its stationary value as compared to 2.6 percent in the benchmark calibration. The persistence of the inflation response, however, does not depend on the degree of habit persistence h. Even without habit persistence, i.e. if we set h close to zero, inflation is still persistent, while the output response is further increased.

5.4 Monetary policy

As his main result, Walsh (2005) shows that policy inertia is the most important factor in accounting for the hump-shaped response of output and the persistent response of inflation. As we already showed above, in the presence of capital, output does not display a hump-shaped response any more. However, we are able to confirm his second result for the economy with capital as soon as we assume capacity utilization to be variable. If the autoregressive parameter of the Taylor rule with respect to the interest rate is reduced from $\rho_R = 0.9$ to $\rho_R = 0$, the impulse response of inflation is flat (not illustrated).

[18]The interested reader can download our programs and validate the results. The link is `http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/pap/hm_lms_gauss.zip`.

5.5 Matching empirical impulse responses

As we have demonstrated in the previous sections, the properties of the model with capital accumulation depart considerable from those of the model with fixed capital. So far we have taken for granted that the latter model provides for a good approximation of the data. The question that we will take up in this paragraph is how close can our model approximate impulse responses from a structural vector autoregressive model if we select the critical parameters h, ω, σ_Φ, and σ_ι so as to minimize the distance between the impulse responses to an interest rate shock implied by the model and those identified empirically. In Appendix D we explain our estimation procedure. Our structural vector autoregressive model (VAR) is similar to the model of Christiano et al. (2005). We exclude from their model the variables about which our model remains silent (the real wage, labor productivity, the growth rate of M2, real profits) and add to it the utilization rate of capital.

Table 4 displays the parameter values for which we obtain the best fit of the model. Both ω and h are at the upper bound of the grid that we have placed over the four dimensional set of possible values for the quadruple $(\omega, h, \sigma_\Phi, \sigma_\iota)$. The high value selected for σ_ι implies an almost constant utilization rate of capital. The also sizeable value for σ_Φ implies a small variance of investment.

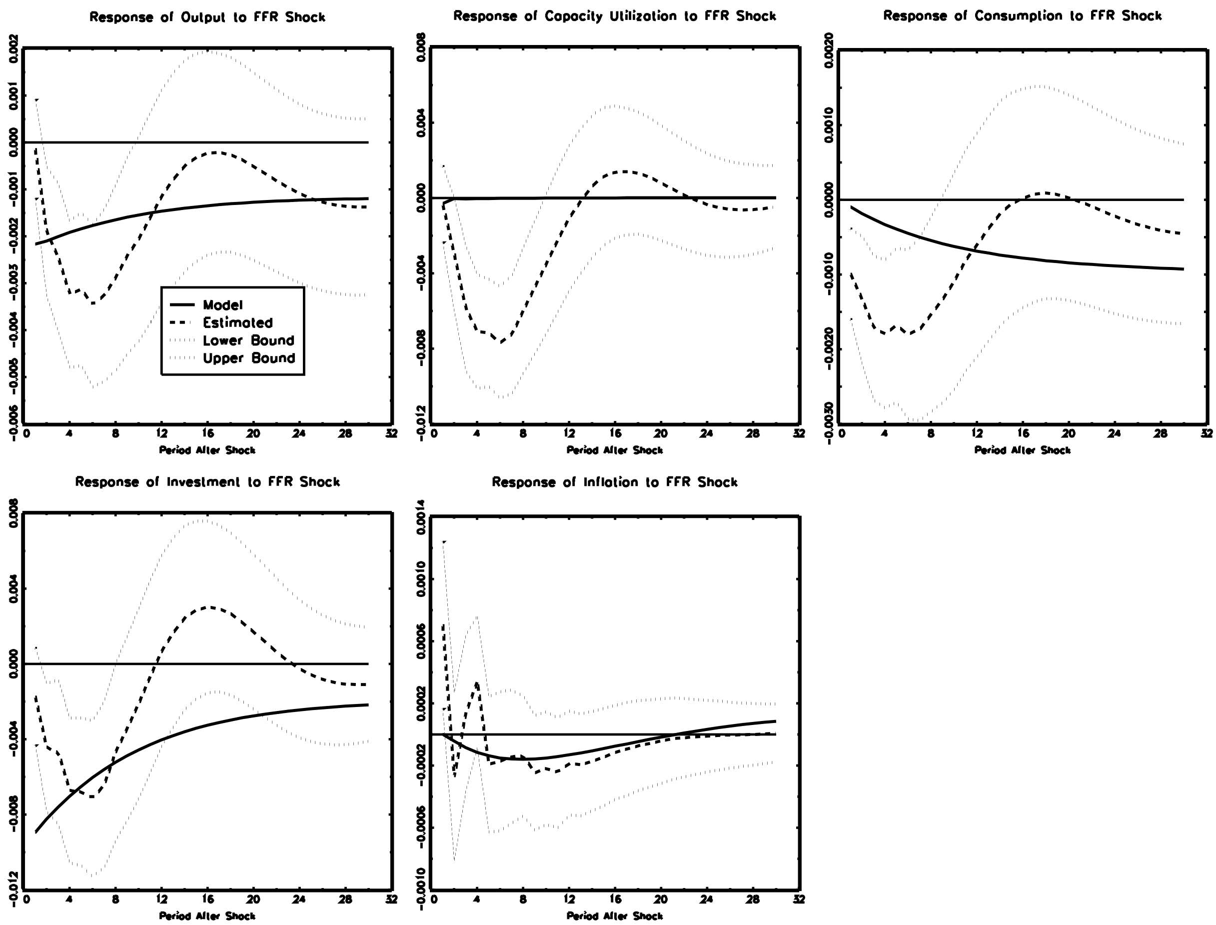
Table 4:

Estimated parameters from optimal adjustment of impulse responses

Preferences	h=0.99	
Production	σ_Φ=17.2	σ_ι=60.2
Price adjustment	ω=0.95	

The impulse responses for this model (solid line) are compared with those estimated by the VAR (broken line) in Figure 5. The dotted lines are error bounds computed from adding (subtracting) twice the estimated standard deviation to the respective impulse response. Even for the best choice of the free parameters the model does a bad job in mimicking the estimated responses. The rate of capacity utilization and consumption are even outside the estimated error bounds for the first several quarters following the arrival of the shock.

Figure 5: Estimated and model implied impulse responses



6 Technology shock

In the previous section, we found that the model with variable capacity utilization helps to explain the persistent response of inflation, even though it cannot account for the hump-shaped response of output. In this section, we analyze if this model is also able to explain the output-inflation dynamics in response to a productivity shock. In comparing the model's predictions with the empirical facts we rely on well-known results from the literature, which uses long-run restrictions to identify technology shocks.[19] Note, however, that Walsh's (2005) model abstracts from stochastic growth so that its impulse responses

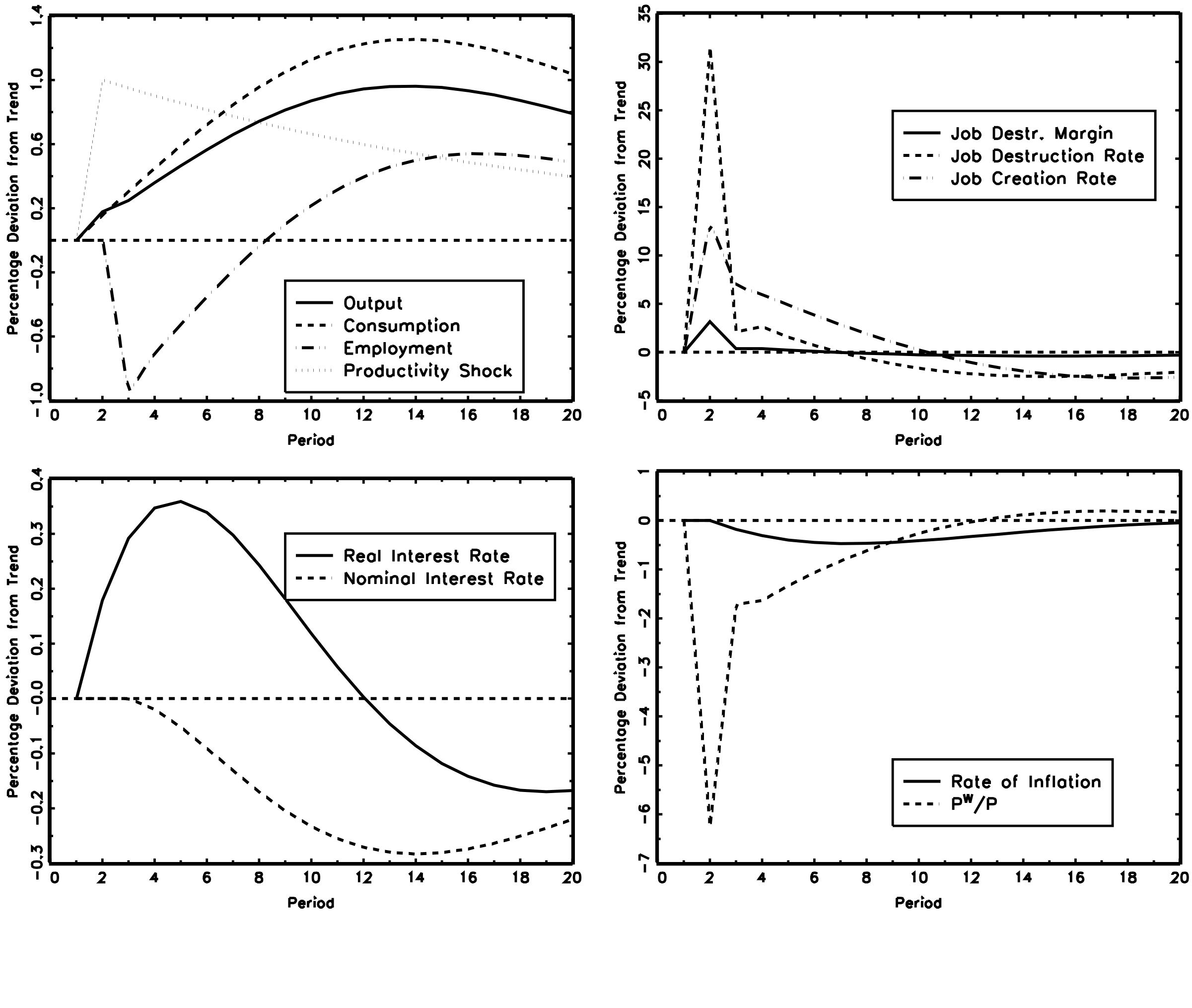
[19]See, among, others, Shapiro and Watson (1988), Galí (1999), Francis and Ramey (2002), and Altig et al. (2005).

cannot be quantitatively compared to those estimated from a structural vector autoregressive model that embeds long-run restrictions. We explain this in Appendix C where we show how to reformulate the model to exhibit exogenous stochastic growth.

6.1 Fixed capital

Figure 6 shows the impulse responses of key variables to a one-time productivity shock in period $t = 2$ of size $\sigma_Z = 0.01$ for the model with fixed capital services (i.e. $\sigma_\Phi = 10,000$ and $\sigma_\iota = 10,000$). All other parameters are calibrated as in Table 1, and prices are preset.

Figure 6: Effects of a technology shock



The responses of output and employment are consistent with the evidence provided by Galí (1999) and Francis and Ramey (2002) who show that a supply shock (the dotted line in Figure 6) raises output but depresses employment in the first few quarters.[20] To understand the mechanism behind this result in our model consider again the relative price of intermediate goods g_t (the line P^W/P in the lower right panel of Figure 6). On impact, the increased productivity entails a lower nominal price of intermediate goods. Since wholesale prices are fixed in the impact period, the relative price of intermediate products falls and counteracts the outward shift of the production function. Thus, the job destruction margin increases and more employment relationships separate endogenously. As soon as prices adjust (see the spikes in the job creation and job destruction rates, and the relative price of intermediate products, P^W/P in Figure 6) the positive effect of the technology shock begins to predominate. Note also that there is a protracted hump-shaped decline of the inflation rate, which is in accordance with the persistent negative impact on inflation found empirically by Galí (1999).

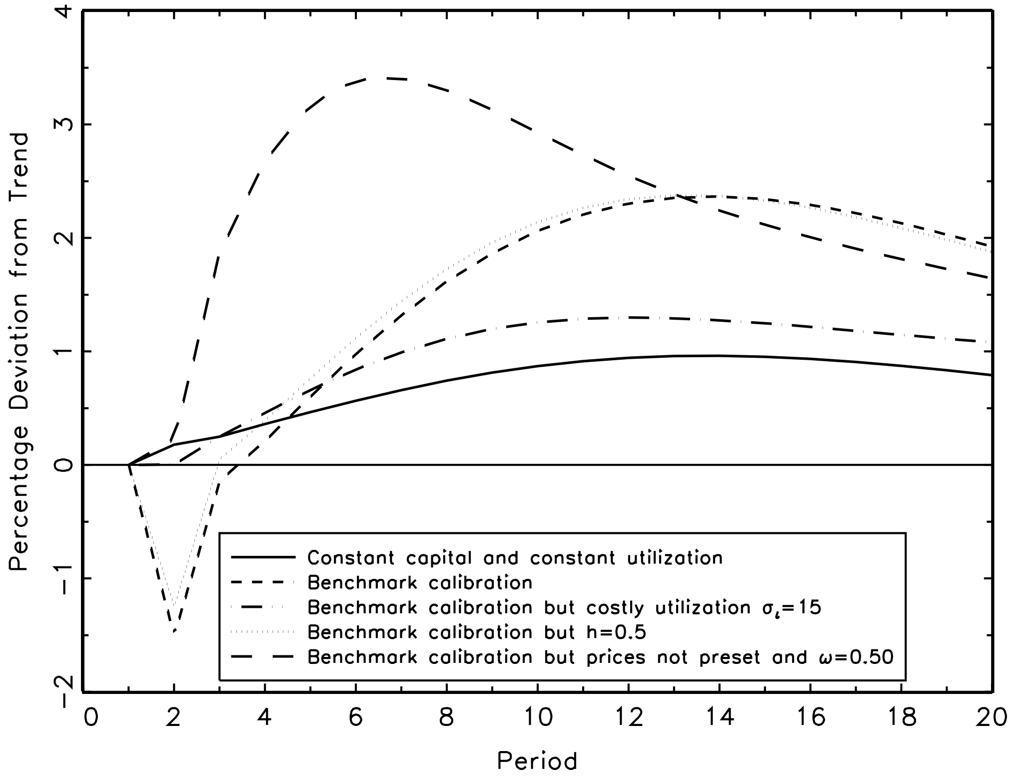
6.2 Variable investment and capacity utilization

As in the case of an interest rate shock the dynamics of output and employment is sensitive with regard to the assumption of fixed capital services. Figures 7 and 8 display the impulse responses of output and employment for different values of the key parameters.

In the case of the benchmark calibration the drop in the relative price of intermediate goods triggers a strong reaction in both the rate of capacity utilization and the endogenous rate of job separations that caused output to decline. A key variable for the size of layoffs is the elasticity of the cumulative distribution $F(\underline{a})$ with respect to $\underline{a}$ which depends in turn on the standard deviation of the log-normal distribution of $f(a)$. For instance, if we reduce σ_a from the benchmark value of 0.13 to $\sigma_a = 0.065$ the decline in output is cut in half. For the given value of $\sigma_a = 0.13$ we must increase the costs of capacity utilization to a value of $\sigma_\iota = 15$ to prevent the decline in output. In addition, we find that consumption habit h has a less important effect on output and employment. A smaller habit in consumption mitigates the negative effect (see the dotted line in Figure 7), yet even if we set h close to zero, output drops in the first period of the shock (not shown in the figure). The role of

[20]The empirical evidence on the effects of technology shocks on employment depends crucially on the question whether hours per worker are stationary or not. In the latter case Christiano et al. (2003, 2004) demonstrate that hours increase after a technology shock.

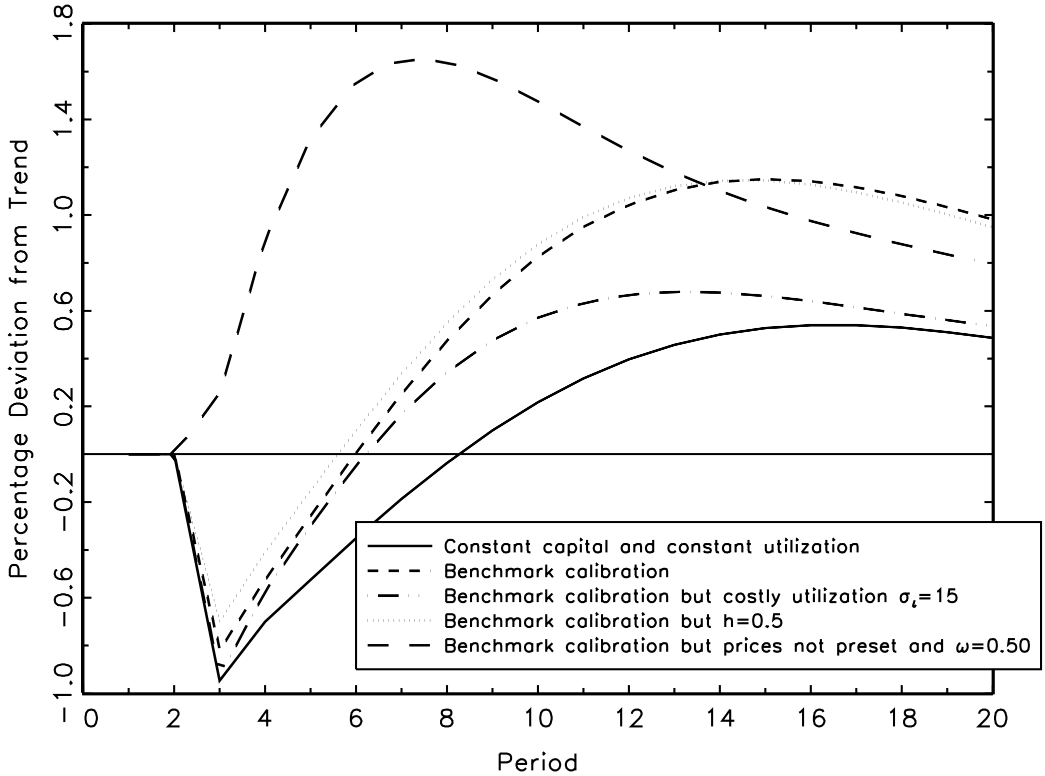
Figure 7: Effects of a technology shock on output



the degree of price flexibility is exemplified by the impulse response with the long dashes in Figure 7. If prices can be changed every second quarter and are set after the technology shock has been observed, the response of output will be strongly positive and hump-shaped, though.

Figure 8 corroborates the finding that the negative effects on output originate in the flexible use of capital services. With fixed capital, employment alone bears the burden of adjustment. The more flexible capital services are, the smaller is the fall of employment (compare the solid line with the dashed line and the line with dots and dashes in Figure 8). However, it requires a substantial amount of price flexibility for employment to increase immediately after a technology shock (see the line with long dashes in Figure 8). Using the parameter values from Table 1 together with $\omega = 0.5$ – so that firms can adjust their prices on average every second quarter – we also need to assume that the firms which are receiving the signal to change their price can do so immediately after the realization of the technology shock.

Figure 8: Effects of a technology shock on employment



In summary, a sharp decrease of output in response to a technology shock is at odds with the empirical findings provided by Galí (1999) and Francis and Ramey (2002). In the present model this puzzle can only be resolved if either the marginal cost function $\iota'(u_t)$ is very elastic or the degree of price rigidity is only modest.

7 Conclusion

In this paper we have studied the inflation and output dynamics in the labor market search model with capital. In the presence of capital adjustment costs, variable capacity utilization helps to reconcile the model's inflation response to a rise in the nominal interest rate with the one that is observed empirically. However, in this case, the magnitude of the output response is much stronger than observed in the US economy and, in particular, the output response is no longer hump-shaped. Therefore, we conclude that the output-inflation dynamics of the labor market search model in response to an interest-rate shock is sensitive with regard to the introduction of capital. This conclusion also applies to the case when we consider the consequences of a technology shock. Contrary to empirical findings, an unexpected productivity increase causes an initial decline in output if capital services are sufficiently flexible and prices are rigid.

Comparing our results with those from the studies of Christiano et al. (2005) and Christoffel et al. (2009), we think that the introduction of wage setting into our model will be a promising direction for future research. In the model of Christiano et al. (2005), there is no search unemployment. Wage rigidity is modeled as in Erceg et al. (2000) such that workers can reset their prices optimally as in the Calvo price-setting model. Therefore, some business-cycle observations from the labor market such as more variation of employment at the extensive rather than the intensive margin cannot be replicated. In Christoffel et al. (2009) this shortcoming is overcome by the introduction of matching frictions and wage bargaining. In their model wages play an allocational role since firms decide about working hours after the wage rate is agreed upon. Wage rigidity arises in this context if not all wage contracts are renegotiated in each period. It dampens the effects of monetary policy on the time path of marginal costs. However, capital is missing as an input factor into production. In conclusion, incorporating the wage bargaining into the present model should allow for a more encompassing and robust study of both business cycle features from the labor market and the dynamics of inflation and output.

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8 Appendix A: The model of Walsh

In this appendix we present the model of Walsh (2005), derive its stationary solution and log-linear approximation which underlies our solution and simulation, and compare our Gauss program to Walsh's Matlab code.

Equilibrium Conditions. The model of Walsh (2005) departs from our model with respect to the production function of the intermediate goods sector presented in equation (9). In his model, labor is the single factor of production. Since each firm i is matched with one worker output is given by

$$y_{it} = a_{it} Z_t. \tag{A.1}$$

Aggregating over all active firms i yields the aggregate production function

$$Y_t = (1-\rho^x) Z_t N_t G(\underline{a}_t), \quad G(\underline{a}_t) := \int_{\underline{a}_t}^{\infty} a f(a) da. \tag{A.2}$$

The job destruction margin $\underline{a}_t$ is determined by

$$l + b = q_t + g_t Z_t \underline{a}_t, \quad q_t := v_t - w_t^u + b, \tag{A.3}$$

which derives from equation (12), if we neglect capital and use (A.1) instead of (9). The surplus of an operating firm can thus be written as

$$s_t = g_t Z_t (a_t - \underline{a}_t). \tag{A.4}$$

This allows us to simplify equation (23) to

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (1-\rho^x)(1-\eta\kappa_t^w) g_{t+1} Z_{t+1} \left[G(\underline{a}_{t+1}) - (1-\rho_{t+1}^n)\underline{a}_{t+1} \right] \right\}. \tag{A.5}$$

The resource constraint (36) simplifies to

$$Y_t = C_t + \gamma V_t. \tag{A.6}$$

The 10 static equilibrium conditions of the model, thus, consist of (14), (A.3), (A.1), (24), (18), (19), (15), (A.6), (25), and (26). The six dynamic conditions are (31a), (31c), (16), (A.5), the interest rate rule (32) and the condition for the optimal price of firms that are allowed to adjust their prices. Since our solution relies on log-linear policy rules, we do not present this latter equation, but only its log-linear approximation (8).

For convenience, we summarize all equilibrium conditions of the model:

$$\rho_t^n = F(\underline{a}_t), F(\underline{a}_t) := \int_0^{\underline{a}_t} f(a) da, \tag{A.7a}$$

$$
\begin{aligned}
l + b &= q_t + g_t Z_t \underline{a}_t, && \text{(A.7b)}\\
Y_t &= (1-\rho^x) Z_t N_t G(\underline{a}_t), && \text{(A.7c)}\\
q_t &= \frac{\gamma(1-\eta\kappa_t^w)}{(1-\eta)\kappa_t^f}, && \text{(A.7d)}\\
\kappa_t^f &= \psi(V_t/U_t)^{-\chi}, && \text{(A.7e)}\\
\kappa_t^w &= \psi(V_t/U_t)^{1-\chi}, && \text{(A.7f)}\\
U_t &= 1-(1-\rho^x)(1-\rho_t^n)N_t, && \text{(A.7g)}\\
Y_t &= C_t + \gamma V_t, && \text{(A.7h)}\\
jdr_t &= \rho_t - \kappa_t^f \rho^x = (1-\rho^x)\rho_t^n + \rho^x(1-\kappa_t^f), && \text{(A.7i)}\\
jcr_t &= \frac{\kappa_t^f (V_t - \rho^x N_t)}{N_t}, && \text{(A.7j)}\\
\lambda_t &= \beta(C_t - hC_{t-1})^{-\sigma} - \beta E_t (C_{t+1} - hC_t)^{-\sigma}, && \text{(A.7k)}\\
\lambda_t &= \beta E_t \left\{ \frac{\lambda_{t+1} R_{t+1}}{\pi_{t+1}} \right\}, && \text{(A.7l)}\\
q_t &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (1-\rho^x)(1-\eta\kappa_t^w) g_{t+1} Z_{t+1} \left(G(\underline{a}_{t+1}) - (1-\rho_{t+1}^n)\underline{a}_{t+1} \right) \right\}, && \text{(A.7m)}\\
N_{t+1} &= (1-\rho^x)(1-\rho_t^n)N_t + \kappa_t^w U_t, && \text{(A.7n)}\\
R_{t+1} &= \bar{\pi}^{(1-\rho_R)(1-\phi_\pi)} \beta^{-(1-\rho_R)} R_t^{\rho_R} \pi_t^{\phi_\pi(1-\rho_R)} e^{\phi_t}. && \text{(A.7o)}
\end{aligned}
$$

The steady state. Consider the model without a technology and an interest rate shock so that $Z_t = 1$ and $\phi_t = 0$ for all t. The steady state of this deterministic model can be derived from (A.7) by dropping all time indices and by solving the ensuing equations for the variables of interest.

Given ρ^n and σ_a we can solve

$$
\rho^n = \int_0^{\underline{a}} f(a)da = \int_0^{\underline{a}} \frac{1}{a\sigma_a\sqrt{2\pi}} e^{-0.5(\ln a/\sigma_a)^2} da,
$$

for $\underline{a}$ as explained in the body of the paper. This allows us to compute

$$
G(\underline{a}) = \int_{\underline{a}}^{\infty} af(a)da = \int_{\underline{a}}^{\infty} \frac{1}{\sigma_a\sqrt{2\pi}} e^{-0.5(\ln a/\sigma_a)^2} da,
$$

numerically.

In the steady state the relative price of the production sector equals $g = (\theta-1)/\theta$. From the steady state version of equation (A.7k) we are thus able to infer q:

$$
q = \beta(1-\rho^x)(1-\eta\kappa^w)\frac{\theta-1}{\theta}\left(G(\underline{a}) - (1-\rho^n)\underline{a}\right). \qquad \text{(A.8)}
$$

In the steady state equations (A.7e) and (A.7f) imply

$$\frac{V}{U} = \frac{\kappa^w}{\kappa^f}, \tag{A.9}$$

and equation (A.7n) reduces to

$$\frac{U}{N} = \frac{1 - (1 - \rho^x)(1 - \rho^n)}{\kappa^w}. \tag{A.10}$$

Note that these equations imply that the steady state rate of job destruction equals the steady state rate of job creation:

$$jcr = \rho - \kappa^f \rho^x = \frac{\kappa^f V}{N} - \rho^x \kappa^f = jdr.$$

Given (A.9) and (A.10), we can solve equation (A.7g) for N:

$$N = \left[\frac{U}{N} + (1 - \rho^x)(1 - \rho^n)\right]^{-1}. \tag{A.11}$$

Thus, the steady state output is given by

$$Y = (1 - \rho^x) N G(\underline{a}).$$

The solution for N allows us to determine $U = (U/N)N$ and $V = (V/U)U$. Given q, we infer γ from equation (A.7d):

$$\gamma = \frac{q(1 - \eta)\kappa^f}{1 - \eta\kappa^w}. \tag{A.12}$$

The budget constraint (A.7h), thus, implies

$$C = Y - \gamma V.$$

The log-linearized system. The system of equations (A.7), log-linearized at the stationary solution, consists of two parts. The first part involves only period t dated variables, whereas the second part determines the model's dynamics.

$$\hat{\rho}_t^n - \varepsilon_{F,a} \hat{\underline{a}}_t = 0, \tag{A.13a}$$

$$\hat{\underline{a}}_t + \hat{g}_t = -\frac{q}{g\underline{a}} \hat{q}_t - \hat{Z}_t, \tag{A.13b}$$

$$\hat{Y}_t - \varepsilon_{G,a} \hat{\underline{a}}_t = \hat{N}_t + \hat{Z}_t, \tag{A.13c}$$

$$\hat{\kappa}_t^f + \frac{\eta\kappa^w}{1 - \eta\kappa^w} \hat{\kappa}_t^w = -\hat{q}_t, \tag{A.13d}$$

$$\hat{\kappa}_t^f + \chi \hat{V}_t - \chi \hat{U}_t = 0, \tag{A.13e}$$

$$\hat{\kappa}_t^w + (\chi - 1)\hat{V}_t + (1-\chi)\hat{U}_t = 0, \tag{A.13f}$$

$$(U/N)\hat{U}_t - (1-\rho^x)\rho^n\hat{\rho}_t^n = -(1-\rho^x)(1-\rho^n)\hat{N}_t, \tag{A.13g}$$

$$\hat{Y}_t - (\gamma V/Y)\hat{V}_t = (C/Y)\hat{C}_t, \tag{A.13h}$$

$$\widehat{jdr}_t - \frac{(1-\rho^x)\rho^n}{jdr}\hat{\rho}_t^n + \frac{\kappa^f\rho^x}{jdr}\hat{\kappa}_t^f = 0, \tag{A.13i}$$

$$\widehat{jcr}_t - \hat{\kappa}_t^f - \frac{V}{V-\rho^x N}\hat{V}_t = -\frac{V}{V-\rho^x N}\hat{N}_t. \tag{A.13j}$$

Log-linearizing equations (A.7k) through (A.7o) yields:

$$\begin{aligned}
&\beta h\Gamma_1 E_t\hat{C}_{t+1} - (1+\beta h^2)\Gamma_1\hat{C}_t && \qquad\text{(A.14a)}\\
&+ h\Gamma_1\hat{C}_{t-1} - \hat{\lambda}_t &= 0, &\\
&E_t\hat{\lambda}_{t+1} + \hat{R}_{t+1} - E_t\hat{\pi}_{t+1} - \hat{\lambda}_t &= 0, & \qquad\text{(A.14b)}\\
&E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{q}_t &= -\hat{g}_{t+1} - \hat{Z}_{t+1} - \Gamma_2 E_t\underline{\hat{a}}_{t+1} - \Gamma_3 E_t\hat{\rho}_{t+1}^n & \qquad\text{(A.14c)}\\
& &+ \frac{\eta\kappa^w}{1-\eta\kappa^w}\hat{\kappa}_t^w, &\\
&\hat{N}_{t+1} - (1-\rho^x)(1-\rho^n)\hat{N}_t &= \kappa^w(U/N)\hat{U}_t + \kappa^w(U/N)\hat{\kappa}_t^w & \qquad\text{(A.14d)}\\
& &- (1-\rho^x)\rho^n\hat{\rho}_t^n, &\\
&\hat{R}_{t+1} - \rho_R\hat{R}_t - \phi_\pi(1-\rho_R)\hat{\pi}_t &= \phi_t, & \qquad\text{(A.14e)}
\end{aligned}$$

$$\Gamma_1 := \frac{\sigma}{(1-\beta h)(1-h)}, \qquad \Gamma_2 := \Delta\left[G(\underline{a})\varepsilon_{G,a} - (1-\rho^n)\underline{a}\right],$$

$$\Gamma_3 := \Delta\rho^n\underline{a}, \qquad \Delta := \left[G(\underline{a}) - (1-\rho^n)\underline{a}\right]^{-1}.$$

The last equation of the log-linear dynamical system is supplied by the Phillips curve. This equation is given by (7) with the time index shifted one period into the future:

$$\hat{\pi}_{t+1} - \frac{1}{1+\beta}\hat{\pi}_t - \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+2} = \Gamma E_t\hat{g}_{t+1}, \tag{A.14f}$$

$$\Gamma = \frac{(1-\omega)(1-\beta\omega)}{(1+\beta)\omega}.$$

The system of equations (A.13) and (A.14) can be put into the canonical form of Heer and Maußner (2009):

$$C_u\mathbf{u}_t = C_{x\lambda}\begin{bmatrix}\mathbf{x}_t\\ \boldsymbol{\lambda}_t\end{bmatrix} + C_z\mathbf{z}_t, \tag{A.15a}$$

$$D_{x\lambda}E_t\begin{bmatrix}\mathbf{x}_{t+1}\\ \boldsymbol{\lambda}_{t+1}\end{bmatrix} + F_{x\lambda}\begin{bmatrix}\mathbf{x}_t\\ \boldsymbol{\lambda}_t\end{bmatrix} = D_uE_t\mathbf{u}_{t+1} + F_u\mathbf{u}_t + D_zE_t\mathbf{z}_{t+1} + F_z\mathbf{z}_t, \tag{A.15b}$$

$$\mathbf{z}_{t+1} = \Pi\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}. \tag{A.15c}$$

For this purpose we define the auxiliary variables

$$\hat{v}_t^1 := \hat{\pi}_{t+1},$$
$$\hat{v}_t^2 := \hat{C}_{t-1},$$

and the vectors

$$\mathbf{u}_t := \left[\hat{Y}_t, \hat{U}_t, \hat{V}_t, \hat{\kappa}_t^w, \hat{\kappa}_t^f, \hat{\rho}_t^n, \underline{\hat{a}}_t, \hat{g}_t\right]',$$
$$\mathbf{x}_t := \left[\hat{N}_t, \hat{R}_t, \hat{\pi}_t, \hat{v}_t^2\right]',$$
$$\boldsymbol{\lambda}_t := \left[\hat{\lambda}_t, \hat{q}_t, \hat{v}_t^1, \hat{C}_t\right]',$$
$$\mathbf{z}_t := \left[\hat{Z}_t, \phi_t\right]'.$$

Differences to Walsh's program. According to our reading of Walsh's Matlab code there are five differences between his solution and our treatment of his model as explained in the previous paragraphs.[21]

First, consider the idiosyncratic shock a (we drop the indices it for convenience) in the production function (A.1). Following den Haan et al. (2000), we assume that a is log-normally distributed with mean $\mu_a = 0$ and standard deviation σ_a. Walsh (2005), p. 834, assumes $E(a) = 1$. Since, for a log-normally distributed variable[22]

$$E(a) = e^{\mu_a + 0.5\sigma_a^2}$$

Walsh had to set $\mu_a = -0.5\sigma_a^2$. Yet, in his Matlab code he puts $\mu_a = -0.5\sigma_a$. Note, however, that this choice of μ_a has no impact on the elasticities $\varepsilon_{F,a}$ and $\varepsilon_{G,a}$ that appear in the log-linear equations. In our program we use the analytical expressions to compute both elasticities, which are given by

$$\varepsilon_{F,a} = \frac{\underline{a} f(\underline{a})}{F(\underline{a})}, \tag{A.16a}$$
$$\varepsilon_{G,a} = \frac{\underline{a}^2 f(\underline{a})}{G(\underline{a})}, \tag{A.16b}$$

whereas Walsh computes the derivative of $F(\underline{a})$ and $G(\underline{a})$ from $100(F(1.01\underline{a}) - 1)$ and $100(G(1.01\underline{a}) - 1)$, respectively, i.e., he uses the slope of the secant to approximate the derivative. Therefore, his elasticities differ from ours.[23]

Second, equations (A.9)-(A.11) show that the parameters ρ_x, ρ_n, κ^f, and κ^w imply unique values for N, V, and U. Walsh (2005), Table 1, however, sets $N = 0.94$, which differs

[21] We would like to thank Carl Walsh for providing us with this MATLAB code.

[22] See, e.g., Sydsæter, Strøm, and Berck (1999), formula 32.13.

[23] We get $\varepsilon_{F,a} = 17.04$ and $\varepsilon_{G,a} = -0.49$. Walsh's numbers are $\varepsilon_{F,a} = 18.18$ and $\varepsilon_{G,a} = -0.50$.

slightly from the value $N = 0.9375$ implied by his choice of $\rho_x = 0.068$, $\rho_n = 0.03433$, $\kappa^f = 0.7$, and $\kappa^w = 0.6$.

Third, instead of using the function $G(\underline{a})$, Walsh's log-linear model builds on the function

$$J(\underline{a}) := \frac{G(\underline{a})}{1-\rho^n} = \frac{G(\underline{a})}{1-F(\underline{a})}.$$

Therefore, his analog to our equation (A.7c) reads

$$Y_t = (1-\rho^x)(1-\rho_t^n)Z_t N_t J(\underline{a}_t). \tag{A.17}$$

Forth, using $J(\underline{a}_{t+1})$ in our equation (A.7m) yields:

$$\begin{aligned} q_t &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t}(1-\rho^x)(1-\rho_{t+1}^n)(1-\eta\kappa_t^w) g_{t+1} Z_{t+1} \big[J(\underline{a}_{t+1}) - \underline{a}_{t+1}\big] \right\}, \\ &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t}(1-\rho^x)(1-\rho_{t+1}^n)(1-\eta\kappa_t^w) \big[g_{t+1} Z_{t+1} J(\underline{a}_{t+1}) + q_{t+1} - l - b\big] \right\}, \end{aligned} \tag{A.18}$$

where the last line follows from $\underline{a}_{t+1} g_{t+1} Z_{t+1} = l + b - q_{t+1}$ (see equation (A.3)). Walsh's Euler equation – derived from his equations (7) and (8) – differs from (A.18) slightly since he defines $q_t = v_t - w_t^u$ whereas we used the definition given in (A.3) to eliminate b from (A.5). Since Walsh (2005), footnote 11, sets $b = 0$, this difference disappears.

Log-linearizing equations (A.17) and (A.18) yields:

$$\hat{Y}_t - \varepsilon_{J,a}\hat{\underline{a}}_t + \frac{\rho^n}{1-\rho^n}\hat{\rho}_t^n = \hat{N}_t + \hat{Z}_t, \tag{A.19}$$

$$\begin{aligned} E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + \xi_1 E_t \hat{q}_{t+1} - \hat{q}_t &= -\xi_2 \hat{g}_{t+1} - \xi_2 \hat{Z}_{t+1} - \xi_2 \varepsilon_{J,a} E_t \hat{\underline{a}}_{t+1} \\ &\quad + \frac{\rho^n}{1-\rho^n} E_t \hat{\rho}_{t+1}^n + \frac{\eta\kappa^w}{1-\eta\kappa^w}\hat{\kappa}_t^w, \end{aligned} \tag{A.20}$$

$$\xi_1 = \beta(1-\rho^x)(1-\rho^n)(1-\eta\kappa^w),$$

$$\xi_2 = \xi_1 \frac{gJ(\underline{a})}{q},$$

where the elasticity of the function $J(\underline{a})$ is related to the elasticity of the function $G(\underline{a})$ according to

$$\varepsilon_{J,a} = \varepsilon_{G,a} + \frac{\underline{a} f(\underline{a})}{1-F(\underline{a})}. \tag{A.21}$$

It is easy to show that this definition, the relation between $\hat{\rho}_t^n$ and $\hat{\underline{a}}_t$ given in equation (A.13a), and equation (A.13b) imply that equations (A.19) and (A.20) reduce to our equations (A.13c) and (A.14c), respectively. However, instead of using $\varepsilon_{J,a}$ in his Matlab code, Walsh uses $\varepsilon_{G,a}$.

Fifth, Walsh (2005) employs the Matlab code of Uhlig (1999) to compute the linear approximations of the model's solution and to obtain second moments. Uhlig uses a frequency domain technique, whereas we rely on simulations of the model in oder to take care of the small sample properties of both the actual time series and their artificial counterparts. To obtain Walsh's result of $\sigma_Y = 1.65$ we had to use time series with 300 data points.

Table A.1 helps to trace the source of the differences between the results of Table 2 of Walsh (2005) and our solution of Walsh's model.[24] The differences in the parameterization (though inconsistent with respect to the choice of N) and the differences in the log-linear structure induce only small differences. What makes Walsh's model to perform rather well vis-a-vis the empirical relative standard deviations is the fact that he employs the elasticity of the function $G(\underline{a})$ instead of the elasticity of $J(\underline{a})$.

Table A.1:
Standard deviations from different versions of Gauss code

Variable	Standard deviations: σ_x/σ_Y					
	US data	I	II	III	IV	V
Output	σ_Y=1.60	σ_Y=1.25	σ_Y=1.25	σ_Y=1.25	σ_Y=1.27	σ_Y=1.65
Employment	0.62	0.90	0.89	0.89	0.89	0.69
Job creation rate	2.89	7.84	8.11	8.02	7.85	3.86
Job destruction rate	4.26	13.69	13.80	13.62	13.40	4.23
Inflation	0.35	0.47	0.47	0.47	0.47	0.42

Notes:
Standard deviations are averages from 300 simulations. The length of the time series in each simulation is 300. All simulated data are HP-filtered with weight $\lambda = 1,600$. The same set of random numbers was employed to compute the figures in columns I-V.
I: our code with our parameterization.
II: our code but with Walsh's choice of μ_a, N, $\varepsilon_{F,a}$ and $\varepsilon_{G,a}$.
III: II and equation (A.13b) replaced by equation (A.19).
IV: III and equation (A.14c) replaced by equation (A.20).
V: IV but $\varepsilon_{J,a}$ replaced by $\varepsilon_{G,a}$, this is Walsh's code.

The model with Walsh's utility function. Assume that instead of C the argument of the utility function u is given by

$$\tilde{C}_h = C_h + (1-\zeta)b - \zeta l,$$

where $\zeta = 1$ if household $h \in [0,1]$ is employed and $\zeta = 0$ if the household is without a job. Aggregating this equation over $[0,1]$ delivers

$$\tilde{C}_t = C_t + bU_t - l(1-U_t), \tag{A.22}$$

[24] You can download our code from
http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/pap/hm_lms_gauss.zip

since all household choose the same level of consumption, irrespective of whether they are employed or jobless. Walsh (2005) assumes $b = 0$ so that the steady state version of equation (12) pins down

$$l = q + g\underline{a},$$

and

$$\tilde{C} = C - (1 - U)l.$$

Equation (A.22) can be used to substitute for $\hat{C}_t$ in the log-linear equation (A.13h), yielding

$$\hat{Y}_t - (\gamma V/Y)\hat{V}_t + (Ul/Y)\hat{U}_t = (\tilde{C}/Y)\hat{\tilde{C}}_t.$$

Replacing $\hat{C}_i$ by $\hat{\tilde{C}}_i$, $i \in \{t-1, t, t+1\}$, in the dynamic equation (A.14a) completes this model. All other equations are not affected.

If we simulate this model using the same parameters that underlie the results displayed in column I of Table A.1 we get a standard deviation of output of $\sigma_Y = 24.87$, which indicates that this version of the model behaves quite different from the versions that use the utility function given in equation (27).

9 Appendix B: the log-linear model

In this appendix we derive the log-linear version of our model presented in the body of the paper.

Equilibrium conditions. The equilibrium conditions of the model consist of two parts. There are 13 contemporaneous equations: (14), (12), (34), (35), (24). (18), (19), (15), (36), (31e), (31b), (25), and (26). We restate these for the readers' convenience:

$$F(\underline{a}_t) = \rho_t^n = \int_0^{\underline{a}_t} f(a)da, \tag{B.1a}$$

$$l + b = q_t + (1-\alpha)\left(g_t\underline{a}_t Z_t\right)^{1/(1-\alpha)}\left(\frac{\alpha}{r_t}\right)^{\alpha/(1-\alpha)}, \tag{B.1b}$$

$$u_t\bar{K}_t = (1-\rho^x)N_t H(\underline{a}_t)\left(\frac{\alpha g_t Z_t}{r_t}\right)^{1/(1-\alpha)}, \tag{B.1c}$$

$$Y_t = Z_t\Big[(1-\rho^x)N_t H(\underline{a}_t)\Big]^{1-\alpha}\left(u_t\bar{K}_t\right)^{\alpha}, \tag{B.1d}$$

$$q_t = \frac{\gamma(1-\eta\kappa_t^w)}{(1-\eta)\kappa_t^f}, \tag{B.1e}$$

$$\kappa_t^f = \frac{M_t}{V_t} = \psi(V_t/U_t)^{-\chi}, \tag{B.1f}$$

$$\kappa_t^w = \frac{M_t}{U_t} = \psi(V_t/U_t)^{1-\chi}, \tag{B.1g}$$

$$U_t = 1 - (1-\rho^x)(1-\rho_t^n)N_t, \tag{B.1h}$$

$$I_t = Y_t - \gamma V_t - C_t - \iota(u_t)\bar{K}_t, \tag{B.1i}$$

$$\xi_t = \frac{1}{\Phi'(I_t/\bar{K}_t)}, \tag{B.1j}$$

$$\iota'(u_t) = r_t, \tag{B.1k}$$

$$jdr_t = \rho_t - \kappa_t^f \rho^x = (1-\rho^x)\rho_t^n + \rho^x(1-\kappa_t^f), \tag{B.1l}$$

$$jcr_t = \frac{\kappa_t^f(V_t - \rho^x N_t)}{N_t}. \tag{B.1m}$$

Note, that we have substituted $v_t - w_t^u = q_t - b$ in (12) to obtain (B.1b) and $u_t\bar{K}_t = K_t$ in (34) to get (B.1c). The second part comprises the 7 dynamic equations (31a), (31c), (31f), (30), (16), (23), and (32):

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t(C_{t+1} - hC_t)^{-\sigma}, \tag{B.2a}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}}, \tag{B.2b}$$

$$\xi_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left\{ r_{t+1}u_{t+1} - \iota(u_{t+1}) - \frac{I_{t+1}}{\bar{K}_{t+1}} + \xi_{t+1}\Big[1 - \delta + \Phi(I_{t+1}/\bar{K}_{t+1})\Big]\right\}, \tag{B.2c}$$

$$\bar{K}_{t+1} = \Phi\left(\frac{I_t}{\bar{K}_t}\right)\bar{K}_t + (1-\delta)\bar{K}_t, \tag{B.2d}$$

$$N_{t+1} = (1-\rho^x)(1-\rho_t^n)N_t + \psi U_t^\chi V_t^{1-\chi}, \tag{B.2e}$$

$$\begin{aligned} q_t = \beta E_t \Bigg\{ & \frac{\lambda_{t+1}}{\lambda_t}(1-\rho^x)(1-\eta\kappa_t^w)(1-\alpha)\left(\frac{\alpha}{r_{t+1}}\right)^{\alpha/(1-\alpha)}(g_{t+1}Z_{t+1})^{1/(1-\alpha)} \\ & \times \Big[H(\underline{a}_{t+1}) - (1-\rho_{t+1}^n)\underline{a}_{t+1}^{1/(1-\alpha)}\Big]\Bigg\}, \end{aligned} \tag{B.2f}$$

$$R_{t+1} = \bar{\pi}^{(1-\rho_R)(1-\phi_\pi)}\beta^{-(1-\rho_R)}R_t^{\rho_R}\pi_t^{\phi_\pi(1-\rho_R)}e^{\phi_t}, \quad \phi_t \sim N(0,\sigma_\phi). \tag{B.2g}$$

Note that equation (B.2f) derives from equations (23) and (13).

The steady state. Consider the model without a technology and an interest rate shock so that $Z_t = 1$ and $\phi_t = 0$ for all t. The steady state of this deterministic model can be derived from (B.1) and (B.2) by dropping all time indices and by solving the ensuing equations for the variables of interest.

Given ρ^n and σ_a we can solve

$$\rho^n = \int_0^{\underline{a}} f(a)da = \int_0^{\underline{a}} \frac{1}{a\sigma_a\sqrt{2\pi}} e^{-0.5(\ln a/\sigma_a)^2} da,$$

for $\underline{a}$ as explained in the body of the paper. This allows us to compute

$$H(\underline{a}) = \int_{\underline{a}}^{\infty} a^{1/(1-\alpha)} \frac{1}{a\sigma_a\sqrt{2\pi}} e^{-0.5(\ln a/\sigma_a)^2} da,$$

numerically.

Our assumptions with respect to $\iota(u)$ and $\Phi(I/\bar{K})$ imply

$$\begin{aligned} \iota(u=1) &= 0, \\ \iota'(u=1) &= r, \\ \Phi(I/\bar{K}) &= \delta, \\ \Phi'(I/\bar{K}) &= 1. \end{aligned}$$

Equation (B.2c), thus, reduces to:

$$r = \frac{1-\beta(1-\delta)}{\beta}.$$

Given the solution for r, we can use equation (B.2f) to obtain

$$q = \beta(1-\rho^x)(1-\eta\kappa^w)(1-\alpha)\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)} g^{1/(1-\alpha)}\left(H(\underline{a}) - (1-\rho^n)\underline{a}^{1/(1-\alpha)}\right),$$

where $g = (\theta-1)/\theta$. Equation (B.1b) thus implies

$$b + l = q + (1-\alpha)\left(g\underline{a}\right)^{1/(1-\alpha)}\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}.$$

In the steady state equations (B.1f) and (B.1g) imply

$$\frac{V}{U} = \frac{\kappa^w}{\kappa^f}.$$

Equations (B.2e) and (B.1g) yield

$$\frac{U}{N} = \frac{1-(1-\rho^x)(1-\rho^n)}{\kappa^w}.$$

Given this, we can solve equation (B.1h) for N:

$$N = \left[\frac{U}{N} + (1-\rho^x)(1-\rho^n)\right]^{-1}.$$

From this solution and the previous equations we obtain $U = (U/N)N$ and $V = (V/U)U$. Equation (B.1e) can be used to infer γ:

$$\gamma = \frac{q(1-\eta)\kappa^f}{1-\eta\kappa^w}.$$

Since $u = 1$, we get

$$\bar{K} = (1-\rho^x)NH(\underline{a})\left(\frac{\alpha g}{r}\right)^{1/(1-\alpha)}$$

from equation (B.1c) so that we are able to solve for Y from (B.1d):

$$Y = \left((1-\rho^x)NH(\underline{a})\right)^{1-\alpha}\bar{K}^{\alpha}.$$

The budget constraint (B.1i), thus, implies

$$C = Y - \delta\bar{K} - \gamma V.$$

The log-linearized system. The system of equations (B.1), log-linearized at the stationary solution, is:

$$\begin{aligned}
\hat{\rho}_t^n - \varepsilon_{F,a}\underline{\hat{a}}_t &= 0, && \text{(B.3a)}\\
\Gamma_1\hat{r}_t - \Gamma_2\underline{\hat{a}}_t &= -\hat{q}_t + \Gamma_2\hat{g}_t + \Gamma_2\hat{Z}_t, && \text{(B.3b)}\\
\varepsilon_{H,a}\underline{\hat{a}}_t - \frac{1}{1-\alpha}\hat{r}_t - \hat{u}_t &= \hat{\bar{K}}_t - \hat{N}_t - \frac{1}{1-\alpha}\hat{g}_t - \frac{1}{1-\alpha}\hat{Z}_t, && \text{(B.3c)}\\
(\alpha-1)\varepsilon_{H,a}\underline{\hat{a}}_t + \hat{Y}_t - \alpha\hat{u}_t &= \alpha\hat{\bar{K}}_t + (1-\alpha)\hat{N}_t + \hat{Z}_t, && \text{(B.3d)}\\
\hat{\kappa}_t^f + \frac{\eta\kappa^w}{1-\eta\kappa^w}\hat{\kappa}_t^w &= -\hat{q}_t, && \text{(B.3e)}\\
\hat{\kappa}_t^f + \chi\hat{V}_t - \chi\hat{U}_t &= 0, && \text{(B.3f)}\\
\hat{\kappa}_t^w + (\chi-1)\hat{V}_t + (1-\chi)\hat{U}_t &= 0, && \text{(B.3g)}\\
(U/N)\hat{U}_t - (1-\rho^x)\rho^n\hat{\rho}_t^n &= -(1-\rho^x)(1-\rho^n)\hat{N}_t, && \text{(B.3h)}\\
\hat{I}_t - (Y/I)\hat{Y}_t + (cV/I)\hat{V}_t + (r/\delta)\hat{u}_t &= -(C/I)\hat{C}_t, && \text{(B.3i)}\\
\sigma_\Phi\hat{I}_t - \hat{\xi}_t &= \sigma_\Phi\hat{\bar{K}}_t, && \text{(B.3j)}\\
\sigma_\iota\hat{u}_t - \hat{r}_t &= 0, && \text{(B.3k)}\\
\widehat{jdr}_t - \frac{(1-\rho^x)\rho^n}{jdr}\hat{\rho}_t^n + \frac{\kappa^f\rho^x}{jdr}\hat{\kappa}_t^f &= 0, && \text{(B.3l)}\\
\widehat{jcr}_t - \hat{\kappa}_t^f - \frac{V}{V-\rho^x N}\hat{V}_t &= -\frac{V}{V-\rho^x N}\hat{N}_t, && \text{(B.3m)}
\end{aligned}$$

$$\Gamma_1 = \frac{\alpha}{1-\alpha}\frac{q-l-b}{q},\ \Gamma_2 = \frac{\Gamma_1}{\alpha},$$

$$\varepsilon_{F,a} = \frac{F'(\underline{a})\underline{a}}{F(\underline{a})}, \quad \varepsilon_{H,a} = \frac{H'(\underline{a})\underline{a}}{H(\underline{a})},$$
$$\sigma_\Phi := -\frac{\phi''(\delta)\delta}{\phi'(\delta)}, \quad \sigma_\iota := \frac{\iota''(\bar{u})\bar{u}}{\iota'(\bar{u})},$$

In (B.3i), we used the steady-state conditions $\iota(u=1)=0$, $\iota'(\bar{u})=r$ and $\bar{K}/I = 1/\delta$.

Log-linearizing equations (B.2) yields

$$\begin{aligned}
&\beta h \Gamma_3 E_t \hat{C}_{t+1} - (1+\beta h^2)\Gamma_3 \hat{C}_t && && \text{(B.4a)}\\
&+ h\Gamma_3 \hat{C}_{t-1} - \hat{\lambda}_t && = 0, &&\\
&E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t && = -\beta r E_t \hat{r}_{t+1} - \beta E_t \hat{\xi}_{t+1} + \hat{\xi}_t, && \text{(B.4b)}\\
&\hat{\bar{K}}_{t+1} + (\delta - 1)\hat{\bar{K}}_t && = \delta \hat{I}_t, && \text{(B.4c)}\\
&\hat{N}_{t+1} - (1-\rho^x)(1-\rho^n)\hat{N}_t && = \kappa^w (U/N)\hat{U}_t + \kappa^w (U/N)\hat{\kappa}^w_t && \text{(B.4d)}\\
& && \quad - (1-\rho^x)\rho^n \hat{\rho}^n_t, &&\\
&E_t\hat{\lambda}_{t+1} + \frac{1}{1-\alpha}E_t \hat{g}_{t+1} - \hat{\lambda}_t - \hat{q}_t && = \frac{\alpha}{1-\alpha}E_t \hat{r}_{t+1} - \Gamma_4 E_t \hat{\underline{a}}_{t+1} - \Gamma_5 E_t \hat{\rho}^n_{t+1} && \text{(B.4e)}\\
& && \quad + \frac{\eta\kappa^w}{1-\eta\kappa^w}\hat{\kappa}^w_t - \frac{1}{1-\alpha}E_t \hat{Z}_{t+1}, &&\\
&E_t\hat{\lambda}_{t+1} + \hat{R}_{t+1} - E_t\hat{\pi}_{t+1} - \hat{\lambda}_t && = 0, && \text{(B.4f)}\\
&\hat{R}_{t+1} - \rho_R \hat{R}_t - \phi_\pi (1-\rho_R)\hat{\pi}_t && = \phi_t, && \text{(B.4g)}\\
&\Gamma_3 := \frac{\sigma}{(1-\beta h)(1-h)}, && \Gamma_4 := \Delta\left[H(\underline{a})\varepsilon_{H,a} - \frac{1-\rho^n}{1-\alpha}\underline{a}^{1/(1-\alpha)}\right], &&\\
&\Gamma_5 := \Delta \rho^n \underline{a}^{(1/(1-\alpha)}, && \Delta := \left[H(\underline{a}) - (1-\rho^n)\underline{a}^{1/(1-\alpha)}\right]^{-1}. &&
\end{aligned}$$

The last equation of our log-linear dynamical system is supplied by the Phillips curve. If prices are set before the interest rate shock is realized, this equation is given by (7) with the time index shifted one period into the future:

$$\hat{\pi}_{t+1} - \frac{1}{1+\beta}\hat{\pi}_t - \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+2} - \Gamma_6 E_t \hat{g}_{t+1} = 0, \tag{B.4h}$$
$$\Gamma_6 = \frac{(1-\omega)(1-\beta\omega)}{(1+\beta)\omega}.$$

If prices are set after the interest rate shock is observed, the final equation of our model is given by (8):

$$\hat{\pi}_t - \frac{1}{1+\beta}\hat{\pi}_{t-1} - \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \Gamma_6 \hat{g}_t = 0. \tag{B.4i}$$

The system of equations (B.3) and (B.4) can be put into the canonical (A.15). In the case of predetermined prices we define the auxiliary variables

$$\hat{v}_t^1 := \hat{\pi}_{t+1},$$
$$\hat{v}_t^2 := \hat{C}_{t-1}$$

and the vectors

$$\begin{aligned}
\mathbf{u}_t &:= \left[\hat{Y}_t, \hat{U}_t, \hat{V}_t, \hat{\kappa}_t^w, \hat{\kappa}_t^f, \hat{\rho}_t^n, \hat{\underline{a}}_t, \hat{r}_t, \hat{I}_t, \hat{\xi}_t, \hat{u}_t, \widehat{jdr}_t, \widehat{jcr}_t\right], \\
\mathbf{x}_t &:= \left[\hat{\bar{K}}_t, \hat{N}_t, \hat{R}_t, \hat{\pi}_t, \hat{v}_t^2\right], \\
\boldsymbol{\lambda}_t &:= \left[\hat{\lambda}_t, \hat{q}_t, \hat{v}_t^1, \hat{g}_t, \hat{C}_t\right], \\
\mathbf{z}_t &:= \left[\hat{Z}_t, \phi_t\right].
\end{aligned}$$

If prices are set after the realization of the interest rate shock, we use

$$\hat{v}_t^1 := \hat{\pi}_{t-1},$$
$$\hat{v}_t^2 := \hat{C}_{t-1}$$

as auxiliary variables and set $\mathbf{x}_t$ and $\boldsymbol{\lambda}_t$ to

$$\begin{aligned}
\mathbf{x}_t &:= \left[\hat{\bar{K}}_t, \hat{N}_t, \hat{R}_t, \hat{v}_t^1, \hat{v}_t^2\right], \\
\boldsymbol{\lambda}_t &:= \left[\hat{\lambda}_t, \hat{q}_t, \hat{\pi}_t, \hat{g}_t, \hat{C}_t\right].
\end{aligned}$$

The solution of (A.15) consists in linear policy functions

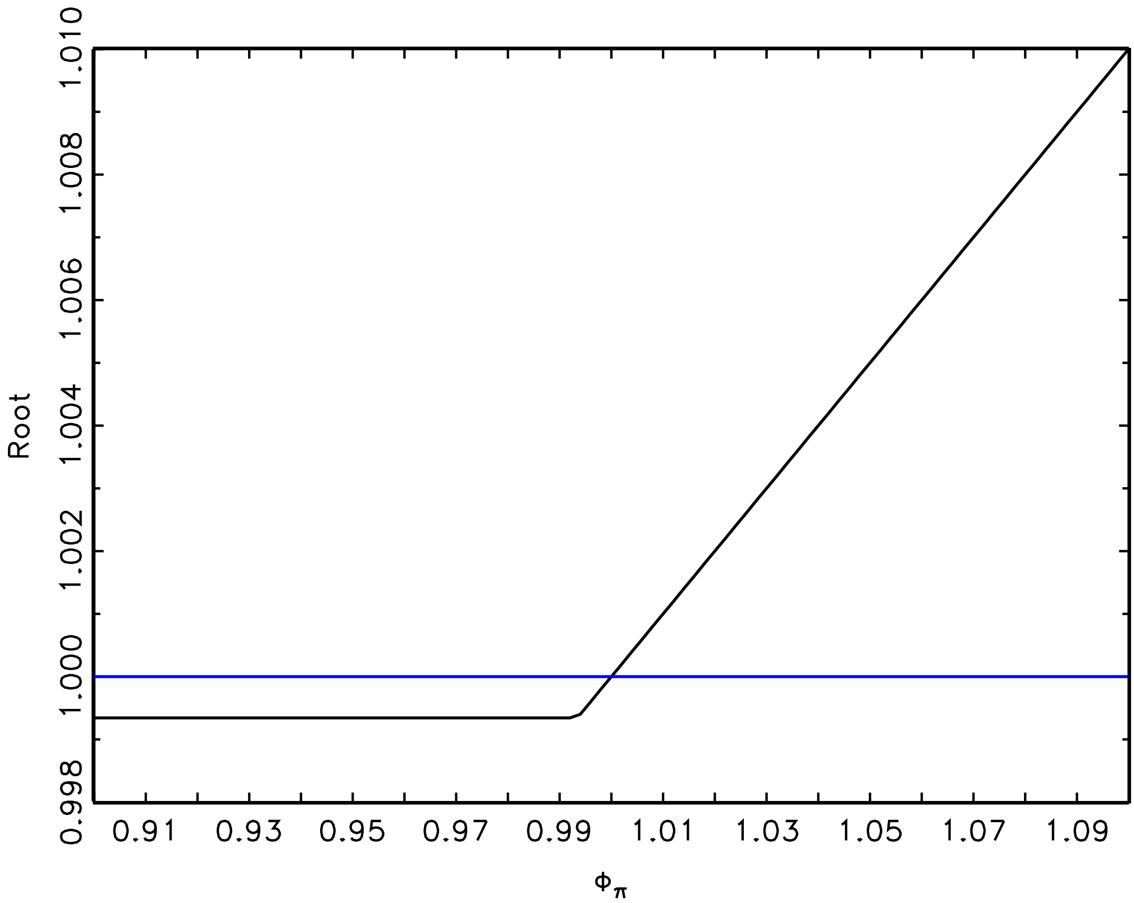
$$\begin{aligned}
\mathbf{u}_t &= L_{ux}\mathbf{x}_t + L_{uz}\mathbf{z}_t, \\
\boldsymbol{\lambda}_t &= L_{\lambda x}\mathbf{x}_t + L_{\lambda z}\mathbf{z}_t, \\
\mathbf{x}_{t+1} &= L_{xx}\mathbf{x}_t + L_{xz}\mathbf{z}_t,
\end{aligned}$$

which can be used to compute impulse response functions and to simulate the model.[25]

Determinacy. The well known Taylor principle states that the feed-back parameter of inflation in the interest rate rule, the parameter ϕ_π in equation (32), must exceed one for a determinate transition path to the stationary equilibrium to exist. Figure 9 displays the behavior of the critical root if ϕ_π varies in the interval $[0.9, 1.1]$. The other parameters of the model are as presented in Table 1. The root crosses one from below at $\phi_\pi = 1$.

[25]You can download the Gauss code that implements the solution from `http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/pap/hm_lms_gauss.zip`

Figure 9: Determinacy



10 Appendix C: the model with technological growth

In this appendix we introduce exogenous economic growth into our model and explain the way in which impulse responses to a technological shock have to be constructed.

Assumptions. We assume that the level of total factor productivity Z_t grows at the gross rate

$$z_t := \frac{Z_t}{Z_{t-1}}$$

and that

$$\hat{z}_t = \ln(z_t/z) = \rho^z \hat{z}_{t-1} + \epsilon_t,$$

where ϵ_t is iid with $E(\epsilon_t) = 0$ and $\sqrt{\text{var}(\epsilon_t)} = \sigma_\epsilon$.

To permit a balanced growth path, along which $z_t = z > 1\,\forall t$, we must modify several equations and introduce new variables that are stationary on that path. As it will become obvious in a moment, it is convenient to choose $Z_{t-1}^{1/(1-\alpha)}$ as scaling factor (except for the marginal utility of wealth) and define

$$\begin{aligned} \tilde{X}_t &= \frac{X_t}{Z_{t-1}^{1/(1-\alpha)}}, \quad X \in \{Y, K, C, I, v, w^u, q, s\}, \\ \tilde{\lambda}_t &= \lambda_t Z_{t-1}^{\sigma/(1-\alpha)}. \end{aligned} \tag{C.1}$$

Consider, first, the surplus of an employment relationship (11). To prevent that the disutility of working becomes asymptotically negligible we index l with the level of total factor productivity. Using the solution for $k(a_t)$ from (10), we write:

$$s_t = (1-\alpha)\left(g_t Z_t \underline{a}_t\right)^{1/(1-\alpha)} \left(\frac{\alpha}{r_t}\right)^{\alpha/(1-\alpha)} - lZ_t^{1/(1-\alpha)} - w_t^u + v_t, \tag{C.2}$$

so that (13) becomes

$$\tilde{s}_t = (1-\alpha)\left(\frac{\alpha}{r_t}\right)^{\alpha/(1-\alpha)} g_t {z_t}^{1/(1-\alpha)}\left[a_t^{1/(1-\alpha)} - \underline{a}_t^{1/(1-\alpha)}\right]. \tag{C.3}$$

Replacing l with $lZ_t^{1/(1-\alpha)}$ and setting $b = 0$ transforms equation (B.1b) to:

$$l = \tilde{q}_t + (1-\alpha)\left(g_t \underline{a}_t\right)^{1/(1-\alpha)} \left(\frac{\alpha}{r_t}\right)^{\alpha/(1-\alpha)}. \tag{C.4}$$

Given the definition of $\tilde{\lambda}_t$ the stochastic discount factor in equations (23), (31c), and (31f) changes to

$$\beta z_t^{-\sigma/(1-\alpha)} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}.$$

and the Euler equation for consumption (31a) becomes

$$\tilde{\lambda}_t = \left(\tilde{C}_t - hz_{t-1}^{-1/(1-\alpha)}\tilde{C}_{t-1}\right)^{-\sigma} - \beta h E_t\left(z_t^{1/(1-\alpha)}\tilde{C}_{t+1} - h\tilde{C}_t\right)^{-\sigma}. \tag{C.5}$$

Second, we must also index the costs of posting a vacancy to the level of technological progress. Therefore we assume

$$\gamma Z_t^{1/(1-\alpha))} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left\{(1-\eta)\kappa_t^f(1-\rho^x)\int_{\underline{a}_{t+1}} s_{t+1} f(a)da\right\}$$

so that this equation can be written in stationary variables as:

$$\gamma = \beta E_t z_t^{-\sigma/(1-\alpha)} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\left\{(1-\eta)\kappa_t^f(1-\rho^x)\int_{\underline{a}_{t+1}} \tilde{s}_{t+1} f(a)da\right\}. \tag{C.6}$$

Equilibrium conditions. Using the definitions in (C.1) and the assumptions made so far it is easy to see that the equilibrium conditions (B.1) and (B.2) change to:

$$
\begin{aligned}
F(\underline{a}_t) &= \rho_t^n = \int_0^{\underline{a}_t} f(a)da, && \text{(C.7a)}\\
l &= \tilde{q}_t + (1-\alpha)\left(g_t\underline{a}_t\right)^{1/(1-\alpha)}\left(\frac{\alpha}{r_t}\right)^{\alpha/(1-\alpha)}, && \text{(C.7b)}\\
u_t\tilde{\bar{K}}_t &= (1-\rho^x)N_tH(\underline{a}_t)\left(\frac{\alpha g_t z_t}{r_t}\right)^{1/(1-\alpha)}, && \text{(C.7c)}\\
\tilde{Y}_t &= z_t\Big[(1-\rho^x)N_tH(\underline{a}_t)\Big]^{1-\alpha}\left(u_t\tilde{\bar{K}}_t\right)^{\alpha}, && \text{(C.7d)}\\
\tilde{q}_t &= \frac{\gamma(1-\eta\kappa_t^w)}{(1-\eta)\kappa_t^f}, && \text{(C.7e)}\\
\kappa_t^f &= \frac{M_t}{V_t} = \psi(V_t/U_t)^{-\chi}, && \text{(C.7f)}\\
\kappa_t^w &= \frac{M_t}{U_t} = \psi(V_t/U_t)^{1-\chi}, && \text{(C.7g)}\\
U_t &= 1-(1-\rho^x)(1-\rho_t^n)N_t, && \text{(C.7h)}\\
\tilde{I}_t &= \tilde{Y}_t - \gamma z_t^{1/(1-\alpha)}V_t - \tilde{C}_t - \iota(u_t)\tilde{\bar{K}}_t, && \text{(C.7i)}\\
\xi_t &= \frac{1}{\Phi'\left(I_t/\bar{K}_t\right)}, && \text{(C.7j)}\\
\iota'(u_t) &= r_t. && \text{(C.7k)}
\end{aligned}
$$

and:

$$
\begin{aligned}
\tilde{\lambda}_t &= \left(\tilde{C}_t - hz_{t-1}^{-1/(1-\alpha)}\tilde{C}_{t-1}\right)^{-\sigma} - \beta hE_t\left(z_t^{1/(1-\alpha)}\tilde{C}_{t+1} - h\tilde{C}_t\right)^{-\sigma}, && \text{(C.8a)}\\
1 &= \beta E_t z_t^{-\sigma/(1-\alpha)}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\frac{R_{t+1}}{\pi_{t+1}}, && \text{(C.8b)}\\
\xi_t &= \beta E_t z_t^{-\sigma/(1-\alpha)}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\Bigg\{r_{t+1}u_{t+1} - \iota(u_{t+1}) - \frac{\tilde{I}_{t+1}}{\tilde{\bar{K}}_{t+1}} && \text{(C.8c)}\\
&\qquad + \xi_{t+1}\Big(1-\delta+\Phi\left(\tilde{I}_{t+1}/\tilde{\bar{K}}_{t+1}\right)\Big)\Bigg\}, &&\\
z_t^{1/(1-\alpha)}\tilde{\bar{K}}_{t+1} &= \Phi\left(\tilde{I}_t/\tilde{\bar{K}}_t\right)\tilde{\bar{K}}_t + (1-\delta)\tilde{\bar{K}}_t, && \text{(C.8d)}\\
N_{t+1} &= (1-\rho^x)(1-\rho_t^n)N_t + \psi U_t^{\chi}V_t^{1-\chi}, && \text{(C.8e)}\\
\tilde{q}_t &= \beta E_t z_t^{-\sigma/(1-\alpha)}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\Bigg\{(1-\rho^x)(1-\eta\kappa_t^w)(1-\alpha)\left(\frac{\alpha}{r_{t+1}}\right)^{\alpha/(1-\alpha)} && \text{(C.8f)}
\end{aligned}
$$

$$\times \left(g_{t+1}z_{t+1}\right)^{1/(1-\alpha)} \left(H(\underline{a}_{t+1}) - (1-\rho^n_{t+1})\underline{a}^{1/(1-\alpha)}_{t+1}\right) \Bigg\},$$

$$R_{t+1} = \bar{\pi}^{(1-\rho_R)(1-\phi_\pi)}\beta^{-(1-\rho_R)}R_t^{\rho_R}\pi_t^{\phi_\pi(1-\rho_R)}e^{\phi_t}, \quad \phi_t \sim N(0,\sigma_\phi). \tag{C.8g}$$

The steady state. Consider the transition equation for capital (C.8e). A stationary solution in a deterministic environment requires $\tilde{\tilde{K}}_{t+1} = \tilde{\tilde{K}}_t$ for all t. This implies

$$\phi(\tilde{I}/\tilde{\tilde{K}}) = z^{1/(1-\alpha)} - 1 + \delta$$

and

$$\frac{\tilde{I}}{\tilde{\tilde{K}}} = z^{1/(1-\alpha)} - 1 + \delta, \tag{C.9}$$

where z denotes the constant average growth rate of technological progress. The Euler equation for capital accumulation (C.8c) implies the stationary real rate of interest:

$$r = \beta^{-1}z^{\sigma/(1-\alpha)} + \delta - 1.$$

Given r we can solve the stationary version of equation (C.8f) for $\tilde{q}$:

$$\tilde{q} = \beta z^{-\sigma/(1-\alpha)}(1-\rho^x)(1-\kappa^w)(1-\alpha)\left(\frac{\alpha}{r}\right)^{\alpha/(1-\alpha)}(gz)^{1/(1-\alpha)}\left(H(\underline{a}) - (1-\rho^x)\underline{a}^{1/(1-\alpha)}\right).$$

Since the equations that determine N, U, V, and M have not changed, there is no need to recompute the stationary values of these variables. The stationary stock of capital derives from (C.7c) for $u = 1$:

$$\tilde{\tilde{K}} = (1-\rho^x)NH(\underline{a})\left(\frac{\alpha g z}{r}\right)^{1/(1-\alpha)}.$$

Equation (C.9), thus, implies the solution for the scaled level of investment:

$$\tilde{I} = \left(z^{1/(1-\alpha)} - 1 + \delta\right)\tilde{\tilde{K}}.$$

Output follows from equation (C.7d)

$$\tilde{Y} = z\left[(1-\rho^x)NH(\underline{a})\right]^{1-\alpha}\tilde{\tilde{K}}^\alpha,$$

and consumption from the economy's resource constraint (C.7i):

$$\tilde{C} = \tilde{Y} - \gamma z^{1/(1-\alpha)}V - \tilde{I}.$$

The log-linearized system. The system of equations (C.7), log-linearized at the stationary solution, is:

$$
\begin{aligned}
\hat{\rho}_t^n - \varepsilon_{F,a}\underline{\hat{a}}_t &= 0, && \text{(C.10a)}\\
\Gamma_1 \hat{r}_t - \Gamma_2 \underline{\hat{a}}_t &= -\hat{\tilde{q}}_t + \Gamma_2 \hat{g}_t, && \text{(C.10b)}\\
\varepsilon_{H,a}\underline{\hat{a}}_t - \frac{1}{1-\alpha}\hat{r}_t - \hat{u}_t &= \hat{\tilde{\tilde{K}}}_t - \hat{N}_t - \frac{1}{1-\alpha}\hat{g}_t - \frac{1}{1-\alpha}\hat{z}_t, && \text{(C.10c)}\\
(\alpha-1)\varepsilon_{H,a}\underline{\hat{a}}_t + \hat{\tilde{Y}}_t - \alpha\hat{u}_t &= \alpha\hat{\tilde{\tilde{K}}}_t + (1-\alpha)\hat{N}_t + \hat{z}_t, && \text{(C.10d)}\\
\hat{\kappa}_t^f + \frac{\eta\kappa^w}{1-\eta\kappa^w}\hat{\kappa}_t^w &= -\hat{q}_t, && \text{(C.10e)}\\
\hat{\kappa}_t^f + \chi\hat{V}_t - \chi\hat{U}_t &= 0, && \text{(C.10f)}\\
\hat{\kappa}_t^w + (\chi-1)\hat{V}_t + (1-\chi)\hat{U}_t &= 0, && \text{(C.10g)}\\
(U/N)\hat{U}_t - (1-\rho^x)\rho^n\hat{\rho}_t^n &= -(1-\rho^x)(1-\rho^n)\hat{N}_t, && \text{(C.10h)}\\
\hat{\tilde{I}}_t - \frac{\tilde{Y}}{\tilde{I}}\hat{\tilde{Y}}_t + cz^{1/(1-\alpha)}\frac{V}{\tilde{I}}\hat{V}_t + r(\tilde{\tilde{K}}/\tilde{I})\hat{u}_t &= -\frac{\tilde{C}}{\tilde{I}}\hat{\tilde{C}}_t - \frac{\gamma z^{1/(1-\alpha)}}{1-\alpha}\frac{V}{\tilde{I}}\hat{z}_t, && \text{(C.10i)}\\
\sigma_\Phi\hat{\tilde{I}}_t - \hat{\xi}_t &= \sigma_\Phi\hat{\tilde{\tilde{K}}}_t, && \text{(C.10j)}\\
\sigma_\iota\hat{u}_t - \hat{r}_t &= 0, && \text{(C.10k)}
\end{aligned}
$$

$$
\Gamma_1 = \frac{\alpha}{1-\alpha}\frac{\tilde{q}-l}{\tilde{q}},\ \Gamma_2 = \frac{\Gamma_1}{\alpha},
$$

$$
\varepsilon_{F,a} = \frac{F'(\underline{a})\underline{a}}{F(\underline{a})}, \quad \varepsilon_{H,a} = \frac{H'(\underline{a})\underline{a}}{H(\underline{a})},
$$

$$
\sigma_\Phi := -\frac{\phi''(\delta)\delta}{\phi'(\delta)}, \quad \sigma_\iota := \frac{\iota''(\bar{u})\bar{u}}{\iota'(\bar{u})}.
$$

Log-linearizing equations (C.8) yields

$$
\begin{aligned}
&\beta h z^{-\sigma/(1-\alpha)}\Gamma_3 E_t\hat{\tilde{C}}_{t+1} && && \text{(C.11a)}\\
&- \left(1+\beta h^2 z^{-(1+\sigma)/(1-\alpha)}\right)\Gamma_3\hat{\tilde{C}}_t && &&\\
&+ h z^{-1/(1-\alpha)}\Gamma_3\hat{\tilde{C}}_{t-1} - \hat{\tilde{\lambda}}_t && = \frac{hz^{-1/(1-\alpha)}}{1-\alpha}\Gamma_3\hat{z}_{t-1} - \frac{\beta h z^{-\sigma/(1-\alpha)}}{1-\alpha}\Gamma_3\hat{z}_t, &&\\
&E_t\hat{\tilde{\lambda}}_{t+1} - \hat{\tilde{\lambda}}_t && = -\beta r z^{-\sigma/(1-\alpha)}E_t\hat{r}_{t+1} && \text{(C.11b)}\\
& && \quad - \beta z^{-\sigma/(1-\alpha)}E_t\hat{\xi}_{t+1} + \hat{\xi}_t + \frac{\sigma}{1-\alpha}\hat{z}_t, &&\\
&z^{1/(1-\alpha)}\hat{\tilde{\tilde{K}}}_{t+1} + (\delta-1)\hat{\tilde{\tilde{K}}}_t && = \frac{\tilde{I}}{\tilde{\tilde{K}}}\hat{\tilde{I}}_t, && \text{(C.11c)}\\
&\hat{N}_{t+1} - (1-\rho^x)(1-\rho^n)\hat{N}_t && = \kappa^w(U/N)\hat{U}_t + \kappa^w(U/N)\hat{\kappa}_t^w && \text{(C.11d)}\\
& && \quad - (1-\rho^x)\rho^n\hat{\rho}_t^n, &&
\end{aligned}
$$

$$
\begin{aligned}
E_t\hat{\hat{\lambda}}_{t+1} + \frac{1}{1-\alpha}E_t\hat{g}_{t+1} - \hat{\hat{\lambda}}_t - \hat{\hat{q}}_t &= \frac{\alpha}{1-\alpha}E_t\hat{r}_{t+1} - \Gamma_4 E_t\hat{\underline{a}}_{t+1} \\
&\quad - \Gamma_5 E_t\hat{\rho}^n_{t+1} + \frac{\eta\kappa^w}{1-\eta\kappa^w}\hat{\kappa}^w_t \\
&\quad - \frac{1}{1-\alpha}E_t\hat{z}_{t+1} + \frac{\sigma}{1-\alpha}\hat{z}_t, && \text{(C.11e)}\\
E_t\hat{\hat{\lambda}}_{t+1} + \hat{R}_{t+1} - E_t\hat{\pi}_{t+1} - \hat{\hat{\lambda}}_t &= \frac{\sigma}{1-\alpha}\hat{z}_t, && \text{(C.11f)}\\
\hat{R}_{t+1} - \rho_R\hat{R}_t - \phi_\pi(1-\rho_R)\hat{\pi}_t &= \phi_t, && \text{(C.11g)}\\
\hat{\pi}_{t+1} - \frac{1}{1+\beta}\hat{\pi}_t - \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+2} - \Gamma E_t\hat{g}_{t+1} &= 0, && \text{(C.11h)}
\end{aligned}
$$

$$\Gamma = \frac{(1-\omega)(1-\beta\omega)}{(1+\beta)\omega},$$

$$\Gamma_3 := \frac{\sigma}{(1-\beta h z^{-\sigma/(1-\alpha)})(1-hz^{-1/(1-\alpha)})}, \Gamma_4 := \Delta\left[H(\underline{a})\varepsilon_{H,a} - \frac{1-\rho^n}{1-\alpha}\underline{a}^{1/(1-\alpha)}\right],$$

$$\Gamma_5 := \Delta\rho^n\underline{a}^{1/(1-\alpha)}, \qquad \Delta := \left[H(\underline{a}) - (1-\rho^n)\underline{a}^{1/(1-\alpha)}\right]^{-1}.$$

To map this system into the canonical form (A.15) we define the auxiliary variables

$$
\begin{aligned}
\hat{v}^1_t &:= \hat{\pi}_{t+1},\\
\hat{v}^2_t &:= \hat{\hat{C}}_{t-1},\\
\hat{v}^3_t &:= \hat{z}_{t-1}
\end{aligned}
$$

and the vectors

$$
\begin{aligned}
\mathbf{u}_t &:= \left[\hat{\hat{Y}}_t, \hat{U}_t, \hat{V}_t, \hat{\kappa}^w_t, \hat{\kappa}^f_t, \hat{\rho}^n_t, \hat{\underline{a}}_t, \hat{r}_t, \hat{\hat{I}}_t, \hat{\xi}_t, \hat{u}_t\right],\\
\mathbf{x}_t &:= \left[\hat{\hat{\hat{K}}}_t, \hat{N}_t, \hat{R}_t, \hat{\pi}_t, \hat{v}^2_t, \hat{v}^3_t\right],\\
\boldsymbol{\lambda}_t &:= \left[\hat{\hat{\lambda}}_t, \hat{\hat{q}}_t, \hat{v}^1_t, \hat{g}_t, \hat{\hat{C}}_t\right],\\
\mathbf{z}_t &:= \left[\hat{z}_t, \phi_t\right].
\end{aligned}
$$

In the case when prices are set after the monetary shock has been observed, i.e., with the Phillips curve (8) instead of equation (C.11e), we use

$$
\begin{aligned}
\hat{v}^1_t &:= \hat{\pi}_{t-1},\\
\hat{v}^2_t &:= \hat{\hat{C}}_{t-1},\\
\hat{v}^3_t &:= \hat{z}_{t-1}
\end{aligned}
$$

as auxiliary variables and set $\mathbf{x}_t$ and $\boldsymbol{\lambda}_t$ to

$$
\begin{aligned}
\mathbf{x}_t &:= \left[\hat{\hat{\hat{K}}}_t, \hat{N}_t, \hat{R}_t, \hat{v}^1_t, \hat{v}^2_t, \hat{v}^3_t\right],\\
\boldsymbol{\lambda}_t &:= \left[\hat{\hat{\lambda}}_t, \hat{\hat{q}}_t, \hat{\pi}_t, \hat{g}_t, \hat{\hat{C}}_t\right].
\end{aligned}
$$

Impulse response functions. In the model with a growing level of total factor productivity we define impulse responses as the time path of a variable X_t after a one-time shock relative to the time path of this variable in the absence of a shock X_t^*.[26] Let

$$Z_t^* = z^t Z_0$$

denote the time t value of the level of technical progress if there has been no shocks to the growth rate of Z_t between time $t = 0$ and t. Furthermore, as before, let $\tilde{X}$ denote the steady state value of $\tilde{X}_t := X_t / Z_{t-1}^{1/(1-\alpha)}$. Then, the path of X_t without a shock is given by

$$X_t^* = (Z_{t-1}^*)^{1/(1-\alpha)} \tilde{X},$$

and

$$x_t := \frac{X_t}{X_t^*} = \frac{Z_{t-1}^{1/(1-\alpha)} \tilde{X}_t}{(Z_{t-1}^*)^{1/(1-\alpha)} \tilde{X}}$$

is the path of X_t relative to the unshocked path X_t^*. Since

$$Z_{t-1} = z_{t-1} z_{t-2} \dots z_1 Z_0 = \Pi_{j=1}^{t-1} z_{t-j} Z_0$$

and

$$Z_{t-1}^* = z^{t-1} Z_0 = \Pi_{j=1}^{t-1} z Z_0,$$

the percentage deviation of X_t from X_t^* is approximately equal to

$$\begin{aligned} \ln(x_t) = \ln(X_t/X_t^*) &= (1-\alpha) \sum_{j=1}^{t-1} \ln(Z_{t-j}/Z_{t-1}^*) + \ln(\tilde{X}_t/\tilde{X}), \\ &= (1-\alpha) \sum_{j=1}^{t-1} \hat{z}_{t-j} + \hat{\tilde{X}}_t. \end{aligned}$$

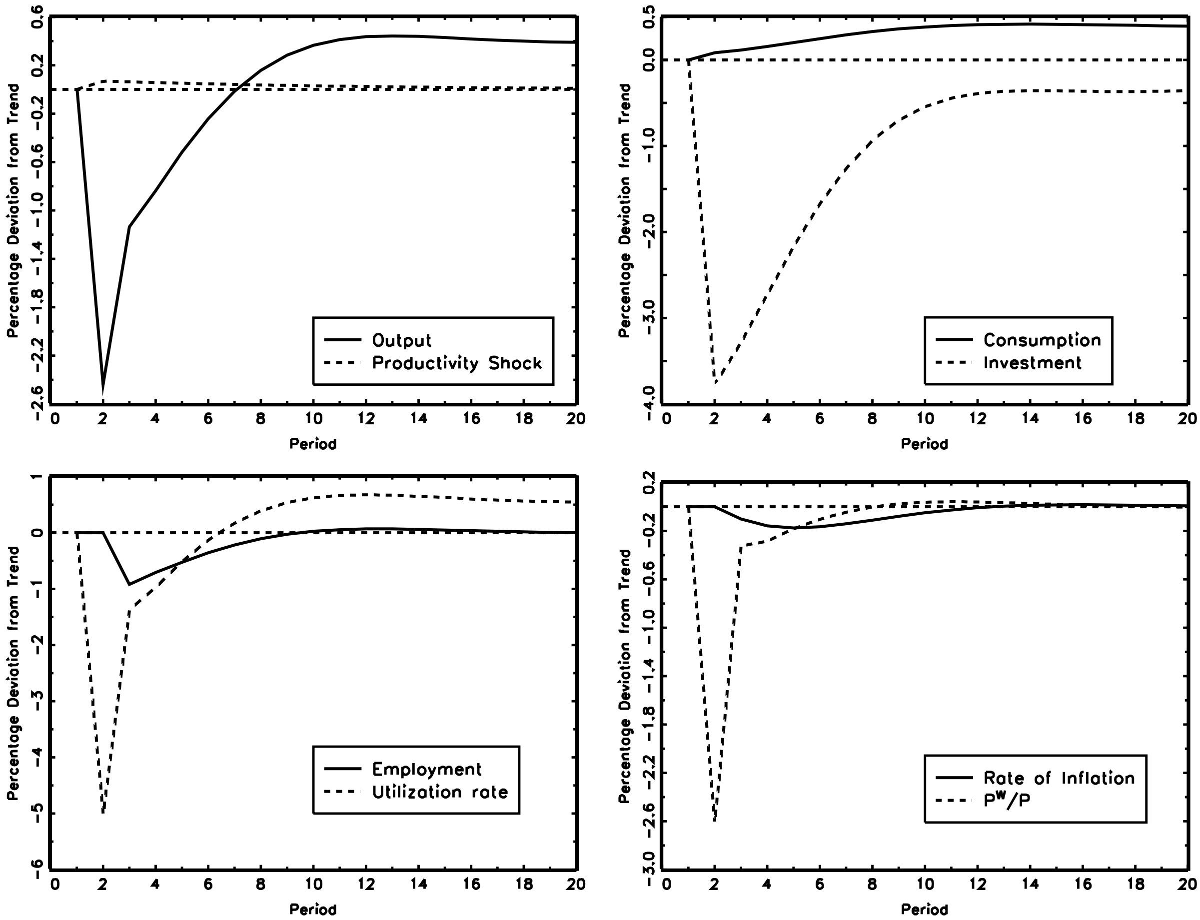
Calibration and response to a technology shock. We employ the estimates by Altig et al. (2005) to calibrate the technology shock in this model. The long-run growth factor is set to $z = 1.0045$, the autocorrelation parameter to $\rho^z = 0.90$, and the standard deviation of the innovations to $\sigma_\epsilon = 0.0007$. All other parameters are set to their benchmark values given in Table 1.

Figure 10 shows that the adverse reaction of output to a positive technology shock is also present in the extended model.[27]

[26] See the technical appendix to Altig et al. (2005), p.53f, available on the web from http://faculty.wcas.northwestern.edu/~lchrist/research/ACEL/acelweb.htm.

[27] You can download the Gauss program that computes these responses from http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/pap/hm_lms_gauss.zip

Figure 10: Impulse respones to a productivty shock in the model with stochastic growth



11 Appendix D: estimation of parameters

In this appendix we explain and document the estimation of those parameters of the model for which no direct evidence is available. These parameters are the habit parameter h, the degree of price rigidity ω, the elasticity of the function Φ, σ_Φ, which determines the size of the adjustment costs of capital, and the elasticity of the function ι, σ_ι, which affects the costs of adjusting the rate of capital utilization. Our approach parallels Christiano et al. (2005). First, we identify the response of key variables to an adverse interest rate shock from an estimated vector autoregressive (VAR) model. Second, we determine the free parameters so that the weighted squared distance between the impulse responses from the model and the estimated VAR is minimized. We use the inverse of the estimated variance of the VAR impulse responses as weights.[28]

[28]You can download the Gauss programs that estimate the impulse responses and estimate the model's parameters from `http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/pap/hm_lms_gauss.zip`

Identification of impulse responses. Consider the following structural vector autoregressive model in the $k = 6$-dimensional vector of variables $\mathbf{y}_t$

$$A_0 \mathbf{y}_t = \mathbf{c} + \sum_{i=1}^{p} A_i \mathbf{y}_{t-i} + \boldsymbol{\epsilon}_t. \tag{D.1}$$

The vector of innovations is identically and independently distributed with $E(\boldsymbol{\epsilon}) = \mathbf{0}_k$ and $E(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}') = I_k$, and A_0 is a lower triangular square matrix. The variables in $\mathbf{y}_t$ are (in this order) output, the rate of capacity utilization, consumption, the rate of inflation, investment, and the federal funds rate. In our model the nominal interest rate at time t is a predetermined variable. Thus, a shock ϵ_{6t} in the interest rate equation has no current effect on the other variables. This is the same identification strategy as pursued by Christiano et al. (2005). Except for the utilization rate (which we take from the data set from Altig et al. (2005)) we use the data from Christiano et al. (2005). These are quarterly data from 1964.ii through 1995.ii.[29] Output is measured by the log of real GDP. Real consumption, real investment, and the rate of capacity utilization are also in logs. Inflation is the difference of the logged price level and the nominal interest rate is the annualized federal funds rate divided by 100.

The model that we estimate is

$$\mathbf{y}_t = B_0 + \sum_{i=1}^{4} B_i \mathbf{y}_{t-i} + \mathbf{u}_t, \quad B_0 = A_0^{-1}, B_i = A_0^{-1} A_i, \mathbf{u}_t = A_0^{-1} \boldsymbol{\epsilon}_t,$$

so that A_0^{-1} equals the Cholesky factor C of the covariance matrix of $\mathbf{u}_t$. The eigenvalues of the characteristic equation are all smaller than one in absolute value so that the estimated VAR is stable. The responses of the variables in $\mathbf{y}_t$ to a one-time, one-unit shock $\epsilon_{nt} = 0.01$ are obtained from

$$\begin{aligned}
\bar{\mathbf{y}}_1 &= \hat{C}\mathbf{e}_6, \quad \mathbf{e}_6 = [0, 0, 0, 0, 0, 0.01]',\\
\bar{\mathbf{y}}_2 &= \hat{B}_1 \bar{\mathbf{y}}_1,\\
\bar{\mathbf{y}}_3 &= \hat{B}_1 \bar{\mathbf{y}}_2 + \hat{B}_2 \bar{\mathbf{y}}_1,\\
\vdots &= \vdots,\\
\bar{\mathbf{y}}_t &= \sum_{i=1}^{4} \hat{B}_i \bar{\mathbf{y}}_{t-i}, \quad t = 5, 6, \ldots,
\end{aligned}$$

where a hat denotes estimated parameters and where $\bar{\mathbf{y}}_t = \mathbf{y}_t - \mathbf{y}^*$, i.e., the deviation of $\mathbf{y}$ from its long-run value $\mathbf{y}^* = \mathbf{c}/(I - \sum_{i=1}^{4} B_i)$.

We compute error bounds for the impulse responses from $\pm$ 2 times the estimated standard errors obtained from a bootstrap procedure: we sample new shocks from the estimated

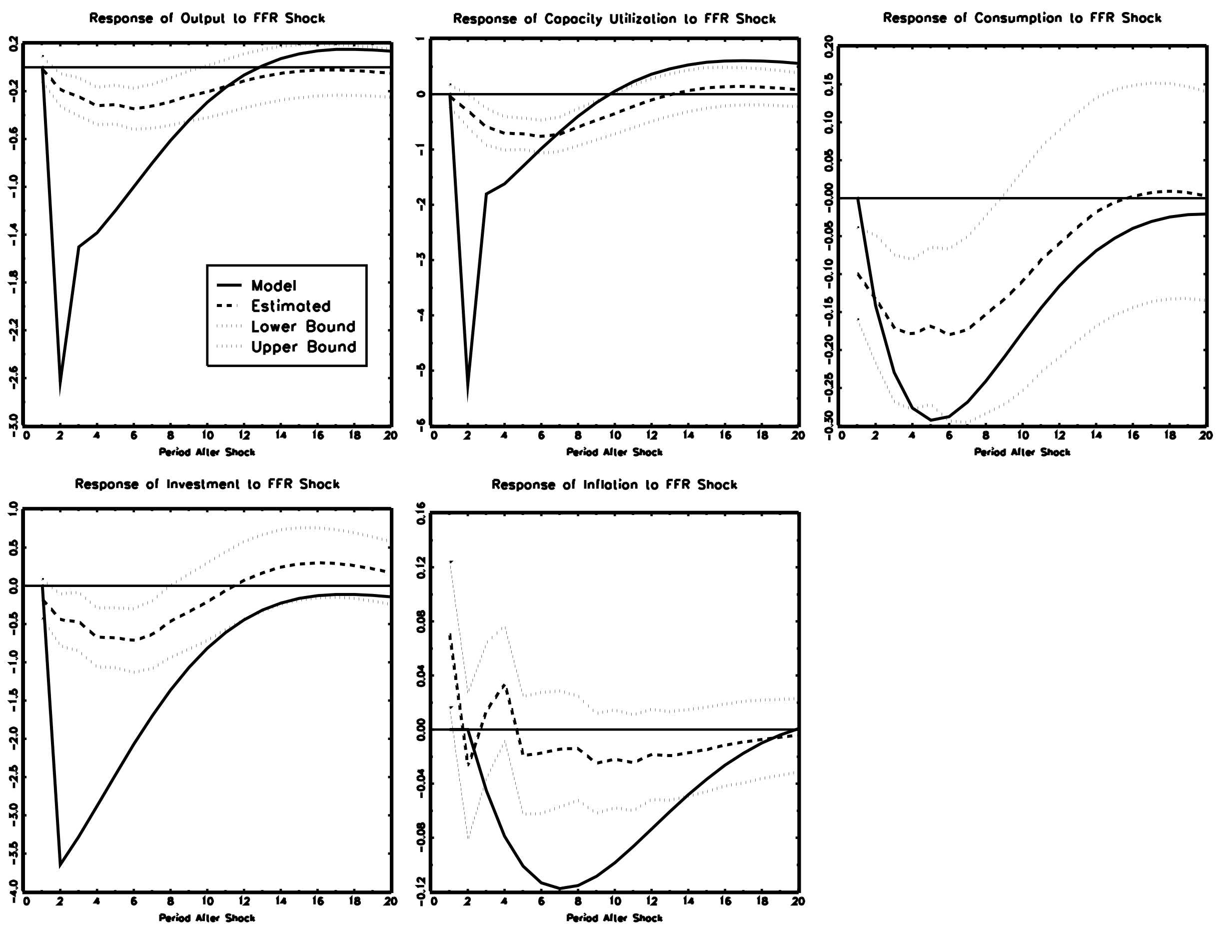
[29] We are grateful to the authors for sharing their data with us.

errors $\hat{\mathbf{u}}_t$ and employ the estimated model to compute new artificial observations from which we obtain new estimates and new impulse responses. We use 1,000 replications.

As it turned out, the estimated impulse responses are insensitive to the ordering of the variables in positions 2-5 in the vector $\mathbf{y}_t$.

Figure D.1 presents the estimated impulse responses, the error bounds, and the impulse response of our model to an interest rate shock. The parameters are those given in Table 1.

Figure D.1: Estimated and Model Implied Impulse Responses



Parameter choice. Let $\boldsymbol{\theta} = [\omega, h, \sigma_\Phi, \sigma_\iota]'$ denote the collection of the parameters that we want to fit to the data. For each collection we solve our model and compute the impulse responses of output, the rate of capacity utilization, consumption, inflation, and investment for n periods after an interest rate shock in period $t = 1$ and store the results in the $5n$ vector $\Psi(\boldsymbol{\theta})$. In the vector $\mathbf{x}$ we store our estimates of the impulse responses, and the

$5n \times 5n$ diagonal matrix $\hat{\Omega}$ is filled with the corresponding estimated variances. We place a grid over the four-dimensional parameter space and seek the collection $\boldsymbol{\theta}$ that provides the minimal value of

$$\big[\Psi(\boldsymbol{\theta}) - \mathbf{x}\big]'\hat{\Omega}\big[\Psi(\boldsymbol{\theta}) - \mathbf{x}\big]. \tag{D.2}$$

The result of this procedure is presented in Table 3.