Signalling in Research Contests

Nick Netzer
University of Konstanz*

Christian Wiermann
University of Konstanz†

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Abstract

In contests, players invest non-retrievable effort to increase the probability of winning a rent. In the context of research, contests arise when departments compete for funds, scientists strive for tenure or firms rival for patents. Such contests are usually dynamic and involve heterogeneity and asymmetric information. This leads to the possibility that players send strategic signals. We model one-sided asymmetric information in a discrete two-stage contest. We show under which circumstances pooling and separating equilibria occur. We also derive the implications for a contest designer, who wants to maximize research activity and can choose between a one- and a two-stage contest, e.g. by deciding on whether intermediate valuation is required before renewal of funds or not.

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*Department of Economics, University of Konstanz, Box D-136, 78457 Konstanz, Germany, Nick.Netzer@uni-konstanz.de

†Department of Economics, University of Konstanz, Box D-138, 78457 Konstanz, Germany, Christian.Wiermann@uni-konstanz.de

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1 Introduction

Among the most intriguing results in economics is the insight of the favorable forces of competition. It helps to unleash productive and innovative forces and reduces the possibility that some few can reap up extraordinary rents at the expense of general welfare. It is therefore not surprising that economists have come to advertise competition in many spheres besides regular markets. One of these areas is research. It is now common practise that research funds are awarded according to criteria such as previous achievements or active solicitation efforts. Tenure is given to those young scientists who have proven for several years to be the best among their competitors.[1] Finally, the race between researchers for patents is believed to foster scientific progress.[2]

A closer look, however, reveals that these examples are not instances of competition in the classical sense but should rather be described as contests. The distinguishing feature is that the investment of effort that affects the probability of winning some advertised rent is non-retrievable. The literature on such contests, starting with Tullock (1980), shows that it is no longer clear whether this kind of competition will lead to a desirable outcome. Aspects such as the nature of the effort, individual characteristics of the rent-seekers and contest rules determine how much effort is wasted, i.e. the degree of rent-dissipation. The understanding of all effects at work is all the more important as the rules of research contests can deliberately be designed along several dimensions such as time structure and the size and division of the rent.[3] Our contribution wants to add to the understanding by examining the possibility of signalling and its impacts in dynamic research contests.

What are the specific features of research contests? First, research is a long-term issue and not a one-shot game with instant results. Furthermore, at least part of the contest effort constitutes research itself. The application of a university department for research funds requires a clear concept and at least some preliminary results. In the case of the patent race, research and contest efforts are largely congruent. This is of importance when it comes to making welfare statements, because inducing high effort levels might be desirable, opposed to other applications of contests such as political rent-seeking.[4]

Second, the effort of competitors can be observable during the run of the game. In some circumstances, this might not be influenceable by the contest designer. For example, assistant professors can simply monitor the publications of their peers. Often, however, the contest rules can directly specify whether efforts are observable or not. They could, for example, either require outside intermediate evaluation before research funds are renewed, or award them once and for all. Abstractly, this amounts to the choice between a one- and a two-stage contest.

[1]See Kolmar and Wagener (2005) for an application of the theory of contests to tenure decisions.

[2]This area is probably the one that has been most investigated. See for example Baye and Hoppe (2003), Tylor (1995) and Che and Gale (2003).

[3]As Moldovanu and Sela (2005) put it, contests ‘are either designed or arise naturally’. In the context of research, most contests are clearly delineated by some governmental or private agency that can set rules at its own discretion. The question of optimal design of contests has recently received much attention. Se for example Singh and Wittman (1988), Gradstein (1998), Gradstein and Konrad (1999), Che and Gale (2003) and Moldovanu and Sela (2005), who all deal with different aspects of contest design.

[4]It is in the interest of the contest designer to *‘induce a general increase of activity in the specific field.’* (Moldovanu and Sela 2005, p. 543)

Third, researchers differ with respect to productivity and talent. While this itself will be important to take into account, even more important is the fact that it can create an information asymmetry. For example, the capabilities of an established Ivy-League university department are well known and have been proven several times. Still one might want to open the possibility for some upcoming young department to compete. By the very nature of the described process, however, neither the contest designer nor the more established competitor will have much information about the new player.

All these aspects create the need to model a *dynamic* contest with *heterogeneous agents* and *asymmetric information*. It is therefore clear that signalling effects are highly relevant in research contests.

The literature on contests has virtually mushroomed since Tullock's first contribution.[5] Various steps have been taken in order to model different economic situations, among them the interaction of heterogeneous players with asymmetric information. Heterogeneity can be introduced both via different price valuations or via different productivities. The first approach is taken by Hillman and Riley (1989), Che and Gale (1998) and Nti (1999). Rosen (1986) uses the second possibility. It turns out that both heterogeneity with respect to valuation and productivity have very similar effects (see for example Nitzan 1994, p. 46). Here we follow the approach taken by Amegashi (2005) who, in the context of elimination tournaments with asymmetric information, models heterogeneity in prize valuation but interprets it as the 'strength' of the players. Other important contributions that deal with heterogeneity and asymmetric information are Harstad (1995) and Wärneryd (2001). They, however, restrict themselves to the static case.

The literature one repeated interaction between contestants can be classified on whether they describe elimination tournaments, in which only the winner of a round proceeds to the next stage (tennis in Wimbledon), or multi-stage contests where the same opponents meet several times (Formula I racing). Amegashi (2005) finds that signalling in elimination contests will, if anything, lead to higher efforts by the players than in the case with perfect information. However, as indicated in the above discussion, we believe that multi-stage contests are more appropriate for the case of research. Within this class one can distinguish between models where players choose efforts simultaneously or sequentially. See for example Yildrim (2003), who models both possibilities and also discusses the preceeding literature such as Leininger (1993), Gradstein (1998) and Gradstein and Konrad (1999). These contributions, however, do not take into account asymmetric information.

We proceed as follows. In the next section we describe and solve a one-stage game under different information assumptions. We then extend the model to a two-stage setting in section 3. Using examples, section 4 illustrates how the new insights can be used by a contest designer. The last section concludes.

[5] For an overview over earlier developments in contest theory see Nitzan (1994).

2 Static Game

2.1 Perfect Information

Consider a one-stage game in which two players simultaneously compete for a prize. In case of winning the prize, the payoff for player 1 (player 2) is V (W). For now, both V and W are assumed to be known by both players. The probability of winning is determined according to a contest success function. Denote the probability of winning for player 1 by $p = p(e^1, e^2)$, where e^i denotes the effort chosen by player $i = 1, 2$. Consequently player 2 is the winner with probability $1 - p(e^1, e^2)$. In contrast to much of the literature we assume that only one of two discrete effort levels can be chosen by each of the players. They can either exert a high effort level e_H or a low effort level $e_L < e_H$. The contest technology is then uniquely defined by four probability values $p_{jk} = p(e_j, e_k)$, where $e_j \in \{e_L, e_H\}$ is the effort chosen by player 1 and $e_k \in \{e_L, e_H\}$ is the respective effort of player 2.

The game will be solved with as few assumptions on the values p_{jk} as possible. Some restrictions, however, have to be taken to capture the nature of a contest. First assume that

$$p_{LL} = p_{HH}, \tag{2.1}$$

which states that the success probability does not vary with a joint increase of effort by both players.[6] Second, we assume that the own probability of winning increases whenever a player increases the effort from low to high. This can be summarized as

$$p_{HL} > p_{LL} = p_{HH} > p_{LH}. \tag{2.2}$$

Because of the unusual discreteness of the available effort levels, we derive the equilibrium of the game in some detail. The optimization problem of player 1 is given by

$$\max_{e^1 \in \{e_L, e_H\}} \pi_1 = p(e^1, e^2)\, V - e^1 \tag{2.3}$$

and analogously for player 2. Reaction functions can easily be derived. Player 1 prefers e_H over e_L given some effort $e^2 = e_k$ by his opponent if $p_{Hk}\, V - e_H \geq p_{Lk}\, V - e_L$ and vice versa.[7] This yields his best response:

$$e^1(e_k) = e_H \Leftrightarrow V \geq \frac{e_H - e_L}{p_{Hk} - p_{Lk}} \equiv \frac{\Delta e}{\Delta p_{\bullet k}}. \tag{2.4}$$

The difference $\Delta p_{\bullet k}$, which is positive due to (2.4), stands for the increase in his success probability resulting from an increase in his own effort level from low to high, given that player 2 supplies the effort

[6] This assumption could easily be based on the standard Tullock contest success function $p(e^1, e^2) = \frac{e^1}{e^1 + \theta e^2}$ where $\theta > 0$ is a parameter measuring the relative effectiveness of player 2 in the contest. Substituting any two effort levels e_L and e_H yields the above given assumption as an implication.

[7] We ignore the fact some player 1 will be indifferent between the two pure actions and might therefore play any mixed strategy. Even in the more advanced game in following section this will not be relevant as the probability that this event occurs is zero.

$e^2 = e_k$. Condition (2.4) reflects the comparison between the positive and negative effects of an increase in effort. If the valuation V is high enough, the utility gain $V\Delta p_{\bullet k}$ is larger than the utility loss Δe. By the same line of reasoning the optimal behavior of player 2 is given by:

$$e^2(e_j) = e_H \Leftrightarrow W \geq \frac{e_H - e_L}{p_{jL} - p_{jH}} \equiv \frac{\Delta e}{\Delta p_{j\bullet}}. \tag{2.5}$$

The critical values defined in (2.4) and (2.5) deserve closer examination. Due to (2.1), their denominators are related as follows:

$$\Delta p_{\bullet L} = \Delta p_{H\bullet} \quad \text{and} \quad \Delta p_{\bullet H} = \Delta p_{L\bullet}. \tag{2.6}$$

Now define the following general critical value function:

$$CV(x) \equiv \frac{\Delta e}{x\Delta p_{H\bullet} + (1-x)\Delta p_{L\bullet}} = \frac{\Delta e}{x\Delta p_{\bullet L} + (1-x)\Delta p_{\bullet H}}. \tag{2.7}$$

Therefore, the critical value that is decisive for player 1's optimal response to high effort of his opponent is $CV(0)$, while the relevant critical value for the best response to low effort is $CV(1)$. Conversely, the best response of player 2 to a high effort level of player 1 is determined by the relative position of his valuation W to $CV(1)$. For the best response to $e^1 = e_L$, the value $CV(0)$ is relevant.

A first natural case to consider would be a situation in which $p_{HH} = p_{LL} = \frac{1}{2}$ and $p_{LH} = 1 - p_{HL}$, i.e. a completely symmetric case.[8] The following proposition summarizes the results for all cases in which the weaker 'symmetry assumption' $p_{HL} - p_{LL} = p_{HH} - p_{LH}$ is satisfied.

Proposition 2.1. *If $p_{HL} - p_{LL} = p_{HH} - p_{LH}$, the game has a unique Nash-equilibrium in strictly dominant strategies. The efforts chosen by the players solely depend on their own valuation of the prize V and W, respectively. Player 1 exerts the high effort if $V \geq CV(0) = CV(1)$ and the low effort otherwise. The same critical value is crucial for player 2.*

The proposition is easily proven since under $p_{HL} - p_{LL} = p_{HH} - p_{LH}$ the gain from increasing one's own effort does not depend on the action of the opponent. Therefore, the two critical values $CV(0)$ and $CV(1)$ coincide.

The equilibrium behavior of the model is summarized in figure 1, where we assume some upper and lower bound on one the valuations V and W. The abscissa (ordinate) measures W (V). Four areas are defined by the critical value $CV(0) = CV(1)$. The first entry in each area denotes the equilibrium effort of player 1. Note that neither asymmetric information about the opponent's valuation nor a dynamic structure would change this result. Signalling effects do not occur if the opponent has a dominant

[8]Such relations would for example emerge if the Tullock function parameter θ mentioned in footnote 6 was set equal to one.

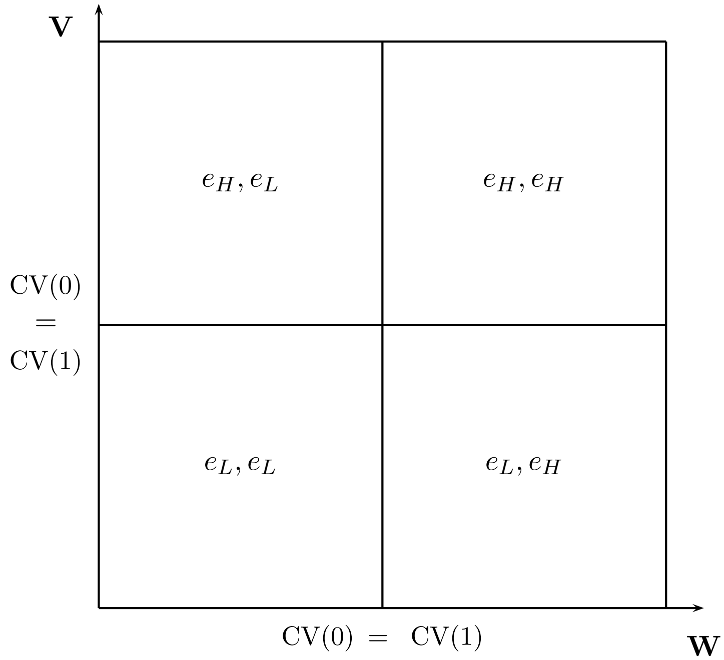

Figure 1: Symmetric One-Stage Game with Perfect Information

strategy and his actions can therefore not be influenced.

Consider now a situation in which the symmetry property does not hold. Specifically, assume without loss of generality that $p_{HL} - p_{LL} > p_{HH} - p_{LH}$.[9] This assumption implies that the gain from increasing effort is larger for player 1 if his opponent chooses the low effort level ($\Delta p_{\bullet L} > \Delta p_{\bullet H}$). Conversely, player 2's additional effort is more productive if his opponent chooses the high effort level ($\Delta p_{H\bullet} > \Delta p_{L\bullet}$). We take this situation to represent an a priori advantage of player 2. To see this note that we have $p_{LL} = p_{HH} < \frac{p_{LH}+p_{HL}}{2}$. Compared to the symmetric case above, in which $p_{LL} = p_{HH} = \frac{p_{LH}+p_{HL}}{2}$ holds, the winning probability of player 1 if both exert the same effort level is smaller. This is most adequately interpreted as some kind of 'bias' in favor of player 2: even if the opponents are equally engaged in the contest, player 2's winning probability is higher. Many circumstances can cause such a bias. In research contests, for example, evaluations and decisions about funds or positions are done by established researchers themselves, such that personal connections, reputation effects or strategic interests will almost certainly influence the decision towards one or the other contestant.

The two critical values derived above now become unequal; the optimal reply of the players can depend on the action undertaken by the opponent. We have $\partial CV(x)/\partial x < 0$, which implies $CV(0) > CV(1)$. The equilibrium of the asymmetric game is characterized in the following proposition.

[9] Reversing the inequality sign would result in an analogous but reverted situation. Note that the current constellation of probability values could be retrieved from a Tullock function in which player 2 has an advantage, i.e. $\theta > 1$.

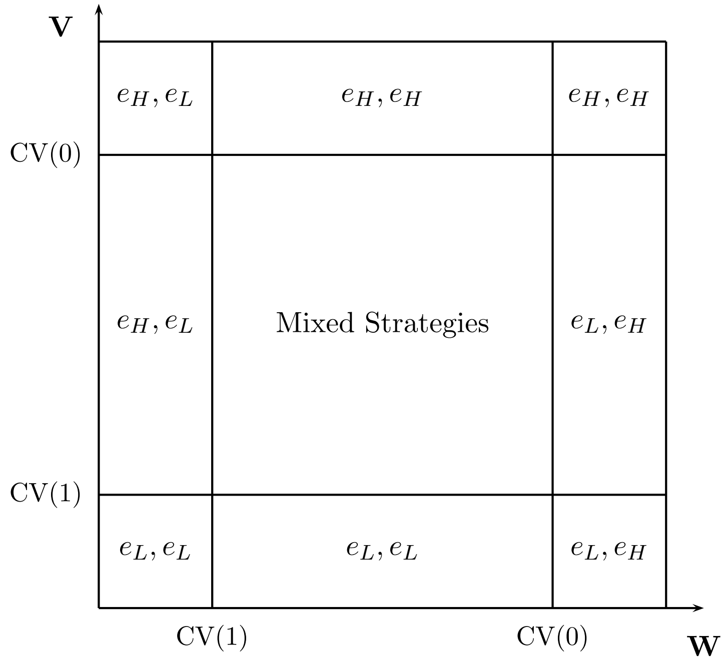

Figure 2: Asymmetric One-Stage Game with Perfect Information

Proposition 2.2. *If $p_{HL} - p_{LL} > p_{HH} - p_{LH}$ there exists a unique Nash-equilibrium in pure strategies if at least one valuation V or W is outside the interval $[CV(1), CV(0)]$. If both valuations lie within the interval there exists a unique equilibrium in mixed strategies. The characteristics of the equilibria are summarized in figure 2.*

Proof. Players below the critical value $CV(1)$ strictly prefer to play the low effort and do not find reaction to the action of their opponent desirable. The same applies for valuations of the prize above $CV(0)$: here players always exert the high effort level. Only in the interval between $CV(1)$ and $CV(0)$ do players have a nonconstant reaction function. If only one of the players is located in this area, he will play his optimal response to the dominant strategy of the opponent as defined in (2.4) and (2.5). If both valuations lie within the respective interval, no equilibrium in pure strategies exists. Consider player 1 exerting the high effort. Player 2 would then also like to exert the high effort, as CV(1) is relevant for his decision. Player 1, however, would then like to deviate to e_L as CV(0) is decisive for him, which in turn alters the best response of player 2 and so on. It can easily be shown that a mixed strategy equilibrium exists in this case. As its features are not relevant for the following, we leave the derivation of the mixing probabilities to the reader. □

2.2 Asymmetric Information

In this section we change our assumption concerning the information structure of the game while we keep the asymmetry assumption $p_{HL} - p_{LL} > p_{HH} - p_{LH}$ for the rest of the paper. We assume that nature first chooses the valuation V of player 1 from a uniform distribution on the support $[\alpha, \beta]$ with

$\alpha < CV(1)$ and $\beta > CV(0)$. While player 1 knows the outcome of this first stage, player 2 is not informed about it but only knows the distribution of V. The valuation W of player 2, on the other hand, is common knowledge and lies within the interval $[CV(1), CV(0)]$, i.e. in the domain in which he does react to the action of player 1. Assuming W to lie in this interval allows for the possibility of signalling.
The assumed structure captures what we believe to be the most important aspects of information in research contests as outlined in the introduction. Player 2, who we assumed to have the a priori advantage, is well-known. He represents an established player whose publicity might be the result of several previously won contests. Player 1 can be interpreted as the newcomer, challenger, or the 'underdog', who is not yet known.

The following two propositions describe the equilibria of the game.

Proposition 2.3. *The one-stage game with asymmetric information admits a unique Nash-equilibrium in pure strategies in which player 2 chooses the effort* e_L *if* $W < CV(q^L)$, *where* $q^L = \frac{\beta - CV(1)}{\beta - \alpha}$, *and* e_H *if* $W > CV(q^H)$, *where* $q^H = \frac{\beta - CV(0)}{\beta - \alpha}$. *Player 1 chooses his effort level according to his best response function* (2.4).

Proof. Examine the choice of the uninformed player 2, whose action depends on the probability q with which he believes player 1 to play e_H. As the reaction function of the informed player 1 is still the same as before, this probability amounts to $q = Pr[V > CV(0)] + Pr[CV(1) < V < CV(0)]\mathbf{1}(e^2 = e_L)$. The indicator function captures the fact that q depends on player 2's equilibrium action via the best-response of a player 1 with V in $[CV(1), CV(0)]$. Player 2 prefers e_H over e_L and vice versa if

$$[q(1 - p_{HH}) + (1 - q)(1 - p_{LH})]W - e_H \geq [q(1 - p_{HL}) + (1 - q)(1 - p_{LL})]W - e_L, \tag{2.8}$$

which reduces to the condition

$$W > \frac{\Delta e}{q\Delta p_{H\bullet} + (1 - q)\Delta p_{L\bullet}} = CV(q). \tag{2.9}$$

Via q, $CV(q)$ still depends on the action of player 2. First assume that $e^2 = e_H$ is optimal. According to (2.9) this does not yield a contradiction if $W > CV(q^H)$ with $q^H = \frac{\beta - CV(0)}{\beta - \alpha}$ from the above specified rational beliefs and the assumption of the uniform distribution of V. Next assume that $e^2 = e_L$ is optimal. This does not yield a contradiction if $W < CV(q^L)$ with $q^L = \frac{\beta - CV(1)}{\beta - \alpha}$. From $\beta > CV(0) > CV(1) > \alpha$ it follows that $1 > q^L > q^H > 0$ and thus $CV(1) < CV(q^L) < CV(q^H) < CV(0)$.

□

Denote the interval $[CV(1), CV(q^L)[$ by Ω_1, the interval $]CV(q^H), CV(0)]$ by Ω_2 and the yet uncovered interval $[CV(q^L), CV(q^H)]$ by Ω_3. Let us now turn to Ω_3 where no equilibrium in pure strategies exists.

Proposition 2.4. *If $W \in \Omega_3$, a unique Nash-equilibrium exists in which player 2 plays the mixed strategy $\sigma^2 = (r, 1-r)$ where the probability r of choosing the low effort, is given by*

$$r(W) = \frac{p_{L\bullet}}{\Delta p_{L\bullet} - \Delta p_{H\bullet}} + \frac{(\Delta e)^2 W}{\Delta e(\alpha - \beta) + W(\beta \Delta p_{H\bullet} - \alpha \Delta p_{L\bullet})}. \tag{2.10}$$

The probability $r(W)$ is decreasing in W and $r(CV(q^L)) = 1$ and $r(CV(q^H)) = 0$. Player 1 plays e_H if $V \geq CV(r)$ and e_L otherwise.

Proof. See Appendix A □

The intuition behind this result is straightforward. By playing a mixed strategy, player 2 creates exactly one type of player 1 ($V = CV(r)$) who is indifferent between the two effort levels. All players 1 with a smaller valuation choose the low, types with higher valuation the high effort level. The equilibrium value of r for player 2 with valuation W is such that he is exactly indifferent between his two pure strategies. The described mixed strategy equilibrium converges to the pure strategy equilibria from proposition 2.3 as the valuation W approaches the boundaries of Ω_3.

Figure 3 summarizes our analysis of the one-stage game with asymmetric information. A valuation W in the interval Ω_3 generates a critical value $CV(r(W))$ that increases in W. In the figure this is indicated by the upwards sloping line.[10]

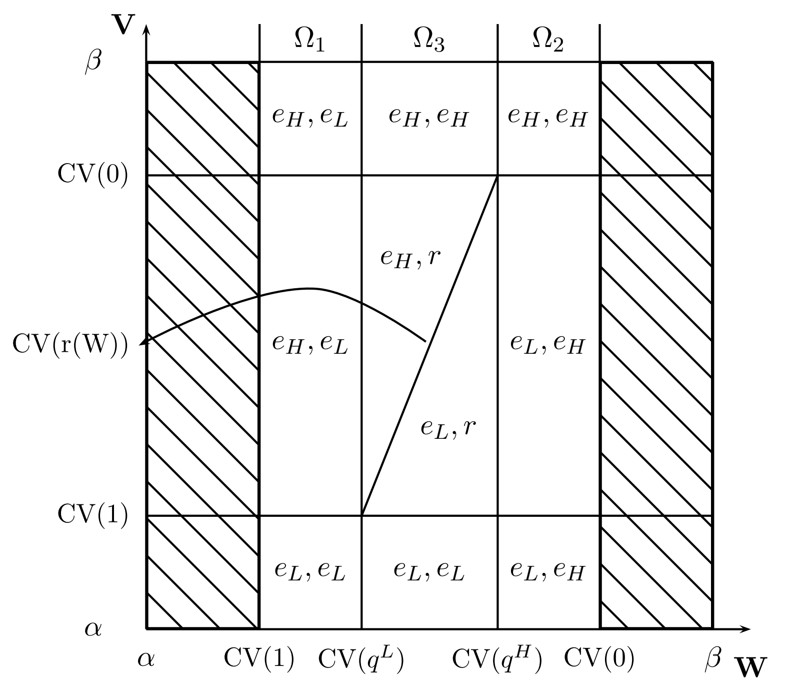

Figure 3: Asymmetric One-Stage Game with Asymmetric Information

[10]The function $CV(r(W))$ is depicted as a straight line only for presentational purposes. In fact it is concave in W.

3 Dynamic Game

3.1 Model Structure

Assume now that the individuals play two times against each other. We assume an additive structure for the probability of winning the prize. The payoff function of player 1 becomes

$$\pi_1 = a[p(e_1^1, e_1^2) + p(e_2^1, e_2^2)]V - e_1^1 - e_2^1, \tag{3.1}$$

where e_j^i denotes the effort player i chooses in period j.[11] The payoff function for player 2 is given analogously. Different situations can be modelled by choosing appropriate values of a. If $a = 1/2$, the probability of winning is determined as the average outcome of two separate contests. A contest designer could for example set up such a dynamic contest instead of a one-stage contest.[12] For $a = 1$, a simple repeated contest is modelled. From now on, only the case with $a = 1$ will be examined. Due to the additive structure, the results can be transferred to other cases by multiplying the valuations V and W by the desired a and applying the following results to the such transformed game.[13]

Players still choose their efforts simultaneously in each period, but first period efforts become observable before period 2 and thus constitute signals. If the action of player 1 in the first period contains some information about his type, in the sense that not all types would choose the same effort level, player 2 will use this information in deciding about his second period move. Equilibria of this kind will be labelled 'separating'.[14] On the other hand, if all possible types choose the same effort e_1^1, no information is revealed. In such 'pooling' equilibria, the second round of the contest takes place as in the one-stage game with asymmetric information since the game is over afterwards and nothing can be gained by deviations from the optimal actions of the one-stage setting.

The number of Perfect Bayesian Equilibria (PBE) in signalling games is usually large. In our model, it is however possible to derive some properties common to all equilibria. Denote the equilibrium strategy of player 2 in the first round by $\sigma_1^2 = (r, 1-r)$, where r represents again the probability of playing the low effort and denote his equilibrium strategy in the second round by $\sigma_2^2(e_1^1) = (s(e_1^1), 1 - s(e_1^1))$, where $s(e_1^1)$ represents the probability of playing the low effort after having observed e_1^1 in the first round. We again restrict ourselves to the examination of pure strategies $e_1^1(V)$ and $e_2^1(V)$ for player 1 since for any strategy of player 2 there will be only one type V that is indifferent between his two pure actions. This type occurs with probability zero.
In equilibrium, no type of player 1 can increase his payoff by unilaterally deviating from his strategy.

[11] Throughout this section, an upper index will signify the player while a lower index will signify the period.

[12] Designing a multi-stage contest is straightforward if people can choose their effort continuously and reduce it as much as they like. In the present model with only two effort levels to choose from, a participation constraint might become binding. We ignore this problem. It could for example be omitted by setting $e_L = 0$.

[13] This transformation, using $a = 1/2$, will for example be necessary when a contest designer wants to compare the results of a one- and a two-stage contest while the same prize is to be awarded in both cases.

[14] The usual difference between perfectly separating and partially separating equilibria is not applicable here, since perfect separation is impossible. As the set of types is infinite while the set of actions is finite, the uninformed player can never infer from the observed action to the type of this opponent perfectly.

Define the 'deviation function' $h(V)$ to measure the payoff change for player 1 of type V of switching from $e_1^1 = e_L$ to e_H, taking the strategy of player 2 as given. It contains a direct first period effect and a second period effect. The second period effect takes into account the induced change in the mixing probability s from $s(e_L)$ to $s(e_H)$ and the optimal reaction of player 1's second period effort to this change. Assume $h(V)$ was strictly increasing in V, for every strategy tuple (σ_1^2, σ_2^2). This would imply that whenever some type V' of player 1 finds it optimal to play $e_1^1 = e_H$, every type $V'' > V'$ would find this optimal as well, as his payoff difference between high and low effort in the first period would even by larger. This in turn would make it possible to restrict attention to pooling equilibria and to separating equilibria characterized by a single value $\tilde{V}$, such that types with a larger V choose the high and types with a lower V choose the low effort in the first period. The value $\tilde{V}$ would be defined by $h(\tilde{V}) = 0$. Unfortunately, $h(V)$ does not in general take such a simple form. Lemma 3.1 summarizes the properties of $h(V)$ that are derived in Appendix B.

Lemma 3.1. *The function $h(V)$ on $[\alpha, \beta]$ is*
(i) continuous,
(ii) stepwise linear, changing the slope at $CV(s(e_L))$ and $CV(s(e_H))$,
(iii) strictly increasing in V if $s(e_L) \leq s(e_H)$.

If $s(e_L) \leq s(e_H)$ we actually obtain the 'monotonicity property' discussed above. If deviation from low to high effort in the first period induces player 2 to play e_L with (weakly) greater probability in the second period, $h(V)$ is strictly increasing. However, $\sigma_2^2(e_1^1)$ is not exogenously given but itself a part of the equilibrium strategy profile. Unfortunately, the monotonicity property does not necessarily hold in equilibria when $s(e_L) > s(e_H)$. In that case, properties (i) and (ii) of Lemma 3.1 still tell us something about the equilibrium. Continuity and stepwise linearity of $h(V)$ on the three subintervals of $[\alpha, \beta]$ separated by the points $CV(s(e_L))$ and $CV(s(e_H))$ implies that $h(V)$ can generically have at most three zero points in $[\alpha, \beta]$.[15] This implies that each equilibrium can be characterized by at most three values $\tilde{V}_k$ at which first period behavior of player 1 changes. In any equilibrium, every zero point at which $h(V)$ actually changes its sign corresponds to one such point $\tilde{V}_k$.[16] Furthermore, a change in sign from minus to plus corresponds to a change in first period effort from low to high and vice versa.
Lemma 3.1 therefore allows us to restrict the search to four categories of equilibria, defined by the number n of points $\tilde{V}_k$. The category of pooling equilibria is characterized by $n = 0$. Within each of the categories, there remain two further possibilities, depending on which effort level e_1^1 the smallest type α chooses, i.e. depending on $e_1^1(\alpha)$. Therefore, the search is restricted to 8 equilibrium *classes*, each uniquely defined by $(n, e_1^1(\alpha))$. Figure 4 illustrates the discussion for the case of a $(0, e_H)$-equilibrium and a $(3, e_L)$-equilibrium.

[15] 'Generically' here means that one could construct effort levels and probabilities p_{ij} such that $h(V) = 0$ on one or more of the three subsets. This infinite set of zero points would, however, vanish if one of the game parameters was varied only slightly.

[16] In the exceptional case that $h(V)$ reaches a local maximum or minimum in a zero point, behavior does not change there and therefore we cannot simply state that each zero point corresponds to a critical value $\tilde{V}_k$; the change in sign is necessary.

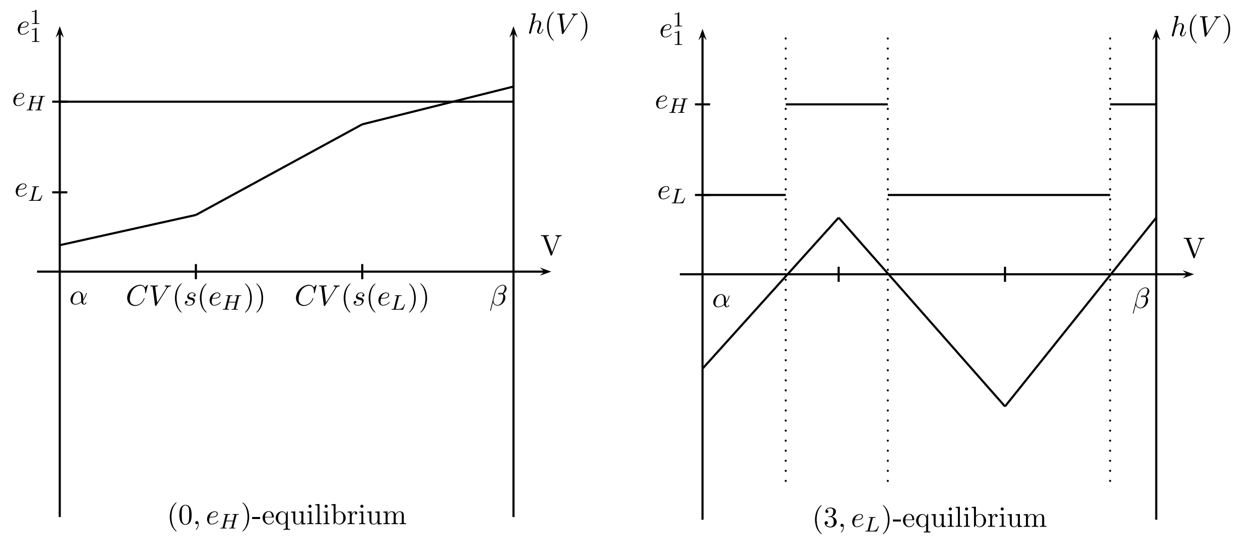

Figure 4: Classes of Perfect Bayesian Equilibria

Besides the two possible pooling equilibria $(0, e_L)$ and $(0, e_H)$, one further class of equilibrium is especially appealing due to its intuitive properties.

Definition 3.2. A PBE of the dynamic game with asymmetric information is said to be *strictly monotonic* if it belongs to the class characterized by $(1, e_L)$.

Strictly monotonic equilibria exhibit one critical value for V in $]\alpha, \beta[$ such that smaller types choose the low and larger types the high effort in the first period. Furthermore:

Definition 3.3. A PBE of the dynamic game with asymmetric information is said to be *monotonic* if it belongs to one of the classes defined by $(0, e_L)$, $(0, e_H)$ or $(1, e_L)$.

A monotonic equilibrium is therefore either strictly monotonic or a pooling equilibrium. It immediately follows that

Corollary 3.4. *Any PBE in which $s(e_L) \leq s(e_H)$ is monotonic.*

Proof. From Lemma 3.1, if $s(e_L) \leq s(e_H)$ the function $h(V)$ is strictly increasing in V. It can therefore have at most one zero point in $[\alpha, \beta]$. Furthermore, if it has one in which $h(V)$ intersects the abscissa, the intersection will be from below. □

Note that $s(e_L) \leq s(e_H)$ is a sufficient but not a necessary condition for monotonicity.

3.2 Monotonic Equilibria

In what follows we restrict attention to monotonic equilibria as defined above. They exhibit the intuitive property that types that have a higher interest in the contest than others do never invest less in the first period.

3.2.1 Pooling Equilibria

Theorem 3.5. *Pooling on e_L.*
If $W \leq W_1^{CV}$ there does exist a PBE with $e_1^1(V) = e_L$ for all $V \in [\alpha, \beta]$ and $e_1^2 = e_L$. The value W_1^{CV} is given by $r^{-1}[(\beta - CV(1))/\beta] \in \Omega_3$, where r^{-1} denotes the inverse of the probability function (2.10). The second stage of the games takes place as described in section 2.2. The equilibrium is supported by any off-equilibrium path belief that implies $s(e_H)$=0.

Proof. See Appendix C □

Proposition 3.5 shows that pooling on the low effort can constitute an equilibrium if on the one hand the maintained lack of information makes player 2 choose e_L with large enough probability in the second period and deviation on the other hand is punished through high effort. Adherence to the first condition is captured by the upper boundary condition on W while the second condition is ensured by the presumed off-equilibrium path belief structure.[17] This illustrates the intuition behind this 'dove' equilibrium: it represents a situation in which even high types V of player 1 understate their interest to induce the opponent to not play hard later.[18]

Theorem 3.6. *Pooling on e_H.*
If $W \leq W_2^{CV}$ and $\alpha \geq \frac{1}{2}\, CV(0)$ there does exist a PBE with $e_1^1(V) = e_H$ for all $V \in [\alpha, \beta]$ and $e_1^2 = e_H$. The value W_2^{CV} is given by $r^{-1}[(CV(0) - \alpha)/\alpha] \in \Omega_3$. The second stage of the games takes place as described in section 2.2. The equilibrium is supported by any off-equilibrium path belief that implies $s(e_L)$=0.

Proof. See Appendix D □

The intuition behind this 'hawk' equilibrium is similar as above. In the present case, however, we assumed a different belief structure to explain what can be interpreted as a typical 'deterrence' situation: all possible types V choose the high effort and pretend to be strong, as this actually intimidates player 2.

The conditions for the existence of such a pooling equilibrium are more restrictive than for the other pooling equilibrium. First, the condition on the relation between α and $CV(0)$ requires that player 1 may not have too small valuations of the prize, because very small types would not be willing to choose

[17]These beliefs are the ones most favorable for existence of the equilibrium. If $s(e_H) \neq 0$, the equilibrium will still exist as long as $s(e_H)$ is small enough for the 'punishment effect' to be present. The boundary condition on W will become more restrictive, however.

[18]Note that the condition $s(e_L) \leq s(e_H)$ is not fulfilled in the described equilibrium since only types $W \in \Omega_2$ would play $s(e_L)$=0. For these types, the described equilibrium does not exist.

the high effort in the first period. Second, under a broad range of parameter assumptions, the critical value that W may not exceed is smaller in the present case than in the case of pooling on e_L.[19] This reflects the fact that pooling on e_L goes with the additional benefit of saving effort.
Note finally that $h'(V) > 0$ holds in the $(0, e_H)$-equilibrium, as illustrated in figure 4, since $s(e_H) \geq s(e_L) = 0$ holds naturally.

3.2.2 Strictly Monotonic Equilibria

Strictly monotonic equilibria can be characterized by a unique value $\tilde{V}_1 \in]\alpha, \beta[$ such that $e_1^1(V) = e_L$ if $V < \tilde{V}_1$ and $e_1^1(V) = e_H$ otherwise. Both information sets (defined by the two possible first period actions of player 1) are therefore on the equilibrium path such that player 2's beliefs can be derived by 'weak consistency'. After having observed $e_1^1 = e_L$, he believes player 1's type to be uniformly distributed on the support $[\alpha, \tilde{V}_1]$. The support of his (uniform) belief if $e_1^1 = e_H$ is $[\tilde{V}_1, \beta]$. Matters become complicated, however, since the two described intervals usually contain types of player 1 with different reaction functions. Appendix E goes into greater detail about this problem. While some general results can be proven, we are not able to derive the strictly monotonic equilibria in a comprehensive fashion.[20] We therefore explore the topic by means of examples in the next section.

4 Examples

4.1 Example 1

The first example gives a case in which only pooling on the low effort constitutes an equilibrium. Neither the other pooling equilibrium nor any strictly monotonic equilibrium do exist. We use this example to show how our findings can be used by a contest designer who can decide on whether to let the opponents meet once or twice.
The following technology parameters are given:

$$p_{HL} = 0.8,\ p_{HH} = p_{LL} = 0.4,\ p_{LH} = 0.1, \tag{4.1}$$

which satisfy (2.1) and (2.2) and the 'asymmetry condition'. It follows that $\Delta p_{H\bullet} = \Delta p_{\bullet L} = 0.4$ and $\Delta p_{L\bullet} = \Delta p_{\bullet H} = 0.3$. Assuming $e_L = 15$ and $e_H = 60$ gives $\Delta e = 45$, $CV(1) = 112,5$ and $CV(0) = 150$. We assume for the valuations $W = 240$, $\alpha = 80$ and $\beta = 320$ and first examine the course of a one-period game with asymmetric information.

Since W lies above the larger critical value $CV(0)$ and therefore in the 'unresponsive' area, player 2 will choose the high effort. The critical value $CV(0)$ will then be decisive for player 1. Types between α and $CV(0)$ occur with probability 0.292. The expected, average valuation of these types, who will choose the low effort, is equal to $(\alpha + CV(0))/2 = 115$. Types between $CV(0)$ and β, who choose the high effort, occur with probability 0.708. Their expected prize valuation will be $(\beta + CV(0))/2 = 235$.

[19] Symmetry $CV(0) - \alpha = \beta - CV(1)$ would be one such assumption.

[20] Among the general results is the fact that no strictly monotonic equilibrium with $\tilde{V}_1 < CV(1)$ does exist.

Putting these figures together, it follows that the expected rent of the one-stage contest equals

$$R^1 = 0.292[p_{LH}115 + (1 - p_{LH})240] + 0.708[p_{HH}235 + (1 - p_{HH})240] \approx 234.93. \tag{4.2}$$

The effort that is spent in expectation is given by

$$E^1 = e_H + 0.292e_L + 0.708e_H \approx 106.86. \tag{4.3}$$

Now turn to the case of a two-stage contest, in which the final winning probability is determined as the average of the winning probabilities of two single contests. Given the notation of the model, this amounts to $a = 0.5$ in equation (3.1). To be able to apply the results from the two-stage model to the current case, the valuations, including the bounds of the support of V, have to be divided by two. This yields $0.5W = 120$, $0.5\alpha = 40$ and $0.5\beta = 160$. The critical values $CV(1)$ and $CV(0)$ remain unchanged but still lie inside the interval $[0.5\alpha, 0.5\beta]$.
First note that the new lower bound $0.5\alpha = 40$ is not larger than half of $CV(0)$ any more, so that a condition for the existence of the $(0, e_H)$-equilibrium is violated (see Proposition 3.6). Furthermore we obtain $q^L = 0.396$ and $q^H = 0.083$ which gives $CV(q^L) = 132.51$ and $CV(q^H) = 145.96$ as the boundaries between the intervals Ω_1, Ω_2 and Ω_3. Therefore, the transformed valuation of player 2 lies in Ω_1 since it is smaller than $CV(q^L)$. The $(0, e_L)$-equilibrium does exist if we use the off-equilibrium path beliefs from Proposition 3.5.[21] As is shown in Appendix F, there does not exist a strictly monotonic equilibrium.
Going through analogous steps as for the one-stage game, we obtain

$$R^2 \approx 226.58 \tag{4.4}$$

and

$$E^2 \approx 77.82. \tag{4.5}$$

To answer the question which of the two options the contest designer will want to choose, one needs to specify his objective. Assume he maximizes the function

$$S(R, E) = bR + cE, \tag{4.6}$$

where b and c are parameters.[22] In the standard context of rent-seeking one would for example set $b = 1$ and $c = -1$, i.e. one assumes that the designer minimizes 'rent dissipation'. Clearly, the two-stage

[21] It is straightforward to show that no participation constraint is violated in this equilibrium, i.e. no player spends more effort in expectation than he expects to earn.

[22] In principle, there might be an objective money value of the rent, for example the value of a professor's salary, that should be included, for example by simply subtracting it. However, since the choice of the size of the prize is not subject of this paper, we take it as given and can as well omit it at this point.

contest would then be preferable. While the expected rent is smaller than in the one-stage game, there is the large benefit of saving effort; the objective takes the value of $S(R^2, E^2) \approx 148.76$ as compared to $S(R^1, E^1) \approx 128.07$. However, in the context of research contests, the designer might have a very different objective. He might want to maximize research effort and might not care about the expected rent of the researchers at all ($b = 0$, $c = 1$). In that case, clearly the one-stage contest would be preferable. Finally, if the valuation of the rent b is normalized to one, the designer will prefer the one-stage contest whenever $c > -0.29$ and the two-stage contest otherwise.

4.2 Example 2

The second example illustrates a case in which a strictly monotonic equilibrium exists, but no pooling equilibrium. The first period action of player 1 therefore constitutes a meaningful signal about his type. We stick to the assumptions about the effort levels and the contest success function from the previous example. The values $CV(1)$ and $CV(0)$ therefore remain unchanged. Furthermore, we assume $\alpha = 40$ and $\beta = 160$ as in the two stage case above. We therefore again have $CV(q^L) \approx 132.51$ and $CV(q^H) \approx 145.96$. However, this time we assume $W = 149 \in \Omega_2$, such that no pooling equilibrium exists. From the general derivations in Appendix E it now follows that a strictly monotonic, separating equilibrium characterized by $\tilde{V}_1 \approx 151.81 > CV(0)$ exists.[23] By definition, types $V < 151.81$ play $e_1^1 = e_L$ and vice versa. Player 2 chooses $e_1^2 = e_H$, which is also derived in the Appendix. The second period mixing probabilities of player 2 are given by $s(e_H) = 0$ and $s(e_L) \approx 0.01$. Accordingly, types $V > \tilde{V}_1$ will then choose $e_2^1 = e_H$ and the action of types $V < \tilde{V}_1$ will depend on their relative position to the critical value induced by $s(e_L)$, which is $CV(0.01) \approx 149.56$. No other strictly monotonic equilibrium exists.[24]
In the described equilibrium, information is revealed in a very intuitive fashion. If player 2 observes e_H, he knows to be confronted with an opponent who has a high valuation and will play hard in the second period as well. This makes him respond with high effort. On the contrary, if he observes e_L, he knows that his opponent's valuation is not that high. By playing an appropriately mixed strategy he makes sure that the probability of facing high effort in the second period is very small (only types V in between 149.56 and 151.81 would do this).
We can again compute expected rents and the amount of effort that is spent in expectation. As before, this is done by considering all possible constellations of types that might meet, taking into account their actions and the resulting success probabilities. We obtain

$$R^2 \approx 288.28 \tag{4.7}$$

[23] Using the notation from Appendix E, we find $W > CV(\mu) \approx 146.66$ and $W \in [CV(\kappa^L), CV(\kappa^H)] = [134.27, 149.20]$. As has been shown in Appendix E, an equilibrium with $\tilde{V}_1 > CV(0)$ exists if $\tilde{V}_1$ is such that it makes player 2 choose the mixing probability $s(e_L) = \frac{CV(1)}{CV(0)} - \frac{CV(1)}{\tilde{V}_1}$ in the second period after having observed e_L in the first period. For this to happen, the beliefs in the equilibrium after having observed e_L have to be such that they make player 2 indifferent between his two pure actions. Given the values from the present example, this can easily be shown to hold.

[24] This follows from the results in Appendix E as well. It can easily be shown that $W = 149$ is always larger than $CV(\theta)$ for all of the remaining strictly monotonic equilibrium candidates, which can be characterized by $CV(1) \leq \tilde{V}_1 \leq CV(0)$. This, however, violates a necessary condition for existence.

and

$$E^2 \approx 156.57. \tag{4.8}$$

We could again interpret the above used valuations as being already transformed, to derive the equilibrium of a one-stage contest with true valuation $2W = 298$ of player 2 and bounds $2\alpha = 80$ and $2\beta = 320$. In this game, player 2 chooses the high effort while the choice of player 1 depends on his relative position to $CV(0) = 150$. This gives an expected rent of

$$R^1 \approx 274.81 \tag{4.9}$$

and expected efforts of

$$E^1 \approx 106.88. \tag{4.10}$$

The fact that $R^2 > R^1$ is not surprising: in a strictly monotonic equilibrium, types with a high interest in the contest choose the high effort level, types with lower interest the low effort level. This makes sure that types with a larger interest win the contest with a higher probability. In this sense, the two-stage contest is the better institution with respect to rent allocation. This goes at the 'expense' of higher effort levels. Therefore, whenever $b, c > 0$, the two-stage contest is preferred by the contest designer; this is the relevant case in research contests. To the contrary, the one-stage contest leads to a higher surplus in the classical sense ($b = 1, c = -1$).

The two examples illustrate the importance of understanding signalling in research contests. For example, whether research funds should be awarded once and for all, or whether there should be an intermediate evaluation before renewal, crucially depends on the type of equilibrium that we expect to arise. The 'danger' of pooling on e_L makes the one-stage option more desirable. However, if different types separate in a strictly monotonic fashion, the two-stage contest with partial information revelation leads to the preferable outcome. This latter case can be expected if the well-known contestant is a very strong player.

5 Conclusions

Contests are games in which players invest non-retrievable effort to increase the probability of winning a rent. In the context of research, contests arise when departments compete for research funds, scientists strive for tenure or firms rival for patents. Research contests usually involve heterogeneous players. This creates asymmetric information and the possibility of signalling. Furthermore, research differs from other applications of contest theory in that a high level of effort invested is not bad but usually desirable, as contest effort is at least in parts also research effort.

Understanding of research contests is not only interesting from a positive, but also from a normative point of view. Mostly, they are deliberately designed by some agency that has to take choices about parameters of the contest, such as the size and division of the rent. Here, we focus on a different choice, namely the time structure of the contest. In several cases, the contest designer can decide between a one- and a two-stage contest, where judging the latter option requires the understanding of signalling.

We model a discrete contest, in which players with differing prize valuations choose one of two possible effort levels. We derive the equilibria of the one-stage game both under perfect and under asymmetric information. An especially interesting case arises if a well-known player with an a priori advantage is challenged by an unknown ‘newcomer’. We believe this to be a relevant situation in research, where contests might even be especially arranged to give upcoming new scientists or research units a chance to prove themselves against established players. In that case, the information contained in the first period actions of a two-stage contest becomes relevant. We prove that eight classes of equilibria can in principle exist. We then focus on the intuitively most appealing case of ‘monotonic’ equilibria in which players with a greater interest in the contest never invest less effort than others in the first period.

We find that both kinds of pooling equilibria can exist, i.e. both the ‘dove’ equilibrium in which all types of the informed player understate their true interest, and the ’hawk’ equilibrium in which they try to intimidate their opponent. Which of the two possibilities will arise crucially depends on the off-equilibrium path believes of the uninformed player. The ‘dove’ equilibrium, however, will occur under a broader range of assumptions as it goes with the benefit of saving effort. Both pooling equilibria will only exist if the valuation of the uninformed player is not too large.
A separating equilibrium in which players with a low valuation choose the low, players with a high valuation the high effort in the first period can also occur. We illustrate in an example that this will happen if the valuation of the uninformed player is large.

For the contest designer who wants to maximize research activity, this implies that a two-stage contest is more attractive if the valuation of the established and well-known player is large; interpreted differently, this is the case if he is a very strong player. Separation will then both lead to higher expected effort levels and to a more efficient allocation of the rent than in the one-stage setting. To the contrary, if the uninformed player is not too strong, the one-stage contest will be preferable, as the possibility of pooling on the low effort makes the two-stage setting less attractive.

The general conditions under which strictly monotonic equilibria exist remain to be derived. This turns out to be sophisticated as the fairly general model assumptions allow for many possible constellations. The problem could possibly be solved by putting more structure on the contest success probabilities and the players’ valuations. Furthermore, five equilibrium classes were not analyzed at all, chiefly because they do not exhibit intuitively appealing properties and might even overstrain the individuals’s rationality.
Future research could examine the contest designer’s problem more rigorously and derive general conditions under which one or the other time structure is preferable. Also, a look at the reverse case in which the privately informed player has the a priori advantage might generate additional insights.

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A Proof of Proposition 2.4

Denote the mixed strategy of player 2 by $\sigma^2 = (r, 1-r)$, where $r \in [0,1]$ is the probability of playing the low effort. The type of player 1 which is then indifferent between playing $e^1 = e_H$ and $e^1 = e_L$ is implicitly determined by

$$[(1-r)p_{HH} + rp_{HL}]V - e_H = [(1-r)p_{LH} + rp_{LL}]V - e_L. \tag{A.1}$$

Solving for V yields the indifferent type

$$\widetilde{V}(r) = \frac{\Delta e}{r\Delta p_{\bullet L} + (1-r)\Delta p_{\bullet H}} = CV(r) \tag{A.2}$$

with $\widetilde{V}(r)$ decreasing in r. Being faced with the mixed strategy $\sigma^2 = (r, 1-r)$, player 1 plays $e^1 = e_L$ if his type is below $\widetilde{V}(r)$ and $e^1 = e_H$ otherwise. Player 2 therefore faces an opponent exerting the low effort with probability $\mu_1 = \frac{\widetilde{V}(r)-\alpha}{\beta-\alpha}$ and a player exerting a high effort with probability $1-\mu_1$. An equilibrium in mixed strategies requires that player 2 is indifferent between playing his two pure strategies. This is the case if

$$[\mu_1(\widetilde{V})(1-p_{LH}) + (1-\mu_1(\widetilde{V}))(1-p_{HH})]W - e_H = [\mu_1(\widetilde{V})(1-p_{LL}) + (1-\mu_1(\widetilde{V}))(1-p_{HL})]W - e_L. \tag{A.3}$$

Solving (A.3) for $\widetilde{V}$ yields

$$\widetilde{V}(W) = \frac{(\alpha-\beta)\Delta e + W(\beta\Delta p_{H\bullet} - \alpha\Delta p_{L\bullet})}{W(\Delta p_{H\bullet} - \Delta p_{L\bullet})}. \tag{A.4}$$

Finally, equating (A.2) with (A.4) and solving for r yields the desired probability function $r(W)$:

$$r(W) = \frac{p_{L\bullet}}{\Delta p_{L\bullet} - \Delta p_{H\bullet}} + \frac{(\Delta e)^2 W}{\Delta e(\alpha-\beta) + W(\beta\Delta p_{H\bullet} - \alpha\Delta p_{L\bullet})}. \tag{A.5}$$

Its derivative with respect to W can be calculated as

$$\frac{\partial r}{\partial W} = \frac{(\Delta e)^2(\alpha-\beta)}{[\Delta e(\alpha-\beta) + W(\beta\Delta p_{H\bullet} - \alpha\Delta p_{L\bullet})]^2} < 0. \tag{A.6}$$

B The Deviation Function

Given the equilibrium strategies of player 2, the effect of switching from $e_1^1 = e_L$ to e_H for player 1 of type V can be decomposed into a first and a second period effect. The first period effect, denoted by $f(V)$, is given by

$$f(V) = [r\Delta p_{\bullet L} + (1-r)\Delta p_{\bullet H}]V - \Delta e. \tag{B.1}$$

The function $f(V)$ is obviously continuous and linearly increasing in V.

The second period effect, denoted by $g(V)$, is more complicated. It depends on the induced change of player 2's second period action from $s(e_L)$ to $s(e_H)$ and on the corresponding optimal adjustment of player 1's second period effort e_2^1.
As a preliminary result, we show which types V will actually change their second period effort in reaction to the changed probability s. After having observed e_L in the first period, player 2 plays the low effort with probability $s(e_L)$ in the second period and therefore makes type $V = CV(s(e_L))$ indifferent between the two effort levels. After the change, player 2 makes the type $V = CV(s(e_H))$ indifferent. Player 1 therefore changes his second period effort if his valuation lies in between the two critical values. It is, however, not yet clear which of the two critical values is larger. Assume first that $s(e_L) = s(e_H)$; the strategy of player 2 in the second round does not vary with what he observes. We now have $CV(s(e_L)) = CV(s(e_H))$ and can conclude that no type will change his second period action. More generally, in this case there will be no second period effect whatsoever and the effect of deviating from low to high effort in the first period boils down to the first period effect $f(V)$.[25] If $s(e_L) < s(e_H)$, we find that $CV(s(e_L)) > CV(s(e_H))$. In this case, types $V \in [CV(s(e_H)), CV(s(e_L))]$ will optimally adjust their second period effort from low to high whenever they deviate from low to high in the first period. All other types will stick to their second period effort. Specifically, no type will invest less in the second period than before. Finally, if $s(e_L) > s(e_H)$, the reverse holds. Types $V \in [CV(s(e_L)), CV(s(e_H))]$ will change their second period effort from high to low while no other types will react.

We can now examine properties of $g(V)$. First, it is continuous. This is only nontrivial at the discussed critical values $CV(s(e_L))$ and $CV(s(e_H))$ at which the first period deviation is accompanied by a discrete second period behavior change. However, behavior changes occur at types who are indifferent between the two actions. Therefore, $\lim_{V' \to V} g(V') = g(V)$ everywhere.

Second, the sign of $g(V)$ can be determined. If $s(e_L) < s(e_H)$, we find that $g(V) > 0, \forall\ V \in [\alpha, \beta]$. This holds since the change from low to high effort induces player 2 to play the low effort with greater probability in the second period, which is unambiguously positive for player 1, independent on whether he adjusts his second period effort or not.[26] If $s(e_L) > s(e_H)$, then $g(V) < 0, \forall\ V \in [\alpha, \beta]$. This is obvious for types V who do not change their second period effort: they unambiguously loose from being faced with high effort with higher probability. To see why $g(V)$ is negative for types $V \in [CV(s(e_L)), CV(s(e_H))]$ as well, consider their second period payoff difference

$$g(V) = [s(e_H)p_{LL} + (1 - s(e_H))p_{LH}]V - e_L - [s(e_L)p_{HL} + (1 - s(e_L))p_{HH}]V + e_H, \tag{B.2}$$

which takes into account their effort adjustment from high to low. (B.2) can easily be shown to be

[25] Formally, this means that $g(V) = 0, \forall\ V \in [\alpha, \beta]$ if $s(e_L) = s(e_H)$.

[26] To see why this is true, assume that the respective player does not change his second period effort. The second period effect is then obviously positive since any player's payoff is increased if his opponent puts less effort into the contest. The possibility of adjusting second period effort will only be used if it further increases player 1's payoff.

negative if

$$V > \frac{\Delta e}{s(e_L)\Delta p_{\bullet L} + (1 - s(e_H))\Delta p_{\bullet H}}. \tag{B.3}$$

The RHS of inequality (B.3) is smaller than $CV(s(e_L))$ such that it is always fulfilled for the relevant types.

Finally, the slope of $g(V)$ is very important. To derive it, it is necessary to explicitly write $g(V)$ in the spirit of (B.2), but for all six possible cases. A first case discrimination results from the possibility that either $s(e_L) > s(e_H)$ or $s(e_L) < s(e_H)$.[27] For each of them, the interval $[\alpha, \beta]$ has to be divided into three subintervals bounded by the critical values for behavior change. It immediately follows that $g(V)$ is linear in V on each of these subintervals. Making use of the 'asymmetry condition' $p_{HL} - p_{LL} > p_{HH} - p_{LH}$ one can also show that $g(V)$ is strictly increasing in V if $s(e_L) < s(e_H)$ and strictly decreasing if $s(e_L) > s(e_H)$. However, $g(V)$ is not linear throughout and therefore not differentiable. The slope of $g(V)$ changes at the values $CV(s(e_L))$ and $CV(s(e_H))$. Furthermore, $g(V)$ is generally neither concave nor convex in V.

We can now state properties of $h(V) = f(V) + g(V)$. First, it is continuous as it is the sum of two continuous functions. Second, since $g(V)$ is not generally differentiable at the two critical values, $h(V)$ is not either.[28] Third, it is linear on each of the three subintervals defined by the critical values, as both $f(V)$ and $g(V)$ are linear there. Fourth, it is strictly increasing in V if $s(e_L) \leq s(e_H)$ as f is strictly increasing and g is nondecreasing in that case. Unfortunately, no such monotonicity condition can be stated if $s(e_L) > s(e_H)$. The function $h(V)$ might be increasing on one and decreasing on another subinterval of $[\alpha, \beta]$, depending on whether the positive slope of f or the negative slope of g dominates on this subinterval.

C Pooling on Low

In the stated equilibrium candidate, player 2 will choose $e_1^2 = e_L$ since we assumed $W \leq CV(0)$ throughout the paper. No information about V will be revealed in the first stage and the second stage of the game will take place as in the one-stage game with asymmetric information. We examine separately the cases of W in Ω_1, Ω_2 and Ω_3.

Assume $W \in \Omega_1$. Player 2 will then choose e_L in the second period ($s(e_L) = 1$), while the choice of player 1 will depend on his valuation of the prize: $e_2^1(V) = e_L$ for $V < CV(1)$ and $e_2^1(V) = e_H$ for $V > CV(1)$. The concept of PBE requires off-equilibrium path beliefs, i.e. player 2's beliefs about V after observing $e_1^1 = e_H$. We assume that this belief will induce him to choose the high effort in the

[27] Note again that $g(V) = 0$ if $s(e_L) = s(e_H)$.

[28] In the special case of $s(e_L) = s(e_H)$, where $h(V) = f(V)$ as mentioned above, $h(V)$ is of course differentiable.

second period ($s(e_H) = 0$), without specifying it explicitly.[29] This implies that $s(e_L) \leq s(e_H)$ is not fulfilled and we can therefore not argue that the deviation function $h(V)$ will be monotone.
However, deviation from the described case is obviously not profitable for $V \in [\alpha, CV(1)]$. Besides the negative payoff effect in the second period, the first period effect is negative as well for such types.[30] The same holds for types $V \in [CV(1), CV(0)]$:

$$\pi = [p_{LL} + p_{HL}]V - e_L - e_H > [p_{HL} + p_{LH}]V - e_H - e_L = \widetilde{\pi}. \tag{C.1}$$

The LHS of inequality (C.1) is the pooling payoff of such types, the RHS equals the payoff in case of deviation. The optimal action in case of deviation is $e_2^1 = e_L$, the best response to a high effort of the opponent. The inequality follows from the fact that $p_{LL} > p_{LH}$. With an analogous argument we find for types $V \in [CV(0), \beta]$:

$$\pi = [p_{LL} + p_{HL}]V - e_L - e_H > [p_{HL} + p_{HH}]V - e_H - e_H = \widetilde{\pi}. \tag{C.2}$$

The RHS of inequality (C.2) again equals the payoff in case of deviation, in which $e_2^1 = e_H$ is optimal. The inequality follows from the fact that $e_H > e_L$. The postulated pooling case is therefore an equilibrium if $W \in \Omega_1$.

Assume $W \in \Omega_3$. Player 2 will play $\sigma_2^2(e_L) = (s(e_L), 1 - s(e_L)) = (r(W), 1 - r(W))$, inducing the critical value $CV(r(W))$ such that smaller types will play $e_2^1 = e_L$ and vice versa. We again check profitability of deviation for all possible types V. The expected payoff in the pooling case for a type $V \in [\alpha, CV(r(W))]$ is

$$\pi = [p_{LL} + rp_{LL} + (1 - r)p_{LH}]V - e_L - e_L. \tag{C.3}$$

The deviation payoff is given by

$$\widetilde{\pi} = [p_{HL} + p_{LH}]V - e_H - e_L \tag{C.4}$$

since types $V \leq CV(r(W)) \leq CV(0)$ respond with a low second period effort in case of deviation. Expression (C.4) is smaller than (C.3) if

$$V < \frac{\Delta e}{\Delta p_{\bullet L} - r\Delta p_{\bullet H}}. \tag{C.5}$$

Therefore, if $CV(r(W))$ is smaller than the RHS of (C.5), none of the examined types will have an

[29] Player 2 has to believe that the probability of facing a high effort in the second period is large enough, to make the high effort an optimal choice for himself as well. First, this seems to be a reasonable belief structure as it simply assumes that player 2 beliefs the strength of his opponent in the contest to be rather high if he observes high effort. Second, it is most likely to support the equilibrium as deviation is followed by a harsh punishment.

[30] This follows directly from the definition of $CV(1)$.

incentive to deviate. Simplification of this condition gives

$$r(W) > 1 - \frac{CV(1)}{CV(0)}. \tag{C.6}$$

Deviation may also not be profitable for types $V \in [CV(r(W)), \beta]$. Their expected pooling payoff is

$$\pi = [p_{LL} + rp_{HL} + (1-r)p_{HH}]V - e_L - e_H. \tag{C.7}$$

When deviating, the payoff becomes

$$\widetilde{\pi} = [p_{HL} + p_{LH}]V - e_H - e_L \tag{C.8}$$

for types $V \in [CV(r(W)), CV(0)]$ and

$$\widetilde{\pi} = [p_{HL} + p_{HH}]V - e_L - e_H \tag{C.9}$$

for types $V \in [CV(0), \beta]$. It is straightforward to show that (C.8) is smaller than (C.7) for all values of $V \in [CV(r(W)), CV(0)]$. (C.9) however is smaller than (C.7) only if $V(1-r) < \Delta e/\Delta p_{\bullet L}$. Deviation is therefore not profitable for any $V \in [CV(0), \beta]$ if it is not profitable for the highest type $V = \beta$. This yields the condition

$$r(W) > 1 - \frac{CV(1)}{\beta}. \tag{C.10}$$

But (C.6) is always fulfilled if (C.10) is fulfilled and only the latter is therefore binding. It states that the probability that player 2 chooses the low effort in the second period has to be high enough to make pooling an equilibrium. If it is not, at least type $V = \beta$ will be harmed more by playing cautiously in the first period than he gains later. The probability $r(W)$ decreases in W, so condition (C.10) is only fulfilled if W is small enough. Inversion of $r(W)$ yields the critical value $W^{CV} \in \Omega_3$. The function r – defined on Ω_3 – is strictly decreasing and therefore invertible. It takes values between zero and one. Since $CV(1) < \beta$, we have $r^{-1}(1 - (CV(1)/\beta)) \in \Omega_3$.

Assume $W \in \Omega_2$. Player 2 will then choose e_H in the second period ($s(e_L) = 0$) and the property $s(e_L) \leq s(e_H)$ holds identically. The deviation function $h(V)$ is therefore strictly increasing as there is no second period effect of deviation. The first period effect $f(V)$ of deviation from e_L to e_H, however, is strictly positive for all types $V > CV(1)$, such that the examined case is not an equilibrium if $W \in \Omega_2$. Note again that the underlying belief structure was the one most likely to support the equilibrium.

D Pooling on High

In the stated equilibrium candidate, player 2 will choose $e_1^2 = e_H$ since we assumed $W \geq CV(1)$ throughout the paper. As no information about V will be revealed, we again examine the cases of W in Ω_1, Ω_2 and Ω_3 separately, using the off-equilibrium path beliefs that are most favorable for the existence of the equilibrium: $s(e_L) = 0$. This immediately implies that $s(e_L) \leq s(e_H)$ and $h(V)$ is therefore strictly increasing. Therefore, the type of player 1 that has the greatest incentive to deviate in the first period is α. Note that in the second period this type will always play the low effort, both under pooling and in case of deviation.

Assume $W \in \Omega_1$ and therefore $e_2^2 = e_L$. Deriving the difference between the resulting payoffs under pooling and deviation, one can easily show that deviation does not pay off if $\alpha > \frac{1}{2}CV(0)$.

Assume $W \in \Omega_3$. Player 2 will play $\sigma_2^2(e_H) = (s(e_H), 1 - s(e_H)) = (r(W), 1 - r(W))$, inducing the critical value $CV(r(W))$. Again, type α will choose $e_2^1 = e_L$ both under pooling and in case of deviation. The condition for not wanting to deviate becomes

$$\alpha > \frac{1}{1+r}\, CV(0), \tag{D.1}$$

Since $r \in [0, 1]$, condition (D.1) requires α to be larger than half of $CV(0)$ for $r = 1$ and is even more restrictive for smaller values of r. Therefore, the restriction $\alpha > \frac{1}{2}CV(0)$ that was derived for the case of $W \in \Omega_1$ and is stated in Theorem 3.6 is always fulfilled in equilibrium if $W \in \Omega_3$. The condition for existence follows from rearrangement of (D.1):

$$r(W) > \frac{CV(0)}{\alpha} - 1. \tag{D.2}$$

Inversion of $r(W)$ gives the critical value W^{CV} in the proposition. Again, $W^{CV} \in \Omega_3$ since $\frac{1}{2}\, CV(0) < \alpha < CV(0)$ and the argument of r^{-1} therefore lies in the interval $[0, 1]$.

Finally assume that $W \in \Omega_2$ and therefore $s(e_L) = 0$. As there is no second period effect of deviation, the type $V = \alpha$ will always want to deviate to $e_1^1 = e_L$ such that pooling on e_H is not an equilibrium.

E Strictly Monotonic Equilibria

Given any equilibrium candidate with $\tilde{V}_1 \in]\alpha, \beta[$, the first period action of player 2 can easily be derived. The uncertainty about his opponent's type makes him face a 'mixed strategy', with probability $\mu = (\beta - \tilde{V}_1)/(\beta - \alpha)$ on the high effort. This induces the critical value $CV(\mu)$ such that he prefers e_L over e_H if $W < CV(\mu)$ and vice versa. His second period beliefs then follow from Bayes Law. After having observed $e_1^1 = e_L$, he believes player 1's type to be uniformly distributed on the support $[\alpha, \tilde{V}_1]$. The support of his uniform belief if $e_1^1 = e_H$ is $[\tilde{V}_1, \beta]$. His optimal second period action then depends on the location of $\tilde{V}_1$. It is convenient to discriminate between three cases: $\tilde{V}_1 < CV(1)$, $CV(1) \leq \tilde{V}_1 \leq CV(0)$ and $CV(0) < \tilde{V}_1$.

First, consider $\tilde{V}_1 < CV(1)$. Player 2 will choose $s(e_L) = 1$ as he knows to be confronted with a low effort in the second period after having observed $e_1^1 = e_L$. This suffices to show that no candidate characterized by such $\tilde{V}_1$ can be an equilibrium. Consider any type $V \in [\tilde{V}_1, CV(1)]$. The first period payoff effect of switching from e_H to e_L will be positive for such a type, by definition of $CV(1)$. The second period effect cannot be negative as $s(e_H) \leq s(e_L) = 1$ will necessarily hold. He therefore has an incentive to deviate.

Consider $CV(0) < \tilde{V}_1$. We immediately obtain $s(e_H) = 0$. Player 2's optimal reaction to having observed $e_1^1 = e_L$ is more complicated. In that case, he cannot predict with certainty his opponent's best response to his own second period action. This gives rise to a consideration exactly in the spirit of Propositions 2.3 and 2.4. It is straightforward to show that he will play as follows:

$$s(e_L) = \begin{cases} 1 & : \quad W \leq CV(\kappa^L) \\ \in]0,1[& : \quad CV(\kappa^L) < W < CV(\kappa^H) \\ 0 & : \quad W \geq CV(\kappa^H) \end{cases} \tag{E.1}$$

with $\kappa^L = (\tilde{V}_1 - CV(1))/(\tilde{V}_1 - \alpha)$ and $\kappa^H = (\tilde{V}_1 - CV(0))/(\tilde{V}_1 - \alpha) < \kappa_L$.
If W is large ($W \geq CV(\kappa^H)$), there will be no second period effect of deviation from the examined equilibrium candidate for player 1. It therefore cannot be an equilibrium since any type $V \in [CV(0), \tilde{V}_1]$ would like to deviate to e_H in the first period, by definition of $CV(0)$. For low values of W ($W \leq CV(\kappa^L)$), the described candidates cannot be an equilibrium either. One can show that all types $V > \tilde{V}_1$ will have an incentive to deviate to $e_1^1 = e_L$, a result that is independent of the effort level of player 2 in the first period. To derive it, simply compare payoffs in the respective equilibrium candidates and in case of deviation, as has been done several times before.
The remaining range of valuations W is $CV(\kappa^L) < W < CV(\kappa^H)$. Only some general results are available for this case. First, if additionally $W > CV(\mu)$, then an equilibrium exists if the valuation W is such that it leads player 2 to choose $s(e_L) = \frac{CV(1)}{CV(0)} - \frac{CV(1)}{\tilde{V}_1}$. This result is found by deriving the conditions under which no type V has an incentive to deviate. It turns out that all types $V > \tilde{V}_1$ do not have this incentive if for them

$$V \geq \frac{\Delta e}{\Delta p_{\bullet H} - s(e_L)\Delta p_{\bullet L}}, \tag{E.2}$$

while no type $V < \tilde{V}_1$ has this incentive if

$$V \leq \frac{\Delta e}{\Delta p_{\bullet H} - s(e_L)\Delta p_{\bullet L}}. \tag{E.3}$$

(E.2) and (E.3) can jointly be fulfilled only if their RHS equals $\tilde{V}_1$. Rearrangement of this conditions yields the above result. It gives a unique value $\tilde{V}_1$ for each valuation W in the admissible interval.
If, on the other hand, $W < CV(\mu)$, the valuation W must be such that player 2 plays $s(e_L) \geq 1 - \frac{CV(1)}{CV(0)}$, a condition that is derived analogously.

Finally consider the candidates $CV(1) \leq \tilde{V}_1 \leq CV(0)$. Both optimal second period strategies $s(e_L)$ and

$s(e_H)$ are sophisticated in that case. With the usual considerations we obtain

$$s(e_L) = \begin{cases} 1 & : \quad W \leq CV(\psi) \\ \in [0,1[& : \quad W > CV(\psi) \end{cases} \tag{E.4}$$

with $\psi = (\tilde{V}_1 - CV(1))/(\tilde{V}_1 - \alpha)$ and

$$s(e_H) = \begin{cases} \in]0,1] & : \quad W \leq CV(\theta) \\ 0 & : \quad W > CV(\theta) \end{cases} \tag{E.5}$$

with $\theta = (\beta - CV(0))/(\beta - \tilde{V}_1)$. Unfortunately, there is no clear relation between ψ and θ. Furthermore, no unambiguous relation exists to the value μ, which is relevant for first period behavior of player 2. This opens up a large amount of different cases that would have to be considered.

It is, however, possible to show that if $W > CV(\mu)$ and therefore $e_1^2 = e_H$, there can only be an equilibrium of the discussed type if $s(e_L) < s(e_H)$. Otherwise, types $V \in [\tilde{V}_1, CV(0)]$ will deviate. From (E.5), a necessary condition for existence is therefore $W \leq CV(\theta)$. We will use this condition in example 2 to show that the respective candidates are no equilibria.

If $W < CV(\mu)$ and therefore $e_1^2 = e_H$, the necessary property is $s(e_L) > s(e_H)$.

F Example 1

This section proves that there does not exist a strictly monotonic equilibrium in the example in section 4.1. We know from Appendix E that equilibria with $\tilde{V}_1 < CV(1)$ do not exist and that equilibria with $CV(0) < \tilde{V}_1$ can only exist if $W \in [CV(\kappa^L), CV(\kappa^H)]$. Here, we obtain $CV(\kappa^L) > 132.5$ for all relevant values $\tilde{V}_1 > CV(0)$ such that the (transformed) valuation of player 2 does not fulfill this condition and only candidates $\tilde{V}_1 \in [CV(1), CV(0)]$ remain to be examined.

With slight abuse of notation denote the transformed valuation of player 2 by $W = 120$ and the transformed valuation of player 1 by V. First note that we obtain $W < CV(\mu)$ for all the remaining values of $\tilde{V}_1$ and therefore $e_1^2 = e_L$ (see first paragraph of Appendix E). It can also be shown easily that $W < CV(\psi)$ always holds. Player 2's optimal choice in response to an observed low effort in the first period is therefore $s(e_L) = 1$ (see (E.4)). The optimal reaction to an observed high effort is more involved. Only for $\tilde{V}_1 > 146.67$ do we obtain $W > CV(\theta)$ and therefore $s(e_H) = 0$ (see (E.5)). Otherwise, the mixing probability $s(e_H)$ can be derived by exploiting the usual 'equalizing property', i.e. be choosing $s(e_H)$ as to make player 2 indifferent between his two pure actions, given the beliefs that are derived by Bayes Law. We obtain $s(e_H) = (33 - 0.225\tilde{V}_1)/(4 + 0.075\tilde{V}_1)$ which induces the critical value $CV(s(e_H)) = 40 + 0.75\tilde{V}_1$ for player 1. Note that $CV(s(e_H)) > \tilde{V}_1$ if $\tilde{V}_1 < 160$, which is fulfilled. Therefore, there are always types $\tilde{V}_1 < V < CV(s(e_H))$ who, in the equilibrium candidates under examination, play $e_1^1 = e_H$ and $e_2^1 = e_L$. In case of deviation ($e_1^1 = e_L$) they would choose $e_2^1 = e_H$

as optimal response to $s(e_L) = 1$. For them, deviation pays off if

$$\pi = [0.9 + 0.3s(e_H)]V - 75 < 1.2V - 75 = \tilde{\pi}, \tag{F.1}$$

which is fulfilled if $s(e_H) < 1$. This in turn is fulfilled for all relevant values of $\tilde{V}_1$, as we have seen above. None of the examined values $\tilde{V}_1$ therefore constitutes an equilibrium.