

Cognitive and Noncognitive Effects of Early Childhood Intervention The case of Perry Program

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Background	Evidence	Beyond Treatment Effects	Conclusion	Appendix

Outline of This Talk

- 1. High/Scope Perry Study Background
- 2. Life Course Impacts of Early Childhood Intervention
- 3. Beyond Treatment Effects: How Early Childhood Intervention works?
 - (A) Explaining Perry: A Parsimonious Model Using Cognitive and Noncognitive Skills
 - (B) Evidence That Perry Operates mainly through its effects on Noncognitive Channels
- 4. Conclusion

Background	Evidence	Beyond Treatment Effects	Conclusion	Appendix
1. Backg	ground: The	Perry Preschool Pre	ogram	

- What: Small Sample Randomized Experiment (123 total children, 58 Treated, 65 Control)
- Where: Conducted in Ypsilanti, Michigan, in the early 1960s.
- Who: Low IQ, Low SES, disadvantage African-American Children
- **Data:** Multiple measurements of life cycle (ages 3-15,19,27,40).

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High/Scope Perry Program

- Early Age: Children were ages 3-5 while treated.
- **Duration:** Program lasted 30 weeks / year mid-October May.
- **Curriculum:** a daily 2-1/2 hour classroom session on weekday mornings.
- Home Visits: Weekly 1-1/2 hour home visits by the teacher.
- **Program cost:** \$9,785 per participant per year (2006 dollars).

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Statistical Challenges and Solutions

- (A) Small sample size
 - Resampling
- (B) **Compromised** Randomization and **Imbalanced** Family Background
 - Account for Perry Randomization Protocol
- (C) "Fishing", biased report of most significant results
 - New *Multiple* hypothesis testing

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2. Lifetime Consequences of the Perry Program

Summary of the Main Findings

Figure 1: Stanford-Binet IQs, Perry Males



• Treatment Effect on IQ fades outs

Figure 2: Stanford-Binet IQs, Perry Females



• Treatment Effect on IQ fades outs

Figure 3: Female and Male Test of Cognitive Skills at 28 Years Old



• Treatment Effects persist on other Cognitive Tests

Figure 4: Female and Male Schooling Achievement



• Treated Females excel on Schooling

Figure 5: Female and Male Employment Status



More Employment and Higher Income

Figure 6: Female and Male Arrests



• Less Crimes for Both Genders

Figure 7: Cost-Benefit Analysis

_	Internal Rates of Return			
_	Females	Males		
Individual	6.2% - 9.9%	3.5% - 7.9%		
Society	7.6% - 11.4%	10.3% - 12.0%		
_				
	Benefit-Cost Ratio			
	Females	Males		
Discount Rate of 3%	4.5 - 11.3	8.2 - 12.7		
Discount Rate of 7%	1.2 - 3.2	2.8 - 4.1		

Crime Costs classified into 9 Categories and Low Murder Cost (4.1 million or 20 thousand). Wage Imputation and Extrapolation methods: Hause Model, NLSY79 Kernel Matching, Various Interpolations. Standard deviations by Bootstrap.

- $\bullet\,$ Internal Rate of Return around $10\%\,$
- Robust estimation of IRR and its standard deviation
- Results are significantly different from zero

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3. Beyond Treatment Effects

- Mechanism through which Early Childhood Interventions work
 - Treatment enhance *Skills*
 - Some Skills are Unobserved abilities
 - Skills can be *proxied* by cog. and noncog. *Tests*
 - Skills affect all Outcomes, but in different ways
 - Treatment effects operates on outcomes *only* through enhancing Skills

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Factor Models

- Skills are Latent Factors
- Treatment improve Skills
- Skills impact Outcomes
- Skills, as cog. and noncog. factors, are proxied by Tests
- Treatment Effects and Skills Impacts are *jointly* estimated using Outcomes and Tests

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Main Fa	ctor Model	Faultion		

$$Y_j = \mu_j(X) + \alpha_j F + \varepsilon_j, j = 1, \dots, J$$

 $F = F_1$ if Treated, a random variable with PDF $g_1(F_1)$

 $F = F_0$ if Control, a random variable with PDF $g_0(F_0)$

- Same factor loadings α_j between treatment status
- Treatment effect operates through the shift of distributions $F_0 \rightarrow F_1$

•
$$F$$
 is a vector: $F = [F^{Cog}, F^{NonCog}]$

• ε_j are mutually independent

Extensions: Big Five, Dynamic Factors F_t , Categorical Latent Models.



Cognitive and Noncognitive Measures

- Cognitive Measures: Stanford-Binet, PPVT, Leitner (ages 4 to 10).
- Noncognitive Measures: YRS (many measures), PBI (many measures), interviewer scales of self-esteem, loneliness, friendliness,leadership...(ages 6 to 19).
- All measure factor loadings are positive and significant at 0.01 level.

Cognitive and Noncognitive Factor Distribution Estimates

Cognitive

Noncognitive



Marriage Duration at age 40 for Males

Combined Effect of Skills



Monthly Income at age 40 for Males

Combined Effect of Skills



Highest Degree Completed at age 19 for Males

Combined Effect of Skills



Months Unemployed at age 27 for Males

Combined Effect of Skills



Total Arrests at age 27 for Males

Combined Effect of Skills



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4. Conclusions Early Childhood Effects through Skills Dynamics

- Strong Evidence that Early Childhood Intervention Matters
- Early Childhood Intervention operates mainly through its effects on Noncognitive Channels
- Cognitive and Noncognitive Skills are positively Correlated
- Neglecting Noncognitive channel *produces* upward biased effect of the Cognitive Ability

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Appendix: Evidence of Program Effects



Empirical Findings: Employment and Income

- All treated individuals experience less unemployment.
- Treated males and females earn more.
- Employment effect for males occurs later than for females.
- The sharpest gender difference in employment outcomes occurs at age 19.

Outcome	Age	Ctl. Mean	Effect	p-val Single	ues Adj.ª
Savings Account	40	0.36	0.37	.001	.007
Car Ownership	40	0.50	0.30	.002	.008
Car Ownership	27	0.59	0.15	.053	.215
Credit Card	40	0.36	0.11	.241	.610
Savings Account	27	0.46	-0.01	.391	.748
Checking Account	40	0.39	0.01	.435	.661
Checking Account	27	0.23	-0.04	.537	.537

Table 1: Economic Activity, Males

Outcome	Age	Ctl. Mean	Effect	p-val Single	u es Adj.ª
Savings Account	27	0.45	0.27	.026	.123
Car Ownership	27	0.59	0.13	.082	.280
Credit Card	40	0.50	0.04	.200	.494
Checking Account	40	0.50	0.08	.214	.493
Car Ownership	40	0.77	0.06	.343	.621
Checking Account	27	0.27	0.01	.382	.580
Savings Account	40	0.73	0.06	.534	.534

Table 2: Economic Activity, Females

Age	Ctl. Mean	Effect	p-values Single Ad	j.ª
19	0.41	0.14	.170 .30	01
19	3.82	-1.47	.778 .83	34
19	0.13	-0.11	.841 .84	41
27	8.79	3.66	.032 .06	68
27	0.31	0.07	.199 .30	09
27	0.56	0.04	.224 .22	24
40 40 40	0.50 10.75 0.46	0.20 3.52 0.10	.011 .02 .017 .02 .067 .06	25 26 67
	Age 19 19 19 27 27 27 27 40 40 40	Age Ctl. Mean 19 0.41 19 3.82 19 0.13 27 8.79 27 0.31 27 0.56 40 0.50 40 10.75 40 0.46	Age Ctl. Mean Effect 19 0.41 0.14 19 3.82 -1.47 19 0.13 -0.11 27 8.79 3.66 27 0.31 0.07 27 0.56 0.04 40 0.50 0.20 40 10.75 3.52 40 0.46 0.10	Age Ctl. Mean Effect p-values Single Ad 19 0.41 0.14 .170 .30 19 3.82 -1.47 .778 .83 19 0.13 -0.11 .841 .84 27 8.79 3.66 .032 .00 27 0.31 0.07 .199 .30 27 0.56 0.04 .224 .22 40 0.50 0.20 .011 .00 40 10.75 3.52 .017 .00 40 0.46 0.10 .067 .00

Table 3: Employment at Ages 19, 27, and 40: Males

Age	Ctl. Mean	Effect	p-values Single Adj.ª
19	0.58	0.34	.000 .001
19	0.15	0.29	.016 .029
19	10.42	5.20	.016 .016
27	0.54	0.29	.023 .044
27	0.55	0.25	.038 .056
27	10.45	4.21	.141 .141
40 40 40	0.41 5.05 0.82	0.25 1.05 0.02	.052 .106 .585 .673 .720 .720
	Age 19 19 19 27 27 27 27 40 40 40	Age Ctl. Mean 19 0.58 19 0.15 19 10.42 27 0.54 27 0.55 27 10.45 40 0.41 40 5.05 40 0.82	Age Ctl. Mean Effect 19 0.58 0.34 19 0.15 0.29 19 10.42 5.20 27 0.54 0.29 27 0.55 0.25 27 0.55 0.25 27 10.45 4.21 40 0.41 0.25 40 5.05 1.05 40 0.82 0.02

Table 4: Employment at Ages 19, 27, and 40: Females

	Model 1	Males Model 2	Model 3	Model 1	Females Model 2	Model 3
Cog. Treatment Shift	_	4.364** 0.035	3.941* 0.051	_	5.916*** 0.008	5.480** 0.012
Non-Cog. Treatment Shift	0.333 0.101	_	0.325 0.109	0.458** 0.030	_	0.503** 0.024
Cog. Non-Cog. Covariance	_	_	7.137*** 0.000	_	_	4.302*** 0.002
Cognitive Factor Variance	_	93.126*** 0.000	92.491*** 0.000	_	65.747*** 0.001	64.730*** 0.001
Non-Cog. Factor Variance	1.121*** 0.000	_	1.134*** 0.000	0.615*** 0.008	_	0.662*** 0.005
Correlation	_	_	0.697	_	_	0.657

Table 5: Treatment Effect Estimation on Factors

Notes: The treatment effect in all models is estimated as shifts in the cognitive and non-cognitive factor distributions. The model is $Y_{i,j} = \chi_i \beta_j + \alpha_j [f_e T_i + F_C (1 - T_i)] + \varepsilon_{i,j}$; The measure Y is indexed by outcome j and observation by i. Treatment label T is 1 for treatment and 0 for control; both labels share factor loading α_j . Single-tailed p-values are presented under the estimated coefficients.

Table 6: Output Factor Loadings for Non-Cog. Factors

Outcome	A	Ma	les	Fema	Females		
Outcome	Age	Model 1	Model 3	Model 1	Model 3		
HS. Graduate	19	0.636*** 0.000	0.415** 0.024	0.962*** 0.001	0.989*** 0.003		
Monthly Earnings	27	0.694*** 0.000	0.698*** 0.008	0.880*** 0.001	1.146*** 0.002		
Monthly Earnings	40	11.453*** 0.000	6.496* 0.077	7.201** 0.038	5.874 0.150		
Marriage Dur.	\leq 40	20.426*** 0.009	13.103 0.157	36.945** 0.011	26.724* 0.098		
Jobless Months in Past 2 Yrs.	27	-4.324*** 0.000	-3.262** 0.023	-5.551*** 0.002	-7.127*** 0.004		
Adult Arrests	≤ 27	-2.591*** 0.000	-2.610*** 0.003	-0.510 0.163	-0.742 0.161		
Felony Arrests	≤ 27	-1.376*** 0.000	-1.422*** 0.004	-0.019 0.431	0.044 0.384		
Total Arrests	≤ 40	-5.140*** 0.000	-5.329*** 0.005	-0.663 0.232	-1.138 0.192		

Notes: The treatment effect in all models is estimated as shifts in the cognitive and non-cognitive factor distributions. The model is $Y_{i,j} = X_i \beta_j + \alpha_j [F_t T_i + F_C (1 - T_i)] + \varepsilon_{i,j}$; The measure Y is indexed by outcome j and observation by i. Treatment label T is 1 for treatment and 0 for control; both labels share factor loading α_j . Single-tailed p-values are presented under the estimated coefficients.

Table 7: Output Factor Loadings for Cog. Factors

Outcomo	100	Mal	es	Fema	Females		
Outcome	Age	Model 2	Model 3	Model 2	Model 3		
HS. Graduate	19	0.071*** 0.000	0.032* 0.077	0.064*** 0.003	-0.009 0.386		
Monthly Earnings	27	0.057*** 0.003	-0.005 0.432	0.041** 0.044	-0.042 0.105		
Monthly Earnings	40	1.358*** 0.000	0.721* 0.078	0.708** 0.025	0.243 0.326		
Marriage Dur.	\leq 40	2.273*** 0.009	1.047 0.237	3.311** 0.010	1.292 0.259		
Jobless Months in Past 2 Yrs.	27	-0.446*** 0.000	-0.145 0.203	-0.268* 0.059	0.249 0.148		
Adult Arrests	≤ 27	-0.227*** 0.000	0.012 0.450	-0.025 0.306	0.035 0.319		
Felony Arrests	≤ 27	-0.118*** 0.001	0.013 0.407	-0.009 0.181	-0.011 0.239		
Total Arrests	≤ 40	-0.443*** 0.001	0.048 0.413	-0.028 0.371	0.065 0.307		

Notes: The treatment effect in all models is estimated as shifts in the cognitive and non-cognitive factor distributions. The model is $Y_{i,j} = X_i \beta_j + \alpha_j [F_t T_i + F_C (1 - T_i)] + \varepsilon_{i,j}$; The measure Y is indexed by outcome j and observation by i. Treatment label T is 1 for treatment and 0 for control; both labels share factor loading α_j . Single-tailed p-values are presented under the estimated coefficients.



Empirical Findings: Cognitive Tests

- The treatment group scores better on both IQ tests (ages 3–10): Stanford-Binet and PPVT.
- On the California Achievement Test, males and females show significant results across multiple ages;
- Female APL effect is stronger than male effect (similar gender pattern).

Cognitive Tests

	Α	ge 19	Ag	ge 27
Measure	Male	Female	Male	Female
Community Resources	.566	.040	.814	.318
Occupational Knowledge	.004	.004	.194	.034
Consumer Economics	.174	.217	.141	.650
Health	.058	.123	.088	.005
Government & Law	.056	.494	.227	.213
ID of Facts and Terms	.175	.115	.690	.135
Reading	.053	.019	.665	.013
Writing	.067	.093	.158	.098
Computation	.068	.439	.244	.664
Problem Solving	.292	.091	.019	.053
Joint Test	.032	.027	.101	.034

Table 8: APL Scores, Ages 19 & 27

p-values are from a one-sided test of treatment effect using Mann-Whitney *U*-statistics, conditioning on maternal employment and paternal presence, and restricting permutations on SES and IQ percentiles, maternal employment, and permuting siblings as a block.

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Empirical Findings: Education

- Three varieties of educational outcome are tested:
 - High-school graduation, GED attainment, and college attendance;
 - Primary and Secondary School Outcomes;
 - and Primary School Problem Indicators.
- All of these outcomes show a pattern of significant results for females at multiple ages and no or few significant results for males.

Education

Table 9: Educational Attainment, Males

Outcome	Age	Ctl. Mean	Effect	p-valu Single	es Adj.ª
Highest Grade Completed	19	11.28	0.08	.292	.582
GPA	19	1.79	0.02	.387	.648
HS Graduation	19	0.51	-0.03	.414	.581
# Years Held Back	\leq 19	0.39	-0.08	.796	.796
Vocational Training Certificate	\leq 40	0.33	0.06	.334	.334

Education

Table 10: Educational Attainment, Females

Outcome	Age	Ctl. Mean	Effect	p-values Single A	¦ ∖dj.ª
HS Graduation GPA Highest Grade Completed # Years Held Back	$19 \\ 19 \\ 19 \\ 19 \\ \leq 19$	0.23 1.53 10.75 0.41	0.61 0.89 1.01 0.20	.000 . .002 . .005 . .075 .	001 005 009 075
Vocational Training Certificate	\leq 40	0.08	0.16	.119 .	119

Table 11: Educational Problems, Males

Outcome	A	Ctl.	Effect	p-value:	p-values	
Outcome	Age	Mean	Ellect	Single	Adj.ª	
Mentally Disabled?	\leq 19	0.33	0.13	.040	.139	
Yrs. in Disciplinary Program	\leq 19	0.42	0.12	.172	.418	
Yrs. of Special Services	≤ 14	0.46	0.04	.257	.433	
Learning Impairment?	\leq 19	0.08	-0.08	.810	.810	

Education

Table 12: Educational Problems, Females

Outcomo	٨٣٥	Ctl.	Effect	p-values	
Outcome	Age	Mean	Ellect	Single Adj.ª	
Mentally Disabled?	\leq 19	0.36	0.28	.010 .034	
Yrs. of Special Services	≤ 14	0.46	0.26	.018 .049	
Learning Impairment?	\leq 19	0.14	0.14	.026 .046	
Yrs. in Disciplinary Program	\leq 19	0.36	0.24	.075 .075	

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Empirical Findings: Crime

- Treatment reduces crime incidence in males and females at all ages.
- Male impact is bigger due to greater crime activity.

_	i anci (a) crime as rifesis					
		Females			Males	
	Treat. Mean	Control Mean	Single Pvalue	Treat. Mean	Control Mean	Single Pvalue
Ever Arrested as a minor	0.00	0.04	0.119	0.03	0.23	0.004
Ever Arrested, Ages 20 to 27	0.48	0.50	0.464	0.61	0.85	0.054
Ever Arrested, Ages 28 to 40	0.52	0.54	0.503	0.79	0.92	0.014
-		Joint Test	0.223		Joint Test	0.013
-						
-		Pa	nel (b) Crime Intensi	ty as Number of Arre	sts	
_		Females			Males	
_	Treat. Mean	Control Mean	Single Pvalue	Treat. Mean	Control Mean	Single Pvalue
Total Arrests at Age 19	0.08	0.79	0.030	1.33	1.46	0.389
Total Arrests at ages 20 to 27	0.28	1.88	0.003	3.03	5.36	0.013
Total Arrests at ages 28 to 40	1.88	2.54	0.298	4.42	6.36	0.078
-		Joint Test	0.006		Joint Test	0.033

Table 13: Crime

Dan I (a) Caime an America

p-Values are computed using the Freedman and Lane (1983) procedure for permutation, conditioning on paternal presence and maternal employment (measured at study entry). Permutations are restricted by these two outcomes as well as quartiles of SES and IQ (measured at study entry).

Figure 8: Cost-Benefit Analysis

Return to	Individual		Society					
Victimization Ratio				arate	Separate		Aggregated	
Murder Victim Cost			Hi	gh	L	DW	L	DW
	Male	Female	Male	Female	Male	Female	Male	Female
Internal Rate of R	eturn							
Min.	3.5%	6.2%	7.9%	15.2%	10.3%	7.6%	9.8%	9.6%
	(2.1%)	(3.5%)	(3.5%)	(5.1%)	(2.6%)	(2.4%)	(3.5%)	(4.1%)
	. ,	, ,						
Max.	7.9%	9.9%	9.3%	16.5%	12.0%	11.4%	11.6%	12.6%
	(2.9%)	(2.5%)	(3.6%)	(1.9%)	(2.4%)	(1.7%)	(3.5%)	(3.0%)
Benefit-Cost Ratio								
w/ 3% discount rat	te							
Min.	-	-	9.6	11.8	8.2	4.5	16.7	6.7
Max.	-	-	14.1	18.6	12.7	11.3	21.2	13.5
w/ 7% discount rate								
Min.	-	-	2.2	4.6	2.8	1.2	5.5	2.2
Max.	-	-	3.5	6.6	4.1	3.2	6.8	4.2

• Robust and Significant Internal Rate of Return

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3. Beyond Treatment Effects How Early Childhood Intervention works?

Estimation of *Skills Enrichment* and the Respective *Outcome Impacts*

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6. Appendix

- A. Reduce dimensionality of hypothesis testing problem
- B. Develop a scientific model for interpreting channels through treatment effects operates
- C. Gives interpretation for estimated factors

Background	Evidence	Beyond Treatment Effects	Conclusion	Appendix
Factor M	lodels			

- Treatment enrich non-observable skills (factors)
- Skills are the channel through which treatment operates on all outcomes
- Skills, as cognitive and non-cognitive factors, operate on all outcomes
- Treatment operates only through shifts in cognitive and noncognitive skills that affect outcomes but not necessarily in the same way
- Skills are the channel through which treatment operates on all outcomes

Figure 9: Full Scale Cognitive Development, ABCDarian Females







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Notation:

- Outcome *j*.
- Y_{0j} is outcome if not treated; Y_{1j} is outcome if treated;
- "0" is for Control (untreated)
- "1" is for Treated

Background	Evidence	Beyond Treatment Effects	Conclusion	Appendix

- $Y_{0j} = \mu_j + \alpha_j f_0 + \varepsilon_j, j = 1, \dots, J$ (for Control)
- $Y_{1j} = \mu_j + \alpha_j f_1 + \varepsilon_j, j = 1, \dots, J$ (for Treatment)
- Treatment effect operates through the shift of distributions $f_0 \rightarrow f_1$ and factor loadings α_j .
- Alternative Representation (Two Level Factor Model with Random Coefficient)

•
$$Y_j = \mu_j + \alpha_j f + \varepsilon_j, j = 1, \dots, J$$

- $f = f_0 + T(f_1 f_0)$ are vectors of factors
- ε_j are mutually independent

Figure 11: Factor Diagram (CFA with Random Coefficient)



Figure 4: PDF of Factors



Notes: Graphs are PDFs of factors for the model $Y_j = X\beta_j + \alpha_j F_D + \epsilon_j$, where α_j is a 1 × 3 vector of factor loadings, F_D is a 3 × 1 vector of factors, which differ for control and treatment group. Factors are modeled as a mixture of two normal distributions. D is the treatment assignment, F_0 denotes control group factors, and F_1 denotes treatment group factors; ϵ_j is an iid error term. The three factors are Cognitive, CNO (Conscientiousness/Neuroticism/Openness), and EA (Extraversion/Agreeableness).



Notes: Graphs are PDFs of model predictions for the model $Y_j = X\beta_j + \alpha_j F_D + \epsilon_j$, Graph (a) shows the distribution of $X\beta_j + \alpha_j F_D$ for the treatment and control cases. Graph (b) shows the distributions of $\alpha_j^{cog} F_D^{cog}$, $\alpha_j^{CNO} F_D^{CNO}$, and $\alpha_j^{EA} F_D^{EA}$. In the model α_j is a 1 × 3 vector of factor loadings, F_D is a 3 × 1 vector of factors, which differ for control and treatment group. Factors are modeled as a mixture of two normal distributions. D is the treatment assignment, F_0 denotes control group factors, and F_1 denotes treatment group factors (in the present model F_1 and F_0 differ by a shift in the mean *delta*—a $k \times 1$ vector of treatment effects); ϵ_j is an iid error term. The three factors are Cognitive, CNO (Conscientiousness/Neuroticism/Openness), and EA (Extraversion/Agreeableness).



(a) Factors Combined

(b) Factors Decomposed

Notes: Graphs are PDFs of model predictions for the model $Y_j = X\beta_j + \alpha_j F_D + \epsilon_j$. Graph (a) shows the distribution of $X\beta_j + \alpha_j F_D$ for the treatment and control cases. Graph (b) shows the distributions of $\alpha_j^{cog} F_D^{cog}$, $\alpha_j^{CNO} F_D^{CNO}$, and $\alpha_j^{EA} F_D^{EA}$. In the model α_j is a 1 × 3 vector of factor loadings, F_D is a 3 × 1 vector of factors, which differ for control and treatment group. Factors are modeled as a mixture of two normal distributions. D is the treatment assignment, F_0 denotes control group factors, and F_1 denotes treatment group factors (in the present model F_1 and F_0 differ by a shift in the mean *delta*—a $k \times 1$ vector of treatment effects); ϵ_j is an iid error term. The three factors are Cognitive, CNO (Conscientiousness/Neuroticism/Openness), and EA (Extraversion/Agreeableness).



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Figure 18: PDF of Treatment Effect on # of Arrests, through Age 40



Notes: Graphs are PDFs of model predictions for the model $Y_j = X\beta_j + \alpha_j F_D + \epsilon_j$, Graph (a) shows the distribution of $X\beta_j + \alpha_j F_D$ for the treatment and control cases. Graph (b) shows the distributions of $\alpha_j^{cog} F_D^{cog}$, $\alpha_j^{CNO} F_D^{CNO}$, and $\alpha_j^{EA} F_D^{EA}$. In the model α_j is a 1 × 3 vector of factor loadings, F_D is a 3 × 1 vector of factors, which differ for control and treatment group. Factors are modeled as a mixture of two normal distributions. D is the treatment assignment, F_0 denotes control group factors, and F_1 denotes treatment group factors (in the present model F_1 and F_0 differ by a shift in the mean *delta*—a $k \times 1$ vector of treatment effects); ϵ_j is an iid error term. The three factors are Cognitive, CNO (Conscientiousness/Neuroticism/Openness), and EA (Extraversion/Agreeableness).



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