Time-varying Noise Trader Risk and Asset Prices*

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Abstract

By introducing time varying noise trader risk in De Long, Shleifer, Summers, and Waldmann (1990) model, we predict that noise trader risk significantly affects time variations in the small-stock premium. Consistent with the theoretical predictions, we find that when noise trader risk is high, small stocks earn lower contemporaneous returns and higher subsequent returns relative to large stocks. In addition, noise trader risk has a positive effect on the volatility of the small-stock premium. After accounting for macroeconomic uncertainty and controlling for time variation in conditional market betas, we demonstrate that systematic risk provides an incomplete explanation for our results. Noise trader risk has a similar impact on the distress premium.

Keywords: Noise trader risk; Small-stock premium; Investor sentiment

JEL Classification: G10; G12; G14

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1 Introduction

In this paper we explore the theoretical and empirical time series relationship between noise trader risk and asset prices. We extend the De Long, Shleifer, Summers, and Waldmann (1990) model to allow noise trader risk to vary over time, and predict its impact on both the returns and volatility of risky assets. Using monthly U.S. data since 1960s, we provide empirical evidence consistent with the model.

Our paper contributes to the debate on whether noise trader risk is priced. The corner stone of modern finance theory, the efficient market hypothesis, rules out price impact generated by noise traders. Even when their beliefs are biased in a systematic way, competition among rational arbitrageurs immediately eliminates any mispricing. However, when noise traders' misperception is both systematic and unpredictable, betting against their misperception is risky and can incur substantial loss. De Long et al. (1990) show that risk averse rational arbitrageurs will demand a risk premium for bearing such risk, and hence noise trader risk is priced.

While De Long et al. (1990) argue that noise trader risk is the heart of their model, surprisingly few empirical studies test directly whether noise trader risk is priced.[1] This might be explained by the fact that noise trader risk in De Long et al. (1990) is time invariant, making it impossible to test its role in time series. Recent efforts have focussed on the cross sectional implications of investor sentiment given the difference in noise trader risk (and other limits to arbitrage) across different types of stocks. For example, Baker and Wurgler (2006) and Lemmon and Portniaguina (2006) test whether investor sentiment affects the small stock premium, the relative returns between small stocks and large stocks, where small stocks are more costly to arbitrage than large stocks.

As noted above, we incorporate time varying noise trader risk into De Long et al. (1990). Intuitively, when rational investors face more difficulty in predicting irrational investors' future biased beliefs, they are more reluctant to hold risky assets. Our model predicts the higher the noise trader

[1] Indirect attempts have been made by Sias, Starks, and Tinic (2001), Gemmill and Thomas (2002), and Scruggs (2007), among others. A notable exception is Lee, Jiang, and Indro (2002). They include the lagged squared change of investor sentiment in the volatility equation of GARCH in mean model, and examine its asymmetric effect on conditional volatility and returns depending on whether the increment in investor sentiment is positive or negative. They find the magnitude of bullish (bearish) changes in sentiment leads to downward (upward) revisions in volatility and higher (lower) future excess returns.

risk (i.e. the higher the expected future volatility of investor sentiment), the higher the risk premium demanded by rational investors which, in turn, depresses the contemporaneous price of risky assets. Since current period returns are determined by both opening and closing current period prices, contemporaneous returns will therefore be negatively correlated with the change in expected noise trader risk. And because a higher level of expected noise trader is associated with lower prices today, future returns will be higher. The model also predicts that prices of risky assets will fluctuate with noise traders' misperception - the price volatility of risky assets will move in line with noise trader risk. We expect these predictions to be stronger for stocks that are predominantly held by noise traders and for those stocks which are more difficult to arbitrage.

In the empirical tests, we focus on the small stock premium for three reasons: first, small stocks are predominately held by individual investors, and individual investors are more likely to trade on noise (Lee, Shleifer, and Thaler (1991), Foucault, Sraer, and Thesmar (2011)); second, small stocks are more difficult to value, which allows unsophisticated investors to defend a wide spectrum of valuations (Baker and Wurgler (2006)); third, small stocks are less liquid, have financial distress risk (Chan and Chen (1991)) and high idiosyncratic risk, making arbitrage especially risky (Wurgler and Zhuravskaya (2002)). These limits to arbitrage, together with noise trader risk itself, deter rational arbitrageurs from aggressively betting against noise traders. Relying on these cross-sectional differences between small and large stocks, we are thus more likely to capture the role of noise trading on asset prices. Lemmon and Portniaguina (2006) and Baker and Wurgler (2006) take the same approach to study the time series relationship between investor sentiment and the small stock premium. While we also examine the effect of investor sentiment, our main aim is to study the time-series relationship between noise trader risk and small stock premium given these cross-section differences.

To derive empirical proxies of noise trader risk, we model the volatility of three commonly used measures of investor sentiment: the closed-end fund discount (CEFD), the Baker and Wurgler (2006) composite sentiment index, and the consumer confidence index from the Michigan Survey Research Center.

We first document the small stock premium conditional upon the contemporaneous change and lagged level of noise trader risk. When the contemporaneous change in noise trader risk is low (below sample average), small stocks earn particularly high contemporaneous returns and low subsequent returns. When the contemporaneous change in noise trader risk is high (above sample average), small stocks earn particularly low contemporaneous and high subsequent returns. We then conduct regression analysis where we control for Fama and French (1993) factors (except SMB), momentum factors, flow of funds, and a set of macroeconomic fundamental variables and their uncertainty. The coefficients of noise trader risk are significant with a hypothesized sign predicted by the theory.

The empirical evidence provides no support for the notion that noise trader risk reflects compensation either for time variation in systematic market-wide risk or for changes in the business cycle that are correlated with changes in the market beta of small stocks. Instead, the findings provide strong support for the notion that sophisticated investors demand a higher risk premium for holding small stocks relative to large stocks when noise trader risk is high and vice versa.

Also consistent with the theory, the volatility of the small stock premium comoves with the noise trader risk over time. We augment a GARCH(1, 1) model to allow the investor sentiment and noise trader risk to enter both the mean and volatility equations. We find that when noise trader risk is high, the small stock premium is more volatile. A number of papers investigate the role that investor sentiment plays in determining asset price volatility - with mixed results (Brown (1999) Lee et al. (2002) and Wang, Keswani, and Taylor (2006)). Our paper adds to this literature by showing theoretically and empirically that it is noise trader risk, not investor sentiment, that affects time varying volatility. Our results also complement the findings of Foucault et al. (2011) that retail trading activity has a positive effect on the volatility of stock returns.

We also provide evidence that the contemporaneous change in investor sentiment is positively correlated with the small stock premium, consistent with the prediction of De Long et al. (1990). Furthermore, we provide evidence that higher investor sentiment predicts lower future returns of small stocks relative to large stocks. More importantly, by focusing on noise trader risk, our paper complements the large literature of investor sentiment that noise trading matters for prices of assets that are difficult to arbitrage.

In addition to the small stock premium, we also examine the effect of noise trader risk on the distress premium (measured as the difference in returns between firms in the top book-to-market decile and firms in the middle book-to-market decile). Firms in the top decile are thought to be more distressed (and more difficult to arbitrage) while firms in the middle decile are considered more stable. Our empirical findings indicate that high noise trader risk is associated with a high contemporaneous distress premium and a lower subsequent distress premium. After controlling for the time variation of beta, we show that these results are not fully explained by systematic risk. We also find noise trader risk contributes to the volatility of distress premium, echoing our findings for the small stock premium.

The reminder of the paper is organized as follows. Section 2 extends the model of De Long et al. (1990) and discusses the theoretical predictions. Section 3 describes and summarizes our data set. Our empirical results are provided in Section 4. Section 5 concludes.

2 Theoretic Effect of Time-varying Noise Trader Risk

2.1 A Model with Time-varying Noise Trader Risk

We incorporate time-varying noise trader risk in the model of De Long et al. (1990). The model presented here is similar to the original model, except that the volatility of noise trader misperceptions is allowed to change over time which allows us to derive testable predictions of the time series relationship between noise trader risk and asset prices. Therefore we only briefly describe the model setup but focus on how we incorporate the time varying noise trader risk into the original model and what is its effect on equilibrium excess returns. More details on derivations of the model are given in the appendix.

The model is a overlapping generations model with two-period-lived agents. There are two type of agents in the economy. One type of agents are noise traders (denoted as "n"), who hold biased beliefs and trade on noise (biased beliefs). The other type of agents are sophisticated investors (denoted as "i"), who have rational expectations. The percentage of noise traders in the market is given as μ, while the percentage of sophisticated investors is $1-\mu$. There are two assets each period: a risk free asset s and an unsafe asset u. Both assets pay a fixed real rate of r. The difference between the risk free asset and unsafe asset is their supply. The risk free asset is in perfectly elastic supply, which implies the price of this asset is also fixed. The supply of the unsafe asset is fixed (normalized to be one unit), which means the price of the asset p_t will fluctuate along with the change in demand.

De Long et al. (1990) assume that the representative noise trader's misperception is an i.i.d normally distributed random variable: $\rho_t \sim N(\rho^*, \sigma_\rho^2)$. The key innovation of this paper is to allow the distribution of noise trader's misperception to change over time.

$$\rho_t \sim N(\rho^*, \sigma_{\rho,t}^2) \tag{1}$$

We further assume

$$\sigma_{\rho,t}^2 = \theta\sigma_{\rho,t-1}^2 + \eta_t \tag{2}$$

where η_t is an *i.i.d.* random variable with zero mean and volatility of σ_η^2, and $0 < \theta < 1$. Therefore $\sigma_{\rho,t}^2$ is weakly stationary, and the volatility of $\sigma_{\rho,t}^2$ is a constant.[2] In the data, the assumption that $0 < \theta < 1$ is well supported by all three measures of $\sigma_{\rho,t}^2$.

The assumption in (2) implies that $E_t(\sigma_{\rho,t+k}^2) = \theta^k\sigma_{\rho,t}^2$. Higher noise risk today leads to an increase in expected noise trader risk in the future. Such an assumption allows us to match the unobservable expected future noise trader risk with the current period noise trader risk and facilitate our empirical tests.

Define the excess return from date $t-1$ to date t as: $R_t = r + p_t - p_{t-1}(1+r)$, in equilibrium we

[2] One criticism of the assumption 3 is that the volatility can be negative. Alternatively, we can assume:

$$\sigma_{\rho,t}^2 = \theta\sigma_{\rho,t-1}^2(1+\eta_t)$$

where η_t is an *i.i.d.* random variable with zero mean and $\sigma_{\eta,t}^2 = \frac{1-\theta^2}{\theta^2}$ with $-1 < \theta < 1$. We can impose an assumption $\eta_t > -1$ to ensure the volatility is positive. Assumptions on $\sigma_{\eta,t}^2$ is to have a constant volatility of $\sigma_{\rho,t}^2$, such that we can obtain an analytic solution. These alternative specifications lead to the same model predictions as the assumptions 1 and 2.

obtain:

$$R_t = \frac{\mu[\rho_t - (1+r)\rho_{t-1}]}{1+r} - \frac{2\nu\gamma}{r}\frac{\theta\mu^2[\sigma^2_{\rho,t} - (1+r)\sigma^2_{\rho,t-1}]}{(1+r)^2} + \zeta \tag{3}$$

where μ is a positive constant, and ζ is a constant.

We can also obtain the volatility of excess returns:

$$\sigma^2_{R,t} = \frac{\mu^2[\sigma^2_{\rho,t} + (1+r)^2\sigma^2_{\rho,t-1}]}{(1+r)^2} + \kappa \tag{4}$$

where κ is a constant.

Equations (3) and (4) provide testable implications of noise trader risk on excess returns and their volatility of risky assets.[3] We now turn to the discussion of our model predictions.

2.2 Model Predictions

An immediate observation from Equation 3 is that current period's excess return is negatively related to $\sigma^2_{\rho,t} - (1+r)\sigma^2_{\rho,t-1}$. Therefore our first hypothesis is:

Hypothesis 1. *An increase in current period noise trader risk is negatively correlated with contemporaneous excess returns.*

Equation 3 shows that, ceteris paribus, an increase in $\sigma^2_{\rho,t-1}$ is related to a higher R_t. In another words, when noise trader risk ($\sigma^2_{\rho,t}$) is higher, the current price will be lower since rational and risk averse investors demand a higher risk premium, which predicts higher future excess returns. This leads to our second hypothesis:

Hypothesis 2. *The level of noise trader risk in the current period predicts future excess returns.*

[3] Excess returns can be interpreted as the relative returns between the sentiment-prone asset ("u") and the sentiment-neutral asset ("s"). In the empirical tests, we examine the relative returns and their volatility between more sentiment-prone risky assets and less sentiment-prone risky assets. Therefore we also solve a model with one risk free asset and two risky assets. The risky assets differ in their exposure to the market wide sentiment. We obtain similar predictions as in Section 2.2. We do not present the more general model since the one risky asset model delivers our main message already.

Equation 4 predicts the volatility of excess returns is positively related to $\sigma^2_{\rho,t} + (1+r)^2\sigma^2_{\rho,t-1}$. Note that the risky asset in the model has constant cash flows, therefore its fundamental volatility is zero. We hypothesize that

Hypothesis 3. *An increase in the sum of the current and the last period noise trader risk is positively correlated with excess return volatility.*

Although our focus is on the pricing implication of time varying noise trader risk, Equation (3) provides also testable predictions on the role of investor sentiment. We hypothesize that increasingly positive sentiment in the presence of noise trader risk leads to a rise in the current price,

Hypothesis 4. *An increase in current period investor sentiment is positively correlated with contemporaneous excess returns.*

Higher current sentiment in the presence of noise trader risk, while leading to a rise in the current price, will be associated with lower excess future returns when the mispricing is corrected. That is:

Hypothesis 5. *The level of investor sentiment in the current period predicts future excess returns.*

Hypothesis 5 has been empirically tested by Baker and Wurgler (2006) and Lemmon and Portniaguina (2006). We examine both predictions in our empirical analysis.

In the empirical tests of the Hypothesis 1, 3 and 4, we ignore the $1+r$ and $(1+r)^2$ terms. This should have only very limited effect on our results since r is typically small at the monthly frequency. The empirical results do not change with inclusion or exclusion of these terms.

3 Data

3.1 Data Sources

We collect data from several different sources. Portfolio returns of different size and book-to-market deciles, Fama-French factors and momentum factors are from Kenneth French's web page.

We calculate the small stock premium as the difference between the returns on the smallest and the largest size decile portfolios. The distress premium is the return differences between the highest (the 10th) and the 6th book-to-market decile portfolios. We consider both equally and value weighted size portfolios.

Data on the closed-end-fund discount (cefd) and the Baker and Wurgler (2006) composite sentiment index (hereafter "sent_bw") are downloaded from Jeffrey Wurgler's web page. Both variables are available at the monthly frequency during 07/1965-12/2010. Baker and Wurgler (2006) calculate the cefd as the average difference between the net asset values of closed-end funds and their market prices. To make it easier to compare cefd with sentiment indicators, we multiply their original cefd by -1, such that the discount is measured as a negative number. If cefd measures the sentiment, discounts widen when sentiment is low. "sent_bw" is based on the first principal component of six underlying sentiment proxies (the closed-end fund discount, NYSE share turnover, the number and average first-day returns on IPOs, the equity share in new issues, and the dividend premium). It is then orthogonalized to several macroeconomic conditions (growth in the industrial production index, growth in consumer durables, nondurables, and services, and a dummy variable for NBER recessions). Our third sentiment indicator, the consumer confidence index (hereafter "cci"), is downloaded from the Michigan Survey Research Center. It is available at the monthly frequency since 01/1978.

We follow the approach of Schwert (1989) and Beber, Breedon, and Buraschi (2010) to estimate the variance of misperception. This method allows the conditional mean to vary over time and different weights for lagged absolute unexpected shocks. We first regress each sentiment indicator on its first 12 lags and monthly dummies D_j. Denoting the absolute residuals from these regressions with $|\hat{\varepsilon}|$, we estimate a regression of the following specification:

$$|\hat{\varepsilon}_t| = \sum_{j=1}^{12} \gamma_j D_{jt} + \sum_{i=1}^{12} \rho_i |\hat{\varepsilon}_{t-i}| + u_t \qquad (5)$$

The fitted value of Equation (5), $\tilde{\varepsilon}_t$, estimates the standard deviation of investor sentiment. We use the squared $\tilde{\varepsilon}_t$ as our measure of noise trader risk, which is denoted as σ^2_{cefd}, $\sigma^2_{sent\_bw}$ and σ^2_{cci} for the corresponding investor sentiment proxies respectively.

To control for fundamental economic risk, we download monthly macroeconomic variables from Datastream. These variables include growth in the industrial production (IP), 3-month interest rate (SR), consumer price index (CPI), unemployment rate (UE) and growth in the broad money (BM). We also consider a set of variables known to have predictive power for stock returns. Using data from Datastream, the term spread is calculated as the difference between the yield of 10 year and 3 month treasuries. Data on monthly dividend yield and price earnings ratio are downloaded from Robert Shiller's web page.[4]

3.2 Summary Statistics

Table I provides the summary statistics of the variables. The sample period covers 02/1967-04/2013. The average equally weighted small stock premium is 0.54% per month and the average equally weighted distress premium is 0.46% per month. Both variables have large kurtosis. The average cefd is -8.96%. For the ease of tabulating regression results later, we rescale sent_bw by 10, and estimate its volatility based on rescaled sent_bw. It has a mean of zero due to its orthogonalization. The consumer confidence index (cci) has a mean of 85.31.

[Insert Table I about here]

Panel A in Table II presents the correlation coefficient matrix of investor sentiment and noise trader risk proxies. Consistent with Baker and Wurgler (2006), we find cefd is highly correlated with sent_bw with a correlation coefficient of 62%. cefd and cci has a correlation coefficient of 33%. cci is weakly correlated with sent_bw, with a coefficient of 10%. The correlation coefficients among the noise trader risk measures are always positive and statistically significant. While σ^2_{cci} is not highly correlated with σ^2_{cefd} (14%) and $\sigma^2_{sent\_bw}$ (20%), the correlation between σ^2_{cefd} and $\sigma^2_{sent\_bw}$ is higher (41%). Interestingly, sent_bw is positively correlated with $\sigma^2_{sent\_bw}$ (23%) while cci is negatively correlated with σ^2_{cci} (-29%). cefd is uncorrelated with σ^2_{cefd}. Noise trader risk measures are not highly correlated with other sentiment measures.

[4] We thank Kenneth French, Robert Shiller and Jeffrey Wurgler for providing their data through their web page.

We also report the first order serial correlation of investor sentiment and noise trader risk proxies in Panel B of Table II. Consistent with the literature, we find the sentiment measures are highly persistent, with an AR(1) coefficient of 97%, 99% and 95% for cefd, sent_bw, and cci respectively. The persistence of the noise trader risk measures is smaller, with an AR(1) coefficient of 77%, 65% and 23% for σ^2_{cefd}, $\sigma^2_{sent\_bw}$ and σ^2_{cci} respectively.[5] The first difference of these variables typically has little persistence (less than 20%), with the exception of σ^2_{cci}, which has an AR(1) coefficient of -48%.

[Insert Table II about here]

4 Empirical Results

4.1 A First Look at the Small Stock Premium, Noise Trader Risk and Investor Sentiment

Before discussing the results of regression analysis, we first document the small stock premium conditional upon changes in the level of noise trader risk. The changes in the noise trader risk measures are first classified into low and high sub-samples. "low" ("high") is defined as the changes in noise trader risk proxies that are below (above) their median value in the full sample. We do this for both value- and equally-weighted size decile portfolios. The average contemporaneous small stock premiums for the two sub-samples are shown in Panel A1 of Table III for the corresponding sub-sample.

Panel A1 shows that when the change in noise trader risk is low, the contemporaneous average small stock premium is positive. When the change in noise trader risk is high, the contemporaneous

[5] Ferson, Sarkissian, and Simin (2003) demonstrate a spurious regression bias can arise in predictive regressions if regressors are highly persistent. Stambaugh, Yu, and Yuan (2012) conduct simulations to show the predictive power of investor sentiment cannot be explained by its persistence. Ferson et al. (2003) shows no serious spurious regression bias if the AR(1) coefficient of a predictor is 0.90 or less. Therefore the smaller AR(1) coefficient of the noise trader risk measures alleviates the concerns about spurious regressions due to persistent regressors.

average small stock premium is negative. In periods when the change in noise trader risk is low, the small stock premium is, on average, 0.42% to 1.55% higher than in periods when the change in noise trader risk is high. The average small stock premium differs more when the size decile portfolios are equally weighted than value weighted.

We also sort by the level of noise trader risk (rather than by the change in noise trader risk). The "low" ("high") sub-sample is defined as when the level of the noise trader risk proxies are below (above) their median value in the full sample. Panel A2 shows the small stock premium for the month following. When noise trader risk is low in the current month, the next month's average small stock premium is always negative, regardless of whether the size portfolios are equally or value weighted. By contrast, when current noise trader risk is high, next period returns are always positive. The difference in the subsequent month's average size premium between low and high noise trader risk periods is economically large, and ranges from -0.69% to -1.27% per month. We find the average small stock premium differs more when the size decile portfolios are equally weighted rather than value weighted.

These findings are consistent with the theoretical prediction that the change in noise trader risk is negatively correlated with contemporaneous returns and that the level of noise trader risk is positively correlated with subsequent returns.

We also tabulate the average contemporaneous (and next period) small stock premium conditional upon the change in, and level of, investor sentiment. Results in Panel B1 show that when the change in investor sentiment is low, the contemporaneous small stock premium is low compared to the small stock premium when the change in the investor sentiment is high. This is consistent with the theoretical prediction that changes in sentiment are positively correlated with contemporaneous returns, and confirms the similar empirical findings in Lee et al. (2002). We also find that when the current level of investor sentiment is higher, the average one-month-ahead small stock premium is lower (Panel B2). Our findings are consistent with the findings of existing literature that predict and empirically confirm that an increase in sentiment forecasts a lower future small stock premium (Baker and Wurgler (2006) and Lemmon and Portniaguina (2006)).

[Insert Table III about here]

4.2 Regression Analysis

While the results of the average small stock premium conditional on noise trader risk and investor sentiment in Table III reveal interesting patterns, the univariate sorts are subject to omitted variables bias. We now turn to regression analysis which allows us to control for Fama-French factors, macroeconomic fundamentals, etc.

We first run contemporaneous OLS regressions of the following specification:

$$\begin{aligned} Small\_Stock\_Premium_t &= a + bSentiment\_d_t + cNoise\_Risk\_d_t + \beta_{Rm-Rf}(Rm_t - Rf_t) \\ &\quad + \beta_{HML}HML_t + \beta_{Mom}Mom_t + \beta_{Term\_spread}Term\_spread_t + \epsilon_t \end{aligned} \quad (6)$$

where “sentiment_d” and “noise_risk_d” are the first difference of the sentiment and noise trader risk proxies respectively. $Rm_t - Rf_t$ (market excess return), hml, and mom are the Fama-French factors and the momentum factor. Term spread is the difference between the yield on 10 year and 3 month treasuries.

Columns two to four in Table IV present results of monthly contemporaneous regressions of the small stock premium on proxies of investor sentiment and noise trader risk proxies. The small stock premium is based on equally weighted size decile portfolios. Since results based on value weighted size decile portfolios are very similar to those based on equally weighted size decile portfolios, we report the results here (and in subsequent sections) for only the equally weighted size decile portfolios. We report Newey-West autocorrelation and heteroscedasticity consistent standard errors in parentheses.

We find that the coefficients of “noise_risk_d” are negative and statistically significant no matter which measure of noise trader risk is used. This implies that an increase in the noise trader risk is related a decline in the contemporaneous small stock premium, consistent with the prediction of time varying noise trader risk. Also consistent with the theory, we find the coefficients of "sentiment_d" are positive and statistically significant - an increase in investor sentiment is positively related to the contemporaneous small stock premium. Although not reported in the table, we consider alternative specifications by excluding all control variables, including only Fama-French (without SMB) and momentum factors, and including contemporaneous dividend to price ratio and price to earnings ratio. The sign and significance of the coefficients of "noise_risk_d" and "sentiment_d" do not change.

[Insert Table IV about here]

To test the hypothesis that noise trader risk predicts the future small stock premium, we run one-month ahead predictive OLS regressions of the following specification:

$$\begin{aligned} Small\_Stock\_Premium_t &= a + bSentiment_{t-1} + cNoise\_Risk_{t-1} + \beta_{Rm-Rf}(Rm_t - Rf_t) \\ &\quad + \beta_{HML}HML_t + \beta_{Mom}Mom_t + \beta_{dp}dp_{t-1} \\ &\quad + \beta_{Term\_spread}Term\_spread_{t-1} + \beta_{pe\_ratio}pe\_ratio_{t-1} + \epsilon_t \end{aligned} \tag{7}$$

where dp is the dividend to price ratio and pe_ratio is the price to earnings ratio.

Columns five to seven in Table IV report results of the predictive regressions. The coefficients of noise trader risk are positive and statistically significant at the 1% level with an exception of σ^2_{cci}, which is positive and significant at the 10% level. This is consistent with the theoretical prediction that an increase in noise trader risk reduces the current price and hence predicts higher future returns. Investor sentiment, on the other hand, has a negative impact on the small stock premium across all three measures, consistent with the theoretical prediction that a temporary rise in investor sentiment in the presence of limits to arbitrage predicts subsequent return reversal. The coefficients are statistically significant with the exception of cefd. Among the control variables, we find evidence that the term spread, the dividend price ratio and the price earnings ratio have some predictive power for one month ahead small stock premium. Again, the coefficients on sentiment and noise trader risk are unchanged when including or excluding some or all of these control variables.

We also run similar predictive OLS regressions as in Equation (7) to forecast the 3-month ahead small stock premium. This leads to serial correlation of residuals due to overlapping observations. We use Newey-West autocorrelation and heteroscedasticity consistent standard errors to address this issue. We find consistent results, with larger and more statistically significant coefficients on the noise trader risk and investor sentiment measures than those in the 1-month predictive regressions.

We conduct four robustness checks. First, we do not include SMB in our regressions since the small stock premium is highly correlated with SMB. Inclusion of SMB in the regressions do not, however, change the role of the noise trader risk on the premium. Second, we consider an alternative definition of small stock premium as the difference between the returns on the bottom three and the top three size decile portfolios. This is in the spirit of SMB in Fama-French factors, although their benchmark factor is value weighted. Our results are again unchanged.. Third, we examine whether our results hinge upon the frequency we have considered. We therefore run quarterly contemporaneous and one quarter ahead predictive regressions as in Equation (15) and (16) and find our results do not change qualitatively. Fourth, individual investors are often taken as noise traders.[6] We include the change in the percentage of quarterly aggregate fund flows in the regressions. Aggregate fund flows are the total equity holdings of certain type of investors. We consider individual investors, foreigners, mutual funds, closed end funds, brokers and dealers, private pension funds, life insurance companies. The percentage of fund flows of each type of investors is calculated as the percentage of its equity holdings to the total equity holdings in U.S.. In untabulated regressions we find the role of noise trader risk remains unchanged when the fund flows are controlled for.

4.3 Systematic Risk

An alternative explanation for our findings is that noise trader risk measures reflect the compensation for either time variation in market wide risk or for changes in the business cycle that are

[6] Lee et al. (1991) argue that investor sentiment should affect stock returns for assets predominantly held by noise traders. Kelly (1997) finds that high participation of noise traders (proxied by low-income households) is a negative predictor of one year returns. Lemmon and Portniaguina (2006) find investor sentiment predicts negatively the returns on stocks with low institutional ownership relative to the returns on stocks with high institutional ownership.

correlated with changes in the market beta of small stocks. We test this possibility in two ways.

The first approach we take is to regress sentiment measures on a number of contemporaneous and the lagged macroeconomic variables. These macroeconomic variables include the log difference in industrial production, the 3-month interest rate, the consumer price index, the unemployment rate, the log difference in broad money, the term spread, the dividend to price ratio and the price to earnings to ratio.[7] We denote them as "cefd_2", "sent_bw_2" and "cci_2". We follow Schwert (1989) again to estimate their volatility (denoted as $\sigma^2_{cefd\_2}$, $\sigma^2_{sent\_bw\_2}$, and $\sigma^2_{cci\_2}$ respectively).

[Insert Table V about here]

Table V reports the results with our new measures of noise trader risk. We find some reduction in the statistical significance of the coefficients of noise trader risk in both the contemporaneous regression and the one-month ahead predictive regression. cci_2 becomes insignificant in the contemporaneous regression but more significant in the 1-month predictive regressions. The statistical significance of investor sentiment, however, becomes smaller and mostly insignificant.

The second and more rigorous approach is to allow the conditional market beta to be a function of noise trader risk. Baker and Wurgler (2006) and Lemmon and Portniaguina (2006) use a similar model to examine whether the predictive role of investor sentiment for cross sectional returns can be explained by its correlation with systematic risk.

More concretely, we run the following contemporaneous regression:

$$\begin{aligned} Small\_Stock\_Premium_t &= a + bSentiment\_d_t + cNoise\_Risk\_d_t \\ &\quad + d(e + \beta Sentiment\_d_t + \gamma Noise\_Risk\_d_t)(Rm_t - Rf_t) + \epsilon_t \end{aligned} \quad (8)$$

We divide both interaction terms, $Sentiment\_d_t \times (Rm_t - Rf_t)$ and $Noise\_Risk\_d_t \times (Rm_t - Rf_t)$ by 100 for the ease of reporting coefficients.

[7] When constructing sent_bw_2, we omit the contemporaneous growth in the industrial production index since sent_bw has been orthogonalized to that variable.

If variation in the small stock premium is due solely to the fundamentals, we expect $d\beta$ to be statistically positive and $d\gamma$ is statistically negative. If, however, investor sentiment and noise trader risk affect the small stock premium, we expect b to be statistically positive and c to be statistically negative.

[Insert Table VI about here]

Columns two to four in Table VI report the results of contemporaneous conditional market beta regressions. It shows that $d\gamma$, the coefficient of $Noise\_Risk\_d_t \times (Rm_t - Rf_t)$, is statistically insignificant for all three noise trader risk measures. The coefficient of the change in noise trader risk, c, is still negative and remains statistically significant. Even the size of c changes little compared to Table IV. The results suggest that the change in noise trader risk, unrelated to economic fundamentals, affects the small stock premium. We also find that $d\beta$, the coefficient of $Sentiment\_d_t \times (Rm_t - Rf_t)$, is significant only for one of the three sentiment indicators, and the sign and significance of b do not change much.

We also estimate a predictive regression of the following specification,

$$\begin{aligned} Small\_Stock\_Premium_t &= a + bSentiment_{t-1} + cNoise\_Risk_{t-1} \\ &\quad + d(e + \beta Sentiment_{t-1} + \gamma Noise\_Risk_{t-1})(Rm_t - Rf_t) + \epsilon_t \end{aligned} \tag{9}$$

An inspection into the columns five to seven in Table VI shows that with only one exception, neither interaction terms of investor sentiment nor noise trader risk is statistically significant. In addition, the coefficients of noise trader risk (c) are similar to those in Table IV, and are positive and statistically significant at the 1% level. The coefficients of investor sentiment, b, becomes less significant, although they still have the predicted sign.

These results indicate that our findings cannot be fully explained by the idea that noise trader risk and investor sentiment reflect investors' rational forecasts of changes in business cycle that are related to changes in the market betas of small stocks.

To further disentangle the effect of macroeconomic uncertainty, we run similar contemporaneous and predictive regressions as in 9 and 8, but replace the noise trader risk measures with the ones that have been orthogonalized to contemporaneous volatility of macroeconomic variables. Results in Table VII confirm our findings in Table VI.

[Insert Table VII about here]

4.4 Noise Trader Risk and the Volatility of Small Stock Premium

Our theory of time varying noise trader risk predicts that volatility of risky assets co-moves with the noise trader risk. We test this prediction by augmenting a GARCH(1, 1) model to allow the investor sentiment and noise trader risk to enter both the mean and volatility equations of the small stock premium. Taking this approach enables us to model simultaneously the effect of investor sentiment and noise trader risk on expected returns and conditional volatility.

More specifically, we estimate a model of the following specification:

$$\begin{aligned} Small\_Stock\_Premium_t &= a + bSentiment\_d_t + cNoise\_Risk\_d_t + \epsilon_t \\ \sigma_t^2 &= exp(\lambda_0 + \lambda_1 Sentiment_t + \lambda_2 Noise\_risk\_sum_t) + \gamma\sigma_{t-1}^2 + \delta\epsilon_{t-1}^2 \end{aligned}$$

where $\epsilon_t = \sigma_t z_t$ and $z_t \overset{\text{iid}}{\sim} N(0, 1)$. “noise_risk_sum” is $\sigma^2_{\rho,t} + \sigma^2_{\rho,t-1}$, the sum of the current and last period noise trader risk. λ_1 is the coefficient of investor sentiment and λ_2 is the coefficient of noise_risk_sum in the conditional variance equation. γ is the coefficient of the GARCH term σ^2 and δ is the coefficient of the ARCH term ϵ^2.

[Insert Table VIII about here]

To control for Fama-French (except the SMB) and momentum factors, we include them in both mean and volatility equations. Column two to four in Table VIII show that the coefficients of noise_risk_sum is positive and statistically significant at the 1% level for all three measures of noise trader risk. In contrast, investor sentiment is insignificant in all three regressions. These findings support our model prediction that it is noise trader risk, not investor sentiment, that has a positive effect on the conditional volatility.

We also consider an alternative specification to include the lagged sentiment and noise trader risk in the mean equation instead of their change. Results in column five to seven in Table VIII indicate that the effect of investor sentiment and noise trader risk on the conditional volatility of the small stock premium remains. The only exception is that $\sigma^2_{sent\_bw}$ has an insignificant coefficient in the volatility equation.

Inspection of the results from the mean equation confirms our previous findings that the small stock premium increases with a positive change in investor sentiment and decreases with the change in noise trader risk, with the exception of insignificant coefficients of σ^2_{cci}.

Instead of the level of investor sentiment, we include the change in investor sentiment in the variance equation as a robustness check and find similar results. In untabulated regressions, we run ARCH(1) and EGARCH(1, 1) models and find the role of noise trader risk on conditional volatility of the small stock premium unchanged.

4.5 Noise Trader Risk and Distress Premium

Although our focus is primarily on the small stock premium, we examine here whether noise trader risk affects the distress premium.[8] Firms with extreme high book-to-market ratio have higher distress risk and command a distress premium relative to "stable" firms. Baker and Wurgler (2006) suggest that these firms are prone to speculation and are also difficult to arbitrage. They also find that investor sentiment predicts a lower future distress premium.

We measure the distress premium as the return differences between the highest (the 10th) and the 6th book-to-market decile portfolios.[9] We run similar contemporaneous and predictive regression-

[8] We also examine the noise trader risk on the momentum premium and returns of extreme growth firms and find little evidence. Lemmon and Portniaguina (2006) find that investor sentiment does not forecast them either.

[9] We consider alternative measures of the distress premium, including return differences between 10th and 5th, 10th and 4th, or average 8th-10th and average 4th-7th book-to-market decile portfolios. The results are qualitatively

s. Table IX shows that an increase in the change of noise trader risk is associated with a low contemporaneous distress premium. This is because high noise trader risk makes it more risky for rational investors to trade against noise traders, therefore they demand a high risk premium to hold the assets that are more subject to noise trader risk. This in turn drives down the contemporaneous prices of distressed firms relative to those firms regarded as more stable, leading to a low contemporaneous distress premium. Table IX also shows that a high level of current noise trader risk predicts higher subsequent distress premium, consistent with Hypothesis 2 that high noise trader risk lowers the current relative prices of distress firms, therefore the future distress premium increases. These findings are consistent with the theoretical predictions.

[Insert Table IX about here]

To explore whether noise trader risk affects the distress premium due to the time variation of the beta of the distressed firms, we run similar conditional beta models of Equation (8) and (9). Table X show that the coefficients of noise trader risk remain statistically significant, supporting the hypothesis of time varying noise trader risk. Two of the six regression coefficients of interaction terms with both investor sentiment and noise trader risk are significant, suggesting some evidence of time-varying betas.

[Insert Table X about here]

Finally, we run GARCH(1, 1) models of the distress premium and allow investor sentiment and noise trader risk to enter both the mean and volatility equation. To control for Fama-French (except the SMB) and momentum factors, we include them in both mean and volatility equations. We find noise trader risk significantly contributes to the volatility of distress premium in all six regressions. However, the coefficients of noise trader risk in the mean equation become less significant, especially in the predictive regressions.

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similar.)

[Insert Table XI about here]

5 Conclusion

We examine the role of noise trader risk on asset prices. We do this by incorporating time varying noise trader risk into the De Long et al. (1990) model. Our theory predicts that noise trader risk affects both contemporaneous and subsequent returns as well as return volatility over time. We estimate the volatility of three investor sentiment indicators to proxy for noise trader risk. We test whether they have time series relationship with the small stock premium as predicted by the theory. We find that when the increment in current noise trader risk is high, small stocks earn low contemporaneous returns and high subsequent returns relative to large stocks. In addition, noise trader risk is associated with the conditional volatility of the small stock premium. These findings support the time varying noise trader risk theory. We find similar roles of noise trader risk on the distress premium. Our results are robust to controls of macroeconomic variables and their uncertainty, and investor flows. Additional tests with conditional beta models show that systematic risk provides an incomplete explanation for our results.

6 Tables and Figures

Table I
Summary Statistics

This table presents the summary statistics for the sample period 07/1965-12/2012. Small stock premium, the difference between the returns on the smallest and the largest equally weighted size decile portfolios; Distress premium, the difference between the returns on the highest and the 6th equally weighted book-to-market decile portfolios; "cefd", the year-end, value-weighted average discount on closed-end mutual funds; sent_bw, composite sentiment index from Baker and Wurgler (2006); cci, the consumer confidence index; cci_2, the consumer confidence index that are orthogonalized to a number of contemporaneous and lagged macroeconomic variables; σ^2_{cefd}, $\sigma^2_{sent\_bw}$ and σ^2_{cci}, the estimated volatility of the corresponding investor sentiment proxies respectively; Rm-Rf, smb, hml, and mom, the Fama-French factors and the momentum factor; ip, change in natural log of industrial production; ir, 3-month interest rate; cpi, consumer price index; ur,unemployment rate; mb, broad money; term_ spread, the difference between the yiled of 10 year and 3 month treasuries; dp, dividend yield; pe_ratio, price earnings ratio; cay, consumption-to-wealth ratio; different types of investors refers to the percentage of equity holding of different type of investors to total equity holdings in U.S.

	Mean	Std.Dev.	Min.	Max.	Skewness	Kurtosis	NOBS
Small stock premium	0.54	5.25	-15.26	26.19	0.97	6.13	570
Distress premium	0.46	2.67	-8.14	22.68	1.74	13.56	570
cefd	-8.96	7.45	-25.28	10.91	0.05	2.46	546
sent_bw	0.00	10.00	-25.78	26.91	0.10	3.19	546
cci	85.31	13.24	51.70	112.00	-0.33	2.31	420
σ^2_{cefd}	2.20	1.41	0.18	9.85	1.80	7.85	522
$\sigma^2_{sent\_bw}$	1.43	0.98	0.07	6.06	1.53	5.50	522
σ^2_{cci}	8.46	2.81	2.96	21.76	0.87	4.53	396
Rm-Rf	0.46	4.56	-23.00	16.01	-0.50	4.75	570
smb	0.28	3.05	-11.60	14.62	0.32	4.61	570
hml	0.31	3.30	-20.79	19.72	-0.11	9.79	570
mom	0.72	4.38	-34.74	18.39	-1.40	13.43	570
ir	5.32	3.30	0.01	15.92	0.49	3.40	492
mb	0.01	0.02	-0.09	0.24	7.89	97.38	570
cpi	122.77	62.11	31.55	231.62	0.04	1.73	570
ip	0.00	0.01	-0.04	0.02	-1.12	7.85	570
ur	6.13	1.69	3.40	10.80	0.57	2.68	570
term_spread	1.57	1.30	-2.65	4.42	-0.33	2.51	570
dp	0.03	0.01	0.01	0.06	0.40	2.37	570
pe_ratio	19.39	8.33	6.64	44.20	0.80	3.53	570

Table II
Correlation of Sentiment and Noise Trader Risk

This table presents the correlation coefficient matrix (Panel A) and serial correlation coefficients (Panel B) of monthly and quarterly investor sentiment and noise trader risk proxies. "cefd" is the year-end, value-weighted average discount on closed-end mutual funds. "sent_bw" is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. "cci" is the consumer confidence index from Michigan Survey Research Center. σ^2_{cefd}, $\sigma^2_{sent\_bw}$ and σ^2_{cci} are the noise trader risk proxies estimated from the corresponding investor sentiment indicators. p-value is in the brackets. Note that "sent_bw" is re-scaled by 10 times for the ease of reporting coefficients in the regression studies.

Variables	cefd	sent_bw	cci	σ^2_{cefd}	$\sigma^2_{sent\_bw}$	σ^2_{cci}
	Panel A: Correlation Matrix					
cefd	1.00					
sent_BW	0.62	1.00				
	(0.00)					
cci	0.33	0.10	1.00			
	(0.00)	(0.04)				
σ^2_{cefd}	0.02	0.10	-0.15	1.00		
	(0.59)	(0.03)	(0.00)			
$\sigma^2_{sent\_BW}$	0.14	0.23	-0.10	0.41	1.00	
	(0.00)	(0.00)	(0.04)	(0.00)		
σ^2_{cci}	-0.02	-0.05	-0.29	0.14	0.20	1.00
	(0.63)	(0.36)	(0.00)	(0.01)	(0.00)	
	Panel B: First-order Serial Correlations					
Level	0.97	0.99	0.95	0.77	0.65	0.23
First difference	-0.14	0.08	-0.03	-0.04	-0.19	-0.48

Table III
Small Stock Premium Breakdown by sorts of sentiment and noise trader risk

This table reports the average contemporaneous and one-month-ahead small stock premium conditional on proxies of noise trader risk and investor sentiment. The small stock premium is the difference between the returns on the smallest and largest size decile portfolios. The proxies for investor sentiment are "cefd" (the year-end, value-weighted average discount on closed-end funds), "sent_BW" (the investor sentiment indicator in Baker and Wurgler (2006), and "cci" (the consumer confidence index from the Michigan Survey Research Center). The three monthly proxies for noise trader risk (σ^2) are estimated from the corresponding investor sentiment indicators.

Panel A1 shows the average (value- and equally-weighted) small stock premium for the months in which there are "high" and "low" changes in noise trader risk.

Panel A: By Noise Trader Risk												
	Panel A1: Contemporaneous small stock premium						Panel A2: One-month-ahead small stock premium					
	Value Weighted			Equally Weighted			Value Weighted			Equally Weighted		
	σ^2_{cefd}	$\sigma^2_{sent\_bw}$	σ^2_{cci}	σ^2_{cefd}	$\sigma^2_{sent\_bw}$	σ^2_{cci}	σ^2_{cefd}	$\sigma^2_{sent\_bw}$	σ^2_{cci}	σ^2_{cefd}	$\sigma^2_{sent\_bw}$	σ^2_{cci}
low	0.73	0.73	0.28	1.15	1.26	0.72	-0.19	-0.41	-0.27	-0.15	-0.14	-0.35
high	-0.20	-0.20	-0.14	-0.19	-0.29	-0.25	0.71	0.93	0.42	1.12	1.11	0.83

The change is "high" ("low") when the change in the monthly measure of the individual proxy for noise trader risk is above (below) its median value. Panel A2 shows the average one-month-ahead small stock premium where the level of noise trader risk in the current month is "high" ("low"). Noise trader risk is "high" ("low") when the monthly value of the individual proxy for noise trader risk is above (below) its median value.

Panel B reports the average contemporaneous and one-month-ahead small stock premium conditional on monthly proxies of investor sentiment.

Panel B: By Investor Sentiment												
	Panel B1: Contemporaneous small stock premium						Panel B2: One-month-ahead small stock premium					
	Value Weighted			Equally Weighted			Value Weighted			Equally Weighted		
	cefd	sent_bw	cci	cefd	sent_bw	cci	cefd	sent_bw	cci	cefd	sent_bw	cci
low	0.04	-0.21	-0.34	0.04	-0.06	-0.27	0.56	0.84	0.63	0.89	1.19	0.79
high	0.73	0.98	0.71	1.16	1.25	0.96	0.20	-0.08	-0.29	0.31	0.00	-0.12

Panel B1 shows the average monthly small stock premium for periods in which there are "high" and "low" changes in investor sentiment. The change in investor sentiment is "high" ("low") when the change in the monthly measure of the individual proxy is above (below) its median value. Panel B2 shows the average one-month-ahead small stock premium where the level of investor sentiment in the current month is "high" ("low"). Investor sentiment is "high" ("low") when the monthly value of the individual proxy is above (below) its median value.

Table IV
Monthly Regressions of the Small Stock Premium

Note: This table presents monthly contemporaneous and predictive regressions of the small stock premium on proxies of investor sentiment and noise trader risk proxies. The small stock premium is the difference between the returns on the smallest and the largest size decile portfolios. “cefd” is the year-end, value-weighted average discount on closed-end mutual funds. “sent_bw” is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. “cci” is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies (σ^2) are estimated from the corresponding investor sentiment indicators. “L.” before a variable means that variable has been lagged by one period. Newey-West standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	Contemporaneous Regressions			1-month Forecasts			3-month Forecasts		
	cefd	sent_bw	cci	cefd	sent_bw	cci	cefd	sent_bw	cci
sentiment_d	0.43***	0.42***	0.31***						
	(0.13)	(0.16)	(0.06)						
noise_risk_d	-0.88***	-1.13***	-0.11*						
	(0.23)	(0.31)	(0.06)						
L.sentiment				-0.06	-0.10***	-0.04*	-0.14	-0.24***	-0.20***
				(0.04)	(0.02)	(0.02)	(0.10)	(0.06)	(0.06)
L.noise_risk				0.71***	1.20***	0.16*	0.93***	1.98***	0.59***
				(0.16)	(0.24)	(0.09)	(0.34)	(0.46)	(0.15)
term_spread	0.33*	0.34**	0.34*						
	(0.17)	(0.17)	(0.20)						
Rm- Rf	0.04	0.02	-0.13*	0.03	0.01	-0.06	0.49***	0.45***	0.53***
	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.07)	(0.09)	(0.10)	(0.10)
HML	-0.07	-0.05	-0.19	-0.02	-0.05	-0.13	-0.06	-0.10	-0.06
	(0.21)	(0.22)	(0.23)	(0.22)	(0.22)	(0.23)	(0.20)	(0.22)	(0.25)
Mom	-0.10	-0.13	-0.11	-0.11	-0.15	-0.11	-0.04	-0.09	-0.10
	(0.09)	(0.10)	(0.08)	(0.10)	(0.10)	(0.08)	(0.15)	(0.15)	(0.15)
L.dp				63.31	106.12**	64.64	196.47	302.75**	253.91*
				(56.45)	(45.71)	(45.42)	(146.82)	(118.14)	(130.84)
L.term_spread				0.47***	0.27*	0.54***	0.99**	0.63	1.26***
				(0.17)	(0.15)	(0.18)	(0.45)	(0.43)	(0.46)
L.pe_ratio				0.13*	0.17**	0.16**	0.32	0.44**	0.58***
				(0.08)	(0.07)	(0.07)	(0.21)	(0.19)	(0.18)
Constant	0.03	0.03	-0.22	-6.74**	-8.09***	-3.76	-15.72*	-20.13***	-8.88
	(0.37)	(0.36)	(0.46)	(3.12)	(2.74)	(3.56)	(8.23)	(7.16)	(9.62)
Adj. R^2	0.05	0.05	0.07	0.04	0.07	0.03	0.08	0.13	0.19
N	521	521	395	522	522	395	522	522	392

Table V
Monthly Regressions of the Small Stock Premium: Controlling for Macroeconomic Variables

Note: This table presents monthly contemporaneous and predictive regressions of the small stock premium on proxies of investor sentiment and noise trader risk proxies. The small stock premium is the difference between the returns on the smallest and the largest size decile portfolios. “cefd” is the year-end, value-weighted average discount on closed-end mutual funds. “sent_bw” is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. “cci” is the consumer confidence index from Michigan Survey Research Center. “_2.” after the sentiment variables indicate these variables are orthogonalized to a number of contemporaneous and lagged macroeconomic variables. The noise trader risk proxies (σ^2) are estimated from the corresponding investor sentiment indicators. “L.” before a variable means that variable has been lagged by one period. Newey-West standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	Contemporaneous Regressions			Predictive Regressions		
	cefd_2	sent_bw_2	cci_2	cefd_2	sent_bw_2	cci_2
sentiment_d	0.17	-0.04	0.14**			
	(0.11)	(0.10)	(0.05)			
noise_risk_d	-0.54***	-0.32***	0.02			
	(0.13)	(0.11)	(0.03)			
L.sentiment				0.02	0.02	-0.08**
				(0.06)	(0.05)	(0.04)
L.noise_risk				0.27**	0.21**	0.13***
				(0.12)	(0.10)	(0.05)
term_spread	0.36**	0.31*	0.45**			
	(0.18)	(0.18)	(0.20)			
Rm- Rf	-0.04	-0.01	-0.06	-0.03	-0.02	-0.06
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
HML	0.01	0.04	-0.17	0.05	0.05	-0.13
	(0.24)	(0.24)	(0.23)	(0.24)	(0.25)	(0.23)
Mom	0.01	-0.02	-0.12	-0.01	-0.02	-0.12
	(0.09)	(0.09)	(0.08)	(0.10)	(0.09)	(0.08)
L.dp				42.57	30.95	50.55
				(60.69)	(61.45)	(45.15)
L.term_spread				0.35*	0.40**	0.53***
				(0.20)	(0.19)	(0.17)
L.pe_ratio				0.07	0.04	0.12*
				(0.09)	(0.09)	(0.07)
Constant	-0.06	0.03	-0.45	-3.45	-2.57	-5.88**
	(0.38)	(0.37)	(0.46)	(3.54)	(3.43)	(2.73)
Adj. R^2	0.04	0.02	0.03	0.01	0.01	0.04
N	442	442	395	443	443	395

Table VI
Monthly Conditional Market Betas

Note: This table presents monthly contemporaneous regressions of the small stock premium on proxies of investor sentiment and noise trader risk proxies while allowing the conditional market beta to be a function of noise trader risk (Equation 8 and 9). The small stock premium is the difference between the returns on the smallest and the largest size decile portfolios. "cefd" is the year-end, value-weighted average discount on closed-end mutual funds. "sent_bw" is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. "cci" is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies are estimated from the corresponding investor sentiment indicators. "L." before a variable means that variable has been lagged by one period. Newey-West standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	Contemporaneous Regressions			Predictive Regressions		
	cefd	sent_bw	cci	cefd	sent_bw	cci
sentiment_d	0.45***	0.39***	0.31***			
	(0.13)	(0.14)	(0.05)			
noise_risk_d	-0.87***	-1.14***	-0.11*			
	(0.24)	(0.32)	(0.06)			
L.sentiment				-0.03	-0.08***	-0.01
				(0.04)	(0.03)	(0.02)
L.noise_risk				0.67***	1.20***	0.19***
				(0.18)	(0.23)	(0.07)
$Sentiment\_d_t \times (Rm_t - Rf_t)$	4.81***	1.22	0.58			
	(1.86)	(3.38)	(0.93)			
$Noise\_Risk\_d_t \times (Rm_t - Rf_t)$	-4.23	-7.02	0.70			
	(5.56)	(7.98)	(1.30)			
$Sentiment_{t-1} \times (Rm_t - Rf_t)$				-0.43	-0.06	-0.71*
				(0.91)	(0.52)	(0.40)
$Noise\_Risk_{t-1} \times (Rm_t - Rf_t)$				4.60	4.42	-3.03
				(3.07)	(7.12)	(2.52)
Rm- Rf	0.09	0.07	-0.08	-0.09	-0.01	0.81*
	(0.05)	(0.05)	(0.05)	(0.12)	(0.12)	(0.46)
Constant	0.48*	0.45*	0.26	-1.26**	-1.20***	-0.15
	(0.27)	(0.27)	(0.29)	(0.57)	(0.42)	(1.95)
Adj. R^2	0.06	0.04	0.06	0.03	0.05	0.01
N	521	521	395	522	522	395

Table VII
Conditional Market Betas After Accounting for Macroeconomic Uncertainty

Note: This table presents one month ahead forecasting regressions of the small stock premium on proxies of investor sentiment and noise trader risk while allowing the conditional market beta to be a function of noise trader risk (Equation 8 and 9). Proxies of investor sentiment and noise trader risk have been orthogonalized to the volatilities of a set of macroeconomic variables and are denoted by "_res". The small stock premium is the difference between the returns on the smallest and the largest size decile portfolios. "cefd" is the year-end, value-weighted average discount on closed-end mutual funds. "sent_bw" is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. "cci" is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies are estimated from the corresponding investor sentiment indicators. "L." before a variable means that variable has been lagged by one period. Newey-West standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	Contemporaneous Regressions			Predictive Regressions		
	cefd_res	sent_bw_res	cci_res	cefd_res	sent_bw_res	cci_res
sentiment_d	0.14	0.13*	0.09			
	(0.11)	(0.07)	(0.06)			
noise_risk_d	-0.69***	-1.14***	-0.10			
	(0.22)	(0.35)	(0.06)			
L.sentiment				-0.06	-0.08**	-0.03
				(0.05)	(0.03)	(0.03)
L.noise_risk				0.76***	1.31***	0.21**
				(0.22)	(0.28)	(0.08)
$Sentiment\_d_t \times (Rm_t - Rf_t)$	2.47	-0.12	0.31			
	(1.53)	(1.84)	(0.78)			
$Noise\_Risk\_d_t \times (Rm_t - Rf_t)$	-3.05	-13.91*	0.89			
	(5.61)	(7.70)	(1.51)			
$Sentiment_{t-1} \times (Rm_t - Rf_t)$				-1.63	-0.02	-0.38
				(1.04)	(0.65)	(0.42)
$Noise\_Risk_{t-1} \times (Rm_t - Rf_t)$				4.09	6.72	-3.69
				(6.36)	(8.67)	(2.78)
Rm- Rf	-0.01	-0.01	-0.03	-0.01	-0.01	-0.04
	(0.06)	(0.05)	(0.06)	(0.06)	(0.05)	(0.06)
Constant	0.59**	0.56**	0.26	0.61**	0.57**	0.32
	(0.28)	(0.28)	(0.29)	(0.28)	(0.24)	(0.29)
Adj. R^2	0.05	0.06	0.01	0.03	0.06	0.01
N	443	443	395	444	444	395

Table VIII
GARCH(1, 1) Model with Noise Trader Risk

Note: This table presents GARCH(1, 1) model of the small stock premium with investor sentiment and noise trader risk. The small stock premium is the difference between the returns on the smallest and the largest size decile portfolios. “cefd” is the year-end, value-weighted average discount on closed-end mutual funds. “sent_bw” is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. “cci” is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies are estimated from the corresponding investor sentiment indicators. “noise_risk_sum” is the sum of current and last period noise trader risk. “L.” before a variable means that variable has been lagged by one period. Robust standard errors are in brackets. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	cefd	sent_bw	cci	cefd	sent_bw	cci
Mean Equation						
sentiment_d	0.34***	0.36***	0.24***			
	(0.10)	(0.12)	(0.05)			
noise_risk_d	-1.05***	-1.12***	-0.03			
	(0.22)	(0.25)	(0.06)			
L.sentiment				-0.03	-0.14***	-0.02
				(0.03)	(0.02)	(0.02)
L.noise_risk				0.59***	1.17***	0.10
				(0.18)	(0.22)	(0.07)
Rm- Rf	-0.02	0.04	-0.04	0.08**	0.07*	0.01
	(0.04)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
HML	-0.01	-0.02	0.06	0.05	0.14**	0.11
	(0.07)	(0.06)	(0.08)	(0.07)	(0.06)	(0.08)
Mom	-0.07	-0.20***	-0.08	-0.12***	-0.07	-0.08
	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
Constant	0.28	0.25	-0.10	-1.10**	-1.29***	0.59
	(0.22)	(0.19)	(0.21)	(0.47)	(0.35)	(1.79)
Volatility Equation						
sentiment	-0.01	-0.02	0.00	0.00	0.00	0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
noise_risk_sum	0.11***	0.16**	0.07***	0.16***	0.03	0.06**
	(0.03)	(0.06)	(0.02)	(0.04)	(0.11)	(0.02)
Rm- Rf	0.00	0.19***	0.21***	0.16***	0.19***	0.21***
	(0.01)	(0.03)	(0.03)	(0.02)	(0.04)	(0.03)
HML	-0.02	-0.01	-0.29***	-0.03	-0.25***	-0.29***
	(0.02)	(0.03)	(0.05)	(0.03)	(0.04)	(0.05)
Mom	-0.06***	0.19***	-0.06	0.16***	-0.03	-0.07
	(0.01)	(0.03)	(0.05)	(0.03)	(0.03)	(0.04)
Constant	2.48***	0.40	-1.20	0.70**	0.16	-1.01
	(0.22)	(0.35)	(0.99)	(0.31)	(0.37)	(0.99)
ARCH						
L.arch	0.32***	0.10**	0.02	0.08	0.09***	0.01
	(0.06)	(0.04)	(0.03)	(0.05)	(0.03)	(0.03)
L.garch	-0.15*	0.67***	0.83***	0.62***	0.78***	0.83***
	(0.08)	(0.06)	(0.04)	(0.06)	(0.03)	(0.04)
N	521	521	395	521	521	395

Table IX
Monthly Regressions of Distress Premium

Note: This table presents monthly contemporaneous and predictive regressions of the distress premium on proxies of investor sentiment and noise trader risk proxies. The distress premium is the difference between the returns on the highest and the 6th equally weighted book-to-market decile portfolios. “cefd” is the year-end, value-weighted average discount on closed-end mutual funds. “sent_bw” is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. “cci” is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies (σ^2) are estimated from the corresponding investor sentiment indicators. “L.” before a variable means that variable has been lagged by one period. Newey-West standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	Contemporaneous Regressions			1-month Forecasts			3-month Forecasts		
	cefd	sent_bw	cci	cefd	sent_bw	cci	cefd	sent_bw	cci
sentiment_d	0.12*	0.04	0.11***						
	(0.06)	(0.08)	(0.03)						
noise_risk_d	-0.40***	-0.36**	-0.12***						
	(0.12)	(0.15)	(0.04)						
L.sentiment				-0.01	-0.03***	0.02	-0.04	-0.10***	0.03
				(0.01)	(0.01)	(0.02)	(0.04)	(0.03)	(0.05)
L.noise_risk				0.19**	0.22**	0.12**	0.73***	0.84***	0.48***
				(0.08)	(0.10)	(0.06)	(0.23)	(0.27)	(0.10)
term_spread	0.20**	0.20**	0.26**						
	(0.10)	(0.10)	(0.11)						
Rm- Rf	-0.05	-0.06	-0.09*	-0.05	-0.06	-0.08	0.27***	0.26***	0.30***
	(0.04)	(0.04)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
SMB	0.29***	0.30***	0.19**	0.30***	0.29***	0.22***	0.07	0.05	0.17**
	(0.06)	(0.06)	(0.07)	(0.06)	(0.07)	(0.07)	(0.08)	(0.07)	(0.09)
Mom	-0.08**	-0.09**	-0.08*	-0.09**	-0.10**	-0.10**	0.00	-0.03	-0.04
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
L.dp				-7.17	5.64	2.27	-49.68	-1.64	-17.73
				(27.10)	(25.93)	(25.45)	(74.45)	(67.49)	(66.27)
L.term_spread				0.20**	0.16*	0.34***	0.45**	0.30	0.81***
				(0.09)	(0.09)	(0.11)	(0.23)	(0.23)	(0.22)
L.pe_ratio				-0.00	0.02	0.00	-0.07	-0.02	-0.04
				(0.04)	(0.04)	(0.04)	(0.11)	(0.10)	(0.09)
Constant	0.13	0.15	-0.04	-0.08	-0.54	-3.33	1.35	0.01	-5.88
	(0.17)	(0.17)	(0.21)	(1.62)	(1.61)	(2.30)	(4.47)	(4.17)	(5.93)
Adj. R^2	0.14	0.13	0.14	0.15	0.16	0.11	0.11	0.11	0.18
N	521	521	395	522	522	395	522	522	392

Table X
Monthly Conditional Market Betas: Distress Premium

Note: This table presents monthly contemporaneous regressions of the distress premium on proxies of investor sentiment and noise trader risk proxies while allowing the conditional market beta to be a function of noise trader risk (Equation 8 and 9). The distress premium is the difference between the returns on the highest and the 6th book-to-market equally weighted decile portfolios. "cefd" is the year-end, value-weighted average discount on closed-end mutual funds. "sent_bw" is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. "cci" is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies are estimated from the corresponding investor sentiment indicators. "L." before a variable means that variable has been lagged by one period. Newey-West standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	Contemporaneous Regressions			Predictive Regressions		
	cefd	sent_bw	cci	cefd	sent_bw	cci
sentiment_d	0.18***	0.05	0.15***			
	(0.07)	(0.08)	(0.03)			
noise_risk_d	-0.51***	-0.48***	-0.14***			
	(0.14)	(0.16)	(0.05)			
L.sentiment				0.00	-0.04***	0.01
				(0.02)	(0.01)	(0.02)
L.noise_risk				0.26***	0.35***	0.14**
				(0.08)	(0.10)	(0.06)
$Sentiment\_d_t \times (Rm_t - Rf_t)$	0.17	-0.81	-0.00			
	(1.02)	(1.61)	(0.56)			
$Noise\_Risk\_d_t \times (Rm_t - Rf_t)$	-3.17	-6.63***	0.45			
	(2.19)	(2.28)	(0.63)			
$Sentiment_{t-1} \times (Rm_t - Rf_t)$				-0.62**	-0.86***	-0.39
				(0.26)	(0.21)	(0.46)
$Noise\_Risk_{t-1} \times (Rm_t - Rf_t)$	-3.17	-6.63***	0.45	5.59***	1.94	-0.63
	(2.19)	(2.28)	(0.63)	(1.58)	(3.46)	(1.28)
Rm- Rf	0.03	0.03	-0.05	-0.17***	0.00	0.36
	(0.04)	(0.04)	(0.05)	(0.06)	(0.07)	(0.45)
Constant	0.43***	0.44***	0.42**	-0.10	-0.06	-1.65
	(0.14)	(0.14)	(0.17)	(0.28)	(0.21)	(1.38)
Adj. R^2	0.05	0.02	0.07	0.05	0.05	0.01
N	521	521	395	522	522	395

Table XI
GARCH(1, 1) Model with Noise Trader Risk: Distress Premium

Note: This table presents GARCH(1, 1) model of the distress premium with investor sentiment and noise trader risk. The distress premium is the difference between the returns on the highest and the 6th book-to-market equally weighted decile portfolios. "cefd" is the year-end, value-weighted average discount on closed-end mutual funds. "sent_bw" is the investor sentiment indicator in Baker and Wurgler (2006) which is a composite sentiment index based on the first principal component of several sentiment proxies and has been orthogonalized to several macroeconomic conditions. "cci" is the consumer confidence index from Michigan Survey Research Center. The noise trader risk proxies are estimated from the corresponding investor sentiment indicators. "noise_risk_sum" is the sum of current and last period noise trader risk. "L." before a variable means that variable has been lagged by one period. Robust standard errors are in brackets. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	cefd	sent_bw	cci	cefd	sent_bw	cci
Mean Equation						
sentiment_d	0.04	-0.04	0.11***			
	(0.05)	(0.06)	(0.02)			
noise_risk_d	-0.31***	-0.34***	-0.06**			
	(0.11)	(0.11)	(0.03)			
L.sentiment				0.02	-0.02	0.01
				(0.01)	(0.01)	(0.01)
L.noise_risk				0.06	0.12	0.07*
				(0.09)	(0.12)	(0.04)
Rm- Rf	-0.08***	-0.08***	-0.13***	-0.08***	-0.09***	-0.11***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
SMB	0.23***	0.24***	0.10**	0.24***	0.24***	0.13***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Mom	-0.05**	-0.07***	-0.03	-0.05**	-0.06**	-0.03
	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)
Constant	0.39***	0.40***	0.29***	0.46**	0.28	-1.22
	(0.11)	(0.11)	(0.10)	(0.23)	(0.19)	(0.90)
Volatility Equation						
sentiment	0.01	-0.02**	0.01	0.01	-0.02**	0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
noise_risk_sum	0.09***	0.10**	0.17***	0.10***	0.09**	0.18***
	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)	(0.04)
Rm- Rf	-0.02*	-0.03**	0.31***	-0.02	-0.03**	0.32***
	(0.01)	(0.01)	(0.06)	(0.01)	(0.01)	(0.06)
SMB	0.17***	0.17***	0.14**	0.16***	0.16***	0.11*
	(0.01)	(0.01)	(0.06)	(0.01)	(0.01)	(0.06)
Mom	-0.05***	-0.06***	0.02	-0.04***	-0.05***	0.01
	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.03)
Constant	1.05***	1.17***	-4.89***	1.08***	1.22***	-5.82***
	(0.16)	(0.14)	(1.82)	(0.15)	(0.13)	(2.01)
ARCH						
L.arch	0.09*	0.09**	0.22***	0.09**	0.09**	0.20***
	(0.05)	(0.04)	(0.05)	(0.04)	(0.04)	(0.05)
L.garch	0.05	0.03	0.62***	0.01	0.02	0.66***
	(0.07)	(0.07)	(0.05)	(0.06)	(0.06)	(0.05)
N	521	521	395	521	521	395

Appendix: Derivations of the Model Predictions

In this appendix, we derive the equilibrium excess returns and their volatility in Equations (3) and (A.8).

Both the noise trader's and sophisticated investor's utility function is a CARA (constant absolute risk aversion) utility function of wealth:

$$U = -e^{-(2\gamma)\omega} \tag{A.1}$$

where $\gamma = -\frac{u''(c)}{u'(c)}$ is the coefficient of absolute risk aversion.

If the holding period return is normally distributed, the expected utility optimization problem is equivalent to maximizing

$$\overline{\omega} - \gamma\sigma_{\omega}^2 \tag{A.2}$$

where $\overline{\omega}$ is expected final wealth, and σ_{ω}^2 is one period ahead of variance of wealth.

As in the model of De Long et al. (1990), the sophisticated investor chooses portfolio λ_t^i of holding risky asset u to maximize his expected utility

$$E(U) = c_0 + \lambda_t^i[r + E_t(p_{t+1}) - p_t(1+r)] - \gamma(\lambda_t^i)^2 E_t(\sigma_{p_{t+1}}^2)$$

The time t expectation of $t+1$ price volatility of unsafe asset is $E_t(\sigma_{p_{t+1}}^2) = E_t[p_{t+1} - E_t(p_{t+1})]^2$.

The noise trader, on the other hand, chooses his portfolio λ_t^n of holding risky asset u given his misperception to maximize his expected utility

$$E(U) = c_0 + \lambda_t^n[r + E_t p_{t+1} - p_t(1+r)] - \gamma(\lambda_t^n)^2 E_t(\sigma_{p_{t+1}}^2) + \lambda_t^n(\rho_t) \tag{A.3}$$

Solving the above optimization problem using the first order condition yields the portfolio holding of unsafe asset u:

$$\lambda_t^i = \frac{r + E_t(p_{t+1}) - p_t(1+r)}{2\gamma E_t(\sigma^2_{p_{t+1}})} \tag{A.4}$$

$$\lambda_t^n = \frac{r + E_t(p_{t+1}) - p_t(1+r)}{2\gamma E_t(\sigma^2_{p_{t+1}})} + \frac{\rho_t}{2\gamma E_t(\sigma^2_{p_{t+1}})} \tag{A.5}$$

Equilibrium price requires the holdings of risky assets from noise traders and sophisticated investors sum to one. Equations (A.4) and (A.5) imply that:

$$p_t = \frac{1}{1+r}[r + E_t(p_{t+1}) - 2\gamma E_t(\sigma^2_{p_{t+1}}) + \mu\rho_t] \tag{A.6}$$

To calculate equilibrium prices, we iterate p_{t+1}. Given our assumptions on noise traders' misperception in Equation 1 and 2, we conjecture that $\sum_{k=1}^{\infty} \frac{1}{(1+r)^k} E_t(\sigma^2_{p_{t+k}}) = \frac{\nu}{r} E_t(\sigma^2_{p_{t+1}})$, where ν is a positive constant (we verify this conjecture below). This results in:

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\nu\gamma}{r} E_t(\sigma^2_{p_{t+1}}) \tag{A.7}$$

Since ρ^*, γ, r, μ and ν are all constants, the above equation shows that only the second term and the last term on the right hand side are stochastic. The volatility of the price is therefore given as:

$$\sigma^2_{p_t} = \frac{\mu^2 \sigma^2_{\rho,t}}{(1+r)^2} + \frac{4\nu^2\gamma^2}{r^2} var[E_t(\sigma^2_{p_{t+1}})]$$

Iterate $E_t(\sigma^2_{p_{t+1}})$ and rule out the exploding price ($\sigma^2_{p,\infty} \neq \infty$), we obtain

$$E_t(\sigma^2_{p_{t+1}}) = \frac{\theta\mu^2\sigma^2_{\rho,t}}{(1+r)^2} + c$$

where c is a positive constant. Therefore our conjecture that $\sum_{k=1}^{\infty} \frac{1}{(1+r)^k} E_t(\sigma^2_{p_{t+k}}) = \frac{\nu}{r} E_t(\sigma^2_{p_{t+1}})$ can be verified by solving for the constant ν.

We obtain the equilibrium pricing equation:

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\nu\gamma}{r}\frac{\theta\mu^2\sigma^2_{\rho,t}}{(1+r)^2} - \frac{2\nu\gamma}{r}c \tag{A.8}$$

Our focus is on the pricing implication of time varying noise trader risk $\sigma^2_{\rho,t}$. Recall that the excess return from date $t-1$ to date t is defined as: $R_t = r + p_t - p_{t-1}(1+r)$. Plugging in the equilibrium price in Equation (A.8), we obtain:

$$R_t = \frac{\mu[\rho_t - (1+r)\rho_{t-1}]}{1+r} - \frac{2\nu\gamma}{r}\frac{\theta\mu^2[\sigma^2_{\rho,t} - (1+r)\sigma^2_{\rho,t-1}]}{(1+r)^2} + \zeta$$

where ζ is a constant. This is Equation 3 that leads to our Hypothesis 1, 2, 4 and 5.

Given our assumptions on independently distributed ρ_t and a constant volatility of $\sigma^2_{\rho,t}$, Equation 3 implies the volatility of excess returns is:

$$\sigma^2_{R,t} = \frac{\mu^2[\sigma^2_{\rho,t} + (1+r)^2\sigma^2_{\rho,t-1}]}{(1+r)^2} + \kappa$$

where κ is a constant. This is Equation 4 that provides our Hypothesis 3.

REFERENCES

Baker, M., and J. Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *The Journal of Finance* 61, 1645–1680.

Beber, A., F. Breedon, and A. Buraschi, 2010, Differences in beliefs and currency risk premiums, *Journal of Financial Economics* 98, 415–438.

Brown, G.W., 1999, Volatility, sentiment, and noise traders, *Financial Analysts Journal* 82–90.

Chan, KC., and N. Chen, 1991, Structural and return characteristics of small and large firms, *The Journal of Finance* 46, 1467–1484.

De Long, J.B., A. Shleifer, L. Summers, and R. Waldmann, 1990, Noise Traders Risk in Financial Markets, *Journal of Political Economy* 98, 703–738.

Fama, E.F., and K.R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Ferson, Wayne E, Sergei Sarkissian, and Timothy T Simin, 2003, Spurious regressions in financial economics?, *The Journal of Finance* 58, 1393–1414.

Foucault, T., D. Sraer, and D.J. Thesmar, 2011, Individual investors and volatility, *The Journal of Finance* 66, 1369–1406.

Gemmill, G., and D.C. Thomas, 2002, Noise trading, costly arbitrage, and asset prices: Evidence from closed-end funds, *The Journal of Finance* 57, 2571–2594.

Kelly, M., 1997, Do noise traders influence stock prices?, *Journal of Money, Credit, and Banking* 351–363.

Lee, C.M.C., A. Shleifer, and R.H. Thaler, 1991, Investor sentiment and the closed-end fund puzzle, *Journal of Finance* 46, 75–109.

Lee, WY., C.X. Jiang, and D.C. Indro, 2002, Stock market volatility, excess returns, and the role of investor sentiment, *Journal of Banking & Finance* 26, 2277–2299.

Lemmon, M., and E. Portniaguina, 2006, Consumer confidence and asset prices: Some empirical evidence, *Review of Financial Studies* 19, 1499.

Schwert, G.W., 1989, Why does stock market volatility change over time?, *The Journal of Finance* 44.

Scruggs, J.T., 2007, Noise trader risk: Evidence from the siamese twins, *Journal of Financial Markets* 10, 76–105.

Sias, R. W., L. T. Starks, and S. M. Tinic, 2001, Is noise trader risk priced?, *Journal of Financial Research* 24, 311–330.

Stambaugh, Robert, Jianfeng Yu, and Yu Yuan, 2012, The long of it: Odds that investor sentiment spuriously predicts anomaly returns, *NBER Working Paper* .

Wang, Y., A. Keswani, and S.J. Taylor, 2006, The relationships between sentiment, returns and volatility, *International Journal of Forecasting* 22, 109–123.

Wurgler, J., and E. Zhuravskaya, 2002, Does arbitrage flatten demand curves for stocks?, *The Journal of Business* 75, 583–608.