# Learning by Suing: Structural Estimates of Court Errors in Patent Litigation

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**Abstract**

This paper presents structural estimates of the probability of validity, and the probability of Type I and Type II errors by courts in patent litigation. Patents are modeled as uncertain property rights, and implications of the model are tested using stock market reactions to patent litigation decisions. The estimation quantifies beliefs about patent validity and court errors in a Bayesian context.

I estimate that the underlying beliefs about validity range from 0.6 to 0.7 for litigated patents. Market beliefs about courts show that Type I errors (finding a valid patent invalid) occur very frequently—an estimated probability of 0.45. However, Type II errors (finding an invalid patent valid) occur with near zero probability. Additional implications of the model address patent value.

My results are the first structural estimates of court errors. Additionally, this study is the first to perform event studies on patent litigation.

**Keywords**: patents, uncertainty, litigation, innovation, event study

**JEL codes**: L19, L29, O32, O34, K41

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## 1 Introduction

Courts make mistakes. That is a fact. It is plainly accepted by legal scholars and laymen alike; so much so that it is pointless to argue it. Appellate courts would not even exist if it were not so. Furthermore, the theoretical consequences of court errors have been well known at least since Priest and Klein (1984). For instance, uncertain legal standards or awards can lead to changes in observed selection rates and win rates (Waldfogel 1995, Rasmusen 1995). Furthermore, an uncertain legal standard can lead to over-precaution on the part of an injurer. At their worst, severely error-prone courts can diminish or eliminate any intended incentives from the justice system; if violators and non-violators are equally likely to be found in violation, the deterrence mechanism fails completely and courts become deadweight. Less egregious errors will soften deterrence effects.

On the other hand, the *frequency* with which courts err is another matter entirely. There is little or no evidence on error rates because error rates are inherently unobservable (were it otherwise, courts would presumably correct their mistakes immediately). Thus, the extent to which courts err is an empirical question that has not yet been addressed. This paper attempts to remedy that by investigating court errors in the context of patent litigation.

Legal uncertainty is inevitable in a patent rights system. Uncertainty over whether a "title" to property can be enforced will undermine its market value: the title is only as good as the ability to enforce it.[1] Legal uncertainty is especially pervasive in emerging technology areas (or emerging *patenting* areas, like business methods and software patents). Where uncertainty is prevalent, the

[1] Illustrative examples of the importance of property rights enforcement can be found in television portrayals of the "Old West." In 1859, a title to land in Virginia was more valuable than one in Nevada, in part because of the "underlying value of the land,"—closer to transportation and markets, more fertile, etc. However, Virginia land was also more valuable because better enforcement mechanisms were in place there. On the TV series Bonanza, the value of the Ponderosa ranch was due in part to the quality of the land for grazing cattle, and in part to the ability of the Cartwrights to enforce their title—whether through formal institutions (the local constabulary) or self-help (the number of able-bodied Cartwrights available during the episode). See Ellickson (1991) for an excellent discussion of formal versus informal dispute resolution.

effects on appropriation and firm behavior can be dramatic. Since the purpose of a patent system is to provide incentives for research, innovation, and diffusion by creating rewards, an inability to appropriate those rewards diminishes the very incentives for which the system was designed.

Intellectual property managers face decisions about whether to patent innovations (Lerner 1995, Hall and Ham-Ziedonis forthcoming 2001, Grindley and Teece 1997), and how to manage market transactions in intellectual property. If property rights are well-defined, firms may organize transactions through arm's length negotiations. In uncertain legal environments, we expect to see more integrated transactions ranging from cross-licensing, to strategic alliances, to consolidation. To the extent that uncertainty affects or drives these decisions, it is of great strategic importance to firms. And, to the extent that policy makers have some control over the amount of legal uncertainty, or legal "quality" as coined by Merges (1999), it is an important and understudied policy instrument. Simulation estimates (Lanjouw 1994, Lanjouw 1998) find that changes in patent law or the legal environment can significantly change the value of patent protection, not just for litigated patents, but for all patents even if none are ever litigated.[2] Additionally, one can expect the value of patent rights to evolve as information about the validity and scope of a patent evolve (Sherry and Teece 2004), especially through court decisions.

Legal uncertainty is introduced into a patent system by the administrative agency (the Patent and Trademark Office—PTO—in the US) and by legal institutions. Because of the importance of enforcement on the value of intellectual property, many researchers in the US have pointed to the establishment of the Court of Appeals for the Federal Circuit (CAFC) in 1982 as a watershed in the rights of patent holders. The CAFC established—among other things—a single court that would hear the appeals of patent cases from all federal district courts (state courts do not hear patent cases). It is claimed that the CAFC strengthened the rights of patent holders—that the court is more "pro-patent" than its district peers (Lerner 1994, Lanjouw 1994, Lanjouw and Shankerman 1997, Kortum and Lerner 1999), so that we can expect a shift in the legal standard in the early 1980s, perhaps increasing beliefs that a patent will be held valid and infringed, and perhaps decreasing the rate at which mistakes were made with regard to validity and infringement suits.

It is these two sources of uncertainty—the PTO and the courts—that I will examine in the model and the estimation below. The PTO alters beliefs about the validity of patents. Courts are believed

[2] For example, Lanjouw estimates that if the underlying probability of success for a plaintiff fell from 75% to 50%, and legal fees doubled, then the average patent value would be halved in her simulation, even if no cases were litigated.

to err with some frequency, which alters beliefs about whether the patent will *win* a case on validity. (Note that when courts err at all, the probability of winning a validity ruling is not generally equal to the probability that the patent is valid).

Changes in the institutions governing patents can increase or decrease the uncertainty over the scope and validity of patents, and we must recognize that this uncertainty will have effects on firms' incentives to litigate, license, do R&D, and to patent in certain areas. Lerner (1994) finds that the "shadow" of litigation may change the patenting behavior of firms; in particular, high-litigation-cost firms may target "less crowded" technology areas in order to avoid disputes. These effects may be large, and may be an important part of the patent system. For this reason, it is important to have an understanding of the quantitative impact of uncertainty on the value of patent rights.

The current political attention on tort reform in the US is evidence that policy-makers recognize the policy dimension of legal uncertainty on a broad scale. Since it is expensive for the administrative agency to authenticate every patent, it may want to depend on individual firms to enforce their own patents: it need not investigate each patent in depth. It may be more cost effective to introduce some degree of uncertainty into the system as to the validity and scope of patents. In this way, expenditure on each granted patent will be reduced, and only those which are in dispute will be investigated (in court) at further cost. One can therefore expect the socially optimal amount of uncertainty to be positive.

This paper presents an empirical estimation of court errors and beliefs about patent validity in the US patents. I make use of stock market reactions to court decisions in order to estimate the magnitude of changes in beliefs about patent validity. It is from litigating that market participants "learn" about the validity of patents from the court, and update their beliefs accordingly. To my knowledge, this is the only study that measures stock market reactions to legal outcomes of patent cases. The results are very provocative. I find that litigated patents are believed to be valid approximately 64% of the time. Further, I find that the win rate for valid patents is believed to be 54% and the win rate for invalid patents is negligible. So, while Type II errors (false positives) are rare, Type I errors (false negatives) are abundant. I make use of these estimates to compare the value of patent grants to the value of patent litigation. I find that litigated patents are worth over $20 million dollars at birth, and that patent litigation is worth $3 million to $5 million on average.

In the next section, I present a simple model of patent litigation and uncertainty that yields an estimable structural equation. Section 3 lays out the econometric specification and Section 4 describes the patent data, litigation data, and event study results. In Section 5, I estimate several models that explain the size of the market reactions, and that compare the effects of infringement suits (where the patent holder brings the suit) to defensive suits (where the patent holder is the defendant). Section 6 concludes.

# 2 Model

I develop a simple model that enables me to interpret the market reactions to news about patent issuance and patent litigation. In particular, by examining the differential impacts of market reactions to wins and losses, I can infer the implicit beliefs that the market has regarding patent validity, patent value, and court errors.

I assume that the market has belief $\alpha$ that a given patent is valid. Additionally, I assume that courts err with a probability known to market participants. When a patent is litigated, parties will update their beliefs about $\alpha$ according to Bayes' Rule (by which they incorporate the error rates of the courts).

More formally, let the prior belief about validity be $\alpha_0$. Further, let $\omega_1 = \Pr(win|valid)$ be the probability that a valid patent is found valid and $\omega_2 = \Pr(win|invalid)$ be the probability that an invalid patent is found valid. Thus, the court makes a Type I error with probability $1 - \omega_1$ and a Type II error with probability $\omega_2$. I assume throughout that $\omega_1 > \omega_2$ so that the signal by the court is meaningful.

The initial belief that the patent will win in court is given by

$$p_0 = \omega_1\alpha_0 + \omega_2(1 - \alpha_0) = \omega_2 + \alpha_0 (\omega_1 - \omega_2). \quad (1)$$

If a patent is litigated, the court will announce a binary decision: either the patent "wins" or "loses." It may seem odd to examine validity independent from infringement. I do so for several reasons. First, validity decisions are clearly defined, whereas infringement decisions contain more noise. That is, infringement decisions depend not only upon the patent itself, but also the technology used by the infringer. In contrast, validity decisions are a function only of patent characteristics. Further, research has shown that patent litigation can be viewed as a unilateral decision on the part of the patent holder (Marco 2005 (forthcoming)), and that validity is a common defense raised by infringers

(Allison and Lemley 1998, Marco 2004). Thus, a patent holder knows that it will risk invalidity when it litigates a patent, and it should expect a decision on validity.

Beliefs about validity are updated based on the court's known propensity to err. The updated belief, $\alpha_1$, can take on one of two values, depending on whether the patent wins or loses on validity. According to Bayes' Rule:

$$\alpha_1 = \begin{cases} \frac{w_1}{p_0}\alpha_0 & \text{with probability} \quad p_0 \\ \frac{1-w_1}{1-p_0}\alpha_0 & \text{with probability} \quad 1-p_0 \end{cases} \tag{2}$$

The updated belief about validity generates two possible values for the updated belief about winning:

$$p_1 = \begin{cases} w_2 + \frac{w_1}{p_0}\alpha_0\left(w_1 - w_2\right) & \text{with probability} \quad p_0 \\ w_2 + \frac{(1-w_1)}{1-p_0}\alpha_0\left(w_1 - w_2\right) & \text{with probability} \quad 1-p_0 \end{cases}. \tag{3}$$

Note that prior to the ruling, $E\alpha_1 = \alpha_0$ and $Ep_1 = p_0$. This is important when considering the empirical results below. The day before a decision, beliefs are $\alpha_0$, which reflect the expected updated beliefs about validity. Thus, when a decision is made by the court, beliefs change discretely—unless someone has rigged the jury or bribed a judge.

Changes in beliefs about winning can be written as

$$\Delta p = p_1 - p_0 = w_2 + \frac{w_1}{p_0}\left(w_1 - w_2\right)\alpha_0 - p_0 \tag{4}$$

if the patent holder wins, and

$$\Delta p = p_1 - p_0 = w_2 + \frac{1-w_1}{1-p_0}\left(w_1 - w_2\right)\alpha_0 - p_0 \tag{5}$$

if the patent holder loses.

To investigate changes in patent value based on market responses I need to model patent value. I assume that the value of the patent at time $t$ is

$$v_t = p_t z_t \tag{6}$$

where $p$ is defined above, and $z$ is the scope of the patent, measured as the discounted stream of profits accruing to the patent holder assuming the patent is valid. The value $pz$ represents a straightforward valuation of the property right to the patent holder. If $p$ and $z$ are common knowledge, then the Nash bargaining solution of the license negotiation game is $pz$. This specification assumes away signaling and litigation costs. Other valuation methods may represent the value as a non-linear function of $p$ and $z$ (see Marco (2005 (forthcoming)) for a more sophisticated analysis of patent value in a real options setting).

At the time of a court decision on validity, only $p$ will change. That is, the underlying value of the technology is unlikely to change overnight, and any change in value must be attributed to updated beliefs about validity. Thus, for a validity decision at time $\tau$

$$\Delta v_\tau = \Delta p \cdot z_\tau. \tag{7}$$

# 3 Empirical Specification

I use Equation (7) to interpret changes in firm value on the day of a court decision. Patent litigation is an especially useful area of law in which to examine market responses. First—as mentioned earlier—validity is a binary decision.[3] Second, there is little or no leakage prior to the announcement of the decision. Third, all the new information about the patent pertains to changes in beliefs about validity as opposed to the patented technology.

For patent $i$ born at time 0 and litigated at time $\tau$, the formal econometric specification becomes

$$\Delta f_{\tau,i} = \Delta p_{\tau,i} \cdot z_{\tau,i} + \varepsilon_{\tau,i} \tag{8}$$

where the change in the probability of winning is given by Equations (4) and (5)

$$\Delta p_{\tau,i} = \left( w_2 + \frac{w_1 (w_1 - w_2) \alpha_{0,i}}{p_{0,i}} - p_{0,i} \right) D_V + \left( w_2 + \frac{(1 - w_1)(w_1 - w_2) \alpha_0}{1 - p_0} - p_0 \right) D_{NV} \tag{9}$$

and the patent value is

$$z_{\tau,i} = (\beta_0 + \beta_1 \Delta f_{0,i}) \, e^{-r\tau}. \tag{10}$$

The $Ds$ are indicator variables for different kinds of decisions: valid and not valid. The parameters to be estimated are $\alpha_0$, $w_1$, $w_2$, $\beta_0$, $\beta_1$, and $r$.

Estimating the model requires four pieces of information:

[3] There are a handful of decisions where a patent is found valid in part and not valid in part. Those cases are excluded from my sample.

1. The change in patent value at time $\tau$ ($\Delta f_\tau$). I measure this below using an event study methodology to calculate the excess returns to patent litigation. This will be the dependent variable.

2. An estimate of the value of the patent prior to litigation. I estimate this by calculating the excess returns at the time the patent was granted ($\Delta f_0$).

3. An estimate of the probability of winning in court ($p_0$). I estimate this using a probit in Section (4).

4. The age of the patent at the time of litigation.

The patent value at the patent's birth is estimated to be a linear function of the change in the value of the firm on the day of patent grant, with unknown parameters $\beta_0$ and $\beta_1$. This specification acknowledges that some information about the patent may leak into the market prior to patent issuance and that market valuations reflect only *changes* in expectations about the value of the patent. Thus, I estimate the initial value of the patent as a linear function of the size of the response on the date of issuance, $z_0 = \beta_0 + \beta_1 \Delta f_0$. The scope at time $\tau$ is depreciated by $e^{-r\tau}$ to account for changes in the value of the patent right because of either an aging patent right or obsolescence.[4]

The specification is agnostic as to whether beliefs about validity and court errors evolve over time. The notation indicates that beliefs about validity and the win rate the day before the decision are equal to the beliefs at time 0 (as indicated by the subscript 0). However, the win rate is measured empirically as of the day of the decision ($\tau - 1$), so that I measure only the change in beliefs due to the court's decision.

## 3.1 Multiple Patents-in-Suit

Before discussing the data and the calculation of excess returns and the probability of validity, one econometric problem must be dealt with. I am not able to observe changes in patent value, but rather

[4] The depreciation rate $r$ is not constrained to be positive. A negative estimated value for $r$ would indicate that the patent has appreciated, perhaps because new uses have been discovered for the technology. While this is possible, or even probable, while the technology is young, the value of the *patent right* (as opposed to the patented technology) will certainly decline at some point prior to expiration.

changes in the value of a firm. So, if there are multiple "patents-in-suit" I observe only the aggregate market reaction. That is, if there are $N$ patents-in-suit that are adjudicated simultaneously,

$$\Delta f_{\tau,i} = \Delta v_{\tau,i,1} + \Delta v_{\tau,i,2} + ... + \Delta v_{\tau,i,N} \tag{11}$$

where $\Delta v_{\tau,i,n}$ represents the change in the value of patent $n$ of firm $i$ at time $\tau$. So, while we observe $\Delta f$, what we seek is the expectation of $\Delta v$ given $\Delta f$, or $E(\Delta v|\Delta f)$. In cases where $N = 1$ there is no difficulty in the estimation. Removing cases where $N > 1$ leaves information from multiple patents-in-suit unexploited (and possibly biases the results). Instead, I use an application of the Expectation-Maximization (EM) Algorithm to make use of the data when there are "missing" $\Delta v$'s.[5] The EM Algorithm in this application is described in detail in Appendix C. The intuition is that I estimate Equation (8) to predict values of $\Delta v$ for multiple patents-in-suit. These predicted values are used in a new iteration of the estimation, and the process is repeated until the parameter estimates converge.

# 4 Data

In the following subsections I describe the sample of litigated patents, the description of court decisions, the calculation of excess returns (on adjudications and patent grants), and the estimation of the probability of validity.

## 4.1 Adjudication data

My data begin with a database compiled by researchers at the National Bureau of Economic Research (NBER) and Case Western Reserve University (CWRU) (Hall, Jaffe and Trajtenberg 2000).[6] My sample consists of over 417,000 patents owned by publicly traded US manufacturing firms. The patents are assigned Cusip identifiers using the 1989 ownership structure of the patent holder.

Litigation data were hand-collected from the United States Patents Quarterly (USPQ) 1977-1997.[7] The USPQ publishes annual indices containing patents on which adjudications were made

[5] For a good overview of the EM Algorithm with applications to economics, see Ruud (1991); McLachlan and Krishnan (1997) provide an extensive treatment of the subject.

[6] My thanks to Bronwyn Hall for permitting access to the data.

[7] See Allison and Lemley (1998). I thank Mark Lemley for the reference to USPQ.

in that year. USPQ contains only "published" adjudications, which is a subset of all adjudications. However, the advantage of the USPQ is that it contains clear information on the disposition of the case with regard to validity and infringement. The USPQ data were merged with the NBER/CWRU data to obtain a list of litigated patents owned by publicly traded firms.

The merged data contain 701 case citations involving 670 patents. I entered the disposition data for each adjudication containing decisions relevant to validity or infringement. Adjudications involving preliminary motions about discovery, jurisdiction, etc. were discarded. Also, PTO interference proceedings and examination proceedings were not used. When a USPQ citation made explicit reference to an earlier related decision, I incorporated that case into the database.

The final adjudication data consist of 390 decisions involving 413 patents owned by 158 publicly traded firms. An observation in my data is a "patent-decision." For example, a single case may involve four patents. Of the patent-decisions, 385 involved a distinguishable decision on validity. About half of the cases involve only one patent-decision. The implied litigation rates are given in Table 1, where case filing data was calculated using data obtained from LitAlert.[8]

## 4.2 Excess returns

In order to be able to analyze the adjudications using my methodology, I require an estimate of stock market reactions to news about the patents. I obtained CRSP data on daily stock returns from the Wharton Research Data Service (WRDS). I use cumulative abnormal returns—or excess returns—as measured by event studies to measure the stock market reactions to patent decisions and patent issuance.

Event studies are appropriate for several reasons. First, the model provides a way to interpret the probability that a patent is valid as a function of changes in patent value. Changes in value are precisely what event studies are designed to measure. Second, while event studies have been used by researchers to investigate the effects of other types of litigation (Bhagat, Brickley and Coles 1994), no study has concentrated on patent litigation.[9] Market reactions provide information that has not been previously incorporated into the patent value (and consequently, the value of the firm). Third, litigation events are well identified: court records for published decisions identify the date of the decision. Last, litigation events can be directly associated with changes in beliefs about the legal

[8] It is likely that both filing data and adjudication data are under-reported.

[9] Austin (1993) uses event studies to examine market reactions to patent issuance.

patent right. If a patent is ruled to be valid, nothing about the decision affects the value of the underlying technology, so the change in value reflects changes in beliefs about the uncertainty over property rights.

In order to estimate Equation 8, I need to calculate a measure for the market reaction to the litigation event, and to patent issuance. So, the output of the event studies forms the dependent variable and an independent variable for the estimation equation. Appendix A describes the empirical methodology for event studies in detail.

I was able to calculate excess returns for 324 of the 385 patent-decisions. Of that number, I was able to calculate excess returns for 198 patent grants (issue dates) and 181 application dates. The excess returns are summarized in Table (2) for 2-day event windows (the day of the event and the day after) and for 11-day event windows (from 5 days prior to 5 days after the event).[10]

We expect the market returns to be somewhat noisy despite the precision of the event date. First, firms differ in size, so reactions to good or bad news about patents will vary not only according to revision in beliefs, but also according to the firm's market capitalization. Large firms will have smaller responses, *ceteris paribus*. In the estimation I use both excess returns and dollar amounts. The results are fairly consistent across specifications, but there are some differences that I describe in Section (5). However, it is useful here to get a bearing on the dollar amount of the excess returns.[11]

First, note that the 11-day returns make more sense with regard to sign than the 2-day returns. That is, with 11-day returns invalidity decisions are unambiguous bad news, and validity decisions and patent grants are unambiguous good news. The implication is that both adjudication information and patent grant information take some time to filter into the markets. The mean 11-day return to a validity decision is $61.5 million. Similarly, the mean 11-day return to a patent application is $31.1 million. Issuance is similar.

To put this in context, compare these reactions to the CAR estimates of patent issuance done by Austin (1993). He finds that excess returns range from a mean of about $500,000 for the full sample, to a mean of $33 million for those patents mentioned in the Wall Street Journal.

[10] I calculate excess returns of litigation for 1, 2, 3, 5, 7, 9, and 11 day event windows. Patent issuance and application returns are estimated for the same windows, except that I leave out the 1-day returns.

[11] Dollar values are obtained by multiplying the abnormal return by the market value of the firm.

### 4.3 Probability of validity

To calculate the probability of validity, I run a simple probit on my sample in the spirit of Marco (2004) and Lanjouw and Schankerman (2001). Appendix (B) describes the estimation in detail. From the estimated probit, I predict the probability that a patent will win and use this as a dependent variable in the estimation.

Note that the estimated probability of validity is conditional upon having been litigated; it does not control for self-selection. Because my model estimates market reactions on a sample of patents that are known to be in court, the conditional probability is appropriate. This implies that the resulting estimation for the probability of validity will also need to be interpreted as conditional on selection. See the Appendix for more details.

## 5 Results

Recall that I measure returns to adjudication events and to patent grant events using multiple windows. Also, the event date for patent grants can be measured two ways: patent application date or patent issuance date. I use excess returns at both of these dates as a proxy for patent value; additionally, I use the sum of the excess returns on both of these dates.

In total I define 7 dependent variables (the seven different event windows for excess returns for adjudications) and 18 different proxies for returns to patent grants (six event windows for three definitions for initial patent value). Matching all pairs gives me a total of 126 possible ways to estimate Equation (8) using excess returns. Using the dollar value of excess returns yields an additional 126 ways to estimate Equation (8), for a total of 252 regressions.

Because of the high degree of non-linearity in the estimating equation, most of the estimations did not converge.[12] In total, only 9 of the excess returns estimations converged, and 25 of the dollar value estimations converged. Table (5) shows the results of estimating Equation (8) for a representative set of excess returns. Column 1 shows the results using excess returns and Column 2 shows the results using dollar values of the excess returns. Both estimations rely on the EM Algorithm correction for multiple patents-in-suit.[13]

[12] The lack of convergence is the result of the non-linear estimation as opposed to the application of the EM Algorithm.

[13] Recall that in these specifications a firm may litigate, e.g., two patents, and receive good news on one patent and

Columns 1 and 2 show several interesting results. First, the estimated beliefs from the model (about validity and the propensity for courts to favor the patent holder) are all estimated between zero and one. Because each belief can be interpreted as a probability, this should be expected from a theoretical perspective. However, it is important to underscore that the coefficients in the estimation are *unconstrained* and can thus lie anywhere on the real line. In fact, all the significantly estimated coefficients in all specifications yielded coefficients between zero and one. This fact alone lends credibility to the model.

Looking at the quantitative estimates the probability of validity is measured inconsistently between the two specifications. The returns equation yields a low estimate of 0.191, and the dollar specification yields an estimate of 0.639. I return later to the consistency of the results. At this point it is sufficient to point out that both 0.19 and 0.64 are low values for the probability of validity. However, one should note that these coefficients represent only the patents selected for trial. These patents cannot be presumed to be representative of the population of patents. In fact, one may expect them to represent patents believed to have a low probability of validity. The patent holder itself may desire to litigate these patents more frequently in order to increase the patent value more significantly (in the event of a win)—a double or nothing approach.

The estimated win rate for a valid patent is also measured inconsistently between 0.875 to 0.437. The corresponding win rate for invalid patents is 0.331 or zero (measured insignificantly). As one would expect (or at least hope) the win rate for valid patents is significantly higher than that for invalid patents.

The scaling parameters (on patent value) are rarely significant for individual estimations, although $\beta_1$ tends to be greater than zero in most estimations. The depreciation rate for the patent right is generally statistically insignificant (although it may come out either significantly positive or negative depending on the specification). In this case, the positive (but insignificant) values for depreciation indicates that patent rights lose value—on average—as they age. However, the insignificant result shows that there are forces counteracting the depreciation, most likely an appreciation of the underlying technology that offsets the depreciation of the patent right.

Since the results are inconsistent, it is prudent to investigate the parameter estimates from all 34 estimations. Figures (1) to (3) show the distributions of the estimated parameters $\alpha$, $\omega_1$,

bad news on the other. Without correcting the excess returns for this, one would expect the two pieces of news to negate each other in the market.

and $\omega_2$. It can be seen that there are some outliers that fall outside the zero to one range for each of the parameters (most notably for $\omega_2$). However, none of the outliers is significant. The distributions make clear that $\omega_1 > \omega_2$ on average. Unfortunately, the graphs cannot distinguish between significant and insignificant coefficients.

Table (6) shows confidence intervals for all the parameters across all specifications. The confidence intervals are calculated for each parameter across all specifications, weighted by the inverse of the variance of each parameter estimate. By exploiting information on the precision of the results from each estimation, I can obtain a tighter distribution on the parameter estimates in aggregate.

From Table (6), the confidence interval for $\alpha$ ranges from about 0.6 to 0.7, with a mean of 0.64. Again, this should be interpreted as representing litigated patents only. However, the relatively low value of 0.64 indicates that low-$\alpha$ patents are being selected for litigation. This result is consistent with Priest and Klein (1984) but inconsistent with Waldfogel (1995) who finds that the win rate for patents is biased upwards relative to the population win rate.[14]

The win rates are very telling. For valid patents the confidence interval for $\omega_1$ ranges from about 0.52 to 0.61, with a mean of 0.57. This indicates that Type I errors occur frequently, with a 0.40 probability. The corresponding win rate for invalid patents is 0.034 at the mean, but is not measured significantly. Thus, there is a negligible Type II error rate.

The scaling parameters are best measured separately by excess returns and dollars, because the magnitudes change based on the units. For dollars, both scaling parameters come in positively. Depreciation is not measured precisely for the dollar estimates, but is significantly positive for the returns estimates.

It is instructive at this point to examine the implications of the estimates on patent value. The average probability of winning, given $\alpha$, $\omega_1$, and $\omega_2$ is given by Equation (1)

$$p_0 = \omega_2 + \alpha_0 (\omega_1 - \omega_2) = 0.376.$$

Following a win in court, this probability would be updated to $p_1 = 0.55$, and $\alpha$ would be updated to $\alpha_1 = \frac{w_1}{p_0}\alpha_0 = 0.97$. However, following a loss, beliefs would be updated to $p_1 = 0.27$ and $\alpha_1 = \frac{1-w_1}{1-p_0}\alpha_0 = 0.44$. These values can be used to evaluate Equation (8) at the means of the dependent variables: at the average excess return of \$20 million for a patent grant, a win on validity

[14] It should be noted that both Priest and Klein and Waldfogel discuss the "probability of winning" without regard to court error, so that $\alpha$ and $p$ are conflated in those models.

would be worth approximately $5 million to the patent holder. A loss would cost approximately $3 million. Thus, the resolution of uncertainty is worth 15%-25% of the value of the patent. Were Type I errors smaller, this value would certainly rise.

## 6 Conclusion

That courts err is not news to anyone—legal scholars and laymen alike. However, to this point there has been no empirical estimates investigating the frequency with which courts err. The reason for this empirical omission is obvious: error rates are inherently unobservable. However, by utilizing information from stock market reactions to patent litigation decisions and to patent grants, I am able to structurally estimate court errors, as well as patent value. In interpreting the results it is important to remember that the results on patent validity and patent value are subject to self-selection. However the court error rates are not subject to self-selection in that they are treated as exogenous to the decision-making of the litigating parties.

Most importantly, I find that Type I errors (false negatives) occur far more frequently than Type II errors (false positives). In particular, a valid patent will be errantly found invalid about 45% of the time. In contrast, an invalid patent will be found valid very infrequently (close to 0). These results are of import to anyone interested in legal reform, including policy makers interested in tort reform. Any positive error rates by courts will necessarily dampen—to a greater or lesser extent—the impacts of reforms. In this case the Type I error is very high, so that positive signals by the court will be very meaningful to the market, but negative signals will not. Negative signals then cause ambiguity and confusion, making appeal more likely.

With regard to patents in particular, litigated patents appear to be self-selected from a pool of fairly "low $\alpha$" patents, with the belief about validity being about 0.64.

I find that the average patent value in my sample is $23.9 million dollars at birth.[15] Interestingly, resolving some uncertainty about validity through "learning by suing" can lead to changes in patent value in the order of $3 to $5 million dollars. That is, resolving some uncertainty about patent validity is worth about 15%-25% of the patent value, on average. This change would be much higher if the Type I error rate were lower.

[15] The average market reaction at patent grant is $20 million dollars. Using the estimates for $\beta_0$, $\beta_1$, and assuming the age is zero, the estimated patent value at birth is $23.9 million.

Future research in this area attempts to separate appellate and lower court errors, as well as differences among jurisdictions.

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# A Event studies

The market model is the model most frequently used in event studies. The estimation equation is

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \varepsilon_{i,t}$$

where

$$\begin{aligned} R_{i,t} &= \text{proportionate return on the stock of firm } i \text{ from time } t-1 \text{ to time } t. \\ R_{m,t} &= \text{proportionate return on the overall market from time } t-1 \text{ to time } t. \end{aligned}$$

Abnormal returns are calculated by estimating the parameters of the market model in some pre-event equilibrium. Essentially, the abnormal return is the forecast error. The cumulative abnormal returns are given by

$$CARi = \sum_{t=-\tau}^{\tau} u_{i,t}$$

That is, cumulative abnormal returns are the summation of abnormal returns over the event window. For the analysis below the pre-event equilibrium is (-300, -20), measured in trading days, and the abnormal returns are calculated for event windows of 1, 3, 5, and 11 trading days around the event date. I use the Equal Weighted Market Return for $R_m$, as defined by CRSP.

# B Probability of Validity

The specification for estimating the probability of validity is based on Marco (2004) and Lanjouw and Schankerman (2001). I base the probability of validity on observable patent characteristics as well as observable case characteristics. Patent data for my sample were obtained from the NBER Patent Citations Data Files, described in Hall, Jaffe and Trajtenberg (2001). The variables used are described in Table (3).

Forward citations are citations received by a patent from subsequently issued patents. Similarly, backward citations are those made by the patent to previously issued patents. Patent applicants are required to cite "prior art" including previously issued patents. Failure to do so is grounds for invalidity if the omission is later discovered (Allison and Lemley). Hall et al. describe the creation of several indices in the NBER data using patent citations, including *generality* and *originality* (Jaffe, Trajtenberg and Henderson 1993).

Generality for patent $i$ is defined as

$$1 - \sum_{j}^{n_i} s_{ij}^2$$

where $s_{ij}$ refers to the proportion of forward citations to patent $i$ from patents in technology class $j$. The higher the index, the more spread out are the patents that cite it, technologically speaking (Hall et al.). Originality is similarly defined, except that $s$ refers to backward citations rather than forward citations. Higher originality indicates that a wider array of technologies were utilized in creating the innovation.

Both generality and originality are undefined if the number of citations is zero. In my sample, undefined values are replaced by zero. In the case of generality this assignment makes sense in that an uncited patent is not applicable to *any* patented technologies (yet), and therefore receives a low score for generality. Alternatively, the patent may just be very young. In the case of originality, one could imagine that highly original patents might cite *no* previous patents. However, from an empirical standpoint, Hall et al. observe that higher numbers of citations tend to be associated with higher originality and generality indices; thus, assigning zero to undefined values seems the logical choice.

Self-citations are another important measure in the patent literature. The NBER dataset defines two self-citation variables, reflecting the proportion of forward citations that are made by the patent holder itself. Since the identity of patent holders is subject to error, an upper bound and a lower bound are defined. Missing values for the self-citation variables will obtain whenever there are no forward citations. In these cases, I replace missing lower (upper) bound values with zero (one), since this is the theoretical lower (upper) bound.

Table (4) shows the results of a simple probit on the probability of validity. Since the cases are subject to selection, the predicted probability of validity must be interpreted as the probability of validity conditional on being litigated (Marco 2004). For the current application, the conditional probability is appropriate because I want to measure the probability the *day before* the court's decision. At that point in time it is known that the patent has been litigated. Thus, using a selection-corrected unconditional probability would cause a bias.

The estimates are used to construct a predicted probability of validity. That variable is used as a dependent variable in the structural estimations in Section (5).

# C EM Algorithm

The EM Algorithm enables me to estimate the change in the value of a particular patent given that I know the change in the value of the firm, and the disposition of the patent in question and other simultaneously adjudicated patents. That is, the EM Algorithm enables me to estimate values for $v$ conditional on $f$ for the special case where the $v$'s are missing. Since multiple patents may be adjudicated simultaneously, the excess returns for the firm's stock price must be apportioned across the $v$'s. To do so, we require $E(v_{ijn}|f_i, \mathbf{h}_{ijn} \cdot \mathbf{d}_{ijn})$ where $\mathbf{h}_{ijn}$ is a vector of coefficients on the disposition of the case $\mathbf{d}_{ijn}$. Let

$$v_{ijn} = \mathbf{h}_{ijn} \cdot \mathbf{d}_{ijn} + \varepsilon_{ijn}$$

where

$$\varepsilon_{ijn} \sim N\left(0,\sigma^2\right)$$

so that the error term is normal and the $\varepsilon_{ijn}$'s are independently and identically distributed. The assumption of independence is convenient but not innocuous. We can imagine that patents that are litigated together may not be independent, but instead be part of a larger system. The validity of any component may rise and fall by the validity of the system. The potential dependence of component patents warrants investigation; however, for simplicity I will assume independence in this paper.

Since I assume that $f = \sum_N v_n$, I can write

$$f_{ij} \sim N\left(\sum_N \mathbf{h}_{ijn} \cdot \mathbf{d}_{ijn},N\sigma^2\right)$$

and

$$\begin{aligned} Var(v_{ijn}) &= \sigma^2 \\ Var(f_{ij}) &= N\sigma^2 \\ Cov(v_{ijn}, f_{ij}) &= \sigma^2. \end{aligned}$$

Generally if two random variables $A$ and $B$ are correlated, the expectation of $A$ given $B$ can be written as $E\left(A|B\right) = E\left(A\right) + \frac{Cov(A,B)}{Var(B)}\left(B - E\left(B\right)\right)$. Applying this formula to the case at hand yields

$$\begin{aligned} E\left(v_{ijn}|f_{ij}\right) &= E(v_{ijn}) + \frac{Cov(v_{ijn}, f_{ij})}{Var(f_{ij})}\left(f_{ij} - E(f_{ij})\right) \\ &= E(v_{ijn}) + \frac{1}{N}\left(f_{ij} - E(f_{ij})\right). \end{aligned}$$

Using predicted values this can be approximated by

$$E\left(v_{ijn}|f_{ij}\right) = \widehat{v_{ijn}} + \frac{1}{N}\left(f_{ij} - \sum_{N}\widehat{v_{ijn}}\right) \tag{12}$$

Implementing the EM Algorithm involves using a predicted value of the vector $v$ to obtain a parameter estimate, which is used to get a better prediction for $v$:

$$v^{(0)} \rightarrow \beta^{(0)} \rightarrow \widehat{v}^{(1)}$$

In this case $v^{(0)}$ is the starting value. In my application $v^{(0)}$ consists of only single-patent cases from which we obtain a parameter vector $\beta^{(0)}$ (this is the maximization step because the EM algorithm is a maximum likelihood technique). I use $\beta^{(0)}$ to predict $\widehat{v}^{(1)}$. This prediction does not incorporate any information from $f$. In particular, for multi-patent cases, the sum of $\sum_N \widehat{v}^{(1)}_{ijn}$ is likely to be a poor predictor of $f_{ij}$. Instead a new value $v^{(1)}_{ijn}$ can be given by

$$v^{(1)}_{ijn} = E(v_{ijn}|f_{ij}) = \widehat{v}^{(1)}_{ijn} + \frac{1}{N}\left(f_{ij} - \sum_{N}\widehat{v}_{ijn}\right)$$

(this is the expectation step). $v^{(1)}$ is regressed on the explanatory variables to determine $\beta^{(1)}$ and the process is iterated until the sequence $\beta^{(0)}, \beta^{(1)}, \ldots$ converges to a fixed point, $\beta^{EM}$.

Table 1: Adjudication data

| | Total | Filed | | Decided | |
|---|---|---|---|---|---|
| Firms | 2,699 | 568 | *21.0%* | 158 | *5.9%* |
| Patents | 417,735 | 1,252 | *0.3%* | 413 | *0.1%* |
| Cases | | | | 390 | |
| Patent-decisions | | 2,252 | | 610 | |

Table 2: Excess returns

| Event | Event Window | Obs | Mean | Std.Err. | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| Adjudications | | | | | | |
| Valid | | | | | | |
| Returns (%) | 2 | 55 | -0.19 | 0.28 | -0.76 | 0.38 |
| | 11 | 55 | 1.52 | 0.81 | -0.11 | 3.15 |
| Dollars ($millions) | 2 | 55 | -23.0 | 11.7 | -46.4 | 0.3 |
| | 11 | 55 | 61.5 | 46.1 | -30.8 | 153.8 |
| Not Valid | | | | | | |
| Returns (%) | 2 | 45 | -0.64 | 0.30 | -1.23 | -0.04 |
| | 11 | 45 | -0.82 | 0.54 | -1.92 | 0.28 |
| Dollars ($millions) | 2 | 45 | 10.2 | 28.2 | -46.6 | 67.0 |
| | 11 | 45 | -10.7 | 58.6 | -128.9 | 107.4 |
| Patent issuance | | | | | | |
| Application date | | | | | | |
| Returns (%) | 2 | 181 | -0.05 | 0.17 | -0.38 | 0.28 |
| | 11 | 181 | 1.10 | 0.39 | 0.32 | 1.87 |
| Dollars ($millions) | 2 | 181 | -14.1 | 11.8 | -37.4 | 9.3 |
| | 11 | 181 | 31.1 | 22.2 | -12.7 | 75.0 |
| Issue date | | | | | | |
| Returns (%) | 2 | 198 | -0.31 | 0.19 | -0.69 | 0.07 |
| | 11 | 198 | 1.41 | 0.60 | 0.23 | 2.59 |
| Dollars ($millions) | 2 | 198 | 6.5 | 7.8 | -9.0 | 22.0 |
| | 11 | 198 | 30.0 | 13.3 | 3.9 | 56.2 |

Notes:
Adjudication data excludes multiple patents-in-suit adjudications

Table 3: Variables used to calculate the probability of validity

| | | |
|---|---|---|
| Dependent indicator variables | | |
| | V | Indicates a positive validity ruling |
| Independent | | |
| Court indicator variables | | |
| | DEFENSIVE | Indicates the patent holder is the defendant |
| | APPEAL | Indicates an appellate decision |
| | PRIORPOS | Indicates there was a prior positive decision on validity (infringement) |
| | PRIORNEG | Indicates there was a prior negative decision on validity (infringement) |
| | PRE82CASE | Indicates case filed prior to 1982 |
| Citation | | |
| | BACKWARD | Number of backward citations per claim |
| | FORWARD | Average number of forward citations per claim per year |
| | SELF | Proportion of forward citations that are self-citations |
| | GENERAL | NBER "Generality" index. Undefined values set to 0 |
| | ORIGINAL | NBER "Originality" index. Undefined values set to 0 |
| Scope | | |
| | NUMIPC | Number of 4-digit International Patent Classes |
| | LOGCLAIM | Number of patent claims |
| Technology | | |
| | CHEM | Chemicals. NBER technology category = 1 |
| | COMP | Computers and communication. NBER technology category = 2 |
| | MED | Drugs and medical. NBER technology category = 3 |
| | ELEC | Electronics. NBER technology category = 4 |
| | MECH | Mechanical. NBER technology category = 5 |
| Other | | |
| | PATDELAY | Time between patent application and patent grant |
| | FOREIGN | Indicates non-US patentee |
| | PRE82PAT | Indicates patent application dated prior to 1982 |
| | AGE | Age of the patent from patent application to case filing |

Table 4: Probability of validity

| Variable | Coef. | | Std. Err. |
|---|---|---|---|
| DEFENSIVE | -0.099 | | (0.287) |
| APPEAL | -0.535 | ** | (0.217) |
| PRIORPOS | 1.244 | *** | (0.314) |
| PRIORNEG | -0.448 | | (0.273) |
| PRE82CASE | -0.483 | * | (0.263) |
| BACKWARD | -0.481 | | (0.301) |
| BACKWARD$^2$ | 0.034 | | (0.038) |
| FORWARD | 0.667 | | (1.879) |
| FORWARD$^2$ | 0.182 | | (2.492) |
| SELF | -0.455 | | (0.482) |
| GENERAL | -0.247 | | (0.350) |
| ORIGINAL | 0.281 | | (0.452) |
| NUMIPC | 0.161 | | (0.177) |
| LOGCLAIM | -0.189 | | (0.208) |
| CHEM | 0.188 | | (0.267) |
| COMP | 0.620 | * | (0.372) |
| MED | -0.425 | | (0.290) |
| ELEC | 0.004 | | (0.263) |
| MECH | -0.157 | | (0.256) |
| PATDELAY | 0.045 | | (0.055) |
| FOREIGN | 0.590 | | (0.680) |
| PRE82PAT | -0.446 | | (0.310) |
| AGE | 0.143 | ** | (0.063) |
| AGE$^2$ | -0.006 | ** | (0.003) |
| CONSTANT | 0.827 | | (0.856) |
| Observations | 303 | | |
| Pseudo R2 | 0.188 | | |
| LR chi2(24) | 77.5 | | |

Notes:

Standard errors in parentheses

* signif. at 10%; ** signif. at 5%; *** signif. at 1%

Table 5: Structural estimations

| Parameter | | Returns (1) | | Dollars (2) | |
|---|---|---|---|---|---|
| Prob(valid) | $\alpha$ | 0.191 | * | 0.639 | *** |
| | | (0.102) | | (0.102) | |
| Win rate (valid) | $\omega_1$ | 0.875 | *** | 0.437 | *** |
| | | (0.109) | | (0.073) | |
| Win rate (invalid) | $\omega_2$ | 0.331 | *** | -1.211 | |
| | | (0.092) | | (0.844) | |
| Scaling parameter 1 | $\beta_0$ | 0.015 | | 2127 | |
| | | (0.018) | | (8462) | |
| Scaling parameter 2 | $\beta_1$ | -1.210 | | 0.416 | |
| | | (0.810) | | (0.326) | |
| Depreciation rate | $r$ | 0.133 | | 0.068 | |
| | | (0.083) | | (0.044) | |
| Observations | | 181 | | 181 | |
| R-squared | | 0.09 | | 0.25 | |

Notes:
Standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%
Dependent variable: returns/dollars on adjudication date (one day window)
Base value: returns/dollars on application date (11 day window)

Table 6: Structural estimates

| Parameter | | Sample | Obs | Mean | Std. Err. | [95% confidence interval] | |
|---|---|---|---|---|---|---|---|
| Prob(valid) | $\alpha$ | Returns | 9 | 0.375 | 0.076 | 0.198 | 0.551 |
| | | Dollars | 25 | 0.674 | 0.027 | 0.619 | 0.729 |
| | | Total | 34 | **0.640** | 0.030 | **0.579** | **0.701** |
| Win rate (valid) | $\omega_1$ | Returns | 9 | 0.773 | 0.054 | 0.648 | 0.898 |
| | | Dollars | 25 | 0.548 | 0.021 | 0.505 | 0.591 |
| | | Total | 34 | **0.568** | 0.022 | **0.524** | **0.613** |
| Win rate (invalid) | $\omega_2$ | Returns | 9 | 0.289 | 0.029 | 0.221 | 0.357 |
| | | Dollars | 25 | -0.171 | 0.030 | -0.234 | -0.108 |
| | | Total | 34 | **0.034** | 0.045 | **-0.058** | **0.126** |
| Scaling parameter 1 | $\beta_0$ | Returns | 9 | **0.000** | 0.001 | **-0.001** | **0.002** |
| | | Dollars | 25 | **12534** | 2490 | **7394** | **17673** |
| | | Total | 34 | 0.000 | 0.001 | -0.001 | 0.002 |
| Scaling parameter 2 | $\beta_1$ | Returns | 9 | **0.038** | 0.098 | **-0.187** | **0.264** |
| | | Dollars | 25 | **0.568** | 0.160 | **0.237** | **0.899** |
| | | Total | 34 | 0.163 | 0.088 | -0.016 | 0.342 |
| Depreciation rate | $r$ | Returns | 9 | **0.081** | 0.028 | **0.017** | **0.146** |
| | | Dollars | 25 | **-0.014** | 0.020 | **-0.054** | **0.026** |
| | | Total | 34 | -0.004 | 0.017 | -0.039 | 0.031 |

Figure 1: Distribution of estimates of probability of validity

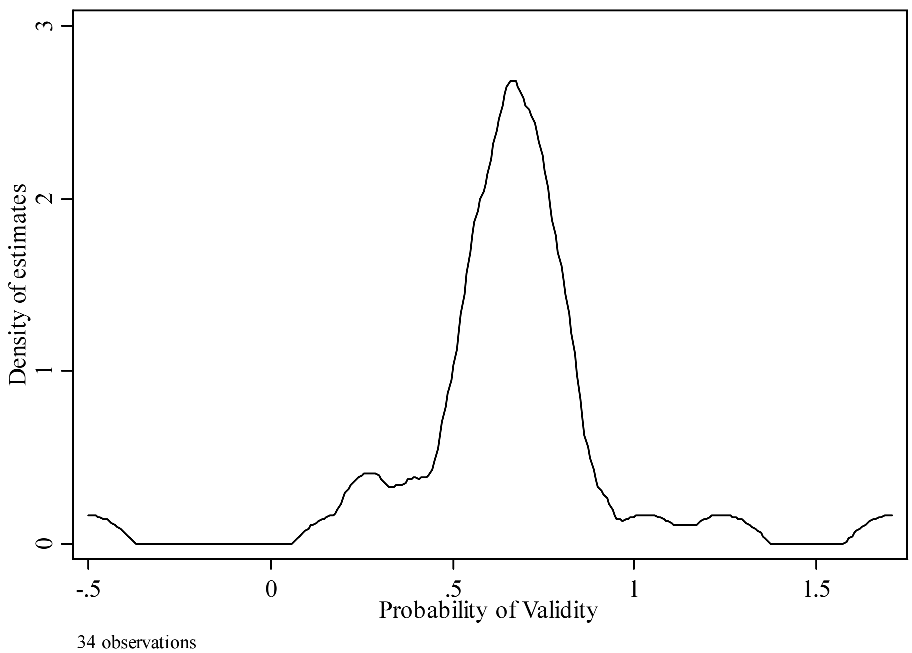


Figure 2: Distribution of estimates of win rate for valid patents

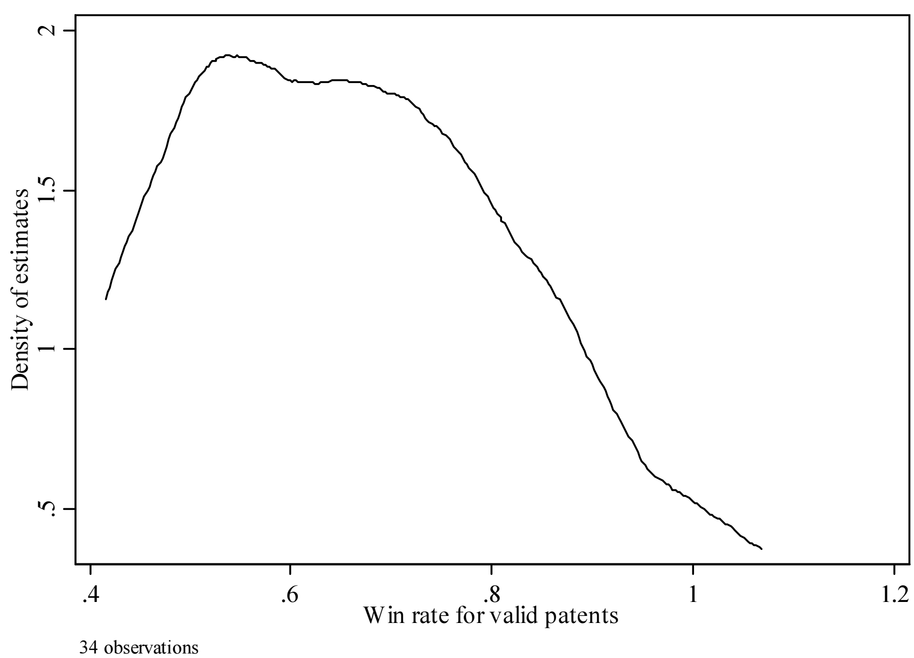
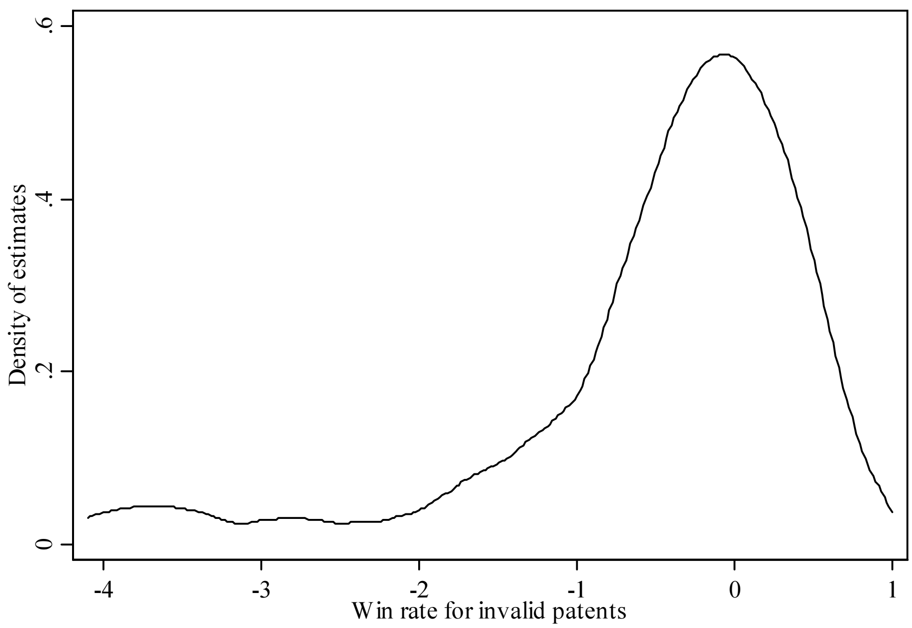


Figure 3: Distribution of estimates of win rate for invalid patents




31 observations. 3 observations excluded.