Backward integration, forward integration, and vertical foreclosure

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Abstract

I show that partial vertical integtaion may either alleviates or exacebate the concern for vertical forclosure and I examine the circumstances under which it enhances or harms welfare relative to full vertical integration.

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1 Introduction

One of the main antitrust concerns that vertical mergers raise is that the merged entity may wish to foreclose either upstream or downstream rivals. The European Commission defines "foreclosure" as "any instance where actual or potential rivals' access to supplies or markets is hampered or eliminated as a result of the merger, thereby reducing these companies' ability and/or incentive to compete... These instances give rise to a significant impediment to effective competition..."[1] While most of the literature on vertical foreclosure has focused on full vertical mergers, in reality, there are many cases of partial vertical integration, where a firm acquires less than 100% of the shares of a supplier (partial backward integration) or a buyer (partial forward integration). This begs the question of whether partial vertical integtaion alleviates, or rather exacerbates, the concern for vertical forclosure, and if so under which circumstances.

To illustrate, consider the carbonated soft drinks industry. From their inception, the Coca Cola Company (Coke) and PepsiCo (Pepsi) manufactured beverage concentrates and syrups and sold them to authorized "bottlers," which produced and marketed finished beverage products. Beginning in the late 1970s, Coke and Pepsi started to integrate foreward into bottling by acquiring some of their large independent bottlers.[2] Coke formed Coca-Cola Enterprises ("CCE") in 1986 as a publicly owned bottling operation, in which it now owns 34%.[3] Pepsi integrated foreward through the "Pepsi-Cola Bottling Group" ("PBG"), its largest bottler. In 1999, Pepsi spun PGB off, although it retained around a 40% stake in the newly public company.[4] Pepsi also held a stake of around 40% in its second largest bottler, PepsiAmericas (PAS). In 2010, Pepsi fully mergered with PBG and with PAS.[5] Foreward integration by Coke and Pepsi into bottling raises competitive concerns that (i) bottlers that are fully or partially owned by Coke or Pepsi may refuse to bottle

[1] See "Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings," Official Journal of the European Union, (O.J. 2008/C 265/07), at §78. available at *http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:52008XC1018%2803%29:EN:NOT*

[2] For an overview of vertical integration in the carbonated soft drinks industry, see Muris, Scheffman, and Spiller (1992), and Saltzman, Levy, and Hilke (1999).

[3] See The Coca Cola Company, 2009 Annual Report On Form 10-K, available at http://www.thecoca-colacompany.com/investors/pdfs/form_10K_2009.pdf

[4] See http://www.fundinguniverse.com/company-histories/The-Pepsi-Bottling-Group-Inc-Company-History.html

[5] Prior to the merger, Pepsi owned approximately 32% of PBG's outstanding common stock, 100% of PBG's class B common stock and approximately 7% of the equity of Bottling Group, LLC, PBG's principal operating subsidiary. At year-end 2009, Pepsi also owned approximately 43% of the outstanding common stock of PAS. See PepsiCo INC., 10-K, Annual report pursuant to section 13 and 15(d), Filed on 02/18/2011.

rival's carbonated soft drinks, such as Dr. Pepper, Crush, and Schweppes, or will market them less aggressively ("downstream foreclosure") and (ii) Coke and Pepsi will refuse to sell concentrates and syrups to independent bottlers or will sell to independent bottlers at a higher prices or on worse conditions ("upstream foreclosure").[6] The question that I ask in this paper is whether the concerns for upstream and downstream foreclosure are alleviated or exacerbated by the fact that Coke now owns only 34% in CCE (rather than more), and whether these concerns are alleviated or exacerbated by the fact that Pepsi has now fully merged with PBG and PAS.

To address this question and examine the welfare implications of partial vertical integration, I consider a model with a single upstream manufacturer, U, that sells an input to two downstream firms, D_1 and D_2, which use it to produce a final product. There are four stages: in stage 1, D_1 and D_2 invest in order to boost the quality of their final products. In stage 2, D_1 and D_2 simultaneously bargain with U over the price of the input; the resulting input price increases with D_i's investment because investments boosts the expected profit of D_i in the final market. In stage 3, the quality of the final products of D_1 and D_2 is realized, and in stage 4, D_1 and D_2 compete in the final market by seeting prices.

Vertical integration between the U and one of the downstream firms, D_1, creates three effects: (i) following vertical integration, D_1 internalizes the positive externality of its investment on U and hence it invests more, (ii) investments are strategic substitutes so the higher investment of D_1 lowers the investment of D_2, and (iii) following vertical integration, D_2 pays a higher price for the input since it needs to compensate U for the erosion of D_1's downstream profits; this higher input price lowers D_2's profit on the margin and hence weakens D_2's incentive to invest. Downstream foreclosure arises in my model because the larger investment of D_1 and the lower investment of D_2 investment mean that in expectations, D_1 gains market share at D_2's expense. When D_1 holds only a fraction α of U's shares (partial backward integration), D_2 must pay a higher price for the input to ensure that a fraction α of this price compensates D_1 for the erosion in its downstream profit due to competition with D_2. Hence, D_2 invests less than it does under full vertical integration. D_1 in turn invests more due to the fact that investments are strategic substitutes. Overall then, D_2 is more likely to be foreclosed in the downstream market. Under partial forward integration, the opposite is true since U gets only a fraction α in D_1's profit and hence does not fully internalizes the negative effect of D_2 on D_1's profit. Consequently the wholesale price that D_2 pays is lower than

[6] Downstream foreclosure of rival manufacturers is often referred to as "customer foreclosure" and upstream foreclosure of rival downstream clients is often referred to as "input foreclosure."

it is under full vertical integration. In sum, my analysis shows that partial backward integration exacerbates the concern for downstream foreclosure while partial forward integraton alleviates it.

The rest of the paper proceeds as follows: Section 2 discuss the vertical foreclosure literature. Section 3 presents the model and Section 4 characterizes the non intergated equilibrium benchmark. In Section 5, I solve for the equilibrium under full vertical integration and evaluate its welfare effects. In Section 6, I turn to partial backward and partial foreward integration and evaluate their welfare effects. In section 7, I consider a modified version of the model that features two upstream firms and one downstream firm and I examine the effects of upstream foreclosure. Concluding remarks are in Section 8. All proofs are in the Appendix.

2 Related literature

There is a sizeable literature on vertical foreclosure.[7] In this section, I review this literature in order to place my own paper in context. Roughly speaking, there are three main strands of the literature. One strand, pioneered by Ordover, Saloner and Salop (1990) and Salinger (1988), considers models in which the vertically integrated firm deliberately forecloses downstream rivals in order to raise their costs and thereby boost the profits of its own downstream unit. Ordover, Saloner, and Salop (1990), consider a model with two identical upstream firms U_1 and U_2 and two downstream firms D_1 and D_2. Following vertical integration between U_1 and D_1, the merged entity commits not to sell to D_2. While this commitment hurts U_1's upstream profit, it boosts the downstream profit of D_1 because U_2 is now the exclusive supplier of D_2, and hence it charges D_2 a higher wholesale price. This makes D_2 softer in the downstream market.[8] Salinger (1988) obtains a similar result in

[7] See Rey and Tirole (2007) and Riordan (2008) for literature surveys.

[8] The assumption that U_1 can commit not to supply D_2 following integration with D_1 was criticized as being problematic: see Hart and Tirole (1990) and Reiffen (1992), and see Ordover, Salop, and Saloner (1992) for a response. Several papers have proposed models that are immune to this criticism. Ma (1997) shows that when U_1 and U_2 offer differentiated inputs, it is in U_1's interest, once it integrates with D_1, to foreclose D_2 (U does not have to commit ex ante to foreclose). He shows that the foreclosure allows D_1 to monopolize the downstream market, although the resulting welfare implications are ambiguous. Chen (2000) shows that when D_1 and D_2 can choose which upstream firm to buy from, then once U_1 and D_1 integrate, D_2 will choose to buy from U_1 (even if it charges a higher wholesale price than U_2) because this choice induces D_1 to be less aggressive in the downstream market in order to protect D_2's sales and hence U_1's profits from selling to D_2. The result then is a de facto foreclosure of U_2. Choi and Yi (2001) assume that U_1 and U_2 need to choose which input to produce. Absent integration, U_1 and U_2 choose to produce a generalized input that fits both firms, but following integration with D_1, U_1 produces a specialized input that fits only D_1. This de facto foreclosure of D_2 allows U_2 to charge D_2 a higher wholesale price and confers a

a successive Cournot oligopoly model. He shows vertical integration between one upstream and one downstream firm creates two conflicting effects: on the one hand, vertical integration eliminates double marginlaization within the integrated entity and boosts its downstream output. On the other hand, the integrated upstream firm stops selling to nonintegrated downstream firms, and hence, these firms end up paying a higher wholesale price and therefore cut their output levels.[9] My model differs from these papers in several important respects: first, I consider a model with a single upstream firm. Second, in my model there is a unit demand function for the final product, so there is no double marginalization problem. Third, foreclosure in my model is a by-product of the effect of vertical integration on the incentives of D_1 and D_2 to invest, rather than an outright refusal to sell to non-integrated rivals. In fact, in my model D_2 continues to buy from U even when the latter integrates with D_1.[10]

Building on the logic of the raising rivals' costs models of foreclosure, Baumol and Ordover (1994) argue that partial backward integration can lead to foreclosure even when full vertical integation does not. Specifically, they argue that under full integration between a bottleneck owner, B, and one of several competing downstream firms, V, B will continue to deal with V's downstream rivals so long as this is efficient. But when V controls B with a partial ownership stake, then it has an incentive to divert business to itself, even if downstream rivals are more efficient. Doing so entails a loss of profits to B, which V only partially internalizes, and allows V to increase its downstream profit. Extending this logic implies that whenever downstream firm D has a controlling stake of less than 100% in upstream firm U (partial backward integration), then it has a stronger incentive to foreclose downstream rivals and thereby shift profits from U, where it owns less than 100%, to D. Conversely, if U has a controlling stake of less than 100% in D (partial foreward integration), then

strategic advantage on D_1 in the downstream market. Church and Gandal (2000) study vertical integration between a hardware and a software firm, and show the integrated firm may choose to make software which is incompatible with the hardware of the nonintegrated hardware firm. If both hardware firms still have positive market shares, this foreclosure harms consumers.

[9] Gaudet and Van Long (1996) show that the intergrated firm may in fact wish to buy inputs from nonintegrated upstream suppliers in order to further inflate the wholesale price that nonintegrated downstream rivals pay. This strategy increases the integrated firm's strategic advantage in the downstream market. Riordan (1998) shows that backward vertical integration by a dominant firm into an upstream competitive industry reduces its monopsonistic power in the upstream market and hence leads to a higher input price. This price increase hurts downstream rivals and leads to a higher retail price in the downstream market. Loertscher and Reisinger (2010) consider a similar model and show that if the downstream firms are Cournot competitors, then, under fairly general conditions, vertical integration is procompetitive because efficiency effects tend to dominate foreclosure effects.

[10] Since U always deals with D_2, my model does not feature a "committment problem."

it has a weaker incentive to shift profits from U to D by foreclosing downstream rivals. Relative to full vertical merger then, partial backward integration exacerbates the concern for downstream foreclosure while partial foreward integration alleviates this concern. Similarly, noting that foreclosure of upstream rivals shifts profits from D (which now earns less from dealing with U's rivals) to U (which now enjoys a strategic advantage over upstream rivals), partial backward integration alleviates the concern for upstream foreclosure, while partial foreward integration exacerbates this concern.[11]

A second strand of the literature, due to Hart and Tirole (1990), views foreclosure as an instrument that allows U to extract monopoly profits from the downstream market. Specifically, Hart and Tirole (1990) consider a setting where U faces two competing downstream firms, D_1 and D_2. Ideally, U would like to supply only one downstream firms, say D_1, in order to eliminate competition downstream. If U can use a two-part tariff, it can then extract the entire monopoly downstream profits from D_1 via a fixed fee. However, D_1 fears that after it accepts the two-part tariff, U will secretely sell to D_2 and thereby make even more money at D_1's expense. Hart and Tirole show that due to this fear, U cannot make more than the duopoly profit in a nonintegrated equilibrium. But if U integrates with D_1, it can credibly commit not sell with D_2 as such sales erode its downstream profit. Hence, integration leads to a foreclosure of D_2 and to a higher retail price.[12] This theory differs from mine because, as in the first strand of the literature, it also views foreclosure as a deliberate refusal to sell to D_2 in order to boost the downstream profit of D_1.

My paper is closely related to the third strand of the literature, due to Bolton and Whinston (1991, 1993). This strand shows that foreclosure can be a by-product of the effect of vertical

[11] Reiffen (1998) builds on this logic and examines the stock market reaction to Union Pacific (UP) Railroad's attempt in 1995 to convert a 30% nonvoting stake in Chicago Northwestern (CNW) Railroad to voting shares. A group of competing railroads argued that since the remaining 70% of CNW's shares were held by dispersed shareholders, UP would gain effective control over CNW, and would use it to foreclose them from some of CNW's transportation routes. Reiffen finds however that CNW's stock price reacted positively, rather than negatively, to events that made the merger more likely to take place. This is inconsistent with the idea that UP would have diverted profits from CNW to itself by foreclosing competing railaroads.

[12] Baake, Kamecke, and Normann (2003), consider a related model in which U faces $n \geq 2$ downstream rivals and needs to make a cost-reducing investment before offering contracts to the downstream firms. They show that vertical integration between U and one of the downstream firms leads to downstream foreclosure, which is ex post inefficient, but it induces U to invest efficiently ex ante. Vertical integration is welfare enhancing in their model when n is suficiently large. White (2007) shows that when U's cost is private information, U has a strong incentive to signal to D_1 and D_2 that its cost is high (and consequently that sales to the rival is limited) by cutting its output below the monopoly level. Vertical integratation restores the monopoly output and hence is welfare enhancing.

integration on the incentives of downstream firms to invest rather than a deliberate refusal to sell by the upstream firm. Bolton and Whinston consider a setting with one upstream firm, U, and two downstream firms, D_1 and D_2, which do not compete with each other downstream. Rather, with some probability, there is excess demand for the upstream input, so D_1 and D_2 compete for a limited input supply. The two firms invest ex ante in order to boost their profits from using the upstream input. Following integration between U and D_1, D_1 internalizes the externality of its investment on U's profit and hence it invests more. Since investments are strategic substitutes, D_2 invests less. In equilibrium then, D_2 is less likely to buy the input whenever there is supply shortage. My model builds on Bolton and Whinston, but unlike in their model , there is no supply shortage in my model, and the strategic interaction between D_1 and D_2 arises because the two firms compete in the downstream market. Moreover, integration in my model affects the wholesale price that D_2 pays and hence creates a new effect that is not present in Bolton and Whinston.

Similarly to my model, Allain, Chambolle, and Rey (2010) also consider a model in which two competing downstream firms first invest in order to boost the value of their final product, and then buy a homogenous input in an upstream market. Unlike in my model, there are two upstream firms in their model and moreover, in order to buy the input, a downstream firm needs to share technical information with its upstream supplier. As a result, D_2 may hesitate to deal with U_1 when the latter is integrated with D_1, because U_1 may leak some of D_2's technical information to D_1 and thereby diminish D_2's potential advantage in the downstream market. The result is equivalent to a de facto foreclosure of D_2 and it weakens its incentive to invest; consequently, vertical integration harms consumers and reduces total welfare.

There is some empirical evidence for the foreclosure effect of vertical mergers. Waterman and Weiss (1996) find that relative to average nonintegrated cable TV systems, cable systems owned by Viacom and ATC (the two major cable networks that had majority ownership ties in the four major pay networks, Showtime and the Movie Channel (Viacom) and HBO and Cinemax (ATC)) tend to (i) carry their affiliated networks more frequently and their rival networks less frequently, (ii) offer fewer pay networks in total, (iii) "favor" their affiliated networks in terms of pricing or other marketing behavior. Chipty (2001) finds that integrated cable TV system operators tend to exclude rival program services, although vertical integration does not seem to harm, and may actually benefit, consumers because of the associated efficiency gains. Hastings and Gilbert (2005) find evidence for vertical foreclosure in the U.S. gasoline distribution industry by showing that a vertically integrated refiner (Tosco) charges higher wholesale prices in cities where it competes more with independent gas stations.

To the best of my knowledge, apart from Baumol and Ordover (1994) and Reiffen (1998), only Greenlee and Raskovitch (2006) and the FCC (2004) consider the competitive effects of partial vertical integration. Greenlee and Raskovitch (2006) consider n downstream firm which buy an input from a single upstream supplier, U, and hold partial passsive ownership stakes in U. An increase in the ownership stake of downstream firm i in U, means that i pays a larger share of the input price to "itself" and hence demands more input. U responds to the increased demand by raising the input price. Greenlee and Raskovitch show that in a broad class of homogeneous Cournot and symmetrically differentiated Bertrand settings, the two effects cancel each other out, so aggregate output and consumer surplus remain unaffected. In my model by contrast, partial backward integration affects consumers in general because it changes the incentives of the downstream firms to invest and therefore the likelihood that coneumers will be able to buy high quality products at low prices.

Finally, in its review of News Corp.'s acquisition of a 34% stake in Hughes Electronics Corporation in late 2003, the FCC (2004) has advanced another theory on the foreclosure effect of partial vertical integration. The acquisition gave News Corp. (a major owner of TV broadcast stations and national and regional cable programming networks) a de facto control over Hughes's wholly-owned subsidiary DirecTV Holdings, LLC, which provides direct broadcast satellite service in the U.S. The FCC argued that News Corp.'s ability to gain programming revenues via its ownership stake in DirecTV would make it easier for News Corp. to temporarily foreclose, or threaten to foreclose, competing cable TV operators during carriage negotiations (i.e., temporarily withdraw regional sports programming networks and local broadcast television station signals) and thereby secure higher prices for its programming. The FCC was concerned that higher programming rates would likely harm consumers by leading to higher prices for cable TV services.

3 The Model

Consider two downstream firms, D_1 and D_2, that purchase an input from an upstream supplier U and use it to produce a final product. Each downstream firm is a monopoly in one market, where its revenue is $\overline{R}$. [how do I use $\overline{R}$?] In addition, the downstream firms compete in a third market where they face a unit mass of identical final consumers, each of whom is interested in buying at most one unit. The utility of a final consumer if he buys from D_i is $V_i - p_i$, where V_i is the quality of the final product and p_i is its price. If a consumer does not buy, his utility is 0.

I assume that initially $V_1 = V_2 = \underline{V}$. By investing, D_i can try to increase V_i to $\overline{V}$; the probability that D_i succeeds to raise V_i to $\overline{V}$ is q_i. The cost of investment is increasing and convex. To obtain closed form solutions, I will assume that the cost of investment is given by $\frac{kq_i^2}{2}$, where $k > \overline{V} - \underline{V} \equiv \Delta$.[13] The total cost of each D_i is then equal to the sum of $\frac{kq_i^2}{2}$ and the price that D_i pays U for the input. The upstream supplier U incurs a constant cost c if it serves only one downstream firm and $2c$ if it serves both downstream firms. To avoid uninteresting cases where foreclosure arises because the cost of serving a downstream firm is too high, I will assume that $0 \leq c < \overline{R} - \underline{V}$.

The sequence of events is as follows:

- **Stage 1:** D_1 and D_2 simultaneously choose how much to invest in the respective qualities of their final products.
- **Stage 2:** Given q_1 and q_2, the two downstream firms buy the input from U. The price that each D_i pays U is determined by bilateral bargaining. Following Bolton and Whinston (1991) and Rey and Tirole (2007), I will assume that the bargaining between D_i and U is such that with probability 1/2, D_i makes a take-it-or-leave-it offer to U, and with probability 1/2, U makes a take-it-or-leave-it offer to D_i.
- **Stage 3:** The qualities of the final products of D_1 and D_2 are realized and become common knowledge.
- **Stage 4:** D_1 and D_2 simultaneously set their prices, p_1 and p_2.

4 The nonintegrated equilibrium

Since p_1 and p_2 are set simultaneously after D_1 and D_2 have already sunk their costs (the cost of investment in quality and the cost of the input), the Nash equilibrium prices are equal to 0 if $V_1 = V_2 = \overline{V}$ or $V_1 = V_2 = \underline{V}$ and to $p_i = \overline{V} - \underline{V} \equiv \Delta$ and $p_j = 0$ if $V_i = \overline{V}$ and $V_j = \underline{V}$.[14] Together

[13] The assumption that the cost function is quadratic is only made for convinience. All the results go through for any increasing and convex cost function that satisfies appropriate restrictions (needed to ensure that the equilibrium is well-behaved). The assumption that $k > \Delta$ ensures that the equilibrium values of q_1 and q_2 do not exceed 1 (q_1 and q_2 are probabilities).

[14] To simplify matters, I assume that when indifferent, consumers buy from the high quality firm. If $V_1 = V_2$, consumers randomize between D_1 and D_2.

with the revenue $\overline{R}$ that each downstream firm makes in its monopoly market, the downstream revenues of D_1 and D_2 are summarized in the following table (the left entry in each cell is D_1's revenue and the right entry is D_2's revenue):

Table 1: The downstream revenues

	$V_2 = \overline{V}$	$V_2 = \underline{V}$
$V_1 = \overline{V}$	$\overline{R}, \overline{R}$	$\overline{R} + \Delta, \overline{R}$
$V_1 = \underline{V}$	$\overline{R}, \overline{R} + \Delta$	$\overline{R}, \overline{R}$

Notice that D_i makes a positive revenue Δ in the competitive downstream market only when $V_i = \overline{V}$ and $V_j = \underline{V}$ (D_i succeeds to raise V_i to $\overline{V}$ while D_j fails); the probability of this event is $q_i (1 - q_j)$. The variable Δ reflects the premium that D_i gets when it is the sole provider of high quality in the competitive downstream market. Also notice that with probability $\phi_i \equiv q_j (1 - q_i)$, $V_i = \underline{V}$ and $V_j = \overline{V}$, in which case D_i makes no sales in the competitive downstream market. Hence, ϕ_i can serve a measure of "downstream foreclosure."[15]

Next, consider stage 2 of the game in which each D_i bargains with U over the input price. When D_i makes a take-it-or-leave-it offer to U, it offers a price c for the input which is the minimal price that U will accept. When U makes a take-it-or-leave-it offer, it offers a price equal to the entire expected revenue of D_i, which is $q_i (1 - q_j) \Delta + \overline{R}$.[16] The expected price that D_i pays for the input is therefore

$$w_i^* = \frac{q_i (1 - q_j) \Delta + \overline{R} + c}{2}.$$

Given w_i^* and given a pair of investments in quality, q_i and q_j, the expected profit of D_i is

$$\begin{aligned} \pi_i &= q_i (1 - q_j) \Delta + \overline{R} - w_i^* - \frac{kq_i^2}{2} \\ &= \frac{q_i (1 - q_j) \Delta + \overline{R} - c}{2} - \frac{kq_i^2}{2}. \end{aligned}$$

In stage 1 of the game, D_1 and D_2 choose q_1 and q_2 to maximize their respective profits. The best response functions of D_1 and D_2 are defined by the following pair of first order conditions:

$$\pi_i' = \frac{(1 - q_j) \Delta}{2} - kq_i = 0, \qquad i = 1, 2. \tag{1}$$

[15] Notice that foreclosure in my model is not a "refusal to deal" - rather I identify foreclosure with the diminished expected sales of the nonintegrated downstream firm.

[16] The assumption that $c < \overline{R} - \underline{V}$ ensures that $q_i (1 - q_j) \Delta + \overline{R} > c$, so U earns a positive profit on the sale of the input. Without this assumption, D_i would not be able to purchase the input whenever $q_i (1 - q_j)$ is small and this would introduced uninteresting technical complications that I am able to avoid by assuming that $c < \overline{R} - \underline{V}$.

The equilibrium levels of investment are defined by the intersection of the two best-response functions and are given by

$$q_1^* = q_2^* = \frac{\Delta}{\Delta + 2k}. \tag{2}$$

Figure 1 illustrates the best-response functions of D_1 and D_2 and the Nash equilibrium levels of investment.

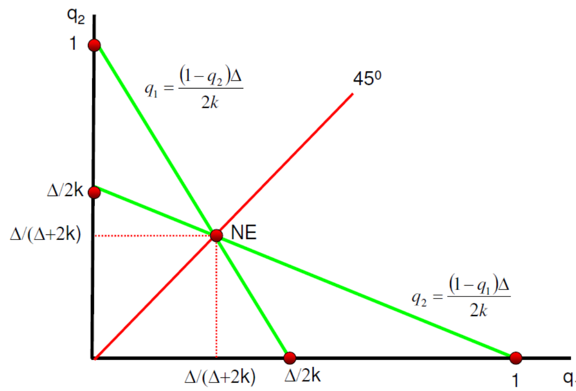

Figure 1: The Nash equilibrium levels of investment under non integration

Using (2), the probability that D_i makes no sales in the competitive downstream market is

$$\phi_i^* \equiv q_j^* \left(1 - q_i^*\right) = \frac{2\Delta k}{\left(\Delta + 2k\right)^2}. \tag{3}$$

5 The vertically integrated equilibrium

Suppose that D_1 and U merge and choose the strategy of the vertically integrated entity, VI, to maximize their joint profit. The merger does not affect the outcome in stages 3 and 4 of the game; in particular, the downstream revenues are still given by Table 1.

Moving to stage 2 in which VI and D_2 bargain over the input price, note that when D_2 makes a take-it-or-leave-it offer, it offers an input price, w, that leaves VI indifferent between selling to D_2 and foreclosing it:

$$\underbrace{q_1\overline{V} + \left(1 - q_1\right)\underline{V} + \overline{R} - c}_{VI\text{'s profit if } D_2 \text{ is foreclosed}} = \underbrace{q_1\left(1 - q_2\right)\Delta + \overline{R} + w - 2c}_{VI\text{'s profit if it sells to } D_2} \quad \Rightarrow \quad w = q_1 q_2 \Delta + c + \underline{V}.$$

D_2 is willing to make this offer since its resulting expected profit is $q_2\left(1 - q_1\right)\Delta + \overline{R} - w = q_2\left(1 - 2q_1\right)\Delta + \overline{R} - \underline{V} - c$, which is positive since, as we shall see later, in equilibrium $q_2\left(1 - 2q_1\right) \geq$

0, and since by assumption, $c < \overline{R} - \underline{V}$. When VI makes a take-it-or-leave-it offer, it offers $q_2(1-q_1)\Delta + \overline{R}$, which is equal to the entire expected revenue of D_2. The expected input price that D_2 will pay U is therefore

$$w_2^{VI} = \frac{q_2(1-q_1)\Delta + \overline{R}}{2} + \frac{q_1 q_2 \Delta + \underline{V} + c}{2} = \frac{q_2\Delta + \overline{R} + \underline{V} + c}{2}.$$

Notice that if we hold q_1 and q_2 fixed, then $w_2^{VI} > w_2^*$: following the integration of D_1 and U, D_2 pays U a higher price for the input. The reason is that U's reservation payoff absent vertical integration is c, whereas under vertical integration it is $c + q_1q_2\Delta + \underline{V}$, where $q_1q_2\Delta + \underline{V}$ represents the erosion of D_1's downstream profit due to competition with D_2. D_2 must compensate U for this amount to induce it to sell the input, since following integration, U internalizes the negative competitive externality it imposes on D_1 when it deals with D_2.[17]

Given w_2^{VI}, the expected profits of VI and D_2 are

$$\pi_{VI} = \underbrace{q_1(1-q_2)\Delta + \overline{R} - \frac{kq_1^2}{2}}_{\text{Downstream profit}} + \underbrace{w_2^{VI} - 2c}_{\text{Upstream profit}},$$

and

$$\begin{aligned}\pi_2 &= q_2(1-q_1)\Delta + \overline{R} - w_2^{VI} - \frac{kq_2^2}{2} \\ &= \frac{q_2(1-2q_1)\Delta + \overline{R} - c - \underline{V}}{2} - \frac{kq_2^2}{2}.\end{aligned}$$

The equilibrium investment levels under vertical integration, q_1^{VI} and q_2^{VI}, are defined by the following pair of first order conditions:

$$\pi'_{VI} = (1-q_2)\Delta - kq_1 = 0, \tag{4}$$

and

$$\pi'_2 = \frac{(1-2q_1)\Delta}{2} - kq_2 = 0. \tag{5}$$

Notice from (5) that $q_2 = 0$ whenever $q_1 \geq 1/2$; hence $q_1 < 1/2$ in every interior equilibrium, implying that $q_2(1-2q_1) \geq 0$, as I have assumed above.

[17] Note that if the bargaining between VI and D_2 was asymmetric in the sense that VI made a take-it-or-leave offer with probability $\gamma \neq 1/2$ and D_2 made a take-it-or-leave offer with probability $1-\gamma$, then w_2^{VI} would be equal to $\gamma\left(q_2\Delta + \overline{R}\right) + (1-2\gamma)q_1q_2\Delta + (1-\gamma)(c + \underline{V})$. Here, w_2^{VI} increases with q_1 if $\gamma < 1/2$ and decreases with q_1 if $\gamma > 1/2$, so in choosing q_1, VI would also take into account its effect on w_2^{VI} and would invest more if $\gamma < 1/2$ and invest less if $\gamma > 1/2$.

The best response functions of the integrated firm VI and of D_2 are illustrated in Figures 2 and 3. Figure 2 shows the best response functions when $\Delta < \frac{k}{2}$ (D_i gets a limited premium from being the sole provider of high quality in the competitive downstream market). In this case, the Nash equilibrium is interior. The best response functions in the non-integrated case are shown by the dotted lines.

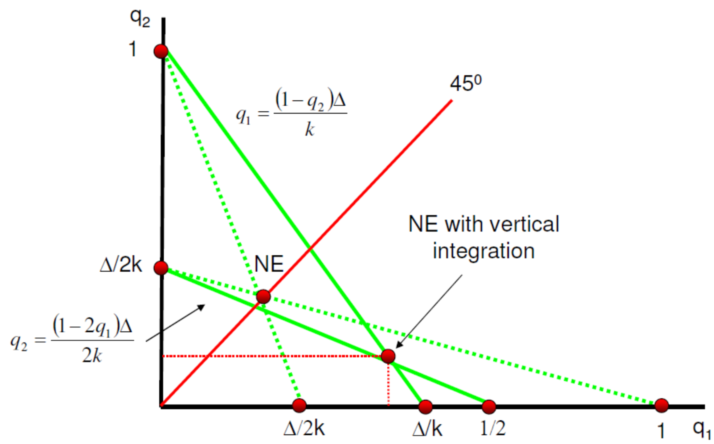

Figure 2: The Nash equilibrium levels of investment under vertical integration - an interior equilibrium

Notice that relative to the non-integration case, the best-response function of D_1 rotates counterclockwise around its vertical intercept, while the best-response function of D_2 rotates clockwise around its vertical intercept. The intuition for these rotations is as follows: absent vertical integration, U captures some of the benefits from D_i's investment. As a result, both D_1 and D_2 underinvest in quality. Vertical integration induces D_1 to internalize the positive externality of its investment on U's profit. The counterclockwise rotation in D_1's best-response function reflects this internalization of the investment externality. The clockwise rotation in the best response of D_2 reflects the increase in the input price that D_2 pays U, which, as mentioned earlier, is due to the fact that under vertical integration, U internalizes the negative externality that the input sale to D_2 imposes on D_1's downstream profit.

Figure 3 shows that when $\Delta \geq \frac{k}{2}$ (D_i gets a large premium from being the sole provider of high quality in the downstream market), the rotations of the two best response functions are relatively strong, so the best-response function of D_1 now lies everywhere above the best-response function of D_2. The resulting Nash equilibrium is such that $q_2^{VI} = 0$.

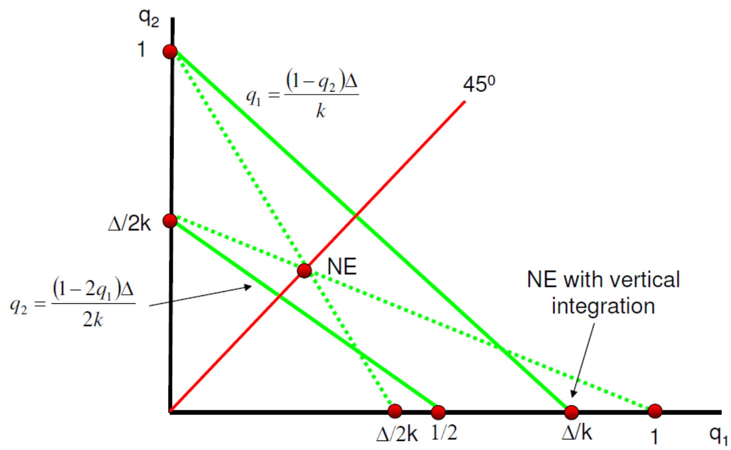

Figure 3: The Nash equilibrium levels of investment under vertical integration - firm 2 does not invest

Solving (4) and (5), the equilibrium levels of investment are

$$q_1^{VI} = \begin{cases} \frac{\Delta(2k-\Delta)}{2(k^2-\Delta^2)} & \text{if} \quad \Delta < \frac{k}{2}, \\ \frac{\Delta}{k} & \text{if} \quad \Delta \geq \frac{k}{2}, \end{cases} \tag{6}$$

and

$$q_2^{VI} = \begin{cases} \frac{\Delta(k-2\Delta)}{2(k^2-\Delta^2)} & \text{if} \quad \Delta < \frac{k}{2}, \\ 0 & \text{if} \quad \Delta \geq \frac{k}{2}. \end{cases} \tag{7}$$

It is easy to check that $q_1^{VI} > q_i^* > q_2^{VI}$: following vertical integration, D_1 invests more while D_2 invests less. This result is due to a combination of 3 effects: (i) following vertical integration, D_1 internalizes the positive externality of its investment on U and hence it invests more, (ii) investments are strategic substitutes so the higher investment of D_1 lowers the investment of D_2, and (iii) following vertical integration, D_2 pays a higher price for the input and hence makes a smaller profit on the margin; this in turn lowers D_2's benefit from investing.

Given q_1^{VI} and q_2^{VI}, the probability that D_2 makes no sales in the competitive downstream market is

$$\phi_2^{VI} \equiv q_1^{VI}\left(1 - q_2^{VI}\right) = \begin{cases} \frac{\Delta k(2k-\Delta)^2}{4(k^2-\Delta^2)^2} & \text{if} \quad \Delta < \frac{k}{2}, \\ \frac{\Delta}{k} & \text{if} \quad \Delta \geq \frac{k}{2}. \end{cases} \tag{8}$$

Notice that ϕ_2^{VI} is continuous, increases with Δ, equal to $\frac{1}{2}$ when $\Delta = \frac{k}{2}$ and equal to 1 when $\Delta = k$. Comparing (8) with (3) reveals that $\phi_2^{VI} > \phi_2^*$: D_2 is more likely to be foreclosed when D_1

is vertically integrated with U. The reason for this is that vertical integration induces D_1 to invest more and induces D_2 to invest less.

The analysis so far established that integration between D_1 and U boosts the investment of D_1, lowers the investment of D_2, and increases the probability that D_2 is foreclosed. But will D_1 and U find it optimal to vertically integrate in the first place? The next proposition proves that the answer is "yes": vertical integration increases the joint profit of D_1 and U relative to the no integration case.

Proposition 1: *Vertical integration is profitable for the upstream supplier U and downstream firm D_1. Whenever it occurs, vertical integration*

(i) *boosts the investment of D_1,*

(ii) *lowers the investment of D_2,*

(iii) *raises the probability that D_2 makes no sales in the competitive downstream market.*

5.1 The welfare effects of vertical integration

To examine how vertical integration affects welfare, recall that the Nash equilibrium prices are equal to 0 if $V_1 = V_2 = \overline{V}$ or $V_1 = V_2 = \underline{V}$ and to $p_i = \overline{V} - \underline{V} \equiv \Delta$ and $p_j = 0$ if $V_i = \overline{V}$ and $V_j = \underline{V}$. Hence, consumer surplus in the competitive downstream market is given by the following table:[18]

Table 2: Consumer surplus

	$V_2 = \overline{V}$	$V_2 = \underline{V}$
$V_1 = \overline{V}$	$\overline{V}$	$\overline{V} - \Delta = \underline{V}$
$V_1 = \underline{V}$	$\overline{V} - \Delta = \underline{V}$	$\underline{V}$

Expected consumer surplus in the competitive downstream market is therefore

$$S(q_1, q_2) = q_1 q_2 \overline{V} + (1 - q_1 q_2)\underline{V} = \underline{V} + q_1 q_2 \Delta. \tag{9}$$

Absent integration, expected consumer surplus is $S^* \equiv S(q_1^*, q_2^*)$, while under vertical integration it is $S^{VI} \equiv S\left(q_1^{VI}, q_2^{VI}\right)$. Comparing S^* and S^{VI} yields the following result:

[18] Consumer surplus in the two monopoly markets is constant and hence I will ignore it.

Proposition 2: *Vertical integration benefits consumers when* $\frac{\Delta}{k} < 0.326$*, but harms consumers otherwise.*

Intuitively, equation (9) shows that vertical integration affects consumers in the competitive downstream market only through its effect on $q_1 q_2$, which is the probability that both firms offer high quality; in that case (and only then), consumers enjoy high quality at a low price. Equation (2) shows that $q_1^* q_2^*$ is strictly increasing with $\frac{\Delta}{k}$. Equations (6) and (7) show that q_1^{VI} is strictly increasing with $\frac{\Delta}{k}$, while q_2^{VI} is an inverse U-shaped function of $\frac{\Delta}{k}$, so that $q_1^{VI} q_2^{VI}$ is first increasing and then decreasing with $\frac{\Delta}{k}$. Not surprisingly then, vertical integration harms consumers when $\frac{\Delta}{k}$ is sufficiently large.

6 Partial vertical integration

So far I have assumed implicitly that under vertical integration, D_1 and U fully merge. In reality though, vertical integration is often partial: the acquiring firm (D_1 in the case of backward integration and U in the case of forward integration) buys only a controlling stake in the target firm which gives it the right to choose the target's strategy, but only a fraction of the target's profit. In this section, I explore the effects of partial integration on foreclosure and on welfare. I will start in subsection 6.1 by considering the case where D_1 buys a controlling stake $\alpha < 1$ in U, and then, in subsection 6.1, I will examine the reverse case where U buys a controlling stake $\alpha < 1$ in D_1.

6.1 Partial backward integration by $\mathbf{D}_1$

Suppose that D_1 acquires a controlling share $\alpha < 1$ in U and chooses the strategy of D_1 and U, with the objective of maximizing the sum of D_1's downstream profit and D_1's stake in U's upstream profit. As in the full merger case, the equilibrium prices and downstream revenues are given by Table 1.

Moving to the bargaining between D_2 and U (which is now controlled by D_1), note first that since D_1 only has a partial stake in U, it will obviously wish to pay as little as possible for the input it buys from U. But if the input price is below c, the minority shareholders of U effectively subsidize the shareholders of D_1. Assuming that such a transfer of wealth violates the fiduciary duties of D_1 towards the minority shareholders of U, I will assume that D_1 pays c for the input; this implies that U simply breaks even on the input sale to D_1. When D_2 makes a take-it-or-leave-it offer for the input, it offers a price, w_2, that leaves D_1 (which controls U) indifferent between selling the input to D_2 at w_2 and foreclosing D_2:

$$\underbrace{q_1\overline{V} + (1-q_1)\underline{V} + \overline{R} - c}_{\substack{D_1\text{'s profit if } D_2 \\ \text{is foreclosed}}} = \underbrace{q_1(1-q_2)\Delta + \overline{R} - c}_{\substack{D_1\text{'s profit if } U \\ \text{sells to } D_2}} + \underbrace{\alpha(w_2 - c)}_{\substack{D_1\text{'s share} \\ \text{in } U\text{'s profit}}}, \quad \Rightarrow \quad w_2 = \frac{q_1 q_2 \Delta + \alpha c + \underline{V}}{\alpha}.$$

When D_1 makes a take-it-or-leave-it offer, it offers D_2 a price $q_2(1-q_1)\Delta + \overline{R}$, which is equal to the entire expected revenue of D_2. The expected input price that D_2 pays under partial backward integration (denoted BI) is therefore

$$w_2^{BI} = \frac{q_2(1-q_1)\Delta + \overline{R}}{2} + \frac{q_1 q_2 \Delta + \alpha c + \underline{V}}{2\alpha}.$$

Notice that w_2^{BI} is a decreasing function of α and is equal to w_2^{VI} when $\alpha = 1$ (full integration). Hence, $w_2^{BI} > w_2^{VI}$ for all $\alpha < 1$. The reason why the input price is higher when α is small is that D_2 must compensate D_1 for the erosion in D_1's downstream profit due to competition with D_2. Since D_1 gets only a fraction α of U's profits, the input price must be high enough so that a fraction α of it will cover the entire erosion of D_1's downstream profit.

Given w_2^{BI}, the expected profits of D_1 and D_2 are

$$\begin{aligned} \pi_1 &= \underbrace{q_1(1-q_2)\Delta + \overline{R} - c - \frac{kq_1^2}{2}}_{D_1\text{'s profit}} + \underbrace{\alpha\left(w_2^{BI} - c\right)}_{U\text{'s profit}} \\ &= \frac{((2-(1+\alpha)q_2)q_1 + \alpha q_2)\Delta + (2+\alpha)\left(\overline{R} - c\right) + \underline{V}}{2} - \frac{kq_1^2}{2}, \end{aligned}$$

and

$$\begin{aligned} \pi_2 &= q_2(1-q_1)\Delta + \overline{R} - w_2^{BI} - \frac{kq_2^2}{2} \\ &= \frac{q_2(\alpha - (1+\alpha)q_1)\Delta + \alpha\left(\overline{R} - c\right) - \underline{V}}{2\alpha} - \frac{kq_2^2}{2}. \end{aligned}$$

The equilibrium levels of investment under partial backward integration, q_1^{BI} and q_2^{BI}, are defined by the following first order conditions:

$$\pi_1' = \left(1 - \frac{(1+\alpha)}{2} q_2\right)\Delta - kq_1 = 0, \tag{10}$$

and

$$\pi_2' = \frac{(\alpha - (1+\alpha)q_1)\Delta}{2\alpha} - kq_2 = 0. \tag{11}$$

Figure 4 shows the interior Nash equilibrium which obtains when $\Delta < \frac{\alpha k}{1+\alpha}$. Compared with full vertical integration, now the best-response function of D_1 rotates clockwise around its

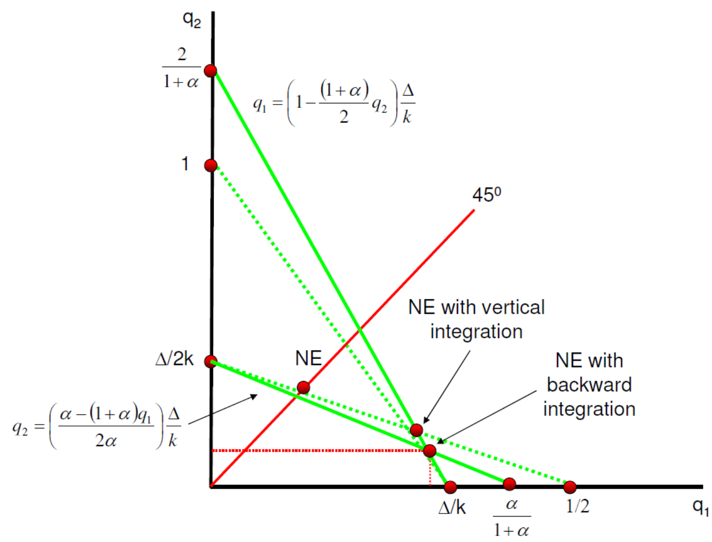

Figure 1: Figure 4: The interior Nash equilibrium levels of investment under partial backward integration

horizontal intercept, while the best-response function of D_2 rotates clockwise around its vertical intercept. The upward rotation in the best-response function of D_1 is due to the fact that now, D_1 internalizes only a fraction α of the negative effect of its investment on U's revenue from selling the input to D_2. The downward shift in the best-response function D_2 reflects the higher price that it pays U for the input.

Solving (10) and (11), the equilibrium levels of investment are

$$q_1^{BI} = \begin{cases} \frac{\alpha\Delta(4k-(1+\alpha)\Delta)}{4\alpha k^2-(1+\alpha)^2\Delta^2} & \text{if} \quad \Delta < \frac{\alpha k}{1+\alpha}, \\ \frac{\Delta}{k} & \text{if} \quad \Delta \geq \frac{\alpha k}{1+\alpha}, \end{cases} \tag{12}$$

and

$$q_2^{BI} = \begin{cases} \frac{2\Delta(\alpha k-(1+\alpha)\Delta)}{4\alpha k^2-(1+\alpha)^2\Delta^2} & \text{if} \quad \Delta < \frac{\alpha k}{1+\alpha}, \\ 0 & \text{if} \quad \Delta \geq \frac{\alpha k}{1+\alpha}. \end{cases} \tag{13}$$

Note that q_1^{BI} is continuous and equals $\frac{\alpha}{1+\alpha}$ when $\Delta = \frac{\alpha k}{1+\alpha}$.

When $\alpha = 1$, the equilibrium under backward integration coincides with the equilibrium under full vertical integration. In the next propossition I examine what happens when $\alpha < 1$.

Proposition 3: *An increase in α (the controlling stake of D_1 in U increases) leads to*

(i) *lower investment by D_1,*

(ii) *higher investment by* D_2,

(iii) *lower* $\phi_2^{BI} \equiv q_1^{BI}\left(1 - q_2^{BI}\right)$ *(*D_2 *is less likely to be foreclosed in the competitive downstream market),*

(iv) *higher consumer surplus.*

Proposition 3 shows that under partial backward integration D_1 invests more while D_2 invests less than they do under full vertical integration. The fact that $q_2^{BI} < q_2^{VI}$ is intuitive given that the price that D_2 pays for the input increases when α decreases. Less obvious is the result that $q_1^{BI} > q_1^{VI}$, because there are two conflicting forces at work: (i) investments are strategic substitutes, so the decrease in D_2's investment when $\alpha < 1$ encourages D_1 to invest more, (ii) when α is lower, D_1 internalizes a smaller fraction of the positive externality of its investment on U's profit, and hence has a weaker incentive to invest. Proposition 3 shows that the first positive effect outweighs the second negative effect. Since q_1^{BI} decreases and q_2^{BI} increases with α, ϕ_2^{BI} decreases with α; given that $\phi_2^{BI} = \phi_2^{VI}$ when $\alpha = 1$, it follows that $\phi_2^{BI} > \phi_2^{VI}$ for all $\alpha < 1$: D_2 is more likely to be foreclosed when D_1 has only a partial controlling stake in U. Finally, Proposition 3 implies that partial backward integration harms consumers more than full vertical integration. The reason is that the decrease in q_2^{BI} has a bigger effect on the probability that consumers will enjoy a high quality product at a relatively low price than the increase in q_1^{BI}.

The next step is to examine D_1's incentive to backward integrate with U. Proposition 1 shows that full vertical integration is profitable for the initial owners of D_1 and U. The question is whether partial backward integration is also profitable. To address this question, I will assume that initially, D_1 and U are each controlled by a single shareholder, whose equity stakes in their respective firms are γ_1 and γ_U. Proposition 1 shows that if $\gamma_U = 1$, then it is profitable for D_1 to acquire all of γ_U and become the sole owner of U. The next proposition shows that this is still true when $\gamma_U < 1$, and moreover it shows that D_1 may prefer to acquire only part of the equity stake of U's controlling shareholder rather than all of it.

Proposition 4: *Suppose that initially, U is controlled by a single shareholder who holds an equity stake γ_U in U and suppose that D_1 offers a price T to the controlling shareholder of U for an equity stake $\alpha \leq \gamma_U$. Then,*

(i) *backward integration is always profitable;*

(ii) *if* $\gamma_U > \frac{\Delta}{k-\Delta}$ *(in which case* $\Delta < \frac{\gamma_U k}{1+\gamma_U}$*, so* D_2 *is not foreclosed in the competitive downstream market when* D_1 *holds a controlling stake* γ_U *in* U*), then* D_1 *may prefer to acquire less than the entire controlling stake of* U*'s initial controlling shareholder if* k *is sufficiently small or if* γ_U *is sufficiently close to* $\frac{\Delta(1+\gamma_U)}{\gamma_U}$*;*

(iii) *if* $\gamma_U \leq \frac{\Delta}{k-\Delta}$ *(in which case* $\Delta \geq \frac{\gamma_U k}{1+\gamma_U}$*, so* D_2 *is foreclosed in the competitive downstream market when* D_1 *holds a controlling stake* γ_U *in* U*), then* D_1 *may prefer to acquire the smallest equity stake in* U *possible subject to gaining control over* U*.*

6.2 Partial forward integration by U

Here I assume that U_1 acquires a controlling share $\alpha < 1$ in D_1 and then chooses the strategy of both U and D_1 with the objective of maximizing the sum of U's upstream profit and its α stake in D_1's downstream profit. As before, the equilibrium prices and downstream revenues are given by Table 1.

Moving to the bargaining between D_2 and U, note that since U gets the full upstream profit but only part of the downstream profit of D_1, it will prefer to charge D_1 a relatively high input price. Denote this price by w_1. Now consider the bargaining between and D_2 over the input price that D_2 pays U. When D_2 makes a take-it-or-leave-it offer for the input, it offers a price, w_2, that leaves U indifferent between selling to D_2 at w_2 and foreclosing D_2:

$$\underbrace{w_1 - c}_{\substack{U\text{'s profit if } D_2 \\ \text{is foreclosed}}} + \underbrace{\alpha\left(q_1\overline{V} + (1-q_1)\underline{V} + \overline{R} - w_1\right)}_{\substack{U\text{'s share} \\ \text{in } D_1\text{'s profit}}} = \underbrace{w_1 + w_2 - 2c}_{\substack{U\text{'s profit if it} \\ \text{sells to } D_2}} + \underbrace{\alpha\left(q_1(1-q_2)\Delta + \overline{R} - w_1\right)}_{\substack{U\text{'s share} \\ \text{in } D_1\text{'s profit}}},$$

$$\Rightarrow \quad w_2 = \alpha\left(\underline{V} + q_1 q_2 \Delta\right) + c.$$

When U makes a take-it-or-leave-it offer, it offers D_2 a price $q_2(1-q_1)\Delta + \overline{R}$, which is equal to the entire expected revenue of D_2. The expected input price that D_2 pays under partial forward integration (denoted FI) is therefore

$$\begin{aligned} w_2^{FI} &= \frac{q_2(1-q_1)\Delta + \overline{R}}{2} + \frac{\alpha\left(\underline{V} + q_1 q_2 \Delta\right) + c}{2} \\ &= \frac{q_2\left(1-(1-\alpha)q_1\right)\Delta + \alpha\underline{V} + \overline{R} + c}{2}. \end{aligned}$$

Notice that w_2^{FI} increases with α and is equal to w_2^{VI} when $\alpha = 1$ (full integration). Hence, $w_2^{FI} < w_2^{VI}$ for all $\alpha < 1$. The reason for this is that when U owns only part of D_1, it requires only partial compensation for the negative competitive effect of D_2 on D_1's downstream profit. Given w_2^{FI}, the expected profits of U (which now chooses D_1's strategy) and D_2 are

$$\begin{aligned}
\pi_1 &= \underbrace{w_1 + w_2^{FI} - 2c}_{U\text{'s profit}} + \alpha \underbrace{\left(q_1 \left(1 - q_2\right) \Delta + \overline{R} - c - \frac{kq_1^2}{2} \right)}_{D_1\text{'s profit}} \\
&= w_1 + \frac{\left(q_2 + q_1 \left(2\alpha - \left(1+\alpha\right) q_2\right)\right) \Delta + \alpha \underline{V} + \left(1 + 2\alpha\right) \left(\overline{R} - c\right) - 2c}{2} - \alpha \frac{kq_1^2}{2}.
\end{aligned}$$

and

$$\begin{aligned}
\pi_2 &= q_2 \left(1 - q_1\right) \Delta + \overline{R} - w_2^{FI} - \frac{kq_2^2}{2} \\
&= \frac{q_2 \left(1 - q_1\right) \Delta - \alpha \left(\underline{V} + q_1 q_2 \Delta\right) + \overline{R} - c}{2} - \frac{kq_2^2}{2}.
\end{aligned}$$

The equilibrium levels of investment under partial backward integration, q_1^{BI} and q_2^{BI}, are defined by the following first order conditions:

$$\pi_1' = \left(1 - \frac{\left(1+\alpha\right)}{2\alpha} q_2\right) \Delta - kq_1 = 0, \tag{14}$$

and

$$\pi_2' = \frac{\left(1 - \left(1+\alpha\right) q_1\right) \Delta}{2} - kq_2. \tag{15}$$

Figure 5 shows the interior Nash equilibrium which obtains under forward integration when $\Delta < \frac{k}{1+\alpha}$. Relative to the full vertical integration case, the best-response function of D_1 rotates counterclockwise around its horizontal intercept, while the best-response function of D_2 rotates counterclockwise around its vertical intercept. The downward rotation in the best-response function of D_1 is due to the fact that U, who chooses q_1, captures only a fraction of D_1's downstream profit but bears the full negative impact of q_1 on w_2^{FI} which is the price at which it sells the input to D_2. Hence, U has an incentive to restrict q_1 in order to keep w_2^{FI} high. The outward hift in the best-response function of D_2 reflects the fact that under forward integration it pays a lower price for the input relative to what it pays under full vertical integration.

Solving (10) and (11), the equilibrium levels of investment are

$$q_1^{FI} = \begin{cases} \frac{\Delta(4\alpha k - (1+\alpha)\Delta)}{4\alpha k^2 - (1+\alpha)^2 \Delta^2} & \text{if} \quad \Delta < \frac{k}{1+\alpha}, \\ \frac{\Delta}{k} & \text{if} \quad \Delta \geq \frac{k}{1+\alpha}, \end{cases} \tag{16}$$

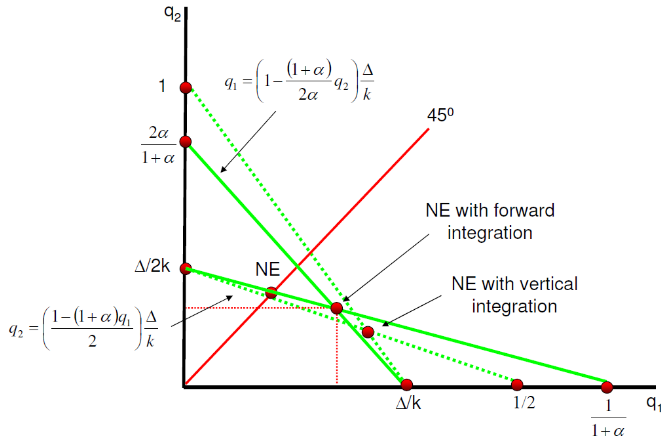

Figure 2: Figure 5: The interior Nash equilibrium levels of investment under partial forward integration

and

$$q_2^{FI} = \begin{cases} \frac{2\alpha\Delta(k-(1+\alpha)\Delta)}{4\alpha k^2-(1+\alpha)^2\Delta^2} & \text{if} \quad \Delta < \frac{k}{1+\alpha}, \\ 0 & \text{if} \quad \Delta \geq \frac{k}{1+\alpha}. \end{cases} \tag{17}$$

Note that q_1^{FI} is continuous and equals $\frac{1}{1+\alpha}$ when $\Delta = \frac{k}{1+\alpha}$. Also note that when $\alpha = 1$, the equilibrium under forward integration coincides with the equilibrium under full vertical integration. In the next propossition, I examine what happens as α drops below 1.

Proposition 5: *An increase in α (the controlling stake of U in D_1 increases) leads to*

(i) *higher investment by D_1,*

(ii) *lower investment by D_2 for all $\alpha > 1/4$,*

(iii) *higher $\phi_2^{FI} \equiv q_1^{FI}\left(1 - q_2^{FI}\right)$ (D_2 is more likely to be foreclosed in the competitive downstream market),*

(iv) *lower consumer surplus for all $\alpha > 1/2$.*

Proposition 5 implies that relative to the full integration case where $\alpha = 1$, under partial forward integration where $\alpha < 1$, D_1 invests less, while D_2 invests more. Intuitively, under partial forward integration, U internalizes only a fraction of the erosion in D_1's downstream profits due to its dealings with D_2. Hence, U charges D_2 a lower price for the input than under full vertical integration. Hence, q_2^{FI} is higher than it is under full vertical integration. Since investments are strategic substitutes this effects induces D_1 to invests less. This effect is compounded by the fact that U captures the full profit from selling the input to D_2, but captures only a fraction of D_1's profits. Consequently, U has an incentive to restrict q_1 in order to keep the price at which it sells the input to D_2 high. Given that q_1^{FI} is lower than under full vertical integration while q_2^{FI} is higher implies that the probability that D_2 is foreclosed in the downstream market, ϕ_2^{FI}, is lower than under full vertical integtaion. Finally, Proposition 5 implies that so long as $\alpha > 1/2$, consumers are better off under partial forward integration than they are under full vertical integration. The reason is that the increase in q_2^{FI} has a bigger effect on the probability that consumers will enjoy a high quality product at a relatively low price than the decrease in q_1^{FI}.

The next step is to examine U's incentive to acquire a controlling stake in D_1. To address this question, I will assume, as in the previous section, that D_1 and U are each initially controlled by a single shareholder, and the equity stakes of these shareholders in thier respective firms are γ_1 and γ_U. Proposition 1 shows that if $\gamma_1 = 1$, then it is profitable for U to acquire all of γ_1 and become the sole owner of D_1. The next proposition shows that.

Proposition 6: *Suppose that initially, D_1 is controlled by a single shareholder who holds an equity stake γ_1 in D_1 and suppose that U offers a price T to the controlling shareholder of D_1 for an equity stake $\alpha \leq \gamma_1$. Then,*

(i) *backward integration is always profitable;*

(ii) *if $\gamma_U > \frac{\Delta}{k-\Delta}$ (in which case $\Delta < \frac{\gamma_U k}{1+\gamma_U}$, so D_2 is not foreclosed in the competitive downstream market when D_1 holds a controlling stake γ_U in U), then D_1 may prefer to acquire less than the entire controlling stake of U's initial controlling shareholder if k is sufficiently small or if γ_U is sufficiently close to $\frac{\Delta(1+\gamma_U)}{\gamma_U}$;*

(iii) *if $\gamma_U \leq \frac{\Delta}{k-\Delta}$ (in which case $\Delta \geq \frac{\gamma_U k}{1+\gamma_U}$, so D_2 is foreclosed in the competitive downstream market when D_1 holds a controlling stake γ_U in U), then D_1 may prefer to acquire the smallest equity stake in U possible subject to gaining control over U.*

7 Upstream foreclosure

So far the model considered the welfare effects of downstream foreclosure. In this section I show that vertical foreclosure can also take place at the upstream market. To this end, suppose now that the there are two upstream suppliers U_1 and U_2 which sell a homogenous input to two downstream firms, D_1 and D_2, which use the input to produce a final product. To simplify matters, I will assume that D_1 and D_2 do not compete with each other, and each operates as a monopoly in a separate downstream market. As before, there is a unit mass of identical final consumers in each downstream market, and each consumer is interested in buying at most one unit and his utility if he buys is $V - p$, where V is the quality of the final good and p is the downstream price.

In this section, I will assume that V depends on the quality of the input. The quality of the input that U_i provides is equal to $\overline{V}$ with probability q_i and to $\underline{V}$ with probability $1 - q_i$, where q_i is chosen by U_i at a cost $\frac{kq_i^2}{2}$, where k is a positive constant.[19]

The total cost of each downstream firm is then equal to the sum of $\frac{kq_i^2}{2}$ and the price that D_i pays U for the input. The upstream firm U incurs a constant cost c per each unit of the input that it produces.

8 Conclusion

`To be written`

[19]This assumption that the cost of investment is quadratic is not essential and is made only for convinience. All the results go through for any increasing and convex cost function that satisfies appropriate restrictions (needed to ensure that the equilibrium is well-behaved).

9 Appendix

Following are the proofs of Propositions 1-6.

Proof of Proposition 1: Absent vertical integration, the joint profit of D_1 and U is

$$\pi_1^* + \pi_U^* = \underbrace{\frac{q_1^*(1-q_2^*)\Delta + \overline{R} - c}{2} - \frac{k(q_1^*)^2}{2}}_{\pi_1} \quad (18)$$
$$+ \underbrace{\frac{q_1^*(1-q_2^*)\Delta + \overline{R} + c}{2} + \frac{q_2^*(1-q_1^*)\Delta + \overline{R} + c}{2} - 2c}_{\pi_U}.$$

Substituting for q_1^* and q_2^* from (2) into (18) and simplifying,

$$\pi_1^* + \pi_U^* = \frac{5\Delta^2 k}{2(\Delta+2k)^2} + \frac{3(\overline{R}-c)}{2}. \quad (19)$$

On the other hand, substituting q_1^{VI} and q_2^{VI} into π_{VI} and rearranging, the profit of the vertically integrated firm is

$$\pi_{VI} = \begin{cases} \frac{\Delta^2(6k^3 - 8\Delta k^2 - \Delta^2 k + 4\Delta^3)}{8(k^2-\Delta^2)^2} + \frac{3(\overline{R}-c)+\underline{V}}{2} & \text{if} \quad \Delta < \frac{k}{2}, \\ \frac{\Delta^2}{2k} + \frac{3(\overline{R}-c)+\underline{V}}{2} & \text{if} \quad \Delta \geq \frac{k}{2}. \end{cases}$$

Comparing the two expressions reveals that,

$$\pi_{VI} - (\pi_1 + \pi_U) = \begin{cases} \frac{\Delta^2\left[4k^4(k-2\Delta) + 5\Delta^2 k(2k^2-\Delta^2) + 4\Delta^3(k^2+\Delta^2)\right]}{8(k^2-\Delta^2)^2(\Delta+2k)^2} + \frac{\underline{V}}{2} & \text{if} \quad \Delta < \frac{k}{2}, \\ \frac{\Delta^2(\Delta^2 + k(4\Delta - k))}{2k(\Delta+2k)^2} + \frac{\underline{V}}{2} & \text{if} \quad \Delta \geq \frac{k}{2}. \end{cases}$$

The sign of the expression in the top line of the equation is positive given that $\Delta < \frac{k}{2}$. The expression in the bottom line is also positive since $\Delta \geq \frac{k}{2}$. Altogether then, vertical integration is profitable for U and for D_1. ■

Proof of Proposition 2: Substituting q_1^* and q_2^* from (2) into (9) yields

$$S^* = \underline{V} + \frac{\Delta^3}{(\Delta+2k)^2}.$$

Substituting q_1^{VI} and q_2^{VI} from (6) and (7) into (9) yields

$$S^{VI} = \begin{cases} \underline{V} + \frac{\Delta^3(2k-\Delta)(k-2\Delta)}{4(k^2-\Delta^2)^2} & \text{if} \quad \Delta < \frac{k}{2}, \\ \underline{V} & \text{if} \quad \Delta \geq \frac{k}{2}. \end{cases}$$

Now,

$$S^{VI} - S^* = \begin{cases} \frac{\Delta^3 k^4 T\left(\frac{\Delta}{k}\right)}{4(k^2-\Delta^2)^2(\Delta+2k)^2} & \text{if} \quad \Delta < \frac{k}{2}, \\ -\frac{\Delta^3}{(\Delta+2k)^2} & \text{if} \quad \Delta \geq \frac{k}{2}, \end{cases} \quad (20)$$

where

$$T\left(\frac{\Delta}{k}\right)=4-12\left(\frac{\Delta}{k}\right)-2\left(\frac{\Delta}{k}\right)^{2}+3\left(\frac{\Delta}{k}\right)^{3}-2\left(\frac{\Delta}{k}\right)^{4}.$$

It turns out that $T'\left(\frac{\Delta}{k}\right)<0$, and $T\left(\frac{\Delta}{k}\right)>0$ when $\frac{\Delta}{k}<0.326$ and $T\left(\frac{\Delta}{k}\right)<0$ otherwise. Hence, $S^{VI}>S^{*}$ for all $\frac{\Delta}{k}<0.326$ and $S^{VI}<S^{*}$ for all $\frac{\Delta}{k}>0.326$. ■

Proof of Proposition 3: First, recalling that $\Delta<k$ and $\alpha<1$,

$$\begin{aligned}\frac{\partial q_1^{BI}}{\partial\alpha} &= \frac{\Delta^2\left((1+\alpha)^2\Delta^2-4k\left(\alpha^2k+\left(1-\alpha^2\right)\Delta\right)\right)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2}\\ &< \frac{\Delta^2\left((1+\alpha)^2\Delta^2-4\Delta\left(\alpha^2\Delta+\left(1-\alpha^2\right)\Delta\right)\right)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2}\\ &= \frac{\Delta^4\left((1+\alpha)^2-4\right)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2}<0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial q_2^{BI}}{\partial\alpha} &= \frac{2\Delta^2\left(k\left(4k-\left(1-\alpha^2\right)\Delta\right)-(1+\alpha)^2\Delta^2\right)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2}\\ &> \frac{2\Delta^2\left(\Delta\left(4\Delta-\left(1-\alpha^2\right)\Delta\right)-(1+\alpha)^2\Delta^2\right)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2}\\ &= \frac{4\Delta^4(1-\alpha)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2}>0.\end{aligned}$$

Given that q_1^{BI} decreases and q_2^{BI} increases with α, $\phi_2^{BI}\equiv q_1^{BI}\left(1-q_2^{BI}\right)$ decreases with α. Since $\phi_2^{BI}=\phi_2^{VI}$ when $\alpha=1$, it follows that $\phi_2^{BI}>\phi_2^{VI}$ for all $\alpha<1$: D_2 is foreclosed more often when D_1 and U only partially integrate.

Substituting q_1^{BI} and q_2^{BI} in (9), consumer surplus under partial backward integration is

$$S^{BI}=\begin{cases}\underline{V}+\frac{2\alpha\Delta^3(4k-(1+\alpha)\Delta)(\alpha k-(1+\alpha)\Delta)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2} & \text{if} \quad \Delta<\frac{\alpha k}{1+\alpha},\\ \underline{V} & \text{if} \quad \Delta\geq\frac{\alpha k}{1+\alpha}.\end{cases}$$

Now, when $\Delta<\frac{\alpha k}{1+\alpha}$

$$\frac{\partial S^{BI}}{\partial\alpha}=\frac{2\Delta^4\left[(1+\alpha)^2\Delta^2\left(\left(4-6\alpha-\alpha^2\right)k-\left(1-\alpha^2\right)\Delta\right)+4\alpha k^2\left(\left(4-\alpha^2\right)k-3\left(1-\alpha^2\right)\Delta\right)\right]}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^3}.$$

The denominator of $\frac{\partial S^{BI}}{\partial \alpha}$ is positive since $\Delta < \frac{\alpha k}{1+\alpha}$ implies

$$4\alpha k^2 - (1+\alpha)^2 \Delta^2 > 4\alpha k^2 - \alpha^2 k^2 = \alpha k^2 (4-\alpha) > 0.$$

As for the numerator of $\frac{\partial S^{BI}}{\partial \alpha}$, note that

$$\begin{aligned}
& (1+\alpha)^2 \Delta^2 \left(\left(4-6\alpha-\alpha^2\right) k - \left(1-\alpha^2\right) \Delta\right) + 4\alpha k^2 \left(\left(4-\alpha^2\right) k - 3\left(1-\alpha^2\right) \Delta\right) \\
> \; & (1+\alpha)^2 \Delta^2 \left(\left(4-6\alpha-\alpha^2\right) k - (1-\alpha)\alpha k\right) + 4\alpha k^2 \left(\left(4-\alpha^2\right) k - 3(1-\alpha)\alpha k\right) \\
= \; & k\left[(1+\alpha)^2 \Delta^2 (4-7\alpha) + 4\alpha k^2 \left(4-3\alpha+2\alpha^2\right)\right] \\
> \; & k\left[(1+\alpha)^2 \Delta^2 (4-7\alpha) + 4\alpha \left(\frac{(1+\alpha)\Delta}{\alpha}\right)^2 \left(4-3\alpha+2\alpha^2\right)\right] \\
= \; & \frac{(1+\alpha)^2 (4-\alpha)^2 \Delta^2 k}{\alpha} > 0.
\end{aligned}$$

■

Proof of Proposition 4: Suppose that D_1 offers a price T to the controlling shareholder of U for an equity stake $\alpha \leq \gamma_U$ in U. The controlling shareholder of U would accept the offer if it increases his payoff relative to the no integration case, i.e., if

$$(\gamma_U - \alpha)\, \pi_U^{BI} + T \geq \gamma_U \pi_U^*,$$

where $\pi_U^{BI} \equiv w_2^{BI} - c$ and $\pi_U^* \equiv \frac{q_1^*\left(1-q_2^*\right)\Delta + \overline{R} + c}{2} + \frac{q_2^*\left(1-q_1^*\right)\Delta + \overline{R} + c}{2} - 2c$. The minimal acceptable offer is then

$$T = \gamma_U \pi_U^* - (\gamma_U - \alpha)\, \pi_U^{BI}.$$

The controlling shareholder of D_1 would agree to make this offer only if his share in the resulting post-merger cash flow of D_1 (D_1's profit minus the payment T plus D_1's share in U's profit) exceeds his share in D_1's profit absent integration, i.e., only if

$$\gamma_1 \left(\pi_1^{BI} - T + \alpha \pi_U^{BI}\right) \geq \gamma_1 \pi_1^*,$$

where $\pi_1^{BI} \equiv q_1^{BI}\left(1-q_2^{BI}\right)\Delta + \overline{R} - c - \frac{k\left(q_1^{BI}\right)^2}{2}$ and $\pi_1^* \equiv \frac{q_1^*\left(1-q_2^*\right)\Delta + \overline{R} - c}{2} - \frac{k\left(q_1^*\right)^2}{2}$. Substituting for T and rearranging, it follows that partial backward integration is an equilibrium provided that

$$\pi_1^{BI} + \gamma_U \left(\pi_U^{BI} - \pi_U^*\right) \geq \pi_1^*. \tag{21}$$

Notice that γ_1 (the controlling stake of D_1's controlling shareholder), is irrelevant: only γ_U, which is the controlling stake of U's initial controlling shareholder matters.

To prove part (i) of the proposition, I will prove that condition (21) holds whenever $\alpha = \gamma_U$. To this end, let me first consider the case where $\gamma_U > \frac{\Delta}{k-\Delta}$. Then $\Delta < \frac{\gamma_U k}{1+\gamma_U}$, so D_2 is not foreclosed in the competitive downstream market when D_1 holds an equity stake γ_U in U. Evaluated at $\alpha = \gamma_U$,

$$\pi_1^{BI} + \gamma_U \left(\pi_U^{BI} - \pi_U^*\right) - \pi_1^* = \frac{\gamma_U^2 k z^2 H}{2\left(1+\gamma_U\right)^2\left(2+2\gamma_U+\gamma_U z\right)^2\left(4-\gamma_U z^2\right)^2} + \left(1-\gamma_U\right)\left(\overline{R}-c\right) + \frac{V}{2},$$

where $z \equiv \frac{\gamma_U k}{(1+\gamma_U)\Delta}$. Since $\gamma_U > \frac{\Delta}{k-\Delta}$, $z < 1$. Together with the fact that $\gamma_U \leq 1$, it follows that

$$\begin{aligned}
H &> 64 + 80\gamma_U - 5\gamma_U^5 z^4 + \gamma_U^4 z^2\left(36+4z-3z^2+2z^3\right) \\
&\quad -\gamma_U^2\left(32+32z+12z^2-8z^3\right) - \gamma_U^3\left(48+32z-16z^2+4z^3-3z^4\right). \\
&> 64\left(1-\gamma_U z\right) - 5\gamma_U^4 z^2 + \gamma_U^4 z^2\left(36+4z-3z^2+2z^3\right) \\
&\quad -\gamma_U\left(12z^2-8z^3\right) - \gamma_U\left(-16z^2+4z^3-3z^4\right) \\
&= 64\left(1-\gamma_U z\right) + \gamma_U^4 z^2\left(31+4z-3z^2+2z^3\right) + \gamma_U z^2\left(4z+4+3z^2\right) \\
&> 64\left(1-\gamma_U z\right) + \gamma_U^4 z^2\left(35+8z+2z^3\right) > 0.
\end{aligned}$$

Hence, acquiring the entire equity stake of U's initial controlling shareholder is profitable for D_1 when $\gamma_U > \frac{\Delta}{k-\Delta}$.

If $\gamma_U \leq \frac{\Delta}{k-\Delta}$, then $\Delta \geq \frac{\gamma_U k}{1+\gamma_U}$, so D_2 is foreclosed in the competitive downstream market when D_1 holds an equity stake γ_U in U. Now,

$$\begin{aligned}
\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right) - \pi_1^* &= \frac{\gamma_U^2 k z^2\left[\left(3-4\gamma_U\right)\left(1+\gamma_U\right)^2 + \gamma_U z(4+4\gamma_U+\gamma_U z)\right]}{2\left(1+\gamma_U\right)^2\left(2+2\gamma_U+\gamma_U z\right)^2} + \frac{\left(1-\gamma_U\right)\left(\overline{R}-c\right)}{2} + \frac{V}{2} \\
&> \frac{\gamma_U^2 k z^2\left[\left(3-4\gamma_U\right)\left(1+\gamma_U\right)^2 + \gamma_U(4+4\gamma_U+\gamma_U)\right]}{2\left(1+\gamma_U\right)^2\left(2+2\gamma_U+\gamma_U z\right)^2} + \frac{\left(1-\gamma_U\right)\left(\overline{R}-c\right)}{2} + \frac{V}{2} \\
&= \frac{\gamma_U^2 k z^2\left[3+6\gamma_U-4\gamma_U^3\right]}{2\left(1+\gamma_U\right)^2\left(2+2\gamma_U+\gamma_U z\right)^2} + \frac{\left(1-\gamma_U\right)\left(\overline{R}-c\right)}{2} + \frac{V}{2} \geq 0,
\end{aligned}$$

where the first inequality follows because $\gamma_U \leq 1$. Once again, its is profitable for D_1 to acquire the entire equity stake of U's initial controlling shareholder.

To prove part (ii) of the proposition, note that the post-merger cash flow of D_1 is $\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right)$. Now assume that $\gamma_U > \frac{\Delta}{k-\Delta}$, so $\Delta < \frac{\gamma_U k}{1+\gamma_U}$. Differentiating $\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right)$ with respect to α and evaluating the derivative at $\alpha = \gamma_U$ yields

$$\frac{\partial}{\partial\alpha}\left(\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right)\right)\Big|_{\alpha=\gamma_U} = \frac{z^4\left(1-z\right)\gamma_U^3 k\left(\gamma_U\left(4-z^2\right) + 4z\left(1-\gamma_U\right)\right)}{\left(1+z\right)^3\left(4-z^2\gamma_U\right)^3} - \frac{V}{2\gamma_U}.$$

Since $\gamma_U > \frac{\Delta}{k-\Delta}$, $z \equiv \frac{\gamma_U k}{(1+\gamma_U)\Delta} < 1$. Hence, the first term in the derivative is positive, but goes to 0 when z goes to 0 or goes to 1. Since the second term is negative, the derivative is negative when

z goes to 0 or goes to 1. Part (ii) of the porposition follows by noting that z goes to 0 when k is small and goes to 1 when γ_U approaches $\frac{\Delta}{k-\Delta}$.

Finally, to prove part (iii) of the proposition, suppose that $\gamma_U \leq \frac{\Delta}{k-\Delta}$. Now, D_2 is foreclosed in the competitive downstream market when D_1's stake in U is γ_U. The resulting post-merger cash flow of D_1 is

$$\pi_1^{BI} + \gamma_U \left(\pi_U^{BI} - \pi_U^*\right) = \frac{\Delta^2 \left(\Delta^2 + 4\Delta k + 4k^2 - 4\gamma_U k^2\right)}{2k\left(2k+\Delta\right)^2} + \frac{\left(2-\gamma_U\right)\left(\overline{R}-c\right)}{2} + \frac{\gamma_U \underline{V}}{2\alpha}.$$

This expression is decreasing with α, implying that D_1 would wish to obtain a minimal controlling stake in U, subject to being able to obtain control over U. ■

Proof of Proposition 5: First, recalling that $\Delta < \frac{k}{1+\alpha}$,

$$\begin{aligned} \frac{\partial q_1^{FI}}{\partial \alpha} &= \frac{\Delta^2 \left(4k\left(k + \left(1-\alpha^2\right)\Delta\right) - \left(1+\alpha\right)^2 \Delta^2\right)}{\left(4\alpha k^2 - \left(1+\alpha\right)^2 \Delta^2\right)^2} \\ &> \frac{\Delta^2 \left(4k\left(k + \left(1-\alpha^2\right)\Delta\right) - k^2\right)}{\left(4\alpha k^2 - \left(1+\alpha\right)^2 \Delta^2\right)^2} > 0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial q_2^{FI}}{\partial \alpha} &= \frac{2\Delta^2 \left(\left(1+\alpha\right)\Delta\left(\left(1+\alpha\right)\Delta - \left(1-\alpha\right)k\right) - 4\alpha^2 k^2\right)}{\left(4\alpha k^2 - \left(1+\alpha\right)^2 \Delta^2\right)^2} \\ &< \frac{2\Delta^2 \left(\left(1+\alpha\right)\Delta\left(k - \left(1-\alpha\right)k\right) - 4\alpha^2 k^2\right)}{\left(4\alpha k^2 - \left(1+\alpha\right)^2 \Delta^2\right)^2} \\ &< \frac{2\Delta^2 \alpha k^2 \left(1-4\alpha\right)}{\left(4\alpha k^2 - \left(1+\alpha\right)^2 \Delta^2\right)^2}, \end{aligned}$$

where the last line is negative for $\alpha > 1/4$. Given that q_1^{FI} increases and q_2^{FI} decreases with α, $\phi_2^{FI} \equiv q_1^{FI}\left(1-q_2^{FI}\right)$ increases with α. Since $\phi_2^{FI} = \phi_2^{VI}$ when $\alpha = 1$, it follows that $\phi_2^{FI} > \phi_2^{VI}$ for all $\alpha < 1$: D_2 is foreclosed more often when U has a partial ownership stake in D_1 than it is when U and D_1 are fully vertically integrated.

Substituting q_1^{FI} and q_2^{FI} in (9), consumer surplus under partial forward integration is

$$S^{FI} = \begin{cases} \underline{V} + \frac{2\alpha\Delta^3 (k-(1+\alpha)\Delta)(4\alpha k-(1+\alpha)\Delta)}{\left(4\alpha k^2-(1+\alpha)^2\Delta^2\right)^2} & \text{if } \Delta < \frac{k}{1+\alpha}, \\ \underline{V} & \text{if } \Delta \geq \frac{k}{1+\alpha}. \end{cases}$$

Now,

$$\frac{\partial S^{FI}}{\partial \alpha} = \frac{2kr^4 \left[4\alpha\left(1-4\alpha^2\right) - 12\alpha\left(1-\alpha\right)r + \left(1+6\alpha-4\alpha^2\right)r^2 - \left(1-\alpha\right)r^3\right]}{\left(1+\alpha\right)^4\left(4\alpha - r^2\right)^3},$$

where $r \equiv \frac{k}{(1+\alpha)\Delta}$. Since $\Delta < \frac{k}{(1+\alpha)}$, $r < 1$. It turns out that $\frac{\partial S^{FI}}{\partial \alpha} < 0$ for all $\alpha \in [0.5, 1]$ and $r \in (0,1)$. ■

Proof of Proposition 6: Analogously to Proposition 4, here U will be able to make an acceptable offer T to the initial controlling shareholder of D_1 in return for a controlling equity stake of $\alpha \leq \gamma_1$ provided that

$$\pi_U^{FI} + \gamma_1 \left(\pi_1^{FI} - \pi_1^*\right) \geq \pi_U^*, \tag{22}$$

where $\pi_U^{FI} \equiv w_1^{FI} + w_2^{FI} - 2c$ and $\pi_1^{FI} \equiv q_1^{FI}\left(1 - q_2^{FI}\right)\Delta + \overline{R} - w_1^{FI} - \frac{k\left(q_1^{FI}\right)^2}{2}$. Since after the merger U controls D_1, it will have an incentive to set w_1^{FI} as high as possible in order to transfer profits from D_1 where its equity stake is α to U where it captures the profits fully. Assuming that w_1^{FI} can be set such that $\pi_1^{FI} = 0$, substituting for w_2^{FI} and rearranging terms, the condition becomes

$$\underbrace{q_1^{FI}\left(1 - q_2^{FI}\right)\Delta + \overline{R} - \frac{k\left(q_1^{FI}\right)^2}{2}}_{w_1^{FI}} + \underbrace{\frac{q_2^{FI}\left(1 - (1-\alpha)\, q_1^{FI}\right)\Delta + \alpha\underline{V} + \overline{R} + c}{2}}_{w_2^{FI}} - 2c \geq \pi_U^* + \gamma_1 \pi_1^*.$$

To prove the part (i) of the proposition, I will prove that condition (22) holds for $\alpha = \gamma_U$. To this end, let me first consider the case where $\gamma_U > \frac{\Delta}{k-\Delta}$. Then $\Delta < \frac{\gamma_U k}{1+\gamma_U}$ so D_2 is not foreclosed in the competitive downstream market when D_1 holds an equity stake γ_U in U. Evaluated at $\alpha = \gamma_U$,

$$\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right) - \pi_1^* = \frac{\gamma_U^2 k z^2 H}{2\left(1+\gamma_U\right)^2\left(2 + 2\gamma_U + \gamma_U z\right)^2\left(4 - \gamma_U z^2\right)^2} + \left(1 - \gamma_U\right)\left(\overline{R} - c\right) + \frac{\underline{V}}{2},$$

where $z \equiv \frac{\gamma_U k}{(1+\gamma_U)\Delta}$. Since $\gamma_U > \frac{\Delta}{k-\Delta}$, $z < 1$. Together with the fact that $\gamma_U \leq 1$, it follows that

$$\begin{aligned}
H &> 64 + 80\gamma_U - 5\gamma_U^5 z^4 + \gamma_U^4 z^2\left(36 + 4z - 3z^2 + 2z^3\right) \\
&\quad -\gamma_U^2\left(32 + 32z + 12z^2 - 8z^3\right) - \gamma_U^3\left(48 + 32z - 16z^2 + 4z^3 - 3z^4\right). \\
&> 64\left(1 - \gamma_U z\right) - 5\gamma_U^4 z^2 + \gamma_U^4 z^2\left(36 + 4z - 3z^2 + 2z^3\right) \\
&\quad -\gamma_U\left(12z^2 - 8z^3\right) - \gamma_U\left(-16z^2 + 4z^3 - 3z^4\right) \\
&= 64\left(1 - \gamma_U z\right) + \gamma_U^4 z^2\left(31 + 4z - 3z^2 + 2z^3\right) + \gamma_U z^2\left(4z + 4 + 3z^2\right) \\
&> 64\left(1 - \gamma_U z\right) + \gamma_U^4 z^2\left(35 + 8z + 2z^3\right) > 0.
\end{aligned}$$

Hence, acquiring the entire equity stake of U's initial controlling shareholder is profitable for D_1 when $\gamma_U > \frac{\Delta}{k-\Delta}$.

If $\gamma_U \leq \frac{\Delta}{k-\Delta}$, then $\Delta \geq \frac{\gamma_U k}{1+\gamma_U}$, so D_2 is foreclosed in the competitive downstream market when D_1 holds an equity stake γ_U in U. Now,

$$\begin{aligned}
\pi_1^{BI} + \gamma_U \left(\pi_U^{BI} - \pi_U^*\right) - \pi_1^* &= \frac{\gamma_U^2 k z^2 \left[(3-4\gamma_U)(1+\gamma_U)^2 + \gamma_U z(4+4\gamma_U+\gamma_U z)\right]}{2(1+\gamma_U)^2(2+2\gamma_U+\gamma_U z)^2} + \frac{(1-\gamma_U)\left(\overline{R}-c\right)}{2} + \frac{\underline{V}}{2} \\
&> \frac{\gamma_U^2 k z^2 \left[(3-4\gamma_U)(1+\gamma_U)^2 + \gamma_U(4+4\gamma_U+\gamma_U)\right]}{2(1+\gamma_U)^2(2+2\gamma_U+\gamma_U z)^2} + \frac{(1-\gamma_U)\left(\overline{R}-c\right)}{2} + \frac{\underline{V}}{2} \\
&= \frac{\gamma_U^2 k z^2 \left[3+6\gamma_U - 4\gamma_U^3\right]}{2(1+\gamma_U)^2(2+2\gamma_U+\gamma_U z)^2} + \frac{(1-\gamma_U)\left(\overline{R}-c\right)}{2} + \frac{\underline{V}}{2} \geq 0,
\end{aligned}$$

where the first inequality follows because $\gamma_U \leq 1$.

To prove part (ii) of the proposition, note that the post-merger cash flow of D_1 is $\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right)$. Now assume that $\gamma_U > \frac{\Delta}{k-\Delta}$, so $\Delta < \frac{\gamma_U k}{1+\gamma_U}$. Differentiating $\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right)$ with respect to α and evaluating the derivative at $\alpha = \gamma_U$ yields

$$\frac{\partial}{\partial \alpha}\left(\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right)\right)\Bigg|_{\alpha=\gamma_U} = \frac{z^4(1-z)\gamma_U^3 k\left(\gamma_U\left(4-z^2\right) + 4z(1-\gamma_U)\right)}{(1+z)^3(4-z^2\gamma_U)^3} - \frac{\underline{V}}{2\gamma_U}.$$

Since $\gamma_U > \frac{\Delta}{k-\Delta}$, $z \equiv \frac{\gamma_U k}{(1+\gamma_U)\Delta} < 1$. Hence, the first term in the derivative is positive, but goes to 0 when z goes to 0, i.e., when k is small, or when z goes to 1, i.e., γ_U approaches $\frac{\Delta}{k-\Delta}$. Since the second term is negative, the derivative is negative when k is small, or when γ_U approaches $\frac{\Delta}{k-\Delta}$.

Finally, to prove part (iii) of the proposition, suppose that $\gamma_U \leq \frac{\Delta}{k-\Delta}$. Now, D_2 is foreclosed in the competitive downstream market when D_1's stake in U is γ_U. The resulting post-merger cash flow of D_1 is

$$\pi_1^{BI} + \gamma_U\left(\pi_U^{BI} - \pi_U^*\right) = \frac{\Delta^2\left(\Delta^2 + 4\Delta k + 4k^2 - 4\gamma_U k^2\right)}{2k(2k+\Delta)^2} + \frac{(2-\gamma_U)\left(\overline{R}-c\right)}{2} + \frac{\gamma_U \underline{V}}{2\alpha}.$$

This expression is decreasing with α, implying that D_1 would wish to obtain a minimal controlling stake in U, subject to being able to obtain control over U. ■

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