An Empirical Analysis of Digital Advertising

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Analyzing the Interdependence between Web and Mobile Advertising

ABSTRACT

As companies divert more funds from traditional media towards digital advertising, they are interested in understanding what effects the two channels of advertising—web advertising and mobile advertising—have on consumer choices. Any analysis that measures the effects of web and mobile advertising only separately remains incomplete. In this paper, we model and estimate the interrelationship between web and mobile advertisements. First, we design and execute a randomized field experiment. Our findings indicate that implementing web and mobile ads simultaneously improves web click-through rates, mobile click through rates and web conversion rates but decreases mobile conversion rates. This happens primarily because consumers are disproportionately more likely to click on a display ad from a mobile device but subsequently make a purchase through a PC. The cross-channel conversion rate from mobile to web is 2.7 times higher than that from web to mobile. Despite this, the net change in sales (revenues) is positive when both web and mobile advertising are available to consumers. We present results from policy simulations regarding the optimal level of web and mobile advertising using both CPC (cost per click) and CPM (cost per thousand impressions) based pricing. To generalize our experiment results, we utilize a massive panel dataset based on advertisement (product)-level responses to display ads on both web and mobile channels for multiple product categories. The results from the archival data analysis corroborate our results from the field experiment. These synergistic relationships run counter to the single-click methodology in use and suggest that, for a market in which advertising dollars are allocated based on their influence on purchase behavior, new methods must be developed to insure efficient market functioning.

Keywords: Mobile Advertising, Web Advertising, Interdependence, Randomized Field Experiment, Econometric Models, Hierarchical Bayesian, Policy Simulations

INTRODUCTION

In the increasingly advertising-filled, multi-channel environment, consumers are exposed to more than one advertising message from a marketer through different channels. As mobile devices such as smartphones and tablets become popular ecommerce channels, consumers can browse for products and make purchases anywhere and anytime. According to Google's recent report on consumer behavior in the new multi-screen world (2012a), nine out of ten people use multiple screens sequentially to accomplish a task over time. For example, a consumer may click on a display advertising about a new cosmetics product on a smartphone first, but she does not need to finish her purchase task on the same device on which she clicks; she can wrap up the purchase decision on the same site subsequently through a laptop or a PC while sitting at work or home. Hence, any analysis that gives web advertising all the credit for conversions or mobile advertising all the credit for click-throughs remains incomplete and flawed.[1]

The purpose of this paper is to analyze the relationship between web display advertising and mobile display advertising to examine how their interaction affects click-throughs and conversion rates. In particular, we examine whether exposures to display advertising through both web and mobile channels are likely to increase click-throughs and conversion rates more, as compared to advertising on the web channel or a mobile channel alone.

There are, on the one hand, a number of likely reasons for positive synergies between web and mobile display advertising. Display advertising can generate brand awareness and increase purchase intent. As smartphones and tablets are frequently used throughout the day, mobile display advertising can reinforce acquisition and brand messages that users may receive from web display advertising. Thus, it can serve as a valuable initial step into the purchase process across multiple devices. On the other hand, there are also likely reasons for negative synergies between web and mobile advertising. Brand messaging through both can provide redundant brand messages and possibly lead to inefficiency in marketing resource allocation if the message gets through irrespective of the channel.

A few papers have looked at the direction and magnitude of the interdependence between two advertising media. Yang and Ghose (2010) have shown that organic and sponsored search listings have a positive interdependence, such that the presence of an advertiser in the organic listings increases its click-through and conversion rates in the paid listings, and vice-versa. In contrast, there is evidence that display ads can have a negative impact on sponsored search advertising when consumers get exposed to them before seeing the search ads (Ghose et al. 2011). In addition, display ads can increase searches for a company's own and competitor brands (Lewis and Nyugen 2012), offline direct marketing substitutes for paid search advertising (Goldfarb and Tucker 2011a) and offline billboard advertising substitutes for web display advertising (Goldfarb and Tucker 2011b). Therefore, it is difficult to simply draw on prior results and infer whether web and mobile advertisements are complements or substitutes. It is an empirical question that can be context-specific.

To examine this question, we first design and execute a controlled field experiment. In the context of a digital-products retailer, we conduct a randomized field experiment in which we periodically switch on and switch off web and mobile ads in order to assess the effect that ads in each channel have on user behavior, separately and collectively. We find that when both web and mobile advertising are switched on: 1) the web click-through rate is 34-percent higher than when only web advertising is on; 2) the mobile click-through rate is 23-percent higher than when only mobile advertising is on; 3) the web conversion rate is 36-percent higher than when only web advertising is inactive; and, notably, 4) the mobile conversion rate is 16-percent lower than when only mobile advertising is present, though it is not statistically significant. Moreover, we find the cross-channel conversion rate from mobile to web is 2.7 times higher than that from web to mobile. Despite this, the net change in sales (revenues) is positive when both web and mobile advertising are switched on compared to when only one channel is switched on, suggesting the presence of a reinforcement effect in consumers' minds from seeing both ads.

Further, based on the experiment results, we present a number of policy simulation results on the optimal level of web and mobile advertising. Specifically, we examine the profit-maximizing ratio between web and mobile advertising impressions when costs per impression and cost per click are given to the advertiser as well as the maximum amount an advertiser should pay for per-unit mobile advertising impressions and click-throughs for a given ratio of web and mobile advertising impressions. Insights into such questions can provide useful guidelines for marketers who deal with resource-allocation decisions between web and mobile advertising channels.

Finally, to generalize our results from the field experiment, we employ a large-scale integrated archival dataset from one of the largest web and mobile advertising network companies in Asia. The dataset contains information on advertisement impressions, consumer click-throughs, and the conversion activities for various kinds of products encompassing more than $33 billion in advertising transactions from 265 advertisers (products) for more than a year. In the dataset, we observe conversions, click-throughs, and advertising impressions from both mobile and web channels. Using these data, we estimate a simultaneous equation model of consumer click-through and conversion behavior. In order to control for product-level heterogeneity, we characterize our model in a hierarchical Bayesian framework and estimate it with Markov Chain Monte Carlo methods.

Overall, we find that the results from the experiment are nicely corroborated by this archival data analysis. Examining clicks on web and mobile display ads and conversion activities through each channel generates our main results. Like in the field experiment, we find that the cross-channel interdependence has a positive impact on click-throughs regardless of the advertising channel. However, while cross-channel interdependence has a positive impact on web conversions, it has a negative impact on mobile conversions. These findings suggest the importance of accounting for the cross-channel interdependence between web and mobile advertising. For example, if only the same-channel effect were accounted for, the combined conversion effect of mobile advertising would be underestimated by 48 percent, while that of web advertising would be underestimated by 17 percent. That is, mobile advertising results in higher cross-channel impacts on combined conversion through indirect interdependence than web advertising does. To summarize, we show that web and mobile work together and affect each other. Optimal decisions in one need to take account of the effects in the other.

The rest of this paper is organized as follows. In Section 2, we provide related literature to build the theoretical framework. Section 3 presents the randomized field experiment results and discusses the policy simulation results. Section 4 describes the archival data, describes the econometric models and provides the results. Section 5 discusses the implications of the results and concludes.

RELATED LITERATURE

In this section, we discuss the literature that has examined the interdependence between different kinds of advertising channels and platforms. We also discuss related literature on mobile marketing and user behavior.

Interdependence between Advertising Channels

An emerging stream of literature has examined the interdependence between advertising channels/platforms. The outcome of such research has important managerial implications for whether a firm should invest in both channels/platforms (if there exists a synergistic effect) or in just one of the two (if there is no synergistic effect). Our paper is closely related to a stream of work that examines the interdependence between online advertising channels. For example, in the literature on online search advertising, Rutz and Bucklin (2011) show that there are spillovers between search advertising on branded and generic keywords; some customers may start with a generic search to gather information, but they later use a branded search to complete their transaction. Ghose and Yang (2009) build a model to map consumers' search–purchase relationship in sponsored search advertising. They provide evidence of horizontal spillover effects from search advertising that result in purchases across other product categories. Yang and Ghose (2010), conducting both an empirical investigation and a randomized field experiment on the impact of paid and organic search for several product categories, demonstrate their interdependence. They find that click-throughs on organic listings have a positive interdependence with click-throughs on paid listings, and vice versa. Agarwal et al. (2012) provide quantitative insights into the impact of organic search results on the sponsored search, especially when there is an overlap in the results. They find that competing organic listings in higher positions have a negative impact on conversion performance for generic keywords, but may help conversion performance for more specific keywords.

Goldfarb and Tucker (2011c) conduct a field experiment and show that targeted advertising and highly visible display advertising work better separately than they do together. They find that, due to the advertising viewers' privacy concerns, display advertising that both matches website content and is obtrusive does worse at increasing purchase intent than advertising that does only one or the other. Their results suggest that two single advertising strategies that are effective on their own do not always work well in combination when negative interdependence exists.

There is also an emerging stream of literature that examines the interdependence between online and offline advertising channels. For example, Goldfarb and Tucker (2011a) conduct a natural experiment to explore substitution patterns between online and offline advertising channels. They find that offline direct marketing substitutes for paid search advertising for legal services. Goldfarb and Tucker (2011b) combine field and natural experiments to show that online display advertising is most effective in places that ban offline advertising for alcoholic beverages. Thus, offline billboard advertising substitutes for online display advertising. Their results suggest that online advertising could reduce the effectiveness of attempts to regulate other advertising channels because online advertising substitutes for (rather than complements) offline advertising.

Mobile Marketing and User Behavior in the Mobile Internet

Our paper builds on and relates to the literature on mobile marketing. An emerging stream of literature has discussed the role of mobile technologies in marketing. Shankar and Balasubramanian (2009) provide an extensive review of mobile marketing. Shankar et al. (2010) develop a conceptual framework for mobile marketing in the retailing environment and discuss retailers' mobile marketing practices. For example, retailers can communicate with consumers near their stores via mobile phones by transmitting relevant information—such as the store's location, product availability, quality, price, and coupons—in response to customers' mobile-phone-initiated requests. Sinisalo (2011) examines the role of the mobile medium among other channels within multichannel CRM communication. Moreover, specific consumer segments, such as the Gen Y youth market, increasingly use mobile phones as single-source communication devices (Sultan et al. 2009) to gain greater access to social circles, location-based information and content. Bart et al. (2012) study mobile advertising campaigns and find that they are effective at increasing favorable attitudes and purchase intentions for higher (versus lower) involvement products, and for products that are seen as more utilitarian (versus more hedonic).

Recently, mobile couponing and location-based advertising have gained increasing interest as a marketing tool. Dickinger and Kleijnen (2008) find that a segment of "value seekers" are more prone to mobile-coupon redemption. Molitor et al. (2012) show that the higher the discount from mobile coupons and the closer the consumers are to the physical store offering the coupon, the more likely they are to download the mobile coupons. The research on location-based advertising is still in its nascent stage. Previous studies have examined consumer perceptions and attitudes towards mobile location-based advertising (e.g., Brunner and Kumar 2007; Xu et al. 2009). Gu (2012) examines both the short-term and long-term sales effects of location-based advertising. There is also an emerging stream of literature on consumer behavior on the mobile Internet. For example, Ghose and Han (2011) find that there is a negative and statistically significant temporal interdependence between content generation and usage on the mobile Internet. This is because, on the mobile Internet, users not only invest time, but also incur explicit transmission charges to generate and use content in certain countries. Ghose et al. (2012) explore how Internet browsing behavior varies between mobile-phone and PC users in a natural experimental setting. They show that search costs are higher and the benefit of browsing for geographically close matches with retail stores is higher on the mobile internet compared to the PC internet.

In summary, the literature has shown that whether different types of advertising are complements or substitutes depends on the context. Hence, the direction and the overall magnitude of the mobile advertising effect above and beyond that of web advertising is an important empirical question.

A RANDOMIZED FIELD EXPERIMENT

In this section, we describe the randomized field experiment and thereafter discuss policy simulations to discuss optimal policies for web and mobile advertising.

Economic Impact of Cross-Channel Interdependence

A field experiment was designed to examine the impact of the simultaneous presence of web and mobile advertising on the click-through and conversion performances in web and mobile channels, respectively. We conduct the experiment in collaboration with a digital products retailer in South Korea and focus on consumer responses to display advertisement for e-books. The e-book company advertises its products by displaying ads on the front page of its website and on its mobile site. During the experimental period, we worked with the company to periodically, display only web advertising, only mobile advertising, both web and mobile advertising, or no advertising. For example, the company displays book-cover advertising for an e-book through only a web channel from Monday to Wednesday in week 1. Then, the company displays the same book- cover advertising through both web and mobile channels from Thursday to Saturday in week 1. Next, the company pauses its advertising in the web channel while continuing to advertise through the mobile channel from Monday to Wednesday in week 2. Finally, the company pauses its advertising through both web and mobile channels from Thursday to Saturday in week 2. We track and measure click-throughs and conversion results from both web and mobile channels throughout the experiment period. This is because consumers can purchase an e-book through either of the two channels regardless of whether the e-book ad is displayed on the web channel or on the mobile channel, or both.

We conduct the field experiment over a six week period over the months of June and July 2012. The company randomly selected a sample of 30 e-books to conduct this experiment. Table 5 demonstrates the advertising schedule in our field experiment. In the first period (Week 1 – Week 2), we conduct the experiment for the first ten e-books (i.e., A1 – J1), and then in the second period (Week 3 – Week 4), we conduct the experiment for another ten e-books (i.e., A2 – J2). Lastly, in the third period (Week 5 – Week 6), we conduct the experiment for the last ten e-books (i.e., A3 – J3). In each period, we randomly assign five e-books to Cohort 1 and the remaining five to Cohort 2. The only difference in terms of manipulation between Cohorts 1 and 2 is the order in which a particular advertising channel is used during the experimental period. We find that our results remain qualitatively the same, regardless of this order effect. Moreover, each treatment runs for three days in our experiment. We find that the time gap between advertising click-throughs and conversions is short. Nearly 90 percent of purchases are made within two days after clicking on the advertising. Hence, any potential carry-over effects of advertising are unlikely in our setting.

<< Insert Table 1 about here>>

The dataset for the field experiment includes approximately 26 million advertising transaction records during the six-week period. The set of books include personal-development books, history and arts books, business books, and literature and fiction books. Table 2 shows the descriptive statistics of e-book profiles used in the field experiment.

<< Insert Table 2 about here>>

Based on the analysis of the field experimental data, we find that when both web and mobile advertising are available to consumers, the web click-through rate is 34-percent higher than when only web advertising is present (see Figure 1(a)). A two-sample t-test reveals that the difference is statistically significant at the five-percent level. We find that when both web and mobile advertising are available to consumers, the mobile click-through rate is 23-percent higher than when only mobile advertising is present (see Figure 1(b)). The difference is statistically significant at the ten-percent level.

<< Insert Figure 1(a) and Figure 1(b) about here>>

We next examine changes in conversion rates. We find that when both web and mobile advertising are available to consumers, the web conversion rate is 36-percent higher than when only web advertising is present (see Figure 2(a)). The difference is statistically significant at the five-percent level. However, we find that when both web and mobile advertising are available to consumers, the mobile conversion rate is 16-percent lower than when only mobile advertising is present (see Figure 2(b)). Although the difference is not statistically significant (p-value 0.213), this result suggests that some consumers click on mobile advertising but prefer to purchase through a web channel.

<< Insert Figure 2(a) and Figure 2(b) about here>>

We also examine changes in total conversion in terms of total sales (revenues) from both web and mobile channels. We find that when both web and mobile advertising are available to consumers, the total sales amount is 97-percent higher than when only web advertising is present (see Figure 3(a)). The difference is statistically significant at the five-percent level. Similarly, we find that when both web and mobile advertising are available to consumers, the total sales amount is 48-percent higher than when only mobile advertising is present (see Figure 3(b)). The difference is also statistically significant at the five-percent level. Note that the two figures show relative increase in sales amounts when the company runs the ad in both channels as compared to in a single channel (either web only or mobile only). This result suggests that when both web and mobile advertising are available to consumers even though web conversions increase and mobile conversions decrease, the net change in sales (revenues) would still be positive. Thus a reinforcement effect of seeing both ads on web and mobile media would increase the overall conversions in total.

<< Insert Figure 3(a) and Figure 3(b) about here>>

To further shed light on our understanding of consumers' cross-channel conversion behaviors, we examine the kinds of multi-channel paths (or multi-device paths) consumers take before they finally purchase. Table 3 presents cross-channel conversion rates in our experiment data. That is, percentage numbers in the diagonal cells denote the same-channel conversion rates, and percentage numbers in the off-diagonal cells denote the cross-channel conversion rates. First, we find that a majority of consumers prefer to purchase from the same channel in which they click through, as one would expect. For example, 95.8 percent of users who click on web advertising finally purchase through the same web channel. Similarly, 88.5 percent of users who click on mobile advertising finally purchase through the same mobile channel. Second, we find that the cross-channel conversion rate from mobile to web is 2.7 times higher than that from web to mobile (i.e., 11.6 percent versus 4.2 percent). This finding lends further support to previous evidence of consumers' preference to purchase in a PC/laptop environment as compared to a mobile environment.

<< Insert Table 3 about here>>

The randomized nature of the experiment suggests a causal interpretation such that: 1) web and mobile display advertising work better together than separately in terms of improving click-throughs; 2) implementing both web and mobile advertising improves web conversions, but it does not improve mobile conversions due to the asymmetric cross-device conversion rates; and 3) the net change in total sales from both web and mobile advertising is positive.

Optimal Policies for Web and Mobile Advertising

We next use the field experiment results and conduct simulations to evaluate and recommend optimal policies for web and mobile advertising. Specifically, we address the following questions: (1) What is the profit-maximizing ratio between web and mobile advertising impressions when costs for per-unit impressions and click-throughs are given to the advertiser? (2) What is the maximum amount an advertiser should pay for per-unit mobile advertising impressions and click-throughs for a given ratio of web and mobile advertising impressions? The answers to these questions will provide useful guidelines for marketing practitioners who deal with resource-allocation decisions between web and mobile advertising channels. Our simulation results for optimal advertising policies are based on cost-per-click (CPC) and cost-per-mile (CPM) pricing, respectively. Table B1 in Appendix B provides notations and parameter descriptions.

<< Insert Table B1 about here>>

CPC-based Optimal Advertising Policies: First, we consider a scenario in which an advertiser sets its target impressions for each channel—web and mobile—given the values of web CPC and mobile CPC. We assume that the advertiser maximizes its profit while satisfying budget constraints. An advertiser will use both web and mobile advertising if and only if the profit from both is greater than that from only web advertising. Table B2 in Appendix B summarizes an advertiser's cost, advertising budget constraints, revenue, total profit, and decision criteria when only web advertising is active and when both web and mobile advertising are active, respectively. We provide an optimal solution of the target impression ratio between web and mobile advertising.

<< Insert Table B2 about here>>

We define three parameters – δ, λ, and π. First, δ refers to the ratio of web ad impressions between when web and mobile ads are active, $I_w^{(1,1)}$, and when only web ads are active and mobile ads are inactive, $I_w^{(1,0)}$. That is, $\delta = I_w^{(1,1)} / I_w^{(1,0)}$. For example, $\delta = 0.9$ means that an advertiser spends ten-percent less on web ad impressions in the presence of mobile advertising than in the absence of mobile advertising. Second, λ refers to the ratio of web ad impressions relative to mobile impressions (i.e., $\lambda = I_m / I_w^{(1,1)}$). When $\lambda = 1$, an advertiser advertises an equal amount of web and mobile ad impressions. Lastly, π refers to the ratio between CPC_m and CPC_w. As π increases, the mobile ad cost becomes higher than the web counterpart.

Table 4 presents the optimal ratio of mobile and web ad impressions. Although we have examined results based on different values for δ, we report results based on $\delta = 0.9$ for brevity. However, the simulation results qualitatively remain the same, irrespective of the value for δ. The vertical axis denotes web CPC, and the horizontal axis denotes mobile CPC. For example, when $CPC_w = \$1$ and $CPC_m = \$0.5$, the profit-maximizing ratio of mobile and web impressions, λ, is 0.39. That is, an advertiser should go with 39,000 mobile impressions for every 100,000 web impressions.

<< Insert Table 4 about here>>

Next, we consider another scenario in which an advertiser determines the maximum willing-to-pay for mobile CPC, for a given ratio of web ad impressions relative to mobile impressions (λ) and a given ratio of CPC_m and CPC_w (π). We also examine results based on different values for δ; however, we report results based on $\delta = 0.9$ for brevity. Table 5 presents the upper bound for CPC_m. For example, when $\lambda = 1$ and $\pi = 0.5$, the profit-maximizing CPC_m upper bound is 0.31.

<< Insert Table 5 about here>>

CPM-based Optimal Advertising Policies: Next, we conduct policy simulations based on cost per thousand impressions (CPM) pricing. Table B3 in Appendix B summarizes an advertiser's cost, advertising budget constraints, revenue, total profit, and decision criteria when only web advertising is active and when web and mobile advertising are active, respectively. Table 6 presents the optimal ratio of mobile and web ad impressions. While we have examined results based on different values for δ, we report results based on $\delta = 0.9$ for brevity. We find that, for example, when $CPM_w = \$5$ and $CPM_m = \$2$, the profit-maximizing ratio of mobile and web impressions, λ, is 2.33. That is, an advertiser should have 233,000 mobile impressions when there are 100,000 web impressions. Table 7 presents the upper bound for CPM_m. For example, when $\lambda = 1$ and $\pi = 0.5$, the profit-maximizing CPM_m upper bound is 5.13.

<< Insert Table B3, 6, 7 about here>>

ARCHIVAL DATA ANLAYSIS

The randomized field experiment was conducted for one set of products: e-books. In this section, we describe an archival data analysis that was conducted to generalize results from the field experiment. We provide detail on our empirical setting and describe the data, describe our econometric model, and provide analysis results.

Empirical Setting

We negotiated access to a massive archival data from an advertising network company that works with a number of different retailers and advertisers in South Korea. The company provided us with details of the mobile and web advertising campaigns of 265 distinct products and their performance metrics, such as the number of advertising impressions and the number of click-throughs. The key role of an ad network company is linking advertisers to media that want to host advertising by aggregating the advertising media supply from various kinds of websites (e.g., Youtube) and mobile apps (e.g., Angry Birds) or mobile websites and matching the supply with advertiser (e.g., Samsung, Walmart) demand.[2] To be specific, an ad network company uses an advertising server to deliver advertisements from advertisers to individuals who are browsing a web site or using mobile applications. When a user views an advertisement through her PC or mobile device, the company's advertising server tracks and stores detailed records, including advertising profiles (e.g., advertiser, product), user profiles (e.g., age, gender, device type), user actions (e.g., clicks-throughs), and so on. We supplement our ad network archival data with web and mobile advertising archival data from various retailers who work with the ad network company. An important thing in this data is that consumers are exposed to the same set of display advertising regardless of whether they use the company's website or the mobile site.

Data Description

We have data on clicks and conversions on mobile and web advertising for a large variety of products, including movie tickets, travel products, e-books, electronic gadgets, and cosmetics. The advertising campaign and performance data span from December 2010 to November 2011. It includes daily information on advertisement (product)-specific impressions and click-throughs from both web and mobile channels encompassing 33 billion observations and on advertisement (product)-specific impressions, click-throughs, and conversions from both web and mobile channels encompassing more than 12 million advertising transaction records. The conversion data include information about the sale price, product or content size, length of the manufacturer-provided product description, age of the product since release into the market, product category, and the average valence and volume of user ratings for the product. Note that we can distinguish users' access channels, such that our advertisement (product)-specific data include web advertising impressions, mobile advertising impressions, web click-throughs, mobile click-throughs, web conversions, and mobile conversions. Table 8 provides variable descriptions and shows summary statistics of the key variables in the dataset.

<< Insert Table 8 about here>>

Econometric Model

To formally characterize our econometric model for the archival data analysis, for a given product, we model the number of click-through activities from each channel (i.e., a PC and a mobile device) in terms of web impression, mobile impression, and the product of the two. Similarly, we model the number of conversion activities from each channel in terms of web clicks, mobile clicks, and the product of the two. To control for unobserved product-level heterogeneity, we characterize our model in a hierarchical Bayesian framework and estimate it using Markov Chain Monte Carlo methods.

Our model consists of two distinct user activities: 1) click-throughs and 2) conversions.

Click-through Equations: We specify that the number of click-through activities for product *i* at time *j* from the web is a function of web advertising impressions, clicks on mobile advertising impressions, the product of the two, and other factors as follows, for the web click-throughs:

$$(1)\ \mathrm{Click\_web}_{ij}^{(L)} = \theta_{i0} + \theta_{i1}\mathrm{Imp\_web}_{ij}^{(L)} + \theta_{i2}\mathrm{Imp\_Mobile}_{ij}^{(L)} + \theta_{i3}\mathrm{Imp\_web}_{ij}^{(L)} \times \mathrm{Imp\_Mobile}_{ij}^{(L)} + \eta_{ij}$$

where *web* denotes channels such as a PC/desktop and *Mobile* denotes a smartphone. $^{(L)}$ denotes the logarithm of the variable. Regarding the interdependence between web and mobile advertising for product *i*, θ_{i2} captures the direct impact of mobile impression on web click-throughs in the absence of any web impression. θ_{i3} captures the indirect impact of mobile impression on web click-throughs in the presence of web impression.

The advertising for each product may have an inherent propensity to click-through. Hence, the likelihood of click-throughs will be associated with the product advertising-specific characteristics. In equation (2), we allow θ_{i0} to vary by product category and time and also capture the product-level unobservable heterogeneity with a random coefficient. Similarly, we allow $\theta_{i1} - \theta_{i3}$ to capture the product-level unobservable heterogeneity with a random coefficient as follows:

$$(2)\ \theta_{i0} = \kappa_0 + \textstyle\sum_k \kappa_{1k}\mathrm{Time}_k + \sum_k \kappa_{2k}\mathrm{Category}_{ik} + \lambda_i^{\theta_0}$$

$$\theta_{i1} = \kappa_3 + \lambda_i^{\theta_1}$$

$$\theta_{i2} = \kappa_4 + \lambda_i^{\theta_2}$$

$$\theta_{i3} = \kappa_5 + \lambda_i^{\theta_3}$$

The covariance among the random coefficients is specified as follows:

$$\underline{\lambda}_i^{\theta} = \begin{bmatrix} \lambda_i^{\theta_0} \\ \lambda_i^{\theta_1} \\ \lambda_i^{\theta_2} \\ \lambda_i^{\theta_3} \end{bmatrix} \sim \mathrm{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\theta} & \Sigma_{12}^{\theta} & \Sigma_{13}^{\theta} & \Sigma_{14}^{\theta} \\ \Sigma_{21}^{\theta} & \Sigma_{22}^{\theta} & \Sigma_{23}^{\theta} & \Sigma_{24}^{\theta} \\ \Sigma_{31}^{\theta} & \Sigma_{32}^{\theta} & \Sigma_{33}^{\theta} & \Sigma_{34}^{\theta} \\ \Sigma_{41}^{\theta} & \Sigma_{42}^{\theta} & \Sigma_{43}^{\theta} & \Sigma_{44}^{\theta} \end{bmatrix} \right).$$

where $\mathrm{Category}_{ik}$ takes 1 if product *i* belongs to category *k*, and 0 otherwise. Time_{jk} takes 1 if $j = k$, and 0 otherwise.

Similarly, we specify the number of click-through activities from mobile as follows:

$$(3)\ \mathrm{Click\_Mobile}_{ij}^{(L)} = \omega_{i0} + \omega_{i1}\mathrm{Imp\_Web}_{ij}^{(L)} + \omega_{i2}\mathrm{Imp\_Mobile}_{ij}^{(L)} + \omega_{i3}\mathrm{Imp\_Web}_{ij}^{(L)} \times \mathrm{Imp\_Mobile}_{ij}^{(L)} + \nu_{ij}$$

We further specify the random coefficients as follows:

$$(4)\ \omega_{i0} = \delta_0 + \sum_k \delta_{1k}\mathrm{Time_k} + \sum_k \delta_{2k}\mathrm{Category_{ik}} + \lambda_i^{\omega_0}$$

$$\omega_{i1} = \delta_3 + \lambda_i^{\omega_1}$$

$$\omega_{i2} = \delta_4 + \lambda_i^{\omega_2}$$

$$\omega_{i3} = \delta_5 + \lambda_i^{\omega_3}$$

$$\underline{\lambda}_i^{\omega} = \begin{bmatrix} \lambda_i^{\omega_0} \\ \lambda_i^{\omega_1} \\ \lambda_i^{\omega_2} \\ \lambda_i^{\omega_3} \end{bmatrix} \sim \mathrm{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\omega} & \Sigma_{12}^{\omega} & \Sigma_{13}^{\omega} & \Sigma_{14}^{\omega} \\ \Sigma_{21}^{\omega} & \Sigma_{22}^{\omega} & \Sigma_{23}^{\omega} & \Sigma_{24}^{\omega} \\ \Sigma_{31}^{\omega} & \Sigma_{32}^{\omega} & \Sigma_{33}^{\omega} & \Sigma_{34}^{\omega} \\ \Sigma_{41}^{\omega} & \Sigma_{42}^{\omega} & \Sigma_{43}^{\omega} & \Sigma_{44}^{\omega} \end{bmatrix} \right).$$

Conversion Equations: We specify that the number of conversion activities for product *i* at time *j* from the web is a function of clicks on web display advertising, clicks on mobile display advertising, the product of the two, and other factors as follows:[3]

$$(5)\ \mathrm{Conv\_Web}_{ij}^{(L)} = \beta_{i0} + \beta_{i1}\mathrm{Click\_Web}_{ij}^{(L)} + \beta_{i2}\mathrm{Click\_Mobile}_{ij}^{(L)} + \beta_{i3}\mathrm{Click\_Web}_{ij}^{(L)} \times \mathrm{Click\_Mobile}_{ij}^{(L)} + \varepsilon_{ij}.$$

Regarding the interdependence between web and mobile advertising for product *i*, β_{i2} captures the direct impact of mobile click-throughs on web conversions in the absence of any web advertising. β_{i3} captures the indirect impact of mobile click-throughs on web conversions in the presence of web advertising.

In addition, each product may have an inherent propensity to convert. Hence, the likelihood of conversion will be associated with the product-specific characteristics. In equation (6), we allow β_{i0} to vary by observable product characteristics, such as price, product or content size, description length, age, user review count, review rating, category, and time. We capture the product-level unobservable heterogeneity with a random coefficient as follows:

$$(6)\ \beta_{i0} = \alpha_0 + \alpha_1\mathrm{Price}_{ij}^{(L)} + \alpha_2\mathrm{Size}_{ij}^{(L)} + \alpha_3\mathrm{Desc}_{ij}^{(L)} + \alpha_4\mathrm{Age}_{ij}^{(L)} + \alpha_5\mathrm{Review}_{ij}^{(L)}$$

$$+\alpha_6\mathrm{Rating_{ij}} + \sum_k \alpha_{7k}\mathrm{Category_{ik}} + \sum_k \alpha_{8k}\mathrm{Time_{jk}} + \lambda_i^{\beta_0}$$

$$\beta_{i1} = \alpha_9 + \lambda_i^{\beta_1}$$

$$\beta_{i2} = \alpha_{10} + \lambda_i^{\beta_2}$$

$$\beta_{i3} = \alpha_{11} + \lambda_i^{\beta_3}$$

where Category_{ik} takes the value of 1 if product *i* belongs to category *k*, and 0 otherwise. Time_{jk} takes the value of 1 if $j = k$, and 0 otherwise. The covariance among the random coefficients is specified as follows:

$$\underline{\lambda}_i^{\beta} = \begin{bmatrix} \lambda_i^{\beta_0} \\ \lambda_i^{\beta_1} \\ \lambda_i^{\beta_2} \\ \lambda_i^{\beta_3} \end{bmatrix} \sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\beta} & \Sigma_{12}^{\beta} & \Sigma_{13}^{\beta} & \Sigma_{14}^{\beta} \\ \Sigma_{21}^{\beta} & \Sigma_{22}^{\beta} & \Sigma_{23}^{\beta} & \Sigma_{24}^{\beta} \\ \Sigma_{31}^{\beta} & \Sigma_{32}^{\beta} & \Sigma_{33}^{\beta} & \Sigma_{34}^{\beta} \\ \Sigma_{41}^{\beta} & \Sigma_{42}^{\beta} & \Sigma_{43}^{\beta} & \Sigma_{44}^{\beta} \end{bmatrix} \right)$$

Similarly, we specify the number of conversion activities for product *i* at time *j* from mobile as follows:

$$(7)\ \text{Conv\_Mobile}_{ij}^{(L)} = \pi_{i0} + \pi_{i1}\text{Click\_Web}_{ij}^{(L)} + \pi_{i2}\text{Click\_Mobile}_{ij}^{(L)} + \pi_{i3}\text{Click\_Web}_{ij}^{(L)} \times \text{Click\_Mobile}_{ij}^{(L)} + \psi_{ij}$$

$$(8)\ \pi_{i0} = \varphi_0 + \varphi_1\text{Price}_{ij}^{(L)} + \varphi_2\text{Size}_{ij}^{(L)} + \varphi_3\text{Desc}_{ij}^{(L)} + \varphi_4\text{Age}_{ij}^{(L)} + \varphi_5\text{Review}_{ij}^{(L)}$$

$$+\varphi_6\text{Rating}_{ij} + \textstyle\sum_k \varphi_{7k}\text{Category}_{ik} + \sum_k \varphi_{8k}\text{Time}_{jk} + \lambda_i^{\pi_0}$$

$$\pi_{i1} = \varphi_9 + \lambda_i^{\pi_1}$$

$$\pi_{i2} = \varphi_{10} + \lambda_i^{\pi_2}$$

$$\pi_{i3} = \varphi_{11} + \lambda_i^{\pi_3}$$

$$\underline{\lambda}_i^{\pi} = \begin{bmatrix} \lambda_i^{\pi_0} \\ \lambda_i^{\pi_1} \\ \lambda_i^{\pi_2} \\ \lambda_i^{\pi_3} \end{bmatrix} \sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^{\pi} & \Sigma_{12}^{\pi} & \Sigma_{13}^{\pi} & \Sigma_{14}^{\pi} \\ \Sigma_{21}^{\pi} & \Sigma_{22}^{\pi} & \Sigma_{23}^{\pi} & \Sigma_{24}^{\pi} \\ \Sigma_{31}^{\pi} & \Sigma_{32}^{\pi} & \Sigma_{33}^{\pi} & \Sigma_{34}^{\pi} \\ \Sigma_{41}^{\pi} & \Sigma_{42}^{\pi} & \Sigma_{43}^{\pi} & \Sigma_{44}^{\pi} \end{bmatrix} \right).$$

Finally, to model the unobserved covariance among all these user actions, we let the error terms be correlated in the following manner:

$$(9)\ \begin{bmatrix} \eta_{ij} \\ \nu_{ij} \\ \varepsilon_{ij} \\ \psi_{ij} \end{bmatrix} \sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{bmatrix} \right).$$

Our econometric model closely resembles the triangular system in standard econometric textbooks (Lahiri and Schmidt 1978; Greene 1999) and the identification approach is similar to Yang and Ghose (2010). To see this more clearly, we model click-throughs from both PC and mobile devices as exogenously determined (modeled as a function of the number of advertising impressions in both web and mobile channels). Click-throughs from both channels, in turn, affect conversion activities through a PC and a mobile device, respectively, making this resemble a triangular system. As shown in Lahiri and Schmidt (1978) and discussed in Greene (1999), a triangular system of simultaneous equations can be identified without any further identification constraint, such as nonlinearity or correlation restriction. In particular, as Hausman (1975) notes, a generalized least- squares-based estimation (GLS) leads to uniquely identified estimates in a triangular system with a full covariance on the error term (Lahiri and Schmidt 1978).

Results

We cast our model in a hierarchical Bayesian framework and estimate it using Markov Chain Monte Carlo methods. We run the MCMC chain for 100,000 iterations and use the last 20,000 iterations to compute the mean and standard deviation of the posterior distribution of the model parameters. A more-detailed description of the MCMC algorithm is provided in Appendix A.

Web and Mobile Click-throughs: We present the estimates from the click-through equations in Table 9(a). Examining the interdependence between web and mobile advertising impressions generates our main results. A one-percent increase in web impressions is associated with a 0.628-percent increase in web click-throughs in the absence of any mobile advertising. A one-percent increase in mobile impressions is associated with a 0.501-percent increase in mobile click-throughs in the absence of web advertising. This implies that advertising impressions and click-throughs are positively associated with each other within the *same channel*.

Second, we find that advertising impressions and click-throughs are also positively associated with each other between *different channels*. Web click-throughs are positively associated with mobile advertising impressions (i.e., the coefficient is 0.235) in the absence of any web impressions. A one-percent increase in mobile impressions is associated with a 0.124-percent increase in web click-throughs.[4] Similarly, a one-percent increase in web impressions is associated with a 0.070-percent increase in mobile click-throughs. These findings indicate that the cross-channel interdependence effect of mobile advertising on web click-through is approximately twice that of web advertising on mobile click-through. Hence, our findings suggest that mobile display advertising reinforces the purchase intent of users who view web display advertising, and vice versa. The statistically significant results on unobserved heterogeneity variance-covariance estimates in Table 9(b) suggest that controlling for unobserved heterogeneity is important in our setting.

<< Insert Table 9(a) and Table 9(b) about here>>

Web and Mobile Conversions: We present the results on the coefficients of conversion equations in Table 10(a). First, we find that advertising click-throughs and conversions are positively associated with each other within the *same channel*. For example, a one-percent increase in web click-throughs is associated with a 0.071-percent increase in web conversions in the absence of any mobile click-throughs. Similarly, a one-percent increase in mobile click-throughs is associated with a 0.086-percent increase in mobile conversions in the absence of any web click-throughs.

In contrast to the results from web and mobile advertising impressions, the marginal effect of cross-channel interdependence on conversions is positive in the web channel but negative in the mobile channel. For example, mobile clicks are negatively associated with web conversions (i.e., the coefficient is -0.033) in the absence of any web click-throughs. When web and mobile click-throughs work together, a one-percent increase in mobile click-throughs is associated with a 0.217-percent *increase* in web conversions. However, a one-percent increase in web click-throughs is associated with a 0.016-percent *decrease* in mobile conversions. Hence, our results suggest that consumers are more likely to click on display advertising from mobile devices but make final purchase decisions through PCs. In addition, the statistically significant results on unobserved heterogeneity variance-covariance estimates in Table 10(b) suggest that controlling for unobserved heterogeneity is important in our setting.

<< Insert Table 10(a) and 10(b) about here>>

Lastly, the statistically significant results on the unobserved covariance among web conversions, mobile conversions, web click-throughs, and mobile click-throughs in Table 11 suggest that it is important to simultaneously model the consumer's click-throughs and purchase behaviors for each channel.

<< Insert Table 11 about here>>

Economic Impact of Cross-Channel Interdependence

We quantify the economic impact of cross-channel interdependence by comparing the impact of web advertising and mobile advertising on combined conversion rates when the cross-channel interdependence is considered versus when it is ignored. More specifically, we calculate the total derivative of combined conversion with respect to web impressions and mobile impressions, when the cross-channel interdependence is considered as follows:

$$(10)\quad \left(\frac{\mathrm{dConv\_Combined}}{\mathrm{dImp\_Web}}\right)^{\mathrm{Interdependence}} = \frac{\mathrm{dConv\_Web}}{\mathrm{dImp\_Web}} + \frac{\mathrm{dConv\_Mobile}}{\mathrm{dImp\_Web}}.$$

The total derivative incorporates the indirect interdependence and captures the overall dependency of the combined conversion on web and mobile advertising by allowing web click-throughs and mobile click-throughs to depend on web and mobile impressions. We calculate the total derivative of web conversion and mobile conversion with respect to web impression in the following manner:

$$\frac{\mathrm{dConv\_Web}}{\mathrm{dImp\_Web}} = \frac{\partial\mathrm{Conv\_Web}}{\partial\mathrm{Click\_Web}} \times \frac{\mathrm{dClick\_Web}}{\mathrm{dImp\_Web}} + \frac{\partial\mathrm{Conv\_Web}}{\partial\mathrm{Click\_Mobile}} \times \frac{\mathrm{dClick\_Mobile}}{\mathrm{dImp\_Web}} \quad \text{and}$$

$$\frac{\mathrm{dConv\_Mobile}}{\mathrm{dImp\_Web}} = \frac{\partial\mathrm{Conv\_Mobile}}{\partial\mathrm{Click\_Web}} \times \frac{\mathrm{dClick\_Web}}{\mathrm{dImp\_Web}} + \frac{\partial\mathrm{Conv\_Mobile}}{\partial\mathrm{Click\_Mobile}} \times \frac{\mathrm{dClick\_Mobile}}{\mathrm{dImp\_Web}}.$$

Next, we calculate the total derivative of the combined conversion with respect to web impressions when the cross-channel interdependence is ignored. Since any partial and total derivative between different channels becomes zero, we calculate the impact of web impressions on combined conversion as follows:

$$(11)\ \left(\frac{\mathrm{dConv\_Combined}}{\mathrm{dImp\_Web}}\right)^{\text{No Interdependence}} = \frac{\mathrm{dConv\_Web}}{\mathrm{dImp\_Web}} = \frac{\partial\mathrm{Conv\_Web}}{\partial\mathrm{Click\_Web}} \times \frac{\mathrm{dClick\_Web}}{\mathrm{dImp\_Web}}.$$

We present the results on changes in the combined conversion rates. The first column in Figure 4 shows that a one-percent increase in web advertising impressions leads to a 0.16-percent increase in the combined conversion when the cross-channel interdependence is considered. However, a one-percent increase in the web advertising impressions leads to a 0.13-percent increase in the combined conversion when the cross-channel interdependence is ignored. Thus, the combined conversion effect of web advertising would be underestimated by 17 percent if only the same-channel effect were accounted for.

<< Insert Figure 4 about here>>

Similarly, we calculate the total derivative of combined conversion with respect to mobile advertising and compare it when the cross-channel interdependence is considered with when the cross-channel interdependence is ignored. We do so in the following manner:

$$(12)\ \left(\frac{\mathrm{dConv\_Combined}}{\mathrm{dImp\_Mobile}}\right)^{\text{Interdependence}} = \frac{\mathrm{dConv\_Web}}{\mathrm{dImp\_Mobile}} + \frac{\mathrm{dConv\_Mobile}}{\mathrm{dImp\_Mobile}} \quad \text{and}$$

$$(13)\ \left(\frac{\mathrm{dConv\_Combined}}{\mathrm{dImp\_Mobile}}\right)^{\text{No Interdependence}} = \frac{\mathrm{dConv\_Mobile}}{\mathrm{dImp\_Mobile}} = \frac{\partial\mathrm{Conv\_Mobile}}{\partial\mathrm{Click\_Mobile}} \times \frac{\mathrm{dClick\_Mobile}}{\mathrm{dImp\_Mobile}}$$

where

$$\frac{\mathrm{dConv\_Web}}{\mathrm{dImp\_Mobile}} = \frac{\partial\mathrm{Conv\_Web}}{\partial\mathrm{Click\_Web}} \times \frac{\mathrm{dClick\_Web}}{\mathrm{dImp\_Mobile}} + \frac{\partial\mathrm{Conv\_Web}}{\partial\mathrm{Click\_Mobile}} \times \frac{\mathrm{dClick\_Mobile}}{\mathrm{dImp\_Mobile}} \quad \text{and}$$

$$\frac{\mathrm{dConv\_Mobile}}{\mathrm{dImp\_Mobile}} = \frac{\partial\mathrm{Conv\_Mobile}}{\partial\mathrm{Click\_Web}} \times \frac{\mathrm{dClick\_Web}}{\mathrm{dImp\_Mobile}} + \frac{\partial\mathrm{Conv\_Mobile}}{\partial\mathrm{Click\_Mobile}} \times \frac{\mathrm{dClick\_Mobile}}{\mathrm{dImp\_Mobile}}.$$

The second column in Figure 4 shows that a one-percent increase in the mobile advertising impressions leads to a 0.29-percent increase in combined conversions when the cross-channel interdependence is factored in. However, a one-percent increase in the mobile advertising impressions leads to only a 0.15% increase in the combined conversions when the cross-channel interdependence is ignored. Thus, the combined conversion effect of mobile advertising would be underestimated by 48 percent if one were to account only for the same-channel effect and not for the cross-channel effect. These findings highlight the importance of accounting for the cross-channel interdependence between web and mobile advertising.

DISCUSSION AND IMPLICATIONS

As consumers increasingly use mobile devices to access the internet, they are exposed to more than one advertising message from marketers through web and mobile channels. This paper provides an understanding of how the interplay between web and mobile display advertising affects click-throughs and conversions on both channels. We show that the results from the randomized field experiments are corroborated by the econometric model based archival data analyses. We demonstrate that there exists interdependence between mobile and web display advertising and that their impact varies (or sometimes is negated) by the channel through which users are exposed to these two types of advertising. Thus, we show that web and mobile work together and affect each other. Optimal decisions in one need to take account of the effects in the other.

First, and most directly, our results can provide companies and advertisers with insights about how they can generate more traffic to their websites or mobile apps by using web and mobile advertising channels together. Our results suggest that companies can improve digital advertising click-throughs by using both the web and mobile channels simultaneously, rather than separately. This can have implications for increasing brand awareness and purchase intent in a multichannel environment. The mobile channel seems to generate disproportionately more traffic than the web channel from the positive cross-channel interdependence. Increasingly, companies are spending more dollars on mobile advertising. This result suggests that the mobile advertising channel can significantly increase even the web channel's effectiveness by improving total click-throughs. According to eMarketeer (2011), while ten percent of the average U.S. adult's day is now spent on mobile, mobile accounts for only one percent of firms' advertising spending, which suggests that there is a material upside to more mobile-advertising spending. Moreover, our policy-simulation results provide practical guidelines for advertisers and companies in making their resource-allocation decisions between web and mobile advertising channels.

These results provide advertisers with insights about how they can quantify the impact of mobile advertising on click-throughs and conversions in an increasingly multiscreen world. Our results show that consumers exhibit asymmetric cross-device conversion patterns by purchasing a product through PCs and laptops after clicking on mobile ads disproportionately more than the other way around. While there is a positive "reinforcement effect" of seeing the same ad twice on two different media for web conversions, it seems this positive effect is counterbalanced by other forces that end up reducing mobile conversions. Hence, when evaluating the effectiveness of advertising channels, it is critical that marketers not measure their effects separately but rather incorporate the cross-channel/cross-device interdependence effects we have identified in this paper. We show that if only the same-channel effect were accounted for, the combined conversion effect of mobile advertising would be underestimated by 48 percent, and the combined conversion effect of web advertising would be underestimated by 17 percent. Hence, practitioners and researchers can reach the wrong conclusion on the economic value of web advertising and mobile advertising if they consider only the same-channel effect and they neglect the cross-channel interdependence effect.

Moreover, many industry reports indicate that there is a major mobile monetization gap, with conversion rates and CPMs much lower on mobile than on the PC Internet. Some consumers hesitate to purchase products through mobile devices due to smaller screen sizes, security concerns about sending credit-card information over wireless networks, among others (Marinsoftware 2012). This study can provide companies with insights about how they can improve their mobile conversion rates. For example, when sponsored messages are accompanied by mobile display advertising, companies should allow users not only to make a purchase immediately (i.e., 1-click ordering), but also to have a quick access to business information (i.e., contact numbers, product information). However, despite the fact that 61 percent of consumers would quickly move on to another site if they did not find what they were looking for right away on a mobile site, 79 percent of large web advertisers still do not have a mobile optimized site (Google 2012b). Thus, companies should develop a mobile transaction-friendly website or app to win their mobile consumers from the competition. For example, they can improve their mobile conversion rates by providing mobile-friendly features such as faster loading (less than five seconds), large buttons, easy search, limited scrolling and pinching, etc. (Google 2012b).

Data availability issues suggest that some caution is warranted in interpreting our key findings. For example, our data on mobile advertising come from advertising that appeared on mobile phones only. It does not address tablets such as iPads, which have somewhat larger screens than phones but are somewhat heavier and less mobile. Future research can examine interdependence among web, tablet, and smartphone advertising. In addition, our analysis assumes that all clicks are intentional. It is possible that some clicks are accidental, especially with mobile devices, due to smaller screen sizes and touchscreen input errors (Gigaom 2012). Future research may model the interdependence between web and mobile advertising by figuring out a way to screen out accidental clicks. Notwithstanding these limitations, our analysis documents that web and mobile display advertising work better together than separately in terms of improving click-throughs. It also demonstrates that implementing both web and mobile advertising improves web conversions but can reduce mobile conversions. To the extent that consumers use both desktops/laptops and mobile devices seamlessly in their searches and purchases, the increasing size of the mobile Internet may have profound implications for the future direction of the mobile economy.

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FOOTNOTES

1. In this paper we use the term web advertising to refer to display ads shown online to PC/desktop users.
2. Source: http://en.wikipedia.org/wiki/Advertising_network.
3. We assume that the conversion is a function of clicks, and not impressions. This is because in our archival data 98.5 percent of those consumers who made a purchase decision had clicked on a display ad and in our experimental setting 99.1 percent of those consumers who made a purchase decision had clicked on a display ad.
4. When firms engage in both displaying web and mobile advertising impressions simultaneously, we can compute the marginal effect of mobile impressions on web click-throughs by calculating the partial derivative of web click-throughs with respect to mobile impressions: $\frac{\partial \mathrm{Click\_web}}{\partial \mathrm{Imp\_Mobile}} = 0.235 - 0.018 \cdot \overline{\mathrm{Imp\_web}^{(\mathrm{L})}}$.

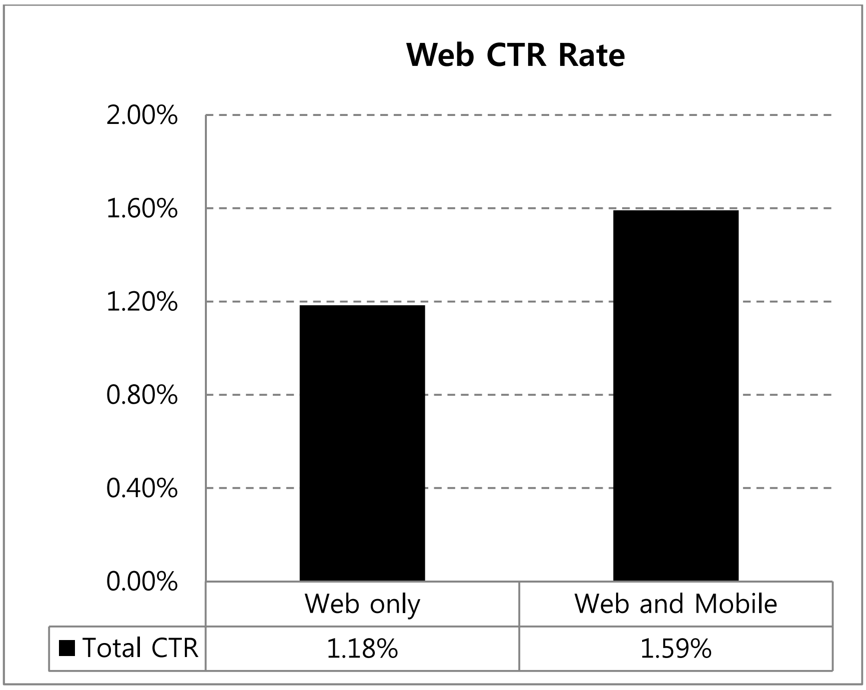


Figure 1(a). Web CTR Comparison

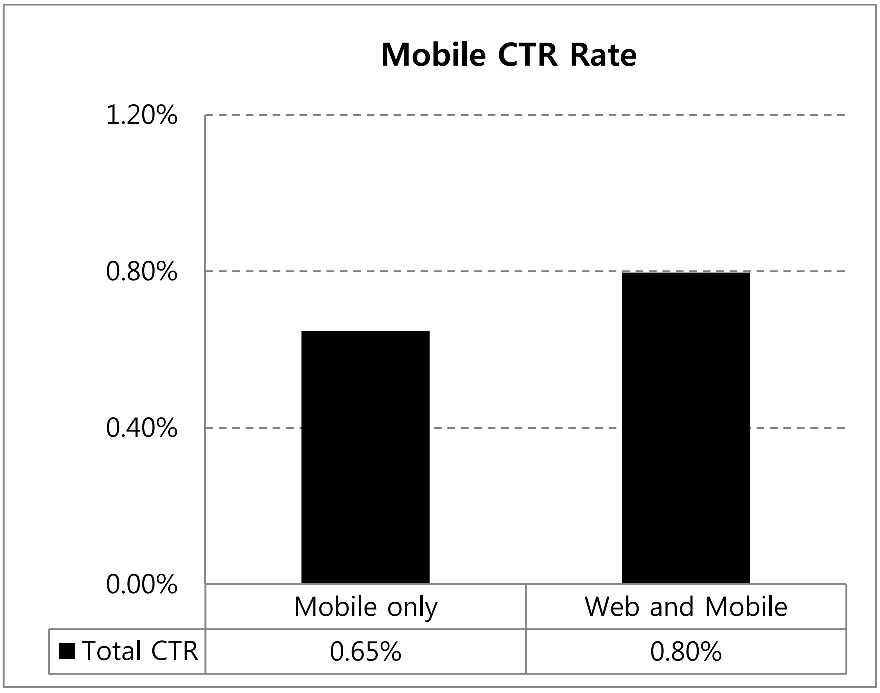


Figure 1(b). Mobile CTR Comparison

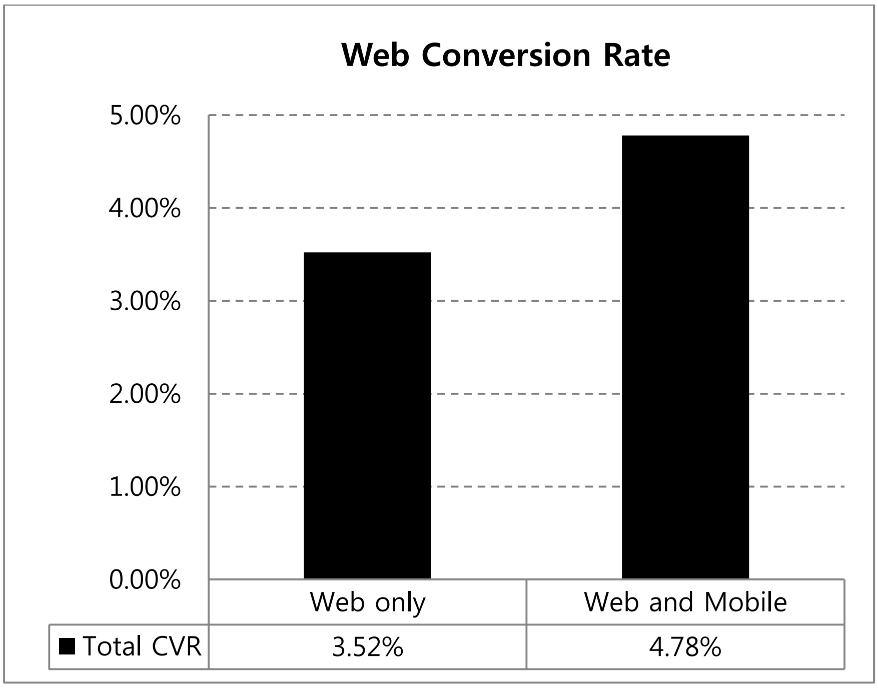


Figure 2(a). Web conversion rate comparison

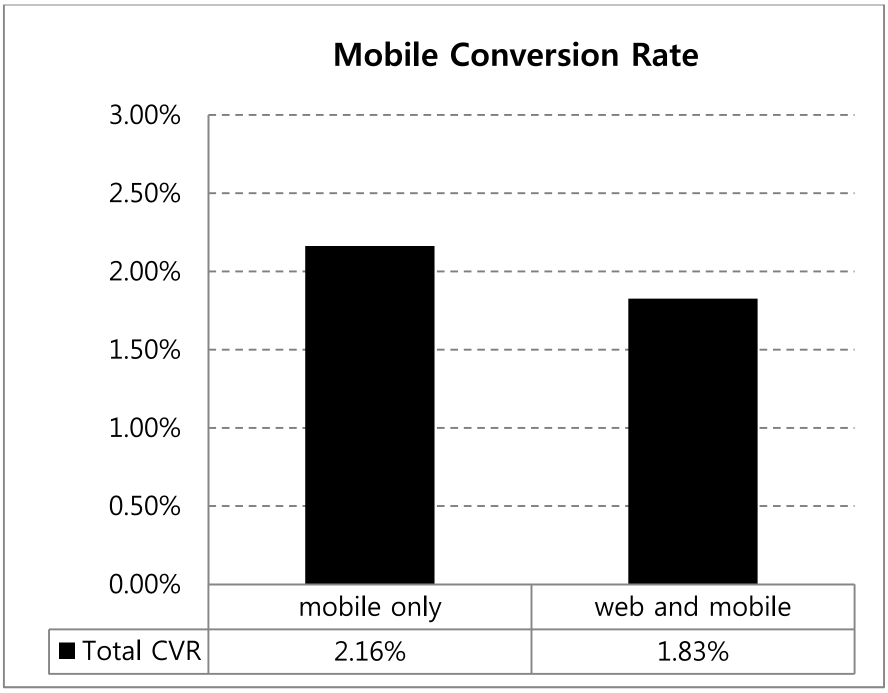


Figure 2(b). Mobile conversion rate comparison

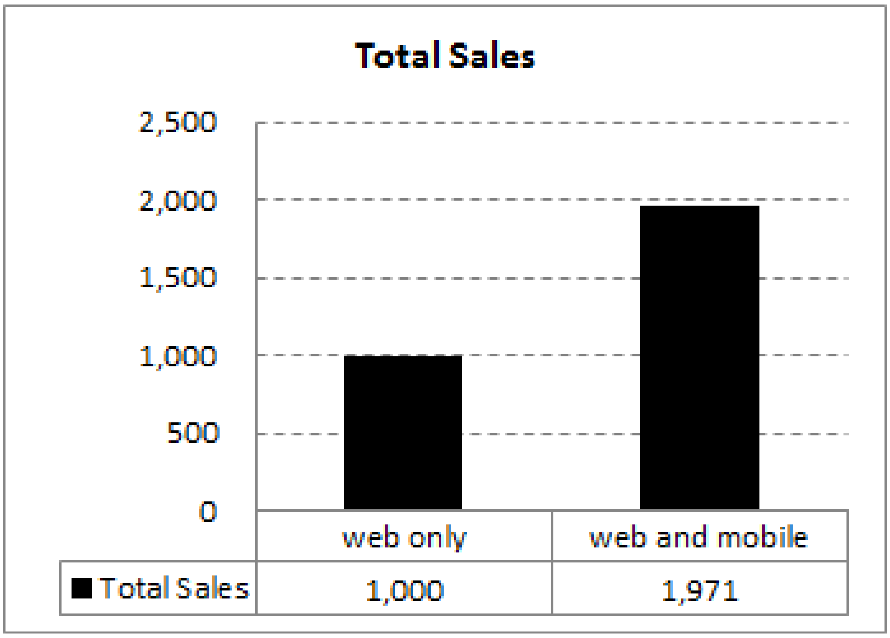


Figure 3(a). Total sales comparison: Web only vs. Web and mobile. Note that we used the sales amount from the single channel as a baseline by fixing it at 1,000.

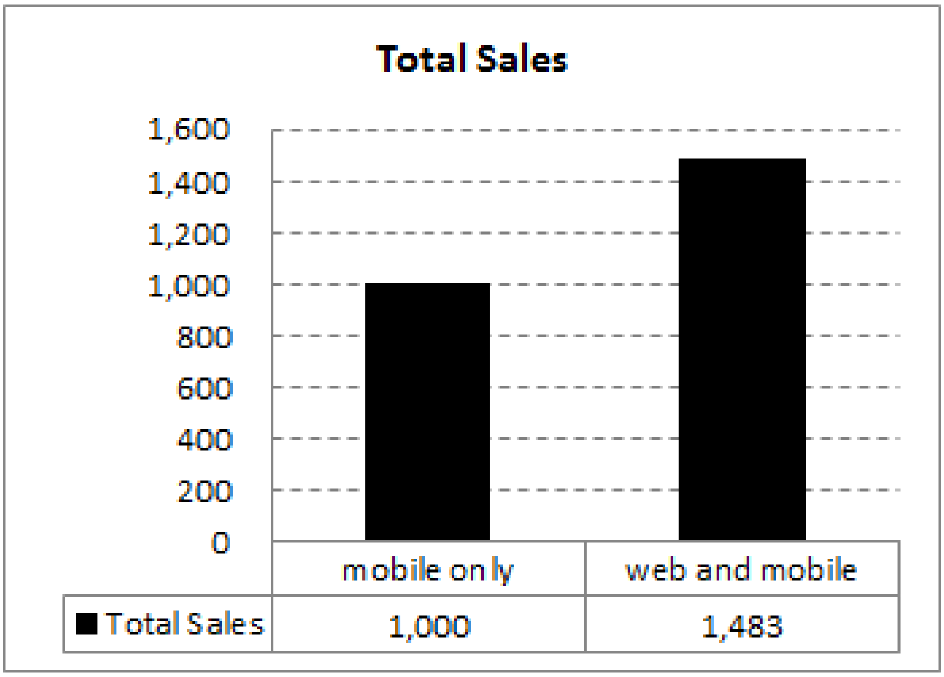
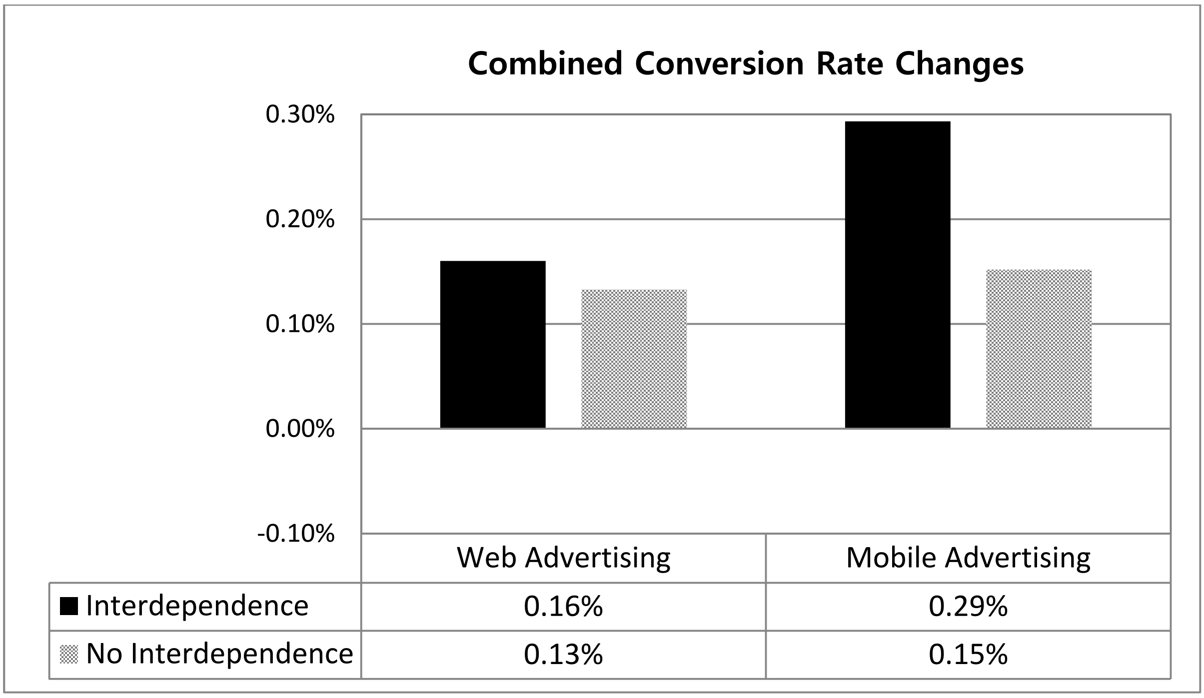


Figure 3(b). Total sales comparison: Mobile only vs. Web and mobile. Note that we used the sales amount from the single channel as a baseline by fixing it at 1,000.



	Web Advertising	Mobile Advertising
Interdependence	0.16%	0.29%
No Interdependence	0.13%	0.15%

Figure 4. The impact of 1% increase in web and mobile advertising on combined conversion rates when the cross-channel interdependence is considered vs. when it is ignored

Table 1. Field Experiment Design: web and Mobile Advertising Schedule

E-books	Time	Cohort 1 {A, B, C, D, E}	Cohort 2 {F, G, H, I, J}
10 e-books {A1, B1, C1, D1, E1, F1, G1, H1, I1, J1}	Mon – Wed in Week 1	web	Mobile
	Thur – Sat in Week 1	web & Mobile	None
	Mon – Wed in Week 2	Mobile	web
	Thur – Sat in Week 2	None	web & Mobile
10 e-books {A2, B2, C2, D2, E2, F2, G2, H2, I2, J2}	Mon – Wed in Week 3	web	Mobile
	Thur – Sat in Week 3	web & Mobile	None
	Mon – Wed in Week 4	Mobile	web
	Thur – Sat in Week 4	None	web & Mobile
10 e-books {A3, B3, C3, D3, E3, F3, G3, H3, I3, J3}	Mon – Wed in Week 5	web	Mobile
	Thur – Sat in Week 5	web & Mobile	None
	Mon – Wed in Week 6	Mobile	web
	Thur – Sat in Week 6	None	web & Mobile

Table 2. Descriptive Statistics of E-book Profile in the Experiment

Variables	Statistics			
	Mean	**Std. dev.**	**Min**	**Max**
Sales Price (US$)	6.66	2.16	1.65	10.43
Content Size (Mega Byes)	5.10	5.20	0.57	20.56
Days Since Release (Days)	180.90	210.72	20	867

Table 3. Cross-Channel Conversion Rates

Conversions: from (row) to (column)	Web	Mobile
Web	95.8%	4.2%
Mobile	11.6%	88.4%

Table 4. CPC-based Simulation Results: Optimal Ratio of Mobile and Web Ad Impressions

Marketing Mix: $I_w^{(1,1)} = \delta I_w^{(1,0)}$ ($\delta = 0.9$) and $I_m = \lambda I_w^{(1,1)}$											
λ ($I_m / I_w^{(1,1)}$)		**CPC_m ($)**									
		$1.0	**$0.9**	**$0.8**	**$0.7**	**$0.6**	**$0.5**	**$0.4**	**$0.3**	**$0.2**	**$0.1**
CPC_w ($) "fixed"	**$1.0**	*0.19*	*0.22*	*0.24*	*0.28*	*0.33*	*0.39*	*0.49*	*0.66*	*0.99*	*2.04*
	$0.9	*0.23*	*0.25*	*0.28*	*0.32*	*0.38*	*0.45*	*0.57*	*0.76*	*1.15*	*2.37*
	$0.8	*0.26*	*0.29*	*0.32*	*0.37*	*0.43*	*0.52*	*0.65*	*0.87*	*1.32*	*2.70*
	$0.7	*0.29*	*0.32*	*0.36*	*0.41*	*0.48*	*0.58*	*0.73*	*0.98*	*1.48*	*3.03*
	$0.6	*0.32*	*0.36*	*0.40*	*0.46*	*0.54*	*0.64*	*0.81*	*1.08*	*1.64*	*3.36*
	$0.5	*0.35*	*0.39*	*0.44*	*0.50*	*0.59*	*0.71*	*0.89*	*1.19*	*1.80*	*3.69*
	$0.4	*0.38*	*0.43*	*0.48*	*0.55*	*0.64*	*0.77*	*0.97*	*1.29*	*1.96*	*4.02*
	$0.3	*0.42*	*0.46*	*0.52*	*0.59*	*0.69*	*0.84*	*1.05*	*1.40*	*2.12*	*4.35*
	$0.2	*0.45*	*0.50*	*0.56*	*0.64*	*0.75*	*0.90*	*1.13*	*1.51*	*2.28*	*4.68*
	$0.1	*0.48*	*0.53*	*0.60*	*0.69*	*0.80*	*0.96*	*1.21*	*1.61*	*2.44*	*5.01*

Table 5. CPC-based Advertising Simulation Results: CPCm Upper Bound

Marketing Mix: $I_w^{(1,1)} = \delta I_w^{(1,0)}$ ($\delta = 0.9$) and $I_m = \lambda I_w^{(1,1)}$											
CPC_m upper bound ($)		π (CPC_m / CPC_w)									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
λ ($I_w^{(1,1)}$:I_m)	**3 (3:1)**	*0.15*	*0.27*	*0.37*	*0.46*	*0.53*	*0.59*	*0.65*	*0.70*	*0.75*	*0.79*
	2.5 (2.5:1)	*0.14*	*0.26*	*0.35*	*0.43*	*0.50*	*0.55*	*0.60*	*0.64*	*0.68*	*0.71*
	2 (2:1)	*0.14*	*0.25*	*0.33*	*0.40*	*0.45*	*0.50*	*0.54*	*0.57*	*0.60*	*0.63*
	1.5 (1.5:1)	*0.13*	*0.23*	*0.30*	*0.35*	*0.39*	*0.43*	*0.46*	*0.48*	*0.50*	*0.52*
	1.00 (1:1)	*0.12*	*0.20*	*0.25*	*0.29*	*0.31*	*0.34*	*0.35*	*0.37*	*0.38*	*0.39*
	0.67 (1:1.5)	*0.11*	*0.17*	*0.20*	*0.23*	*0.24*	*0.25*	*0.26*	*0.27*	*0.28*	*0.28*
	0.5 (1:2)	*0.10*	*0.15*	*0.17*	*0.19*	*0.20*	*0.21*	*0.21*	*0.22*	*0.22*	*0.22*
	0.4 (1:2.5)	*0.09*	*0.13*	*0.15*	*0.16*	*0.17*	*0.17*	*0.18*	*0.18*	*0.18*	*0.18*
	0.33 (1:3)	*0.09*	*0.11*	*0.13*	*0.14*	*0.14*	*0.15*	*0.15*	*0.15*	*0.16*	*0.16*

Table 6. CPM-based Simulation Results: Optimal Ratio of Mobile and Web Ad Impressions

Marketing Mix: $I_w^{(1,1)} = \delta I_w^{(1,0)}$ (δ = 0.9) and $I_m = \lambda I_w^{(1,1)}$											
λ (I_m / I_w)		CPM$_m$ ($)									
		$10	$9.0	$8.0	$7.0	$6.0	$5.0	$4.0	$3.0	$2.0	$1.0
CPM$_w$ ($) "fixed"	$10	*0.51*	*0.56*	*0.64*	*0.73*	*0.85*	*1.02*	*1.28*	*1.71*	*2.58*	*5.27*
	$9.0	*0.50*	*0.55*	*0.62*	*0.71*	*0.83*	*1.00*	*1.25*	*1.68*	*2.53*	*5.17*
	$8.0	*0.49*	*0.54*	*0.61*	*0.70*	*0.82*	*0.98*	*1.23*	*1.64*	*2.48*	*5.06*
	$7.0	*0.48*	*0.53*	*0.60*	*0.68*	*0.80*	*0.96*	*1.20*	*1.61*	*2.43*	*4.96*
	$6.0	*0.47*	*0.52*	*0.59*	*0.67*	*0.78*	*0.94*	*1.18*	*1.57*	*2.38*	*4.85*
	$5.0	*0.46*	*0.51*	*0.57*	*0.66*	*0.77*	*0.92*	*1.15*	*1.54*	*2.33*	*4.75*
	$4.0	*0.45*	*0.50*	*0.56*	*0.64*	*0.75*	*0.90*	*1.13*	*1.51*	*2.28*	*4.65*
	$3.0	*0.44*	*0.49*	*0.55*	*0.63*	*0.73*	*0.88*	*1.10*	*1.47*	*2.22*	*4.54*
	$2.0	*0.43*	*0.48*	*0.54*	*0.61*	*0.71*	*0.86*	*1.08*	*1.44*	*2.17*	*4.44*
	$1.0	*0.42*	*0.46*	*0.52*	*0.60*	*0.70*	*0.84*	*1.05*	*1.41*	*2.12*	*4.33*

Table 7. CPM-based Advertising Simulation Results: CPM$_m$ Upper Bound

Marketing Mix: $I_w^{(1,1)} = \delta I_w^{(1,0)}$ (δ = 0.9) and $I_m = \lambda I_w^{(1,1)}$											
CPM$_m$ upper bound ($)		π (CPM$_m$ / CPM$_w$)									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ ($I_w^{(1,1)}$:I_m)	3 (3:1)	-	-	-	*48.88*	*30.55*	*24.44*	*21.39*	*19.55*	*18.33*	*17.46*
	2.5 (2.5:1)	-	-	-	*27.2*	*20.38*	*17.5*	*15.9*	*14.8*	*14.1*	*13.6*
	2 (2:1)	-	-	*24.48*	*16.32*	*13.60*	*12.24*	*11.42*	*10.88*	*10.49*	*10.20*
	1.5 (1.5:1)	-	*24.5*	*12.3*	*9.81*	*8.76*	*8.17*	*7.8*	*7.54*	*7.36*	*7.21*
	1.00 (1:1)	-	*8.20*	*6.15*	*5.47*	*5.13*	*4.92*	*4.78*	*4.69*	*4.61*	*4.56*
	0.67 (1:1.5)	*8.24*	*4.12*	*3.53*	*3.30*	*3.17*	*3.09*	*3.04*	*3.00*	*2.97*	*2.94*
	0.5 (1:2)	*4.14*	*2.76*	*2.48*	*2.37*	*2.30*	*2.26*	*2.23*	*2.21*	*2.19*	*2.18*
	0.4 (1:2.5)	*2.77*	*2.08*	*1.92*	*1.85*	*1.81*	*1.78*	*1.76*	*1.75*	*1.74*	*1.73*
	0.33 (1:3)	*2.09*	*1.67*	*1.57*	*1.52*	*1.49*	*1.48*	*1.46*	*1.45*	*1.45*	*1.44*

Table 8. Variable Description and Summary Statistics of Archival Data

Variable	Description	Mean	Std. dev.	Min	Max
Advertisement Campaign and Performance Data					
Imp_Web	Number of web ad impressions	1,455,890	6,228,920	0	235,011,871
Imp_Mobile	Number of mobile ad impressions	122,858	533,443	0	13,538,164
Click_Web	Number of web clicks	2,169	11,046	0	271,114
Click_Mobile	Number of mobile clicks	228	1,691	0	43,157
Conv_Web	Number of web purchases	1.20	3.30	0	81
Conv_ Mobile	Number of mobile purchases	0.71	2.07	0	43
Product Profile Data					
Price	Sale price	6.63	3.17	0.26	41.39
Size	File size (kilo bytes)	7,120	7,963	4	62,420
Age	Days since product release	48.93	134.21	1	958
Desc	Length of product description	9,511	5,524	90	34,668
Review	User review count	4.79	16.90	0	144
Rating	Average user rating	2.11	2.44	0	5

Notes: Imp_web denotes advertising impression through the web channel and Imp_Mobile denotes advertising impression through a mobile channel. Similarly, Click_web denotes advertising clicks through the web channel and Click_Mobile denotes advertising clicks through a mobile channel. Conv_web denotes conversions through the web channel and Conv_Mobile denotes conversions through a mobile channel. *web* denotes channels such as a PC, a desktop, etc., and *Mobile* denotes channels such as a smartphone.

Table 9(a). Estimation Results: Web and Mobile Click-throughs

Independent Variable	Dependent Variable	
	Web Click-throughs	**Mobile Click-throughs**
Main Variables		
Imp_Web$^{(L)}$	0.628*** (0.027)	0.060*** (0.020)
Imp_Mobile$^{(L)}$	0.235*** (0.046)	0.501*** (0.047)
Imp_Web$^{(L)}$ × Imp_Mobile$^{(L)}$	-0.018*** (0.006)	0.002 (0.006)
Controls		
Category	Yes	Yes
Time	Yes	Yes

Table 9(b). Unobserved Heterogeneity Estimates in the Click-Through Model

web Click-throughs	Constant	Imp_Web	Imp_Mobile	Imp_Web x Imp_Mobile
Constant	0.204** (0.100)	-0.120* (0.077)	0.146* (0.089)	0.465 (0.328)
Imp_Web		0.129** (0.058)	-0.112* (0.066)	-0.302 (0.190)
Imp_Mobile			0.145** (0.072)	0.366* (0.214)
Imp_Web x Imp_Mobile				1.145* (0.620)

Mobile Click-throughs	Constant	Imp_Web	Imp_Mobile	Imp_Web x Imp_Mobile
Constant	4.071** (2.025)	0.176 (0.255)	0.633 (0.473)	-1.756 (1.121)
Imp_Web		0.119*** (0.030)	0.021 (0.042)	-0.051 (0.106)
Imp_Mobile			0.121** (0.060)	-0.272 (0.198)
Imp_Web x Imp_Mobile				0.807** (0.403)

Notes: Posterior means and posterior deviations (in parentheses) are reported. *** denotes significant at 0.01, ** denotes significant at 0.05, * denotes significant at 0.1. Imp_web denotes advertising impression through the web channel and Imp_Mobile denotes advertising impression through a mobile channel. *web* denotes channels such as a PC, a desktop, etc., and *Mobile* denotes channels such as a smartphone. $^{(L)}$ denotes logarithm of the variable.

Table 10(a). Estimation Results: Web and Mobile Conversions

Independent Variable	Dependent Variable: Web Conversion	Dependent Variable: Mobile Conversion
Main Variables		
Click_Web$^{(L)}$	0.071*** (0.028)	-0.165*** (0.054)
Click_Mobile$^{(L)}$	-0.033*** (0.009)	0.086*** (0.020)
Click_Web$^{(L)}$ × Click_Mobile$^{(L)}$	0.075*** (0.029)	0.063*** (0.019)
Controls		
Price$^{(L)}$	-0.193** (0.088)	-0.187*** (0.063)
Size$^{(L)}$	0.007 (0.014)	0.001 (0.008)
Description Length$^{(L)}$	0.001 (0.091)	0.007 (0.057)
Content Age$^{(L)}$	0.073*** (0.024)	0.038*** (0.010)
User Review Count$^{(L)}$	0.014** (0.007)	0.026*** (0.006)
User Review Rating	0.017 (0.015)	0.011 (0.009)
Category	Yes	Yes
Time	Yes	Yes

Table 10(b). Unobserved Heterogeneity Estimates in the Conversion Model

web Conversion	Constant	Click_Web	Click_Mobile	Click_Web x Click_Mobile
Constant	0.084** (0.040)	0.009 (0.014)	-0.012** (0.007)	-0.008 (0.030)
Click_Web		0.024*** (0.007)	-0.001 (0.005)	-0.011 (0.013)
Click_Mobile			0.019** (0.009)	0.022 (0.021)
Click_Web x Click_Mobile				0.119** (0.058)

Mobile Conversion	Constant	Click_Web	Click_Mobile	Click_Web x Click_Mobile
Constant	0.035** (0.017)	-0.019* (0.011)	0.037 (0.029)	-0.020 (0.019)
Click_Web		0.065*** (0.022)	0.018 (0.038)	-0.030 (0.037)
Click_Mobile			0.121** (0.050)	-0.067*** (0.023)
Click_Web x Click_Mobile				0.077*** (0.029)

Notes: Posterior means and posterior deviations (in parentheses) are reported. *** denotes significant at 0.01, ** denotes significant at 0.05, * denotes significant at 0.1. Click_web denotes advertising clicks through the web channel and Click_Mobile denotes advertising clicks through a mobile channel. *web* denotes channels such as a PC, a desktop, etc., and *Mobile* denotes channels such as a smartphone. $^{(L)}$ denotes logarithm of the variable.

Table 11. Estimated Covariance Across Web Conversion, Mobile Conversion, Web Click-throughs, and Mobile Click-throughs (Ω)

	Web Conversion	Mobile Conversion	Web Click-throughs	Mobile Click-throughs
Web Conversion	0.880** (0.398)	0.349* (0.201)	0.603 (1.009)	0.782 (0.993)
Mobile Conversion		0.333* (0.196)	0.338 (0.472)	0.483 (0.540)
Web Click-throughs			3.826** (1.862)	0.833* (0.470)
Mobile Click-throughs				4.142* (2.192)

Notes: Posterior means and posterior deviations (in parentheses) are reported. *** denotes significant at 0.01, ** denotes significant at 0.05, * denotes significant at 0.1.

Appendix A. MCMC Estimation

We ran the MCMC chain for 80,000 iterations and used the last 20,000 iterations to compute the mean and standard deviation of the posterior distribution of the model parameters. We report below the MCMC algorithm for the simultaneous model of web conversion, mobile conversion, web click-throughs, and mobile click-throughs.

We rewrite our main equations (1) – (8) as follows:

$$\text{(A1)}\ \underline{y}_{ij} = \underline{X}_{ij}\,\underline{b}_i + \underline{e}_{ij}$$

where

$$\underline{y}_{ij} = \begin{bmatrix} \text{Conv\_web}_{ij}^{(L)} \\ \text{Conv\_Mobile}_{ij}^{(L)} \\ \text{Click\_web}_{ij}^{(L)} \\ \text{Click\_Mobile}_{ij}^{(L)} \end{bmatrix},\ \underline{e}_{ij} = \begin{bmatrix} \varepsilon_{ij} \\ \psi_{ij} \\ \eta_{ij} \\ \nu_{ij} \end{bmatrix},$$

$$\underline{X}_{ij}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \text{Click\_web}_{ij}^{(L)} & 0 & 0 & 0 \\ \text{Click\_Mobile}_{ij}^{(L)} & 0 & 0 & 0 \\ \text{Click\_web}_{ij}^{(L)} \times \text{Click\_Mobile}_{ij}^{(L)} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \text{Click\_web}_{ij}^{(L)} & 0 & 0 \\ 0 & \text{Click\_Mobile}_{ij}^{(L)} & 0 & 0 \\ 0 & \text{Click\_web}_{ij}^{(L)} \times \text{Click\_Mobile}_{ij}^{(L)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \text{Imp\_web}_{ij}^{(L)} & 0 \\ 0 & 0 & \text{Imp\_Mobile}_{ij}^{(L)} & 0 \\ 0 & 0 & \text{Imp\_web}_{ij}^{(L)} \times \text{Imp\_Mobile}_{ij}^{(L)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \text{Imp\_web}_{ij}^{(L)} \\ 0 & 0 & 0 & \text{Imp\_Mobile}_{ij}^{(L)} \\ 0 & 0 & 0 & \text{Imp\_web}_{ij}^{(L)} \times \text{Imp\_Mobile}_{ij}^{(L)} \end{bmatrix}$$

and $\underline{b}_i = (\beta_{i0}, \beta_{i1}, \beta_{i2}, \beta_{i3}, \pi_{i0}, \pi_{i1}, \pi_{i2}, \pi_{i3}, \theta_{i0}, \theta_{i1}, \theta_{i2}, \theta_{i3}, \omega_{i0}, \omega_{i1}, \omega_{i2}, \omega_{i3})$.

The corresponding mixed model is as follows:

$$\text{(A2)}\ \underline{y}_{ij} = \underline{X}_{ij}\,\underline{\mu} + \underline{Z}_{ij}\,\underline{\lambda}_i + \underline{e}_{ij}$$

where $\underline{\mu} = (\alpha_0, \dots, \alpha_{8K}, \dots, \alpha_{11}, \varphi_0, \dots, \varphi_{8K}, \dots, \varphi_{11}, \kappa_0, \dots, \kappa_{1K}, .., \kappa_4, \delta_0, \dots, \delta_{1K}, \dots \delta_4)'$,

$\underline{\lambda}_i = \left(\underline{\lambda}_i^{\beta\prime}, \underline{\lambda}_i^{\pi\prime}, \underline{\lambda}_i^{\theta\prime}, \underline{\lambda}_i^{\omega\prime}\right) = \left(\lambda_i^{\beta_0}, \lambda_i^{\beta_1}, \lambda_i^{\beta_2}, \lambda_i^{\beta_3}, \lambda_i^{\pi_0}, \lambda_i^{\pi_1}, \lambda_i^{\pi_2}, \lambda_i^{\pi_3}, \lambda_i^{\theta_0}, \lambda_i^{\theta_1}, \lambda_i^{\theta_2}, \lambda_i^{\theta_3}, \lambda_i^{\omega_0}, \lambda_i^{\omega_1}, \lambda_i^{\omega_2}, \lambda_i^{\omega_3}\right)'$, and $\underline{Z}_{ij} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$.

Hence, the full conditionals are:

(A) $\Pr\left(\underline{\lambda}_i | \underline{\mu}, \Sigma^{\beta}, \Sigma^{\theta}, \Sigma^{\omega}, \Omega^{-1}, \underline{y}_{ij}\right)$,

(B) $\Pr\left(\underline{\mu} | \Omega^{-1}, \left\{\underline{\lambda}_i\right\}_{i=1}^{I}, \left\{\left\{\underline{y}_{ij}\right\}_{i=1}^{I}\right\}_{j=1j}^{n_i}\right)$,

(C) $\Pr\left(\Omega^{-1} | \underline{\lambda}_i, \underline{\mu}, \left\{\left\{\underline{y}_{ij}\right\}_{i=1}^{I}\right\}_{j}^{n_i}\right)$, and

(D) $\Pr\left(\Sigma^{\beta} | \left\{\lambda_i^{\beta}\right\}_{i=1}^{I}\right)$, $\Pr\left(\Sigma^{\pi} | \{\lambda_i^{\pi}\}_{i=1}^{I}\right)$, $\Pr\left(\Sigma^{\theta} | \left\{\lambda_i^{\theta}\right\}_{i=1}^{I}\right)$, and $\Pr\left(\Sigma^{\omega} | \{\lambda_i^{\omega}\}_{i=1}^{I}\right)$

where *I* is the total number of advertisements in the sample.

Step 1. Draw $\underline{\lambda}_i$

$$\Pr\left(\underline{\lambda}_i | \underline{\mu}, \Sigma^{\beta}, \Sigma^{\pi}, \Sigma^{\theta}, \Sigma^{\omega}, \Omega^{-1}, \textstyle\sum_{j=1}^{n_i} \underline{y}_{ij}\right) = N\left(\widehat{\underline{\lambda}_i}, V_{\underline{\lambda}_i}\right)$$

where $$V_{\underline{\lambda}_i}^{-1} = \begin{bmatrix} \Sigma^{\beta} & \cdots & & 0 \\ \vdots & \Sigma^{\pi} & & \\ & & \Sigma^{\theta} & \vdots \\ 0 & & \cdots & \Sigma^{\omega} \end{bmatrix}^{-1} + \textstyle\sum_{j=1}^{n_i} \underline{Z}_{ij}' \Omega^{-1} \underline{Z}_{ij} \quad \text{and}$$

$$\widehat{\underline{\lambda}_i} = V_{\underline{\lambda}_i}\left(\begin{bmatrix} \Sigma^{\beta} & \cdots & & 0 \\ \vdots & \Sigma^{\pi} & & \\ & & \Sigma^{\theta} & \vdots \\ 0 & & \cdots & \Sigma^{\omega} \end{bmatrix}^{-1} (0) + \textstyle\sum_{j=1}^{n_i} \underline{Z}_{ij}' \Omega^{-1} \underline{\tilde{y}}_{ij}\right)$$

$$\text{with } \Pr\left(\underline{\lambda}_i\right) = MVN\left(0, \begin{matrix} \Sigma^{\beta} & \cdots & & 0 \\ \vdots & \Sigma^{\pi} & & \\ & & \Sigma^{\theta} & \vdots \\ 0 & & \cdots & \Sigma^{\omega} \end{matrix}\right) \text{ and } \underline{\tilde{y}}_{ij} = \underline{y}_{ij} - \underline{X}_{ij}\underline{\mu}.$$

Step 2. Draw $\underline{\mu}$

$$\Pr\left(\underline{\mu} | \Omega^{-1}, \left\{\underline{\lambda}_i\right\}_{i=1}^{I}, \left\{\left\{\underline{y}_{ij}\right\}_{i=1}^{I}\right\}_{j=1}^{n_i}\right) = N\left(\hat{\underline{\mu}}, V_{\underline{\mu}}\right)$$

where $V_{\underline{\mu}}^{-1} = C^{-1} + \sum_{i=1}^{I} \sum_{j=1}^{n_i} \underline{X}_{ij}' \Omega^{-1} \underline{X}_{ij}$ and $\hat{\underline{\mu}} = V_{\underline{\mu}}\left[C^{-1}\underline{\mu}_0 + \sum_{i=1}^{I} \sum_{j=1}^{n_i} \underline{X}_{ij}' \Omega^{-1} \underline{\tilde{\tilde{y}}}_{ij}\right]$

with $\underline{\tilde{\tilde{y}}}_{ij} = \underline{y}_{ij} - \underline{Z}_{ij}\underline{\lambda}_i$, $\Pr\left(\underline{\mu}\right) = N\left(\underline{\mu}_0, C\right)$, $\underline{\mu}_0 = 0$, and C=100*I*.

Step 3. Draw Ω^{-1}

$$\Pr\left(\Omega^{-1} \mid \sum_{j=1}^{n_i} \underline{\lambda}_i, \underline{\mu}, \left\{\left\{\underline{y}_{ij}\right\}_{i=1}^{I}\right\}_{j=1}^{n_i}\right) =$$

$$W\left(\rho^{\Omega} + \sum_{i=1}^{I} n_i, \left[\sum_{i=1}^{I} \sum_{j=1}^{n_i} \left(\underline{y}_{ij} - \underline{X}_{ij}\underline{\mu} - \underline{Z}_{ij}\underline{\lambda}_i\right)\left(\underline{y}_{ij} - \underline{X}_{ij}\underline{\mu} - \underline{Z}_{ij}\underline{\lambda}_i\right)' + R^{-1}\right]^{-1}\right)$$ with $\Pr(\Omega^{-1}) =$ $W(\rho^{\Omega}, R^{\Omega})$, $\rho^{\Omega} = 18$ (i.e., 2+ number of random coefficients) and R^{Ω} is 10*I*.

Step 4. Draw Σ^{β}, Σ^{π}, Σ^{θ}, and Σ^{ω}

$$\Pr\left(\Sigma^{\beta} \mid \left\{\underline{\lambda}_i^{\beta}\right\}_{i=1}^{I}\right) = W\left(\rho^{\beta} + \sum_{i=1}^{I} n_i, \left[\sum_{i=1}^{I} \underline{\lambda}_i^{\beta}\underline{\lambda}_i^{\beta\prime} + R^{\beta^{-1}}\right]^{-1}\right)$$

with $\Pr(\Sigma^{\beta}) = W(\rho^{\beta}, R^{\beta})$, $\rho^{\beta} = 6$ and R^{β} is 10*I*.

$$\Pr\left(\Sigma^{\pi} \mid \left\{\underline{\lambda}_i^{\pi}\right\}_{i=1}^{I}\right) = W\left(\rho^{\pi} + \sum_{i=1}^{I} n_i, \left[\sum_{i=1}^{I} \underline{\lambda}_i^{\pi}\underline{\lambda}_i^{\pi\prime} + R^{\pi^{-1}}\right]^{-1}\right)$$

with $\Pr(\Sigma^{\pi}) = W(\rho^{\pi}, R^{\pi})$, $\rho^{\pi} = 6$ and R^{π} is 10*I*.

$$\Pr\left(\Sigma^{\theta} \mid \left\{\underline{\lambda}_i^{\theta}\right\}_{i=1}^{I}\right) = W\left(\rho^{\theta} + \sum_{i=1}^{I} n_i, \left[\sum_{i=1}^{I} \underline{\lambda}_i^{\theta}\underline{\lambda}_i^{\theta\prime} + R^{\theta^{-1}}\right]^{-1}\right)$$

with $\Pr(\Sigma^{\theta}) = W(\rho^{\theta}, R^{\theta})$, $\rho^{\theta} = 6$ and R^{θ} is 10*I*.

$$\Pr\left(\Sigma^{\omega} \mid \left\{\underline{\lambda}_i^{\omega}\right\}_{i=1}^{I}\right) = W\left(\rho^{\omega} + \sum_{i=1}^{I} n_i, \left[\sum_{i=1}^{I} \underline{\lambda}_i^{\omega}\underline{\lambda}_i^{\omega\prime} + R^{\omega^{-1}}\right]^{-1}\right)$$

with $\Pr(\Sigma^{\omega}) = W(\rho^{\omega}, R^{\omega})$, $\rho^{\omega} = 6$ and R^{ω} is 10*I*.

Appendix B. Optimal Policies for Web and Mobile Advertising

Table B1. Notations and Parameter Descriptions

Parameters	Description	Value
$I_w^{(1,0)}$	web impressions when web ads are active	
$I_w^{(1,1)}$	web impressions when web and mobile ads are active	
I_m	mobile impressions	
$CTR_w^{(1,0)}$	web click-through rate when web ads are active	1.1837%
$CTR_w^{(1,1)}$	web click-through rate when web and mobile ads are active	1.5916%
$CTR_m^{(0,1)}$	mobile click-through rate when mobile ads are active	0.6470%
$CTR_m^{(1,1)}$	mobile click-through rate when web and mobile ads are active	0.7965%
CPC_w	cost per web click (US dollar)	0.1 to 1.0
CPC_m	cost per mobile click (US dollar)	0.1 to 1.0
CPM_w	cost per thousand web ad-views (US dollar)	1 to 20
CPM_m	cost per thousand mobile ad-views (US dollar)	1 to 20
$CR_w^{(1,0)}$	web conversion rate when web ads are active	3.5228%
$CR_w^{(1,1)}$	web conversion rate when web and mobile ads are active	4.7807%
$CR_m^{(0,1)}$	mobile conversion rate when web ads are active	2.1619%
$CR_m^{(1,1)}$	mobile conversion rate when web and mobile ads are active	1.8256%
p	web users' cross-channel conversion rate (e.g., click on web, but purchase on mobile)	4.2%
q	mobile users' cross-channel conversion rate (e.g., click on mobile, but purchase on web)	11.6%
R	average revenue per conversion (or sale)	\$2
M	available ad budget in dollar (US dollar)	\$30,000
λ	$\lambda = I_m / I_w^{(1,1)}$	1/3 to 3
π	CPC_m / CPC_w	0.1 to 1.0
δ	$I_w^{(1,1)} / I_w^{(1,0)}$	1.0 to 0.5

Table B2. Summary of Costs, Revenues and Profits: CPC-based advertising

	Web ad only	**Web and Mobile ads**
Cost	$I_w^{(1,0)} CTR_w^{(1,0)} CPC_w$	$I_w^{(1,1)} CTR_w^{(1,1)} CPC_w$ $+ I_m CTR_m^{(1,1)} CPC_m$
Budget Constraint	Total cost should be less than or equal to M: $I_w^{(1,1)} CTR_w^{(1,1)} CPC_w + I_m CTR_m^{(1,1)} CPC_m \leq M$ (if web ad only I_m=0)	
Revenue	$R \times I_w^{(1,0)} CTR_w^{(1,0)} CR_w^{(1,0)}$	$R[I_w^{(1,1)} CTR_w^{(1,1)} (1\text{-}p)CR_w^{(1,1)}$ $+ I_m CTR_m^{(1,1)} qCR_m^{(1,1)}]$
Profit	$I_w^{(1,0)} CTR_w^{(1,0)} [R \times CR_w^{(1,0)} - CPC_w]$	$I_w^{(1,1)} CTR_w^{(1,1)} [R(1\text{-}p)CR_w^{(1,1)} - CPC_w]$ $+ I_m CTR_m^{(1,1)} [RqCR_m^{(1,1)} - CPC_m]$
Decision Criteria	Profit ≥ 0 if $R \geq CPC_w / CR_w^{(1,0)}$	Profit ≥ 0 if $R \geq CPC_w / [(1\text{-}p)\, CR_w^{(1,1)}]$ and $R \geq CPC_m / [q\, CR_m^{(1,1)}]$
	web profit when web and mobile ads are active should be greater than web profit when web ads are active and mobile ads are inactive. i.e., choose $I_m = \lambda I_w^{(1,1)}$ for any λ such that $\lambda < [(0.0067\delta - 0.004)R - (0.0159\delta - 0.0118)CPC_w]/(0.008CPC_m - 0.00002R)$, or choose CPC_m such that $CPC_m < R(0.0067\delta - 0.004 + 0.00002\lambda) / [(0.0159\delta - 0.0118)/\pi + 0.008\lambda]$ For example, when $\delta = 1$ (i.e., $I_w^{(1,1)} = I_w^{(1,0)}$), choose $I_m = \lambda I_w^{(1,1)}$ for any λ such that $\lambda < (0.0027R - 0.0041CPC_w) / (0.008CPC_m - 0.00002R)$, or choose CPC_m such that $CPC_m < R(0.0027 + 0.00002\lambda) / (0.0041/\pi + 0.008\lambda)$.	

Table B3. Summary of Costs, Revenues and Profits: CPM-based advertising

	Web ad only	Web and Mobile ads
Cost	$I_w^{(1,0)}$ (CPM_w/1,000)	$I_w^{(1,1)}$ (CPM_w/1,000) + I_m(CPM_m/1,000)
Budget Constraint	total cost should be less than or equal to M: $I_w^{(1,1)}$ (CPM_w/1,000) + I_m(CPM_m/1,000) ≤ M (if web ad only, set I_m=0)	
Revenue	R x $I_w^{(1,0)}$ $CTR_w^{(1,0)}$ $CR_w^{(1,0)}$	R[$I_w^{(1,1)}$ $CTR_w^{(1,1)}$ (1-p)$CR_w^{(1,1)}$ + I_m $CTR_m^{(1,1)}$ q$CR_m^{(1,1)}$]
Profit	$I_w^{(1,0)}$ [R $CTR_w^{(1,0)}$ $CR_w^{(1,0)}$ – CPM_w/1,000]	$I_w^{(1,1)}$ [R $CTR_w^{(1,1)}$(1-p)$CR_w^{(1,1)}$ – CPM_w/1,000] + I_m [R $CTR_m^{(1,1)}$ q$CR_m^{(1,1)}$ – CPM_m/1,000]
Decision Criteria	Profit ≥ 0 if R ≥ CPM_w/ [1,000$CTR_w^{(1,0)}$ $CR_w^{(1,0)}$]	Profit ≥ 0 if R ≥ CPM_w/[1,000 $CTR_w^{(1,1)}$(1-p) $CR_w^{(1,1)}$] and R ≥ CPM_m/[1,000$CTR_m^{(1,1)}$ q $CR_m^{(1,1)}$]
	web profit when web and mobile ads are active should be greater than web profit when web ads are active and mobile ads are inactive. i.e., choose $I_m = \lambda I_w^{(1,1)}$ for any λ such that $\lambda < [(0.0067\delta - 0.004)R - (CPM_w/1{,}000)(\delta-1)] / (CPM_m/1{,}000 - 0.00002R)$, or choose CPC_m such that $CPM_m < (0.0067\delta + 0.00002\lambda - 0.004)R \{1{,}000 /[(\delta-1)/\pi + \lambda]\}$. For example, when δ = 1 (i.e., $I_w^{(1,1)} = I_w^{(1,0)}$), choose $I_m = \lambda I_w^{(1,1)}$ for any λ such that $\lambda < 0.0027R / (CPM_m/1{,}000 - 0.00002R)$, or choose CPC_m such that $CPC_m < (2.7R/\lambda + 0.02R)$.	