Flea Market and Bazaar: Profit-Maximizing Platform Mechanisms for Matching and Search

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Abstract

We consider optimal pricing mechanisms of a profit maximizing platform running a dynamic search and matching market. Buyers and sellers enter continuously, meet and bargain under private information. Two market archetypes are investigated: flea markets, which clear at discrete times, and bazaars, which clear continuously. The optimal allocation rule has no delay and can be decentralized through participation fees charged by the intermediary to both sides. In a flea market the sum of buyers' and sellers' fees equals the sum of semi-elasticities and their ratio equals the ratio of bargaining weights. Fees are the same in a bazaar as in a flea market, except for an adjustment by the matching elasticities. A flea market is balanced, in a bazaar the side with the higher matching elasticity is more abundant. Finally, we show revenue equivalence between flea markets and market-makers' bid-ask spreads; how per period, per match, and per transaction fees compare; and that a monopolistic intermediary may be welfare enhancing in a search market.

Keywords: Dynamic random matching, two-sided private information, intermediaries

JEL Codes: D82, D83

1 Introduction

Search market intermediaries that charge participation fees to traders and let them search for and bargain with trading partners play an important role in many markets. Examples include online trade and auction web sites, job search platforms, and markets created by credit card issuers. Such search market intermediaries have been at the center of attention for quite some time and have received increased attention recently, in areas as diverse as labor markets,[1]

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[1]There have been diverging views on private labor market intermediaries for over a century: the International Labor Organization of the United Nations passed a convention in 1949 that banned private fee-charging em-

competition policy,[2] and international trade and development;[3] attention that seems justified by the money at stake: as an example, intermediation services account for over a quarter of GDP in the US (Spulber, 1999, p. 21).

Some markets are best approximated by discrete times at which trades may occur: flea markets typically open once or twice a year, traditional markets for agricultural goods once a week,[4] other markets once a day.[5] In other markets, buyers and sellers search for each other continuously, such as traditional bazaars, NASDAQ, and online trading platforms.

We consider two archetypes of markets: flea markets and bazaars, capturing two main types of clearing observed in reality: discrete and continuous time. A *flea market* opens at regular periods of time, and the intermediary charges participation fees to the traders. Buyers and sellers meet randomly and bargain bilaterally over the price. All potential traders enter simultaneously at the market opening date and the mass of matches is the minimum of the mass of buyers and sellers. In a *bazaar*, there is less centralization: it is open at all times, buyers and sellers search continuously for each other, meet randomly, bargain bilaterally, and pay participation fees to the intermediary. Search frictions arise because of a lack of coordination of timing and the matching rate is determined by the stocks of buyers and sellers waiting in the market.

In this paper, we develop a mechanism design based theory of optimal pricing by profit maximizing intermediaries in a search market. Buyers and sellers arrive to the intermediary's platform continuously over time, match and bargain either at discrete periods or continuously.

ployment agencies, only to be revoked by a second convention by the International Labor Organization in 1997. See conventions C96 and C181 of the International Labor Organization, C96 Fee-Charging Employment Agencies Convention (Revised), 1949, http://www.ilo.org/ilolex/cgi-lex/convde.pl?C096, C181 Private Employment Agencies Convention, 1997, http://www.ilo.org/ilolex/cgi-lex/convde.pl?C181. Similar developments were also present in the US: in 1914 a referendum in Washington state banned private labor market intermediaries, a law that was later overruled by the US supreme court. See Adams v. Tanner, 244 U.S. 590 (1917) and a description of the controversy in Foner (1965, p. 177-185).

[2] For example, there is a controversy on the level and structure of credit card fees and their regulation. There have been more than 50 lawsuits filed by merchant associations against credit card issuers. More than 20 countries and areas around the world have started regulating or investigating credit card fees. See e.g. Shy and Wang (forthcoming) for a discussion of current regulatory issues and a double marginalization perspective on credit card fees. See Rochet and Tirole (2006) for a two-sided markets perspective.

[3] See Antràs and Costinot (forthcoming) and the references therein for a discussion of the recent debate on the role of profit-maximizing intermediaries in international trade, especially involving less-developed countries.

[4] Sometimes, market opening times even shape the evolution of a language: the Hungarian word for "Sunday" is derived from "market day", since in the middle ages markets were held on Sundays.

[5] One example is the Smithfield Market for meat in London, which opens and clears in the early hours every day and has been operating for over 800 years. A similar example is the famous Tsukiji fish market in Tokyo.

Sellers have one unit of an indivisible homogeneous good, buyers have unit demand. We focus on steady-state equilibria in the search market. The matching technology is constant returns to scale. To keep matters as simple as possible, the bargaining protocol is assumed to be a random proposer one: a buyer makes a take-it-or-leave-it offer with some probability, and the seller makes such an offer with a complementary probability.

We focus on three aspects of such markets. First, traders have private information about their valuation for the good *before* they decide to join the platform. Second, we consider dynamics in such markets. Hence buyers and sellers have an option value of future trade and distributions in the market are endogenous, since inefficient traders may need longer time to trade and may be overrepresented in the market. Third, prices are formed through bargaining among buyers and sellers.

The main concern of this article is the optimal pricing by monopolistic flea market and bazaar intermediaries in markets with these aspects. We focus on the following questions in the context of stationary pricing. Can the profit maximizing allocation rule be implemented with simple participation fees? What is the profit maximizing price structure?

For both markets, the flea market and the bazaar, we first analyze optimal direct revelation mechanisms as a benchmark. We restrict attention to stationary and anonymous mechanisms. In the flea market, this is a centralized mechanism: in each market opening, buyers and sellers in the market report their types to the intermediary, who then determines the allocation of the goods and transfers. In the bazaar, the mechanism has to satisfy the constraints of the continuous time search technology.

In the optimal flea market, the most efficient traders (i.e. buyers above and sellers below a certain threshold) enter and trade immediately upon matching. All other traders do not enter. We call this the full trade property. The spread between the marginal entering buyer and seller is equal to the sum of the inverse semi-elasticities of demand and supply at the margin. The platform will ensure that there is the same number of buyers and sellers in the market (balanced market) and that traders trade immediately when they get matched.

The optimal bazaar also has a full trade property, and the spread between the marginal buyer and marginal seller is equal to the sum of inverse semi-elasticities at the margin. However, the platform will shift the marginal types up or down depending on the search technology: the platform takes into account the matching elasticities on both sides of the market when choosing the optimal marginal types. It will no longer desire a balanced market; instead, it will make the side with the higher matching elasticity more abundant. At the same time, the trading mechanism for the bazaar still has the full-trade property: traders who enter trade the first time they are matched.

The maximal revenue from the optimal mechanisms provides upper bounds on the intermediary's profit in the bargaining markets. We then show that these upper bounds are attainable by setting appropriate participation fees in both the flea market and the bazaar. Therefore, pricing through participation fees is in fact an optimal mechanism.

We show that the optimal allocation can be implemented with different types of fees: per period participation fees (which correspond to "membership fees" in the two-sided markets literature), per match fees ("per usage fees"), and per transaction fees. Our results on the per period fee structure in flea markets are the following. The sum of the buyer and seller fees is equal to the spread of the optimal mechanism and hence equal to the sum of the inverse semi-elasticities of demand and supply at the margin. The ratio of fees for the buyer and seller is equal to the ratio of the bargaining weights. The fee structure in a bazaar is the following. The sum of fees is equal to the sum of semi-elasticities multiplied by the weighted average of the matching rates of buyers and sellers, where the weights are equal to the bargaining weights. The ratio of buyer and seller fees are equal to the ratio of the bargaining weights, multiplied by the ratio of the inverse matching elasticities of demand. This multiplier is equal to the ratio of matching rates and to the equilibrium market tightness, such that the scarcer side of the market pays higher fees. The intermediary makes sure that traders are more abundant on the side of the market which exhibits the higher elasticity of matching.

The optimal flea market is balanced and has the full trade property, so buyers and sellers trade in the first market opening upon entry, and hence per period fees and per match fees are equivalent. For the optimal bazaar, the ratio of per match buyer's fees to seller's fees is equal to the ratio of bargaining weights of buyers and sellers, i.e. the same as for flea markets.

To gain some intuition for why the fee must be small when the bargaining power is small, consider a marginal buyer. He makes a positive expected profit only when he is chosen as a proposer. If his bargaining power is small, he can only break even if his participation fee is also small. Otherwise a market with endogenous entry would unravel, resulting in an equilibrium with no traders entering.

Note that both for per match and per period fees, there is a non-neutrality of the fee structure: not only the sum, but also the composition of fees matters. This stands in contrast to models in the two-sided markets literature, which typically imply fee neutrality under transferable utility and need the assumption of non-transferable utility to get non-neutrality. In our setup, fee neutrality is restored with per transaction fees: here the optimal sum of fees is given by the sum of semi-elasticities of demand and supply of marginal traders. The optimal fee breakdown between buyers and sellers is indeterminate: a change of the fee structure will be bargained away by traders.

Our framework provides a tractable way of thinking about markets run by profit maximizing intermediaries with selective entry and dynamics. We illustrate this by applying our model to several practically relevant questions. How do different levels of centralization of an intermediated market (market makers, flea markets, and bazaars) compare? What are the welfare effects of a monopolistic search market intermediary?

Literature This paper relates to three strands of literature: dynamic random matching (see e.g. Rubinstein and Wolinsky (1985); Gale (1987); Wolinsky (1988); Satterthwaite and Shneyerov (2007, 2008); Atakan (2007a,b); Shneyerov and Wong (2010a,b); Lauermann (2011); Lauermann, Merzyn, and Virág (2011)), intermediaries (see e.g. Rubinstein and Wolinsky (1987); Gehrig (1993); Spulber (1996); Rust and Hall (2003); Loertscher and Niedermayer (2008)), and two-sided markets (see e.g. Caillaud and Jullien (2003); Armstrong (2006); Rochet and Tirole (2006)).

It is closest to the dynamic random matching literature. We depart from this strand of literature by assuming that the search and matching platform is owned by a profit maximizing intermediary. It turns out that if the search costs incurred by traders are endogenously determined, as fees charged by the intermediary, the equilibrium becomes simpler in that it has the full trade property. This enables us to obtain our characterization of the optimal fee structure and a simple, tractable equilibrium characterization that is a helpful starting point for further analysis.

Our contribution to the intermediation literature is that we let the intermediary design a mechanism that takes into account the possibility of delay, of changing the steady state distributions in the market, and traders' option values of future trade, and lets buyers and sellers meet randomly and bargain bilaterally. In particular, the monopolistic intermediating platform in this paper differs from the middlemen in papers such as Rubinstein and Wolinsky (1987); Rust and Hall (2003); Loertscher and Niedermayer (2008) that do not have market power and take the market equilibrium as given. Further, we provide a justification for the monopolistic market maker considered in Spulber (1996) and Rust and Hall (2003) by showing that setting bid-ask prices is indeed an optimal mechanism in the class of anonymous, stationary mechanisms.[6] We also show revenue equivalence between the mechanism of a market maker and of a flea market intermediary.

Our contribution to the two sided markets literature is that we provide a comprehensive analysis of a dynamic search market where prices are determined through bargaining by buyers and sellers, analyze the entry decision of traders who know their private types *before* choosing whether to enter the market, and again, by allowing the platform owner to influence distributions and option values.[7] We find that the participation fee structure depends on relative bargaining weights rather than elasticities of demand, that trade occurs in equilibrium in every match, and that the fee structure is non-neutral even with transferable utility. We also show uniqueness of the non-trivial equilibrium in a static setup even if the intermediary is restricted to simple participation fees.

The paper is organized as follows. Section 2 describes the setup. Sections 3 and 4 consider the flea market and the bazaar. In both sections, we first investigate the optimal mechanisms, and then show that their outcomes can be implemented by imposing participation fees in the corresponding markets with bargaining. Section 5 discusses different levels of centralization; the comparison of profits for flea markets and bazaars; the relation between per period, per match, and per transaction fees; exogenous participation costs and entry subsidies; and welfare

[6] If one assumes that the intermediary is restricted to setting bid-ask prices, the possibility of delay and the endogeneity of distributions in the market are not an issue, since traders either trade immediately or never enter by assumption. In this article, this is a result rather than an assumption.

[7] This is the difference to the part of the two-sided markets literature closest to our paper, where buyers and sellers have private information about their valuations of the good and payments between them are possible and unrestricted. Many papers have a reduced form model of buyers' and sellers' utilities, which can be justified by a random matching and bargaining microfoundation.

effects of profit maximizing intermediaries. Section 6 concludes.

2 Setup

We consider an infinite horizon model of a market for a homogeneous, indivisible good. Buyers and sellers enter continuously with an inflow rate 1. There are frictions of exogenous exit from the market.[8] The exit rate is $\delta > 0$ for both buyers and sellers. Each buyer has a unit demand for the good, while each seller has unit supply. All traders are risk neutral.

Potential buyers are heterogeneous in their valuations v of the good. Potential sellers are also heterogeneous in their costs c of providing the good. In the following, we will also refer to valuations and costs as types. The types of new potential buyers are private information and are drawn independently from the continuously differentiable cumulative distribution function $F_B(v)$ for buyers and $F_S(c)$ for sellers. Densities are denoted with $f_B(v)$ and $f_S(c)$ and have support $[0, 1]$. We make the standard assumption that Myerson's regularity condition holds, that is the virtual type functions

$$J_B(v) := v - \frac{1 - F_B(v)}{f_B(v)}, \qquad J_S(c) := c + \frac{F_S(c)}{f_S(c)},$$

are increasing. Each trader's type will not change once it is drawn. Entry (or participation, or being active) is voluntary. Potential traders decide whether to enter the market once they are born. Those who do not enter will get zero payoff. We consider stationary equilibria in an infinite horizon setup.

3 Flea Market

The market opens and closes at times $t = ..., -\tau, 0, \tau, ...$ Because of the exogenous exit rate δ, traders that did not trade in the current opening exit the market before the next opening with probability $\epsilon := 1 - e^{-\delta\tau}$. This follows from the fact that the time until exit is exponentially distributed with mean δ^{-1}.

The net mass of entrants between two subsequent market openings, i.e. the total mass of entrants minus entrants that drop out before the market reopens, is $(1 - e^{-\delta\tau})/(\delta\tau)$. The reason

[8] See Satterthwaite and Shneyerov (2008) regarding the relevance of the exogenous exit model, where some examples are also provided.

for this is the following. A trader entering at some time $n\tau + t$ after the market opening at $n\tau$ with some $t \in [0, \tau]$ will still be in the market at the next opening at $(n+1)\tau$ with probability $e^{-\delta(\tau - t)}$. Since the inflow rate is 1, the net mass of entrants is

$$\int_0^\tau e^{-\delta(\tau-t)} dt = \frac{1 - e^{-\delta\tau}}{\delta}.$$

To avoid carrying around a multiplying constant, we will compute profits per unit of net mass of entrants per market opening in the following section. Profits per time interval of length 1 can be obtained by multiplying these profits by $(1 - e^{-\delta\tau})/(\delta\tau)$. Dropping this constant multiplier can be seen as temporarily normalizing both the net mass of entrants $(1 - e^{-\delta\tau})/\delta$ and the time between market reopenings τ to 1 in the section on flea markets. We will return to this constant multiplier in Section 5 when comparing profits between a flea market and a bazaar.

In the next section, we characterize an optimal anonymous direct trading mechanism for the flea market. The profit from this mechanism is an upper bound for the profit that can be obtained in the flea market where buyers and sellers bargain and the intermediary charges per period fees. After deriving the optimal mechanism, we will describe the assumptions of the flea market in more detail and show that this upper bound can indeed be attained.

3.1 Optimal Mechanism

Consider a mechanism in which all entering buyers and sellers report their valuations and costs, respectively, and the intermediary can arbitrarily match buyers and sellers. Since the good is homogeneous, this can be expressed in the following equivalent way: the goods of sellers who sell enter a pool, buyers get their goods from this pool. The only constraint is that the same number of sellers sell as buyers buy. A buyer's or seller's probability of trade and transfers are determined by his report. The highest profits that can be achieved in such a centralized market will serve as an upper bound for the profits in the decentralized market, where the intermediary cannot influence how buyers and sellers are matched.

To fit our matching and bargaining market's setting, we restrict attention to stationary anonymous mechanisms. By the revelation principle we can focus our attention to direct mechanisms, in which buyers and sellers report their types truthfully and the intermediary designs the mechanism such that traders have an incentive to tell the truth. In the direct mechanism design by the intermediary a buyer's expected probability of trade in any market opening is $q_B(v)$, his per period transfer to the intermediary is $t_B(v)$. In parallel, the mechanism prescribes $q_S(c)$, $t_S(c)$ for the sellers.[9] Denote the expected net present value of his transfers as $T_B(v)$. As the total probability of exit by the end of the next market opening, either exogenously or due to trading, is $\epsilon + (1-\epsilon)q_B(v)$, we have

$$T_B(v) = \frac{t_B(v)}{\epsilon + (1-\epsilon)q_B(v)}, \tag{1}$$

and analogously the "ultimate discounted probability of trade" is

$$Q_B(v) = \frac{q_B(v)}{\epsilon + (1-\epsilon)q_B(v)}. \tag{2}$$

Denote analogously for the seller, $Q_S(c)$, and $T_S(c)$.

The utilities of buyers and sellers can be expressed in terms of expected net present values:

$$U_B(v,\hat{v}) = vQ_B(\hat{v}) - T_B(\hat{v}), \qquad U_S(c,\hat{c}) = T_S(\hat{c}) - cQ_S(\hat{c}),$$

where v, c are the true and $\hat{v}$, $\hat{c}$ are the reported types. As usual, the incentive compatibility constraints ensure that participants have an incentive to report their types truthfully,

$$U_B(v,v) \geq U_B(v,\hat{v}), \quad \forall v,\hat{v} \in [0,1],$$
$$U_S(c,c) \geq U_B(c,\hat{c}), \quad \forall c,\hat{c} \in [0,1],$$

and the individual rationality constraints ensure that everyone is willing to participate in the mechanism,[10]

$$U_B(v,v) \geq 0, \quad \forall v \in [0,1], \qquad U_S(c,c) \geq 0, \quad \forall c \in [0,1].$$

An envelope-theorem type of argument applied to the incentive compatibility constraints shows that the trading probabilities Q_B and Q_S pin down the transfers T_B and T_S.

Lemma 1. *For any incentive compatible mechanism, transfers are given by*

$$T_B(v) = vQ_B(v) - \int_0^v Q_B(x)dx - U_B(0,0), \tag{3}$$

$$T_S(c) = \int_c^1 Q_S(x)dx - cQ_S(c) + U_S(1,1). \tag{4}$$

[9] See Lauermann (2011) and Shneyerov and Wong (2010b) for foundations of such mechanisms.

[10] We distinguish between an agent "participating" in the mechanism design problem and being "active" when considering the dynamic random matching implementation later on. A participating agent reports his type, but may or may not trade. An active agent trades with positive probability.

Proof. This is a standard revenue equivalence result. The only difference to standard results is that we have net present values of transfers and probabilities of trade (T_B, T_S, Q_B, Q_S) rather than transfers and probabilities of trade themselves (t_B, t_S, q_B, q_S), but mathematically this is the same. See for example Myerson and Satterthwaite (1983) for a proof. $\square$

A profit maximizing intermediary will always want to design the mechanism such that the rents of the most inefficient types are fully extracted, $U_B(0,0) = U_S(1,1) = 0$. From now on, we drop the terms $U_B(0,0)$ and $U_S(1,1)$.

The mechanism induces steady state distributions of active buyers and sellers in the market, which we denote as Φ_B and Φ_S, and the steady state masses of active traders B and S. The densities are denoted by ϕ_B and ϕ_S. The steady state condition for the buyers is that for each value of v the mass of entering buyers is equal to the mass of exiting buyers. Since the net entering mass of buyers between the openings has been normalized to 1, the probability of a buyer trading is $q_B(v)$, and the probability of exiting without trading is $\epsilon(1 - q_B(v))$, for any buyer type v who enters the market,

$$f_B(v) = [q_B(v) + \epsilon(1 - q_B(v))]B\phi_B(v).$$

Note that the expression in brackets can be rewritten as $q_B(v)/Q_B(v)$. Using the rewritten condition and the analogous condition for the seller we get

$$f_B(v)Q_B(v) = Bq_B(v)\phi_B(v), \tag{5}$$

$$f_S(c)Q_S(c) = Sq_S(c)\phi_S(c). \tag{6}$$

Because the same mass of the good has to be sold and bought,

$$B \int_{\underline{v}}^{1} q_B(v)d\Phi_B(v) = S \int_0^{\bar{c}} q_S(c)d\Phi_S(c).$$

For any incentive compatible and individually rational mechanism, the intermediary's profit is given by

$$\bar{T} = B \int t_B(v)d\Phi_B(v) - S \int t_S(c)d\Phi_S(c). \tag{7}$$

The following lemma is our key result in this section, showing that $\bar{T}$ admits a simple representation in terms of exogenous objects: the distributions of incoming trader types F_B and F_S and virtual type functions J_B and J_S.

Lemma 2. *The intermediary's profit is equal to*

$$\bar{T} = \int_0^1 J_B(v)Q_B(v)dF_B(v) - \int_0^1 J_S(c)Q_S(c)dF_S(c). \tag{8}$$

Proof. First, from the steady state conditions (5) and (6), the endogenous stock distributions are related to exogenous incoming flow distributions according to

$$Bd\Phi_B(v) = \frac{Q_B(v)}{q_B(v)}dF_B(v), \qquad Sd\Phi_S(c) = \frac{Q_S(c)}{q_S(c)}dF_S(c).$$

Substituting these expressions in (7), we obtain

$$\bar{T} = \int t_B(v)\frac{Q_B(v)}{q_B(v)}dF_B - \int t_S(c)\frac{Q_S(c)}{q_S(c)}dF_S. \tag{9}$$

Next, from (1) and the analogous expression for T_S,

$$t_B(v) = (\epsilon + (1-\epsilon)q_B(v))T_B(v), \qquad t_S(c) = (\epsilon + (1-\epsilon)q_S(c))T_S(c).$$

Substituting the expressions for $T_B(v)$ and $T_S(c)$, given by equations (3) and (4) in Lemma 1, into (9), we further obtain

$$\begin{aligned}\bar{T} = \int \frac{\epsilon + (1-\epsilon)q_B(v)}{q_B(v)}Q_B(v)\left[vQ_B(v) - \int_0^v Q_B(x)dx\right]dF_B(v) \\ - \int_0^1 \frac{\epsilon + (1-\epsilon)q_S(c)}{q_S(c)}Q_S(c)\left[\int_c^1 Q_S(x)dx - cQ_S(c)\right]dF_S(c)\end{aligned} \tag{10}$$

From (2) and the analogous expression for the sellers,

$$\frac{\epsilon + (1-\epsilon)q_B(v)}{q_B(v)}Q_B(v) = 1, \qquad \frac{\epsilon + (1-\epsilon)q_S(c)}{q_S(c)}Q_S(c) = 1.$$

Substituting these identities into the expression for $\bar{T}$ above, we obtain

$$\bar{T} = \int_0^1 \left[vQ_B(v) - \int_0^v Q_B(x)dx\right]dF_B(v) - \int_0^1 \left[\int_c^1 Q_S(x)dx - cQ_S(c)\right]dF_S(c),$$

The above equation combined with Theorem 3 in Myerson and Satterthwaite (1983) implies the statement in the proposition. (Note that Myerson and Satterthwaite (1983)'s Theorem 3 is stated in terms of the probabilities of trade q_B and q_S in a static model, but the same reasoning also applies to the "discounted net present value of future trade" Q_B and Q_S.) $\square$

The intermediary maximizes his profit $\bar{T}$ with respect to the constraint[11] that the same number of buyers and sellers trade:

$$\int_0^1 Q_B(v)dF_B(v) = \int_0^1 Q_B(c)dF_S(c).$$

Note that both the objective function $\bar{T}$ and the constraint can be expressed in terms of Q_B and Q_S alone. And there is a one-to-one mapping between Q_B and q_B. The same applies to Q_S and q_S. Hence the intermediary can maximize profits by choosing Q_B and Q_S.

The Lagrangian of this maximization problem is

$$\int_0^1 J_B(v)Q_B(v)dF_B(v) - \int_0^1 J_S(c)Q_S(c)dF_S(c) - \mu \left[\int_0^1 Q_B(v)dF_B(v) - \int_0^1 Q_S(c)dF_S(c)\right]$$

with the Lagrange multiplier μ.

The profit maximization problem can be solved by pointwise maximization with respect to $Q_B(v)$ and $Q_S(c)$. The derivative with respect to $Q_B(v)$ is

$$\frac{\partial L}{\partial Q_B(v)} = J_B(v) - \mu,$$

and with respect to $Q_S(c)$

$$\frac{\partial L}{\partial Q_S(c)} = \mu - J_S(c).$$

Since derivatives are constant with respect to Q_B and Q_S, this clearly gives a bang-bang solution, which implies that the intermediary sets $Q_B(v) = 1$ for $J_B(v) \geq \mu$ and $Q_B(v) = 0$ otherwise. Similarly, $Q_S(c) = 1$ for $J_S(c) \leq \mu$ and $Q_S(c) = 0$. Denote the marginal buyer and seller with $\underline{v}$ and $\bar{c}$, where buyers with $v \geq \underline{v}$ and sellers with $c \leq \bar{c}$ trade. This leads us to the following proposition.

Proposition 1. *The optimal allocation rule is to allow trade for buyers above and sellers below a certain threshold. Formally,*

$$Q_B(v) = \begin{cases} 1 & \text{if } v \geq \underline{v}, \\ 0 & \text{otherwise,} \end{cases} \qquad Q_S(c) = \begin{cases} 1 & \text{if } c \leq \bar{c}, \\ 0 & \text{otherwise.} \end{cases}$$

[11] Note that this constraint is different from Myerson and Satterthwaite (1983)'s in two aspects. First, in Myerson and Satterthwaite (1983) the probability of trade for a buyer and the seller matched to him has to be the same, $q_B(v, c) = q_S(v, c)$ for each realization of v and c. For us, they only have to be equal in expectation. Second, and connected to the previous point, since we are dealing with a centralized mechanism, we do not need to care about which buyer is matched to which seller. Sold goods are simply put into a common pool, buyers get their goods from this pool, without having to care which seller sold it. Hence a buyers probability of trade will only depend on his own valuation v. The same applies for the seller.

The thresholds (or marginal types) $\underline{v}$ *and* $\bar{c}$ *are determined by*

$$J_B(\underline{v}) = J_S(\bar{c}), \qquad 1 - F_B(\underline{v}) = F_S(\bar{c}).$$

The following intuition can be given for this proposition. The optimal price p_S set by a profit maximizing seller with cost c is given by $J_B(p_S) = c$ according to Myerson (1981).[12] Similarly, a profit maximizing buyer would set the price given by $J_S(p_B) = v$. Putting the optimal buying and selling strategy and the goods market balance condition together determines the marginal types.[13] Another intuition following in spirit Bulow and Roberts (1989)'s characterization of auction theory results in terms of marginal revenues is the following. J_B can be seen as marginal revenue and J_S as marginal cost. The intermediary increases the quantity traded, $q = 1 - F_B(\underline{v}) = F_S(\bar{c})$, as long as marginal revenue is higher than marginal cost, $J_B(F_B^{-1}(1 - q)) > J_S(F_S^{-1}(q))$, and stops as soon as it is lower.

This allocation rule can be implemented by charging price $\underline{v}$ to buyers and offering price $\bar{c}$ to sellers. Since traders either never enter or trade immediately after entering, the steady state distributions Φ_B and Φ_S are the static distributions F_B and F_S, truncated at $\underline{v}$ and $\bar{c}$.

Remark 1. *The implied profit maximizing spread* $\theta^* = \underline{v} - \bar{c}$ *is given by*

$$\theta^* = \frac{1 - F_B(\underline{v})}{f_B(\underline{v})} + \frac{F_S(\bar{c})}{f_S(\bar{c})} = \frac{\underline{v}}{\eta_B(\underline{v})} + \frac{\bar{c}}{\eta_S(\bar{c})}, \tag{11}$$

where the $\eta_B(v) = v f_B(v)/(1-F_B(v))$ *and* $\eta_S(c) = -c f_S(c)/F_S(c)$ *are the elasticities of demand and supply, respectively, and* $v/\eta_B(v)$ *and* $c/\eta_S(c)$ *are the inverse semi-elasticities. This shows that the mechanism of setting a bid and an ask price considered in Spulber (1996) and Rust and Hall (2003) is the optimal stationary mechanism in a dynamic random matching model.*[14]

Obviously, the profit generated by this centralized mechanism is an upper bound for the decentralized mechanism with random matching. This is because decentralization adds a further

[12] While the argument in Myerson (1981) is more subtle, a simplified version is that $J_B(p_S) = c$ is the first order condition when maximizing $(p_S - c)(1 - F_B(p_S))$ with respect to p_S.

[13] See also Baliga and Vohra (2003) and Loertscher and Niedermayer (2008) for the optimal mechanism with a discrete number of traders in a one-period model.

[14] While this appears intuitive, it this is not completely obvious a priori. The intermediary is willing to accept efficiency losses by excluding buyers and sellers who would trade in a Walrasian equilibrium. It is not obvious at first sight that he would not also incur some efficiency losses by delaying trade for some of the traders and thus extract rents through price discrimination.

constraint to the maximization problem: trade can only occur between a buyer and a seller who are randomly matched.

An obvious way to implement the same allocation rule and the same profits with a decentralized matching and centralized bargaining (or exchange) protocol is the following. Buyers and seller are matched randomly. A matched buyer has to pay the price $\underline{v}$ and a seller gets the price $\bar{c}$. The intermediary keeps the spread $\underline{v} - \bar{c}$. A trader cannot get a better deal in the future, hence he has no interest to delay. Buyers below $\underline{v}$ and seller above $\bar{c}$ do not enter, since they would have zero utility from participation. Hence buyers and sellers trade straight away and we get the same allocation rule and the same payments as in the centralized mechanism.

Corollary 1. *The optimal mechanism can be implemented by random matching with ask and bid prices given by $\underline{v}$ and $\bar{c}$.*

3.2 Optimal Implementation with Participation Fees

After having derived the optimal profit maximizing mechanism, we can turn to a specific mechanism the intermediary can use: charge buyers and sellers per period participation fees K_B and K_S in a dynamic random matching environment with bargaining. We will show that such a mechanism can generate the same profits as the optimal mechanism. For the moment, we focus on per period fees. In Section 5.3 we show how results are changed when looking at per match or per transaction fees. First, we characterize the fees for which the marginal types given by the optimal mechanism are just indifferent between entering and not, provided that a balanced full-trade equilibrium exists. Then, using this fee structure, we show that a balanced full-trade equilibrium exists and implies the optimal allocation rule.

Active buyers and sellers meet *randomly* in each market opening. The mass of matches is given by the constant elasticity of scale matching function $M(B, S) = \min\{B, S\}$, where B and S are the masses of active buyers and active sellers currently in the market.

Upon being matched, the buyer gets to make a take-it-or-leave-it price offer with probability α_B. With probability $\alpha_S = 1 - \alpha_B$ the seller gets to make the offer. If a type v buyer and a type c seller trade at a price p, then they leave the market with payoff $v - p$ and $p - c$, respectively. If bargaining between the matched pair breaks down, both traders can either stay in the market waiting for another match as if they were never matched, or simply exit and never come back.

We assume the market is anonymous, so the traders do not know their partners' market history, e.g. how long they have been in the market, what they proposed previously, and what offers they rejected previously.

It can be shown that in any equilibrium, buyers above some threshold $\underline{v}$ and sellers below some threshold $\overline{c}$ are active. Non-active traders stay out of the market. This is stated formally in the following Lemma.

Lemma 3. *The set of active buyers is* $[\underline{v}, 1]$ *and the set of active sellers is* $[0, \overline{c}]$ *for some* $\underline{v}, \overline{c} \in [0, 1]$ *in any equilibrium.*

Proof. The proof follows the same logic as Lemma 1 in Shneyerov and Wong (2010b) and therefore omitted. □

Lemma 3 allows us to focus our analysis on the marginal types $\underline{v}$ and $\overline{c}$.

The marginal buyer $\underline{v}$ gets a zero net expected utility from participating, hence his option value of future trade is also zero. Further, a seller would never set a price below $\underline{v}$, hence the buyer's utility if the seller makes the offer is zero as well. Therefore, when looking for the marginal buyer we only need to consider the buyer's utility in case he makes the offer, which gives us

$$\frac{M(B,S)}{B}\alpha_B \Phi_S(\underline{p}_B)(\underline{v} - \underline{p}_B) = K_B, \tag{12}$$

where $\underline{p}_B$ the optimal price set by the marginal buyer. The left-hand side of the equation is the gross utility of participation of the marginal buyer: the product of the probability of being matched M/B, the probability of making the offer α_B, of the seller accepting the offer $\Phi_S(\underline{p}_B)$, and the buyer's utility when buying at the price the marginal buyer sets $\underline{v} - \underline{p}_B$. The right-hand side is the cost of participation K_B. Similarly, for the marginal seller $\bar{c}$

$$\frac{M(B,S)}{S}\alpha_S(1 - \Phi_B(\overline{p}_S))(\bar{c} - \overline{p}_S) = K_S. \tag{13}$$

While deriving the market equilibrium is complicated in general in such setups, since there may be multiple equilibria, a full trade equilibrium may not exist for all values of K_B and K_S (see e.g. Satterthwaite and Shneyerov (2007)), and almost everything in (12) and (13) is endogenous, it turns out that the analysis is strongly simplified by focussing on the profit maximizing equilibrium.

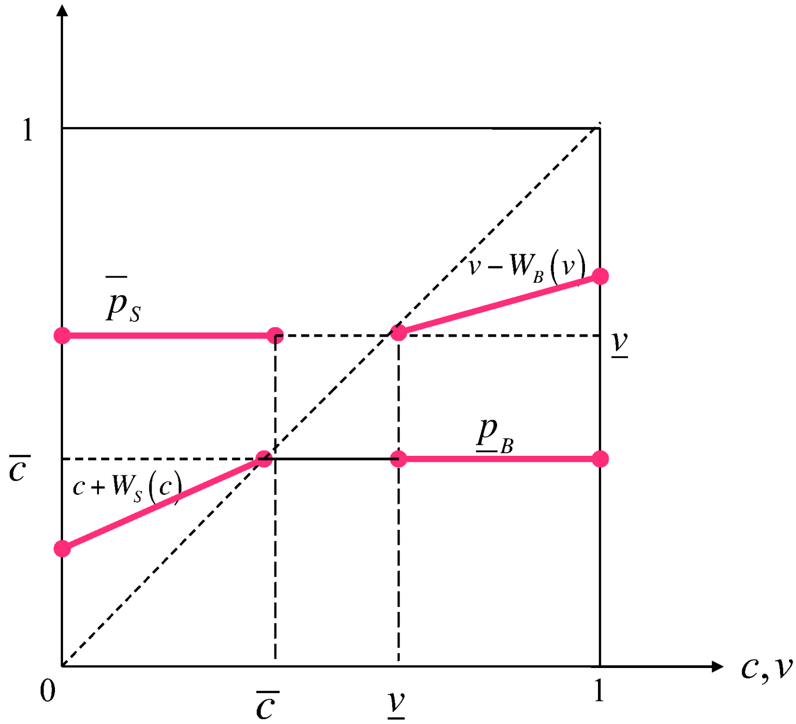

Figure 1: A full trade equilibrium: every offer is accepted. $W_B(v)$ and $W_S(c)$ are the option values of future trade. $\underline{p}_B$ is the price set by the marginal buyer, $\overline{p}_S$ is the price set by the marginal seller.

We know from the analysis in the previous section that if an intermediary can implement the allocation rule of the centralized mechanism in the decentralized dynamic random matching setup, then he cannot do better. In the following we will show that choosing K_B and K_S indeed enables the intermediary to do this. We will hence focus our attention on the optimal allocation rule, which has the properties that there is full trade (anyone who gets matched, trades with probability 1, here: $\underline{p}_B = \bar{c}$ and $\overline{p}_S = \underline{v}$) and that the market is balanced $1 - F_B(\underline{v}) = F_S(\bar{c})$ (or, in this case also, $M = B = S$). See Figure 1 for an illustration of full trade. We will first characterize the fee structure that is implied by the marginal types in a balanced full trade equilibrium. Then we will show, in Proposition 2 below, that this fee structure indeed induces a balanced full trade equilibrium.

Denote the optimal fees as K_B^* and K_S^*. For full trade balanced market equilibria, the conditions for the marginal types (12) and (13) reduce to

$$\alpha_B \theta = K_B^*, \tag{14}$$

$$\alpha_S \theta = K_S^*, \tag{15}$$

where $\theta = \underline{v} - \bar{c}$ is the spread. This is due to the balanced market $M/B = M/S = 1$ and full trade $\Phi_S(\underline{p}_B) = 1 - \Phi_B(\overline{p}_S) = 1$. Dividing the first with the second equation gives us

$$\frac{\alpha_B}{\alpha_S} = \frac{K_B}{K_S}. \tag{16}$$

Adding the two equations gives us

$$\theta^* = K_B^* + K_S^*,$$

where θ^* is the profit maximizing spread chosen by the intermediary. Following Remark 1 this can also be written in terms of the elasticities

$$K_B^* + K_S^* = \frac{\underline{v}}{\eta_B(\underline{v})} + \frac{\bar{c}}{\eta_S(\bar{c})}, \tag{17}$$

where $\underline{v}$ and $\bar{c}$ are given by the optimal allocation rule. These results mean that the sum of the fees is equal to the sum of the semi-elasticities of demand and supply for marginal traders. It further means that the ratio of the fees (the price structure) is independent of the elasticities and is equal to the ratio of the bargaining weights. Therefore, the side of the market with the stronger bargaining power will be charged a higher fee. Take e.g. the special case where the sellers set the price (i.e. $\alpha_S = 1$). In this case participation is free for buyers, $K_B^* = 0$, and sellers bear the full burden of the fee, $K_S^* = \theta^*$.

Existence of a full trade equilibrium. Before showing existence of a balanced full trade equilibrium in a dynamic setup, the following intuition for existence can be given in the simpler static setup with $\epsilon = 1$. A seller faces the following trade-off when considering raising the price: a higher price increases profits in case of trade, but it also decreases the probability of trade. An intermediary faces the same trade-off, but can additionally lower his costs as he raises the price for buyers. This is because less entry on the buyer side means that he can decrease the number of sellers entering while keeping the trade volume constant, which lowers his cost. Hence, if the intermediary is not willing to deviate from full trade in the centralized optimal mechanism, neither are the sellers in the decentralized setup. The same reasoning applies to the buyers.

Returning to the dynamic case (arbitrary ϵ), recall that the profit maximizing spread of the intermediary is given by

$$J_B(\underline{v}) = J_S(\bar{c}). \tag{18}$$

It can be shown that if the spread between $\underline{v}$ and $\bar{c}$ is large enough, then a full trade equilibrium exists. Intuitively, the reason is that the marginal seller $\overline{c}$ does not have an interest to set a price above $\underline{v}$ if the difference $\underline{v} - \overline{c}$ is large enough. The same applies to the marginal buyer $\underline{v}$. We show this formally in the following Lemma, the proof of which is in the Appendix.

Lemma 4. *A full trade equilibrium exists if the marginal types satisfy the following conditions:*

$$\epsilon J_B(\underline{v}) + (1-\epsilon)\underline{v} \geq \bar{c}, \tag{19}$$

$$\epsilon J_S(\bar{c}) + (1-\epsilon)\bar{c} \leq \underline{v}. \tag{20}$$

We can use Lemma 4 to show existence of a profit maximizing equilibrium. Using the profit maximizing spread in (18) and $J_S(c) \geq c$ we can find a lower bound for the left-hand side of (19) in Lemma 4:

$$\epsilon J_B(\underline{v}) + (1-\epsilon)\underline{v} = \epsilon J_S(\bar{c}) + (1-\epsilon)\underline{v} \geq \epsilon\bar{c} + (1-\epsilon)\underline{v}$$

which is greater or equal $\bar{c}$, since $\underline{v} \geq \bar{c}$ for a profit maximizing intermediary. Hence, condition (19) is always fulfilled for the profit maximizing spread. By an analogous reasoning, condition (20) is also always satisfied. Since the ratio of the fees K_B/K_S is chosen such that a balanced market is achieved if there is full trade, we have shown existence of a full trade balanced market equilibrium that maximizes the intermediary's profit.

Proposition 2. *The intermediary's profit maximizing per period fees K_B^* for the buyer and K_S^* for the seller in the flea market setup are given by*

$$K_B^* + K_S^* = \frac{1-F_B(\underline{v})}{f_B(\underline{v})} + \frac{F_S(\overline{c})}{f_S(\overline{c})},$$
$$\frac{K_B^*}{K_S^*} = \frac{\alpha_B}{\alpha_S},$$

where the marginal types $\underline{v}$ and $\overline{c}$ are given by Prop. 1. For these fees, a balanced full trade market equilibrium exists in the flea market setup, i.e. an equilibrium such that $M(B,S)/B = M(B,S)/S = 1$ and $\Phi_S(\underline{p}_B) = 1 - \Phi_S(\overline{p}_S) = 1$, where $\underline{p}_B$ is the lowest price set by a buyer and $\overline{p}_S$ is the highest price set by a seller.

Uniqueness It is well known that there is typically a multiplicity of equilibria in such dynamic random matching setups. There is always a no-trade equilibrium, where neither buyers nor sellers enter and, therefore, not entering is an optimal best response. In the dynamic random matching literature this is called the trivial no-trade equilibrium. In the two-sided markets literature, such issues are called the "chicken-and-egg-problem": if merchants are not willing to accept credit cards, consumers will not use them and vice versa. Note, however, that a small change of the mechanism by the platform can make sure that any other equilibrium than the profit maximizing one is destroyed. For the no-trade equilibrium it is e.g. sufficient to offer a small net reward to entering sellers if there are no (or only few) sellers in the market. In case of an equilibrium with an abundance of sellers, the intermediary can commit in advance to charge a small penalty to sellers (or a small reward for exiting), so that the marginal seller in the original equilibrium is not indifferent between entering or not any more. If all equilibria are destroyed except the profit maximizing one, the platform will never have to pay these rewards (or charge these penalties) on the equilibrium path. Destroying the no-trade equilibrium can be interpreted as the story of Diner's Club initially subsidizing participation by restaurants and consumers to jump start their credit card system.

Despite the fact that the platform could easily construct off equilibrium payments additionally to the simple participation fees in order to make sure that its preferred equilibrium is unique, it is remarkable that for a static setup ($\epsilon = 1$) simple participation fees by themselves lead to a unique non-trivial equilibrium, as shown in the next Proposition, the proof being relegated to the Appendix.

Proposition 3. *The profit maximizing balanced full trade equilibrium is the unique non-trivial equilibrium in the static setup ($\epsilon = 1$) given the profit maximizing fees K_B^* and K_S^*.*

The proof is by contradiction. It shows that if more than the traders $[0, \overline{c}]$ and $[\underline{v}, 1]$ were to enter, then the marginal entering traders would have strictly negative utility. If less were to enter, then they would have strictly positive utility.

4 Bazaar

In a bazaar, a profit maximizing intermediary runs a continuous time search market. We make the standard assumption of constant returns to scale for the search technology:

$$M(B,S) = BM(\frac{B}{S},1) = SM(1,\frac{S}{B}).$$

Denote market tightness with $\zeta = B/S$ and the relative matching rate with $m(\zeta) = M(\zeta,1) = \zeta M(1,1/\zeta)$. Further, denote search intensity $\lambda = M(1,1) = m(1)$. The search intensity parameter scales up the matching rate by a constant factor; $\lambda \to \infty$ means instantaneous matching. Denote the elasticities of the matching function as

$$\sigma_B(\zeta) = \frac{dM/M}{dB/B} = \frac{\zeta m'(\zeta)}{m(\zeta)}, \qquad \sigma_S(\zeta) = \frac{dM/M}{dS/S} = \left(1 - \frac{\zeta m'(\zeta)}{m(\zeta)}\right).$$

An example of constant returns to scale functions is the Cobb-Douglas search technology:

$$M(B,S) = \lambda B^{\bar{\sigma}_B} S^{\bar{\sigma}_S},$$

with $\bar{\sigma}_B + \bar{\sigma}_S = 1$ where $\bar{\sigma}_B$ and $\bar{\sigma}_S$ are the (constant) elasticities of the matching function.[15] The matching function for a Cobb-Douglas search technology is $m(\zeta) = \zeta^{\sigma_B}$. Another example is the min search function $M(B,S) = \lambda \min\{B,S\}$.[16]

We will further use the matching rates for buyers and sellers

$$l_B(\zeta) = \frac{m(\zeta)}{\zeta}, \qquad l_S(\zeta) = m(\zeta).$$

Note that for Cobb-Douglas, $l_B(\zeta) = \zeta^{-\sigma_S}$ and $l_S(\zeta) = \zeta^{\sigma_B}$.

We will first consider an intermediary that chooses an optimal trade mechanism subject to matching occurring according to the search technology. Similarly to Section 3, the profit from this mechanism will serve as an upper bound for the one that can be obtained in the bazaar. Also here, we will then show that this upper bound can indeed be attained.

[15] The Cobb-Douglas matching technology is often used in labor economics to model the search of workers for jobs; see e.g. Rogerson, Shimer, and Wright (2005).

[16] It arises as a limiting case of constant returns to scale and constant elasticity of substitution functions in our setup.

4.1 Optimal Mechanism

The intermediary designs a trade mechanism for a buyer and a seller that meet, but we assume it cannot directly influence the matching technology. We depart from the previous sections in that now buyers and sellers are not constrained to meet at discrete matching periods that occur after time τ passes. Instead they meet continuously according to a continuous time search technology.

Note that a mechanism design problem in continuous time without restrictions imposed by the search technology would not be helpful, since it would be optimal to match buyers and sellers instantaneously after they enter the market (according to the allocation rule derived in the previous mechanism). Such an allocation rule could only be achieved by a search technology with an infinite search intensity. Therefore, we will solve the mechanism design problem under the constraint of the continuous time matching technology, without the market microbalance constraint of the original problem. No market microbalance constraint means that a buyer and a seller are not restricted to trade with their matching partner, but can trade with anyone. Consequently, the solution of the mechanism design problem delivers an upper bound to the search problem; one that can be reached, as we will show.

Denote by $q_B(v)$ and $q_S(c)$ the probability that a buyer or seller with valuation v or cost c trades in the mechanism *in a particular match*. Denote by $t_B(v)$ and $t_S(c)$ the transfer *per match* that a buyer pays or a seller gets.

The exposition is simplified by introducing the (continuous time version of) the expected net present values of transfers $T_B(v)$ and $T_S(c)$. Since the effective "discount rate" is $\delta+l_B(\zeta)q_B(v)$ for the buyers and $\delta + l_S(\zeta)q_S(c)$ for the sellers, we have

$$T_B(v) = \frac{l_B(\zeta)t_B(v)}{\delta + l_B(\zeta)q_B(v)}, \qquad T_S(c) = \frac{l_S(\zeta)t_S(c)}{\delta + l_S(\zeta)q_S(c)}.$$

Analogously, the expected net present value of the probability of trade $Q_B(v)$ and $Q_S(c)$ (the "ultimate discounted probability of trade") is:

$$Q_B(v) = \frac{l_B(\zeta)q_B(v)}{\delta + l_B(\zeta)q_B(v)}, \tag{21}$$

$$Q_S(c) = \frac{l_S(\zeta)q_S(c)}{\delta + l_S(\zeta)q_S(c)}. \tag{22}$$

In contrast to the discrete time setup, the upper bounds of $Q_B(v)$ and $Q_S(c)$ are not 1, but are constrained by the matching technology: e.g. if there are much more buyers than sellers, a buyer is matched only infrequently and will ultimately drop out for exogenous reasons without having been matched. These upper bounds are given by setting $q_B(v)$ and $q_S(c)$ to 1:

$$\overline{Q}_B(\zeta) := \frac{l_B(\zeta)}{\delta + l_B(\zeta)}, \qquad \overline{Q}_S(\zeta) := \frac{l_S(\zeta)}{\delta + l_S(\zeta)}.$$

Similar to the discrete time setup, the profit maximization problem of the intermediary can be shown to be

$$\begin{aligned}
\max_{Q_B(v), Q_S(c), \zeta} & \int_0^1 J_B(v) Q_B(v) dF_B(v) - \int_0^1 J_S(c) Q_S(c) dF_S(c) \\
\text{s.t.(i)} & \int_0^1 Q_B(v) dF_B(v) = \int_0^1 Q_S(c) dF_S(c) \\
\text{(ii)} & \int_{\underline{v}}^1 (1 - Q_B(v)) dF_B(v) = \zeta \int_0^{\bar{c}} (1 - Q_S(c)) dF_S(c) \\
\text{(iii)} & \;\; 0 \le Q_B(v) \le \overline{Q}_B(\zeta), \qquad 0 \le Q_S(c) \le \overline{Q}_S(\zeta), \qquad \forall v, c
\end{aligned}$$

The constraints and the notation used are described in the following. (i) is the balanced goods constraint, i.e. the number of buyers who buy and the sellers who sell is equal. (ii) is the market tightness constraint. It ensures that the market tightness (ratio of buyers to sellers) implied by the mechanism is equal to the market tightness assumed when choosing the controls. Since buyers of type v exit with the overall rate $\delta + l_B(\zeta) q_B(v)$, and sellers of type c exit with the overall rate $\delta + l_S(\zeta) q_S(c)$, the steady-state stocks of buyers and sellers are

$$B = \int_{\underline{v}}^1 \frac{dF_B(v)}{\delta + l_B(\zeta) q_B(v)}, \qquad S = \int_0^{\bar{c}} \frac{dF_S(c)}{\delta + l_S(\zeta) q_S(c)}.$$

In view of (21) and (22), these stocks can be alternatively written as

$$B = \frac{1}{\delta} \int_{\underline{v}}^1 (1 - Q_B(v)) dF_B(v), \qquad S = \frac{1}{\delta} \int_0^{\bar{c}} (1 - Q_S(c)) dF_S(c).$$

Noting that $B/S = \zeta$, we obtain constraint (ii). Finally, (iii) are the boundaries of the ultimate probabilities of sale.

We can use the following reasoning to simplify the problem. Similarly as for flea markets, we can show that the solution has a bang-bang property by showing that the derivatives of the Lagrangian with respect to $Q_B(v)$ and $Q_S(c)$ are constant:

$$\frac{\partial L}{\partial Q_B(v)} = J_B(v) - \mu - \gamma, \qquad \frac{\partial L}{\partial Q_S(c)} = J_S(c) - \mu - \gamma\zeta, \tag{23}$$

where μ and γ are the Lagrange multipliers of constraints (i) and (ii) respectively.

Combining this with Q_B being increasing and Q_S decreasing, we get $Q_B(v) = \overline{Q}_B(\zeta)$ for $v \in [\underline{v}, 1]$ and $Q_B(v) = 0$ otherwise. Similarly for sellers. Hence, constraint (i) becomes

$$\overline{Q}_B(\zeta)(1 - F_B(\underline{v})) = \overline{Q}_S(\zeta)F_S(\bar{c}), \tag{24}$$

and constraint (ii) becomes $(1 - \overline{Q}_B(\zeta))(1 - F_B(\underline{v})) = \bar{\zeta}(1 - \overline{Q}_S(\zeta))F_S(\bar{c})$. Dividing (ii) by (i) gives

$$\frac{1 - \overline{Q}_B(\zeta)}{\overline{Q}_B(\zeta)} = \zeta \frac{1 - \overline{Q}_S(\zeta)}{\overline{Q}_S(\zeta)}.$$

Note that this is always satisfied by the definitions of $\overline{Q}_B$ and $\overline{Q}_S$. Hence, constraint (ii) is redundant and $\gamma = 0$.

Also note that it is not optimal for the intermediary to allow entry of a positive mass of traders who will not trade with positive probability. Doing so could only be beneficial if it increased the range of market tightness values ζ that are implementable. But the trade balance constraint (24) implies $\overline{Q}_B(\zeta)/\overline{Q}_S(\zeta) = F_S(\bar{c})/(1 - F_B(\underline{v}))$, so any value of $\zeta > 0$ can be implemented with a suitable choice of $\bar{c}, \underline{v} \in (0, 1)$.

By this reasoning, the intermediary's problem is reduced to maximizing $\int_{\underline{v}}^{1} J_B(v)\bar{Q}_B(\zeta)dF_B(v) - \int_0^{\overline{c}} J_S(c)\bar{Q}_S(\zeta)dF_S(c)$ subject to the constraint $\bar{Q}_B(\zeta)(1 - F_B(\underline{v})) = \bar{Q}_S(\zeta)F_S(\overline{c})$ using the control variables $\underline{v}$, $\overline{c}$, and ζ. The following Lemma characterizes the optimal ζ.

Lemma 5. *The profit maximizing market tightness ζ^* satisfies*

$$\zeta^* = \frac{\sigma_B(\zeta^*)}{\sigma_S(\zeta^*)} \frac{f_B(\underline{v})}{f_S(\overline{c})}$$

The proof of the Lemma is in the Appendix. Intuitively, Lemma 5 describes the intermediary's trade-off when choosing how to balance the market. The intermediary is more willing to tilt the market towards more buyers (i.e. $\zeta = B/S$ larger) if the elasticity of the matching function $\sigma_B(\zeta)$ is large for buyers, since this will have a large impact on the number of matches. A large marginal density $f_B(\underline{v})$ has similar effects, since lowering $\underline{v}$ has a large effect on additional entry by buyers.

Using (23), (24), $\gamma = 0$, and Lemma 5 leads us to the following characterization of the optimal mechanism.

Proposition 4. *The optimal allocation rule is to allow trade for buyers above and sellers below certain thresholds $\underline{v}$ and $\bar{c}$, as in Proposition 1. The thresholds are now determined by the system of equations*

$$J_B(\underline{v}) = J_S(\bar{c}),$$
$$\overline{Q}_B(\zeta^*)(1 - F_B(\underline{v})) = \overline{Q}_S(\zeta^*)F_S(\bar{c}),$$

where ζ^* satisfies

$$\zeta^* = \frac{\sigma_B(\zeta^*)}{\sigma_S(\zeta^*)}\frac{f_B(\underline{v})}{f_S(\bar{c})}.$$

A few remarks are in order before moving to the search market implementation of the optimal mechanism. Note that for a Cobb-Douglas search technology the elasticities are constant, $\sigma_B(\zeta) = \bar{\sigma}_B$, $\sigma_S(\zeta) = \bar{\sigma}_S$ and hence $\zeta^* = \bar{\sigma}_B f_B(\underline{v})/(\bar{\sigma}_S f_S(\bar{c}))$. Note further that for the min search technology $M(B,S) = \lambda \min\{B,S\}$ (as the limiting case of a CES-function) $\zeta^* = 1$ and the allocation rule is the same as for discrete matching. Further, as search frictions disappear, i.e. search intensity goes to infinity ($\lambda \to \infty$), the allocation rule converges to the discrete time allocation. However, market tightness remains the same as for a finite λ.

Under the symmetry assumptions $\sigma_B(1) = \sigma_S(1)$ and $f_B(x) = f_S(1-x)$ (i.e. f_B is a mirror image of f_S) we get $\zeta^* = 1$, i.e. it is optimal to make sure that the market is balanced ($B = S$). This in turn implies $\overline{Q}_B(\zeta^*) = \overline{Q}_S(\zeta^*)$ and hence the marginal types $\underline{v}$ and $\bar{c}$ in Prop. 4 are the same as for the discrete time matching setup in Prop. 1.

4.2 Optimal Implementation with Participation Fees

In a bazaar, a matched buyer and seller bargain according to the same take-it-or-leave-it protocol as in the flea market. In this section, we show that the outcome of the optimal continuous time mechanism can be attained in a bazaar where buyers and sellers bargain. We continue to assume that the intermediary prices its services through per period fees, K_B and K_S. As for flea markets, we defer the discussion of per match and per transaction fees to Section 5.3. We can use a similar logic for marginal types and fees as for flea markets. The optimal allocation rule also has the full trade property in this setup, but the optimal market is not balanced in general: the probability of being matched in a certain period is given by the matching rates M/B and M/S rather than 1. The zero utility condition for marginal types under full trade is hence

$$\frac{M(B,S)}{B}\alpha_B\theta^* = K_B, \qquad \frac{M(B,S)}{S}\alpha_S\theta^* = K_S.$$

Hence, the sum of fees is

$$K_B + K_S = \left(\frac{\underline{v}}{\eta_B(\underline{v})} + \frac{\overline{c}}{\eta_S(\overline{c})}\right)\left(\alpha_B\frac{M(B,S)}{B} + \alpha_S\frac{M(B,S)}{S}\right),$$

and their ratio

$$\frac{K_B}{K_S} = \frac{\alpha_B}{\alpha_S}\frac{M(B,S)/B}{M(B,S)/S}.$$

Alternatively, this can be written as

$$\frac{K_B}{K_S} = \frac{\alpha_B}{\alpha_S}\frac{1/B}{1/S} = \frac{\alpha_B/[\sigma_B(\zeta^*)f_B(\underline{v})]}{\alpha_S/[\sigma_S(\zeta^*)f_S(\overline{c})]}.$$

Hence, a larger proportion of the fee is charged to the side that has the larger bargaining weight and that is more abundant in the market in equilibrium;[17] which is equivalent to the ratio of bargaining weights being multiplied by the ratio of the inverse matching elasticities of demand.

For the example of a Cobb-Douglas matching function with uniform distributions over $[0,1]$, we have $S/B = \bar{\sigma}_S/\bar{\sigma}_B$, and therefore the above equation reduces to

$$\frac{K_B}{K_S} = \frac{\alpha_B/\bar{\sigma}_B}{\alpha_S/\bar{\sigma}_S}.$$

The Hosios (1990) condition is equivalent to the right-hand side being equal to 1. If the Hosios condition is satisfied, the market is constrained efficient in absence of the intermediary.[18] Under this condition, the intermediary's optimal fee structure is balanced: $K_B/K_S = 1$. For general distributions, the fee ratio in a constrained efficient market is $K_B/K_S = f_S(\overline{c})/f_B(\underline{v})$.

In order to show existence of a full trade equilibrium, we need the following lemma, in parallel to Lemma 4 for the flea market. The proof is in the Appendix.

Lemma 6. *A full trade equilibrium exists if the marginal types satisfy the following conditions:*

$$\left(1 - \overline{Q}_B(\zeta)\right) J_B(\underline{v}) + \overline{Q}_B(\zeta)\underline{v} \geq \bar{c}, \tag{25}$$

$$\left(1 - \overline{Q}_S(\zeta)\right) J_S(\bar{c}) + \overline{Q}_S(\zeta)\bar{c} \leq \underline{v}. \tag{26}$$

[17] Here, one has to be careful with the interpretation of this ratio, since B/S is endogenous. It can also be seen as the platform making sure that the side with the larger bargaining weight is tighter.

[18] Strictly speaking, we need a small exogenous search cost to claim that the market is constrained efficient without the intermediary. See Mortensen and Wright (2002).

Note that while the optimal marginal types $\underline{v}$ and $\overline{c}$ in Propositions 1 and 4 are different, for both cases they satisfy the condition $J_B(\underline{v}) = J_S(\overline{c})$. Hence, we can use the same logic as for Proposition 2 to show that the profit maximizing fees indeed induce an equilibrium which implements the allocation rule given by the optimal mechanism: $J_B(\underline{v}) = J_S(\overline{c})$ implies the conditions in Lemma 6 and hence we get the following Proposition.

Proposition 5. *The intermediary's profit maximizing per period fees K_B^* for the buyer and K_S^* for the seller in the bazaar setup are given by*

$$K_B^* + K_S^* = \left(\frac{1 - F_B(\underline{v})}{f_B(\underline{v})} + \frac{F_S(\overline{c})}{f_S(\overline{c})}\right)\left(\alpha_B \frac{M(B,S)}{B} + \alpha_S \frac{M(B,S)}{S}\right),$$

$$\frac{K_B^*}{K_S^*} = \frac{\alpha_B / [\sigma_B(\zeta^*) f_B(\underline{v})]}{\alpha_S / [\sigma_S(\zeta^*) f_S(\overline{c})]},$$

where the marginal types $\underline{v}$ and $\overline{c}$ are given by Prop. 4. For these fees, a full trade market equilibrium exists in the bazaar setup, i.e. an equilibrium such that $\Phi_S(\underline{p}_B) = 1 - \Phi_S(\overline{p}_S) = 1$, where $\underline{p}_B$ is the lowest price set by a buyer and $\overline{p}_S$ is the highest price set by a seller.

As for the discrete time setup, the intermediary can achieve uniqueness by a slight modification of the mechanism.

5 Discussion

5.1 Different Levels of Centralization

Our paper also touches a more general question in economics: what level of centralization is desirable? Our article gives insights about this question in the context of profit maximizing intermediaries rather than social welfare, i.e. what level of centralization is desirable for a profit maximizing intermediary running a market?

The relation to centralization becomes more evident if we consider the third most wide spread archetype of intermediaries that was briefly alluded to before: market makers. Market makers take orders from buyers and sellers and decide on a centralized mechanism that determines how the good is exchanged multilaterally. A prominent example are specialists at the New York Stock Exchange. Market makers, flea markets, and bazaars can be seen as decreasing levels of centralization: a market maker opens the market at discrete periods of time and centrally determines the exchange mechanism; in a flea market the intermediary determines the discrete opening times but leaves matching and bargaining to traders; in a bazaar the intermediary does not even determine the opening time, traders enter the market at times not coordinated.

The discussion in Section 3.1 shows that the mechanism typically used by market makers, namely setting an ask and bid price $\underline{v}$ and $\overline{c}$, is indeed optimal in the class of stationary anonymous mechanisms. It further shows revenue equivalence between a market maker's bid-ask prices and a flea market: expected profits of the intermediary and (interim) expected utilities of all buyers and sellers are the same in both types of markets. This is due to the fact that the intermediary makes sure that there is full trade in a flea market, i.e. every offer is accepted in equilibrium, and that the market is balanced. This leads to the same allocation rule and the same profits as with a market maker. Whether an intermediary wants to run a market as a market maker or as a flea market hence depends on factors not explicitly modeled here. Such factors can be the cost of setting up a centralized system, the costs of search and bargaining, and how well the intermediary is informed about the details of the market.[19] Further decentralizing the market, namely moving to a bazaar, does change the profits of the intermediary as outlined below.

5.2 Profit Comparison Flea Market versus Bazaar

In this subsection, we compare the revenue performance of the flea market and bazaar for different levels of frictions δ. For tractability, we assume that buyers and sellers are symmetric: $f_B(x) = f_S(1-x)$ and $\sigma_B(1) = \sigma_S(1)$. In a flea market, ratio $1-(1-e^{-\delta\tau})/(\delta\tau)$ of traders drop out without trading between two subsequent market openings. The remainder gets matched and trades with probability 1. In a bazaar, the meeting rates are $l_B = l_S = \lambda$ and the drop out rate δ under the symmetry assumption (which leads to $\zeta^* = 1$). Hence, ratio $\delta/(\delta+\lambda)$ drop out without trading. When frictions vanish ($\delta \to 0$) nobody drops out without trading both for discrete and for continuous time. Since the allocation rule is the same for discrete and continuous time under the symmetry assumption, profits will be the same for discrete and continuous time as $\delta \to 0$. Denote these profits as Π. Since losses only occur through drop-outs,

[19]A market maker would typically need better information about the distributions of traders' valuations to get the mechanism right than the organizer of a flea market.

profits in the flea market are

$$\Pi_F = \frac{1}{\delta\tau}(1 - e^{-\delta\tau})\Pi, \tag{27}$$

and in a bazaar

$$\Pi_B = \frac{\lambda}{\delta + \lambda}\Pi, \tag{28}$$

where

$$\Pi := \underline{v}(1 - F_B(\underline{v})) - \bar{c}F_S(\bar{c}),$$

and $\underline{v}$ and $\bar{c}$ are determined in Proposition 1.

As mentioned before, profits are the same for the flea market and the bazaar as frictions vanish, $\lim_{\delta\to 0}\Pi_F = \lim_{\delta\to 0}\Pi_B = \Pi$. Further, for search frictions becoming large $\lim_{\delta\to\infty}\Pi_F/\Pi_B = 1/(\lambda\tau)$.

Search frictions δ can be seen as costs of decentralization. This makes it all the more surprising that a decentralized bazaar can become more attractive for the intermediary as δ increases. The ratio of flea market versus bazaar profits Π_F/Π_B can be non-monotone in the exit rate δ. Rearranging the expressions for Π_F and Π_B reveals that Π_F/Π_B increases in δ at $\delta = 0$ iff $\lambda < 2/\tau$. For intermediate values of the search intensity $\lambda \in (1/\tau, 2/\tau)$, $\Pi_F > \Pi_B$ for δ close to 0 and $\Pi_F < \Pi_B$ for δ close to ∞. A graphical illustration is given in Figure 2. For $\tau = 1$ and $\lambda = (1-e^{-10})/(0.1e^{-10}+0.9) \approx 1.111$ in Fig. 2, a flea market is preferred for $\delta < 10$ and a bazaar is preferred for $\delta > 10$.

We can shed more light on the issue at hand by normalizing the time between rematchings to $\tau = 1$. For any value of $\delta > 0$, the flea market yields a higher profit than the bazaar if the search intensity λ is sufficiently low. For a given $\delta > 0$, $\Pi_F > \Pi_B$ if and only if

$$\lambda < \bar{\lambda}(\delta) := \frac{\delta(1 - e^{-\delta})}{e^{-\delta} - 1 + \delta}.$$

The function $\bar{\lambda}(\delta)$ can be shown to be decreasing, with $\lim_{\delta\to 0}\lambda(\delta) = 2$ and $\lim_{\delta\to\infty}\bar{\lambda}(\delta) = 1$. See Figure 3. Furthermore, we can compare (27) and (28) for a fixed level of λ across different levels of δ. For $\lambda \in (0, 1]$, profits for a flea market are larger, $\Pi_F > \Pi_B$ for all $\delta > 0$. For $\lambda \in (1, 2)$ profits in the flea market are larger, $\Pi_F > \Pi_B$, if and only if $\delta \in (0, \bar{\delta}(\lambda))$, where

where $\bar{\delta}(\cdot)$ is the inverse of $\bar{\lambda}(\cdot)$. For $\lambda \geq 2$, profits in the bazaar are larger, $\Pi_B > \Pi_F$ for all $\delta > 0$.

For $\lambda \in (1, 2)$, the intermediary prefers a greater degree of centralization with *smaller* frictions δ. Intuitively, in a market with patient traders (low exit rate δ), the intermediary is willing to wait until enough traders gather and then opens a flea market. If, on the other hand, traders are impatient, most would exit before the next opening of the flea market. Therefore, it is worth incurring the search frictions of the search market.

The result that more frictions make decentralization more attractive of course depends on the specific kind of frictions: the exit rate. The result is driven by the different ways the exit rate enters the profit function: close to exponentially for the flea market, close to hyperbolically for the bazaar.

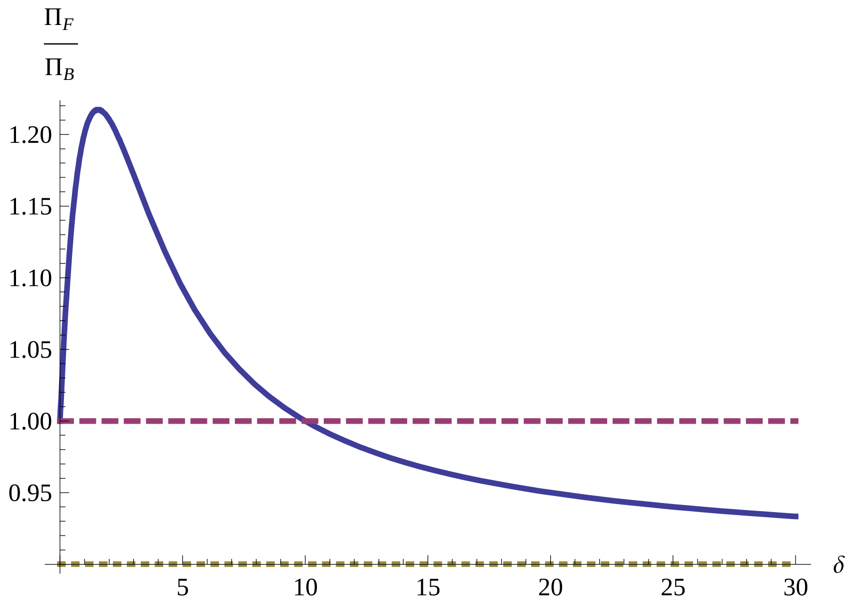

Figure 2: Ratio of flea market profits Π_F to bazaar profits Π_B as a function of the exit rate δ (solid line) for parameters $\tau = 1$ and $\lambda = (1 - e^{-10})/(0.1e^{-10} + 0.9) \approx 1.111$. The dashed line is at $\Pi_F/\Pi_B = 1$ and intersects the solid line at $\delta = 0$ and $\delta = 10$. The dotted line is at $1/(\lambda\tau)$ and is the asymptotic value of Π_F/Π_B as δ goes to infinity.

An issue we have not dealt with, but which can be addressed by a straightforward extension of our model, is that the time between subsequent openings of a flea market τ and the search intensity in a bazaar λ might be endogenously chosen. Assume that the intermediary incurs per period costs $\kappa_\tau(\tau)$ of running the market, more frequent market opening times being more

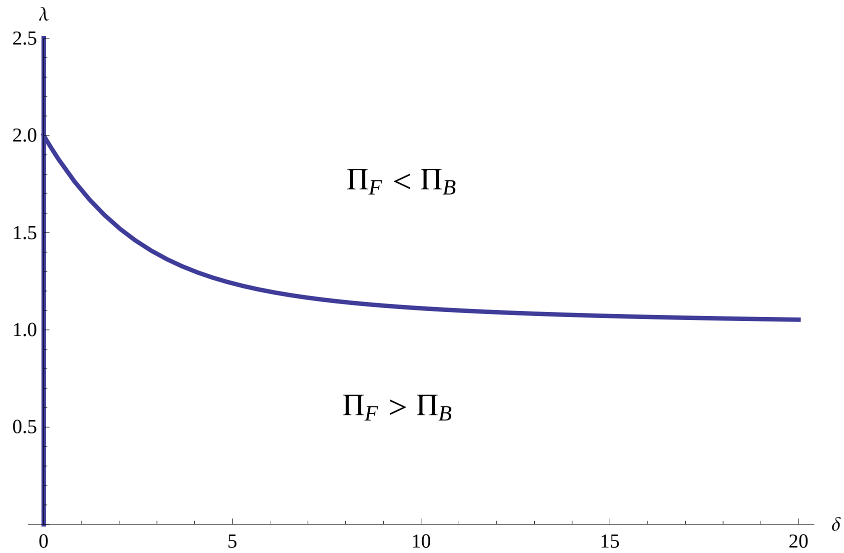

Figure 3: Profit comparison of flea market and bazaar. The curve indicates the values of the search intensity $\lambda = \bar{\lambda}(\delta)$ such that $\Pi_F = \Pi_B$ for $\delta > 0$. The profit is higher for the flea market above the curve, and lower below. $\Pi_F = \Pi_B$ for all λ if $\delta = 0$.

costly. Likewise, it is costly to increase search intensity, per period costs of running a bazaar being $\kappa_\lambda(\lambda)$. A flea market intermediary will choose τ^* to maximize $[(1-e^{-\delta\tau})/(\delta\tau)]\Pi - \kappa_\tau(\tau)$. A bazaar intermediary chooses λ^* to maximize $[\lambda/(\delta+\lambda)]\Pi - \kappa_\lambda(\lambda)$. The above profit comparison is the same, except that τ^* and λ^* are endogenously chosen values and the costs $\kappa_\tau(\tau^*)$ and $\kappa_\lambda(\lambda^*)$ of running the market also have to be taken into account.

5.3 Other Types of Fees

Per Match Fees. In a flea market, a trader gets matched immediately in the period he enters the market and his probability of being matched is independent of his actions. Hence, per match fees and per period participation fees are equivalent.

In a bazaar, per period and per match fees are different, since traders do not get matched once a period, but according to the matching rates M/B and M/S, respectively. Further, the market is not necessarily balanced. Therefore, if per match rather than per period fees are charged, the condition for the marginal types is that gross utility *per match* is equal to fees *per match*. Retaining the same notation for the fees, we have

$$\alpha_B\theta = K_B, \qquad \alpha_S\theta = K_S,$$

where K_B and K_S are now per match fees. Since this condition is the same as in a flea market, optimal fees are also the same, i.e. given by $K_B + K_S = \theta^*$ and $K_B/K_S = \alpha_B/\alpha_S$.

Transaction fees. Once again, we retain the same notation for per transaction fees, K_B and K_S. Since transaction fees are only paid in case of a transaction, a buyer only accepts a price p_S if it is at most $v - K_B - (1-\epsilon)W_B(v)$, where K_B is the transaction fee and $(1-\epsilon)W_B(v)$ is his option value of future trade. For the marginal buyer $\underline{v}$ the option value of future trade is zero, $W_B(\underline{v}) = 0$, hence in a full trade equilibrium he will get the price offer $p_S = \underline{v} - K_B$. By a similar logic, the marginal seller will get the price offer $p_B = \overline{c} + K_S$. A profit maximizing intermediary will set $K_B + K_S = \underline{v} - \overline{c}$. Combining this with the expressions for p_B and p_S we get $p_B = p_S = \underline{v} - K_B = \overline{c} + K_S$. Hence with transaction fees we get fee neutrality: the sum of the fees is given by the semi-elasticities $K_B + K_S = \underline{v}/\eta_B(\underline{v}) + \overline{c}/\eta_S(\overline{c})$, but the composition of the fees does not matter for the profits of the platform, it will merely move the transaction price $p = p_B = p_S$, the ratio of the fees satisfying $K_B/K_S = (\underline{v} - p)/(p - \overline{c})$.

5.4 Exogenous participation costs

The model can be easily extended to include both participation fees K_B and K_S set by the intermediary and exogenously given search costs x_B and x_S per period. The conditions for the marginal types are

$$\alpha_B\theta^* = K_B + x_B \qquad \alpha_S\theta^* = K_S + x_S,$$

which is essentially the same as before, except that total participation costs $K_B + x_B$ and $K_S + x_S$ are decomposed to fees and exogenous search costs. The optimal spread is $\underline{v} - \overline{c} = \theta^* = \underline{v}/\eta_B(\underline{v}) + \overline{c}/\eta_S(\overline{c}) + x_B + x_S$ and the market balancing condition becomes $\alpha_B/\alpha_S = (K_B + x_B)/(K_S + x_S)$. As an example, in a market where sellers always get to make take-it-or-leave-it offers to buyers ($\alpha_S = 1$) and buyers have positive exogenous search costs ($x_B > 0$), the platform should charge high fees to sellers ($K_S = \theta^* - x_S + x_B$) and subsidize buyers ($K_B = -x_B$).

5.5 Welfare Effects of Intermediaries

Our model can be used to shed light on the debate whether intermediaries are welfare decreasing (see conventions of the International Labor Organization and the referendum in Washington state mentioned in the Introduction). While a detailed analysis is outside of the scope of this paper, we add a brief discussion of these issues in light of our theoretical framework. Would welfare increase or decrease if the intermediary was removed from the market?[20]

In this section, we compare a flea market without fees with the same flea market run by a for-profit intermediary. (A similar reasoning holds for continuous time search markets without search fees.) We will use the Walrasian outcome as a benchmark, in which buyers above and sellers below the Walrasian price p^* trade and the Walrasian price is given by $1 - F_B(p^*) = F_S(p^*)$. The payoffs of buyers, sellers, and the intermediary are the same for a flea market and a market maker setting bid and ask prices $\underline{v}$ and $\overline{c}$. We can hence simplify the analysis by looking at payoffs of the market maker. Here, buyers above $\underline{v}$ and sellers below $\overline{c}$ enter and trade occurs at price $\underline{v}$ and $\overline{c}$. The marginal types satisfy $1 - F_B(\underline{v}) = F_S(\overline{c})$ and $J_B(\underline{v}) = J_S(\overline{c})$.

To simplify the exposition, we look at the extreme case in which search frictions are so high, that traders only consider one trade opportunity and then exit, that is $\epsilon = 1$. This is essentially a static model. To get an analytically tractable example, we also assume that traders' types follow power distributions, $F_B(v) = 1 - (1 - v)^\beta$ and $F_S(c) = c^\beta$ with $\beta > 0$. Power distributions result in linear virtual types, $J_B(v) = (1 + \frac{1}{\beta})v - \frac{1}{\beta}$ and $J_S(c) = (1 + \frac{1}{\beta})c$.[21]

The Walrasian price is $p^* = \frac{1}{2}$ because of symmetry and welfare of buyers and sellers in a Walrasian market is

$$W_S^* = \int_0^{p^*} (p^* - c)dF_S(c) = \frac{1}{(\beta + 1)2^{\beta+1}}, \qquad W_B^* = \int_{p^*}^{1} (v - p^*)dF_B(v) = \frac{1}{(\beta + 1)2^{\beta+1}},$$

Welfare with a profit maximizing intermediary for buyers W_B^P, sellers W_S^P, and the intermediary

[20] There are several other questions that arise when dealing with these issues. How should costs of running a platform be recouped? Is it optimal to choose participation fees to be positive rather than zero from a social planner's point of view? How does competition between a for-profit and a non-profit platform look like and what are the welfare and policy implications? Our results may be a helpful starting point for an analysis dealing with these questions.

[21] A higher β can be seen as a higher elasticity of demand and supply, since the elasticities are $\eta_B(v) = \beta v/(1-v)$ and $\eta_S(c) = \beta$. Further, as $\beta \to 0$, informational asymmetries vanish, since in the limit buyers have valuation 1 with probability 1 and seller have valuation 0 with probability 1.

W_I^P is

$$W_S^P = \int_0^{\overline{c}} (\overline{c} - c) dF_S(c) = \left(\frac{\beta}{\beta+1}\right)^{\beta+1} W_S^*, \qquad W_B^P = \int_{\underline{v}}^1 (v - \underline{v}) dF_B(v) = \left(\frac{\beta}{\beta+1}\right)^{\beta+1} W_B^*,$$

$$W_I^P = (\underline{v} - \overline{c}) F_S(\overline{c}) = \frac{\beta^\beta}{(\beta+1)^{\beta-1}} (W_S^* + W_B^*).$$

In a flea market with free entry and no profit maximizing intermediary, all sellers enter. This is because a seller with cost c close to 1 still has a positive probability of making the price offer. Since we are in an essentially static setup, a buyer with $v \geq c$ will accept the offer, which leads to a positive expected utility of sellers with $c < 1$. By the same reasoning, all buyers enter. Welfare for a seller with cost c is $W_S^N(c) = \alpha_S (P_S(c) - c)(1 - F_B(P_S(c))) + \alpha_B \int_0^1 \max\{0, P_B(v) - c\} dF_B(v)$, where the price set by a seller is $P_S(c) = \arg\max_p (p - c)(1 - F_B(p)) = J_B^{-1}(c)$ and by a buyer $P_B(v) = J_S^{-1}(v)$. The analogous expression holds for the welfare $W_B^N(v)$ of a buyer with valuation v. Total welfare for sellers $W_S^N = \int_0^1 W_S^N(c) dF_S(c)$ and buyers $W_B^N = \int_0^1 W_B^N(v) dF_B(v)$ can be computed as

$$W_S^N = \frac{\sqrt{\pi}\beta^\beta(\alpha_S + \beta)\Gamma(\beta+1)}{2^{2\beta+1}(\beta+1)^{\beta+1}\Gamma\left(\frac{3}{2}+\beta\right)}, \qquad W_B^N = \frac{\beta^\beta(\alpha_B + \beta)\Gamma(\beta+1)^2}{(\beta+1)^{\beta+1}\Gamma(2(\beta+1))},$$

where Γ is the gamma function.

Figure 4 shows how buyer and seller welfare as a ratio of Walrasian welfare compares for different values of β. The solid line is welfare with an intermediary. The dashed line is total welfare without an intermediary for large search frictions $\epsilon \to 1$. What is striking in Figure 4 is that welfare with an intermediary may be higher – even if excluding the intermediary's profits and only looking at buyer and seller welfare. The reason for this result is that in a search market, there may be excessive entry: inefficient buyers and sellers who would not trade in a Walrasian equilibrium ($v < p^*$ and $c > p^*$) enter, hoping to find a very efficient trading partner who prefers accepting an unattractive offer to incurring search costs to find another trading partner. An intermediary excludes some of the traders, which has the effect of reducing excessive entry additionally to the effect of the deadweight loss of monopoly. In some cases, efficiency gains resulting from the reduction of excessive entry dominate efficiency losses from the standard deadweight loss of exclusion.[22]

[22] The example provided was chosen for the sake of simplicity, since it allows for a closed form solution and there is a unique non-trivial equilibrium. For this particular example, overall welfare with an intermediary (including the intermediary's profits) is always higher in a static setup ($\epsilon \to 1$). However, one can easily construct examples, in which there is ambiguity even in the static setup.

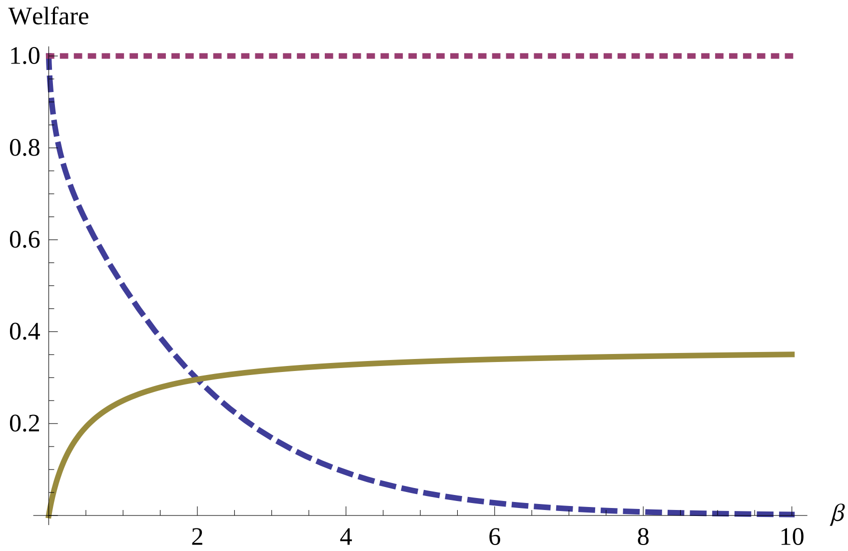

Figure 4: Welfare of buyers and sellers as a ratio of Walrasian (i.e. first-best) welfare $W^* = W_B^* + W_S^*$ for different setups: with an intermediary $(W_B^P + W_S^P)/W^*$ (solid), without an intermediary and large search frictions, $\epsilon \to 1$, $(W_B^N + W_S^N)/W^*$ (dashed), without an intermediary and small search frictions, $\epsilon \to 0$, $(W_B^N + W_S^N)/W^*$ (dotted).

When search frictions are small (the exit probability between two subsequent matches vanishes, $\epsilon \to 0$), it is known from the literature (see Satterthwaite and Shneyerov (2007) and Shneyerov and Wong (2010b)) that a dynamic random matching market converges to a Walrasian outcome (for any distribution). The allocation rule and hence welfare with an intermediary are independent of ϵ and still given by the marginal types $\underline{v}$ and $\overline{c}$ satisfying $1 - F_B(\underline{v}) = F_S(\overline{c})$ and $J_B(\underline{v}) = J_S(\overline{c})$. Therefore, welfare is clearly lower with an intermediary, since buyers and sellers that would have traded in a Walrasian equilibrium are excluded by a profit maximizing intermediary (since $\overline{c} < p^* < \underline{v}$). This is the standard result of the deadweight loss of monopoly. Figure 4 illustrates the welfare of buyers and sellers with (solid line) and without (dotted line) an intermediary, for small search frictions $\epsilon \to 0$.

6 Conclusions

We have investigated two types of search and bargaining markets organized by a profit-maximizing platform: periodically clearing flea markets and continuously clearing bazaars. In both markets, the optimal allocation rule takes the form of two thresholds: buyers above the upper threshold trade with probability 1, below they trade with probability 0. Similarly, sellers below the lower threshold trade for sure and never trade above it. The thresholds, however, are different in flea markets and bazaars.

The optimal allocation can be achieved by setting per match participation fees in a bargaining market, in which the ratio of buyers' and sellers' fees is proportional to their bargaining weights. This result is obtained in a class of simple equilibria that have a full trade property: every meeting in the market results in trade. Even though a full trade equilibrium may not exist for certain values of fees, we show that it does exist for the optimal level of fees. This allows us to develop a tractable model of intermediation for markets with bargaining, with clear implications on total fees and fee structure. The intermediary can achieve the same outcome by charging per period or per transaction fees. The structure of per period fees is the same for a flea market as that of per match fees. For a bazaar, per period fees have to be adjusted by the relative abundance of traders on the buyer or seller side. For transaction fees, fee neutrality holds: only the sum of the buyer's and the seller's fee matters, not their composition.

This simple and tractable solution allows us to investigate several other important questions. How much centralization is desirable for the intermediary? Which market structure, flea market or bazaar, generates more profits for the intermediary? It turns out that the flea market is optimal for small frictions, while the bazaar is optimal when frictions are large. Next, focusing on markets with small frictions, we show that the presence of the intermediary may actually *increase* the welfare of the traders. The intuition is that the intermediary excludes inefficient buyers and sellers, which reduces search frictions.

This tractable framework also points to an intriguing issue for future research concerning the regulation of intermediaries: in our setup, buyer fees and seller fees are complements. This is because lower fees for sellers lead to additional entry by sellers, which make more entry by buyers (and lower fees) more attractive for the intermediary. Hence, putting a regulatory price cap on fees on one side of the market can lead to *lower* fees for the other side of the market, provided the price cap is not too far from the unregulated fee level. This stands in contrast to the markets analyzed in the two-sided markets literature, in which buyer and seller fees are substitutes and price caps on one side lead to higher fees on the other side.

Appendix: Proofs

Proof of Lemma 4. We only need to show that sellers do not have an incentive to deviate to an ask price higher than $\underline{v}$, and buyers do not have an incentive to deviate to a bid price lower than $\bar{c}$. We will only prove the result for sellers; for buyers, the argument is parallel.

Consider a static setup first, $\epsilon = 1$. The expected profit of a seller of type c in any match where she deviates from equilibrium and proposes an ask price p_S is

$$\begin{aligned}\pi_S(c, p_S) :&= \alpha_S(p_S - c)\Phi_B(p_S) + \alpha_B(\bar{c} - c) \\ &= \alpha_S(p_S - c)\frac{1 - F_B(p_S)}{1 - F_B(\underline{v})} + \alpha_B(\bar{c} - c),\end{aligned} \tag{29}$$

and its slope is[23]

$$\begin{aligned}\frac{\partial \pi_S(c, p_S)}{\partial p_S} &= \frac{1 - F_B(p_S) - f_B(p_S)(p_S - c)}{1 - F_B(\underline{v})} \\ &= \frac{f_B(p_S)}{1 - F_B(\underline{v})}\left(c - p_S + \frac{1 - F_B(p_S)}{f_B(p_S)}\right), \\ \frac{\partial \pi_S(c, p_S)}{\partial p_S} &\propto c - J_B(p_S).\end{aligned}$$

At $p_S = \underline{v}$, this slope is nonpositive for all $c \leq \bar{c}$ if $J_B(\underline{v}) \geq \bar{c}$, which implies that the seller will not prefer such a deviation. This is our condition (19) in the statement of the lemma when $\epsilon = 1$.

In a dynamic setup, $\epsilon < 1$, the probability that an offer p_S is accepted depends on the market distribution of buyers' *reservation values.* Consider a buyer of type v. His expected profit in the next market opening, discounted back to the current one, is equal to

$$\begin{aligned}W_B(v) &= (1 - \epsilon)\left(\alpha_B(v - \bar{c}) + \alpha_S(v - \underline{v}) - K_B^*\right), \\ &= (1 - \epsilon)(\alpha_B + \alpha_S)(v - \underline{v}) + (1 - \epsilon)\alpha_B(\underline{v} - \bar{c}) - (1 - \epsilon)K_B^* \\ &= (1 - \epsilon)(v - \underline{v}),\end{aligned} \tag{30}$$

where the equality in the last line follows from the fact that, in a full trade equilibrium, $\alpha_B(\underline{v} - \bar{c}) = \alpha_B\theta = K_B$. In the current period, he will accept any price p_S if $v - p_S \geq W_B(v)$. Equivalently, he will accept any p_S at or above his reservation value

$$\tilde{v}(v) := v - W_B(v) = \epsilon v + (1 - \epsilon)\underline{v}. \tag{31}$$

[23]Here and below, we use the notation "$\propto$" for "has the same sign as".

Going in the reverse direction, the buyer's type v as a function of his reservation value $\tilde{v}$ is

$$v = \frac{\tilde{v} - (1-\epsilon)\underline{v}}{\epsilon},$$

and therefore the market distribution of reservation values is

$$\tilde{\Phi}_B(\tilde{v}) = \Phi_B\left(\frac{\tilde{v} - (1-\epsilon)\underline{v}}{\epsilon}\right).$$

The expected seller's profit in any match is of the same form as in (29), with $\Phi_B(p_S)$ replaced with $\tilde{\Phi}_B(p_S)$. The corresponding virtual valuations are

$$\begin{aligned}
\tilde{J}_B(\tilde{v}) :&= \tilde{v} - \frac{1-\tilde{\Phi}_B(\tilde{v})}{\tilde{\phi}_B(\tilde{v})} \\
&= \epsilon v + (1-\epsilon)\underline{v} - \epsilon \frac{1-\Phi_B(v)}{\phi_B(v)} \\
&= \epsilon v + (1-\epsilon)\underline{v} - \epsilon \frac{1-F_B(v)}{f_B(v)} && (32) \\
&= \epsilon J_B(v) + (1-\epsilon)\underline{v}. && (33)
\end{aligned}$$

In parallel to the static setup,

$$\begin{aligned}
\frac{\partial \pi_S(c, \underline{v})}{\partial p_S} &\propto c - \tilde{J}_B(\underline{v}) \\
&= c - \epsilon J_B(\underline{v}) + (1-\epsilon)\underline{v}
\end{aligned}$$

so (33) implies (19) in the lemma. □

Proof of Proposition 3. First, we will show that no other equilibrium with expansion on at least one side of the market exists (i.e. there is either more entry by buyers, more entry by sellers, or both). Then we will show that there is no other equilibrium with contraction (i.e. less entry by both buyers and sellers).

In the following we will take a non-full-trade-equilibrium where buyers $v \in [\tilde{v}, 1]$ and sellers with $c \in [0, \tilde{c}]$ enter. As a comparison, we will denote the marginal types in the full-trade-equilibrium that maximizes the intermediary's profits as $\underline{v}$ and $\bar{c}$. We will show that any other equilibrium leads to a contradiction.

Recall that for full trade, the marginal types are given by

$$\alpha_B(\underline{v} - \bar{c}) = K_B$$

$$\alpha_S(\underline{v} - \bar{c}) = K_S$$

Similarly for the alternative non-full-trade-equilibrium

$$\alpha_B(\tilde{v} - p_B)\frac{M}{B}\tilde{\Phi}_S(p_B) = K_B$$
$$\alpha_S(p_S - \tilde{c})\frac{M}{S}(1 - \tilde{\Phi}_B(p_S)) = K_S.$$

The endogenous distributions $\tilde{\Phi}_B$ and $\tilde{\Phi}_S$ are given by truncation of non-entering types in the static model:

$$\tilde{\Phi}_B(v) = \begin{cases} (F_B(v) - F_B(\tilde{v}))/(1 - F_B(\tilde{v})) & \text{if } v \geq \tilde{v}, \\ 0 & \text{otherwise,} \end{cases}$$
$$\tilde{\Phi}_S(c) = \begin{cases} F_S(c)/F_S(\tilde{c}) & \text{if } c \leq \tilde{c}, \\ 1 & \text{otherwise.} \end{cases}$$

In the following we will show for all cases of $\tilde{v} \neq \underline{v}$ or $\tilde{c} \neq \bar{c}$ that there is a contradiction, which gives us uniqueness.

First, expansion on both sides of the market ($\tilde{v} < \underline{v}$ and $\tilde{c} > \bar{c}$) is not possible.

Lemma 7. *There cannot be an equilibrium with marginal types $\tilde{v}$ and $\tilde{c}$ where $\tilde{v} < \underline{v}$ and $\tilde{c} > \bar{c}$.*

Proof. In the full trade equilibirum the utility of a type $\underline{v}$ buyer is

$$\max_{p_B} \alpha_B(\underline{v} - p_B)\{1\}\left\{\overline{\Phi}_S(p_B)\right\} = K_B \tag{34}$$

where $\overline{\Phi}_S$ is the distribution of sellers when truncating sellers above $\bar{c}$.

Compare this with the utility of the $\underline{v}$ buyer in the non-full-trade-equilibrium:

$$\max_{p_B} \alpha_B(\underline{v} - p_B)\left\{\frac{M}{B}\right\}\left\{\tilde{\Phi}_S(p_B)\right\} \tag{35}$$

Clearly, the two expressions in curly braces are both weakly less than the corresponding expressions in the previous equation, hence the maximized function in (35) is weakly less than the maximized function in (34) for all p_B, hence (35) is weakly less than K_B. Therefore, the marginal seller in the non-full-trade-equilibrium, $\tilde{v} < \underline{v}$, will have a utility strictly less than K_B, which is a contradiction. Intuitively, $\underline{v}$ is less likely to be matched in the non-full-trade-equilibrium and in case of being matched he is less likely to sell at a given price. Hence entering is less attractive to him. The same reasoning applies to the marginal seller $\tilde{c}$. ◻

Second, expansion on one and contraction on the other side cannot be an equilibrium.

Lemma 8. *There cannot be an equilibrium with marginal types $\tilde{v}$ and $\tilde{c}$ where either of the two conditions holds: (i) $\tilde{v} > \underline{v}$ and $\tilde{c} > \bar{c}$ or (ii) $\tilde{v} < \underline{v}$ and $\tilde{c} < \bar{c}$*

Proof. We will only consider case (i), the same reasoning applies for case (ii). For case (i), the sellers are the long side of the market, i.e. $S > B$, which is equivalent to $F_S(\bar{c}) > 1 - F_B(\tilde{v})$.

Consider the profit of seller $\bar{c}$ in the full trade equilibrium if he makes the offer:

$$\max_{p_S}(p_S - \bar{c})(1 - \overline{\Phi}_B(p_S)) = \max_{p_S}(p_S - \bar{c})\frac{1 - F_B(\underline{v})}{F_S(\bar{c})}\frac{1 - F_B(p_S)}{1 - F_B(\underline{v})} = \max_{p_S}(p_S - \bar{c})\frac{1}{F_S(\bar{c})}(1 - F_B(p_S)) \tag{36}$$

where the first equality follows from $1 - F_B(\underline{v}) = F_S(\bar{c})$ in the full trade equilibrium and the definition of the truncated distribution $\overline{\Phi}_B$.

The profit of the seller $\bar{c}$ in the non-full-trade-equilibrium if he makes the offer is

$$\max_{p_S}(p_S - \bar{c})\frac{M}{S}(1 - \tilde{\Phi}_B(p_S)) = \max_{p_S}(p_S - \bar{c})\frac{1 - F_B(\tilde{v})}{F_S(\tilde{c})}\frac{1 - F_B(p_S)}{1 - F_B(\tilde{v})} = \max_{p_S}(p_S - \bar{c})\frac{1}{F_S(\tilde{c})}(1 - F_B(p_S)) \tag{37}$$

where the first equality follows from the definitions of M/S when sellers are on the long side of the market and the truncated distribution $\tilde{\Phi}_B$. Comparing the maximized functions on the RHS of (36) and (37) reveals that profits are lower in the non-full-trade-equilibrium, since $\tilde{c} > \bar{c}$.

The profit of the marginal seller in the non-full-trade-equilibrium $\tilde{c}$

$$\max_{p_S}(p_S - \tilde{c})\frac{1}{F_S(\tilde{c})}(1 - F_B(p_S))$$

are even lower than in (37). Neither the $\bar{c}$ seller in the full trade equilibrium nor the $\tilde{c}$ seller in the non-full-trade-equilibrium make any profits in case the buyer makes the offer. Hence, the profit of the $\tilde{c}$ seller in the non-full-trade-equilibrium are below K_S, which is a contradiction. □

Third, contraction on both sides of the market cannot be an equilibrium.

Lemma 9. *There cannot be an equilibrium with marginal types $\tilde{v}$ and $\tilde{c}$ where $\tilde{c} < \bar{c}$ and $\tilde{v} > \underline{v}$.*

Proof. We will show that the marginal utility of increasing the price is positive for the marginal buyer in any equilibrium with two sided contraction. Since the same argument holds also for sellers, we know that buyers price at $\tilde{c}$ and sellers at $\tilde{v}$. Then we will show that an equilibrium with this pricing cannot exist for a spread less than the profit maximizing one, i.e. $\tilde{v} - \tilde{c} < \underline{v} - \bar{c}$.

The marginal buyer's utility when setting price p_B is

$$\pi_B(p_B) = (\tilde{v} - p_B)\frac{F_S(p_B)}{F_S(\tilde{c})}\frac{M}{B}$$

Looking at the marginal utility of pricing, leaving aside constants, we get

$$\begin{aligned}
\pi_B'(p_B) &\propto -F_S(\tilde{c}) + (\tilde{v} - \tilde{c})f_S(\tilde{c}) \\
&= (\tilde{v} - J_S(\tilde{c}))f_S(\tilde{c}) \\
&> (\underline{v} - J_S(\tilde{c}))f_S(\tilde{c}) \\
&\geq (\underline{v} - J_S(\bar{c}))f_S(\tilde{c}) \\
&\geq 0
\end{aligned}$$

where the first two inequalities follow from $\tilde{v} > \underline{v}$ and $\tilde{c} < \bar{c}$ and the third from the fact that there is full trade at $\underline{v}$, $\bar{c}$. A positive π_B' means that the buyer will set a price equal to the cost of the marginal seller $\tilde{c}$. By monotonicity, all other buyers (who have $v > \tilde{v}$), will also price at $\tilde{c}$. By an analogous argument, all sellers will price at $\tilde{v}$. Since the probability of trading conditional on being matched is 1, the utility of the marginal types is

$$\alpha_B(\tilde{v} - \tilde{c})\frac{M}{B} = K_B \tag{38}$$

$$\alpha_S(\tilde{v} - \tilde{c})\frac{M}{S} = K_S \tag{39}$$

Dividing the two equations and using the fact that the intermediary makes sure that $\alpha_B/\alpha_S = K_B/K_S$ we get $B/S = 1$. Substituting this back into (38) and (39) gives us a contradiction to the full trade marginal type conditions

$$\alpha_B(\underline{v} - \bar{c}) = K_B, \qquad \alpha_S(\underline{v} - \bar{c}) = K_S,$$

since $\tilde{v} - \tilde{c} > \underline{v} - \bar{c}$. Hence, a contraction equilibrium cannot exist. □

Putting Lemmas 7, 8, and 9 together, we get the result in Proposition 3 that the full trade equilibrium is unique in the static model. □

Proof of Lemma 5. The Lagrangian of the intermediary's problem is

$$L = \int_{\underline{v}}^{1} J_B(v)\overline{Q}_B(\zeta)dF_B(v) - \int_0^{\bar{c}} J_S(c)\overline{Q}_S(\zeta)dF_S(c) - \mu\left[\overline{Q}_B(\zeta)(1 - F_B(\underline{v})) - \overline{Q}_S(\zeta)F_S(\bar{c})\right].$$

Since it can be shown by partial integration that

$$\int_{\underline{v}}^{1} J_B(v)dF_B(v) = \underline{v}(1 - F_B(\underline{v})), \qquad \int_0^{\bar{c}} J_S(c)dF_S(c) = \bar{c}F_S(\bar{c}),$$

the Lagrangian can be rewritten as

$$\begin{aligned} L &= \overline{Q}_B(\zeta)(\underline{v} - \mu)(1 - F_B(\underline{v})) + \overline{Q}_S(\zeta)(\mu - \bar{c})F_S(\bar{c}) \\ &= \overline{Q}_B(\zeta)\frac{1 - F_B(\underline{v})}{f_B(\underline{v})}(1 - F_B(\underline{v})) + \overline{Q}_S(\zeta)\frac{F_S(\bar{c})}{f_S(\bar{c})}F_S(\bar{c}), \end{aligned}$$

where the second line follows from $J_B(\underline{v}) = \mu = J_S(\bar{c})$ and substituting in the definitions of J_B and J_S. The first order condition with respect to ζ is

$$-\overline{Q}'_B(\zeta)\frac{(1 - F_B(\underline{v}))^2}{f_B(\underline{v})} = \overline{Q}'_S(\zeta)\frac{F_S(\bar{c})^2}{f_S(\bar{c})}.$$

Dividing by constraint (i) twice to get rid of $1 - F_B(\underline{v})$ and $F_S(\bar{c})$ we get

$$-\frac{\overline{Q}'_B(\zeta)}{\overline{Q}_B(\zeta)^2}\frac{1}{f_B(\underline{v})} = \frac{\overline{Q}'_S(\zeta)}{\overline{Q}_S(\zeta)^2}\frac{1}{f_S(\bar{c})}. \tag{40}$$

Using the definitions of $\overline{Q}_B$, $\overline{Q}_S$, σ_B, and σ_S, some algebra reveals that

$$\frac{\overline{Q}'_B(\zeta)}{\overline{Q}_B(\zeta)^2} = -\delta\frac{m'(\zeta)\zeta - m(\zeta)}{m(\zeta)^2} = -\frac{\delta\sigma_S(\zeta)}{m(\zeta)}, \qquad \frac{\overline{Q}'_S(\zeta)}{\overline{Q}_S(\zeta)^2} = \delta\frac{m'(\zeta)}{m(\zeta)^2} = \frac{\delta\sigma_B(\zeta)}{\zeta m(\zeta)}.$$

Substituting this back into (40) yields

$$\frac{\sigma_S(\zeta^*)}{f_B(\underline{v})} = \frac{\sigma_B(\zeta^*)}{\zeta^* f_S(\bar{c})},$$

which completes the proof. □

Proof of Lemma 6. As in the proof of the parallel result for discrete time matching, Lemma 4, it is sufficient to verify that buyers do not have an incentive to bid less than $\bar{c}$, and marginal sellers do not have an incentive to ask more than $\underline{v}$. As in Lemma 4, we only prove the result for sellers, i.e. (25); the proof of (26) is parallel.

In a full trade equilibrium, the type v buyer's profit in a given match is $\alpha_B(\bar{c}-v)+\alpha_S(v-\underline{v})$, as in discrete time. Since the effective "discount rate" is $\delta + l_B(\zeta)$, the net present value of the participation fee flow is $K_B/\left(\delta + l_B(\zeta)\right)$, and the market continuation value is

$$\begin{aligned} W_B(v) &= Q_B(v)\left(\alpha_B(v - \bar{c}) + \alpha_S(v - \underline{v})\right) - \frac{K_B}{\delta + l_B(\zeta)}. \\ &= \frac{l_B(\zeta)\left(\alpha_B(v - \bar{c}) + \alpha_S(v - \underline{v})\right) - K_B}{\delta + l_B(\zeta)} \end{aligned}$$

where we substituted $Q_B(v)$, the "ultimate discounted probability of trade", from (21). The numerator in the above expression simplifies to

$$\begin{aligned} l_B(\zeta)\left(\alpha_B(v-\bar{c})+\alpha_S(v-\underline{v})\right)-K_B &= l_B(\zeta)\left(v-\underline{v}+\alpha_B(\underline{v}-\bar{c})\right)-K_B \\ &= l_B(\zeta)(v-\underline{v}), \end{aligned}$$

where we have used the fact that in a full trade equilibrium, $l_B(\zeta)\alpha_B(\underline{v}-\bar{c}) = K_B$. Therefore

$$\begin{aligned} W_B(v) &= \frac{l_B(\zeta)}{\delta + l_B(\zeta)}(v-\underline{v}) \\ &= \overline{Q}_B(\zeta)(v-\underline{v}), \end{aligned}$$

and the reservation value is

$$\begin{aligned} \tilde{v} &= v - W_B(v) \\ &= (1-\overline{Q}_B(\zeta))v + \overline{Q}_B(\zeta)\underline{v}. \end{aligned}$$

The rest of the proof proceeds exactly as in Lemma 4 following (31), with ϵ replaced by $1-\overline{Q}_B(\zeta)$. □

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