

Corporate taxes and the location of intellectual property

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Abstract

The income derived from intellectual property is highly mobile; firms can and do locate patents separately from other aspects of their operations. This continues to raise challenges for governments in determining how to tax such income. We estimate the impact of corporate taxes on where innovative European multinationals choose to hold patents. We consider both source and residence country taxes, and control for the potential non-tax benefits associated with different locations. We allow heterogeneity across industries, firm size and, most importantly, we use a random coefficients model to capture unobservable patent specific heterogeneity in the responsiveness of patent location choice to tax. Our results suggest that, on average, corporate tax rates have a negative impact on the likelihood of a firm choosing a location, and that there is important heterogeneity in responses. We simulate the impact of recent reforms that apply a lower tax rate to patent income, finding that they attract patent income but result in non-trivial losses in government revenues.

Keywords: corporate tax; intellectual property; multinational firms; Patent Box

JEL classification: F21; F23; O3; H3

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1 Introduction

The taxation of corporate income is an issue that has vexed academics and policymakers for some considerable time. The growing importance of intellectual property as a factor in production¹ presents increased challenges. In particular, intellectual property is more mobile than other forms of capital, making it more difficult for governments to ascertain the associated tax burden and easier for firms to relocate taxable income. As noted by a tax lawyer quoted in the New York Times, "... most of the assets that are going to be reallocated as part of a global repositioning are intellectual property ... that is where most of the profit is".² There are, however, both costs and benefits to be traded off when selecting where to hold intellectual property meaning that firms do not always select the lowest tax countries. For example, anti-profit shifting rules may diminish the tax benefits of selecting a low tax location while the relative costs of establishing a subsidiary to hold intellectual property may make a higher tax country more attractive. Indeed, relatively little intellectual property is held in tax havens.

In this paper we estimate the impact of corporate income tax on innovative European multinationals' choices over where to hold patents, an important form of intellectual property. We expect there to be considerable heterogeneity in where patents are located and how responsiveness such choices are to tax. An important contribution of this paper is to set out a model of firm behaviour which, in addition to controlling for observable firm characteristics and location specific factors, explicitly accounts for unobservable patent heterogeneity in the

¹ NESTA (2009) estimates that in the UK knowledge investment overtook fixed capital investment in the mid-1990s and is now about 50% higher. OECD (2006, p34) describes the growing significance of intellectual property and its simultaneous use by many different parts of a firm as "*one of the most important commercial developments in recent decades.*"

² See "Key Company Assets Moving Offshore," New York Times, Nov. 22, 2002

responsiveness of location choices to tax. We do this through estimating a discrete choice demand model with random coefficients that capture unobservable heterogeneity and allow the model to produce realistic substitution patterns that do not exhibit the restrictive Independence of Irrelevant Alternatives (IIA) property. The use of these models is now common in many other literatures in economics, for example, transport (McFadden (1974) and Train (2003)) and demand (Berry, Levinsohn and Pakes (1995, 2004), Nevo (2001)).³ Our structural model yields estimates of the elasticity of location choice to tax that are both location specific and a function of the tax rates firms will face in every possible location. This is distinct from most of the current literature on firms' responsiveness to tax which estimate a single elasticity.⁴

We simulate the impact of recently introduced Patent Boxes that give preferential tax treatment to income arising from patents. We find that they have significant effects on the location of intellectual property, and are likely to lead to non-trivial reductions in tax revenue. Patents are legal documents that grant a firm the exclusive rights to use (or license) a novel technology for a specified period of time. While tax laws constrain the extent to which firms can lower their tax bill through location decisions, patents can be held at a distance to a firm's headquarters, to the location where the underlying technology was created and to the location where it will be applied.⁵ Indeed, the share of European Patent Office (EPO) patent applications that UK multinationals hold in offshore subsidiaries had increased from 8% in 1985 to 30% by 2005. The location of the subsidiary in which a firm holds a patent will in part determine where the income from the patent is taxed, providing firms with an

³ For a recent application of a discrete choice model to firms' location decisions see Cohen (2009).

⁴ That is, papers in the current literature tend to estimate (or use) a single estimate of the responsiveness of location choice (usually of capital) to tax that does not vary across countries and that is indifferent to tax policy changes.

⁵ Lipsey (2008) notes that, in multinational firms, intangible assets "have no clear geographical location, but only a nominal location determined by the parent company's tax or legal strategies".

opportunity to reduce tax payments by holding a patent in a lower tax jurisdiction. For example, Microsoft reportedly saved at least \$500m in taxes by licensing its intellectual property from an Irish subsidiary.⁶ There are also important interactions between tax jurisdictions as a result of Controlled Foreign Company (CFC) regimes operated in multinational firms' home countries to prevent tax motivated profit shifting. Our model explicitly accounts for the presence of CFC regimes, which can remove the tax benefits of locating a patent in a lower tax country.

We expect the responsiveness of each individual patent location decision to tax to be heterogeneous. For instance, this will vary across patents depending on the expected value of the patent. The benefit of a lower tax rate and therefore the incentives to achieve this by, for example, attempting to circumvent any anti-profit shifting rules, will be increasing in the expected net present pre-tax value of a patent. At the same time, there may be costs associated with locating a patent, some of which may be related to the patent's expected value. For example, the higher a patent's expected value, the larger the incentive for tax authorities to enforce transfer pricing rules thereby potentially diminishing the firm's ability to transfer income to a lower tax country. These patent characteristics are unobservable.

Tax will not be the only factor influencing location choice. Firms may also incur both costs (in addition to their tax liability) and benefits depending on where a patent is held. That is, non-tax characteristics of countries can make a location a more or less attractive place to hold patent income. Such characteristics include, for example, the costs (both material and legal) of establishing and maintaining an entity to manage intellectual property (for example a subsidiary). This means that some firms may choose a higher tax location if they place a high value on the country's characteristics. Krautheim and Schmidt-Eisenlohr (2010) presents a

⁶ Simpson, Glenn R. "Irish Subsidiary Lets Microsoft Slash Taxes in U.S. and Europe." *The Wall Street Journal*, November 7, 2005, p.A1.

model which, in this context, implies that, when there are fixed costs (net of benefits) to locating patents offshore and profits differ across patents (either as a result of heterogeneous costs of creating patent income or heterogeneous patent values), it will only be beneficial for firms to locate some patents in low tax countries.

There are also important firm characteristics that affect the costs a firm faces when deciding where to locate a patent.⁷ In particular, firms differ in their organisational structure (for example in their network of subsidiaries, the size and proficiency of their legal and tax departments, and existing relationships with tax authorities); in the strategies they employ (for example in relation to how they manage patent income for tax purposes); in the countries in which they are headquartered (which differ in the stringency of their tax rules and effectiveness with which they are applied) and in the markets in which they operate (which will be relevant if, for example, markets with many market-based transactions make it easier for tax authorities to enforce transfer prices that more closely reflect underlying value than if most trade is intra-firm). In addition, both firm size and industry have been highlighted as important in the context of firms making decisions over how to organise their offshore activities. For instance, Graham and Tucker (2006) shows that larger firms use tax haven operations more intensively. Desai, Foley, and Hines (2006) also find this and additionally highlight the importance of other characteristics, notably industry.⁸ Indeed, the values of patents, the relative attractiveness of locations and firms' strategies and organisational structures are likely to vary across industries and, within industries, according to firm size.

⁷ A number of papers have emphasised the importance of incorporating heterogeneity in firms' decisions. See for example Melitz (2003), Bernard, Redding and Schott (2007a,b).

⁸ Desai, Foley, and Hines (2006) show that industries in which a large proportion of total sales are to related parties offshore have a higher presence of subsidiaries in tax havens.

Our paper is novel in directly considering the impact of taxes on the location of intellectual property.⁹ Most of the literature on the impact of tax on firm location choices has focussed on real activity, notably the location of production, with a few considering the location of R&D.¹⁰ The literature on firms shifting income to minimise tax has tended to look at tangible capital or to provide only indirect evidence on the importance of intangibles in firms' strategies, in both cases almost exclusively providing evidence on US firms.¹¹ This evidence suggests a particularly important role for intangible assets. For example, Grubert (2003) formalises how intangible assets can be used to shift income and finds that about half of the income shifted from high-tax to low-tax countries by US manufacturing firms can be accounted for by income from R&D linked intangibles.¹²

The rest of the paper is structured as followed. In Section 2 we describe our model of firms' decisions over where to hold patents. In Section 3 we provide an overview of the data, which is discussed in detail in Appendix A. Section 4 presents the estimation results, including the own- and cross-tax elasticities between locations. Simulations of recent reforms that reduce the tax rate for patent income are presented in Section 5 and a final section summarises.

⁹ Our work relates to the literature on the impact of tax when a project incurs costs and earns profits in the same location, in the Hall and Jorgenson (1967) tradition, including Auerbach (1979), Devereux and Griffith (2003).

¹⁰ Devereux (2006) provides a survey of the empirical literature. See also, inter alia, Hall (1993), Devereux and Griffith (1998, 2000), and Bloom, Griffith and Van Reenen (2002). A recent exception, Karkinsky and Riedel (2009), considers the location of patents within a multinational group in a reduced form model.

¹¹ A review article, Hines (1996), sets out this body of work. See also Hines (1999). On tangible capital see Grubert and Mutti (1991), Hines and Rice (1994), Altshuler and Grubert (2006). The methods of studies considering indirect evidence have included looking at firms' relative profitability in low tax countries; the share of royalty payments associated with low tax countries; the tax liabilities of foreign affiliates.

¹² A recent paper, Dyreng, Lindsey and Thornock (2011), considers how tax rates and rules affect the state in which US subsidiaries incorporate. They find that firms hold intangible assets in Delaware in order to make payments from high tax entities and thereby reduce tax payments.

2 Model

2.1 Firm behaviour

We model firms' choices over where to hold patents. What we mean by where a firm holds a patent is the legal address of the subsidiary that files the patent application. The tax liability on patent income will depend predominately on the tax rate in the country where the patent is held (the source country) and, potentially, on CFC rules operated by firms' home countries.

In the introduction we outlined a number of reasons why the responsiveness of patent location decisions to corporate tax is likely to vary both between firms and, within firms, across patents. We capture this heterogeneity in two ways. Firstly, we allow all coefficients in the model to vary according to firm size and industry classification of a patent. Secondly, and most importantly, we capture unobservable patent specific factors by including random coefficients which allows for variation in the impact of tax across individual patents. We account for the many non-tax factors that influence a firm's decision over where to hold a patent by allowing for fixed costs (and benefits) associated with locating offshore that are common across patents but can differ by country.

Let $i \in (1, \dots, I)$ index firms, t denote time, $p \in (1, \dots, P) \equiv \Omega$ index patents, and $j \in (1, \dots, J)$ index the location of each patent. Suppose at time t firm i faces the decision of where to locate patent p , which has an expected net present pre-tax value \bar{V}_p . \bar{V}_p denotes the value net of any patent specific costs associated with the location decision over patent p . Let τ_{ijt} denote the tax rate at time t levied on patent income attributed to a subsidiary in location j and associated with parent firm i . The inclusion of an i subscript on the tax rate captures the fact that there may be interactions between the tax system of a firm's residence country and that of the country in which the patent is held through operation of CFC rules in the former.

We assume that when firm i chooses the location of patent p it makes its decision under the assumption that the current tax regime will remain unchanged.

We assume that for each patent location decision there is a cost that firm i incurs when locating the patent, $C(\tilde{V}_p, X_i)$. This includes, for instance, attempting to circumvent transfer pricing rules when locating patent p . The size of this cost can depend on the expected net present pre-tax value of the patent, \tilde{V}_p – the higher a patent’s expected value, the larger the incentive for tax authorities to enforce transfer pricing rules and for the firm to attempt to get round them. It can also depend on a vector of firm characteristics, X_i , that influence the cost associated with locating a patent, including firms’ resident country – some countries operate more stringent or more effective transfer pricing rules; firms’ corporate and financial structure – some firms are organised in ways and have certain resources that are more conducive to locating a patent at a relatively low cost; the market that a firm operates in – transfer pricing rules may be stricter in markets with more market based transactions. We assume that $C(\tilde{V}_p, X_i)$ does not vary across locations.

We also assume that there is some net fixed costs to firm i of locating patent p in location j , F_{ipj} . This will capture both location specific costs of locating in j – such as the legal costs associated with setting up a subsidiary – net of any location specific benefits – such as services provided from government tax revenues.

Hence firm i will choose to locate patent p in location j_p^* where:

$$j_p^* = \operatorname{argmax}_{j \in \{1, \dots, J\}} \{(1 - \tau_{ijt})\tilde{V}_p - C(\tilde{V}_p, X_i) - F_{ipj}\}$$

Notice that without the location specific net fixed cost F_{ipj} each firm would locate each of its patents in the location in which they face the lowest tax rate.¹³ However, given the presence of the net fixed costs, F_{ipj} , the location in which a patent is placed depends on both the after tax expected net present value of the patent and the importance of these fixed costs. For example a firm may choose not to locate a patent in a location in which it would face the lowest tax rate (and to instead locate it in a higher tax location) if the expected net pre-tax value of the patent is relatively low and/or the fixed cost associated with locating in the country is relatively high. The determinants of location choice are such that, within firms, not all patents will necessarily be held in the same location.

2.2 *Empirical specification*

To derive an empirical form for this model, firstly we define a partition for the set of patents, $\{R_r \subset \Omega: r \in \{1,2,3\}\}$, where r denotes the industry in which the patent is applied, such that patent p belongs to the subset R_r if it is classified in industry r . Similarly, we define a partition for the set of patents based on firm size, $\{S_s \subset \Omega: s \in \{1,2\}\}$. Thus $p \in S_s$ if firm i , which owns patent p , is of size s .

Secondly, we assume the location net fixed costs can be decomposed into two components:

$$F_{ipj} = \gamma_{rsj} + e_{ipj}$$

γ_{rsj} captures factors that influence locating in location j that are common across patents within a given industry-firm size grouping. e_{ipj} is an idiosyncratic shock which we assume is distributed i.i.d. extreme value.

¹³ Which may not be the country with the lowest statutory tax rate if the firm is subject to binding CFC rules.

Thirdly, we account for the fact that the responsiveness of patent location for any patent p to tax depends on the expected net present value of the patent, \tilde{V}_p , which is unobservable to the modeller, through inclusion of random coefficients. In particular, we assume that δ_p is the parameter governing the responsiveness of location for patent p to tax, where:

$$\delta_p = \mu_{rs} + \sigma_{rs}\eta_p$$

and $\eta_p \sim N(0,1)$. This specification allows the net present value of a patent, and therefore the importance of tax in determining location choice, to vary across patents in a flexible way; for $p \in R_r \cap S_s$, the random coefficient, η_p , allows for variation in responsiveness of location choice to tax along unobservable dimensions. We allow the distribution of responsiveness to differ for each subset of patents $R_r \cap S_s \forall (r, s)$.¹⁴

Together these assumptions imply the following empirical specification for (1):

$$j_p^* = \operatorname{argmax}_{j \in \{1, \dots, J\}} \{ \varphi_{ip} - (\mu_{rs} + \sigma_{rs}\eta_p)\tau_{ijt} - (\gamma_{rsj} + e_{ipj}) \}$$

where:

$e_{ipj} \sim i.i.d. \text{ extreme value}$ and $\eta_p \sim N(0,1)$. φ_{ip} captures both the expected pre-tax net present value of the patent (\tilde{V}_p) and the patent-firm fixed costs associated with locating patent p ($C(\cdot)$). As φ_{ip} does not vary across location it is not identified and differences out when comparisons are made across location. We estimate the model parameters, $(\mu_{rs}, \sigma_{rs}, \gamma_{rsj}) \forall (r, s)$ and j , using maximum likelihood.¹⁵

¹⁴ This means the random coefficient on the tax rate comprises a mixture of normal distributions. This allows the model to more flexibly capture underlying unobserved heterogeneity relative to the standard approach of using one parametric distribution. For further discussion see Burda, et al (2008) and Fiebig (2010).

¹⁵ Evaluation of the log-likelihood function requires numerical integration over the distribution of the random coefficients. We do this using a Gauss-Hermite quadrature rule.

2.3 Elasticities

The coefficient estimates allow us to calculate own and cross tax elasticities of location choice that vary across patents, firms, locations and time. Denote the unconditional probability that firm i chooses to locate patent p (in year t) in location j where it faces a tax rate τ_{ijt} as

$$Pr_{ipjt} = \int L_{ipjt}(\tau_{ijt}, \theta_1, \theta_2) \phi(\theta_2) d\theta_2,$$

where θ_1 is the vector of non-stochastic coefficients $(\mu_{rs}, \sigma_{rs}, \gamma_{rsj})$ and θ_2 the vector of stochastic variables (η_p) . $L_{ipjt}(\tau_{ijt}, \theta_1, \theta_2)$ is the probability that firm i chooses to locate patent p (in year t) in location j conditional on θ_2 :

$$L_{ipjt}(\tau_{ijt}, \theta_1, \theta_2) = \frac{e^{\pi_{ipjt}(\tau_{ijt}, \theta_1, \theta_2)}}{\sum_k e^{\pi_{ipkt}(\tau_{ikt}, \theta_1, \theta_2)}}.$$

The location j tax elasticity for firm i and for patent p (in year t) is given by:

$$\epsilon_{ipjt} = \frac{\partial Pr_{ipjt}}{\partial \tau_{ijt}} \frac{\tau_{ijt}}{Pr_{ipjt}}.$$

This contrasts with the multinomial logit elasticity, which is given by $\mu_{rs} \tau_{ijt} (1 - Lipjt \tau_{ijt}, \theta_1)$, where $Lipjt$ is the (now unconditional) probability of locating patent p in location j . The multinomial logit model imposes that the elasticity varies proportionally with the tax rate, while the random coefficients (or mixed logit) model that we estimate captures variation in substitution patterns between countries.

The cross tax elasticity for location j with respect to the tax rate in location k for firm i and for patent p is given by

$$\epsilon_{ipjkt} = \frac{\partial Pr_{ipjt}}{\partial \tau_{ikt}} \frac{\tau_{ikt}}{Pr_{ipjt}},$$

Like the own tax elasticity, this form for the cross tax elasticity is much more flexible than that implied by the multinomial logit model. Aggregating these elasticities across patents yields the market level or aggregate elasticities.

3 Data

To estimate the model we combine data from a number of sources, including the European Patent Office's PATSTAT data, Bureau van Dijk's Amadeus and Icarus databases and Thomson's Derwent database, to measure the location of firms' patent holdings and the tax rates they faced. The data are described in detail in Appendix A.

We measure where a patent is held using the country of the applicant listed on the patent application. The applicant is the entity which legally holds the patent, and will therefore be liable for any tax on associated income. We have matched the corporate applicants from 14 European countries and the US to their European parent firms.

We classify patents into (one or more of) three broad industry groups – Chemical, Electrical and Engineering – based on the technology embodied in the patent as well the markets in which the technology is used. Within each industry we use data on the population of parent firms excluding those that are below the 20th percentile in terms of the number patents. Our data therefore include 639 parent firms that collectively have 4,740 patenting subsidiaries, which file 233,471 patent applications over the period 1985-2005. These account for 72% of patent applications made by matched corporate entities in these countries. Within each industry we distinguish *large firms* as those with a level of patenting above the 80th percentile of the distribution of number of patents per firm. The resulting numbers of patent applications and firms in each industry-firm size group are shown in Table 1.

Table 1: Number of patent applications and firms by Industry and firm size

Industry	Large firms		Medium firms	
	Patent applications	Firms	Patent applications	Firms
Chemical	81,268	124	23,307	480
Electrical	93,473	125	22,335	490
Engineering	64,811	149	26,063	480

Notes: The total number of patent applications sums to more than the 233,471 cited in the text because patents can be classified into multiple industries. Likewise, firms can operate in multiple industries. Large firm are those associated with a total number of patent applications above the 80th percentile in each industry.

To measure the tax rate a firm faces we consider both the statutory corporate tax rate in the source country (the location of the applicant) and the presence of Controlled Foreign Companies (CFC) rules in the parent firm's home country, which may imply additional tax on source country income if the source country is deemed to be 'low-tax'. We consider the tax regime in place at the time that the firm applies for the patent, thus assuming the firm expects this regime to remain unchanged in the future. Firms can subsequently sell a patent; we assume that at the time of filing the application the firm does not expect to do this.¹⁶ We define the tax variable, τ_{ijt} , at time t as equal to the source country tax rate or the residence country's statutory rate if a binding CFC regime is in place.

In recent years, some European countries have introduced 'Patent Boxes', which levy a lower rate of corporate tax on the income derived from patents. In 2007 Belgium introduced a Patent Box which entails a rate of 6.8% and the Netherlands one with a 10% rate. In 2008 Luxembourg adopted a Patent Box which introduced rate of 5.9% and in 2013 the UK will introduce a 10% rate.¹⁷ We use these tax rates in simulating the effects of Patent Boxes.

¹⁶ There can be tax penalties involved in transferring patents to a different subsidiary. Selling a patent to a foreign subsidiary will generally attract an exit tax, which taxes the gain in value of the patent since it was originally acquired. Broadly, the value at the point of transfer should represent the net present value of the expected revenue stream from that point forward. This means that subsequently transferring a patent to a low tax location will not necessarily reduce the firm's tax liability.

¹⁷ For information on the UK's plan to introduce a Patent Box see HM Treasury (2010) and for the authors' response to the UK's proposals see Griffith and Miller (2010, 2011).

4 Results

To estimate the affect of corporate tax on the location of patents, controlling for other potentially confounding observed and unobserved factors, we estimate a model of the form described in Section 2.1. To highlight the importance of unobserved heterogeneity we report results from a multinomial logit specification, which precludes unobservable heterogeneity in the tax coefficient, alongside our main random coefficient (mixed logit) specification. An observation is a single decision that a firm i made over where to hold the intellectual property for patent p in year t , choosing between locations $j=1,\dots,J$. We consider a firm's decision conditional on having made an innovative discovery. As described above, we estimate the model allowing all coefficients to vary according to which of the three industries (Chemical, Electrical and Engineering) and firm size group (large and medium) the observation (i.e. patent) is classified in.¹⁸ This allows the mean and standard deviation of the tax random coefficient and the location specific factors to vary along these six dimensions.

Table 2 reports our estimates. Column (1) presents coefficient estimates from a multinomial logit model, including tax as the only regressor. We expect there may be other factors (both costs and benefits) that affect firms' location choice, which vary across countries and may be correlated with tax. For example, firms may be attracted by the services countries offer, some of which may be funded from taxation. Many of these non-tax factors are likely to be constant over time. Therefore we control for them through the addition of country fixed effects (which we allow to vary by industry-firm size group) in the regressions reported in columns (2) and (3). We find that the fixed effects are all statistically significant and that their relative size differs across firm size and industry, indicating the importance of allowing heterogeneity in these dimensions. In column (2) the tax coefficients becomes more negative,

¹⁸ This equivalent to estimating 6 separate regressions; one for each industry-firm size grouping.

and are statistically significant, supporting the suggestion that country characteristics are positively correlated with higher statutory tax rates and cause upward bias when not incorporated.

Table 2: Location of Intellectual Property

	<i>Multinomial logit</i>	<i>Multinomial logit</i>	<i>Random coeff. logit</i>
	(1)	(2)	(3)
<i>Chemical Industry</i>			
<i>Large firms</i>			
Tax rate, Mean	-0.04 (0.04)	-1.42 (0.09)**	-4.00 (0.14)**
Tax rate, Std Dev	-	-	8.85 (0.20)**
<i>Medium firms</i>			
Tax rate, Mean	-0.55 (0.08)**	-2.67 (0.18)**	-3.30 (0.22)**
Tax rate, Std Dev	-	-	4.06 (0.39)**
<i>Electrical Industry</i>			
<i>Large firms</i>			
Tax rate, Mean	0.59 (0.04)**	-3.17 (0.09)**	-5.01 (0.12)**
Tax rate, Std Dev	-	-	6.80 (0.16)**
<i>Medium firms</i>			
Tax rate, Mean	-1.11 (.08)**	-4.48 (0.19)**	-5.17 (0.27)**
Tax rate, Std Dev	-	-	3.52 (0.51)**
<i>Engineering industry</i>			
<i>Large firms</i>			
Tax rate, Mean	0.44 (0.05)**	-1.80 (0.11)**	-2.60 (0.13)**
Tax rate, Std Dev	-	-	4.66 (0.23)**
<i>Medium firms</i>			
Tax rate, Mean	-0.15 (0.07)*	-2.98 (0.16)**	-3.76 (0.21)**
Tax rate, Std Dev	-	-	4.20 (0.39)**
Industry-firm size specific country fixed effects	no	yes	yes

Notes: Estimation is conducted using 311,257 observations. Standard errors in parentheses, * significant at 5%; ** significant at 1%. Large firm are those associated with a total number of patent applications above the 80th percentile in each industry. The numbers of observations in each of the six regressions is reported in Table 1. Country fixed effects are not reported, but are available on request.

The multinomial logit specification imposes that the effect of the corporate tax rate is the same across all patents in the industry–firm size group, creating very restrictive substitution patterns. However, we expect different firms and, within firms, different patents to respond differently to tax, depending on unobserved factors. Column (3) reports results when we allow for unobserved heterogeneity through the inclusion of a random coefficient. The estimated mean effects of tax increase in absolute terms compared to the multinomial logit. The standard deviations for the random coefficients are large and statistically significant. That is, there is important variation in the effect of corporate taxes on the probability of a firm locating a patent in a country which is not captured by observable characteristics.

The estimates of the main random coefficient specification suggest important variation in the responsiveness of location choice to tax. Considering the six random coefficients together we calculate that a marginal increase in the tax rate a firms faces in a location will lead to a reduction in the probability of locating there (*ceteris paribus*) for 75% of the patents in our data.¹⁹ This varies by both firm size and industry. The overall effect of a change in the tax rate on the probability of choosing a location is captured by the elasticities, which, as we show below, are negative for all locations.

We use the coefficient estimates from the random coefficient logit model to calculate the own and cross tax elasticities of location choice with respect to tax for all patents, as outlined in Section 2.2. Aggregating over the patent specific elasticities we obtain market level or aggregate elasticities.

Table 3 presents market elasticities for 2005. Each cell shows the change in the share of patents located in the country indicated in row 1 when the tax rate firms face in the country

¹⁹ Within each firm size–industry group we use the estimated parameters for the normally distributed random coefficient to calculate the proportion of patents with a negative response to tax. We compute the total proportion of all patents with negative responsiveness as the average across the six distribution (one from each size–industry group), weighted by the proportion of patents in that group.

indicated in column 1 changes. Cells on the diagonal (in bold) are the own tax elasticities; as expected, these are negative. For the majority of countries they are greater than one, indicating that a 1% increase in the rate of tax faced by firms in a particular location will induce a (slightly) greater than 1% reduction in the share of patents locating there. The off-diagonal cells contain the cross tax elasticities, the majority of which are positive, indicating that the different locations are substitutes as one would expect.²⁰

When we measure the tax rate and compute the elasticities in Table 3 we account for the presence of CFC regimes. Not doing so is akin to incorrectly assuming that all firms face the statutory tax rate in the source country. However, if a country deemed to be ‘low-tax’ by CFC regimes, for instance Ireland, were to reduce its statutory tax rate we would not expect firms subject to a CFC regime to respond: UK resident firms will still face the UK statutory rate if they locate in Ireland.²¹ Not including the effect of CFC regimes in the calculation of the elasticity would result in higher reported elasticities for countries deemed ‘low tax’ in other countries CFC regimes. Since the elasticities of each country are a function of taxes in all other countries, ignoring CFC regimes would also alter the reported elasticities of countries not deemed ‘low tax’ in the prevailing set of CFC regimes.

²⁰ There are five cross-tax elasticities which are negative, but very close to zero.

²¹ To the extent that firms can avoid the CFC regime they may respond to a fall in a low country’s tax rate. However, we would still expect firms subject to a CFC regime to be less responsive than those which are not.

Table 3: Own and cross tax elasticities; random coefficients model, accounting for CFC regimes

<i>Location country</i>	<i>Country changing tax rate</i>														
	Belgium	Denmark	Finland	France	Germany	Ireland	Italy	Luxembourg	Netherlands	Norway	Spain	Sweden	Switzerland	UK	US
Belgium	-1.006	0.031	0.051	0.171	0.026	0.001	0.042	0.006	0.168	0.006	0.004	0.080	0.111	0.143	-0.012
Denmark	0.064	-1.375	0.056	0.261	0.076	0.001	0.089	0.011	0.228	0.011	0.007	0.109	0.193	0.257	0.038
Finland	0.055	0.030	-1.568	0.471	0.112	0.001	0.062	0.005	0.486	0.006	0.004	0.193	0.147	0.202	0.054
France	0.030	0.023	0.077	-0.917	0.035	0.000	0.031	0.003	0.232	0.004	0.002	0.097	0.095	0.124	0.000
Germany	0.011	0.016	0.046	0.087	-0.642	0.000	0.016	0.003	0.109	0.004	0.002	0.060	0.069	0.080	-0.053
Ireland	0.082	0.081	0.083	0.311	0.094	-0.768	0.129	0.017	0.252	0.016	0.014	0.136	0.461	0.318	0.053
Italy	0.028	0.029	0.038	0.117	0.025	0.001	-0.842	0.008	0.089	0.008	0.005	0.064	0.091	0.132	-0.014
Luxembourg	0.058	0.056	0.045	0.194	0.074	0.001	0.124	-1.299	0.129	0.013	0.010	0.089	0.160	0.242	0.028
Netherlands	0.038	0.025	0.103	0.301	0.056	0.000	0.030	0.003	-1.067	0.004	0.002	0.124	0.116	0.148	0.018
Norway	0.061	0.055	0.056	0.249	0.085	0.001	0.115	0.013	0.183	-1.340	0.008	0.105	0.168	0.242	0.039
Spain	0.043	0.041	0.040	0.148	0.052	0.001	0.097	0.012	0.090	0.010	-1.081	0.068	0.099	0.171	0.018
Sweden	0.052	0.035	0.119	0.365	0.090	0.001	0.063	0.006	0.359	0.007	0.004	-1.405	0.146	0.196	0.043
Switzerland	0.069	0.061	0.085	0.336	0.094	0.002	0.087	0.010	0.316	0.011	0.005	0.140	-0.857	0.276	0.052
UK	0.052	0.046	0.069	0.258	0.067	0.001	0.073	0.008	0.239	0.009	0.005	0.109	0.160	-1.181	0.026
US	-0.007	0.012	0.031	-0.001	-0.075	0.000	-0.013	0.002	0.048	0.002	0.001	0.040	0.058	0.044	-0.266

Notes: Each cell contains the elasticity of the share of patents in the country in the row when the tax rate changes in the country in the column. Elasticities are derived from the full random coefficients model and incorporate the presence of CFC regimes.

4.1 Importance of including the random coefficient

A key strength of our model is that, by allowing unobservable characteristics to affect patents' tax responsiveness, we are able to generate flexible substitution patterns. This flexibility is further strengthened by allowing the distribution of responsiveness to vary by industry-firm size category. To date much more restrictive models have been used in the literature on firms' location choice. A common approach has been to estimate a multinomial logit model, which imposes very restrictive substitution patterns. To highlight this, Table 4 presents the market elasticities generated through estimating a standard multinomial logit model on all the patents in our data, with tax and fixed effects as right-hand side variables (i.e. the equivalent column (2) in Table 2 but imposing common coefficients across industry and size class). To accentuate the point we have also shut off the CFC regimes.²²

In Table 4 the cross tax elasticities are the same within each column: cross tax elasticities in the multinomial logit model are a function of the tax rate in the country that is adjusting its tax rate, the share of patents in that country and the coefficient on the tax rate only. In contrast, in the random coefficients model the cross tax elasticities vary across countries, and are functions of the both the countries' characteristics and how close they are to one another. Countries that have more similar characteristics will be seen as closer substitutes by firms, and therefore the cross tax elasticity will be higher.

²² Incorporating CFC regimes into the computation of the multinomial logit elasticities would introduce a limited amount of heterogeneity. In particular the cross tax elasticities for a change in the tax rate in Ireland (or Switzerland) would differ across countries depending on whether they operated a CFC regime that deemed Ireland (or Switzerland) a 'low tax' location, or not.

Table 4: Own and cross tax elasticities; multinomial logit model, not accounting for CFC regimes

<i>Location country</i>	<i>Country changing tax rate</i>														
	Belgium	Denmark	Finland	France	Germany	Ireland	Italy	Luxembourg	Netherlands	Norway	Spain	Sweden	Switzerland	UK	US
Belgium	-0.816	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Denmark	0.031	-0.683	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Finland	0.031	0.015	-0.622	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
France	0.031	0.015	0.027	-0.671	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Germany	0.031	0.015	0.027	0.173	-0.865	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Ireland	0.031	0.015	0.027	0.173	0.090	-0.311	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Italy	0.031	0.015	0.027	0.173	0.090	0.001	-0.879	0.003	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Luxembourg	0.031	0.015	0.027	0.173	0.090	0.001	0.049	-0.755	0.130	0.003	0.003	0.049	0.070	0.090	0.048
Netherlands	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	-0.656	0.003	0.003	0.049	0.070	0.090	0.048
Norway	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	-0.695	0.003	0.049	0.070	0.090	0.048
Spain	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	-0.870	0.049	0.070	0.090	0.048
Sweden	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	-0.649	0.070	0.090	0.048
Switzerland	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	-0.461	0.090	0.048
UK	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	-0.658	0.048
US	0.031	0.015	0.027	0.173	0.090	0.001	0.049	0.003	0.130	0.003	0.003	0.049	0.070	0.090	-0.950

Notes: Each cell contains the elasticity of the share of patents in the country in the row when the tax rate changes in the country in the column. Elasticities are derived from a multinomial logit model and do not incorporate the presence of CFC regimes.

4.2 The effects of Patent Boxes

We use the estimates from the random coefficients logit model to simulate how the share of newly created patents in each of the 15 locations responds to the introduction of the Benelux Patent Boxes and the planned UK Patent Box.²³ We use the final year of our data (2005) as a baseline and consider alternative assumptions about how Patent Boxes might interact with CFC regimes. Specifically, we explore how the results would change if countries operating CFC regimes were to classify countries as ‘low tax’ for the purposes of their CFC regulations based on the Patent Box tax rates.

Table 5 sets out these simulations. Column (1) shows the predicted share of newly created patents across countries in 2005. Columns (2) and (3) report the simulated proportional change in shares and the resulting level when the Benelux Patent Boxes are introduced. Here we assume that countries operating CFC regimes do not use Patent Box rates to determine whether the Benelux countries are subject to CFC rules. The share of new patents locating in the Benelux countries increases, with the proportional increase being largest in Belgium and Luxembourg and the absolute increase largest in the Netherlands. The shares elsewhere fall, with the countries experiencing the largest fall being those that are seen as most closely substitutable for the three Benelux countries.

The use of a random coefficient logit model allows for correlation in unobservables across choices and thereby captures any tendency firms have to perceive countries with similar characteristics as closer substitutes. This means that when a Patent Box is introduced, the greatest substitution is away from other low tax countries; the firms that substitute towards the Benelux countries when Patent Boxes are introduced are those

²³ Existing Patent Boxes regimes do not apply to patents issued prior to the introduction of the Patent Box policy. Therefore, we do not expect firms to relocate pre-existing patents in response. The shares we report can be interpreted as flows – the share of newly created patents that are held in a location.

with ‘preferences’ which favour low tax countries and, as a result, it is low tax countries that lose the greatest share (proportionately).

In a standard multinomial logit model the change in share is proportional to the initial share of the country whose tax changes meaning that the proportional change in share is the same across all countries. Had we used the multinomial logit results, the proportional changes (column (2)) would have been the same for all countries introducing a Patent Box (and the same for all countries that did not) by assumption.²⁴

In Column (4) we assume that countries that operate CFC regimes use Patent Box tax rates to determine whether activity in the Benelux countries is captured by CFC rules.²⁵

This dampens the effects of Benelux Patent Boxes, because firms from countries operating a CFC regime have a reduced incentive to substitute towards these countries.

The remaining columns of Table 5 describe the introduction of a UK Patent Box into a world where the Benelux Patent Boxes are operating. Column (5) shows the proportional change in the share for each country (compared to column (3)) and column (6) the resulting level. The UK share more than doubles to 16.81%, more than offsetting the loss in share which occurred following the Benelux Patent Box introductions. The shares in Belgium, Luxembourg and the Netherlands fall back but remain higher than initially.

²⁴ That is, in column (2) the proportional increase in shares of Belgium, Luxembourg and the Netherlands would be the same and the proportional decrease in the other countries shares would be the same. This highly restrictive pattern would be imposed by assumption.

²⁵ This means, for example, that under the UK CFC rules a subsidiary of a UK parent firm operating in the Netherlands would be subject to the UK tax rate of 28% and not the Dutch Patent Box rate.

Table 5: Impact of introducing a Patent Box on firm location

Country	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Initial share	Patent Box introduced in Belgium, Luxembourg and the Netherlands			Patent Box also introduced in UK			
		% change in share from (1)	Share	<i>Benelux countries low tax in CFC regimes</i>	% change in share from (3)	Share	<i>UK low tax in CFC regimes</i>	<i>(7) plus remove the UK CFC regime</i>
Belgium	3.72%	+157.97%	9.60%	6.82%	-17.97%	7.88%	5.82%	6.01%
Denmark	2.21%	-31.10%	1.52%	1.90%	-9.24%	1.38%	1.75%	1.67%
Finland	4.77%	-43.01%	2.72%	3.96%	-7.05%	2.53%	3.79%	3.69%
France	21.79%	-25.20%	16.30%	19.56%	-3.53%	15.72%	18.99%	18.65%
Germany	7.64%	-15.88%	6.42%	7.14%	-1.96%	6.30%	6.99%	6.92%
Ireland	0.09%	-16.92%	0.07%	0.06%	-16.90%	0.06%	0.05%	0.06%
Italy	5.03%	-16.61%	4.19%	4.66%	-5.53%	3.96%	4.45%	4.33%
Luxembourg	0.39%	+157.14%	1.00%	0.70%	-23.14%	0.77%	0.57%	0.60%
Netherlands	18.35%	+78.35%	32.72%	25.00%	-12.71%	28.56%	22.67%	23.19%
Norway	0.44%	-28.20%	0.32%	0.39%	-10.56%	0.29%	0.36%	0.34%
Spain	0.29%	-18.38%	0.24%	0.27%	-8.05%	0.22%	0.25%	0.24%
Sweden	7.00%	-36.05%	4.47%	5.97%	-6.82%	4.17%	5.68%	7.52%
Switzerland	11.51%	-33.49%	7.66%	8.54%	-11.82%	6.75%	7.80%	8.54%
UK	11.59%	-29.64%	8.15%	10.08%	+106.13%	16.81%	15.93%	13.33%
US	5.18%	-11.22%	4.60%	4.94%	0.08%	4.60%	4.90%	4.89%

Notes: Column (1) shows predicted patent shares from the random coefficients model using 2005 statutory tax rates. The shares refer to the share of patent applications made in 2005. Column (2) shows the proportional change in predicted shares and column (3) the resulting shares when the Benelux countries introduce Patent Boxes (Belgium at 6.8%, Luxembourg at 5.9%, Netherlands at 10%). Column (4) shows predicted shares when the Benelux countries introduce Patent Boxes and it is assumed that income located there may be captured by the CFC regimes of other countries (which may classify Benelux countries as low tax). Columns (5), (6) and (7) repeat (2), (3) and (4) when the UK also introduces a Patent Box at rate 10%. Column (8) removes the UK's CFC regime.

Column (7) considers the case where Patent Box rates in each of the four countries are used to determine whether activity located there is captured by other countries' CFC rules. For the UK this has two affects. First, the move away from the UK is dampened because UK firms cannot access the low taxes offered in the Benelux countries (without being captured by the CFC regime and taxed at the UK rate). Second, the move towards the UK is dampened by the reduced incentive for firms from other CFC countries. In this scenario the UK share increases to 15.93%. This is lower than when no CFC-Patent Box interactions are considered, implying that the lower propensity of UK firms to substitute out of the UK is more than offset by the lower propensity of firms based in other countries operating CFC regimes to come to the UK.

It is unclear how the UK CFC regime will treat offshore patent income when the UK also operates a Patent Box. In column (7) we assumed that the UK continues to operate a CFC regime, where the threshold is based on the full statutory corporate rate, and the full rate (not the Patent Box rate) is applied to any foreign income captured by the CFC regime. Alternatively, the UK may choose to exempt patent income from their CFC regime.²⁶ Column (8) therefore effectively removes the UK CFC regime for patent income.²⁷ We see that the UK share is now 13.33%; higher than initially (column (4)) but lower than when either Patent Box rates are never considered by CFC regimes (column (6)) or all CFC regimes consider Patent Box rates (column (7)). This is because, with no UK CFC regime in place, UK firms face greater incentives to locate patents in low tax countries. The Benelux countries capture an increased share of new patents as a result.

²⁶ This is not an issue for the Benelux countries since they do not operate CFC regimes.

²⁷ An alternative assumption, and one which gives almost identical results, would be to apply the current rules, where low tax is defined as a rate less than 75% of the UK rate, to the 10% tax rate. This gives a threshold of 7.5% which will be binding only for the Patent Boxes of Belgium and Luxembourg and result in only a small amount of additional tax.

In summary, there are potentially important interactions between CFC regimes and Patent Boxes, with the extent to which governments are able to operate a CFC regime which effectively captures patent income held in Patent Box countries greatly affecting the distribution of patent income across countries.

4.3 The impact on tax revenue

We consider the impact that the introduction of Patent Boxes is likely to have on government revenue from patent income. Revenue is a function of the share of patent income held in a country and the tax rate; since a Patent Box leads one to increase and the other to decrease it is not clear what the net effect will be.²⁸ We assume that when Patent Boxes are introduced, CFC regimes continue to operate as they do now, based on the statutory tax rates, so that Patent Box countries are not deemed ‘low tax’ destinations for the purposes of CFC rules. This is the scenario shown in columns (3) and (6) of Table 5 and is the most optimistic assumption about the UK share.

Table 6 shows the effect of introducing Patent Boxes on government tax revenues from new patent applications.²⁹ We assume that the introduction of Patent Boxes does not affect the expected income from each patent, and that the number of new patent applications is not affected. In this sense we are estimating a short-run impact. In the long-run the lower tax rate may lead firms to invest more in research and discovery, leading to either more patent applications or more valuable patents. However, given the long timeline from research inception to commercialisation the effects of this will take some considerable time to feed through to tax revenue.

²⁸ The own tax elasticities reported in Table 3 tell us the impact on government revenue of a *marginal* change in the tax rate (evaluated at the vector of tax observed in the data). This means that for those countries with an own tax elasticity greater than one (in absolute terms) a small reduction in the tax rate would increase revenue. But the elasticity itself is a function of all tax rates, meaning the elasticity evaluated at one vector of tax rates does not inform us about how revenue will change for non-marginal tax changes, such as those induced by the introduction of Patent Boxes.

²⁹ Like the shares presented earlier, we report government revenue raised from the taxation of newly created patents. Patent Boxes do not apply to pre-existing patents.

We index revenue prior to the introduction of any Patent Boxes to 100 (column (1)). Column (2) shows how revenue changes when Patent Boxes are introduced in the Benelux countries.³⁰ Revenue is reduced in all countries. In non-Benelux countries this is driven by the reduction in the share of patent income. In the Benelux countries the increase in share of income is outweighed by the lower tax rate applied to income.

Table 6: Government revenue from new patents as Patent Boxes are introduced

Country	(1) Initial	(2) Patent Box introduced in Belgium, Luxembourg and the Netherlands	(3) Patent Box also introduced in the UK (on top of col (2))
Belgium	100	51.6	42.3
Denmark	100	68.9	62.5
Finland	100	57.0	53.0
France	100	74.8	72.2
Germany	100	84.1	82.5
Ireland	100	83.1	69.0
Italy	100	83.4	78.8
Luxembourg	100	49.9	38.4
Netherlands	100	56.6	49.4
Norway	100	71.8	64.2
Spain	100	81.6	75.1
Sweden	100	63.9	59.6
Switzerland	100	66.5	54.3
UK	100	70.4	48.3
US	100	88.8	88.8
Total	100	70.1	63.7

Notes: Government tax revenue equals the tax rate times the share of patents. We normalise that to 100 in the base year. Columns (2) and (3) show revenue relative to column (1). Shares used are those reported in Columns (3) and (6) of Table 5.

Column (3) shows indexed revenue when the UK additionally introduces a Patent Box.

Recall that we have assumed that the UK does not consider the Benelux Patent Boxes as

low tax for the purposes of its CFC regime, and that other countries do not consider the

³⁰ These revenue figures do not include revenue from imposing the CFC regime on income from other countries. For the UK for example, figures are based on revenue from patents based in the UK and do not include revenue from the patents of UK firms which are held in low tax countries but subjected to the UK tax rate under a CFC regime.

UK Patent Box as low tax for the purposes of their CFC regimes. In this situation revenue also falls in all countries. UK revenue falls because the affect of the lower tax rate (10%) on all income outweighs any income gained from an increased share of patent income. UK revenue from income associated with new patents is halved when all four Patent Boxes are in place compared to its initial level with no Patent Boxes. As shown in Table 5, alternative assumptions about the interaction of Patent Boxes and CFC regimes further reduce the UK share and would therefore lead to further reductions in UK government revenue.³¹

4.4 The effects of further Patent Boxes

It seems unlikely that the UK will be the last country to introduce a favourable tax regime for patent income. There is a considerable literature relating the reduction in corporate tax rates over the 1980s and 1990s to tax competition between countries.³² What would be the consequences of other countries also deciding to introduce Patent Boxes? In Table 7 we simulate the effect on both the shares of patents across countries and tax revenues of the introduction of a Patent Box by Sweden or France (separately).³³ We assume that when Patent Boxes are introduced CFC regimes continue to operate based on the existing statutory tax rates. Column (1) shows the shares of new patents across countries when the Benelux countries and the UK operate Patent Boxes (a repeat of column (6) in Table 5).

³¹ The revenue loss would be mitigated to some extent if the introduction of Patent Boxes leads to an increase in patenting activity. Assuming the UK's share of patent income remains as given in column (6) of Table 5, the aggregate value of patenting activity in the 15 countries considered would have to increase *as a consequence of the introduction of Patent Boxes* by 45% for the UK's revenue to recover to the pre-UK Patent Box level or by over 100% to recover to the level before Benelux Patent Boxes are introduced.

³² See Devereux, Lockwood and Redoano (2008) for a recent discussion of the literature and empirical evidence.

³³ France and Sweden were chosen for illustration, not because there is any suggestion that they are proposing Patent Boxes.

Table 7: Impact of other countries introducing a Patent Box on firm location and government revenue

Country	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Initial share (col. 6, Table 6)	10% Patent Box in Sweden		10% Patent Box in France		Government revenue	
		Proportional change in share	share	Proportional change in share	share	Sweden has Patent Box	France has Patent Box
Belgium	7.88%	-6.74%	7.34%	-22.74%	6.08%	39.5	32.7
Denmark	1.38%	-2.50%	1.35%	-5.51%	1.31%	61.0	59.1
Finland	2.53%	-3.74%	2.43%	-9.20%	2.30%	51.0	48.1
France	15.72%	-1.46%	15.49%	+80.43%	28.37%	71.1	38.5
Germany	6.30%	-0.52%	6.26%	+7.63%	6.78%	82.0	88.8
Ireland	0.06%	-4.77%	0.06%	-14.93%	0.05%	39.4	35.2
Italy	3.96%	-1.37%	3.91%	+1.19%	4.01%	77.7	79.7
Luxembourg	0.77%	-6.12%	0.72%	-18.44%	0.63%	36.0	31.3
Netherlands	28.56%	-7.20%	26.51%	-24.51%	21.56%	45.9	37.3
Norway	0.29%	-2.92%	0.28%	-7.11%	0.27%	62.3	59.7
Spain	0.22%	-1.92%	0.22%	-3.48%	0.21%	73.6	72.4
Sweden	4.17%	+105.05%	8.55%	-6.24%	3.91%	43.6	55.9
Switzerland	6.75%	-3.93%	6.49%	-11.76%	5.96%	52.2	47.9
UK	16.81%	-6.09%	15.79%	-20.18%	13.42%	45.4	38.6
US	4.60%	+0.15%	4.61%	+12.02%	5.16%	89.0	99.5

Notes: see notes to Table 5 and 6. Col.1 repeats column 3, Table 5. Columns 2 and 4 both show the proportional change in share with respect to column 1. Patent Boxes at 10% are added to Sweden or France (and not to France in addition to Sweden). Columns 6 and 7 show an index of government revenue (=tax rate* share of patents). Both show revenue relative to the case in which the UK and Benelux countries have Patent Boxes and can both be compared to column 3 in Table 6.

Columns (2) and (3) show the proportional change in shares and the resulting share if Sweden introduces a 10% Patent Box. Columns (4) and (5) do the same in the case where France introduces a 10% Patent Box. We see that both countries experience large gains in shares. The UK loses share, particularly when France introduces a Patent Box: the UK share falls from 16.81% to 13.42%.

Columns (6) and (7) show the effect on government revenue from patent income (derived from new patents). Each shows the additional revenue effects on top of the impact of the Patent Boxes in the Benelux countries and the UK, and is therefore comparable to column (3) in Table 6. In each case the UK loses both share and revenue.

5 Summary and concluding comments

We specify and estimate a structural model of firms' decisions over where to locate patents, finding that tax is an important determinant of location choice – our estimated own-tax elasticities are negative for all locations and are generally around -1. Our model also highlights that other non-tax factors influence patent location choice. Indeed, such factors are the reason why we do not see firms choosing to hold all patent income in the lowest tax jurisdictions.

We contribute to the literature on firm location choice in a number of ways. We explicitly consider the impact of corporate taxes on where firms choose to hold their intellectual property, an increasingly important part of the debate over governments' design of corporation taxes. In doing so we account for important interactions between source and residence tax jurisdictions that arise through the operation of CFC regimes. Our strategy is to estimate a structural model of firm behaviour which, in addition to controlling for observable firm characteristics and location specific factors, is able to

capture unobservable patent specific factors which affect the responsiveness of location decisions to tax. Our estimates suggest that the extent of this heterogeneity is significant. Using the structural estimates from our model we compute elasticities of location choice with respect to tax. Unlike elasticities estimated in the literature to date, these vary across location and are functions of the tax rates firms face in all potential locations.

In a recent policy development several European countries have introduced or are planning to introduce Patent Boxes, which provide preferential tax treatment for patent income. We simulate the introduction of Patent Boxes and show that while they will succeed in attracting patent income, they also will result in a reduction in tax revenue from patent income. The introduction of Patent Boxes by several European countries in a relatively short space of time has given rise to concerns that countries are engaging in tax competition for patent income. We find that if another European country were to introduce a Patent Box all countries could expect to see a reduction in tax revenues from patent income. In future work we will explicitly consider the optimal response of governments considering the tax rate applicable to intellectual property in the face of Patent Boxes elsewhere.

Appendix A. Data

A.1 Firm and location data

We measure where firms hold patents using data on patent applications made to the European Patents Office (EPO) and recorded in the EPO's Worldwide Patent Statistical Database (PATSTAT).³⁴ EPO patent applications include (by law) the names and residential addresses of the entity filing the patent (the applicant) and the inventors which created the underlying technology.³⁵ The applicant is the entity which legally holds the intellectual property and therefore will be eligible for any resulting income and liable for any associated tax. We use the country of residence of the applicant to identify the location of a patent holding for tax purposes. In our model each patent is associated with a single location indicating where the patent is held.³⁶ This can be distinct from either where the inventors are located or where legal protection is sought (i.e. the countries in which the patent will be protected).³⁷

In order to model firms' location choices, we need to observe the parent (headquarter) company of the patent applicant. This information is not available in patent applications.

The patents data does not record the parent (headquarter) company of the patent applicant. To obtain information on firms' ownership structure we have matched the applicants of EPO patent applications from 14 European countries and the US to firms listed in Bureau van Dijk's Amadeus and Icarus databases as well as other sources such

³⁴ We use all *patent applications* made to the EPO, not only those that go on to be granted. This greatly improves the timeliness of our data since there is a long lag between observing an application and the eventual outcome.

³⁵ Note that, in contrast, patent applications made to the US Patent and Trademark Office (USPTO) list the inventors (i.e. the individuals which created the technology) as 'applicants'.

³⁶ A handful of patents are held in the name of multiple applicants in multiple locations. In this case we randomly select one of these subsidiaries.

³⁷ For more information on the distinctions between the different locations contained in an EPO patent document see section 2.5 of Abramovsky et al (2008).

as business directories and firms' websites. The result is a mapping between European parent firms and their European and US subsidiaries that apply for patent applications.³⁸ These data are described in detail in Abramovsky et al (2008). It is notable that we do not include the set of small countries sometimes described as tax havens. However, in the EPO data, over the period 2001-2005, we observe that fewer than 0.5% of EPO applications are filed from such countries³⁹ and many such applicants are unrelated to European firms.

We use data on 639 firms that collectively have 4,740 patenting subsidiaries that file 233,471 patent applications over the period 1985-2005.⁴⁰ The breakdown of firms and subsidiaries by the country of the parent firm (residence country) is described in Table A.1. Column (1) shows the number of firms headquartered in each country, column (2) the number of subsidiaries associated with these firms, and column (3) the number of patent applications these subsidiaries filed.

We classify patents into three broad industry groups using the Derwent Innovation Index, compiled by Thomson.⁴¹ We consider three industries: Chemical, which includes pharmaceuticals; Electrical, which includes communications and computing; Engineering which includes machinery, vehicles, and construction.

An individual patent application can be classified into multiple industries. In such cases we allow the patent application to enter the estimation for each relevant industry group. Columns (4)-(6) in Table A.1 show the proportion of patents in column (3) that are in

³⁸ We use information on firms headquartered in thirteen European countries: Belgium, Denmark, Finland, France, Ireland, Italy, Luxembourg, Netherlands, Norway, Spain, Sweden, Switzerland and UK and all of their patent applications made by subsidiaries located in these countries, as well as in Germany and the US.

³⁹ This is based on patent applications filed by applicants based in Bahamas, Barbados, Bermuda, Cayman Islands, Gibraltar, Hong Kong, Liechtenstein, Malta, Monaco, Netherlands Antilles, Panama or Singapore.

⁴⁰ As highlighted in section 3, we exclude firms that are below the 20th percentile in terms of the number patent applications in each industry.

⁴¹ For more information on the Derwent Innovation Index and the precise industry definitions, see section 5 of Abramovsky et al (2008).

each of the three industries. Since patents can be in multiple industries, the percentages do not sum to 100 within rows.

Table A.1: Number of firms, subsidiaries and patents, by residence country

Residence country	Firms (1)	Associated subsidiaries (2)	Patent applications (3)	% in		
				Chemicals (4)	Electrical (5)	Engineering (6)
Belgium	28	208	8229	73.3	35.7	53.1
Denmark	27	193	6686	72.8	19.1	35.3
Finland	25	107	10112	16.5	79.6	24.5
France	108	1212	58015	36.4	55.7	42.1
Ireland	4	15	511	94.1	10.8	28.0
Italy	77	297	11153	43.3	40.2	57.8
Luxembourg	8	47	1505	31.0	66.4	39.6
Netherlands	58	330	47084	37.1	65.1	31.9
Norway	7	50	1128	53.4	35.4	52.3
Spain	8	43	697	54.9	18.2	43.6
Sweden	48	456	16560	17.2	67.8	44.5
Switzerland	106	729	37260	60.6	30.9	38.7
UK	135	1053	34531	41.9	52.3	39.5
Total	639	4740	233471	44.8	49.6	38.9

Notes: The residence country refers to the country in which a firm is headquartered. Column 1 shows the number of firms headquartered in each country and column 2 the number of patent application filing subsidiaries that are associated with these firms. Column 3 reports the number of patent applications made by these firms. All figures refer to the period 1985-2005.

In our model we allow behaviour to vary by firm size. In each industry we identify firms patenting above the 80th percentile in terms of the number patent applications filed over the period 1985-2005. Table A.2 shows the number of patents in each industry and firm size category by source country (the location of the subsidiary that applied for the patent).

Table A.2: Patent applications, by industry, size and source country

Industry:	Chemicals		Electrical		Engineering	
Firm size:	Large	Medium	Large	Medium	Large	Medium
Source country						
Belgium	3546	1244	1969	828	2597	1080
Denmark	2216	1142	367	737	1216	725
Finland	1050	869	7270	763	1461	779
France	13690	3553	25461	3233	16645	3484
Germany	7651	1088	7178	1329	5228	1878
Ireland	167	162	35	24	69	58
Italy	2571	2453	1753	2624	2852	3369
Luxembourg	254	305	13	276	49	265
Netherlands	12690	1838	23949	1030	11007	1689
Norway	286	270	76	327	351	205
Spain	47	431	57	185	88	261
Sweden	3698	1228	8517	1492	5031	2216
Switzerland	14042	3251	5466	4018	8299	4483
UK	12728	4582	6813	4650	6538	4644
US	6632	891	4549	819	3380	927
Total	81,268	23,307	93,473	22,335	64,811	26,063

Notes: The country refers to the location of the subsidiary that applied for the patent, i.e. the country in which the patent is held or the source country. In Chemicals and Electrical large firms are those which have applied for more than 300 patents across 1985-2005, in Engineering they are those that have applied for more than 250.

A.2 Taxes

We consider the impact of taxes in the source country (the location of the subsidiary) and the home (residence) country of the parent firm. We use variation in the statutory tax rate and in the rules governing the taxation of foreign source income (i.e. income earned offshore) to identify the sensitivity of firms' location decisions to changes in the tax rate. Statutory tax rates have varied differentially across countries over time. In most countries the statutory rate has fallen over the last two decades, but there is variation in both the trajectories and the levels, as described in Table A.3.

Table A.3: Statutory corporate tax rates, 1985-2005

Source country	Statutory tax rate			Number of changes
	min	max	mean	
Belgium	0.340	0.450	0.400	5
Denmark	0.280	0.500	0.368	7
Finland	0.250	0.602	0.364	7
France	0.333	0.500	0.378	13
Germany	0.383	0.617	0.534	15
Ireland	0.100	0.125	0.104	1
Italy	0.373	0.532	0.455	7
Luxembourg	0.304	0.394	0.373	6
Netherlands	0.315	0.430	0.361	4
Norway	0.280	0.508	0.356	1
Spain	0.350	0.353	0.351	1
Sweden	0.280	0.520	0.351	2
Switzerland	0.213	0.285	0.270	6
UK	0.300	0.400	0.327	5
US	0.340	0.495	0.393	5

Notes: This table shows the minimum, maximum and mean of the statutory corporate tax rates across the period 1985-2005 as well as how many times the rate changed. The rates reflect the corporate income tax rate adjusted to account for surcharges, local taxes, and additional taxes levied on tax bases that are similar to the corporate income tax where such things exist.

There are important interactions between tax jurisdictions which mean that the overall tax liability a firm faces may depend on the tax regime in the parent firm's home country of residence as well as that in the country where a patent is held. In relation to foreign source income countries operate one of two systems – an exemption or a credit system – which set out how income earned offshore is taxed.⁴² Within both systems there are opportunities for firms to locate income in low tax countries in an attempt to avoid taxation in their home country. To counter this many governments operate anti-avoidance legislation, in the form of Controlled Foreign Company (CFC) regimes,

⁴² Under an exemption system income earned in offshore subsidiaries can be remitted to the home country without attracting additional tax liability. In a credit system, income earned in offshore subsidiaries will be liable for corporation tax when remitted back to the home country. A credit is given for any tax already paid in the source country (and as a result there is no additional tax due if the tax rate is higher in the source than residence country).

which make realising the tax benefits of lower tax jurisdictions more difficult.⁴³ CFC legislation defines the set of subsidiaries that are located offshore but deemed to be subject to tax in the parent firm's residence country. The precise form of CFC regimes varies across countries. Broadly, CFC regimes apply when:

- (i) the parent firm owns a sufficiently large share of an offshore subsidiary, and
- (ii) a sufficiently high proportion of the subsidiaries' income arises from 'passive sources'(i.e. income from non-commercial activities, mainly investment income such as interest, dividends and patent royalties), and
- (iii) the subsidiary is located in a country deemed to be a low tax country.

Under these circumstances, the passive income of the subsidiary will be taxed at the statutory corporate tax rate of the residence country, with a credit given for any tax already paid in the source country. This affects the tax liability on income earned in an offshore subsidiary. We assume that the foreign patenting subsidiaries in our data satisfy condition (i) and, since we are considering the location of patents, that a subsidiaries income would be considered passive and therefore fulfil (ii). We use the rules set out in countries CFC regimes to define which source countries would be deemed 'low tax'.⁴⁴ That is, for each residence country that operates a CFC regime we identify the source countries that would be deemed low tax. We assume that, in these cases, patent income earned in the source country would be subject to the statutory tax

⁴³ Firms' location choices may also be affected by withholding taxes and transfer pricing rules. Withholding taxes are levied on royalty payments between some countries. Between the European countries and in the years we consider, these taxes are low or have been negotiated to zero. Transfer pricing rules, which aim to ensure that the prices of goods that are charged between entities within the same group are those that would result from transactions between unrelated parties, limit the size of the payments which can be made between subsidiaries.

⁴⁴ Most countries that operate a CFC regime set a threshold tax rate below which a source country is deemed 'low tax'. This is either an absolute rate, or a proportion of the resident country's rate. Alternatively, some countries use a (black) list which explicitly sets out low-tax countries.

rate in the residence country (i.e. the tax liability is comprised of taxes paid in the source country plus the additional tax which accrues in the residence country).⁴⁵

CFC regimes for each of the 13 residence countries in our data are described in Table A.4. Denmark, Finland, France, Italy, Norway, Spain, Sweden and the UK operate CFC regimes.

In summary, we define the tax variable, τ_{ijt} , as equal to the source country tax rate unless the residence country (of firm i) operates a CFC regime and the source country (j) is deemed low tax, in which case τ_{ijt} is equal to the residence country's statutory rate.

Table 4: Controlled Foreign Company (CFC) regimes

Residence country	Year introduced (1)	Threshold (2)	Threshold rate (3)	Source countries for which CFC ever binds (no. of years*) (4)
Belgium	-	-	-	-
Denmark	1995	Relative	0.75	FI (1 year), IE (all years)
Finland	1995	Relative	0.60	IE (11 years)
France	1980	Relative	0.66	CH (6 years), IE (all years)
Ireland	-	-	-	-
Italy	2002	Blacklist [†]	-	-
Luxembourg	-	-	-	-
Netherlands	-	-	-	-
Norway	1992	Relative	0.66	IE (14 years)
Spain	1996	Relative	0.75	CH (7 years), FI (2 years), IE (all years)
Sweden	1990	Relative	0.55	CH (1 year). IE (11 years)
Switzerland	-	-	-	-
UK	1984	Relative	0.75	CH (2 years), IE (all years)

Notes: The country refers to the country in which a firm is headquartered, the residence country. Where the year of introduction of a CFC regime is missing (-) in column 1, this indicates that no CFC regime is in operation. In cases where the CFC regime was introduced during the period 1985-2005, there is no CFC regime before the year of operation. Column 2 shows the type of threshold in operation and column 3 the rate of the threshold. Column 4 shows that total number of source countries for which the CFC regime is binding. Country codes refer to countries as follows: CH, Switzerland; FI, Finland; IE, Ireland.

[†]Italy operates a blacklist which sets out the countries deemed low tax.

*This is the number of years for which the CFC regime binds between 1985-2000.

⁴⁵ As highlighted in the introduction, there are numerous ways that firms can seek to circumvent these measures.

In recent years, some European countries, including Belgium, Luxembourg and the Netherlands, have introduced ‘Patent Boxes’, under which income from patents is taxed at a lower corporate tax rate. The UK has announced that it will introduce a Patent Box in 2013 which will reduce the rate of corporation tax for the income from patents to 10%. These four Patent Box regimes are described in Table A.5.

Table A.5: Patent Boxes

Country	Year effective from	Patent Box	Rate
Belgium	2007	80% of patent income (net of development costs) is exempt from corporate tax;	6.8%
Netherlands	2007	Net (of development costs) patent income is taxed at a rate of 10%. Extended to ‘Innovation Box’ with wider scope in 2010.	10%
Luxembourg	2008	80% of patent income (net of development costs) is exempt from corporate tax	5.9%
UK	Planned for 2013	Net (of development costs) patent income is taxed at a rate of 10%.	10%

Notes: We do not include Ireland or Spain. Ireland operates a system whereby certain patent royalties are tax exempt and qualifying patents are required to have had an element of the underlying research or processing carried out in Ireland. Spain introduced a Patent Box in 2008 (applied retrospectively to patents for 2007) whereby 50% of revenues from patents are tax exempt with development costs being deducted from the part of revenue that is not tax exempt (which is different to other Patent Boxes that have tax exemptions for revenue net of costs). The Patent Box schemes in Belgium, the Netherlands and Luxembourg are also associated with criteria that define which patents are eligible and thresholds for the total amount of income that can be exempted from tax. The legislation which will finalise the precise details of the UK Patent Box is due to be introduced in 2011.

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