Labor Turnover Costs, Workers' Heterogeneity and Optimal Monetary Policy[*]

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Abstract

We study the design of optimal monetary policy in a New Keynesian model with labor turnover costs in which wages are set according to a right to manage bargaining where the firms' counterpart is given by currently employed workers. Our model captures well the salient features of European labor market, as it leads to sclerotic dynamics of job flows. The coexistence of those types of labor market frictions alongside with sticky prices gives rise to a non-trivial tradeoff for the monetary authority. The design of optimal policy is done in two stages. We first solve for Ramsey policy, then we search for an optimal operational monetary policy rule. Our results can be summarized as follows. Monetary policy must be pro-cyclical in face of adverse shocks with the optimal volatility of inflation being an increasing function of firing costs. The optimal rule should react (negatively) to output and employment on top and above inflation.

JEL Codes: E52, E24

Keywords: optimal monetary policy, hiring and firing costs, labor market frictions, policy trade-off.

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1 Introduction

European labor markets are characterized by two main features. First, labor turnover costs in the form of hiring and firing costs are very high (see next section for stylized facts). Second, in most countries wages are bargained ex-ante at a centralized level and the bargaining process usually takes place between firms and workers currently employed. There is ample empirical evidence that large turnover costs induce much persistence in job flows, a phenomenon which has been labeled eurosclerosis[1] (see among other Kugler and SaintPaul 2004). In addition it has been noticed that labor turnover costs coupled with the inability of wages to adjust promptly to idiosyncratic shocks, e.g. due to collective bargaining processes, induces inefficiently high levels of unemployment (see Bertola and Rogerson 1997). Persistence in job flows and the inability of labor markets to adjust promptly to shocks alongside with the presence of inefficiently high unemployment rates induces severe tradeoffs for monetary policy. The literature has so far neglected them in the analysis of optimal monetary policy. There is a flourishing literature studying optimal monetary policy in New Keynesian settings whose conclusions invariably lead to support the case for the optimality of price stability policy. In most cases those prescriptions are derived in an environment which lacked any significant role for labor market frictions. We move step forward in this direction focusing on a type of labor market friction which fits well the euro area countries.

We study optimal monetary policy in a dynamic stochastic general equilibrium model (DSGE) with sticky prices and labor market frictions. The labor market we consider is characterized by two main features. First, worker/firm relations are subject to idiosyncratic operating costs and turnover costs in the form of hiring and firing costs. As worker/firm pairs are heterogenous the marginal worker is hired only when the future stream of discounted expected profits exceed hiring costs (see also Lechthaler, Merkl, and Snower 2008, for the model structure). The same holds for firing decisions. In the absence of labor turnover costs, a worker's current employment probability is independent of whether she was previously employed or unemployed, so that her retention rate is equal to her job finding rate. In the presence of hiring and firing costs, by contrast, her retention rate exceeds her job finding rate, and thus current employment depends on past employment. Such path dependence allows the model dynamic to show additional persistence compared to standard walrasian labor markets, something in line with empirical evidence. Second, the wage setting mechanism in our model follows a *right to manage bargaining*[2] which takes place between firms and

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[1] Giersch 1985 first introduced this terminology which has then been used by several others (among whom Bentolila and Bertola 1990 and Blanchard and Portugal 2001) to describe European labor markets.

[2] This feature captures well the reality of European labor markets in which wage schedules are typically determined

insider workers. The chosen structure for the wage bargain process has two implications. First, due to the right to manage structure hiring and firing decisions are taken only after the wage schedule has been determined; this implies that, consistently with empirical evidence, shocks have a larger impact on job flows than on wages. Second, if insider workers retain stronger bargaining positions than searching workers, unemployment rates tend to be inefficiently high.

Our economy features two types of distortions. On the one side, sticky prices call for optimality of zero inflation policy. On the other side, the presence of labor market frictions which generate inefficiently low levels of employment call for an active monetary policy with variable inflation rates. Those two forces produce a tradeoff for the policy maker. Importantly, the specific form of labor market frictions employed in our model allows us to highlight novel dimensions in the analysis of monetary policy tradeoffs. First, hiring and firing costs reduce labor turnover at any period in time, therefore inducing a gap compared to the economy characterized by walrasian labor markets. Second, the model features an intertemporal wedge that distorts hiring and firing decision between two subsequent periods. Indeed once workers are inside the firm they are fired only if the discounted stream of future profits is smaller than the firing costs, on the other side firms will hire only if the discounted value of future profits is bigger than the hiring costs. Because of those two wedges the retention rate, defined as the mass of workers who keep their jobs, is always bigger than the firing rate. Second, in the right to manage bargaining wages are not contingent on current employment decisions, therefore they lose part of their allocative role compared to the case with efficient Nash bargaining. Ex-post we observe inefficiently high unemployment rates. This is even more so, as wages are bargained between firms and currently employed workers, who presumably have stronger bargaining power than searching workers.

We analyze monetary policy in three steps. First, we highlight the role of wedges in driving monetary policy tradeoffs. Second, given our distorted economy we study both an unconstrained and a constrained optimal plan. In the first case the design of optimal policy follows the Ramsey approach (Atkinson and Stiglitz 1976, Lucas and Stokey 1983, Chari, Christiano and Kehoe 1991) in which the optimal path of all variables is obtained by maximizing agents' welfare subject to the relations describing the competitive economy and via an explicit consideration of all wedges that characterize both the long run and the cyclical dynamics. Recent studies apply this approach to the analyses of optimal policy in the context of New Keynesian models (Adao et al. 2003, Khan, King, and Wolman 2003, Schmitt-Grohe and Uribe 2004b and Siu 2004). In the second case the

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ex-ante through collective bargaining agreement.

monetary authority solves the same plan but with the addition of an extra constraint represented by an operational monetary policy rule: in this case the solution delivers optimal targeting rules.

We find three main results. Monetary policy should be pro-cyclical. Consider a positive productivity shock. As output and employment are below the Pareto optimal level, the monetary authority should take full advantage of the productivity improvement to push the real economy toward the Pareto frontier. In an economy with sticky prices, it can do that by increasing aggregate demand: such an increase will in turn increase firms profits, increase hiring (reduce firing) and increase employment. Second, we find that the optimal volatility of inflation is an increasing function of the firing costs. The higher the firing costs in this economy, the more pronounced are inefficient unemployment fluctuations. As the monetary authority should tradeoff between stabilizing prices, a goal achieved through zero inflation policies, and reducing inefficient unemployment fluctuations, higher firing costs tend to tilt the balance toward higher deviations from the flexible price allocations. Finally, we find that the optimal rule should target output and employment alongside with inflation. We find that the optimal rule is characterized by negative coefficients on output and employment, confirming once again the optimality of pro-cyclical policies.

Our paper is related to the recent literature that introducing labor market frictions in the form of matching frictions due to cost of posting vacancies into standard New Keynesian models.[3] Within this literature some authors have studied the design of optimal policy (see Blanchard and Galí 2008, Faia 2008, and Thomas 2008). Against this background the literature on labor markets highlighted more complex dimensions of labor market dynamic, such as hiring and firing costs and alternative wage setting mechanisms. The model employed in this paper aims at bridging the gap between the labor market literature and the macro literature. We believe that for the design of optimal monetary policy to be realistic and effective, the more complex dimensions of the labor market cannot be neglected. All in all, the model employed in this analysis adds realism to the policy analysis as its main ingredients are empirically robust and it introduces a novel dimension of monetary policy tradeoffs into DSGE models.

The rest of the paper proceeds as follows. Section 2 shows some stylized facts relating the dynamic of selected macro variable and labor turnover costs. Section 3 presents the model and highlights the role of frictions in this economy. Section 4 presents the full-fledged Ramsey plan while section 5 presents the analysis of the optimal rule. Finally section 6 concludes.

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[3]Several contributions exist. See, e.g., Walsh 2005 and Krause and Lubik 2007.

2 Labor Turnover Costs and Euro Area Labor Markets

There is a vast literature looking at the importance of labor turnover costs and employment protection legislation (with the latter considered as proxy for firing costs) for unemployment dynamics, particularly for euro area labor markets. The literature dates back to Solow 1968, Sargent 1978, Nickell 1978, 1986 who introduce adjustment costs on labor demand. More recently Bentolila and Bertola 1990 and Hopenhayn and Rogerson 1993 have shown that hiring and firing costs reduce labor turnover and make unemployment dynamics more persistent. Moreover, Hopenhayn and Rogerson 1993 and Bertola and Rogerson 1997 find that turnover costs have a sizable negative impact on unemployment, possibly leading to inefficient unemployment fluctuations. Kugler and Saint Paul 2004 show that this is even more so if turnover costs are coupled with asymmetric information. Finally, Alvarez and Veracierto 2001 find that severance payments decrease aggregate productivity and output.

Before turning to our model's implications we highlight a few stylized facts concerning the impact of labor turnover costs on the dynamic of selected macro variables, specifically inflation and output. We focus on euro area countries. Figures 2 and 3 show that there is a negative and significant relation between labor turnover costs and the volatility of output resp. inflation . As argued before, higher labor turnover costs imply that the retention rates exceed job finding rates (see Wilke 2005 for empirical evidence), and thus current employment depends on past employment. The persistence in employment carries over to output, marginal costs and therefore to inflation. The data sample covers years from the 1999 to 2008: we choose this time span for two reasons. First, we are interested in analyzing the recent implications of labor market regulations for macroeconomic dynamics. Second, we want to isolate the dynamic of macro variables from policy regime shifts. Therefore, we choose to focus on the EMU period.

As a proxy for labor turnover costs we use the employment protection legislation index (see OECD 2004), which is a weighted average of indicators capturing protection of regular workers against individual dismissals, requirements for collective dismissals and regulation of temporary employment. We choose this index because it is a more precise measure than alternative employment protection indicators.[4] Table 1 shows values of this index for various countries. It becomes immediately clear that turnover costs are much smaller in the anglo-saxon countries. European

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[4] Compared with other indicators, such as the Employment Legislation Index in Botero et al. (2003), or the hiring and firing costs calculated by the World Bank in its "Doing Business" studies, the OECD's indicator both covers a larger range of relevant aspects of LTC, and has more precise and differentiated sub-indicators. Therefore, it is the best available measure for the relative importance of LTCs in different countries.

	1998	**2003**		**1998**	**2003**
Eastern Europe			*Southern Europe*		
Czech Republic	1.94	1.94	Greece	3.49	2.90
Hungary	1.54	1.75	Italy	3.06	2.44
Poland	1.93	2.14	Portugal	3.66	3.49
Slovak Republic	2.20	1.70	Spain	2.96	3.06
AVERAGE	**1.90**	**1.88**	*AVERAGE*	**3.29**	**2.97**
Northern and Central Europe			*Anglo-saxon countries*		
Austria	2.38	2.15	Australia	1.47	1.47
Belgium	2.48	2.50	Canada	1.13	1.13
Denmark	1.83	1.83	Ireland	1.17	1.32
Finland	2.18	2.12	New Zealand	0.78	1.29
France	2.84	2.89	United Kingdom	0.98	1.10
Germany	2.64	2.47	United States	0.65	0.65
Netherlands	2.27	2.27			
Norway	2.72	2.62	*AVERAGE*	**1.03**	**1.16**
Sweden	2.62	2.62			
Switzerland	1.60	1.60	*Rest of the world*		
			Japan	1.94	1.79
AVERAGE	**2.36**	**2.31**	Korea	2.00	2.00
			Mexico	3.23	3.23
			Turkey	3.40	3.49

Figure 1: Version 2 of the EPL, including protection against collective dismissals. Source: OECD.Stat, originally published in the OECD (1999 and 2004).

countries generally have considerably higher index values than anglo-saxon countries, although even among European countries there is a lot of heterogeneity. Southern European countries tend to have a higher index value than Northern and Central European countries or Eastern European countries. The volatility of real economic activity is calculated based on a quarterly output gap measure. The seasonally adjusted real GDP series (in 2000 prices) is taken from the International Financial Statistics. The output gap is calculated as percentage deviation of output from its trend, namely the Hodrick-Prescott filter with Lambda = 1600. The inflation gap volatility is calculated in the same way.[5]

As we argue later on, the model we are presenting in the next section can account well for the abovementioned stylized facts. We therefore conclude that it is well suited as background model for monetary policy analysis.

3   The Model

Our model grafts a labor market with labor turnover costs, wage bargaining, and employed and unemployed workers onto a New Keynesian framework with Rotemberg adjustment costs. To

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[5]Note that the undetrended inflation rate delivers the same qualitative results and similar significance levels.
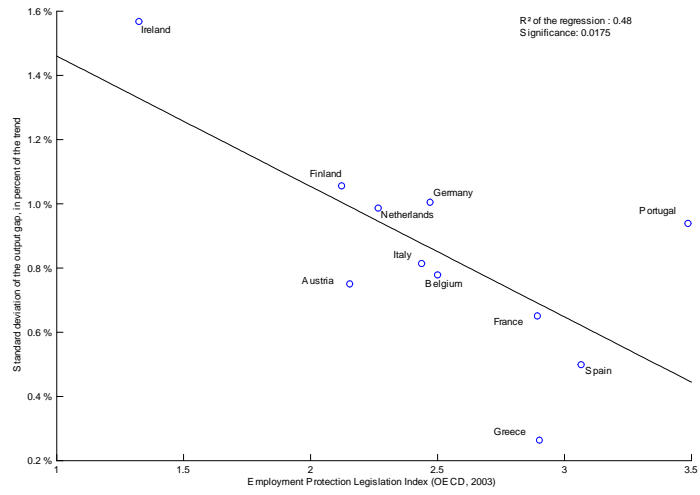
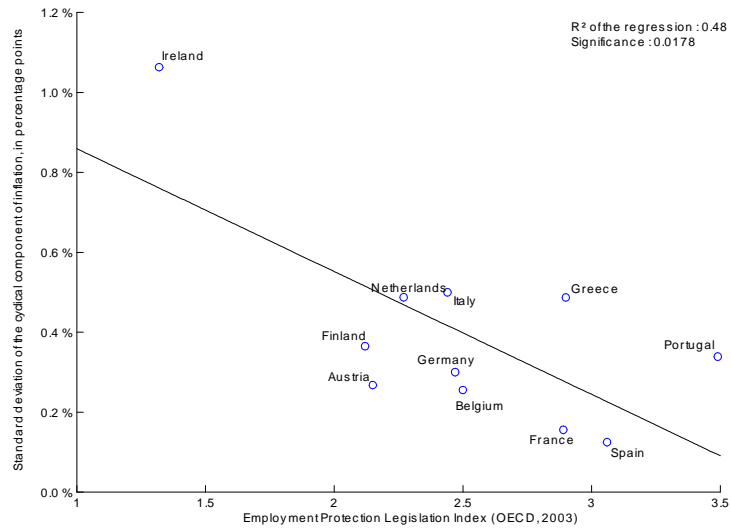Figure 2: Output gap volatility and employment protection legislation.

Figure 3: Inflation volatility and employment protection legislation.

endogenize hiring and firing decisions, it is assumed that the profitability of each worker is subject to an i.i.d. shock each period. Firms can change their price in any period but price changes are subject to quadratic adjustment costs.

3.1 Households

We assume that households have a standard utility function of the form:

$$U_t = \sum_{j=t}^{\infty} \beta^{j-t} E_t \frac{c_j^{1-\sigma}}{1-\sigma}, \tag{1}$$

where β is the household's discount factor, σ the elasticity of inter-temporal substitution, c a consumption aggregate (described below)[6] and E is the expectation operator.

As is common in the literature, we assume that each household consists of a large number of individuals, each individual supplies one unit of labor inelastically and shares all income with the other household members. This implies that consumption does not depend on a worker's employment status. Thus the representative household maximizes its utility subject to the budget constraint:

$$Bo_t + c_t P_t - T_t = W_t N_t + B_t U_t + (1 + i_{t-1})Bo_{t-1} + \Pi_{a,t}, \tag{2}$$

where Bo are nominal holdings of one period discounted bonds, P is the aggregate price level, T are tax payments, i is the nominal interest rate and Π_a are nominal aggregate profits, which are transferred in lump-sum manner, W is the nominal wage, N is the total household labor input, B the income of unemployed workers[7] and U the number of unemployed workers. The inter-temporal utility maximization yields the standard consumption Euler equation:

$$c_t = \beta E_t c_{t+1} \left(\frac{(1 + i_t)}{\pi_{t+1}} \right)^{-\frac{1}{\sigma}}. \tag{3}$$

where $(1 + i_t)$ is the nominal interest rate and π_{t+1} is the expected inflation rate.

3.2 Production and the Labor Market

There are three types of firms. (i) Firms that produce intermediate goods employ labor, exhibit linear labor adjustment costs (i.e. hiring and firing costs) and sell their homogenous products on a

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[6]In what follows capital letters refer to nominal variables and small letters refer to real variables (i.e., detrended by the price level).

[7]B can either be interpreted as home production or as unemployment benefits provided by the government (financed by lump-sum taxes).

perfectly competitive market to the wholesale sector. (ii) Firms in the wholesale sector transform the intermediate goods into consumption goods and sell them under monopolistic competition to the retailers. They can change their price at any time but price adjustments are subject to a quadratic adjustment cost à la Rotemberg 1983. (iii) The retailers, in turn, aggregate the consumption goods and sell them under perfect competition to the households.

3.2.1 Intermediate Goods Producers and Employment Dynamic

Intermediate good firms hire labor to produce the intermediate good z. Their production function is:

$$z_t = a_t N_t, \tag{4}$$

where a is technology and N the number of employed workers. They sell the product at a relative price $mc_t = P_{z,t}/P_t$, which they take as given in a perfectly competitive environment, where P_z is the absolute price of the intermediate good and P is the economy's overall price level. The variable mc_t in this economy plays the role of marginal costs as it represents the lagrange multiplier on the production function.

We assume that every worker (employed or unemployed) is subject to a random operating cost ε, which follows a *logistic* probability distribution $g(\varepsilon_t)$ over the support $-\infty$ to $+\infty$[8]. The operating costs can be interpreted as an idiosyncratic shock to workers' productivity or as a firm-specific idiosyncratic cost-shock. The firms learn the value of the operating costs of every worker at the beginning of a period and base their employment decisions on it, i.e. an unemployed worker with a favorable shock will be employed while an employed worker with a bad shock will be fired. Hiring and firing is not costless, firms have to pay linear hiring costs, h, and linear firing costs, f, both measured in terms of the final consumption good. Wages are determined through Nash bargaining between insiders and the firm. The bargaining process takes the form of a *right to manage*. This assumption leads to the following timing of events. First, the operating cost shock takes place and median insiders and the intermediate goods firm bargain over the wage. Given the wage schedule, firms make their hiring and firing decisions. Thus, firms will only hire those workers who face low operating costs and fire those workers who face high operating costs.

The hiring and firing costs induce two types of distortions (a gap and a wedge). The presence of hiring and firing costs reduce labor turnover at any period in time compared to a walrasian labor

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[8]The logistic distribution was chosen because it is very similar to the normal distribution, but in contrast to the latter there is a neat expression for the cumulative density function.

market, thereby inducing a gap between the perfectly competitive economy and our non-walrasian labor market. Second, our model features an inter-temporal wedge that distorts hiring and firing decisions between two sub-sequent periods. Indeed once workers are inside the firm they are fired only if the discounted stream of future profits is smaller than the firing costs, on the other side firms will hire only if the discounted value of future profits is bigger than the hiring costs. Because of those two wedges the retention rate, defined as the mass of workers who keep their jobs, is always bigger than the firing rate.

The operating costs, ε, are measured in terms of the final consumption good and are assumed to grow at the same rate as productivity [9]. It turns out that this ensures that technological progress does not affect the unemployment rate.

Let's now consider the real profit generated by a firm-worker relation whose operating cost are ε_t:

$$
\begin{aligned}
\tilde{\Pi}_{I,t}(\varepsilon_t) &= a_t mc_t - w_t - \varepsilon_t + \\
&+ E_t \left\{ \sum_{j=t+1}^{\infty} \Delta_{t,j} \left[\left(1-\phi_j\right)^{j-t} \left(a_j mc_j - w_j - \left(\frac{1}{1-\phi_j} \int_{-\infty}^{v_{f,j}} \varepsilon_j g(\varepsilon_j) d\varepsilon_j \right) \right) - \phi_j f_j (1-\phi_j)^{j-t-1} \right] \right\}
\end{aligned}
$$

where w is the real wage, ϕ is the separation probability, $\Delta_{t,j}$ is the stochastic discount factor from period t to j. To simplify the profit function we rewrite it in recursive manner:

$$
\tilde{\Pi}_{I,t}(\varepsilon_t) = a_t mc_t - w_t - \varepsilon_t + E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) \tag{5}
$$

where $\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})$ are future profits.

We now solve for the dynamic of employment and wages. Because of the abovementioned timing of events we solve the model backward and derive the hiring and firing decisions for *given wage schedule*. Let's define the hiring and the firing rate threshold respectively as $v_{h,t}$ and $v_{f,t}$. Hiring decisions are carried as follows. Unemployed workers are hired whenever their operating cost does not exceed a certain threshold such that the profitability of this worker is higher than the hiring cost. Thus, the hiring threshold $v_{h,t}$ is obtained by solving the following zero profit condition:

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[9]For permanent technology shocks it can be assumed the the operating, hiring and firing costs grow at the same rate as the technological progress. This ensures that the hiring and firing rates are independent of long-run technological growth (see Lechthaler, Merkl, and Snower, 2008). As we only consider mean-reverting technology shocks in this paper, we skip this assumption for analytical simplicity.

$$h_t = a_t mc_t - w_t - \upsilon_{h,t} + \tag{6}$$

$$+E_t \left\{ \sum_{j=t+1}^{\infty} \Delta_{t,j} \left[(1-\phi_j)^{j-t} \left(a_j mc_j - w_j - \left(\frac{1}{1-\phi_j} \int_{-\infty}^{\upsilon_{f,j}} \varepsilon_j g(\varepsilon_j) d\varepsilon_j \right) \right) - \phi_j f_j (1-\phi_j)^{j-t-1} \right] \right\}$$

Unemployed workers whose operating cost is lower than this value get a job, while those whose operating cost is higher remain unemployed. The resulting hiring probability is given by:

$$\eta_t = \int_{\upsilon_{h,t}}^{\infty} \varepsilon_t g(\varepsilon_t) d\varepsilon_t \tag{7}$$

Similarly, the firm will fire a worker if current losses are higher than the firing cost. Again a zero profit condition defines the firing threshold as follows:

$$-f_t = a_t mc_t - w_t - \upsilon_{f,t} + \tag{8}$$

$$+E_t \left\{ \sum_{j=t+1}^{\infty} \Delta_{t,j} \left[(1-\phi_j)^{j-t} \left(a_j mc_j - w_j - \left(\frac{1}{1-\phi_j} \int_{-\infty}^{\upsilon_{f,j}} \varepsilon_j g(\varepsilon_j) d\varepsilon_j \right) \right) - \phi_j f_j (1-\phi_j)^{j-t-1} \right] \right\}$$

and the separation rate is defined as:

$$\phi_t = \int_{\upsilon_{f,t}}^{\infty} \varepsilon_t g(\varepsilon_t) d\varepsilon_t \tag{9}$$

We are now in the position to obtain the aggregate employment evolution. The change in employment $(N_t - N_{t-1})$ is the difference between the hiring from the unemployment pool (ηU_{t-1}) and the firing from the employment pool (ϕN_{t-1}), where U_{t-1} and N_{t-1} are the aggregate unemployment and employment levels: $N_t - N_{t-1} = \eta U_{t-1} - \phi N_{t-1}$. Letting $(n_t = N_t / L_t)$ be the employment rate, we assume a constant workforce, L_t, and normalize it to one. Therefore, we obtain the following employment dynamics curve.

$$n_t = n_{t-1}(1 - \phi_t - \eta_t) + \eta_t \tag{10}$$

The unemployment rate is simply $u_t = 1 - n_t$.

3.2.2 Wage Bargaining

For simplicity, let the real wage w_t be the outcome of a Nash bargain between the median insider with operating cost ε^I and her firm. The median insider faces no risk of dismissal at the negotiated wage. The wage is renegotiated in each period t. Under bargaining agreement, the insider receives the real wage w_t and the firm receives the expected profit $(a_t mc_t - w_t)$ in each period t. Under disagreement, the insider's fallback income is b_t, assumed for simplicity to be equal to the real unemployment benefit. The firm's fallback profit is zero as during disagreement there is no production. Assuming that disagreement in the current period does not affect future surpluses, workers' surplus is $(w_t - b_t)$ while the firm's surplus is $a_t^I p_z - w_t - \varepsilon^I$, where ε^I are the operating costs of the median insider. Consequently, the Nash-product is:

$$\Lambda = (w_t - b_t)^\gamma \left(a_t^I p_z - w_t - \varepsilon^I \right)^{1-\gamma} \tag{11}$$

where γ represents the bargaining strength of the insider relative to the firm. Maximizing the Nash-product with respect to the real wage, yields the following equation:

$$(1 - \gamma)(w_t - b_t) = \gamma \left(a_t mc_t - w_t - \varepsilon_t^I \right) \tag{12}$$

which implicitly defines the negotiated wage.

Rearranging yields the following simple formula:

$$w_t = \gamma \left(a_t mc_t - \varepsilon_t^I \right) + (1 - \gamma) b_t \tag{13}$$

Two things are worth noticing. First, due to the right to manage structures wages loose part of their allocative role compared to wages negotiated within efficient Nash bargaining arrangements. In the efficient Nash bargaining individual firms would have to choose wages alongside with hiring and firing decisions. This would allow wages to be contingent on employment decisions and to adjust faster to shocks. Instead, in this context wages are negotiated at an aggregate level and firms make hiring and firing decisions only ex-post[10]. This implies, for instance, that negative shocks can affect job flows more strongly than wages. We believe such a bargaining arrangement can capture well the reality of euro area labor markets in which wages are usually bargained ex-ante

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[10]This is a particular case of a sequential bargaining framework proposed by Manning 1987, as firms and workers fail to internalize the consequences of today´s wage decisions on future hiring and firing decisions. The scope for pre-commitment is barred as neither workers nor firms can credibly commit to a sequence of future wages and employment.

at an aggregate level (collectively), while individual firms make ex-post hiring and firing decision. This type of wage mechanism gives rise to higher volatility of job flows and persistent wage dynamic consistent with data evidence for euro area countries.

Second, as the wage is bargained between current employees and firms a gap arises with the wage that would be bargained by the entire workers´ population. Since typically insiders have stronger bargaining power than searching workers, the wage negotiated by insiders produces inefficient unemployment fluctuations as it distorts at the margin the hiring decisions between two subsequent periods.

3.2.3 Marginal costs

Marginal costs in this model summarize the set of wedges that characterize the labor market. To obtain a measure of the marginal cost we should first characterize the equilibrium conditions for labor market flows. By merging equations 6 and 8 we obtain the following equilibrium condition:

$$v_{h,t} - h_t = v_{f,t} - f_t \tag{14}$$

which states that if hiring costs happen to be higher than hiring costs the hiring threshold should adjust to equilibrate the market. This condition implies that marginal costs can be equally derived from 6 or from 8. The expression for marginal costs will then read as follows:

$$mc_t = \left(w_t + v_{h,t} + h_t - E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) \right) / a_t \tag{15}$$

In this context wages loose part of their allocative role as marginal costs depend also on two additional components. The first component which is given by $-v_{h,t} + h_t$ is an intra-temporal wedge which makes hiring (and firing) deviate from the ones that would arise in a walrasian labor market at any time t. The second component, represented by $E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1}))$, is an inter-temporal wedge which distorts hiring (and firing) decisions between two sub-sequent dates. This second wedge represents the long run value of a worker as by retaining the marginal worker the firm can earn extra profits in the future. Because of this positive externality attached to the marginal worker, retention rates tend to be higher than job finding rates.

3.2.4 Wholesale Sector and Retail Sector

Firms in the wholesale sector are distributed on the unit interval and indexed by i. They produce a differentiated good $y_{i,t}$ using the linear production technology $y_{i,t} = z_{i,t}$, where $z_{i,t}$ is their demand

for intermediate goods. They sell their goods under monopolistic competition to the retailers who use the differentiated goods to produce the final consumption good according to the Dixit-Stiglitz-aggregator:

$$y_t = \left(\int y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{16}$$

which delivers the standard price index (where $P_{i,t}$ and $y_{i,t}$ denote the firm specific price and output level respectively):

$$P_t = \left(\int P_{i,t}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \tag{17}$$

from the cost minimization problem of the aggregating firm. The implied demand function for differentiated products is:

$$y_{i,t} = y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \tag{18}$$

Firms in the wholesale-sector can change their prices every period, facing quadratic price adjustment costs a la Rotemberg. They maximize the following profit function:

$$\tilde{\Pi}_{W,t} = E_t \sum_{j=t}^{\infty} \Delta_{t,j} \left[\frac{P_{i,j}}{P_j} y_{i,j} - mc_j y_{i,j} - \frac{\Psi}{2} \left(\frac{P_{i,j}}{P_{i,j-1}} - \bar{\pi} \right)^2 y_{i,j} \right]$$

where Ψ is a parameter measuring the extent of price adjustment costs and $\bar{\pi}$ is the steady state inflation rate. Taking the derivative with respect to the price yields after some manipulations the standard price-setting rule under Rotemberg adjustment costs:

$$
\begin{aligned}
0 = \ & (1-\varepsilon)\beta c_t^{-\sigma} + \varepsilon \beta c_t^{-\sigma} mc_t - \beta c_t^{-\sigma} \Psi (\pi_t - \bar{\pi}) \pi_t \\
& + \beta E_t \{ c_{t+1}^{-\sigma} \Psi (\pi_{t+1} - \bar{\pi}) \frac{y_{t+1}}{y_t} \pi_{t+1} \}.
\end{aligned}
\tag{19}
$$

3.3 Workers' Heterogeneity and Aggregation

As this economy features firms/workers heterogeneity some assumptions are needed to obtain aggregation. We start by deriving aggregate real profits of intermediate firms which are given by revenues minus wage payments, operating costs and labor turnover costs:

$$\tilde{\Pi}_I = mc_t a_t n_t - w_t n_t - (1 - \phi_t)n_t \Xi_t^i - \qquad (20)$$
$$(1 - n_t)\eta_t \Xi_t^e - n_t \phi_t f_t - (1 - n_t)\eta_t h_t$$

where Ξ_t^i is the expected value of operating costs for insiders, conditional on not being fired and Ξ_t^e is the expected value of operating costs for entrants, conditional on being hired, defined by:

$$\Xi_t^e = \frac{\int_{-\infty}^{v_h} \epsilon_t f(\epsilon_t) d\epsilon_t}{\eta_t}$$
$$\Xi_t^i = \frac{\int_{-\infty}^{v_f} \epsilon_t f(\epsilon_t) d\epsilon_t}{1 - \phi_t}$$

The real profits ($\tilde{\Pi}_W$) of the wholesale sector are given by:

$$\tilde{\Pi}_W = y_t - p_{z,t} a_t n_t - \frac{\Psi}{2} (\pi_t - \bar{\pi})^2 y_t$$

Retailers make zero-profits. Aggregate real profits in this economy therefore are given by:

$$\tilde{\Pi}_{a,t} = y_t - w_t n_t - n_t \phi_t f_t - u_t \eta_t h_t - (1 - \phi_t)n_t \Xi_t^i) - \qquad (21)$$
$$(1 - n_t)\eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - \bar{\pi})^2 y_t$$

We can substitute this into the budget constraint, (2), and after imposing equilibrium in the bond market we obtain the following resource constraint:

$$c_t = y_t - n_t \phi f_t a_t - (1 - n_t)\eta_t h_t - (1 - \phi_t)n_t \Xi_t^i - \qquad (22)$$
$$(1 - n_t)\eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - \bar{\pi})^2 y_t$$

Inspection of the resource constraint makes clear the role of wedges and gaps in this economy.

3.4   Competitive Equilibrium and the Role of Wedges in This Economy

Definition 1. *For a given nominal interest rate $\{i_t\}_{t=0}^{\infty}$ and for a given set of the exogenous processes $\{z_t\}_{t=0}^{\infty}$ a determinate competitive equilibrium for the distorted competitive economy is a sequence of allocations and prices $\{c_t, \pi_t, mc_t, v_{h,t}, v_{f,t}, w_t, y_t\}_{t=0}^{\infty}$ which, for given initial B_0 satisfies equations 3, 6, 8, 10, 12, 19, 22.*

The competitive equilibrium in this economy is distorted by the wedges induced by hiring and firing costs. This induces a monetary policy tradeoff as on the one side the monetary authority would like to keep inflation stable by closing the gap with the flexible price allocation, while on the other it is tempted to run active policy to reduce the inefficiently high unemployment rate. The role of tradeoffs in this economy can be well understood by deriving the reduced form expression for the Phillips curve. By substituting the marginal cost expression, 15, into the Phillips curve, equation 19, we obtain the following expression:

$$0 = (1-\varepsilon) + \frac{\varepsilon}{a_t}(w_t + \upsilon_{h,t} + h_t - E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1}))) - \Psi(\pi_t - \bar{\pi})\pi_t + \beta E_t\{\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\Psi(\pi_{t+1} - \bar{\pi})\frac{y_{t+1}}{y_t}\pi_{t+1}\}$$
(23)

which together with the wage equation, 12, leads to a reduced form Phillips curve.

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{\varepsilon\frac{W}{A}}{\Psi}\hat{w}_t + \frac{\varepsilon\frac{\upsilon_h}{A}}{\Psi}\hat{\upsilon}_{h,t} - \beta\frac{\varepsilon\frac{\tilde{\Pi}_I}{A}}{\Psi}E_t\hat{\Pi}_{I,t+1} - \frac{\varepsilon(\varepsilon-1)}{\Psi}\hat{a}_t$$
(24)

A comparison between the latter reduced form and the one arising in standard New Keynesian models with walrasian labor markets allows to highlight the nature of the tradeoffs. In standard DNK models the Phillips curve takes the following form:

$$0 = (1-\varepsilon) + \varepsilon(\frac{w_t}{a_t}) - \Psi(\pi_t - \bar{\pi})\pi_t + \beta E_t\{\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\Psi(\pi_{t+1} - \bar{\pi})\frac{y_{t+1}}{y_t}\pi_{t+1}\}$$
(25)

In response to a productivity shock the marginal cost is unaffected as w_t moves one to one with the shock. This implies that inflation is unaffected too and that monetary policy should stay passive. In this context price stability policy allows the policy maker to achieve the flexible price allocation, which also corresponds to the Pareto optimal allocation. On the the other side in equation 23 the extra component represented by $-\upsilon_{h,t} + h_t + E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\upsilon_{h,t+1}, \upsilon_{f,t+1}))$ plays the role of an *endogenous cost push shock*. In response to positive productivity shocks both the hiring threshold, $\upsilon_{h,t}$, and future profits, $\tilde{\Pi}_{I,t+1}$, increase. Given that the hiring threshold rises more than future profits, we should expect an increase in marginal costs, which in turn would lead to an increase in inflation. In this context the flexible price allocation is simply not feasible any longer. Depending on whether marginal costs rise or decline, monetary policy would have to be pro-cyclical or counter-cyclical.

The reduced form Phillips curve described in equation 23 allows us to derive the first result concerning the optimal policy design in this set-up.

Proposition 1. For the model economy described in **Definition 1,** a flexible price allocation is not feasible, therefore not implementable under zero inflation policies.

Proof. In a model with monopolistic firms the flexible price allocation requires firms pricing at constant mark-ups:

$$\frac{\varepsilon - 1}{\varepsilon} = \frac{w_t}{a_t} \tag{26}$$

which implies that wages must adjust one to one with productivity. In a walrasian world this is possible. In the model economy described in **Definition 1** wages have limited allocative role. Imposing zero inflation on equation 23 we obtain the following relation:

$$\frac{(\varepsilon_t - 1)}{\varepsilon_t} = \frac{(w_t + v_{h,t} + h_t - E_t(\Delta_{t,t+1}\tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})))}{a_t} \tag{27}$$

which can be verified only with time varying mark-ups. Therefore the flexible price allocation with constant mark-ups cannot be implemented.

In general, the monetary authority in our model is tempted to run active policies to reduce the inefficiently high unemployment rate. The transmission mechanism, through which it can do this, can be well understood by looking at the discounted value of future profits. By forward substitution in equation 5 we obtain:

$$\tilde{\Pi}_{I,t}(\varepsilon_t) = (a_t mc_t - w_t - \varepsilon_t) + E_t \left(\sum_{i=1}^{\infty} \Delta_{t,t+i} \left[(1 - \phi_i)^i (a_{t+i} mc_{t+i} - w_{t+i} - \varepsilon_{t+i+1}) - \phi_i f (1 - \phi_i)^{i-1} \right] \right) \tag{28}$$

Consider now the zero profit condition used to obtain the hiring threshold. If we set $\tilde{\Pi}_{I,t}(\varepsilon_t) = h_t$ we obtain:

$$h_t + v_{h,t} = (a_t mc_t - w_t) + E_t \left(\sum_{i=1}^{\infty} \Delta_{t,t+i} \left[(1 - \phi_i)^i (a_{t+i} mc_{t+i} - w_{t+i} - \varepsilon_{t+i+1}) - \phi_i f (1 - \phi_i)^{i-1} \right] \right) \tag{29}$$

For given hiring costs the monetary authority can increase the hiring threshold (therefore employment) by increasing firms' future profits. This can be done by increasing firms' marginal revenues, $a_{t+i} mc_{t+i}$, which in turn implies by reducing mark-ups. By leveraging on countercyclical mark-ups the monetary authority can boost firms profits and increase the incentive that firms have in hiring.
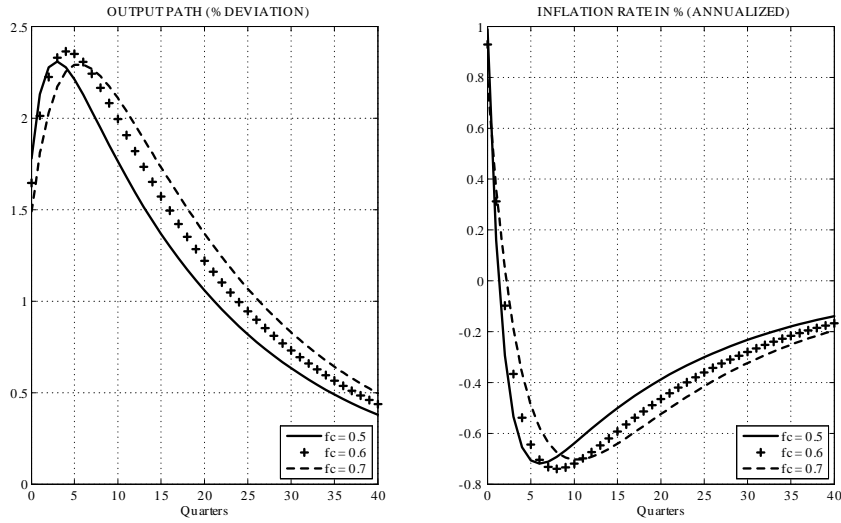
Figure 4: Model Reaction under Different Taylor Rules

3.5 Dynamic Properties of the Model

Before we turn to optimal monetary policy (which is the main focus of this paper), we want to show the ability of our model to replicate the empirical evidence on labor turnover costs and macroeconomic volatilities. We simulate our model with three different firing cost levels (fc=0.5, 0.6, and 0.7) and a rule targeting inflation. Figure 4 shows the model response to a productivity shock with autocorrelation 0.95. The optical inspection shows that the model persistence (volatility) increases (decreases) with the level of firing costs.

To make our model results comparable to the empirical graphs shown further above, we calculate the model's HP-filtered standard deviation for a joint productivity and government spending shock under the different firing cost levels. Table 2 confirms that the volatility of our model economy decreases with the level of firing costs. Thus, our model is in line with the empirical evidence.

Standard Deviations	fc=0.5	fc=0.6	fc=0.7
Inflation	0.68	0.60	0.59
Output	0.40	0.34	0.29

4 Optimal Ramsey Policy

We now turn to the specification of a general set-up for the optimal policy conduct. The optimal policy plan is determined by a monetary authority that maximizes the discounted sum of utilities of

all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey 1983. There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and other frictions, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King, and Wolman 2003 adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe 2004a analyze a problem of joint determination of optimal monetary and fiscal policy.

$$a_t mc_t - w_t - v_{h,t} + E_t(\beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) = h_t \tag{30}$$

$$a_t mc_t - w_t - v_{f,t} + E_t(\beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \tilde{\Pi}_{I,t+1}(\varepsilon_{t+1})) = -f_t \tag{31}$$

$$n_t = n_{t-1}(1 - \phi_t - \eta_t) + \eta_t \tag{32}$$

$$w_t = \gamma \left(a_t mc_t - \varepsilon_t^I \right) + (1 - \gamma) b_t \tag{33}$$

$$0 = (1 - \varepsilon) + \varepsilon(mc_t) - \Psi (\pi_t - \bar{\pi}) \pi_t + \beta E_t \{ \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \Psi (\pi_{t+1} - \bar{\pi}) \frac{y_{t+1}}{y_t} \pi_{t+1} \} \tag{34}$$

$$c_t = y_t - n_t \phi f_t - (1 - n_t)\eta_t h_t - (1 - \phi)n_t a_t \Xi_t^i - \\ (1 - n_t)\eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - \bar{\pi})^2 y_t \tag{35}$$

We do not need to include the government resource constraints among the equilibrium conditions as we assume absence of distortionary taxation.

Definition 2. *Let* $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (30), (31), (32), (33), (34), (35). Let* k_0, *be given. Then for a given stochastic process* $\{z_t\}_{t=0}^{\infty}$, *plans for the control variables* $\Xi_t^n \equiv \{c_t, n_t, w_t, mc_t, \pi_t, v_{h,t}, v_{f,t}\}_{t=0}^{\infty}$ *and for the co-state variables* $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}} Max_{\{\Xi_t^n\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \tag{36}$$

subject to (30), (31), (32), (33), (34), (35).

4.1 Non-Recursivity and Initial Conditions

As a result of constraints (30), (31) and (34) exhibiting future expectations of control variables, the maximization problem as spelled out in (36) is intrinsically non-recursive. As first emphasized in Kydland and Prescott 1980, and then developed by Marcet and Marimon 1999, a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that we denote $\chi_{1,t}$, $\chi_{2,t}$ and $\chi_{3,t}$ for (30), (31) and (34) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:

$$
\begin{aligned}
\frac{\chi_{1,t+1}}{\beta} &= \lambda_{1,t} \\
\frac{\chi_{2,t+1}}{\beta} &= \lambda_{2,t} \\
\frac{\chi_{5,t+1}}{\beta} &= \lambda_{5,t}
\end{aligned}
\tag{37}
$$

Using the new co-state variables so far described we amplify the state space of the Ramsey allocation to be $\{a_t, \chi_{1,t}, \chi_{2,t}, \chi_{5,t}\}_{t=0}^{\infty}$ and we define a new saddle point problem which is recursive in the new state space. Consistently with a *timeless perspective* we set the values of the three co-state variables at time zero equal to their solution in the steady state. We return to this point in the next subsection.

4.2 Model Calibration

Calibration is summarized in table 1 below. In what follows we discuss in detail the calibrated parameters.

Preferences. We use a standard quarterly discount rate of one percent, $\beta = 0.99$, and a log-utility of consumption, $\sigma = 2$. The elasticity of substitution between different product types, ε, is set to 10 (see, e.g., Galí 2008).

Pricing. The parameter of price adjustments, Ψ, is calibrated in line with microeconometric evidence for Europe (see Alvarez et al. 2006).

Labor markets. The bargaining power of workers, γ, is set to the standard value 0.5. We take continental Europe as our reference point. We set the firing costs to 60 percent ($f = 0.6$)

of the annual productivity which amounts to approximately 88 percent of the annual wage[11] and the hiring costs to 10 percent ($h = 0.1$) of annual productivity (see Chen and Funke 2003). We set the unemployment benefits to 50 percent of the level of productivity ($b = 0.5$). This implies, that in steady state the wage replacement rate is roughly 73 percent, which is in line with evidence for continental European countries (see OECD 2007). Operating costs are assumed to follow a logistic distribution. The two parameters of the distribution ($E(\varepsilon)$ and sd) are chosen in such a way that the resulting labor market flow rates match the empirical hiring and firing rates described further below. This yields an expected value of $E(\varepsilon) = 0.23$ and a scale parameter of 0.53. We calibrate our flow rates using evidence for West Germany, as there are only Kaplan-Meier functions for individual countries.[12] Wilke's 2005 Kaplan-Meier functions indicate that about 20 percent of the unemployed leave their status after one quarter. For a steady state unemployment rate of 9 percent, a quarterly job finding rate of 2 percent is necessary. This is roughly in line with Wilke's estimated yearly risk of unemployment. The used flow numbers are in line with the OECD, 2004, numbers for other continental European countries.[13] We conclude that a quarterly job hiring rate of $\eta = 0.20$ and a firing rate of $\phi = 0.02$ are reasonable averages for continental European countries.

Shocks. We simulate the model under productivity shocks which follow $AR(1)$ processes. The autocorrelation is set to 0.95 and the standard error of the shock is 0.008. Government consumption evolves according to the following exogenous process, $\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g$, where the steady-state share of government consumption, g, is set so that $\frac{g}{y} = 0.25$ and ε_t^g is an i.i.d. shock with standard deviation σ_g. Empirical evidence for the US in Perotti 2004 suggests $\sigma_g = 0.0074$ and $\rho_g = 0.9$.

4.3 Optimal Policy in the Long run

Before turning to the analysis of the dynamic path of the optimal policy plan in response to shocks we need to characterize the log-run optimal policy. We now characterize the steady state around which the dynamic optimal plan evolves. In analogy with the Ramsey-Cass-Koopmans

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[11]For the period from 1975 to 1986 Bentolila and Bertola 1990 calculate firing costs of 92 percent, 75 percent and 108 percent of the respective annual wage in France, Germany and Italy respectively. The OECD 2004 reports that many European countries have reduced their job security legislation somewhat from the late 1980 to 2003 (in terms of the overall employment protection legislation strictness). Therefore, we consider $f = 0.6$ to be a realistic number for continental European countries.

[12]We choose the Kaplan-Meier functions for Germany, as it is the largest continental European country.

[13]Although the numbers of the OECD outlook are not directly applicable to our model, since they are built on a monthly basis, it is possible to adjust them using a method described in Shimer 2007.

Table 1:   Parameters of the Numerical Model

Parameter	Description	Value	Source
β	Subjective discount factor	0.99	Standard value
σ	Consumption utility	2	Log-utility
ε	Elasticity of subst.	10	Gali 2003
Ψ	Price adjustment cost	104.85	Equivalent to $\theta = 0.75$
a	Productivity	1	Normalization
γ	Workers' bargaining power	0.5	Standard value
f	Firing cost	0.6	Bentolila and Bertola 1990
h	Hiring cost	0.1	Chen and Funke 2003
b	Unemployment benefits	0.5	OECD 2007
$E(\varepsilon)$	Expected value of op. costs	0.23	To match the flow rates
sd	Distr. scaling parameter	0.53	To match the flow rates

model, such steady state amounts to computing the *modified golden rule* steady state[14]. The unconstrained optimal long-run rate of inflation (arising from the modified golden rule) is the one to which the planner would like the economy to converge to if allowed to undertake its optimization unconditionally. It is obtained by imposing steady state conditions ex-post on the first order conditions of the Ramsey plan. Notice in particular that the first order condition with respect to inflation reads as follows:

$$(\lambda_{5,t} - \chi_{5,t})c_t^{-\sigma}\psi(2\pi_t - 1) - \lambda_{6,t}\psi(\pi_t - 1) = 0 \tag{38}$$

For the whole set of optimality conditions associated with the Ramsey plan to be satisfied in the steady state a necessary condition is that equation (38) is satisfied in the steady state. In that steady-state, we have $\lambda_{5,t} = \lambda_{5,t-1} = \chi_{5,t}$. Hence condition (38) immediately implies:

$$\lambda_6 \psi(\pi - 1) = 0 \tag{39}$$

Since $\lambda_6 > 0$ (the resource constraint must hold with equality), and $\psi > 0$ (we are not imposing *a priori* that the steady-state coincides with the flexible price allocation), in turn (39) must imply $\pi = 1$. Hence the Ramsey planner would like to generate an average (net) inflation rate of zero.

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[14]Notice that an important distinction must be made between the optimal level of inflation characterizing the *modified golden rule* and the one characterizing the *golden rule (*See also King and Wolman 1996*)*. In dynamic economies with discounted utility in fact the two level of inflations do not necessarily coincide. The golden rule level of inflation is the one that maximizes households' instantaneous utility under the constraint that the steady state conditions are imposed ex-ante. The impatience reflected in the rate of time preferences gives rise to a negatively sloped long run Phillips curve which by constraining the optimal policy problem maintains alive the tension between closing the inflation gap on the one side and the inefficient unemployment gap on the other.

The intuition for why the long-run optimal inflation rate is zero is simple. Under commitment, the planner cannot resort to ex-post inflation as a device for eliminating the inefficiency related to the goods and labor markets. Hence the planner aims at choosing that rate of inflation that allows to minimize the cost of adjusting prices as summarized by the quadratic term $\frac{\vartheta}{2}(\pi_t - 1)^2$.

4.4 Response to Shocks and Optimal Volatility of Inflation

To compute dynamic responses of the optimal plan to shocks we resort on second order approximations[15] of the first order conditions of the Lagrangian problem described in definition 2. Technically, we compute the stationary allocation that characterizes the deterministic steady state of the first order conditions to the Ramsey plan. We then compute a second order approximation of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy has been conducted around such a steady state already for a long period of time (under timeless perspective).

Figure 5 shows impulse response functions of the Ramsey plan to positive productivity shocks. In response to an increase in productivity consumption, output and employment go up. The firing threshold goes up implying that there is a reduction in the mass of firings; on the other side the hiring threshold goes down implying that there is an increase in the mass of hiring. Most importantly inflation deviates significantly from zero and falls, implying that monetary policy behaves pro-cyclically. The monetary authority in this context has a tradeoff between stabilizing inflation and reducing inefficient unemployment fluctuations. The latter task can be accomplished by taking full advantage of the improved production possibilities. Therefore in response to an improvement in the production possibilities, the monetary authority reduces the interest rate. The fall in the interest rate by increasing aggregate demand, increases marginal costs, reduces the markups and increases marginal profits. For given hiring and firing costs, the increase in marginal profits (today and in the future) reduces the hiring threshold and increases the firing threshold, as shown by the zero profit condition 28. It is important to notice that contrary to traditional New Keynesian models our model induces deviations from price stability even in response to productivity shocks. This is so since, due to the type of labor market frictions considered, productivity enters the Phillips curve in the same way that traditional costs push shocks do. This can be clearly seen from the loglinear version of the Phillips curve, 24.

The higher are the hiring and firing costs, the higher are the incentives for the policy to deviate

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[15]Second order approximation methods have the particular advantage of accounting for the effects of volatility of variables on the mean levels of the same. See Schmitt-Grohe and Uribe (2004b) among others.

from the zero inflation policy since the welfare costs of inefficient unemployment fluctuations are larger. This can be seen from figure 7 where we show that optimal volatility of inflation, as computed in response to both productivity and government expenditure shocks, increases when firing costs increase.

Finally, figure 6 shows impulse responses of the Ramsey plan in response to government expenditure shocks. An increase in government expenditure crowds out consumption demand. However because of the increase in aggregate demand employment increases, the mass of firings shrinks and the mass of hirings rises. Once again we observe deviations from price stability. This result is consistent with the literature as Adao, Correia and Teles 2003 and Khan, King and Wolman 2003 have shown that shocks to government expenditure cause fluctuations in the ratio between aggregate demand and output which prevent implementability of the flexible price allocation with constant mark-ups. Notice however that consistently with previous studies deviations from zero inflation are rather small under this shock.

5   Optimal Operational Rules

So far our analysis indicated what should be the optimal path of variables if the economy had been run by a Ramsey planner. We follow here a Tinbergen approach and ask what type of operational[16] rules central bankers should follow in the context of our model if they want to maximize agents´ welfare. We therefore solve the problem of a monetary authority which maximizes households welfare subject to the competitive equilibrium conditions and the class of monetary policy rules represented by:

$$
\begin{aligned}
\ln\left(\frac{1+r_t^n}{1+r^n}\right) &= (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_n \ln\left(\frac{n_t}{n}\right)\right) \\
&\quad + \phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right)
\end{aligned}
\tag{40}
$$

where ϕ_π represents a response to the CPI inflation rate, ϕ_y represents a response to the output gap, ϕ_n represents the response to unemployment and ϕ_r represents interest rate smoothing. Within this class of rules, we look for the coefficients which deliver the highest level of welfare. Note that we allow interest rates to react to unemployment alongside with inflation. The reason for that is as follows. In our model the policy maker faces a tradeoff between inflation and unemployment stabilization due to the inefficiencies associated with the labor market. Because of this we expect

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[16]The word operational indicates two things: a) a rule that responds to variables which can be easily observed and b) a rule that delivers real determinacy.

a rule targeting unemployment alongside with inflation to perform better than a rule neglecting labor market variables.

Some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare associated to each monetary policy arrangement. Indeed in an economy with a distorted steady state, stochastic volatility affects both first and second moments of those variables that are critical for welfare. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions.[17] Additionally one needs to focus on the *conditional* expected discounted utility of the representative agent. This allows to account for the transitional effects from the deterministic to the different stochastic steady states respectively implied by each alternative policy rule. Define Ω as the fraction of household's consumption that would be needed to equate conditional welfare $\mathcal{W}_0$ under a generic interest rate policy to the level of welfare $\widetilde{\mathcal{W}}_0$ implied by the optimal rule. Hence Ω should satisfy the following equation:

$$\mathcal{W}_{0,\Omega} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1+\Omega)c_t) \right\} = \widetilde{\mathcal{W}}_0$$

Under a given specification of utility one can solve for Ω and obtain:

$$\Omega = \exp\left\{ \left(\widetilde{\mathcal{W}}_0 - \mathcal{W}_0\right)(1-\beta) \right\} - 1$$

In our simulations we search for the rule that maximizes welfare. In response to both productivity and government expenditure shocks the optimal rule features the following coefficients: $\phi_r = 0.00765794, \phi_\pi = 1.66628, \phi_y = -0.117797, \phi_n = -0.055$. Also figure 8 reports the response of selected macro variables to productivity shocks and shows that inflation deviates significantly from zero. Two main results arise. First, the optimal rule targets two measures of the real activity, namely output and employment. Since the economy features frictions that induce inefficiently low levels of output and employment the monetary authority should play an active role. Second, consistently with the results obtained by solving the global Ramsey plan, we see that optimal rules should be pro-cyclical: In response to a raise in output and employment, the interest rate must fall. This is so since the monetary authority must take full advantage of positive demand and productivity shocks in order to push output and employment toward the Pareto optimal level.

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[17]See Kim and Kim 2003 for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.

6 Conclusions

We study the design of optimal monetary policy in an economy with sticky price, labor turnover costs and insider bargaining. The assumptions we made on the structure of the labor market captures well the reality of euro area labor markets. These types of labor market frictions give rise to non trivial trade-offs for the monetary authority. We show that optimal policy features deviations from price stability and that those deviations are larger the larger the size of firing costs. We also search for the optimal operational rule and find that it should respond to unemployment alongside with inflation.

Our paper provides a theoretical and a policy contribution. From a theoretical point of view our analysis shows that the case for price stability can be challenged if one considers a model with a significant role for real frictions. We stress that optimal monetary policy in the presence of real frictions can be usefully characterized by applying a Ramsey-type analysis. Our analysis carries also some important policy implications. Before the Lisbon agenda is brought to completion and structural reforms in the labor market are implemented, an active role for the monetary authority is needed to overcome some of the welfare costs generated by turnover costs and insider bargaining.

A natural extension of our analysis would be to study the type of labor market frictions considered here within a currency area model. Euro area countries face significant differences in terms of labor market institutions. An analysis of those differences could shed light on the differential response of output and inflation to common monetary policy actions.

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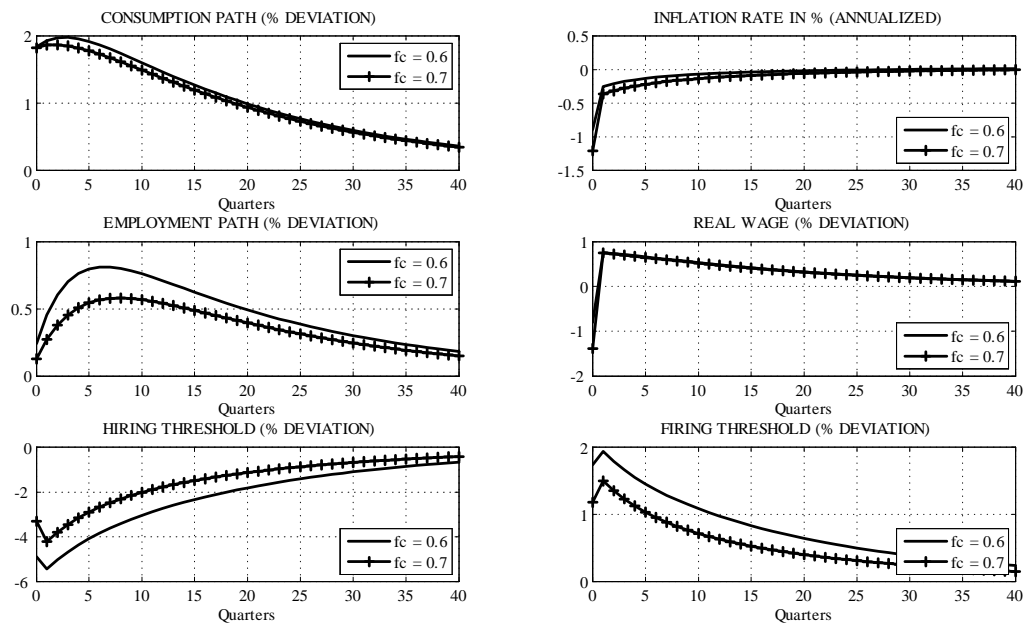
Figure 5: Impulse responses of Ramsey plan to productivity shocks (under two different firing costs).
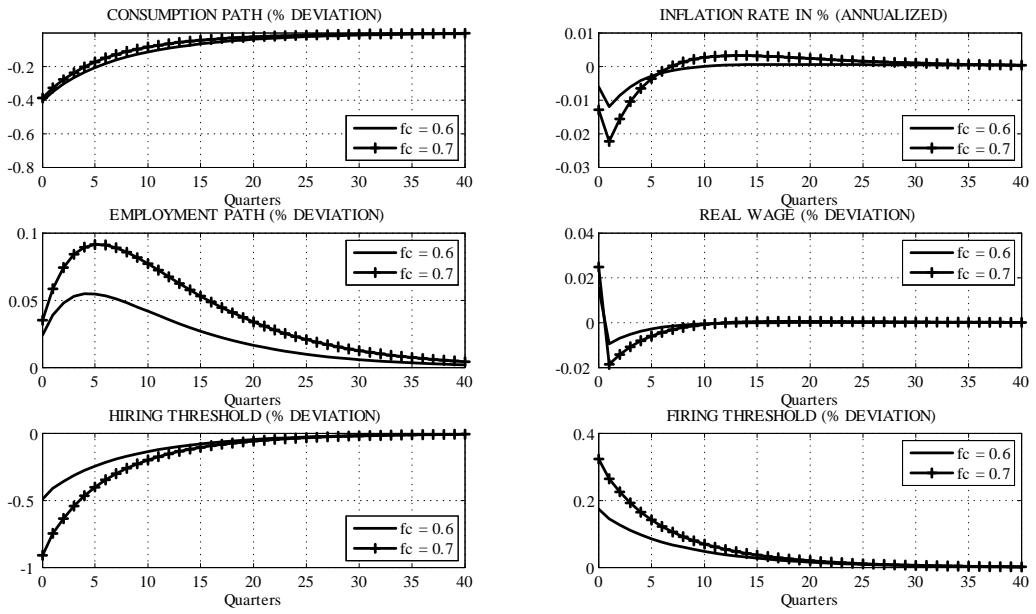
Figure 6: Impulse responses of Ramsey allocation to government expenditure shocks (under two different firing costs)
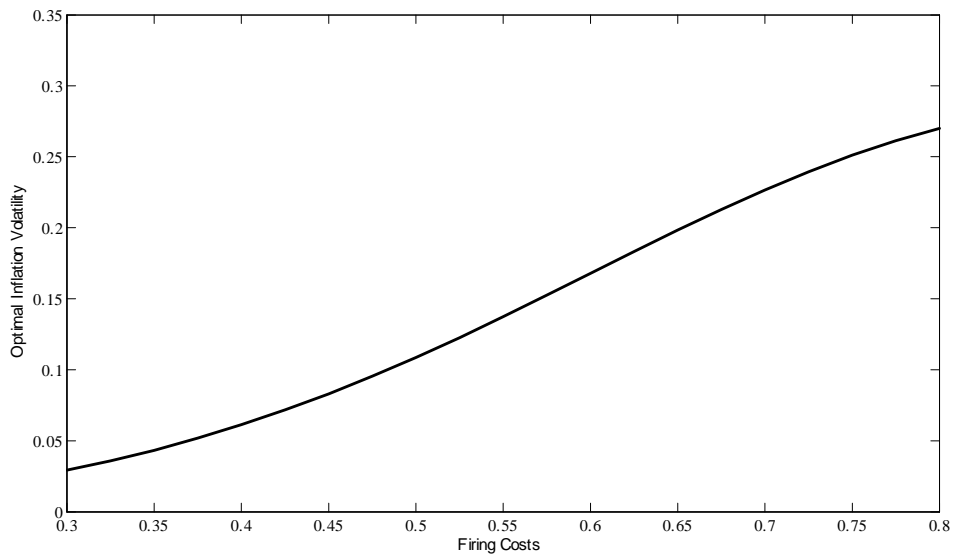

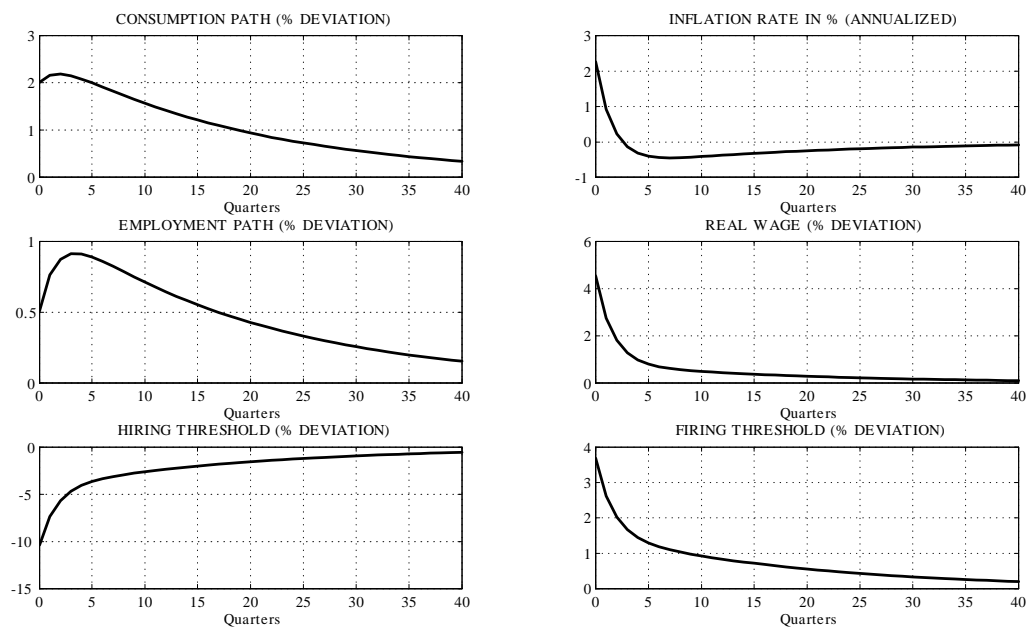Figure 7: Optimal inflation volatility in response to the two shocks.

Figure 8: Impulse response of optimal simple rule to productivity shock.