Anticompetitive Vertical Merger Waves*

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Abstract

This paper develops an equilibrium model of vertical mergers. We show that competition on an upstream market between integrated firms only is less intense than in the presence of unintegrated upstream firms. Indeed, when an integrated firm supplies the upstream market, it becomes a soft downstream competitor to preserve its upstream profits. This benefits other integrated firms, which may therefore choose not to cut prices on the upstream market. This mechanism generates waves of vertical mergers in which every upstream firm integrates with a downstream firm, and the remaining unintegrated downstream firms obtain the input at a high upstream price.

1 Introduction

The anticompetitive effects of vertical mergers have long been a hotly debated issue among economists. Until the end of the 1960s, the traditional vertical foreclosure theory was widely accepted by antitrust practitioners. According to this theory, vertical mergers were harmful to competition, since vertically integrated firms had incentives to raise their rivals' costs. This view was seriously challenged by Chicago school authors in the 1970s, notably Bork (1978) and Posner (1976), on the ground that firms cannot leverage market power from one market to another.

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A more recent strategic approach of the subject, initiated by Ordover, Saloner and Salop (1990), has established conditions under which vertical integration can relax competition. In this strand of the literature, vertical mergers can lead to input foreclosure because upstream competition between vertically integrated firms and unintegrated upstream firms is less tough than upstream competition between unintegrated upstream firms only. This result, however, holds true only under specific assumptions, including extra commitment power for vertically integrated firms (Ordover, Saloner and Salop, 1990), choice of input specification (Choi and Yi, 2000), switching costs (Chen, 2001), tacit collusion (Nocke and White, 2007), exclusive dealing (Chen and Riordan, 2007).

In this paper we argue that a wave of vertical mergers that eliminates all unintegrated upstream firms can have severe anticompetitive effects, even in the absence of the above-mentioned specific assumptions leading to input foreclosure. This is because upstream competition between vertically integrated firms only – a market structure the literature has surprisingly overlooked – can be very ineffective, even when all the usual ingredients of tough Bertrand competition are in place.

In our model there are initially two upstream firms and three downstream firms. First, the downstream firms bid to integrate backward with the first upstream firm. Then, if a merger has taken place, the remaining unintegrated downstream firms bid to acquire the second upstream firm. Upstream firms (integrated or not) then compete in prices to sell the intermediate input to the remaining unintegrated downstream firms. Finally, downstream firms (integrated or not) compete in prices with differentiated products. The upstream market exhibits the usual ingredients of tough competition: upstream firms compete in prices, produce a perfectly homogeneous upstream good, and incur the same constant marginal cost. When there has been zero or one vertical merger, the standard Bertrand logic applies and upstream competition drives the upstream price to the marginal cost.

When two mergers have taken place, however, the Bertrand logic can collapse. The intuition is the following. There are now two vertically integrated firms, called $U1{-}D1$ and $U2{-}D2$, and one unintegrated downstream firm, called $D3$. Assume that $U1{-}D1$ sells the intermediate input to $D3$ at a strictly positive price-cost margin, and consider the incentives of its integrated rival $U2{-}D2$ to corner the upstream market. Notice first that, when $U1{-}D1$ increases its downstream price, it recognizes that some of the final consumers it loses will eventually purchase from $D3$, thereby increasing upstream demand and revenues. Therefore, supplying the upstream market strengthens an integrated firm's incentives to be a soft competitor on the downstream market; we refer to this effect as the *softening effect*. The softening effect benefits $U2{-}D2$, which faces a less aggressive competitor on the final market. Now, if $U2{-}D2$ undercuts $U1{-}D1$ on the upstream market and becomes the upstream supplier, then $U1{-}D1$ stops being a softening competitor on the downstream market. To sum up, integrated firm $U2{-}D2$ faces the following trade-off when deciding whether to undercut: on the one hand, undercutting yields upstream profits; on the other hand, it makes integrated firm $U1-D1$ more aggressive on the downstream market. When the latter effect dominates, the incentives to undercut vanish and the Bertrand logic collapses.

We exhibit equilibria in which there are two vertical mergers, one integrated firm charges its monopoly upstream price, and its integrated rival decides rationally to make no upstream offer. These monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria from the integrated firms' point of view. They are also the only equilibria which do not involve weakly dominated strategies. Besides, we show that partial foreclosure equilibria degrade both consumers' surplus and social welfare.

Using linear demand functions, we show that downstream competition is fiercer, the weaker competition on the upstream market. Intuitively, when downstream products are good substitutes, the softening effect is strong since a firm's downstream price has a large impact on its rivals' demands. Thus, undercutting on the upstream market is not profitable and the monopoly-like outcome is an equilibrium. Conversely, when downstream products are strongly differentiated, the softening effect is weak and undercutting on the upstream market is always profitable.

We then show that the existence of merger-specific synergies does not always make the merger more desirable for consumers. This is because synergies increase the scope for monopoly-like equilibria by putting vertically integrated firms at a cost advantage to unintegrated downstream firms. The reason is that the benefits from selling the input to a relatively inefficient unintegrated downstream firm are low, while the softening effect, which works at the margin, is not affected by the size of the upstream demand. Therefore, an integrated firm's incentives to undercut on the upstream market are weaker when vertical mergers create synergies. We also extend our framework to larger numbers of firms and show that it does not necessarily strengthen competition on the upstream market.

Our analysis can shed light on the recent wave of vertical mergers in the satellite navigation industry. The only two (upstream) firms that provide navigable digital maps, Tele Atlas and Navteq, have been acquired by, respectively, (downstream) TomTom and Nokia. TomTom embed digital maps in its portable navigation devices, Nokia in its mobile handsets with navigation possibilities. Our model suggests that, as long as mobile phones and portable navigation devices remain rather imperfect substitutes, competition between digital map providers should not be harmed. However, when these products become increasingly substitutes over time, as it is envisaged by the European Commission, the softening effect should strengthen and upstream competition might weaken.

Our paper contributes to the literature on the competitive effects of vertical mergers. A strand of the literature can be summarized in a common framework with two upstream firms, two downstream firms, and price competition on both markets.[1] Ordover, Saloner and Salop (1990) show

[1] Exceptions include an early contribution by Salinger (1988) who considers Cournot competition on both markets,

that a vertical integration raises the upstream price when the integrated firm can commit to exiting the upstream market and letting the remaining unintegrated upstream firm monopolize the upstream market. As pointed out by Reiffen (1992), their analysis relies crucially on the assumption that the merged entity has a strong commitment power. Choi and Yi (2000) provide foundations for this commitment power through the choice of input specification. Nocke and White (2007) and Normann (2009) provide another justification for the commitment assumption; they show that the commitment can be enforced in an infinitely repeated game through tacit collusion, and that a vertical merger facilitates the enforcement of the commitment. Chen and Riordan (2007) argue that vertical integration and exclusive contracts complement each other to implement partial foreclosure.

The softening effect that shows up in our model has been unveiled by Chen (2001).[2,3] He shows that when there is one vertical merger, the remaining downstream firm prefers purchasing the input from the integrated firm than from the unintegrated upstream firm in order to benefit from the softening effect. If there are upstream cost asymmetries and upstream switching costs, then the unintegrated upstream firm is unable to undercut the integrated firm on the upstream market and there is partial foreclosure in equilibrium. Our result is different. We show that in the two-merger situation the integrated rival is *able* to undercut since we assume away any cost differential or switching cost, but it is not *willing* to do so.

The rest of the paper is organized as follows. We describe the model in Section 2 and solve it in Section 3. We then extend the model to account for synergies (Section 4) and larger numbers of firms (Section 5). The robustness of the results are discussed in Section 6. We conclude in Section 7 by discussing a recent case.

2 Model

We consider a vertically related industry with two identical upstream firms, $U1$ and $U2$, and three symmetric downstream firms, $D1$, $D2$ and $D3$. The upstream firms produce an homogeneous input at constant marginal cost m and supply it to the downstream firms. The downstream firms transform the intermediate input into a differentiated final product on a one-to-one basis at zero cost. The input can also be obtained from an alternative source at a constant marginal cost

and the strand of the literature initiated by Hart and Tirole (1990) which analyzes the consequences of upstream secret offers and focuses mainly on the commitment problem faced by an upstream monopolist.

[2]Chen (2001) refers to it as the collusive effect. We adopt a different terminology to make it clear that the softening effect does not involve any form of tacit or overt collusion.

[3]See also Fauli-Oller and Sandonis (2002) for an application of the softening effect in a licensing context.

$\overline{m} > m$.[4]

The demand for downstream firm Di's product, $i = 1, 2, 3$, is $q_i(p_1, p_2, p_3)$, where p_j denotes Dj's price. The demand addressed to a firm is decreasing in its own price and increasing in its competitors' prices: $\partial q_i/\partial p_i \leq 0$ (with a strict inequality if $q_i > 0$) and $\partial q_i/\partial p_j \geq 0$ (with a strict inequality if $q_i > 0$ and $q_j > 0$), for $i \neq j$ in $\{1, 2, 3\}$. We assume that these demand functions are twice continuously differentiable. Symmetry between downstream firms implies that Di's demand can be written as $q_i = q(p_i, p_{-i})$, where p_{-i} denotes the set of prices charged by Di's rivals and $q(., .)$ is the same for all downstream firms.

We now describe the four-stage game played by the firms. In the first stage, the three downstream firms can bid to acquire upstream firm $U1$. In the second stage, if a merger has occurred, the remaining unintegrated downstream firms can counter it by bidding to integrate backward with $U2$. In the third stage, each upstream firm (integrated or not) Ui, $i = 1, 2$, announces the price w_i at which it is ready to supply any unintegrated downstream firm.[5,6] Unintegrated downstream firms then choose from which upstream producer to purchase. Downstream prices are set in the fourth stage. Unintegrated downstream firms are allowed to switch to another upstream supplier at zero cost once downstream prices are set, if this is strictly profitable.[7] To avoid trivial situations, we also consider that a firm decides to merge if it is strictly preferred. This would obviously be the case whenever mergers involve transaction costs. We look for subgame-perfect pure strategy Nash equilibria.

For all the market structures studied in this article and for i in $\{1, 2, 3\}$, we denote by π_i the total profit made by Di (including the profit made by its upstream subsidiary if it is integrated). We make the following assumptions:

(i) Firms' best responses on the downstream market are unique and defined by the first order conditions $\partial \pi_i/\partial p_i = 0$.

(ii) There exists a unique Nash equilibrium on the downstream market.

(iii) Prices are strategic complements: for all $i \neq j$ in $\{1, 2, 3\}$, $\partial^2 \pi_i/\partial p_i \partial p_j \geq 0$.

[4] This assumption is also made, e.g., by Ordover, Saloner and Salop (1990) and Hart and Tirole (1990). We relax it in Section 6.3. The alternative source of supply can come from a competitive fringe of inefficient upstream firms.

[5] The internal transfer price for a vertically integrated firm is irrelevant since it cancels out in the expression of its total profit.

[6] Notice that discrimination is not possible on the upstream market and that only linear tariffs are used. We relax these assumptions in Sections 6.3 and 6.2, respectively.

[7] As we explain in Section 6.1, this assumption simplifies the analysis by ensuring that downstream firms always buy the input from the cheapest supplier. This is in contrast to Chen (2001), in which upstream switching costs, together with an upstream cost asymmetry, generate anticompetitive vertical mergers.

Assumption (i) together with (iii) implies that the best response function of a firm is increasing in its rivals' prices. Combining (ii) with (iii), we also get that the unique downstream equilibrium is stable.[8] Finally, we assume that $\overline{m}$ is a relevant outside option: whatever the market structure, an unintegrated downstream firm earns strictly positive profits if it buys the intermediate input at a price lower than or equal to $\overline{m}$.

3 Anticompetitive Vertical Merger Waves

We solve the model backwards. The equilibria in the upstream-downstream competition subgames are presented in Section 3.1; the equilibria of the merger game are in Section 3.2.

3.1 Upstream-Downstream Competition

3.1.1 Zero Merger

Consider that no merger has taken place. Since downstream firms can switch to another supplier at zero cost after downstream prices are set, they always eventually buy from the cheapest supplier. Consequently the upstream equilibrium features both unintegrated upstream firms charging m and making no profit. The three downstream firms compete on a level playing field and earn the same profit, denoted by π^*.[9]

Lemma 1. *When no merger has taken place, the unique equilibrium outcome on the upstream market is the Bertrand outcome: $w_1 = w_2 = m$.*

3.1.2 One Merger

We now consider that exactly one vertical merger has taken place. Without loss of generality, we assume that $D1$ has merged with $U1$ to form integrated firm $U1-D1$. We establish that the upstream market is supplied at marginal cost in equilibrium.

First, it cannot be that both unintegrated downstream firms purchase the upstream good from $U1-D1$ at price $w_1 > m$, or from the alternative source of input at $\overline{m}$. Otherwise $U2$ would obviously undercut since the upstream market is its sole source of profit.

Conversely, if $U2$ supplies the upstream market at price $w_2 > m$, then $U1-D1$ is willing to undercut. First, this brings in upstream profits. Second, becoming the upstream supplier modifies the downstream outcome in a way that is, as we now show, profitable to $U1-D1$. Consider indeed that downstream firms $D2$ and $D3$ buy the input at an upstream price $w > m$. If $U2$ is their

[8] See Vives (1999), p.54.

[9] Formally, $\pi^* = (p^* - m)q(p^*, p^*, p^*)$, where $p^* = \arg\max_p (p - m)q(p, p^*, p^*)$.

upstream supplier, the profits are given by $\pi_1 = (p_1 - m)q_1$ and $\pi_j = (p_j - w)q_j$, for $j \in \{2, 3\}$. If they purchase from $U1{-}D1$, the profit of $U1{-}D1$ becomes $\pi_1 = (p_1 - m)q_1 + (w - m)q_2 + (w - m)q_3$ and the profits of $D2$ and $D3$ are unchanged. Moving from the former to the latter situation, $U1{-}D1$'s first order condition shifts from

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} = 0$$

to

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\frac{\partial q_2}{\partial p_1} + (w - m)\frac{\partial q_3}{\partial p_1} = 0.$$

Then its best response function moves upwards. As already pointed out by Chen (2001), when $U1{-}D1$ supplies the upstream market, it realizes that any customer lost on the downstream market may be recovered *via* the upstream market. This provides it with additional incentives to raise its price. Strategic complementarity, in turn, leads $D2$ and $D3$ to charge higher prices as well. At the end of the day, all downstream prices are higher when $U1{-}D1$ becomes the upstream supplier, and in particular those of $D2$ and $D3$. This makes $U1{-}D1$ better off on the downstream market.

This mechanism has a fat-cat flavor (Fudenberg and Tirole, 1984): being the upstream supplier and thus a soft downstream competitor, $U1{-}D1$ relaxes downstream competition thanks to strategic complementarity. Moreover, the soft behavior of $U1{-}D1$ benefits its downstream rivals. In the following, we shall refer to the consequences of $U1{-}D1$'s soft behavior as the *softening effect*.

Finally, the upstream market cannot be shared between $U1{-}D1$ and $U2$ at $w_1 = w_2 > m$ neither, since both unintegrated downstream firms would prefer purchasing the intermediate input from $U1{-}D1$ to benefits from the softening effect. It is also straightforward to see that the upstream market cannot be supplied at a price $w < m$, and we obtain the following lemma.

Lemma 2. *When exactly one merger has taken place, the unique equilibrium outcome on the upstream market is the Bertrand outcome:* $w_1 = w_2 = m$.

Proof. See Appendix A.2 □

In equilibrium, all firms obtain the input at the same price m, and there are no upstream profits. Since downstream competition is not modified with respect to the no-merger case, the three downstream firms earn the same profits: $\pi_1 = \pi_2 = \pi_3 = \pi^*$.

3.1.3 Two Mergers

In this section we assume that two vertical mergers have taken place. Let us suppose, without loss of generality, that $D1$ has merged with $U1$, and $D2$ has merged with $U2$, giving birth to two integrated firms $U1{-}D1$ and $U2{-}D2$.

$U1{-}D1$ and $U2{-}D2$ compete to sell the input to unintegrated downstream firm $D3$. Since $D3$ can obtain the input at cost $\overline{m}$ and always chooses the cheapest supplier, any upstream offer strictly above $\overline{m}$ is equivalent to no offer. The strategy space on the upstream market can therefore be restricted to $[0, \overline{m}] \cup \{+\infty\}$, where an infinite price stands for no offer.

There is no equilibrium in which firm $D3$ obtains the input from the alternative supplier. Otherwise a vertically integrated firm would undercut to get upstream profits and relax competition on the downstream market through the softening effect. Therefore, the determination of the upstream equilibrium only requires to analyze the downstream equilibrium when firm $D3$ is supplied by a vertically integrated firm.

Downstream equilibrium. We assume, without loss of generality, that firm $D3$ purchases the input from firm $U1{-}D1$ at price $w \in [0, \overline{m}]$. The profit functions can be written as

$$\pi_1 = (p_1 - m)q_1 + (w - m)q_3, \tag{1}$$
$$\pi_2 = (p_2 - m)q_2, \tag{2}$$
$$\pi_3 = (p_3 - w)q_3. \tag{3}$$

The equilibrium downstream prices, denoted by $p_i(w)$ for i in $\{1, 2, 3\}$, solve the set of first order conditions

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\frac{\partial q_3}{\partial p_1} = 0, \tag{4}$$
$$q_2 + (p_2 - m)\frac{\partial q_2}{\partial p_2} = 0, \tag{5}$$
$$q_3 + (p_3 - w)\frac{\partial q_3}{\partial p_3} = 0. \tag{6}$$

We also denote by $\pi_i(w)$ the equilibrium profits. Last, define $\pi_i(+\infty)$ the equilibrium profits when $D3$ gets the input from the alternative source.

The comparison of the first order conditions of both vertically integrated firms indicates that the upstream supplier has more incentives to raise its downstream price. This is again the softening effect. When $U1{-}D1$ charges a higher downstream price, some of the customers it loses will eventually purchase from $D3$, which increases its upstream revenues. Together with the stability of the downstream equilibrium, this implies that upstream supplier $U1{-}D1$ ends up charging a higher downstream price than $U2{-}D2$.

Firm $U2{-}D2$ benefits from firm $U1{-}D1$'s being a soft competitor on the downstream market. As a result, it earns larger downstream profits than the upstream supplier. These insights are summarized in the following lemma.

Lemma 3. *If $w > m$, then $U1{-}D1$ charges a strictly higher downstream price and earns strictly lower downstream profits than $U2{-}D2$,*

$$p_1(w) > p_2(w), \tag{7}$$

$$(p_1(w) - m)q_1(p_1(w), p_2(w), p_3(w)) < (p_2(w) - m)q_2(p_1(w), p_2(w), p_3(w)). \tag{8}$$

Proof. See Appendix A.3. □

An important consequence of this result is that we cannot tell unambiguously which of the integrated firms earns more total profits. On the one hand, the upstream supplier extracts revenues from the upstream market, on the other hand, its integrated rival benefits from larger downstream profits thanks to the softening effect. It may well be that the softening effect is strong enough to outweigh the upstream profit effect and make $U2{-}D2$ earn more total profits than $U1{-}D1$.

Upstream equilibrium. There is an equilibrium in which firm $U1{-}D1$ offers $w_1 \leq \overline{m}$ and $U2{-}D2$ offers $w_2 \geq w_1$ if, and only if, the upstream supplier does not want to supply the upstream market at another price

$$\pi_1(w_1) \geq \max_{w<w_2, w\leq\overline{m}} \pi_1(w),$$

nor to exit the upstream market

$$\pi_1(w_1) \geq \pi_2(w_2),$$

and its vertically integrated rival is not willing to undercut

$$\pi_2(w_1) \geq \max_{w<w_1} \pi_1(w).$$

Note that w_2 can be infinite in the above expressions.

This set of necessary and sufficient conditions generally characterize multiple equilibria. As common sense suggests, there always exists an upstream equilibrium in which the input is priced at its marginal cost.

However, partial foreclosure equilibria with an upstream price strictly above marginal cost can also exist. Consider that firm $U1{-}D1$ supplies the upstream market at the monopoly upstream price $w_m = \arg\max_{w\leq\overline{m}} \pi_1(w)$.[10] w_m is the price that a vertically integrated firm would charge if, for an exogenous reason, its integrated rival had made no upstream offer. Consider the incentives of $U2{-}D2$ to corner the upstream market. On the one hand, this would generate upstream revenues. On the other hand, $U1{-}D1$ would stop being a soft downstream competitor, which would lower $U2{-}D2$'s downstream profits by Lemma 3. When the latter effect dominates the former, the proposed monopoly-like outcome on the upstream market is an equilibrium.

[10] We assume, without loss of generality, that this price is unique. Defining w_m as $\max\{\arg\max_{w\leq\overline{m}} \pi_1(w)\}$, our results would still hold if $\pi_1(.)$ reached its maximum for several values of w.

Proposition 1. *When two mergers have taken place, there is a* monopoly-like *equilibrium in which one integrated firm proposes $w_m > m$ and the other integrated firm makes no offer if, and only if,*

$$\pi_1(w_m) \leq \pi_2(w_m). \tag{9}$$

In all the other equilibria, upstream prices are equal, $w_1 = w_2$, and satisfy $w \leq w_m$ and $\pi_1(w) = \pi_2(w)$. In particular, there is always an equilibrium with the Bertrand outcome on the upstream market, $w_1 = w_2 = m$.

Moreover, from the integrated firms' point of view, the monopoly-like equilibria, when they exist,

- *Pareto-dominate all other equilibria,*
- *are the only equilibria involving no weakly dominated strategies.*

Proof. See Appendix A.4. □

When the softening effect is large enough so that condition (9) holds, the hypothetical situation in which one of the integrated firm has exogenously exited the upstream market, granting a monopoly position to the other integrated firm, is an equilibrium. This might sound somewhat tautological. Yet, our contribution is to show that condition (9) may well be satisfied, because losers on the upstream market become winners on the downstream market. We present such an example in the next subsection. Note also that monopoly-like equilibria come by pairs since the upstream supplier can be either $U1-D1$ or $U2-D2$.

Proposition 1 gives foundations to the classical analysis of Ordover, Saloner and Salop (1990), in which a vertically integrated firm commits to exiting the upstream market in order to let the upstream rival charge the monopoly price. We show that no commitment is actually necessary when the upstream rival is integrated, provided that the softening effect is strong enough.

All other equilibria feature both vertically integrated firms setting the same upstream price (and only one of them actually supplying the market): $w_1 = w_2 = w$. Obviously, $w \leq w_m$, otherwise an integrated would rather undercut to w_m. Such a symmetric outcome is part of an equilibrium only if the softening effect and the upstream profit effect exactly cancel out, so that the upstream supplier earns as much profits as the vertically integrated firm which does not supply the upstream market: $\pi_1(w) = \pi_2(w)$.[11] The Bertrand outcome is one such symmetric equilibrium. Other symmetric equilibria can also feature an upstream price strictly above m, as well as strictly below m.[12]

[11]The conditions $w \leq w_m$ and $\pi_1(w) = \pi(w)$ are necessary but not sufficient to have a symmetric equilibrium with $w_1 = w_2 = w$. In addition, it must be that an integrated firm does not want to undercut by more than ϵ: $\pi_1(w) \geq \max_{\tilde{w}<w} \pi_1(\tilde{w})$. That condition does not necessarily stem from $w \leq w_m$, because $\pi_1(.)$ is not necessarily quasiconvex.

[12]When $w < m$, upstream profits are negative and the softening effect is reversed, with the upstream supplier adopting an aggressive stance on the downstream market to limit its upstream losses.

This multiplicity of equilibria can be resolved using standard selection criteria. First, the monopoly-like equilibria Pareto-dominate all the symmetric equilibria from the integrated firms' standpoint. Indeed, in a symmetric equilibrium, the integrated firms earn less then $\pi_1(w_m)$ by definition of w_m, which is itself lower than $\pi_2(w_m)$ when monopoly-like equilibria exist. Second, the monopoly-like equilibria are the only equilibria involving no weakly dominated strategies. In particular, any symmetric equilibrium strategy is weakly dominated by w_m. Therefore, it seems reasonable to think that integrated firms will coordinate on one of the monopoly-like equilibria.

3.2 Merger Game

In monopoly-like equilibria the profits of the two merged entities are strictly larger than the profit of the remaining unintegrated firm. Therefore, when a monopoly-like outcome is expected to emerge after two mergers, downstream firms all bid to get a share of the profit created by the mergers. Then, a wave of vertical mergers occurs that eliminates all unintegrated upstream firms and implements the monopoly-like outcome.[13]

Although standard selection criteria indicate that the monopoly-like equilibria are likely to emerge when they exist, extra-model considerations may modify that presumption. For instance, a firm may have troubles with the competition authority, it may try to drive some of its rivals out of business, or it may be willing to develop a reputation of tough competitor. Besides, the Bertrand outcome is always an equilibrium of the upstream market. Therefore, if we use no selection criterion, there always exists an anticipation scheme that leads to an equilibrium with no merger. This discussion is summarized in

Proposition 2. *There always exists an equilibrium with no merger.*

If $\pi_1(w_m) \leq \pi_2(w_m)$ and integrated firms

- *do not play weakly dominated strategies on the upstream market*
- *or do not play equilibria that are Pareto-dominated by another equilibrium,*

then, in equilibrium, there are two mergers and the upstream market is supplied at the monopoly price.

Proof. See Appendix A.5. □

We argue that a wave of vertical mergers that leads to partial foreclosure of the remaining downstream firm hurts consumers and lowers social welfare. Indeed, when the upstream price increases above the marginal cost in the two-merger subgame, the best response functions of

[13] We do not discuss the equilibrium bids, since they depend strongly on the assumption that upstream firms have all the bargaining power.

$U1-D1$ and $D3$ shift upwards, which increases all downstream prices by strategic complementarity. This is clearly detrimental to all consumers. This also degrades social welfare, since the total demand is already too low in the Bertrand outcome because of positive markups on the downstream market.

Proposition 3. *Consumers' surplus and social welfare are strictly lower in an equilibrium with a merger wave and partial foreclosure of the remaining downstream firm than in an equilibrium with no merger.*

Proof. See Appendix A.6. □

3.3 Tension Between Upstream and Downstream Competition

We now present an example to illustrate when condition (9) is satisfied. The demand addressed to downstream firm $i \in \{1, 2, 3\}$ is given by $q_i(p_i, p_{-i}) = 1 - p_i - \gamma(p_i - (p_1 + p_2 + p_3)/3)$, where $\gamma \geq 0$ parameterizes the degree of differentiation between final products, which can be interpreted as the intensity of downstream competition. Perfect competition corresponds to γ approaching infinity and local monopolies to $\gamma = 0$. The upstream cost m is equal to zero, and the cost of the alternative source of input $\overline{m}$ is high enough not to constrain the monopoly upstream price of a vertically integrated firm. We solve the model with this specification and find that

Proposition 4. *In the linear case, there exists $\underline{\gamma} > 0$, such that, if $\gamma > \underline{\gamma}$, there exist exactly four upstream equilibria in the two-merger subgame*

- *two monopoly-like equilibria $(w_m, +\infty)$ and $(+\infty, w_m)$,*
- *one symmetric equilibrium (w_s, w_s) with $w_s > m$,*
- *the Bertrand equilibrium (m, m),*

and there are two mergers in equilibrium if integrated firms do not play weakly dominated strategies on the upstream market, or do not play equilibria that are Pareto-dominated by another equilibrium.

If $\gamma < \underline{\gamma}$, the Bertrand outcome is the only upstream equilibrium in the two-merger subgame, and there is no merger in equilibrium.

Proof. See Appendix A.7. □

Proposition 4 unveils a tension between competition on the downstream market and competition on the upstream market. When the downstream market features fierce competition (high γ), there exist monopoly-like equilibria on the upstream market, while the upstream market is perfectly competitive when downstream competition is weak (low γ). To grasp the intuition, suppose that the upstream market is supplied at the monopoly upstream price. When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low a downstream price since this would strongly contract its upstream profit. The other integrated firm benefits from a substantial softening effect and, as a result, is not willing to corner the upstream market. The reverse holds when downstream products are strongly differentiated.

4 Synergies

In the analysis so far vertical mergers were motivated by anticompetitive purposes only. We now allow mergers to create synergies and show that they interact with the anticompetitive effects we have unveiled. We consider two types of synergies, those leading to a reduction of downstream costs and those leading to a reduction of upstream costs. Downstream costs can no longer be normalized to zero. Instead, downstream firms are now assumed to transform the input into the final good at constant marginal cost n. It is straightforward to show that all our results carry on when a downstream cost is added to the model. We denote by $\pi^*(n_i, n_j, n_k)$ the equilibrium profit of a downstream firm when its downstream marginal cost is n_i, its rivals' marginal costs are n_j and n_k, and every downstream firm obtains the input at marginal cost m. We make the standard assumptions that a firm's profit decreases in its own cost and increases in its rivals' costs, $\partial\pi_i^*/\partial n_i < 0$ and $\partial\pi_i^*/\partial n_j = \partial\pi_i^*/\partial n_k > 0$, and decreases when every cost increases, $\partial\pi^*/\partial n_i + \partial\pi^*/\partial n_j + \partial\pi^*/\partial n_k < 0$.

4.1 Reduction of Downstream Costs

We consider that a vertical merger reduces the downstream marginal cost of the newly integrated firm from n to $n - \delta n < n$.

When no merger has taken place, nothing is modified with respect to the previous analysis. At equilibrium, there is a Bertrand outcome on the upstream market and downstream firms earn a profit of $\pi^*(n, n, n)$. When exactly one merger has taken place, the integrated firm has now a cost advantage on the downstream market. This does not affect the equilibrium analysis on the upstream market which is still supplied at marginal cost at equilibrium. The profit of the vertically integrated firm is $\pi^*(n - \delta n, n, n)$ and unintegrated downstream firms earn $\pi^*(n, n - \delta n, n)$. After two vertical mergers, competition on the upstream market is now between two integrated firms. As in Section 3, we denote by $\pi_1(w)$, $\pi_2(w)$, and $\pi_3(w)$ the total profit of, respectively, the upstream supplier, its integrated rival, and the unintegrated downstream firm when the upstream price is w, and by $w_m = \arg\max_{w \leq \overline{m}} \pi_1(w)$ the monopoly upstream price. Monopoly-like equilibria exist if, and only if, $\pi_1(w_m) \leq \pi_2(w_m)$, and they Pareto-dominate all other equilibria from the integrated firms' standpoint. The other equilibria feature both integrated firms making the same upstream offer $w_1 = w_2 = w \leq w_m$, with $\pi_1(w) = \pi_2(w)$. In particular, there is always an equilibrium with a Bertrand outcome on the upstream market. If supra-competitive symmetric equilibria with $w_1 = w_2 < m$ exist, they are Pareto-dominated by the Bertrand equilibrium. Therefore, if Pareto-dominated equilibria are not played, the integrated firms obtain at least $\pi_1(m) = \pi_2(m) = \pi^*(n - \delta n, n - \delta n, n)$.

The above analysis implies that there are always two mergers at equilibrium, because firms are willing to integrate in order to reap the efficiency gains.[14] Now that the wave of vertical mergers occurs even without anticompetitive motives, the welfare analysis becomes less clear-cut. If the upstream market is supplied at marginal cost, then every merger benefits to consumers and improves total welfare. By contrast, if partial foreclosure occurs at equilibrium, then there is a tradeoff between the efficiency gains and the anticompetitive effects. Obviously, larger cost reductions tend to enhance welfare *for a given outcome on the upstream market*. More interestingly, however, larger cost reductions increase the scope for monopoly-like equilibria. Intuitively, as the cost differential between the unintegrated downstream firm and its integrated counterparts widens, the market share of the former declines and profits on the upstream market shrink. Besides, the magnitude of the softening effect, which works at the margin and reflects the willingness of the upstream supplier to boost its upstream demand, is a priori not affected. Therefore, when an integrated firm trades off the potential upstream profits and the softening effect when deciding whether to corner the upstream market, it tends to prefer staying out of the upstream market as δn increases.

The first merger always increases consumers' surplus and total welfare, while the effect of the second merger is ambiguous when it leads to foreclosure on the upstream market. From an antitrust perspective, it is therefore the last integration of the merger wave that calls for scrutiny. We state the welfare effect of the second merger in the proposition below. Before stating the result we note that after two vertical mergers, the remaining downstream firm might be squeezed from the market if it is too inefficient relatively to integrated firms and/or if the upstream price is too high. In the linear case, for all $\gamma \geq 0$, there exists $w_{\max}(\gamma, \delta n)$ such that $\pi_3(w) = 0$ when $w \geq w_{\max}(\gamma, \delta n)$. $\pi_1(w)$ is strictly concave in w and reaches its maximum in $w_m(\gamma, \delta n) < w_{\max}(\gamma, \delta n)$ if, and only if, δn is strictly below a threshold value $\delta n_{\max}(\gamma)$. We hereafter restrict our attention to $\delta n < \delta n_{\max}(\gamma)$ and assume $\overline{m} \in (w_m(\gamma, \delta n), < w_{\max}(\gamma, \delta n))$, so that the monopoly upstream price is $w_m(\gamma, \delta n)$.

[14] Once one merger has occurred, the remaining downstream firms are ready to bid at least up to $\pi^*(n - \delta n, n - \delta n, n) - \pi^*(n, n - \delta n, n - \delta n) > 0$ to engage in a counter-merger. They both obtain a payoff $\pi^*(n, n - \delta n, n - \delta n)$. Going back to the first stage, all the downstream firms are willing to merge, and they bid $\pi^*(n - \delta n, n - \delta n, n) - \pi^*(n, n - \delta n, n - \delta n) > 0$.

Proposition 5. *With downstream synergies $\delta n > 0$, when integrated firms do not play equilibria that are Pareto-dominated by another equilibrium, there are two vertical mergers at equilibrium. Moreover, in the linear case, there exist $0 \leq \overline{\delta n}(\gamma) \leq \overline{\delta n}_w(\gamma) \leq \overline{\delta n}_c(\gamma)$ that are decreasing in γ such that*

- *(region 1) if $\delta n < \overline{\delta n}(\gamma)$, then the second merger leads to a Bertrand outcome on the upstream market and increases consumers' surplus and social welfare,*
- *(region 2) if $\delta n \in [\overline{\delta n}(\gamma), \overline{\delta n}_w(\gamma)]$, then the second merger leads to a monopoly-like outcome on the upstream market and reduces consumers' surplus and social welfare,*
- *(region 3) if $\delta n \in [\overline{\delta n}_w(\gamma), \overline{\delta n}_c(\gamma)]$, then the second merger leads to a monopoly-like outcome on the upstream market, reduces consumers' surplus, and increases social welfare,*
- *(region 4) if $\delta n \geq \overline{\delta n}_c(\gamma)$, then the second merger leads to a monopoly-like outcome on the upstream market and increases consumers' surplus and social welfare.*

There exist $0 < \underline{\gamma}_c < \underline{\gamma}_w < \overline{\gamma}_c < \overline{\gamma}$ such that regions 2 and 3 are empty when $\gamma \leq \underline{\gamma}_c$, region 2 is empty when $\gamma \in [\underline{\gamma}_c, \underline{\gamma}_w]$, region 4 is empty when $\gamma \in [\overline{\gamma}_c, \overline{\gamma}]$, and regions 1 and 4 are empty when $\gamma \geq \overline{\gamma}$.

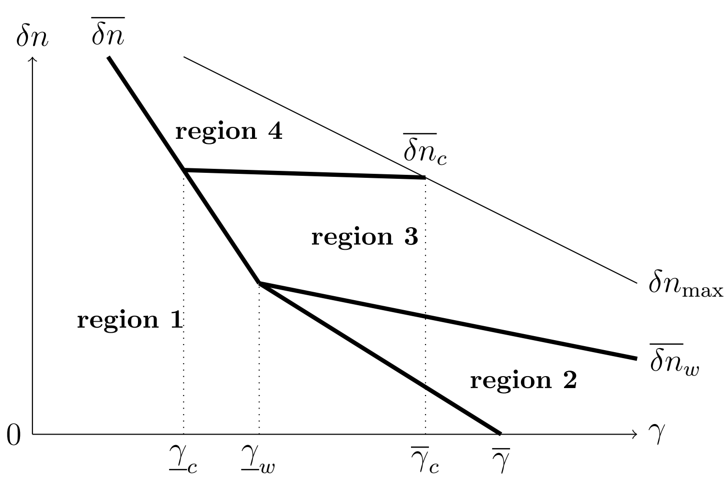

Figure 1: The four regions of Proposition 5

The four regions defined in Proposition 5 are depicted in Figure 1. The figure illustrates that synergies that reduce downstream costs are not necessarily welfare-improving. Consider for instance an industry with a substitutability parameter $\gamma \in (\underline{\gamma}_w, \overline{\gamma}_c)$. As synergies get larger, one switches from a perfectly competitive upstream market (region 1) to a monopolized upstream market (region 2), thereby lowering consumer surplus and social welfare. Only then do synergies start to improve social welfare (region 3) and consumer surplus (region 4) as they become more and more important.

4.2 Reduction of Upstream Costs

We now consider that a vertical merger reduces the upstream marginal cost from m to $m-\delta m < m$. As long as an integrated firm produces the intermediate input for its own downstream division only, the difference between upstream and downstream efficiency gains is immaterial. The difference is that upstream efficiency gains lower the cost of producing the input sold to another downstream firm.

The analysis is not affected if no merger has taken place. After exactly one merger, upstream competition is between a vertically integrated firm with marginal cost $m-\delta m$ and an unintegrated upstream firm with marginal cost m. Since the unintegrated downstream firms choose the cheaper offer, the upstream market is supplied at equilibrium by the integrated firm at price m (minus ϵ). The unintegrated downstream firms earn $\pi^*(n, n-\delta m, n)$ and the integrated firm makes a total profit of $\pi^*(n-\delta m, n, n) + 2q^*(n, n-\delta m, n)\delta m$, where $q^*(n, n-\delta m, n)$ is the equilibrium quantity supplied by each unintegrated downstream firm. After two vertical mergers, upstream competition is between two vertically integrated firms having the same cost structure. Using the same notations as before, there are monopoly-like equilibria if, and only if, $\pi_1(w_m) \leq \pi_2(w_m)$, which Pareto-dominate all other equilibria, and there is always a Bertrand equilibrium with $w_1 = w_2 = m - \delta m$.

Similarly to the case where synergies reduce downstream costs, there are always two mergers in equilibrium when synergies reduce upstream costs. The difference is that the unintegrated downstream firm is no longer at a downstream cost disadvantage. Therefore, the upstream cost reduction does not have the effect of reducing the upstream profit any more. We can then state the following result.

Proposition 6. *With upstream synergies $\delta m > 0$, when integrated firms do not play equilibria that are Pareto-dominated by another equilibrium, there are two vertical mergers at equilibrium. Moreover, in the linear case, there are monopoly-like equilibria if, and only if, $\gamma \geq \overline{\gamma}$ independently on the value of δm.*

Clearly, the merger wave is beneficial when $\gamma < \overline{\gamma}$. When $\gamma \geq \overline{\gamma}$ its welfare effects are negative if the synergies are not large enough, and positive otherwise.

5 Market Structure

This section extends the model to larger numbers of firms. It leads us to exhibit another type of foreclosure outcome following a wave of mergers and to show that increasing the number of firms has an ambiguous impact on the competitiveness of the upstream market.

5.1 More Firms

Consider that the disintegrated industry is populated by $M \geq 2$ upstream firms and $N \geq M+1$ downstream firms. The timing is similar to the one in the basic framework. First, downstream firms bid to integrate with the first upstream firm. Then, if a merger has occurred at the first stage, the remaining downstream firms can bid for the second upstream firm, and so on. At the Ith stage, if the $I-1$ first upstream firms have been acquired, the remaining unintegrated downstream firms bid to merge with the Ith upstream firm. As in Section 4.1 each merger can create synergies that reduce the downstream marginal cost from n to $n - \delta n \leq n$. The case where $\delta n = 0$ corresponds to no synergy.

Once all the upstream firms have been acquired, or after one upstream firm fails to generate any bid, competition takes place on the upstream market. If $I \in \{0, 1, .., M\}$ mergers have occurred, I integrated firms and $M - I$ unintegrated upstream firms compete to supply the input to the $N - I$ unintegrated downstream firms. Upstream offers cannot be discriminatory and there is a relevant outside option $\overline{m} > m$ that effectively puts a cap on the upstream price. Finally, competition takes place on the downstream market. As in the basic framework, unintegrated downstream firms can switch supplier after downstream prices are set, which ensures they all choose the cheapest upstream offer(s) at equilibrium.

As in the case where $M = 2$, there is no equilibrium in which the upstream market is supplied at price $w > m$ as long as there remains at least one unintegrated upstream firm. Indeed, an unintegrated upstream firm does not benefit from letting rivals supplying the market, nor does integrated firms benefit from letting an unintegrated upstream firm supplying the market. Besides, the Bertrand outcome in which all the upstream firms offer the input at marginal cost is an equilibrium.[15]

5.2 Merger Wave and Input Foreclosure

Consider now that M mergers have taken place. Contrary to the basic framework where there remains only one unintegrated downstream firm after every upstream firm has merged, the upstream market can now be supplied simultaneously by several integrated firms. A condition for this to occur is that the lowest upstream price is offered by several integrated firms.

Moreover, all the upstream offers at the lowest upstream price are not equivalent. Consider,

[15]Note that equilibria in which several integrated firms offer $w < m$ can also exist. As in Section 3 these super-competitive equilibria are Pareto-dominated by the Bertrand equilibrium from the upstream firms' point of view.

for instance, that there are two integrated firms, $U1-D1$ and $U2-D2$, which both charge $w > m$, and two unintegrated downstream firms, $D3$ and $D4$. Assume $D3$ purchases the input from $U1-D1$, which will therefore adopt a soft stance on the downstream market, and consider $D4$'s choice of upstream supplier. $D4$ can also buy from $U1-D1$ to make it an even softer downstream competitor, or it can buy from $U2-D2$ to make it a soft competitor as well. It is unclear which strategy is optimal, i.e., whether the choices of upstream supplier are strategic complements or strategic substitutes, but in general $D4$'s optimal choice depends on $D3$'s upstream supplier. As a result, the choices of upstream supplier are a game between the unintegrated downstream firms. As it turns out, the strategic dimension of that game disappears when demand functions are linear:

Lemma 4. *When demand functions are linear, an unintegrated downstream firm's choice of upstream supplier does not depend on the others' choices.*

Proof. See Appendix A.8. □

We now focus on the linear case to streamline the analysis. It implies that any sharing of the upstream demand between the cheapest upstream suppliers is supported by optimal choices of input supplier. Another consequence of this simplification is to generate a lot of potential equilibria with every possible asymmetric outcome on the upstream market. Since a complete characterization of all the equilibria requires cumbersome notations with no conceptual difficulty or meaningful economic interpretation, we focus here on two polar types of foreclosure equilibria.

The first type of foreclosure equilibria features a *monopoly-like outcome* as in the case where $M = 2$: the upstream market is supplied by one vertically integrated firm only at the monopoly price (the upstream price that maximizes its profit conditionally on all unintegrated downstream firms accepting the offer) while the other integrated firms make no offer. If one of these other integrated firms matches the offer of the upstream supplier, all the unintegrated downstream firms choose the same supplier, which indeed corresponds to optimal strategies since they are indifferent between the two offers. The monopoly-like outcome is part of an equilibrium if, and only if, the integrated firms which do not supply the upstream market earn more total profits than the upstream supplier.[16] That condition may be satisfied because these other integrated firms have higher downstream profits than the upstream supplier according to the softening effect. There is a monopoly-like equilibrium if the softening effect outweighs the upstream profit effect.

In the other polar case of foreclosure equilibria, there is a *collusive-like outcome* on the upstream market in which all the integrated firms offer the same upstream price $w > m$ and each one of them supplies a fraction $(N - M)/M$ of the upstream market, where integer constraints are ignored. In

[16]Formally, denoting by $\pi_1(w)$ and $\pi_2(w)$ the profits of the upstream supplier and of the other integrated firms, respectively, when the upstream price is w, there is a monopoly-like equilibrium if, and only if, $\pi_2(w_m) \geq \pi_1(w_m)$, where $w_m = \arg\max_{w \leq \overline{m}} \pi_1(w)$.

this equilibrium, the upstream demand from unintegrated firms is always split equally between all the integrated firms making the cheapest offer. The proposed collusive-like outcome is part of an equilibrium if, and only if, no integrated firm is willing to undercut the upstream market nor to cancel its upstream offer. The former condition is met if an integrated firm's benefits from the soft behavior of the other integrated suppliers on the downstream market outweigh the additional upstream profits from undercutting. The latter condition is met when the upstream profit is large enough to deter an integrated firm to cancel its offer.[17]

We refer to this type of outcomes as collusive-like equilibria because they look like collusion to an outside observer. All the upstream competitors stick to the same upstream price that is strictly higher than what a standard single market analysis would predict. Moreover, the upstream profits are equally shared between the integrated firms. Yet, this poorly competitive outcome is sustained with no agreement among the vertically integrated firms. They do not undercut each other because each integrated firm benefit from the others' soft behaviors on the downstream market.

As already mentioned we state the results with linear demands. Moreover, in order to reduce the number of parameters of the model, we use a generalized Hotelling demand system rather than the demand functions of Section 3.3.[18] We also want to obtain comparative static results on N, which requires to specify how the demand functions are modified when the number of downstream firms changes. A lot of possible assumptions are, of course, possible. We choose the simple specification in which the market size does not depend on N. Finally, normalizing the transportation cost to 1/2, the demand addressed to downstream firm $i = 1, ..., N$ is equal to

$$q_i = \frac{1}{N} - \sum_{j \neq i} (p_i - p_j).$$

Remind that only the comparative static on N in the following proposition depends on the assumption that the market size does not vary with N.

Proposition 7. *In the generalized Hotelling model with downstream synergies $\delta n \geq 0$, $M \geq 2$ upstream firms, and $N \geq M+1$ downstream firms, there exist $\overline{\delta n}(M, N) \geq 0$ and $\overline{\delta n}_{coll}(M, N) \geq 0$ such that:*

[17]Formally, denoting by $\pi_1^{coll}(w)$ the profit of an integrated firm when the upstream market is equally shared between all M integrated firm at price w, and by $\pi_2^{coll}(w)$ the profit of an integrated firm when the upstream market is equally shared between all $M-1$ other integrated firms at price w, there is a collusive-like equilibrium with an upstream price w if, and only if, $\pi_1^{coll}(w) \geq \max\{\sup_{w<w_m} \pi_1(w), \pi_2^{coll}(w)\}$.

Note that when an upstream price w satisfies strictly these two inequalities, all the upstream prices in the neighborhood of w also do by continuity of the profit functions. Therefore, in non degenerated situations, there is a continuum of collusive-like equilibria.

[18]Each pair of firms is linked by a segment of size $2/(N(N-1))$ on which a mass one of consumers are uniformly distributed. Consumers have unit demand, transport costs are linear, and the market is assumed to be covered in every market situation.

- *there is an equilibrium with M mergers and a monopoly-like outcome on the upstream market if, and only if, $\delta n \geq \overline{\delta n}(M,N)$,*

- *there is an equilibrium with M mergers and a collusive-like outcome on the upstream market if, and only if, $\delta n \geq \overline{\delta n}_{coll}(M,N)$.*

Moreover, $\overline{\delta n}(M,N)$ increases as M goes from 2 to 3, then decreases with M. When $M = 2$, it increases as N goes from 2 to 6, then decreases with N; when $M \geq 3$, it decreases with N.

Proposition 7 illustrates that the equilibria in which a wave of mergers leads to a monopoly-like outcome on the upstream market are robust to a modification of the market structure. There are also collusive-like equilibria in which the upstream market is equally shared between all the integrated firms at a price strictly above marginal cost. Consider for instance an industry with $(M,N) = (2,4)$ and where mergers create no synergy. Since $\overline{\delta n}_{coll}(2,4) = 0$, there are equilibria with two vertical mergers in which each integrated firm sells the input to one of the unintegrated downstream firms and they both charge the same price strictly above marginal cost.

Another interpretation of collusive-like equilibria in that of dual sourcing. Consider for instance the case where $(M,N) = (2,3)$ and assume that, after two vertical mergers, the unintegrated downstream firm splits up its upstream demand between the two integrated firms if they offer the same price. This is part of an optimal strategy with linear demands, since the downstream firm is indifferent between any sharing rule. In that case, there exist two-merger collusive-like equilibria in which the unintegrated downstream firm is supplied by both integrated firms strictly above marginal cost.

The last part of Proposition 7 describes how the number of firms modifies the set of downstream cost reductions induced by synergies such that monopoly-like equilibria exist. It highlights that the impact of the market structure on the competitiveness of the upstream market is in general ambiguous. Consider for instance the effect of an increase in the number of downstream firms. On the one hand, upstream profits tend to raise as unintegrated downstream firms account for a larger part of the market. On the other hand, the softening effect also becomes stronger since the upstream supplier tries to preserve an upstream profit coming from more downstream firms. As a result, the tradeoff between upstream profits and the softening effect is affected in an ambiguous way, which is why $\overline{\delta n}(M,N)$ can either increase or decrease with N.

This tension is reminiscent to the dilemma between upstream and downstream competitiveness that we discuss in Section 3.3. As N increases competition becomes fiercer on the downstream market, but competition on the upstream market may well be relaxed.

6 Robustness

The (counter-)examples given in this section are in the case where $(M, N) = (2, 3)$ and, unless otherwise specified, no synergy.

6.1 Softening Effect

The crucial mechanism leading to foreclosure equilibria is the softening effect according to which a vertically integrated firms which sells the input to downstream rivals prices less aggressively on the downstream market to preserve its upstream profits, which benefits to its integrated rivals. The softening effect, in turn, relies on two basic features of the model.

The first feature is that an integrated upstream supplier can enhance its upstream profits by pricing less aggressively on the downstream market. Obviously, if the model does not satisfy that condition, the softening effect disappears. One may then wonder whether the softening effect hinges on the assumption of price competition on the downstream market, for if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its upstream profit, and the softening effect is still at work.[19] To summarize, the issue is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can strategically affect its rivals' quantities on the downstream market.

This discussion also highlights that the assumption of strategic complementarity is not crucial. Indeed, even if downstream strategic variables are substitutes, the upstream supplier prices less aggressively on the downstream market and this benefits to its integrated rivals.

The second feature of the model that generates the softening effect, is that the identity of the upstream supplier is known before downstream competition takes place. Even though downstream firms are free to switch upstream supplier after the downstream competition stage, the identity of the supplier is perfectly anticipated in a subgame perfect equilibrium. To give a better understanding of this second ingredient of the model, we exhibit two configurations in which it is not present. The first one is when upstream offers are secrete and the upstream supplier is chosen after the downstream competition stage. Consider indeed, by contradiction, an equilibrium in which the upstream market is supplied by $U1{-}D1$ at $w > m$. $U2{-}D2$ has then a profitable deviation: he can (secretly) undercut without loosing the softening effect, since $U1{-}D1$ does not know that its offer will eventually be rejected. Another, more drastic counter-example is when upstream offers are

[19] With quantity competition and a linear demand function, if integrated firms are Stackelberg leaders on the downstream market, then the monopoly-like outcome is an equilibrium and there is an anticompetitive wave of mergers.

made after downstream competition. In that case, the integrated firms always want to undercut each other on the upstream market, since it no longer affects the downstream outcome.

6.2 Two-Part Tariffs

We now show that our results still hold when two-part tariffs are used on the upstream market. Denote by w_i (respectively, F_i) the variable (respectively, the fixed) part of the tariff. After zero or one merger, the Bertrand outcome $(w_1, F_1) = (w_2, F_2) = (m, 0)$ is an equilibrium. After two vertical integrations, in a monopoly-like outcome, $U1{-}D1$ sets the variable part which maximizes the sum of its profit and $D3$'s profit, i.e., $w_{tp} = \arg\max_{w \leq \overline{m}} \pi_1(w) + \pi_3(w)$ which is strictly larger than m by strategic complementarity,[20] while $U2{-}D2$ makes no upstream offer. The fixed fee captures $D3$'s profit, i.e., $F_1 = \pi_3(w_{tp})$. This is an equilibrium provided that $U2{-}D2$ does not want to undercut: $\pi_1(w_{tp}) + \pi_3(w_{tp}) \leq \pi_2(w_{tp})$. When this inequality is not satisfied, a candidate equilibrium is the symmetric outcome in which both integrated firms charge the variable part w_{tp} and a fixed fee equal to $\pi_2(w_{tp}) - \pi_1(w_{tp})$, which makes them indifferent between supplying the upstream demand or not. It is an equilibrium if the fixed fee is positive, i.e., $\pi_1(w_{tp}) \leq \pi_2(w_{tp})$, else the downstream firm accepts both offers to pocket the fixed part.[21]

Proposition 8. *With upstream two-part tariffs, there exists an equilibrium with two mergers and a foreclosure outcome on the upstream market if, and only if, $\pi_1(w_{tp}) \leq \pi_2(w_{tp})$. Moreover,*

- *if $\pi_1(w_{tp}) + \pi_3(w_{tp}) \leq \pi_2(w_{tp})$, the foreclosure outcome is monopoly-like,*
- *if $\pi_1(w_{tp}) \leq \pi_2(w_{tp}) \leq \pi_1(w_{tp}) + \pi_3(w_{tp})$, the foreclosure outcome is symmetric.*

In the linear case with downstream synergies, for any $\gamma > 0$, there exist $0 < \overline{\delta n}_{tp}^{mon}(\gamma) < \overline{\delta n}_{tp}^{sym}(\gamma)$ that are decreasing in γ, such that the first condition is equivalent to $\delta n \geq \overline{\delta n}_{tp}^{mon}(\gamma)$, and the second condition to $\overline{\delta n}_{tp}^{sym}(\gamma) \leq \delta n \leq \overline{\delta n}_{tp}^{mon}(\gamma)$.

As in the case where upstream prices are linear, input foreclosure occurs when competition on the downstream market is fierce (γ large) and downstream synergies are large (δn large).

[20]See Bonanno and Vickers (1988).

[21]When $\pi_2(w_{tp}) < \pi_1(w_{tp})$ the symmetric outcome would be an equilibrium if upstream contracts could be exclusive. This is reminiscent to Chen and Riordan (2007)'s result that vertical integration and exclusive dealing complement each other to achieve an anticompetitive effect. Even in that equilibrium with a negative fixed fee the upstream profit is strictly positive, since both integrated firms earn the same total profits and we know that the upstream supplier has lower a downstream profit by the softening effect.

6.3 Other Simplifying Assumptions

We have assumed so far that downstream firms can switch upstream supplier at no cost after downstream prices are set. This assumption is made mainly for convenience to ensure that the cheapest upstream offer(s) is (are) always chosen. It also makes clear that our results do not hinge on upstream switching costs as in Chen (2001). If, instead, we assume that the profit of an unintegrated downstream firm is decreasing in the upstream price, holding fixed its upstream supplier, then the equilibria we have exhibited still exist under the same conditions.[22]

Proposition 9. *When upstream suppliers are definitely chosen before downstream competition, there exists an equilibrium with two mergers and a monopoly-like outcome on the upstream market if, and only if,* $\pi_1(w_m) \leq \pi_2(w_m)$.

We have also ruled out discrimination on the upstream market to ease the resolution of the one merger-subgame. Relaxing that assumption obviously does not modify the equilibrium analysis in the two-merger subgame, since there is only one buyer on the input market. Since the Bertrand outcome is still an equilibrium after zero or one merger, we can state the following result.[23]

Proposition 10. *With discriminatory upstream offers, there exists an equilibrium with two mergers and a monopoly-like outcome on the upstream market if, and only if,* $\pi_1(w_m) \leq \pi_2(w_m)$.

In the model the upstream price is bounded from above by the alternative source of input at price $\overline{m}$. If that outside option disappears, the monopoly upstream price is no longer constrained to be lesser than $\overline{m}$ and, in particular, it could be equal to $+\infty$ if an integrated firm finds it optimal

[22]Note that, under this alternative assumption, we cannot rule out the following pathological equilibrium in the one merger-subgame. Integrated firm $U1{-}D1$ sets the upstream price w_1, and unintegrated upstream firm $U2$ sets the upstream price w_2, where w_1 and w_2 are their monopoly upstream prices when each one of them supplies one unintegrated downstream firm. Formally, $w_1 = \arg\max_{\tilde{w}_1} \pi_{U1-D1}(\tilde{w}_1, w_2)$ and $w_2 = \arg\max_{\tilde{w}_2} \pi_{U2}(w_1, \tilde{w}_2)$, where $\pi_i(\tilde{w}_1, \tilde{w}_2)$ denotes the profit of firm i when firms $D2$ and $D3$ supplied by firms $U1{-}D1$ and $U2$, respectively, at prices $\tilde{w}_1$ and $\tilde{w}_2$. $w_1 > w_2$, which makes sense, since an integrated firm has more incentives than an unintegrated upstream firm to charge a high upstream price. $D2$ purchases from $U1{-}D1$ to make the integrated firm less aggressive on the final market, while $D3$ buys from $U2$ to benefit from a lower upstream price. This equilibrium seems rather unlikely and, indeed, we have not been able to find one such example using standard specifications.

[23]Note that the following pathological equilibrium of the one merger-subgame cannot be excluded. Integrated firm $U1{-}D1$ offers its monopoly price w_1 to unintegrated downstream firm $D2$, and unintegrated upstream firm $U2$ offers its monopoly price w_2 to unintegrated downstream firm $D3$, where w_1 and w_2 are defined in footnote 22. $U2$ prefers not to make an acceptable offer to $D2$, since, if that offer were eventually accepted, integrated $U1{-}D1$ would become more aggressive on the downstream market, which would erode the profit earned by $U2$ on $D3$. Similarly, $U1{-}D1$ prefers not to make an acceptable offer to $D3$, since, if that offer were accepted, $U1{-}D1$ would become less aggressive on the downstream market. By strategic complementarity, $D2$ would increase its downstream price as well, which could lower its demand, and hence, the upstream profit that $U1{-}D1$ makes on $D2$. This equilibrium seems rather unrealistic and, indeed, it never shows up in the standard specifications we have tried.

to make no upstream offer to squeeze the unintegrated downstream firm rather than supplying it at any price. In that case, there is an equilibrium with an anticompetitive wave of mergers with a downstream duopoly between the integrated firms. Otherwise the analysis is unaffected.

7 Conclusion

We show that upstream competition between integrated firms only is weaker than competition between integrated firms and unintegrated upstream firms, or between unintegrated firms only. This provides a rationale for the existence of anticompetitive waves of vertical mergers in which every unintegrated upstream firm is eliminated.

Our analysis can shed light on the recent wave of vertical mergers in the satellite navigation industry investigated by the European Commission.[24] The upstream market is the market for navigable digital map databases, where only Tele Atlas and Navteq are active, with pre-merger market shares of approximately 50% each. At the downstream level, firms embed digital maps in the devices they manufacture in order to provide their customers with navigation solutions. Downstream firms include portable navigation device manufacturers TomTom, Garmin and Mio Tech & Navman, and manufacturers of mobile handsets that incorporate navigation possibilities, Nokia, Motorola, Samsung, Sony Ericsson. The European Commission does not consider portable navigation devices and mobile phones with navigation possibilities to be part of the same market yet, but envisages that, as technology evolves, both markets will increasingly converge.

In October 2007, TomTom announces its acquisition of Tele Atlas; four months later Nokia responds by announcing its acquisition of Navteq. The European Commission has given clearance for these two mergers, thereby allowing the vertical integration of every upstream firm. Our analysis suggests that, as long as mobile phones and portable navigation devices remain rather imperfect substitutes, competition between digital map providers should not be harmed. However, when the two markets converge as it is envisaged by the European Commission, the softening effect should strengthen and upstream competition might weaken.

[24]See European Commission COMP M.4854 *TomTom/Tele Atlas* and COMP M.4942 *Nokia/Navteq*.

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A Appendix

A.1 A Preliminary Lemma

To ease the proofs of all lemmas and propositions, we begin by showing the following technical lemma.

Lemma 0. *If the best response function on the downstream market of at least one firm shifts upwards, then all equilibrium downstream prices increase strictly.*

Assume that firm i's best response shifts upwards. This happens if, and only if, the first derivative of its profit with respect to its price shifts upwards. For all j, let us denote by $\pi_j^{(0)}(.)$ (respectively $\pi_j^{(1)}(.)$) the profit of firm j before (resp. after) the marginal profit shift. The game $(\mathbb{R}; \pi_j^{(k)}(.), k = 0, 1; j = 1, 2, 3)$ is strictly supermodular. For all j, $\pi_j^{(k)}(p_j, p_{-j})$ has increasing differences in (p_j, k), and $\pi_i^{(k)}(p_i, p_{-i})$ has strictly increasing differences in (p_i, k). Since we assume that every configuration analyzed in this paper yields a unique downstream equilibrium, supermodularity theory (see Vives, 1999, p.35) tells us that this equilibrium is strictly increasing in k.

A.2 Proof of Lemma 2

First, if $U1{-}D1$ supplies the market at $w > m$, then $U2$ clearly wants to undercut.

Conversely, assume that $U2$ supplies the market at $w > m$, and let us show that $U1{-}D1$ wants to undercut. If it becomes the upstream supplier at price (arbitrarily close to) w, its first order condition on the downstream market shifts from $q_1 + (p_1 - m)\partial q_1/\partial p_1 = 0$ to $q_1 + (p_1 - m)\partial q_1/\partial p_1 + (w - m)(\partial q_2/\partial p_1 + \partial q_3/\partial p_1) = 0$. Its best response function shifts upwards, while all other best responses remain unaffected, then all downstream prices increase by Lemma 0. Therefore $U1{-}D1$'s profit increases thanks to the upstream revenues and the softening of downstream competition. Formally, denoting with superscript 1 (respectively 2) the outcome variables when firm $U1{-}D1$ undercuts (resp. does not undercut):

$$\begin{aligned}
\pi_1^{(2)} &= p_1^{(2)} q_1(p_1^{(2)}, p_2^{(2)}, p_3^{(2)}) \\
&< p_1^{(2)} q_1(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) \quad \text{by Lemma 0} \\
&< p_1^{(2)} q_1(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) + (w - m)\left(q_2(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) + q_3(p_1^{(2)}, p_2^{(1)}, p_3^{(1)})\right) \quad \text{since } w > m \\
&< p_1^{(1)} q_1(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) + (w - m)\left(q_2(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) + q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(1)})\right) \quad \text{by revealed preference} \\
&= \pi_1^{(1)}.
\end{aligned}$$

Then, we show that if firms $U1{-}D1$ and $U2$ propose the same upstream price $w > m$, then both unintegrated downstream firms purchase from $U1{-}D1$, which shall proves that this situation cannot be an equilibrium since $U2$ would undercut. We have already seen that the best response function of $U1{-}D1$ shifts upwards when $D3$ buys $U1{-}D1$ rather than $U2$, which raises all downstream prices by Lemma 0 and makes $D3$ better off. Formally, denoting with superscript 1 (respectively 2) the outcome variables when firm $U1{-}D1$ (resp. $U2$) supplies $D3$ at price w:

$$\begin{aligned}
\pi_3^{(2)} &= (p_3^{(2)} - w)q_3(p_1^{(2)}, p_2^{(2)}, p_3^{(2)}) \\
&< (p_3^{(2)} - w)q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(2)}) \quad \text{since } p_i^{(1)} > p_i^{(2)} \text{ for all } i \\
&< (p_3^{(1)} - w)q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) \quad \text{by revealed preference} \\
&= \pi_3^{(1)}.
\end{aligned}$$

This implies that $D3$'s dominant strategy is to purchase from $U1{-}D1$. By symmetry, this also holds for $D2$.

It remains to prove that the upstream market cannot be supplied at a price below the marginal cost. It is obvious that $U2$ never sells the input at $w < m$, otherwise it would be better off exiting the market. Assume now that $U1{-}D1$ is the upstream supplier at $w < m$, and denote $U2$'s upstream offer by $w' \geq w$.[25] $U1{-}D1$ is better off exiting the upstream market, since its shifts its best response upwards, which strictly increases all the downstream prices by 0.

A.3 Proof of Lemma 3

Let $w > m$. To show that $p_1(w) > p_2(w)$, we denote by $B_1(p_2, p_3, w)$ (respectively $B_2(p_1, p_3, w)$) firm $U1{-}D1$ (resp. $U2{-}D2$)'s best response when the upstream market is supplied by $U1{-}D1$ at price w. The first order conditions (4) and (5) indicate that $B_1(., ., w) = B_2(., ., w)$. Then

$$p_1(w) = B_1(p_2(w), p_3(w), w) > B_2(p_2(w), p_3(w), w).$$

Besides

$$p_2(w) = B_2(p_1(w), p_3(w), w).$$

By strategic complementarity, B_2 is increasing in its first argument, therefore $p_1(w) > p_2(w)$.

A straightforward revealed preference argument shows that $U1{-}D1$ earns a strictly lower downstream profit than $U2{-}D2$.

A.4 Proof of Proposition 1

Proof that $w_m > m$. We start by showing that $w_m \geq m$ by proving that $\pi_1(w) < \pi_1(m)$ for $w < m$. The proof is along the line of the proof of Lemma 2. When the upstream price decreases from m to $w < m$, then its best response functions of $U1{-}D1$ and $D3$ shift downwards, and all downstream prices decrease by Lemma 0. Therefore, $U1{-}D1$ lose money on the downstream market, as well as on the upstream market.

[25] Actually $w' < w$. Indeed, if $w' = w$, unintegrated downstream firms strictly prefer purchasing from $D2$ since it shifts $U1{-}D1$'s best response function upwards.

To obtain that $w_m > m$, it remains to show that first derivative of $\pi_1(.)$ for $w = m$ is strictly positive. Using the envelope theorem, we get

$$\frac{d\pi_1}{dw}(m) = (p_1 - m)\left(\frac{dp_2}{dw}(m)\frac{\partial q_1}{\partial p_2}(p_1(m), p_2(m), p_3(m)) + \frac{dp_3}{dw}(m)\frac{\partial q_1}{\partial p_3}(p_1(m), p_2(m), p_3(m))\right) + q_3(p_1(m), p_2(m), p_3(m)) > 0,$$

since the downstream prices are strictly increasing in w by Lemma 0.

Proof that monopoly-like equilibria exist if, and only, if $\pi_1(w_m) \leq \pi_2(w_m)$. Assume that $\pi_1(w_m) \leq \pi_2(w_m)$ and let us show that $(w_m, +\infty)$ is an equilibrium. Clearly, firm $U1-D1$ does not want to set another price, by definition of w_m. In addition, since $\pi_1(w_m) \leq \pi_2(w_m)$, and again by definition of w_m, firm $U2-D2$ does not want to undercut its rival.

Conversely, if $\pi_1(w_m) > \pi_2(w_m)$, then monopoly-like outcomes cannot be equilibria, since the integrated firm which does not supply the upstream market would rather undercut its rival.

Proof that all other equilibria are symmetric, $w_1 = w_2 = w$, with $w \leq w_m$ and $\pi_1(w) = \pi_2(w)$. Symmetric equilibria obviously satisfy $w \leq w_m$, else an integrated firm can undercut to w_m to increase its profits. $\pi_1(w) = \pi_2(w)$ must also hold, otherwise an integrated firm would rather undercut or exit the upstream market.

It remains to show that asymmetric equilibria other than monopoly-like equilibria cannot exist. Notice first that $\pi_2(.)$ is strictly increasing since, by the envelope theorem,

$$\frac{d\pi_2}{dw} = (p_2 - m)\left(\frac{\partial q_2}{\partial p_1}\frac{dp_1}{dw} + \frac{\partial q_2}{\partial p_3}\frac{dp_3}{dw}\right) > 0$$

as downstream prices are strictly increasing in w. Let $w_1 < w_2$ and assume, by contradiction, that (w_1, w_2) is an equilibrium. If $\pi_1(w_1) > \pi_2(w_1)$, then firm $U2-D2$ has a strictly profitable deviation: setting $w_1 - \varepsilon$. If $\pi_1(w_1) \leq \pi_2(w_1)$, then $\pi_1(w_1) < \pi_2(w_2)$ since $\pi_2(.)$ is strictly increasing, and firm $U1-D1$ has a strictly profitable deviation: setting $w_2 + \varepsilon$. In both cases we get a contradiction.

Proof that monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria. Consider a monopoly-like equilibrium $(w_m, +\infty)$, and another equilibrium, which we know is symmetric, (w, w), with $\pi_1(w) = \pi_2(w)$. Then we have, by definition of w_m, $\pi_2(w) = \pi_1(w) < \pi_1(w_m) \leq \pi_2(w_m)$, which proves that the monopoly-like equilibria Pareto-dominate all other equilibria.

Proof equilibria other than monopoly-like equilibria involve weakly dominated strategies on the upstream market. These other equilibria are of the form (w, w), with $w \leq w_m$ and $\pi_1(w) = \pi_2(w)$. Let us show that offering $w_i = w_m$ weakly dominates offering $w_i = w$ for integrated firm i. If the integrated rival offers $w_j \leq w$, then both strategies are equivalent. If $w < w_j < w_m$, then offering w_m yields a payoff $\pi_2(w_j)$, which is larger than the payoff when offering w, $\pi_2(w)$, because $\pi_2(.)$ is upward-slopping. If $w_j > w_m$, then offering w_m yields a payoff $\pi_1(w_m)$, which is larger than the payoff when offering w, $\pi_1(w)$, by definition of w_m. If $w_j = w_m$, integrated can supply the upstream market or not; in either situation, the former two cases show that it is also preferable to offer w_m than w.

A.5 Proof of Proposition 2

When the Bertrand outcome arises in every two-merger subgames, firms are indifferent between merging or not. In that case, it is an equilibrium that downstream firms submit no bid.

We now show that there is two mergers in equilibrium if $\pi_1(w_m) \leq \pi_2(w_m)$, and integrated firms do not play weakly dominated strategies on the upstream market or do not play equilibria that are Pareto-dominated by another equilibrium. In that case, the only downstream equilibria in the two-merger subgames are the two monopoly-like equilibria. It cannot be that there is only zero or one merger in equilibrium, since any remaining downstream firm would rather bid a small amount to vertically integrate and increase its profit from π^* to $\pi_1(w_m)$ or $\pi_2(w_m)$. Therefore, in any equilibrium, there are two mergers and the input price is w_m.

A.6 Proof of Proposition 3

We compare an equilibrium with two mergers in which $D3$ purchases the from $U1-D1$ at $w > m$, with an equilibrium with no merger in which all downstream firms access the input at marginal cost. First, all downstream prices are strictly higher in the partial foreclosure equilibrium by Lemma 0, therefore consumers are strictly worse off.

Second, we show that social welfare in also strictly lower in the partial foreclosure equilibrium. Assume that there exists a representative consumer with a quasi-linear, continuously differentiable and quasi-concave utility function $q_0 + u(q_1, q_2, q_3)$, where q_0 denotes consumption of the numeraire and q_k denotes consumption of product $k \in \{1, 2, 3\}$. We can then write the social welfare as

$$W(p_1, p_2, p_3) = u\left(q_1(p_1, p_2, p_3), q_2(p_1, p_2, p_3), q_3(p_1, p_2, p_3)\right) - m \sum_{k=1}^{3} q_k(p_1, p_2, p_3).$$

We have to show that $W(p_1(w), p_2(w), p_3(w)) - W(p_1(m), p_2(m), p_3(m))$, the variation in social welfare when one shifts from the Bertrand outcome from the partial foreclosure outcome, is strictly negative. Since the welfare function is symmetric in its arguments, we can relabel the downstream prices in the partial foreclosure equilibrium by $\{\hat{p}_1, \hat{p}_2, \hat{p}_3\} = \{p_1(w), p_2(w), p_3(w)\}$ such as $\hat{p}_3 > \hat{p}_2 > \hat{p}_1 > p_1(m) = p_2(m) = p_3(m) \equiv p^*$.[26] The variation in welfare then writes as

$$\int_{p^*}^{\hat{p}_1} \sum_{k=1}^{3} \frac{\partial W}{\partial p_k}(r, r, r)dr + \int_{\hat{p}_1}^{\hat{p}_2} \sum_{k=2}^{3} \frac{\partial W}{\partial p_k}(\hat{p}_1, r, r)dr + \int_{\hat{p}_2}^{\hat{p}_3} \frac{\partial W}{\partial p_3}(\hat{p}_1, \hat{p}_2, r)dr.$$

All the integrands are strictly negative. Indeed, for instance,

$$\frac{\partial W}{\partial p_k}(p_1, p_2, p_3) = \sum_{k'=1}^{3} \left(\frac{\partial u}{\partial q_{k'}} - m \right) \frac{\partial q_{k'}}{\partial p_k} = \sum_{k'=1}^{3} (p_{k'} - m) \frac{\partial q_{k'}}{\partial p_k}$$

is strictly negative when $p_k \geq p_{k'}$, $k' \neq k$, since $\partial q_k / \partial p_k < -\sum_{k' \neq k} |\partial q_{k'} / \partial p_k|$. This concludes the proof.

[26] We do so since, in the general case, we cannot order the downstream prices of the unintegrated downstream firm and the upstream supplier.

A.7 Proof of Proposition 4

In the two-merger subgame, when the upstream market is supplied by $U1{-}D1$ at price w, downstream prices are

$$p_1(w) = \frac{18+\gamma(15+w(9+5\gamma))}{2(3+\gamma)(6+5\gamma)} \quad p_2(w) = \frac{3(6+\gamma(5+w+w\gamma))}{2(3+\gamma)(6+5\gamma)} \quad p_3(w) = \frac{3(6+5\gamma)+w(18+7\gamma(3+\gamma))}{2(3+\gamma)(6+5\gamma)},$$

and profits

$$\pi_1(w) = \frac{3(3+\gamma)(6+5\gamma)^2+6w(1+\gamma)(6+5\gamma)(18+\gamma(18+5\gamma))-w^2(1+\gamma)(648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}{4(3+\gamma)^2(6+5\gamma)^2},$$

$$\pi_2(w) = \frac{3(3+2\gamma)(6+\gamma(5+w+w\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2},$$

$$\pi_3(w) = \frac{3(3+2\gamma)(6+5\gamma-w(1+\gamma)(6+\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2}.$$

$\pi_1(.)$ is strictly concave and reaches its maximum value for

$$w_m = \frac{3(6+5\gamma)(18+\gamma(18+5\gamma))}{648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4}.$$

$\pi_3(.)$ is strictly decreasing and strictly positive if it purchases the input from the alternative source at price w_m. We assume that the price of the alternative source of input does not constrain the monopoly upstream price, $\overline{m} > w_m$, and provides $D3$ with positive profits.

Straightforward computations indicate that $\pi_2(w) \geq \pi_1(w)$ if, and only, if $w \in [m, w_s]$, where $w_s \geq w_m$ if, and only if, $\gamma \geq \underline{\gamma} \simeq 41$. The same holds with strict inequalities. As a result, the only equilibrium outcome is the Bertrand outcome when $\gamma < \overline{\gamma}$. When $\gamma > \overline{\gamma}$, there are also the two monopoly-like equilibria $(w_m, +\infty)$ and $(+\infty, w_m)$, and a symmetric equilibrium (w_s, w_s) with $w_s < w_m$. In that case, if integrated firms do not play weakly dominated strategies on the upstream market or do not play equilibria that are Pareto-dominated by another equilibrium, then there are two mergers in equilibrium and a monopoly-like outcome on the upstream market.

A.8 Proof of Lemma 4

We need to show that an unintegrated downstream firm that faces two identical upstream offers from integrated firms i and j is indifferent between buying from i or from j. More precisely, we show that when it switches from i to j, all the downstream prices p_k, $k \notin \{i,j\}$, do not change, neither does $p_i + p_j$, which will conclude the proof, by linearity of the demand functions.

Start from an equilibrium in which the upstream market is supplied at price w, possibly by several integrated firms. Equilibrium downstream prices are given by the first order conditions $\partial \pi_k / \partial p_k = 0$, where the profit functions are, for integrated firms, $\pi_k = (p_k - n + \delta n) D_k + w \sum_\ell \mathbb{I}_{k,\ell} D_\ell$, where the sum is taken over all unintegrated downstream firms and $\mathbb{I}_{k,\ell}$ is a dummy equal to 1 if k sells the input to ℓ, and for unintegrated downstream firms, $\pi_k = (p_k - n - w) D_k$.

The partial derivative of the best response function of an integrated firm k with respect to $\mathbb{I}_{k,\ell}$,

$$\frac{\partial p_k}{\partial \mathbb{I}_{k,\ell}} = \frac{\partial^2 \pi_k / (\partial p_k \partial \mathbb{I}_{k,\ell})}{-\partial^2 \pi_k / \partial p_k^2} = \frac{w \partial D_k / \partial p_k}{-2 \partial D_k / \partial p_k},$$

is a constant. Note that it is a strictly positive constant; this is the softening effect. Therefore, when an unintegrated downstream firm switches from integrated firm i to integrated firm j, i's best response function shifts upwards by that constant, j's best response function shifts downwards by the same amount, and all other firms' best response functions are unaffected, hence we get our result.