Stable and Efficient Electronic Business Networks: Key Players and the Dilemma of Peripheral Firms

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Preliminary and incomplete draft version.

Abstract

This paper studies a spatial model of electronic business network formation where firms build links based on a cost-benefit analysis. Benefits result from directly and indirectly connected firms in terms of knowledge flows, which are heterogeneous: a "key player" (e.g. a firm providing an exchange platform in a business-to-business network) provides a higher level of knowledge flows than "peripheral" firms (e.g. tier 3 suppliers in a vertically differentiated industry). For intermediate cost values of link formation, stable and efficient network structures comprise only a subset of the total set of firms, excluding peripheral firms which are most distantly located to the key player. When link formation implies a certain degree of network congestion, the stable and efficient network size is smaller than in a model with bilateral decisions upon link formation between two firms.

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1 Introduction

The design and organization of network structures play an important role in significant economic and social relationships. Informal social networks are often the means for communicating information and allocating goods and services which are not traded on markets. Such goods do not only comprise invitations to parties and other forms of exchanging friendship but also, e.g. in the context of electronic business networks where co-operation is a central competition factor, information about e.g. job openings, business opportunities or product development. More than ever firms dependent on connecting their abilities and resources with those from external partners. Networks play a fundamental role in providing platforms for research and development and collusive alliances among corporations. Furthermore, they determine how information is exchanged and convey social capital as one of the important determinants in trade.

For several decades, the management of the external environment took a high priority to firms by building stronger relationships with customers and suppliers. Recently, organizations have moved beyond customer/supplier connections to begin to establish alliances even with their competitors which is among other reason due to the revolution in information technology which has brought organizational changes that modify transaction costs, and thereby affect both the horizontal structure and the vertical configuration of industries. In this context, three lines of research can be distinguished:[1] First, there is a reduction in the frequency of hierarchical coordination, which is due to an increasing fragmentation of value chains. The advances in information technology, which cut coordination and monitoring costs, facilitated codification of knowledge, and reduced the importance of geographical distance, at least for some activities. In this wise, the new technology has reduced internalization-based advantages and reversed the process of vertical integration. Market-based transactions have squeezed out some of the ones hitherto coordinated hierarchically. The key players in an increasing num-

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[1] See Szalavetz (2003).

ber of industries have adopted modular organizational structures. Hierarchically coordinated, vertically integrated organizations have thus given way to network organizations marked by horizontal cooperation, reciprocity and mutual trust, instead of hierarchical supervision of work processes.

Second, there is a flattening of vertically integrated organizations due to the mounting importance of distributed knowledge. In intellectual capitalism however, a diminishing proportion of the relevant knowledge base remains internal in many industries, and an increasing part is provided by outside experts. The more specialized the knowledge of an actor, the greater the extent to which hierarchical coordination loses its hold. This is especially the case for multi-component, IT-intensive products like aircraft engines, power stations, residential and office safety systems and so on incorporate a plurality of technologies, and firms cannot develop them all inside. The manufacturers of such products and systems integrate the knowledge and coordinate the activity of various external, specialized suppliers and research institutions.

Eventually, networks as a third form of coordination alongside markets and hierarchies are becoming increasingly common in economic activity. Sustainable competitive advantage is determined by factors other than the traditional determinants of corporate competitiveness. Companies now have to capitalize on their own as well as outside knowledge. Alliance business networks also demonstrate sparsity, decentralization and clustering. Interfirm networks tend to be extremely sparse since forming and maintaining alliances has a cost in terms of time and effort. When firms forge relationships with other organizations for information sharing and exchange of knowledge, they face a variety of search, monitoring, and enforcement costs. Monitoring and managing alliances is also complex and costly, causing the firm's effectiveness at managing its alliances to decline with the number of alliances maintained.[2] Thus, due to cost

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[2] See Deeds and Hill (1996).

constraints in forging and maintaining links, interfirm networks will tend to have far fewer links than if all pairs of firms were directly connected. Hence, although there is a growing importance of maintaining links to competitors, suppliers or clients in electronic business networks, there are cost factors that prevent many links. Moreover, we often observe networks being formed around certain key players, often excluding smaller or peripheral firms because those players do not provide sufficient knowledge for the network. This paper therefore analyzes the interplay between network benefits and hindering costs of network formation, when players provide heterogeneous benefits to external partners.

Related literature:

An excellent overview on related contributions to the theory of network formation provides Jackson (2004). Related models of network formation and collaborative networks comprise e.g. Bala and Goyal (2000), Jackson and Wolinsky (1996), and Kranton and Minehart (2000). Bala and Goyal (2000) follow a significantly different approach to this paper since they consider a directed network model, where individuals are able to connect to others without the consent of the connected individual. Controversially, this paper deals with non-directed networks requiring the consent of both involved individuals to successfully create a link. The work of Kranton and Minehart (2000) deals with networks between vertically related firms. Issues relating to group formation and cooperation have been a central concern in economics, and game theory in particular. The traditional approach to these issues has been in terms of coalitions. In recent years, there has been considerable work on coalition formation in games; see e.g. Jackson and Watts (2002), Bloch (1995). One application of this theory is to the formation of groups in oligopolies. In this literature, group formation is modelled in terms of a coalition structure which is a partition of the set of firms.

The present paper contributes to the theory of network formation by introducing three aspects which are especially observable in electronic business networks. First, we account for the fact that a crucial feature of such electronic business networks is the participation of so-called ‘key players’ which are e.g. crucial value enhancers in value chains or precursors in product development alliances. Accordingly, we account for heterogeneity among firms’ information contribution to networks. Key players provide higher levels of knowledge than ‘ordinary’ firms which could be e.g. tier 2 suppliers in value chains or followers in R&D development consortia. Second, in our model, firms can only connect to their direct neighbors but not to more distant players. This assumption reflects the peculiarities of electronic business networks where it is not necessarily required that every network member has a direct link to all other participants in order to guarantee knowledge exchange flows between all participants. A further intuition behind this assumption is that distances are interpreted in terms of similarities in business activities. That means, if a firm intents to join a network it has to incur costs (e.g. adjustment costs for its database, training of personnel) in order to sample a neighbor which is member of the network. On the other hand, the existing network has to incur corresponding adjustment costs. The utility from a connection to a direct industry competitor might be higher than the utility from a very distant network member say from another industry. Third, we introduce network congestion costs into a model of network formation. A joining member imposes costs on all existing network members in terms of increasing communication costs or adjustment costs causing a firm’s effectiveness at managing its alliances to decline with the number of alliances maintained.[3]

The remainder of the paper is structured as follows: the assumptions of the network model are introduced in section 2. Section 3 determines stability and efficiency in the static model. In section 4 we determine the outcomes of a dynamic

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[3] See Deeds and Hill (1996).

network formation process. Section 5 discusses some extensions to the model and section 6 concludes the paper.

2 The Network Model

The outline of the model follows the common structure of models on network formation (e.g. Jackson and Watts, 2002, Jackson, 2004 or Jackson and Wolinksy, 1996). There is a finite set of $N = 1, ..., n$ *firms* in a market (with $n \geq 3$) which have a fixed location on a circle and are equidistantly located. This spatial dispersion should be interpreted to represent some diversity in terms of professional distance, differences in industry affiliations, etc. between the firms.[4]

A business *network* g is a list of firms which are linked to each other. The network relations among the firms are represented by graphs whose nodes or vertices represent the firms and whose links capture pairwise relations. This paper focusses on non-directed networks where links are bilateral. Every firm can only connect to (one or both of) its two direct neighbors. This can be interpreted as follows: if a firm wants to join a network, than it would have to adjust its database or its information technology infrastructure in such a way that it is compatible with the existing network. This happens by adjusting the database to the firm that is closest to the joining member and which is already in the network, which is one of the direct neighbors. The complete network, denoted g^N, is a chain of all subsets of N of size 2 where every firms has 2 links (one to each of its direct neighbors). The set of possible networks or graphs on N is $\{g | g \subset g^N\}$. The subset of N containing i and j is for simplicity denoted ij and is referred to as the link ij. The interpretation is that if $ij \in g$, then nodes i and j are directly connected,

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[4] Note, that some contributions deal with players that are located on the real line. In these models, players located at the end of the line only have one direct neighbor, which would lead to an ex-ante asymmetry. To rule this out, we use a circular model where every firm has two direct neighbors.

while if $ij \notin g$, then nodes i and j are not directly connected. Let $g + ij$ denote the network obtained by adding link ij to the existing network g and let $g - ij$ denote the network obtained by deleting link ij from the existing network g (i.e., $g + ij = g \cup \{ij\}$ and $g - ij = g \ \{ij\}$). If $g' = g + ij$ or $g' = g - ij$, then g and g' are adjacent. Let $N(g) = \{i | \exists j \text{ s.t. } ij \in g\}$ be the set of firms involved in at least one link and $n(g)$ denotes the cardinality of $N(g)$.

A *chain*[5] in g connecting i_1 and i_n is a set of distinct nodes $\{i_1, i_2, ..., i_n\} \subset N(g)$ such that $\{i_1 i_2, i_2 i_3, ..., i_{n-1} i_n\} \subset g$. A nonempty network $g' \subset g$ is a *component* of g, if for all $i \in N(g')$ and $j \in N(g')$, $i \neq j$, there exists a chain in g' connecting i and j, and for any $i \in N(g')$ and $j \in N(g)$, $ij \in g$ implies $ij \ g'$.

The *value* of a network is represented by $v : \{g | g \subset g^N\} \to \mathbb{R}$, where $v(g)$ represents the total utility or production of the network. The set of all such functions is V. The value function allows for a wide variety of applications and quite general forms of externalities. Here, the value will be an aggregate of individual firms' utilities from the network, $v(g) = \sum_i u_i(g)$, where $u_i : \{g | g \subset g^N\} \to \mathbb{R}$. A network $g \subset g^N$ is *strongly efficient* if $v(g) \geq v(g') \forall g' \subset g^N$. Strong efficiency and Pareto efficiency coincide when value is transferable.

An *allocation rule* $u : \{g | g \subset g^N\} \times V \to \mathbb{R}$ describes how the value associated with each network is distributed to the participating individual firms. $u_i(g, v)$ may be thought of as the payoff to player i from network g under the value function v. For simplicity, if v is fixed, we will simply write $u_i(g)$. The allocation rule may represent several things. When considering a purely social network, the allocation rule may represent the utility that each individual receives from the network and this utility might not be transferable. When considering an exchange or production network, the allocation rule may represent either the trades or

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[5] According to Jackson and Watts (2001), a chain is a sequence of links. Sometimes in the literature chains are referred to as a path, but the term path stands for a sequence of networks in this paper.

production accruing to each individual, the outcome of a bargaining process, or some exogenous redistribution. The following notion of *joint pairwise stability* builds on the concept of *pairwise stability* by Jackson and Watts (2001)[6] which describes a network as stable when no player would benefit by severing an existing link, and no two players would benefit by forming a new link.

Definition 1 *A network g is jointly pairwise stable, if*
(i) $\forall ij \in g$, $u_i(g,v) + u_j(g,v) \geq u_i(g - ij; v) + u_j(g - ij; v)$, *and*
(ii) $\forall ij \notin g$, $u_i(g,v) + u_j(g,v) < u_i(g + ij; v) + u_j(g + ij; v)$.

In words, definition 1 states that a link is *jointly pairwise stable*, so that if this link is formed then the sum of the values of both agents from the link is higher than the sum of their utilities if the link was not formed. Contrarily, if the sum of the utilities of both link establishing firms is less than without forming the link, the notion requires that the link is not formed. When a network g is not *jointly pairwise stable* it is said to be *defeated* by g' if either $g' = g + ij$ and (ii) is violated for ij, or if $g' = g - ij$ and (i) is violated for ij. As in the model by Watts (2001) the approval of two firms is required for the formation of a link, but here, those firms have to be adjacent, and the sum of both their utilities minus the cost of the link creation have to be (weakly) higher than 0. The consideration of the joint utilities of link establishing players accounts for the possibility of interfirm compensation.

This definition of joint pairwise stability is a relatively weak notion among those which account for link formation and it does not dependent on any particular formation process. Accordingly, it admits for a relatively large set of sta-

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[6] Jackson and Watts (2001) describe a network g as *pairwise stable* if (i) $\forall ij \in g$, $u_i(g,v) \geq u_i(g - ij; v)$ and $u_j(g,v) \geq u_j(g - ij; v)$, and (ii) $\forall ij \notin g$, if $u_i(g,v) < u_i(g + ij; v)$ then $u_j(g,v) > u_j(g + ij; v)$. Accordingly, their notion does not allow for possible compensation payments between link establishing partners, which is the crucial difference to our concept of joint pairwise stability.

ble allocations compared to more restrictive definitions or an explicit formation procedure. But for our purposes, it already narrows the set of graphs substantially and therefore such a weak definition provides strong results. One obvious strengthening of this stability notion is to allow decision upon the creation of links to be made by coalitions of network members which include more than two firms (which are connected via the link). This will be discussed as an extension in section 5.[7]

3 Efficiency and Stability in the Static Model

A network on the set of firms $N = 1, ..., n$ creates benefits to the participating firms, which result from the communication of information and from the allocation of goods and services which are not traded in markets (e.g. information about business opportunities, know-how on information technology, etc.). Firms may connect only to firms in their immediate neighborhood. But they also benefit from indirect communication to those firms to whom their direct neighbors are linked, and so on. The value of communication or knowledge flows obtained from other firms diminishes in the distance to those players, represented by a spatial depreciation rate $0 < \delta < 1$, which captures the idea that the value that i derives from a connection to j is proportional to the distance between those two firms. Further, there is an "intrinsic value" $w_{ij} \geq 0$, firm i provides to firm j.[8] In what follows, it is assumed, that all firms are identical except one so-called "key-player" k which provides a higher value than all other firms (this could be interpreted as k being a technology leader, or a platform provider in an electronic business network).[9] Without loss of generality, we assume that $w_{ij} = 1, \forall i \neq k$ and $w_{kj} > 1$. For notational purposes, w_{kj} is labelled w_k in the remainder. The net utility of

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[7] See Jackson and Wolinsky (1996).

[8] See Jackson and Wolinsky (1996) for a similar notion.

[9] Indeed, in many industries or value chains, we observe the existence of one or few key players whose involvement in business networks is crucial for the network's overall prospects of

each firm i from graph g is then given by

$$u_i(g) = \begin{cases} \delta^{l_{ik}} w_k + \sum_{j \neq i} \delta^{l_{ij}}, & \text{for } i \neq j, i \neq k \\ \sum_{j \neq i} \delta^{l_{kj}}, & \text{for } i \neq j, i = k \end{cases} \tag{1}$$

where l_{ij} is the number of links in the shortest path between firm i and firm j. The total cost of a link is between two firms is c.

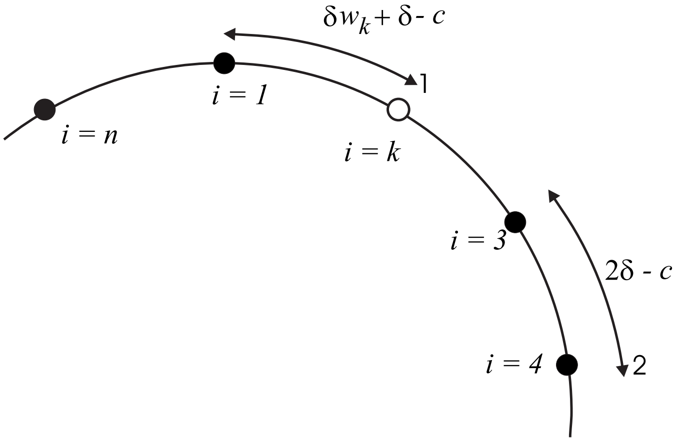

Figure 1: Circular network setup

Figure 1 depicts the setup of the model with a randomly selected position of the key player at $k = i = 2$.[10] Initially all firms are unconnected and numbered from $i = 1$ to $i = n$. The graph also shows the value of two randomly selected links, for the case that those links are the only two existing links in the network. Link 1 between $i = 1$ and the key player $i = k$ has a total value of $\delta(1 + w_k) - c$, where $\delta w_k (> \delta)$ is the net value to player $i = 1$ and δ is the net value to the key player k. Link 2 is a link between two non-key players which has the value $2\delta - c$.

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success.

[10] The line of the partial circle does not resemble any links but only illustrates the circular framework.

Proposition 1 *For all N a stable network exists. Further,*

- *(i) if $c \leq \frac{2\delta-\delta^2}{1-\delta}$ then the complete chain network g^N is stable.*
- *(ii) If $\frac{2\delta-\delta^2}{1-\delta} < c \leq \delta(1+w_k)$, $\exists n$ for a given c such that the network is stable for exactly n firms, with $0 < n < N$ and $\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i \simeq c$.*
- *(iii) If $c > \delta(1+w_k)$, then the empty network is stable.*

Proof. The proof builds on the proof by Jackson and Wolinsky (1996), why it is sketched out only briefly here. If $c > \delta(1+w_k)$ then no link can be stable, because in this case, c is higher than the highest possible benefit from any link (which in this case must be a link between the key-player and one of its neighbors). The value of this link is $\delta(1+w_k) - c$ which necessarily has to be higher than 0, otherwise no link can be stable. If $c < \delta(1+w_k)$ then the value of each link is[11]

$$\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i \quad = \quad \delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \frac{1-\delta^{n+1}}{1-\delta} - 1. \tag{2}$$

Although, we consider N to be finite, this value of a link has a lower bound it converges to, when n approaches infinity:

$$\lim_{n\to\infty} \left(\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \frac{1-\delta^{n+1}}{1-\delta} - 1 \right) \quad = \quad \frac{2\delta-\delta^2}{1-\delta}, \tag{3}$$

such that if $c < \frac{2\delta-\delta^2}{1-\delta}$ the complete chain graph g^N is stable. The most interesting case is $\frac{2\delta-\delta^2}{1-\delta} < c < \delta(1+w_k)$. In this case the network is only stable for a subset n of the total number of N firms. Accordingly, for each c in this region, there is an n such that the network is stable for exactly n but not for $n \pm 1$ firms. In every case this network comprises the key-player as central firm in a symmetric half-circle. Due to symmetry, the corresponding value of n is determined by $\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i \simeq c$. ■

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[11] Note that $\lceil x \rceil$ is the "ceiling" of x, which is defined as $\lceil x \rceil = \min\{m : m \text{ is an integer and } m \geq x\}$.

Note, that the lowest possible value of a link between any unconnected players is 2δ. Accordingly, for $c < 2\delta$ any link is stable. Since we consider a circular model, in the case of a complete chain graph, every player i would have to possible link direction to any other player j but only the utility from the closer connection is taken into account. Due to this property, the complete chain network is also stable for $\frac{2\delta-\delta^2}{1-\delta} > c > 2\delta$.

Figure 2 gives a graphical illustration of the stable network structures.

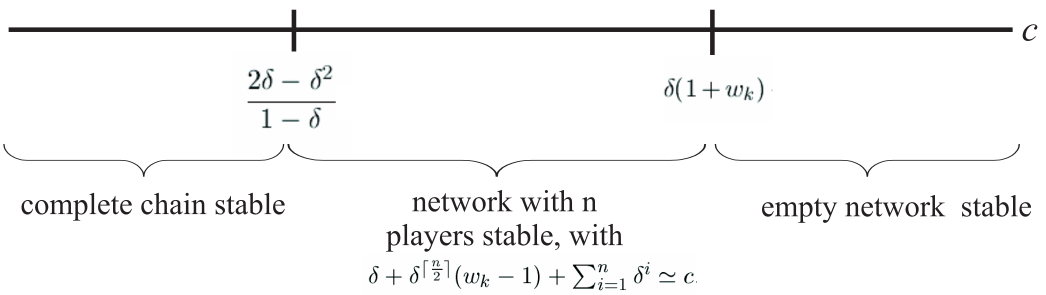

Figure 2: Stability of network structures dependent on c values

This outcome is highly relevant to observations in practice. In the intermediate cost range, peripheral firms which are most distantly located to the key-player cannot be part of a stable network, since the value added (e.g. in terms of know-how on product development or process data, or industry knowledge) is not enough to compensate for the costs their connection to the network would imply. Closely related to the stability of the network structures is efficiency.

Proposition 2 *For all N an efficient network exists. Further,*

(i) if $c \leq \frac{2\delta-\delta^2}{1-\delta}$ then the complete chain network g^N is efficient.

(ii) If $\frac{2\delta-\delta^2}{1-\delta} < c \leq \delta(1+w_k)$, $\exists n$ for a given c such that the network is efficient for n firms, with $0 < n < N$ and $\delta+\delta^{\lceil\frac{n}{2}\rceil}(w_k-1)+\sum_{i=1}^{n}\delta^i \simeq c$.

(iii) If $c > \delta(1+w_k)$, then the empty network is the only efficient network structure.

Proof. The proof is relegated to the appendix. The following corollary verbally summarizes the results from Propositions 1 and 2.

Corollary 1 *For intermediate cost values c, networks with a positive but smaller subgroup of all firms N are stable and efficient. These networks imply the participation of the key-player as central agent and do not imply peripheral firms which are most distantly located to the key-player.*

This conclusion follows directly from Propositions 1 and 2 above. It states that peripheral firms suffer from the dilemma of being distantly located to the key-player, so that their participation in the network does not provide enough utility to compensate the cost for establishing a link to their closest neighbor.

4 Dynamic Network Formation

The description of the dynamic network formation process follows Watts (2001). The n firms are initially unconnected. Over time the firms meet one of their direct neighbors, having the opportunity to form (or in case that this link already exists, to severe) a link with each other. Time, T, is divided into periods, being modelled as a countable, infinite set $T = \{1, 2, ..., t, ...\}$. The network that exists at the end of period t is labelled g_t whereas the payoff each firm i receives at the end of t then reads as $u_i(g_t)$. In each period, a (potential) link $i : i \pm 1$ between two neighbored firms is randomly identified to be updated with uniform probability. If the identified link $i : i \pm 1 \in g_{t-1}$, both firms i and $i \pm 1$ decide whether to sever the link or not. Otherwise, if $i : i \pm 1 \notin g_{t-1}$, then firm i and $i \pm 1$ can form link $i : i \pm 1$ requiring that the sum of both firms' utilities from the link is higher than its cost. Firms are myopic, so that a firm's decision whether to sever or form a link is based on whether or not severing or forming a link increases its payoff in period t. A *stable state* in the network formation process is reached if after some time period t, no additional links are formed or broken. Accordingly, the resulting network must be a stable (static) network. If the process reaches a stable state, the resulting network, by definition, must be a stable (static) network. In Proposition 3 we derive what type of networks the formation process converges to allowing us to determine whether or not the formation process converges to an efficient network.

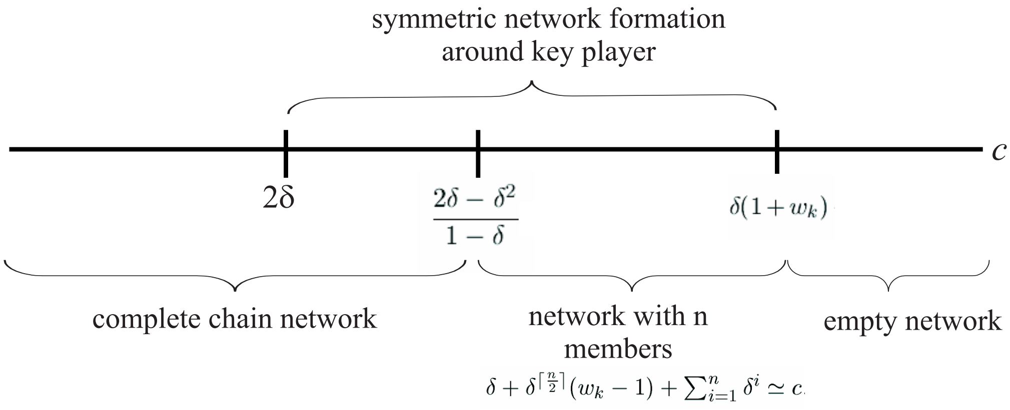

Figure 3: Outcomes of the dynamic network formation process

Proposition 3 *The dynamic network formation process converges to the following network structures:*

(i) If $c \leq 2\delta$, then every link forms (as soon as possible) and remains (no links are ever broken). The network converges to the complete chain g^N.

(ii) If $2\delta < c \leq \delta(1 + w_k)$, links form symmetrically around the key player (starting with a link between k and one of its two neighbors):

a) in case of $2\delta < c \leq \frac{2\delta - \delta^2}{1 - \delta}$, the network converges to the complete chain g^N,

b) in case of $\frac{2\delta - \delta^2}{1 - \delta} < c \leq \delta(1 + w_k)$, the network size reaches $n < N$ members, determined by $\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i \simeq c$.

(iii) If $c > \delta(1 + w_k)$, then no links ever form.

Proof. The proof is in the appendix, a graphical representation of the outcomes of the dynamic network formation process dependent on c is depicted in figure 3 above.

Propositions 3 tells us what type of networks the formation process converges to. This information allows us to determine whether or not the formation process converges to an efficient network. Each agent prefers a direct link to any indirect link. Each period, two agents, say i and $i+1$, meet. If players i and $i+1$ are not yet connected, then they will each gain at least from forming a direct link, if $c < 2\delta$ and so the connection will take place. Using the same reasoning as above, if an agent ever breaks a direct link, his payoff will strictly decrease. Therefore, no direct links are ever broken. Proposition 3 says that if $0 \leq c < \frac{2\delta-\delta^2}{1-\delta}$, then the network formation process always converges to the complete chain network, which is the unique efficient network according to Proposition 2. This network is also the unique stable network.

5 Extensions

The network model could be extended in various ways. Most common in the literature is a distinction between two-sided and one-sided knowledge flows yielding quantitatively slightly differentiated results (see e.g. Bala and Goyal, 2000). Here, we want to focus on an extension that is not common in the network literature: the occurrence of a certain network congestion costs. Instead of modelling a link creation cost that has to be incurred by (at most) the two firms between which the link is created, we could think about a certain network congestion cost. That is, with every joining member, there arises a cost c' to all existing network members in terms of e.g. adjustment costs to the new member or increased administrative effort. This network congestion cost is modelled as an alternative cost to the link establishing cost c from above, such that the utility of a player i from the network

g denotes as:

$$u_i(g) = \begin{cases} \delta^{l_{ik}} w_k + \sum_{j \neq i} \delta^{l_{ij}} - (n-1)c', & \text{for } i \neq j, i \neq k \\ \sum_{j \neq i} \delta^{l_{kj}} - (n-1)c', & \text{for } i \neq j, i = k \end{cases} \tag{4}$$

where n represents the cardinality of g. Accordingly, the higher the number of network members, the higher gets the interest of network members to prevent further firms to join. Furthermore, the decision to accept a link between a network member and a firm outside the network could also be influenced by all existing members of the network. Especially when network congestion costs are present, such a scenario is highly relevant to practice. In such a case the stable and efficient network size is smaller than in the case with only link establishing firms being involved in the carrying the cost burden of a new link.

From that it follows, that the value of a link is now determined by

$$\begin{aligned} &\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i - 2(n-1)c' \\ = &\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \frac{1 - \delta^{n+1}}{1 - \delta} - (1 + 2(n-1)c'). \end{aligned} \tag{5}$$

TO BE COMPLETED!

6 Conclusion

The recent advances of information technology have brought along many organizational changes for firms in always faster changing markets. Together with a reduction in the frequency of hierarchical coordination, an in creasing fragmentation of value chains we observe a flattening of vertically integrated organizations. Furthermore, networks as a form of coordination alongside markets become increasingly common, especially in the electronic business. Recently, organizations have moved beyond customer/supplier relationships to begin to establish alliances with their direct and closely related industry competitors. Typically, these interfirm alliances take the form of formal organizational partnerships, which are of growing importance in the context of electronic business networks. Such competitor alliances formerly focused exclusively on specific joint product development efforts, but tend increasingly to long-term basic research and development collaborations.

The present paper contributes to the theory of network formation by introducing three aspects which are especially observable in electronic business networks. First, we account for the fact that a crucial feature of such electronic business networks is the participation of so-called ‘key players’ which are e.g. crucial value enhancers in value chains or precursors in product development alliances. Accordingly, we account for heterogeneity among firms’ information contribution to networks. Key players provide higher levels of knowledge than ‘ordinary’ firms which could be e.g. tier 2 suppliers in value chains or followers in R&D development consortia. Second, in our model, firms can only connect to their direct neighbors but not to more distant players. This assumption reflects the peculiarities of electronic business networks where it is not necessarily required that every network member has a direct link to all other participants in order to guarantee knowledge exchange flows between all participants. A further intuition behind this assumption is that distances are interpreted in terms of similarities in business activities. That means, if a firm intents to join a network it has to incur costs (e.g. adjustment costs for its database, training of personnel) in order to sample a neighbor which is member of the network. On the other hand, the existing network has to incur corresponding adjustment costs. The utility from a connection to a direct industry competitor might be higher than the utility from a very distant network member say from another industry. Third, we introduce network congestion costs into a model of network formation. A joining member imposes costs on all existing network members in terms of increasing communication costs or adjustment costs causing a firm’s effectiveness at managing its alliances to decline with the number of alliances maintained.

Appendix

Proof of Proposition 2

The proof follow basically the same argumentation as the proof to Proposition 2. Again this builds on the proof by Jackson and Wolinsky (1996). If $c > \delta(1 + w_k)$ then there is no link that provides a (weakly) positive utility. Accordingly, the only efficient network structure is the empty network. If $c < \delta(1 + w_k)$ then the value of each link is as determined in (2). The value of any additional link is then always positive for $n + 1$ firms, as long as $\frac{2\delta-\delta^2}{1-\delta} < c < \delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i$. If $c < \frac{2\delta-\delta^2}{1-\delta}$, the complete graph is efficient due to the argumentation in the proof to Proposition 2. ■

Proof of Proposition 3

The proof takes into account the results from the proofs to Propositions 1 and 2. Dependent on the values of c different links may form. Note that the lowest net value of a link is 2δ, which is a link between two firms which are not the key player.

- If $c < 2\delta$ than even this link forms immediately when those two neighbors are matched. Accordingly, which such a low value for c every link forms.
- If $2\delta < c \leq \frac{2\delta-\delta^2}{1-\delta}$, then only a link between the key player and one of its neighbors is valuable in the first period. Because $c > 2\delta$, no other link will be formed in the first period. Due to this argumentation, in subsequent periods, only links to the already existing network, including the key player can be valuable. Since $c \leq \frac{2\delta-\delta^2}{1-\delta}$, the cost for a link is low enough, that the network converges to the complete chain.
- If $\frac{2\delta-\delta^2}{1-\delta} < c \leq \delta(1 + w_k)$, due to the same argumentation as above, only a

link involving the key player k and one of its neighbors can be valuable in the first period. Again, due to $c > 2\delta$, in subsequent periods, only links to the already existing network, including the key player can be valuable. But now, since $\frac{2\delta-\delta^2}{1-\delta} < c$ the network will not converge to the complete chain g^N but only to a network including just n members (with $0 < n < N$), determined by $\delta + \delta^{\lceil \frac{n}{2} \rceil}(w_k - 1) + \sum_{i=1}^{n} \delta^i \simeq c$.

- If $c > \delta(1 + w_k)$, then no links can ever form. The highest value of a link is $\delta(1 + w_k)$, which is a link between the key player and one of its direct neighbors. If this value is lower than the cost c of establishing a link, there is no incentive to form any link. ■

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