Public Debt, Public Investment and Economic Growth

Alfred Greiner[1]

[1]Department of Business Administration and Economics, Bielefeld University, P.O. Box 100131, 33501 Bielefeld, Germany

1 Introduction

Greiner, Semmler and Gong (2005) *The Forces of Economic Growth: A Time Series Perspective*

Arrow and Kurz (1970), Futagami et al., (1993)

Empirical Evidence: Romp, W. and J. de Haan (2005)

Public Debt: Turnovsky (1995)

Sustainability: Hamilton and Flavin (1986), Afonso (2005)

Bohn (1995, 1998), Greiner et al. (2004)

2 The primary surplus and sustainability of public debt

Government budget constraint:

$$\dot{B}(t) = B(t)r(t) - S(t) = B(t)r(t) - T(t) + I_p(t) \tag{1}$$

$$e^{-\int_0^t r(\tau)d\tau} B(t) + \int_0^t e^{-\int_0^\tau r(\mu)d\mu} S(\tau)d\tau = B(0) \tag{2}$$

Sustainable:

$$B(0) = \int_0^t e^{-\int_0^\tau r(\mu)d\mu} S(\tau)d\tau \leftrightarrow \lim_{t\to\infty} e^{-\int_0^t r(\tau)d\tau} B(t) = 0 \tag{3}$$

Assumption:

$$\frac{T(t) - I_p(t)}{Y(t)} = \phi + \beta \frac{B(t)}{Y(t)} \tag{4}$$

$$\dot{B}(t) = r(t)\,B(t) - T(t) + I_p(t) = (r(t) - \beta)\,B(t) - \phi\,Y(t) \tag{5}$$

$$e^{-\int_0^t r(\tau)d\tau} B(t) = e^{-\beta t}\left(B(0) - \phi\,Y(0)\int_0^t e^{\beta\tau - \int_0^\tau (r(\mu) - \gamma_y(\mu))d\mu} d\tau\right) \tag{6}$$

$r > \gamma_y$: dynamically efficient economy

3 The structure of the growth model

3.1 The household and the productive sector

Household:

$$\max_{C} \int_0^\infty e^{-\rho t} \ln C \, dt \tag{7}$$

subject to

$$(1-\tau)(w + rW + \pi) = \dot{W} + C \tag{8}$$

$W \equiv B + K$

$$\frac{\dot{C}}{C} = -\rho + (1-\tau)(1-\alpha)K^{-\alpha}G^{\alpha} \tag{9}$$

Productive sector:

$$Q = K^{1-\alpha}G^{\alpha}L^{\xi} \tag{10}$$

$L \equiv 1$

$$w = \xi K^{1-\alpha}G^{\alpha} \tag{11}$$

$$r = (1-\alpha)K^{-\alpha}G^{\alpha} \tag{12}$$

3.2 The government

$$\dot{B} + T = rB + I_p \leftrightarrow \dot{B} = (r - \beta)B + T(i_p - 1) \tag{13}$$

$\beta > 0$, $I_p = i_p T - \beta B$: public investment

$$\dot{G} = I_p = i_p T - \beta B \tag{14}$$

3.3 Equilibrium conditions and the SBGP

Economy-wide resource constraint:

$$\frac{\dot{K}}{K} = -\frac{C}{K} + \frac{K^{1-\alpha}G^{\alpha}}{K} - \left(i_p \frac{T}{K} - \beta \frac{B}{K}\right) \tag{15}$$

SBGP: $\dot{K}/K = \dot{G}/G = \dot{B}/B = \dot{C}/C$

$x \equiv G/K$, $b \equiv B/K$ and $c \equiv C/K$

$$
\begin{aligned}
\dot{x} &= x\left(c-\beta b(1+x^{-1})-x^{\alpha}+(1+(1-\alpha)b)i_p\tau x^{\alpha}(1+x^{-1})\right) \quad (16)\\
\dot{b} &= b\left(c-\beta(1+b)+(1-\alpha)x^{\alpha}+(i_p-1)\tau x^{\alpha}((1-\alpha)+b^{-1})-\right.\\
&\quad x^{\alpha}+i_p\tau\, x^{\alpha}(1+(1-\alpha)b)) \quad (17)\\
\dot{c} &= c\,(c-\rho+(1-\tau)(1-\alpha)x^{\alpha}-x^{\alpha}-\beta b+i_p\tau x^{\alpha}\\
&\quad (1+(1-\alpha)b)) \quad (18)
\end{aligned}
$$

4 Implications of the model

$$
\frac{\dot{b}}{b}=(\rho-\beta)+(i_p-1)\tau x^{\alpha}((1-\alpha)+b^{-1})+(1-\alpha)\tau x^{\alpha}\,. \quad (19)
$$

$\beta \leq \rho : i_p < 1 \leftarrow$ SBGP (for $b > 0$)

$\beta > \rho : i_p < 1$ not necessary for SBGP

4.1 The economy on the SBGP

Numerical examples: $\tau = 0.1$, $\alpha = 0.25$ $\rho = 0.3$

Table 1

β	$\beta = 0.15$					$\beta = 0.25$				
i_p	$\frac{\partial \gamma}{\partial i_p}$	$b^\star$	$x^\star$	γ	Stability	$\frac{\partial \gamma}{\partial i_p}$	$b^\star$	$x^\star$	γ	Stability
$i_p = 1.15$	+	-0.05	0.4	0.238	unstable	+	-0.1	0.45	0.252	unstable
$i_p = 1.05$	+	-0.02	0.37	0.226	unstable	+	-0.04	0.39	0.232	unstable
$i_p = 0.9$	+	0.04	0.31	0.204	unstable	+	0.07	0.27	0.186	unstable
$i_p = 0.75$	+	0.09	0.24	0.174	unstable	+	0.16	0.05	0.022	unstable
						-	0.15	0.04	0	unstable
$i_p = 0.45$	+	0.15	0.04	0	unstable	no SBGP for $i_p \leq 0.74$				
	-	0.11	0.01	-0.088	unstable					
	no SBGP for $i_p \leq 0.44$									

Table 2

β	$\beta = 0.35$					$\beta = 0.4$				
i_p	$\frac{\partial\gamma}{\partial i_p}$	$b^\star$	$x^\star$	γ	Stability	$\frac{\partial\gamma}{\partial i_p}$	$b^\star$	$x^\star$	γ	Stability
$i_p = 1.15$	+	-0.47	0.72	0.322	unstable	-	0.16	0.1	0.077	stable
$i_p = 1.05$	+	-0.24	0.56	0.283	unstable	-	0.08	0.26	0.181	stable
	-	0.15	0.04	0	unstable					
$i_p = 1.02$	+	-0.13	0.47	0.258	unstable	-	0.04	0.31	0.205	stable
	-	0.16	0.1	0.08	stable					
$i_p = 0.9$	no SBGP for $i_p < 1$					-	-0.2	0.53	0.275	stable
$i_p = 0$	no SBGP for $i_p < 1$					-	-1.01	1.06	0.384	stable

$i_p \in (1, 1.028)$: stable, $i_p > 1.028$: unstable, $i_p = i_p^{crit} = 1.028651$: Hopf bifurcation

$\rho = 0.32$: $i_p = i_p^{crit} = 0.971824$: supercritical Hopf bifurcation

Figure 1

4.2 Fiscal policy on the transition path

Linearized system

$$x(t) = x^\star + C_1\, v_{11}\, e^{\mu_1 t} + C_2\, v_{21}\, e^{\mu_2 t} \tag{20}$$

$$b(t) = b^\star + C_1\, v_{12}\, e^{\mu_1 t} + C_2\, v_{22}\, e^{\mu_2 t} \tag{21}$$

$$c(t) = c^\star + C_1\, v_{13}\, e^{\mu_1 t} + C_2\, v_{23}\, e^{\mu_2 t} \tag{22}$$

Figure 2

Figure 3

Figure 4

5 The primary surplus and the debt ratio: Empirical results for Germany and Italy

5.1 Theoretical considerations

Intertemporal budget constraint:

$$dB_t(\omega) = (r(t,\omega)B_t - S(t,\omega))dt + \sigma dW_t(\omega), \tag{23}$$

B_t : public debt, $r(t,\omega)$: interest rate, $S(t,\omega)$: primary surplus, W_t : Wiener process, $\sigma \equiv 1$.

Bohn (1995, 1998)

$$s(t,\omega) = \alpha(t,\omega) + \beta(t)b(t,\omega), \tag{24}$$

$$dB_t(\omega) = \left(h(t,\omega)B_t - \alpha(t,\omega)Y(t,\omega)\right)dt + dW_t(\omega), \tag{25}$$

$h(t,\omega) \equiv r - \beta$.

$$B_t = e^{\int_0^t h(\tau)d\tau}\left(B_0 - \int_0^t e^{-\int_0^\tau h(\mu)d\mu}\alpha(\tau)Y(\tau)d\tau + \int_0^t e^{-\int_0^\tau h(\mu)d\mu}dW_\tau\right) \tag{26}$$

Proposition

Assume that the mean of the realized real interest rate is strictly positive for any interval $[t_1, t_2]$ *sufficiently large. Then, we have the following result.*

i) For $\alpha_t = 0$, $\lim_{t\to\infty} \int_0^t \beta(\tau)d\tau = \infty$ *is necessary and sufficient for* $e^{-\int_0^t r(\tau)d\tau} B_t$ *to converge to zero.*

(ii) For $\alpha_t \neq 0$, $\lim_{t\to\infty} \int_0^t \beta(\tau)d\tau = \infty$ *is necessary and*

$$\lim_{t\to\infty} \int_0^t \beta(\tau)d\tau = \infty \quad \text{and} \quad \lim_{t\to\infty} \frac{e^{-\int_0^t (r(\mu)-\gamma_y(\mu)-\gamma_\alpha(\mu))d\mu}}{\beta(t)} = 0 \;\; \text{w.p.1}$$

are sufficient for $e^{-\int_0^t r(\tau)d\tau} B_t$ *to converge to zero, with* γ_y *and* γ_α *the growth rate of* Y *and* α*, respectively.*

5.2 Empirical Evidence for Germany and Italy

$$s_t = \beta_t b_t + \alpha^\top \mathbf{Z_t} + \epsilon_t, \tag{27}$$

$$s_t = \beta_t b_{t-1} + \alpha_0 + \alpha_1 Soc_t + \alpha_2 int_t + \epsilon_t. \tag{28}$$

Germany

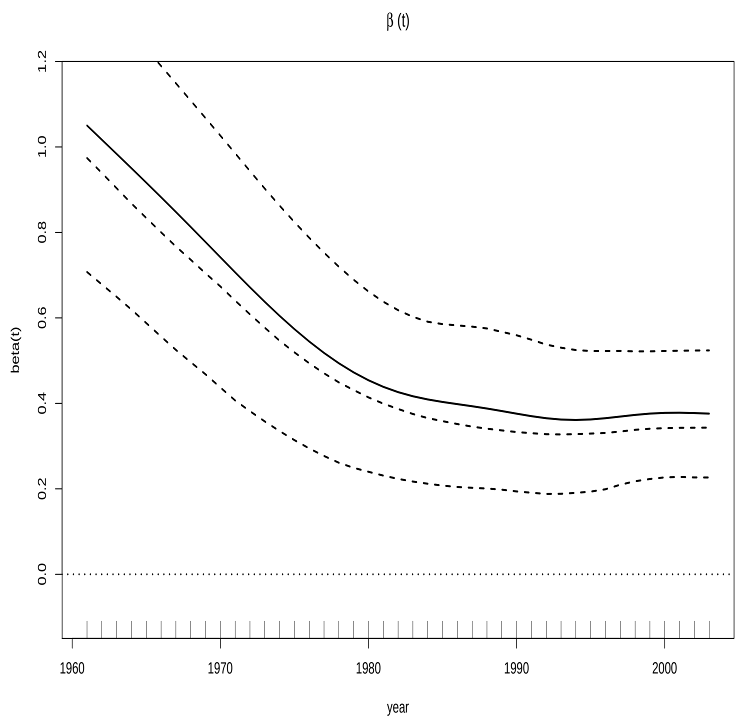

Figure 1: Time varying coefficient β_t for Germany obtained from estimating (28).

variable	estimate	std.err
intercept	-0.070	0.015
Soc	1.621	0.237
int	-0.060	0.167

Table 1: Parameter estimates for Germany

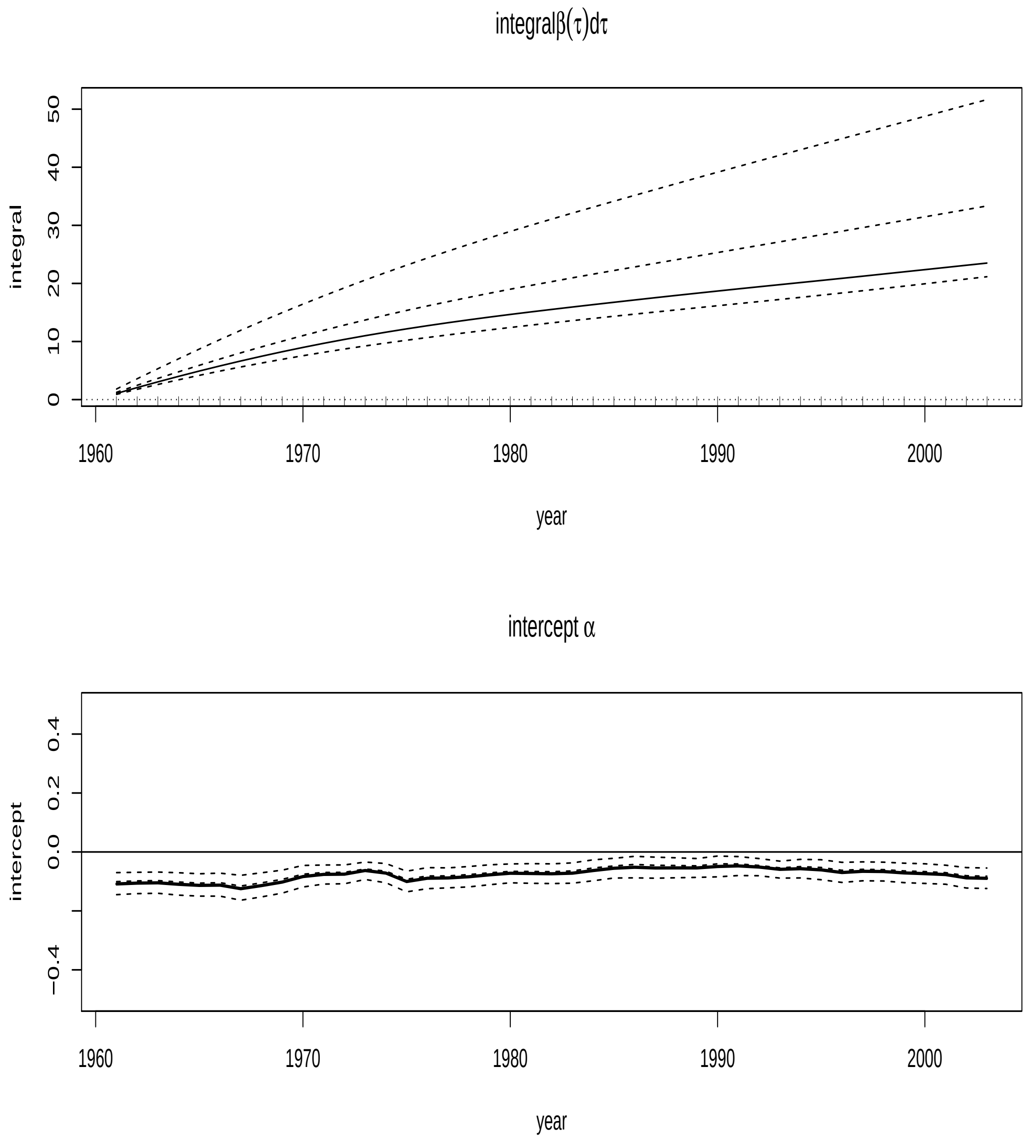

Figure 2: $\int \beta(\tau) d\tau$ and α_t obtained from estimating (28).

Italy

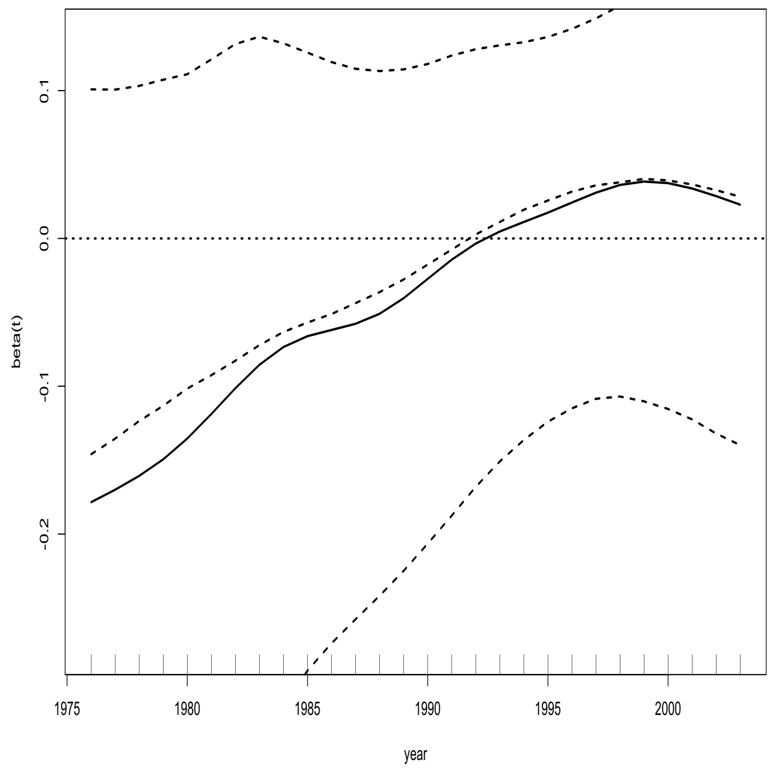

Figure 3: Time varying coefficient β_t for Italy obtained from estimating (28).

variable	estimate	std.err
intercept	0.025	0.063
Soc	0.856	0.360
int	-0.156	0.126

Table 2: Parameter estimates for Italy

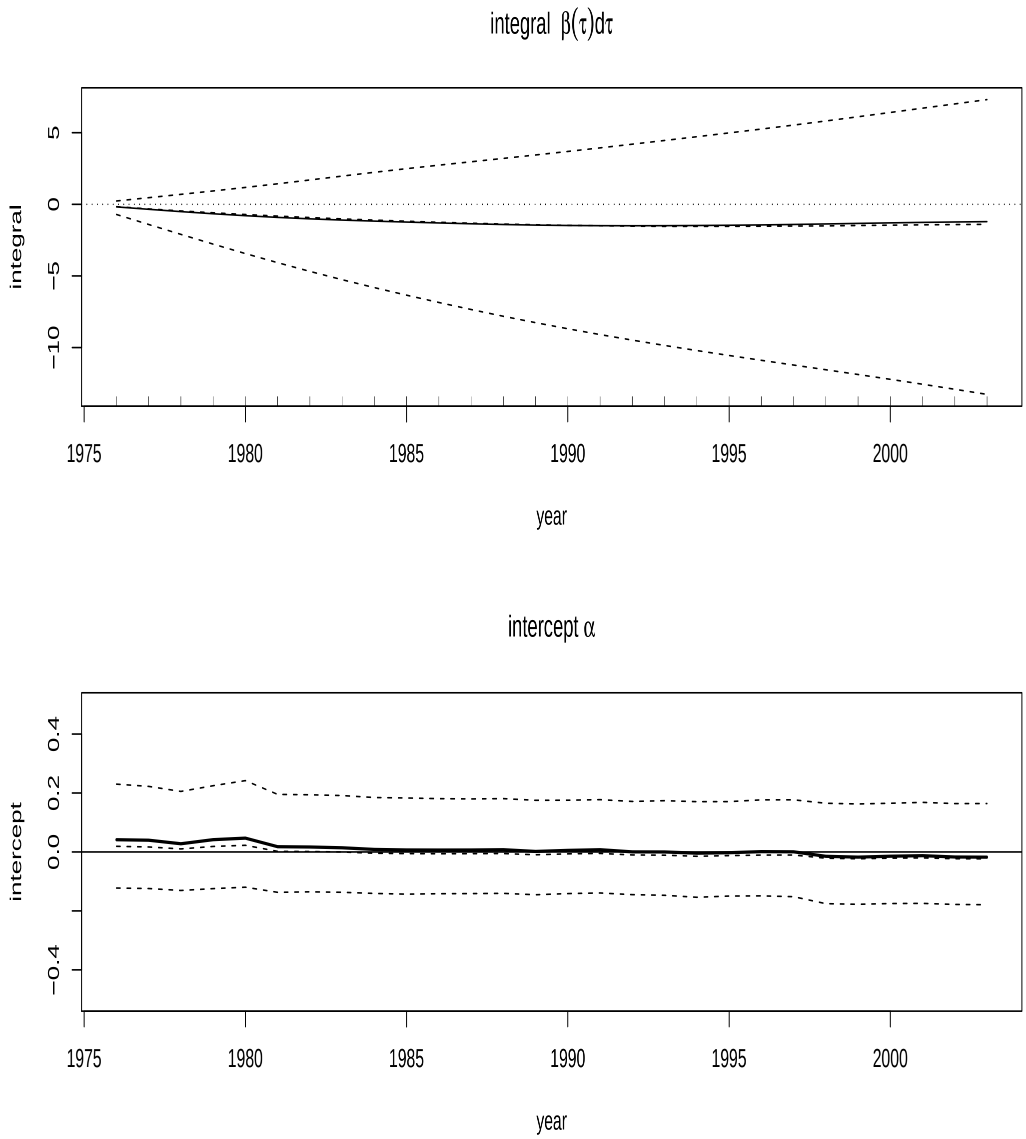

Figure 4: Time varying coefficient β_t for Italy obtained from estimating (28).

6 Conclusion

1. $i_p < 1$, $\beta \leq \rho$: SBGP can exist

   $i_p > 1$: $\beta \leq \rho$, no SBGP, $\beta > \rho$, SBGP can exist ($b > 0$)

2. β decisive as to existence and stability of SBGP, limit cycles

3. β "large": negative growth effects of deficit financed public investment

   β "small": positive growth effects of deficit financed public investment

4. Transition path: overshooting; income effect matters

5. Empirics: reaction of primary surplus to debt ratio not constant

Crowding-out of public investment (Heinemann (2002), Gong et al. (2001))