Mixed Bundling in Two-Sided Markets: Theory and Evidence

Yong Chao and Timothy Derdenger*

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Abstract

We analyze mixed bundling in two-sided markets and find that the pricing structure deviates from traditional bundling as well as the standard two-sided markets literature—we determine prices on both sides fall with bundling. Mixed bundling acts as a price discrimination tool segmenting the market more efficiently and functions as a coordination device helping solve "the chicken or the egg" problem in two-sided markets. After theoretically evaluating the impact mixed bundling has on prices and welfare, we test the model predictions with new data from the portable video game console market. We find empirical support for all theoretical predictions.

1 Introduction

The practice of mixed bundling consists of selling two or more separate products together with a discount, in addition to selling them individually. Mixed bundling is commonly used in the technology and media industries where two-sided market structures are prevalent, both in bundling hardware with software and in bundling different software products.

*Chao: College of Business, University of Louisville, Louisville, KY 40292, email: yong.chao@louisville.edu. Derdenger: Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213, email: derdenger@andrew.cmu.edu. Acknowledgments: We thank the NET Institute for financial support as well as discussions with Juan Carrillo, Jay Pil Choi, Guenter Hitsch, Vineet Kumar, Marc Rysman, Jeff Shulman, Guofu Tan and the conference participants at the 8th Annual International Industrial Organization Conference, the 2010 Summer Institute in Competitive Strategy and the 10th World Congress of the Econometric Society and the 81th Southern Economic Association Annual Meeting

Consider for instance, Sony the originator of the BluRay player and owner of a movie production studio or Nintendo, designer of both video game hardware and software. In these scenarios, Sony and Nintendo can price the hardware (BluRay player or console) and the software (DVD or video game) separately in an attempt to maximize combined profit. Alternatively, they can practice mixed bundling in which it sells two or more separate products together with a discount in addition to selling them individually. They can also sell only the bundle, a practice known as pure bundling. An additional example of mixed bundling is Apple incorporating its iLife software suite as part of its operating system with every new Mac computer as well as making it available for sale through its retail channels.

Besides an efficiency reason, two leading explanations of bundling are price discrimination and entry deterrence. For price discrimination[1], it is the heterogeneity in consumer valuations that frustrates the seller in its ability to extract consumer surplus through one price. Thus, bundling helps reduce the dispersion in valuations which increases a firm's profit. Whinston (1990) proposes other explanations for bundling——changing the market structure by exclusion, and that precommitment matters. Nalebuff (2004) advances the literature by showing that bundling can effectively deter entry even without precommitment.

Although mixed bundling has been widely studied, as is evident from the above literature, it has yet to be studied in the context of a two-sided (or multi-sided) market.[2] This is because two-sided market theory is quite recent and sparse.[3] To the best of our knowledge, only three papers——Rochet and Tirole (2008), Amelio and Jullien (2007) and Choi (2010)——have analyzed an extreme form of bundling——tying. Rochet and Tirole (2008) studies the payment card industry and illustrate how tying can make the pricing structure more balanced and raise social welfare. Amelio and Jullien (2007) consider a platform would like to set negative price on one side of the market but worries about opportunistic risk. Tying can then serve as a tool to implement an implicit subsidy without attracting undesirable customers. Choi (2010) analyzes the effect of tying on two-sided market competition with multi-homing and shows that tying can be welfare enhancing.

[1] See Stigler (1963), Adams and Yellen (1976), Schmalensee (1984), McAfee, McMillan and Whinston (1989), and Bakos and Brynjolfsson(1999).

[2] A two-sided market differs from a "traditional" one-sided market (such as those studied above) because it involves two or more end users which interact via an intermediary. Moreover, each end user's participation is determined by the participation of other types of end users. Examples of such markets are credit cards, media, yellow page phone directories, computer operating systems and video game consoles.

[3] See Caillaud and Jullien (2003), Rochet and Tirole (2006), Armstrong (2006), Carrillo and Tan (2006).

Table 1: Predicted and Observed Correlations Between Bundling and Component Prices

	Traditional Bundling/Two-Sided Literature	Our Results
Standalone Platform Price	+	−
Standalone Content Price	+	+
Content Royalty Rate	?	−

We attempt to fill the gap in the bundling and two-sided markets literatures by presenting a theoretical monopoly model of mixed bundling in the context of a two-sided (or multi-sided) market. In particular, we formulate a generalized theoretical model of mixed bundling to establish a moderator on the existing bundling theory and on the existing two-sided market literature. We then take our theoretical predictions to data on the portable video game console market, where a bundle consists of a game and console sold together for a single price. Specifically, the empirical analysis is intended to lend support to the novel theoretical predictions. We select the video game industry to empirically support our theoretical model because it is a prototypical two-sided market with consumers and game developers interacting with each other through the intermediary console. Furthermore, during a period from mid 2001 through March 2005, there existed only one portable video game console manufacturer, Nintendo. With access to a new data set that tracks sales and revenue of Nintendo's portable consoles, all available software and bundles, we are able to determine whether our theoretical model predictions are consistent with the data.

From our theoretical model we determine results that run counter to both the traditional mixed bundling and two-sided market literatures. In particular, the classical case in traditional bundling literature is that the standalone component prices should rise under bundling. *But in our model, the standalone console price **falls** with the introduction of bundling.* Table 1 presents the predicted price-bundle correlations from the standard case in the traditional bundling literature next to correlations we find from our model.

Additionally, we conclude that the lowered component price under bundling is due to the effects of cross group externalities on bundling. To be more specific, in the presence of indirect network effects, the platform has an incentive to lower its standalone price on the consumer side in order to attract more participation on the content developer side. Since the standalone price for the content is targeted at the installed base, and the installed base is locked-in, the platform will only reduce its standalone price of the platform to new consumers. Hence, a larger number of marginal new consumers are attracted to the platform. The increase in consumers consequently leads to a higher quantity of content being produced; therefore, a further increase in the demand for the platform (a consequence of the presence of cross-group externalities), which compensates the platform more for its lost profit from the lowered standalone platform price to new consumers. Such two-way indirect network effects reinforce each other and give the platform more incentive to lower prices, but profits still increase due to the increased participations on both sides.

Our theoretical results determine that the platform price levied on the other side of the market, the price content developers pay for the right to produce and sell content, declines too. In standard two sided markets literature, the optimal pricing usually involves a cross subsidy from the inelastic side to the elastic side (Rochet and Tirole (2006)), which is in the same spirit as Ramsey pricing. Under bundling, we show that prices on the consumer side are lower due to price discrimination, so according to the cross-subsidization rule, we should expect an inflated price on the content developer side. However, we see the opposite. The intuition behind this result is nonetheless quite simple and consistent with the very cross-subsidization intuition. This is due to a different price discrimination effect in the two-sided markets context——it changes the relative elasticities of two sides with respect to participation. In a two-sided market setting, the offering of a bundle enables consumers to reveal their true type. Specifically, bundling generates two forms of price discrimination. The first segments new potential customers into distinct groups, like a traditional mixed bundling case, while the second is specific to the two-sided markets setting. The second form capitalizes on the fact that by offering the bundle the firm can segment consumers into two additional independent groups, potential consumers and the installed base, and set segment specific prices, the effective content price for potential platform consumers (the difference between the bundle price and the standalone platform price) and the content price for the installed base. Consequently, with the introduction of a bundle, consumers become more inelastic with respect to their participation on the platform from the fact that bundling can target the consumers more accurately. Such a shift changes the relative elasticities between consumers and content developers' platform participations. With relatively more elastic content developers with respect to participation, the platform is required to shift its relative attention away from consumers to content developers. The platform, consequently, lowers its price to content developers in order to attract them to its platform.

We also find total surplus increases with mixed bundling. The introduction of a mixed bundle not only acts as a price discrimination tool to increase a platform's profit, but also as a method to better coordinate the participation of consumers and game developers, which aids in the solving of "the chicken or the egg" problem: "how to attract buyers without a lineup of established sellers and how to obtain the lineup of sellers without first demonstrating a group of willing buyers" (Evans 2002). This is consistent with Amelio and Jullien (2007)'s result that tying in two-sided markets increases social welfare, but their model is different from ours in several important aspects. The mechanisms through which bundling or tying works is different as well as the main themes of the papers—our paper focuses on the comparison of the pricing structure. First, they assume homogeneity among consumers for the tied good, while in our model, the consumer heterogeneity is the primary reason for the firm to bundle. The homogeneity in their model also implies that only the special form of bundling, tying, is relevant. Our relatively more general model has mixed bundling matter thanks to consumer heterogeneity. Second, the optimal price is below zero in their model, but the platform is constrained to set non-negative prices. Thus, tying offers the platform an instrument to provide an implicit subsidy and drive out the opportunistic customers. In our model, there is no such negative pricing issue since all prices are always positive, so the incentive to tie in Amelio and Jullien (2007) disappears in our model. The driving force of bundling in our model is price discrimination resulting from the consumer heterogeneity. In other words, the platform's incentive to bundle in our model is purely based upon price discrimination. The firm wants to lock-in as many consumers as possible and to perfectly price discriminate with respect to those who buy the standalone content only; and it does so with a lower standalone platform price to new consumers and a lower price to content developers. Consequently, as a by-product of this price discrimination, each of the two sides are better coordinated and social welfare is enhanced. We show unambiguously that platform participations increase on each side of the market.

After theoretically evaluating the impact mixed bundling has on prices and welfare, we determine whether the theoretical model predictions are consistent with data from the portable console market in the early to mid 2000s. We employ a reduced form approach to do so and conclude that the proposed model is consistent with the data.

The structure of this paper is as follows. First, we set up the model and describe the game. In Section 3, we present two regimes in a two-sided market structure. To assist in the identification of the impact mixed bundling has in a two-sided market structure, the first regime does not allow for mixed bundling while the second regime does. In this section we also compare prices, profits and welfare between the two regimes. Moreover, due to the

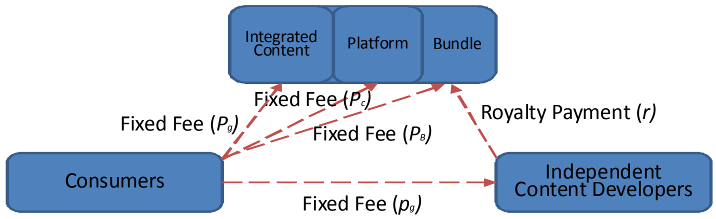


Figure 1: Two-Sided Market Structure

unobservability of the royalty rate on the game developer side in our data, we perform the analyses with the price levied to content developers as both exogenously and endogenously determined. Section 4 discusses our data and presents industry statistics. We present the results of our reduced form regression of the theoretical model in Section 5. Lastly, we conclude. All proofs are relegated to the Appendix.

2 Model Setting

There are three classes of players in the model: two types of agents and a platform. The agents are consumers and content developers. We assume interactions among all three classes of players exist and are illustrated by Figure 1.[4] In this section, we use lower-case letters to denote prices, and in the later part, lower-case letters are used specifically for the independent pricing (IP) regime, while upper-case letters are used to denote the corresponding prices under bundling.

Platform:

There is a monopoly platform that locates at the origin of a unidirectional horizontal line and produces integrated content. For simplicity, we assume the platform has only one piece of integrated content, and the marginal costs of producing its platform and content are both zero. The platform interacts with both agents by charging a fixed fee p_c to consumers for the access to its platform, a fixed fee p_g for the integrated content, and levying a per unit royalty rate r to independent content developers for the right to produce and sell content compatible with the platform. r takes the form of a fixed fee that

[4] See Kaiser (2002), Caillaud and Jullien (2003), Rochet and Tirole (2006), Rysman (2004), Kaiser and Wright (2006), Armstrong (2006), Hagiu (2006) and for general literature on two-sided platform markets.

a platform receives per independent software unit sold, where a unit is not a game title but a copy of a title. More explicitly, the royalty rate is not a rate of revenue or a rate of profit but a rate of sale, or put differently, a simple fixed dollar amount per game sold (e.g. \$8). Likewise, consumers and content developers interact with consumers purchasing content from developers at their corresponding prices.

Consumers:

We implement a modified Hotelling model to analyze the consumers' decisions. There are two groups of consumers ($i = 1, 2$) with total size normalized to one. Group 1, identified as the installed base (with fraction α) locating at the origin, is a pre-existing group who already has purchased access to the platform but has yet to purchase the integrated content. There are several realistic reasons why there might be a set of consumers who have yet to purchase the integrated content but own access to the platform: a) the integrated content was not yet available when the consumers bought their platform or b) some fraction of the installed base did not have enough information about this content to decide if they should purchase it.[5] Regardless of the reason why a set of consumers chooses to own a platform and not the integrated content, the mere fact that there exists such a set is important.

The gross utility a consumer from Group 1 garners from purchasing the integrated content is $u_{installed} = 1$. And Group 2, a continuum of new potential consumers with fraction $1 - \alpha$ population, is uniformly located on a horizontal line and has yet to purchase access to the platform. The utility a Group 2 consumer receives from purchasing access to the platform is dependent upon the quantity of content d provided by the content developers, and the transportation cost equal to $t \cdot x$. Here, t is the transportation cost per unit of length and x is the consumer's distance from the origin. The marginal utility of the content is β ($\beta > 0$). To be more specific, the gross utility associated with a new consumer situated at point x who elects to purchase access to platform only is $(1 - tx) \cdot 1\{\beta d > 0\} + \beta \cdot d$[6], while $1 - tx + v_g + \beta \cdot d$ if he purchases both the platform access and the integrated content, where $1\{\cdot\}$ is the indicator function and 1 and v_g are the new consumers intrinsic values for the platform and integrated content, respectively. For simplicity, we assume that v_g is drawn from the uniform distribution $U(\cdot)$ on $[0, 1]$, which is always weakly less than that of Group 1 users. This higher willingness to pay for the installed base group is to capture the fact that early adopters are more likely to be "locked-in" than

[5] We thank an anonymous reviewer for suggesting this interpretation.

[6] When $\beta d = 0$, the platform itself won't provide any utility to the consumer unless purchased with the integrated content.

new consumers, or they have more experience with the platform, and thus are more likely to purchase the integrated content than those who have never experienced the platform before. At the same time, the homogeneity of the installed base group's valuation on the integrated content is innocuous. As will be discussed later after presenting our main theoretical results, introducing heterogeneity among the installed base will not change our main results, as long as the average willingness to pay from the installed base is higher than that of the new consumers, which is reasonable as explained above. Lastly, note that new users are heterogeneous in two dimensions: in their location x; and in their valuation for the integrated content v_g.[7]

Content Developers:

We assume the platform is essential for consumers to enjoy content. In the case of content developers, they also must join the platform in order for their content to be compatible with the platform. For simplicity, we assume the quantity of content is given by $d(r) \equiv 1 - r$, where r is the per unit royalty rate. Equivalently, we can consider $r = 1 - d$ as the inverse demand curve from content developers, which indicates their willingness to pay to join the platform.[8]

Thus, the royalty revenue the platform receives is $r \cdot (1-r) \cdot [\alpha + (1-\alpha) q_{new}]$, where q_{new} is the number of new consumers who newly purchase the access to the platform, and α is the aggregate number of consumers who have previously purchased access to the platform or what we denote as the installed base. This equation therefore implies that as more consumers join the platform, higher royalty revenue will be received and this is denoted throughout the economic literature as an indirect network effect.

Timing of the Game:

The timing of the game is as follows. First, the monopoly platform chooses either to bundle or not and then sets prices accordingly. Next, after observing the price offers

[7] Empirically, one might construe the new users' valuations for platform and content should be positively correlated, rather than indepedent as assumed here. As we will discuss in Appendix, positively correlated preferences won't change our main results. So we choose the independent preference for the simplicity of illustration.

[8] Here is a detailed microstructure on d. If we assume there is free entry into the market for content and that content developers are heterogeneous in their willingness to pay to reach a consumer then each developer's willingness to pay per consumer can be summarized by θ. For simplicity, we assume θ is i.i.d. according to a uniform distribution $U(\cdot)$ on $[0, 1]$. The total number of potential content developers is therefore normalized to one. With the assumption of free entry into the developer segment, developers do not set content prices. Instead, they decide whether to enter the market and join the platform. Consequently, a type θ developer will create and produce content for platform if and only if $\theta \geq r$. Then the total amount of content available on the platform is also $d = \Pr(\theta \geq r) = 1 - r$.

from the platform, consumers and the developers make their purchase decisions and content quantity supply decisions, respectively. Rational expectations are assumed for the simultaneous equilibrium outcome.[9]

3 Equilibrium Analysis

We begin by looking at a two-sided market model that omits the practice of mixed bundling and then modify the model to allow for its practice. After the introduction and characterization of the equilibrium of both models, we compare the two regimes to determine the effects of mixed bundling on prices and welfare.

As indicated in Section 2, the quantity of independent content is determined as $d = 1-r$. The presence of this independent content has two implications: first, the integrated content is not essential to new potential consumers, since they can enjoy the platform with other independent content; second, indirect network effects are present in this setting, because the royalty revenue from the content developer side depends on the number of total platform owners $\alpha + (1-\alpha)q_{new}$, and the number of total platform owners hinges on the quantity of content too. As a result, the market structure is two-sided.

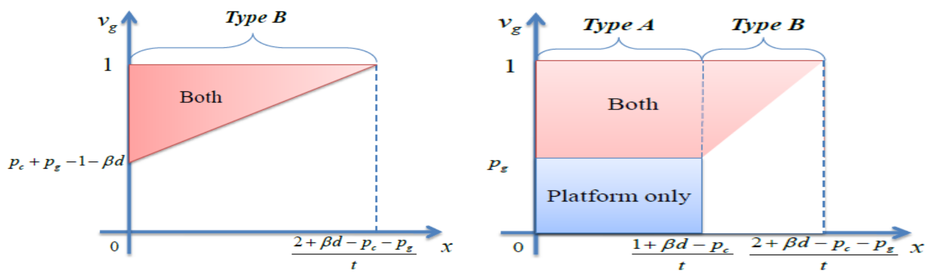
3.1 Independent Pricing (IP) Equilibrium

The IP equilibrium consists of the monopoly platform setting prices (p_c, p_g). The two groups of consumers' decisions are as follows.

For the installed base, they will purchase the integrated content from the platform if and only if $u_{installed} - p_g \geq 0$. Hence, each individual's demand for the content is $n_{installed} = 1\{1 \geq p_g\}$. Aggregating across the installed base yields an aggregate demand of $q_{installed} = \alpha \cdot n_{installed} = \alpha \cdot 1\{1 \geq p_g\}$. Since 1 is the upper bound of all consumers' valuation for the integrated content, we have $1 \geq p_g$ in equilibrium. Thus, $q_{installed} = \alpha$.

The equilibrium number of new platform owners is more challenging to derive given that consumers can either solely purchase access to the platform, purchase access the platform and the integrated content in conjunction or elect to not purchase either platform or content. We thus classify new consumers based upon their different valuations into two

[9] We would also like to note that the model described above presents a nice first approximation to the video game market which is inherently dynamic in nature, given that bundles are formed after a game has been identified as a "hit" and that release decisions are made by software developers and not by the console producer.



(i) $2 - p_g > p_c - \beta d \geq 1$ (ii) $1 > p_c - \beta d$

Figure 2: New Consumer Demand under IP

different types. The first, Type A, values the platform enough on its own to purchase, given the availability of independent content. That is, the consumer's utility from consumption of the platform solely is greater than zero, or $1 + \beta \cdot d - tx - p_c \geq 0$. Hence, these Type A new consumers will buy both the platform and integrated content if $v_g \geq p_g$ and only the platform if $v_g < p_g$. The second, Type B, consumers do not value the platform enough on its own to purchase it, given the availability of independent content. That is, $1+\beta \cdot d - tx - p_c < 0$. Thus, these consumers only purchase access to the platform with the integrated content if the integrated content is valuable enough. In this case the integrated content is a complementary product which makes the console more attractive, although it is not essential. They, therefore, will buy both the platform and the integrated content if $(1+\beta \cdot d - tx - p_c) + (v_g - p_g) \geq 0$; and buy nothing otherwise. There are two possibilities: (i) when $2 - p_g > p_c - \beta d \geq 1$; and (ii) when $p_c - \beta d < 1$. The new consumers' demand is in the figure below.[10]

The aggregate demand for the platform only as well as the demand for both a platform

[10]Type A ($x \leq \max\{0, \frac{1-p_c+\beta d}{t}\}$): Given the availability of independent content, platform itself is attractive enough to justify purchase.

Type B ($x > \max\{0, \frac{1-p_c+\beta d}{t}\}$): given the availability of independent content, platform itself is not attractive enough to justify the purchase.

and the integrated content are

$$
\begin{aligned}
q_{platform-only} &= \begin{cases} 0 & \text{(i)} \\ \frac{p_g(1+\beta d-p_c)}{t} & \text{(ii)} \end{cases} \\
q_{both} &= \begin{cases} \frac{(1+\beta d-p_c+1-p_g)^2}{2t} & \text{(i)} \\ \frac{(1-p_g)(1+\beta d-p_c)+\frac{(1-p_g)^2}{2}}{t} & \text{(ii)} \end{cases},
\end{aligned}
$$

respectively. Thus, the total number of new consumers joining the platform is

$$
\begin{aligned}
q_{new} &= q_{platform-only} + q_{both} \\
&= \begin{cases} \frac{(1+\beta d-p_c+1-p_g)^2}{2t} & \text{(i)} \\ \frac{1+\beta d-p_c+\frac{(1-p_g)^2}{2}}{t} & \text{(ii)} \end{cases}
\end{aligned}
$$

Lemma 1 *Case (i) cannot be the equilibrium.*

So we focus on case (ii): $1 > p_c - \beta d$.

With equilibrium demand for the platform and integrated content as well as the quantity of independent content developers determined, in terms of platform price p_c, integrated content price p_g and royalty rate r, the platform manufacturer maximizes its profit with respect to these strategic variables. The corresponding platform profit under independent pricing becomes a multiproduct monopoly problem with a network externality.

$$
\begin{aligned}
\pi^{IP}(p_c, p_g, r) &= \alpha \cdot p_g \cdot 1 + (1-\alpha) \cdot [p_c \cdot q_{new}(p_c, p_g, r) \\
&\quad + p_g \cdot q_{both}(p_c, p_g, r)] + r \cdot (1-r) \cdot [\alpha + (1-\alpha) \cdot q_{new}(p_c, p_g, r)] \\
&= \alpha \cdot p_g + (1-\alpha) \cdot [(p_c + p_g) \cdot \frac{1 - p_c + \frac{(1-p_g)^2}{2}}{t} - p_g^2 \cdot \frac{1-p_c}{t}] \\
&\quad + r \cdot (1-r) \cdot [\alpha + (1-\alpha) \cdot \frac{1 - p_c + \frac{(1-p_g)^2}{2}}{t}] \\
&\quad + \beta \cdot (1-\alpha) \cdot \frac{p_c + p_g(1-p_g) + r \cdot (1-r)}{t} \cdot (1-r). \qquad (1)
\end{aligned}
$$

Denote $B(p_c, p_g, r) \equiv p_c \cdot q_{new}(p_c, p_g, r) + p_g \cdot q_{both}(p_c, p_g, r) + r \cdot (1-r) \cdot \frac{\alpha+(1-\alpha)\cdot q_{new}(p_c,p_g,r)}{1-\alpha}$ as the per capita profit from new users and independent game developers. Then the platform's profit can be rewritten as

$$\pi^{IP}(p_c, p_g, r) = \alpha \cdot p_g + (1-\alpha) \cdot B(p_c, p_g, r).$$

Since the royalty rates are unobservable in our data, we analyze two possible cases: (1) when the royalty rate is exogenously determined, and (2) when it is endogenously determined.

3.2 Bundling Equilibrium

Our mixed bundling model differs slightly from the above IP model in that new consumers now possess the option of purchasing access to the platform and the integrated content bundled together. Consumers still retain the option of purchasing content and access to the platform separately. Like the above IP model, the platform interacts with both agents by charging a fixed fee P_c to consumers for access to the platform and levying a per unit royalty rate, R, to the independent content developer for the right to produce and sell content compatible with the platform. Consumers and content developer still interact, with consumers purchasing content from the developer. Consumers can purchase the integrated content separately for a fixed fee, P_g. Yet, in the bundling model, the platform also sells its content and access to its platform together at price $P_{B.}$ Prices are thus $\{P_c, P_g, P_B, R\}$.

To begin our equilibrium analysis, first note that in order for the bundle to be effective, we must have $P_c + P_g > P_B$. Hence, if the new consumers elect to purchase the integrated content they will do so via the bundle. And, they will never solely purchase the integrated content at P_g since this content provides zero utility without access to the platform. P_g is thus specifically targeted to the installed base of users who already have access to the platform but have not purchased the integrated content. Hence, it is easy to see that the price of the integrated content is set to $P_g = 1$, since P_g is directed to the installed base. The resulting demand for the content from the installed base is $Q_g = \alpha$.

Note that if we remove the installed base from our model, then there is no need to bundle——for new consumers only two prices (P_c, P_B) matter since they either buy access to the platform only, or buy both the integrated content and access to the platform. In other words, it is the presence of installed base or heterogeneity among consumers making the bundle necessary.

Under bundling, new consumers determine their purchase decisions on two strategic variables, the price of the platform and the effective price of the integrated content $P_g^e \equiv$

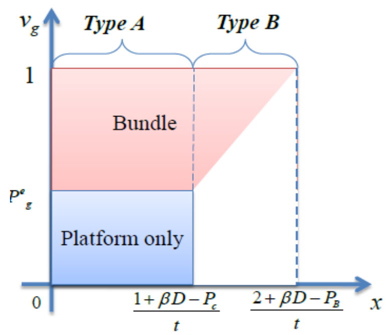


Figure 3: New Consumer Demand under Mixed Bundling

$P_B - P_c$. Our analysis regarding new consumer demand for the platform and the purchase of both the platform and integrated content (the bundle) takes the same structure as the IP equilibrium if we replace p_g there with the effective price P_g^e. Likewise, we have two possible cases: (i') $2 - P_g^e > P_c - \beta D \geq 1$ and (ii') $1 > P_c - \beta D$. Direct computation can eliminate (i') and we can focus on (ii') below.

The standalone demand for the platform and the bundled demand are

$$\begin{aligned} Q_{platform-only} &= \frac{P_g^e(1+\beta D - P_c)}{t} \\ Q_B &= \frac{(1-P_g^e)(1+\beta D - P_c) + \frac{(1-P_g^e)^2}{2}}{t}, \end{aligned}$$

respectively. Thus, the total number of new consumers joining the platform is

$$\begin{aligned} Q_{new} &= Q_{platform-only} + Q_B \\ &= \frac{1+\beta D - P_c + \frac{(1-P_g^e)^2}{2}}{t}. \end{aligned}$$

Given the demand for the platform generated content from the installed base, the demand from new consumers for only the platform and the demand for the bundle, and the quantity of independent content, the monopoly platform's profit under bundling is

$$
\begin{aligned}
\Pi^B(P_c, P_g^e, R) &= \alpha \cdot 1 + (1-\alpha) \cdot [P_c \cdot Q_{new}(P_c, P_g^e, R) \\
&\quad + P_g^e \cdot Q_B(P_c, P_g^e, R)] + R \cdot (1-R) \cdot [\alpha + (1-\alpha) \cdot Q_{new}(P_c, P_g^e, R)] \\
&= \alpha + (1-\alpha) \cdot B(P_c, P_g^e, R) \\
&= \alpha \cdot P_g^e + (1-\alpha) \cdot B(P_c, P_g^e, R) + \alpha \cdot (1 - P_g^e) \\
&= \pi^{IP}(P_c, P_g^e, R) + \alpha \cdot (1 - P_g^e).
\end{aligned}
$$

Notice that the structure of this profit function is identical to the IP model, except that p_g is replaced by P_g^e and now the platform has one more degree of freedom by setting $P_g = 1$ for the installed base. Thus, compared with the platform's profit under IP, the only extra term is the surplus gains extracted from the installed base, that is, $\alpha \cdot (1 - P_g^e)$. Consequently, we determine that bundling is a dominant strategy for the monopoly platform since offering P_B and P_g simultaneously is equivalent to offer P_g^e and P_g to new consumers and the installed base separately while retaining $P_c = P_B - P_g^e$ as the platform price. Offering a bundle, therefore, provides the monopoly platform an additional instrument to extract consumer surplus.

Lemma 2 ***(Mixed Bundling is Profitable)*** *Whenever bundling is possible, mixed bundling is a dominant strategy over no bundling or pure bundling.*

The above lemma is consistent with the existing literature on mixed bundling in traditional one-sided market in that mixed bundling is the optimal strategy for the monopolist. Next, we perform the analyses for both cases in which the royalty rate is exogenously or endogenously determined.

3.3 Prices, Profits and Welfare Comparison

In this subsection, we compare the equilibria of the two regimes—–IP vs Bundling. Interestingly, we find the bundling pricing structure when the royalty rate is exogenously given to differ from when the royalty rate is endogenously determined. Moreover, with an endogenous royalty rate, the pricing structure differs from that of traditional bundling.

3.3.1 When Royalty Rate $\overline{R}$ is Exogenously Determined

Proposition 3 (One-Sided Pricing in Two-Sided Markets) *In two-sided markets, when royalty rate $\overline{R}$ is exogenously determined, under mixed bundling, the pricing structure is the same as the standard bundling pricing in traditional one-sided market: the standalone prices for the access to the platform and the integrated content are higher than their corresponding prices under IP, while the bundle price is lower than the sum price of platform and integrated content under IP. Specifically,*

$$\begin{aligned} P_c^{*\overline{R}} &> p_c^{*\overline{R}} \\ P_g^{*\overline{R}} &= 1 > p_g^{*\overline{R}} > P_g^{e*\overline{R}} = P_B^{*\overline{R}} - P_c^{*\overline{R}} \\ p_c^{*\overline{R}} + p_g^{*\overline{R}} &> P_B^{*\overline{R}}. \end{aligned}$$

We determine from the above proposition that by offering the bundled option it allows the monopolist to increase the standalone price of the integrated content. The above price structure allows consumers to sort into distinct groups and consequently reveal their true preferences. The mixed bundling option thus acts as a price discrimination tool and allows the monopolist to raise standalone prices in search of more efficient and complete extraction of consumer surplus. It is true that when the platform faces a two-sided market and can set prices on both sides, it should employ two-sided pricing. The above one-sided pricing in a two-sided market serves as an intermediate step illustrating the effects of a bundle and two-sided pricing. As shown above, when pricing is one-sided, the bundling pricing structure is the same as the traditional one——both component prices go up while the bundle price goes down, even though the market is two-sided here. By contrast, as we will see next, the pricing structure is different from the traditional structure when the platform performs two-sided pricing. Consequently, this intermediate "one-sided pricing in two-sided markets" step clearly identifies the role of how the two-sidedness impacts the pricing structure.

Lastly, we show that total surplus increases under mixed bundling.

Proposition 4 ***(Total Surplus)*** *When royalty rate $\overline{R}$ is exogenously determined, total surplus under bundling is higher than IP.*

3.3.2 When Royalty Rate R is Endogenously Determined

Proposition 5 (Two-Sided Pricing in Two-Sided Markets) *When royalty rate R is endogenously determined, under mixed bundling, all prices except the standalone price of the integrated content are lower than those under IP. Specifically,*

$$\begin{aligned} r^* &> R^* \\ p_c^* &> P_c^* \\ P_g^* &= 1 > p_g^* > P_g^{e*} = P_B^* - P_c^* \\ p_c^* + p_g^* &> P_B^*. \end{aligned}$$

This is quite a surprising result. Both the standalone platform price and the royalty rate are lower under the mixed bundling equilibrium than their respective counterparts in the IP equilibrium. As stated by Rochet and Tirole (2006), "the price to side i is inversely related to that side's elasticity of demand". In two-sided markets, the optimal pricing scheme is to subsidize the more elastic side of the market and extract rents from the other, more inelastic, side. Or more generally, the optimal price structure is to adjust prices downward by the external benefit a platform receives from attracting an additional side i user. When the platform maker uses mixed bundling they are in affect offering a "subsidy" to consumers which increases demand for its platform by attracting a greater number of marginal consumers. We might expect that by subsidizing consumers, via mixed bundling in our case, the platform maker is increasing the content developer's willingness to participate and thus the ability to raise the royalty rate in which it levies. Yet, this is not what we encounter. We find that the royalty rate is in fact *lower* under the mixed bundling equilibrium. By offering the mixed bundle, the platform becomes more effective in extracting consumer surplus, compared to the IP case. Consequently, by offering the mixed bundle, the consumer side becomes less elastic to platform pricing since it can more efficiently extract consumer surplus without deterring consumer participation. The content developer side therefore becomes relatively more elastic, which creates an incentive for the platform to lower R under mixed bundling.[11]

[11] To be more specific, the price elasticity of demand from content developer side $|\varepsilon_D| = \left|d'(r) \cdot \frac{r}{d(r)}\right|$, which is fixed for a given r. The demand for new consumers $q_{new} = \frac{1+\beta d}{t} - \frac{p_c-(1-p_g)^2}{t}$ depends on both p_c and p_g. So we can define an effective price index for new consumers as $PI \equiv \frac{p_c-(1-p_g)^2}{t}$. Then $q_{new} = \frac{1+\beta d}{t} - PI$. The price elasticity of demand from new consumers $|\varepsilon_N| = \left|\frac{dq_{new}}{dPI} \cdot \frac{PI}{q_{new}}\right| = \frac{1}{\frac{1+\beta d}{t \cdot PI}-1}$ is

There also is an additional argument for the lowering of the royalty rate. We know that the platform would like to increase participation on the side it can more efficiently extract surplus from, since doing so will increase profits. Given that nonlinear pricing is only available to the consumer side, the platform is able to more effectively extract rents from consumers. Given this, the platform has an incentive to increase demand for its platform. How does the console accomplish this? It does so by reducing the content developers' royalty rate R. A reduction in the royalty rate will lead to an increase in content development and attract more consumers through the indirect network, which will consequently lead to higher quantity of the content through the indirect network effect resulting in each of these network effects to reinforce the other.

In addition to a decrease in royalty rate, we also find the standalone platform price is less under a mixed bundling regime. This is in stark contrast with the pricing pattern in the traditional bundling literature or our one-sided pricing in the two-sided markets. When $\overline{R}$ is exogenously given, the pricing structure is parallel to the traditional bundling pricing structure that standalone prices go up while the bundle price goes down, compared with IP case. Nevertheless, in our two-sided pricing in two-sided markets case, when R is endogenously determined, the standalone platform price *falls*, too.

This smaller standalone platform price results from two factors. First, it is a consequence of the mixed bundle segmenting the market into new consumers and the installed base. Under the mixed bundle regime, the standalone integrated content price is specifically targeted to the installed base as oppose to a uniform price under the IP equilibrium. Since the installed base's value of the content is known to all, the platform is able to perfectly price discriminate and set price equal to 1, which is greater than p_g. As a result, the additional profit the platform receives from selling its integrated content and the payment of royalty rates from independent developers is larger under a mixed bundling equilibrium leading to a larger discount of the standalone platform price and hence a smaller price.[12] The second source is the cross-market strategic interactions related to multiproduct pricing. When $\overline{R}$ is exogenously given, only two prices for the new consumers are set. Since

increasing in PI. For any given r, it is shown that PI under IP is larger than PI under bundling (please see the last part of the proof of Proposition 3 in Appendix). Therefore, if we fix r and switch from IP to bundling, then $|\varepsilon_D|$ won't change while $|\varepsilon_N|$ becomes smaller. Relatively, the content developer side becomes more elastic under bundling, which leads the platform to lower the royalty rate under bundling.

[12] Remember that the presence of installed base makes bundling have bite. If we eliminate the installed base from our model, then there won't be any bundling, and hence no such price decline.

the platform and the integrated content are strategic substitutes[13] ($\frac{\partial^2 \pi}{\partial p_c \partial p_g} < 0$), their price changes in opposite direction. When R is endogenously determined, the problem involves one more price to be set—–the royalty rate R. As can be checked from their pairwise cross-derivative, the platform, the integrated content, and the independent content, are all pairwise complements. However, *overall* the platform and the integrated content become strategic complements ($-(\frac{\partial^2 \pi}{\partial p_c \partial p_g}\frac{\partial^2 \pi}{\partial r^2} - \frac{\partial^2 \pi}{\partial p_c \partial r}\frac{\partial^2 \pi}{\partial p_g \partial r}) > 0$). The independent content market here becomes important in the overall effect because the demands are interdependent, but this effect is missing in Bulow et al. (1985) since two markets are independent in their model. Consequently, although the lower integrated content price gives an incentive to increase the standalone platform price as when $\overline{R}$ is exogenously given, the lower royalty rate offers an offsetting power to lower the standalone platform price. And as a result the latter effect dominates the former.

After determining that all prices are lower, with the exception of the standalone integrated content price, we find that new consumers are strictly better off. Yet, the installed base of consumers is strictly worse off, which is a consequence of the installed base being locked-in to the platform and the ability of the platform to segment the market and target the installed base with a segment specific content price that extracts all surplus from them under mixed bundling. This extraction, however, does not cause total surplus to change since it is a transfer from consumers to the platform. Moreover, from Lemma 2, we know that the platform's profits are strictly higher under mixed bundling. We, thus, have the following proposition regarding the comparison of total surplus between regimes.

Proposition 6 *When royalty rate R is endogenously determined, total surplus is higher under bundling than under IP.*

From our theoretical analyses, we show the effects of mixed bundling on prices, surplus and demand for the platform to differ substantially under two different market structures. While the motivations behind the act of offering a bundle are consistent across structures (price discrimination), mixed bundling under a two-sided market structure leads to a very different and unique outcome. We determine that, unlike the single-sided case, all prices with the exception of the standalone platform created content price are lower, and total surplus in a two-sided market structure is definitively larger than the welfare under an IP regime. When a platform is able to optimally set its royalty rate and offer a mixed bundle,

[13] For strategic substitutes and complements, please see Bulow et al. (1985) .

the firm's response is not to increase the standalone platform price but is to lower both the royalty rate and standalone platform price. The decreases in marginal revenues from the decline in platform price and content developer royalty rate are more than overcome by the increases in consumer demand and game developer's quantity supply.

3.4 Discussions

We certainly recognize that any implications from our proposed model may be limited to due to the imposed assumptions and from the fact that they correspond to a particular set of functional forms. Below we provide a discussion on a few of the most disconcerting assumptions and how they may impact the results and implications of our model.

Homogeneity of the Installed Base's Valuation on the Integrated Content

As can be seen from the relationship between platform's profits under IP and bundling—–$\Pi^B(P_c, P_g^e, R) = \pi^{IP}(P_c, P_g^e, R) + \alpha \cdot (1 - P_g^e)$, the platform's incentive to bundle is rooted in the existence of the installed base whose valuation for the integrated content is higher than new consumers. In other words, the incentive to bundle is from the heterogeneity *across* the installed base and new consumer groups, not *within* group.[14] Therefore, our main results do not change even if we allow for heterogeneity among the installed base group or assume only a fraction of the installed base may purchase the integrated content. With or without heterogeneity within the installed base, as long as the average willingness to pay from the installed base is higher than that of new consumers, the platform will have an incentive to bundle. Moreover, our price comparison results do not change. This is because the comparative statics analysis also relies on the presence of the installed base instead of on how their values differ within each setup. Thus our predictions on the direction of price changes will be the same, albeit the magnitude of changes may vary accordingly.[15]

Role of β

[14] Due to the homogeneity of the installed base, there is no price discrimination issue within them. Moreover, there is no way to further price discriminate within the installed base, because the quantity demanded for the integrated content would be either 1 or 0, and the integrated content is the same to all users too.

[15] The driving force for bundling in our paper is the heterogeneity across the installed base and the new consumers. And it hinges on the fact that the former group's average willingness to pay from is higher than the latter's. If the average willingness to pay from the installed base is low, then our results may change. Since under bundling, P_g is specifically targeted to the installed base, while P_c and P_B (or P_g^e) are for the new consumers. In order for bundling to be effective, we need $P_g + P_c > P_B = P_c + P_g^e$, which is equivalent to $P_g > P_g^e$. When the average willingness to pay from the installed base is low enough, which requires P_g to be very low, $P_g > P_g^e$ will become binding and this restriction will force the bundling to be degenerated to the IP case.

In the extreme case when $\beta = 0$, consumers do not receive any utility from independent content, and so the platform purchase decision is independent of quantity of independent content. Consequently, the indirect network effect disappears and the market is reduced to a one-sided market. Moreover, because there is only one piece of content available on the platform, the platform and its integrated content becomes perfect complements with fixed proportion. Every new consumers must buy both in order to make the platform useful. Hence, a pure bundle would suffice for them, and what matters for them is the *sum* of the platform price and the integrated content price. As for the installed base, they only buy the integrated content since they already have access to the platform; the standalone price is all they care about. Therefore, only two prices are enough to segment the installed base and the new consumers. Consequently, in this one-sided market, the Chicago School's "single-monopoly-profit theorem" is restored, and mixed bundling becomes redundant in this extreme case.[16]

Once $\beta > 0$, the market structure switches to a two-sided market. Although the platform and the integrated content remain complements, they no longer are *perfect* complements with fixed proportion——new consumers could buy access to the platform without purchasing the integrated content, for independent content can be consumed with the platform. Consequently, the availability of substitutes to the integrated content ($\beta > 0$) dramatically changes the market structure and invalidates the "single-monopoly-profit theorem".

Furthermore, from the proof of Proposition 5, the comparative statics results are independent of β, so long as $\beta > 0$. This indicates our results in Proposition 5 are robust when the synergies across two sides vary. To be more specific, the drop in royalty rate comes from the extra royalty revenue from the expansion on the consumer side, which changes the relative elasticities of two sides. This feedback effect from consumer side exists as long as $\beta > 0$, no matter how large or small β is. With respect to the fall in standalone platform price, there are two main reasons as pointed out before: (1) the additional profits from better targeting consumers and coordination of two sides give the platform more leeway to entice new consumers through lower platform price; (2) the additional pricing instrument——royalty rate r——transforms the relationship between p_c and p_g from strategic substitutes to strategic complements. While both effects become small when β is small, their signs never change. More importantly, the effect (2) has a component independent of β, which

[16] In an earlier version of this paper, we have a detailed analysis on this extreme case. It is available upon request.

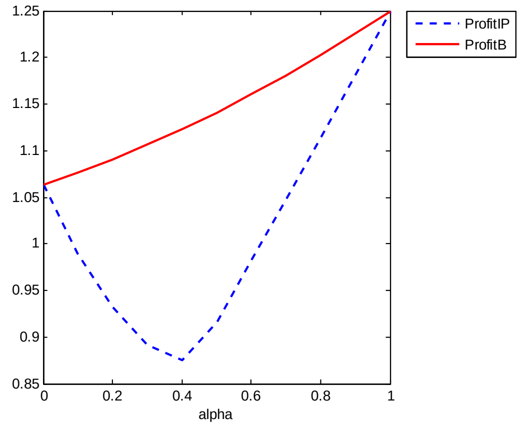


Figure 4: Platform Profit Under IP and Bundling Equilibrium for t=1,$\boldsymbol{\beta}$=0.5

comes from the term $r \cdot (1-r) \cdot [\alpha + (1-\alpha) \cdot \frac{1-p_c+\frac{(1-p_g)^2}{2}}{t}]$ in profit equation (1). Starting from the extreme case of $\beta = 0$, where prices do not change before or after bundling as explained above, prices will move in the direction as predicted in Proposition 5, no matter how tiny β is, so long as $\beta > 0$. This interesting result is rooted in our specific modelling of how royalty revenue kicks in with $\beta > 0$.

Role of α

We have emphasized above that all our results hinge on the existence of the installed base group, whose valuation for the integrated content is higher than that of new consumers. It is worthwhile to look at how the fraction of the installed base affects the platform's incentive to bundle. Figure 4 shows platform's profit under IP and Bundling equilibrium for a set of parameter values, as α varies from 0 to 1.

In the two extreme cases when $\alpha = 0$ or $\alpha = 1$, we either have no installed base or all consumers are installed base. The former, only two prices p_c and p_g are enough to segment the new consumers; while the latter, only one price p_g is enough for the installed base. Therefore, there is no need to bundle in these two extreme cases. Yet, when $\alpha \in (0, 1)$, these two prices are not sufficient to segment the new consumers and target installed base at the same time. Thus, bundling has some bite, as it introduces an extra pricing tool to further segment the market. This confirms the key role of the existence of the installed base for bundling adoption. Note that the profit gain from bundling compared with IP ($\pi_B - \pi_{IP}$) reaches maximum for a certain intermediate value of α. This is because switching from IP to bundling, the major gains include more profit extracted from the installed base via higher integrated content price and better segmentation of consumers, which encourages more participations on both sides. Since the consumer market size is fixed, as α increases, less new consumers means less demand for the platform access. Although the platform still gains from better price discrimination and coordination through bundling, the profit gain from bundling may shrink due to the fact that the market for the platform access becomes saturated as the fraction of new consumers falls. As a result, even with of our model being static, this simple comparative statics analysis provides a dynamic guideline on how much mixed bundle profits increase over IP as the installed base evolves.

This result also aids us in our empirical analysis below. The fact that a firm's decision to offer a bundle is endogenous does complicate our empirical analysis. However, to overcome this complication we leverage the fact that bundling becomes more irrelevant as the installed base or number of new consumers purchasing the console tends toward zero. We, thus, employ lagged measures of the installed base as instruments for when a platform should offer a bundle to control for the bias associated with the endogenous bundling decision in our treatment regressions.

4 Hypotheses and Data

In this section, we provide empirical support for the above theoretical predictions with data from the portable video game console market. Our model above generates three distinct pricing results between two regimes——mixed bundling and independent pricing and one with regard to the impact bundling has on the number of games. They are:

$$H_1 : P_c^* < p_c^*, \quad H_2 : P_g^* > p_g^*, \quad H_3 : R^* < r^*,$$

where H_1 accordingly predicts the standalone console price under a mixed bundling regime is smaller than its price under independent pricing, H_2 states that the standalone component price of the software bundled with the console under a mixed bundling regime is larger than its price associated with an independent pricing model and H_3 states the royalty rate levied on independent game developers is smaller with mixed bundling than without[17]

If all of the above results are present in the data, we by no means should interpret our proposed theoretical model as the only correct model. Instead, the empirical evidence should lead the reader to interpret our theoretical model of mixed bundling in two-sided markets as being consistent with the data.

The data used in this study originates from NPD Funworld. Data from the marketing group NPD Funworld tracks sales and pricing for the video game industry and is collected using point-of-sale scanners linked to over 65% of the consumer electronics retail stores in the United States. NPD extrapolates the data to project sales for the entire country. Included in the data are quantity sold and total revenue for two consoles and three bundles and all of their compatible video games, roughly 700. The data set covers 45 months starting in June 2001 and continues through February 2005, during which Nintendo was a monopolist in the portable video game market and before Sony's PlayStation Portable entered the market.

During the early 2000s through February 2005, Nintendo was a monopolist in the production of portable video game consoles. Specifically, it was a multi-product monopolist producing two versions of its very popular Game Boy Advance (GBA) console as well as a portfolio of games to be played on its console. Each version was internally identical, but the second version dubbed the GBA SP was reoriented with the display lying horizontally rather than vertically. The GBA SP looked like a mini laptop computer and was close to half the size of the original GBA. Moreover, it is usually the case with the introduction of a new device that new games are released which are not backwards compatible, yet with the introduction of the GBA SP, this was not the case since the internal parts of both devices were identical. Consequently, both devices shared the same set of games. The target market of these two devices was toward younger kids rather than teenagers or young adults, which was the targeted demographic for the home console. The portable console market most drastically differs from the traditional home video game console market in that it is extremely portable with the size of the device being no larger than an adult hand. It can easily travel with a consumer and be played in a car or airplane, while a home console is restricted to a location that has a television and electricity.

General statistics of the portable video game industry are provided in the Tables 2 and 3.

[17] We include in the appendix empirical support for the number of games under mixed bundling is larger than without. We move this support to the appendix given our focus on pricing.

Table 2: Portable Console Market Statistics

	Release Date	Units	Months on Console Market
Nintendo			
GBA	June 2001	12,821,233	45
GBA SP	March 2003	13,070,720	24
GBA w/ Mario Kart	November 2001	215,394	29
GBA w/ Mario Advance 2	November 2002	199,225	17
GBA SP w/ Mario Advance 4	November 2003	149,065	4

Table 3: Portable Console and Bundle Prices

	Average Price	Max Price	Min Price	Independent Games Sold
Nintendo				
GBA	$72.00	$94.46	$52.37	
GBA SP	$93.73	$100.30	$70.60	
GBA w/ Mario Kart	$86.17	$150.54	$61.50	2,027,636
GBA w/ Mario Advance 2	$67.33	$71.73	$56.60	2,438,732
GBA SP w/ Mario Advance 4	$97.62	$99.85	$94.92	1,673,304

In Tables 2 and 3 we present statistics regarding the release date, total units sold and the number of months on the console market, average (min and max) prices and total standalone units sold of the bundle games for the two standalone consoles and three bundles. From these tables it is evident that Nintendo elected to release its bundles during the holiday time period but continue to sell such bundles well into the following year(s)—–the first being a GBA device bundled with the hit game Mario Kart in November 2001. Additionally, all bundled games were high quality hit video games with each selling over one and half million standalone units.[18]

Figure 5 illustrates the sales of consoles and bundles over time. The video game industry exhibits a large degree of seasonality in console sales with significant increases in the months of November and December. Therefore, account for the large degree of seasonality in our empirical models is important. Figure 5 also illustrates that sales of pure hardware

[18] This fact provides some support for the lack of modeling independent content price setting behavior in the theoretical model above.

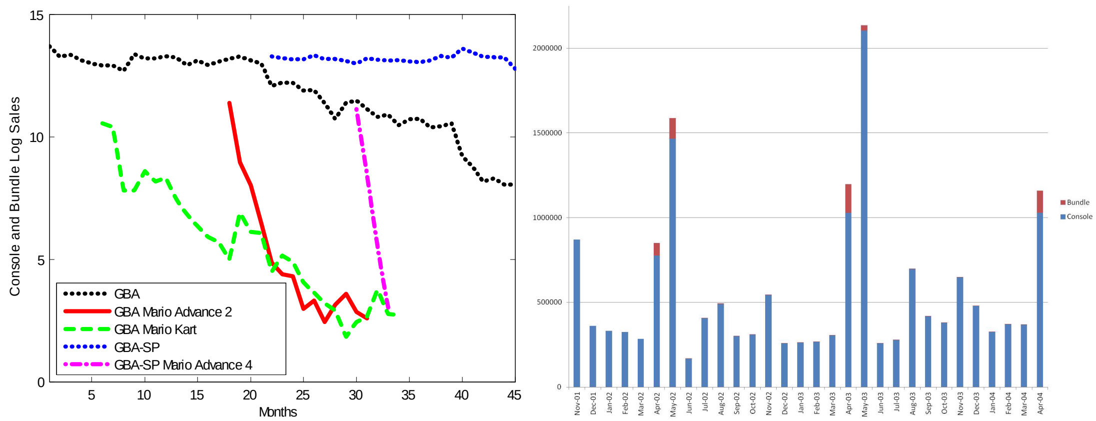


Figure 5: Console and Bundles Sales

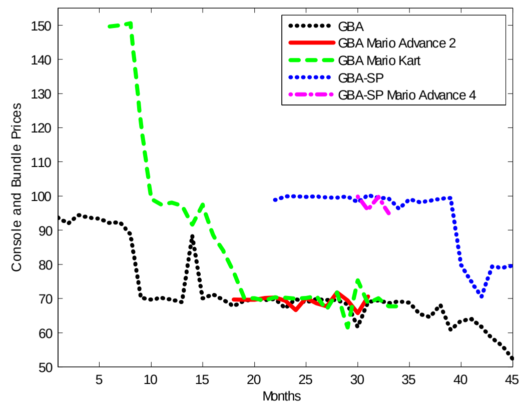


Figure 6: Console and Bundles Prices

dominate sales of bundles in all months including holiday periods and that bundles sales are prevalent throughout the intermediate months. In Figure 6 we present the prices of each console and bundle to illustrate declining prices, which is prevalent in durable goods, and to more clearly compare prices of consoles to bundles.

5 Empirical Support

Before we begin providing support for the above predictions, it is important to discuss our empirical research design(s). We adopt the treatment effect methodology of Rubin (1974, 1978) to determine whether the introduction of mixed bundles has causal effects on market prices. Let $Y_i(1)$ denote the corresponding product price under the existence of mixed bundles and $Y_i(0)$ the price when bundling is not present. Then the average treatment effect is the difference between these potential outcomes :

$$\alpha_i = E[Y_i(1) - Y_i(0)]$$

where E is the expectations operator. However, we are unable to observe all possible outcomes in a given period and thus α_i cannot be determined. To alleviate such a problem, we must generate the counterfactual observations. One such possibility is to gather data from a suitable comparison group that does not experience the introduction of bundles (a control group) and compare the outcomes from these two groups. Data from the home video game console market would be an ideal control given the video games available for the portable consoles and for Nintendo's home console substantially overlap. Unfortunately this data will not suffice given it too includes bundles. Another such alternative is to employ data from the computer video game market as the control given this market of software never receives "treatment" of a bundle. Nonetheless, such a control is only available for the software markets. We therefore implement a regression approach to estimate the average treatment effect for each of our theoretical predictions as well as a differences in difference estimator for the last two predictions. What follows below is a general formulation of the empirical models employed for each of the three predictions.

Assume we observe N units, indexed by $i = 1...N$ and $t = 1...T$ and the existence of each potential outcome $(Y_{it}(0), Y_{it}(1))$, where $Y_{it}(1)$ is the outcome associated with treatment and $Y_{it}(0)$ the outcome for the control–no treatment. Moreover, there exists a set of observable variable or covariates X_{it} which explain the observed outcomes. We observe a triple $(Y_{it}, X_{it}, \tau_{it})$ for each unit where Y_{it} is the realized outcome of i in period t and τ_{it} is the treatment indictor for observation i in period t.

$$Y_{it} \equiv Y_{it}(\tau) = \begin{Bmatrix} Y_{it}(0) \text{ if } \tau = 0 \\ Y_{it}(1) \text{ if } \tau = 1 \end{Bmatrix}$$

To estimate the average treatment effect using a regression approach, we assume a parametric form for the outcome variable. Suppose the control outcome $Y_{it}(0)$ is linear in the observable covariates such that $Y_{it}(0) = X_{it}'\beta + \gamma_i + \varepsilon_{it}$ with $\varepsilon_{it} \perp X_{it}, \gamma_i$ where X_{it} is a matrix of product characteristics and γ_i a product fixed effect. Additionally, suppose $Y_{it}(1) = \alpha_{it}\tau_{it} + X_{it}'\beta + \gamma_i + \varepsilon_{it}$. In order to identify the parameter of interest, the average treatment effect α_{it}, we must make three assumptions:

Assumption 1 (Unconfoundedness) $Y_{it}(0), Y_{it}(1) \perp \tau | X$

Assumption 2 (Constant Treatment Effect) $\alpha_{it} = \alpha \ \forall \ i$ and t.

Assumption 3 (Conditional Mean Independence) $E[Y_0|\tau = 1, X] = [Y_0|\tau = 0, X] = [Y_0|X]$

These three assumptions together assume that τ_{it} can be treated as an exogenous variable, eliminating any simultaneity, selection or omitted variable bias. In each of the three predictions below we report the results of this model under regression (1). However, Assumption 1 and 3 are quite strong, and if invalid they would lead to a selection bias associated with an endogenous treatment variable or selection on unobservables. To correct for this bias we implement an instrumental variable approach given the console manufacturer strategically determines when to offer bundles resulting in correlation between ε_{it} and τ_{it}. In particular, we assume $E[\varepsilon_{it}|Z_{it}] = 0$ where Z_{it} are instruments for the causal effect of τ_{it} on Y_{it}. Moreover, for Z_{it} to be valid instruments it must be correlated with τ_{it} but independent of the model error term, $\varepsilon_{it} \perp Z_{it}$. Our exclusion restriction to properly identify the treatment effect is the measure of the installed base before any new consoles are sold in period t and $t-1$ and there corresponding squared values. We employ these measures as instruments from our comparative statics in Figure 4 concerning the installed base above. Since bundling becomes more irrelevant as the installed base or number of new consumers purchasing the console tends toward zero, we therefore should see bundling occur during the middle rather than at the infancy or end of the hardware life cycle.

Additionally, we adapt the parametric forms above to account for state dependence in pricing by including one period lagged outcomes as a robustness check to the IV and differences-in-difference estimators.

$$Y_{it}(\tau) = \rho Y_{it-1} + \alpha\tau_{it} + X_{it}'\beta + \gamma_i + \varepsilon_{it} \tag{2}$$

We perform each test using the Arellano and Bond (91) GMM estimator .

In summary, we implement instrumental variable estimators to determine whether the above theoretical model is consistent with the data from the portable video game industry. In addition to the instrumental variable methodology, we implement difference-in-differences and dynamic panel data methodologies as robustness checks, when applicable. Specifically, when employing a difference-in-differences estimator we use the computer video game market as the control group given the inability of this market to offer hardware and software bundles. Determining whether our estimates are unbiased in our difference-in-differences estimator depends on the identifying assumption that the time effects in the two markets are identical while the instrumental variable methodology hinges on determining a proper exclusion restriction.

Our first empirical model determines whether the presence of bundles lead to lower standalone console prices. We analyze the impact mixed bundles have on standalone console prices by restricting the data to consist only of the two standalone consoles, the GameBoy Advance and the GameBoy Advance SP. Table 5 presents the result of three regressions. The first column report results from simple regression without correcting for any endogeneity bias. Column (2) corrects for this bias with the use of an IV estimator and is followed by the results from a dynamic panel data method. Unfortunately, we are able to employ a difference-in-differences estimator for this first empirical model given that we only have supplemental computer video game data. To determine whether console fixed effects or a pooled model should be accepted we implement a Hausman test, which rejects the regression with fixed effects for the pooled regression. This test result should not be surprising as we discussed above that the internal parts of both devices were identical. Column (3) addresses the issue of endogeneity of bundles by the console manufacturer and state dependence in prices with the use of the Arellano and Bond dynamic panel data estimator. The results of this regression are presented as an additional robustness check. This estimator uses an unbalanced set of instruments to correct for the correlation between prices in t and $t-1$ as well as the endogenous bundle indicator variable. For instance, for data of three periods long $t = 3$ one can use y_{i1}, for $t = 4$ one can use y_{i1} and y_{i2}, and so on as instruments. Furthermore, we can reject the second-order autocorrelation in the residuals (test stat presented below in table); otherwise the Arellano and Bond estimator would be inconsistent.

There may also be concern that numerous high price sensitivity shoppers enter the market during the holiday months of November and December and cause prices to fall during these months (e.g. Chevalier, Kashyap and Rossi (2003), Sudhir, Chintagunta and Kadiyali (2005) and Meza and Sudhir (2006)). To control for this effect we include a seasonal indicator variable in each of the three price regressions. The results of all three regression illustrate that such an effect is not present in the console market. The identification of such an effect (we'll discuss only the OLS regression for simplicity) originates from the variation between the bundle and seasonal indicator. The seasonal indicator again only picks up the November and December effects for each year while the bundle indicator certainly incorporates these months but also includes many other months over a stretch of some years. Thus, this variation in the data allows us to separately identify both effects.

It is also important to discuss why we elect not to include the number of video games in any of the follow regressions. Including the number of video games might seem like a natural covariate for our regressions to control for either the indirect network effect associated with the platform or for software competition. We, however, do not include the number of games as a control from the fact that the number of games is affected by the treatment and with its inclusion the unconfoundedness assumption above would be violated. Imbens (2004) addresses the concern of including intermediate outcomes, like that of number of games, in a treatment regression and the pitfalls associated with it. He states the only suitable control variables are those that are unaffected by treatment. Nonetheless, the effect of treatment on the intermediate outcome of number of games in an important prediction of our model and is worth analyzing.

Lastly, the coefficient of interest that corresponds to the presences of a bundle is negative and significant in all three regressions indicating that our first theoretical prediction is consistent with the data. However, in order for us to claim that our theoretical model is consistent with the observed data, we need to further analyze the remaining predictions.

Next we analyze prediction two, whether the component bundled software price increases when the bundle is introduced, by restricting the data set to only include software which was bundled with a console to determine whether the theoretical prediction is consistent with the data. We follow a similar methodology to the above analysis to analyze the average treatment effect, but as previously noted, we also present a difference-in-differences estimator.

With the availability of computer video game data from July 2002 through February 2005, we implement a difference-in-differences estimator to recover the average treatment effect in the following two tests, in addition to the IV and dynamic panel data methods.[19]

[19] Given the computer game data does not orginate until July 2002, we restrict the treated groups date to also orginate in July 2002 in the each of the two difference-in-differences estimates.

Table 4: Standalone Console Price Regression

Price	1-OLS	2-IV	3-Dynamic
I(Bundle)	-5.534**	-5.254**	-1.495*
	(1.898)	(2.220)	(0.863)
Presence of Additional Console	14.479**	14.999**	5.506**
	(1.806)	(1.790)	(0.398)
Age	-1.431**	-1.721**	-0.655**
	(0.215)	(0.256)	(0.143)
Age^2	0.003	0.008*	0.003**
	(0.005)	(0.005)	(0.001)
Seasonal	0.832	0.500	0.688**
	(2.098)	(1.838)	(0.245)
Price_{t-1}			0.572**
			(0.119)
Console FE's	No	No	-
Number of Obs.	69	65	65

Column (1) and (2) include an unreported constant **significant at 95% *significant at 90%

Hausman Test Stat: 2.37 (Prob>Chi2=0.667) A&B Test for 0 autocorr in FD of error: Z=1.408 (Prob>Z=0.159)

We are certainly aware that the implementation of a difference-in-differences estimator does not require means to match only that the trends in the absence of the intervention are the same for both groups. Consequently, we test whether such trends are equivalent by restricting the data to only the non-treatment period and allowing month fixed effects to be control and treatment specific. If we cannot reject the null hypothesis that the month effects are equivalent, then the computer video game data can be used as a proper control. We determine we cannot reject the null.

We now discuss the results. Column (1) is an OLS regression that does not correct for the bundle indicator endogeneity, but it does include software fixed effects. Like above, we perform a Hausman test to determine whether fixed effects or a pooled regression is more appropriate. We determine that we cannot reject the fixed effects regression. Column (2) corrects for the endogeneity bias associated with treatment variable while column (3) presents the results of the difference-in-differences estimator. Column (4) instruments for the endogenous bundling decision with the incorporation of dynamic panel techniques where we reject the second-order autocorrelation in the residuals (test stat presented below in table).

Table 5: Bundle Software Price Regression

Price	1-OLS	2-IV	3-DID	4-Dynamic
I(Bundle)	0.318*	0.787**	1.213**	0.234**
	(0.179)	(0.389)	(0.604)	(0.050)
Age	0.067**	0.022	-0.213**	0.019
	(0.024)	(0.034)	(0.064)	(0.024)
Age^2	-0.001	-0.001	0.002**	-0.0002
	(0.001)	(0.001)	(0.0006)	(0.0004)
Seasonal	-0.417**	-0.489**	–	-0.337**
	(0.167)	(0.186)		(0.156)
$Price_{t-1}$				0.419 **
				(0.079)
Game FE's	Yes	Yes	Yes	–
Number of Obs.	97	91	6,685	91

All models include an unreported constant and standard errors are clustered around software id. **significant at 95%

Test Statistic of Month FE of Control=Month FE of Treated: Model 5: F(20, 4652) = 1.26 Prob > F =0.2026

Hausman Test Stat: 53.1 (Prob>Chi2=0.00) A&B Test for 0 autocorr in FD of error: Z=0.236 (Prob>Z=0.813)

As the table illustrates, the OLS estimates of column (1) underestimates the treatment effect due to the correlation between the error term and the treatment effect. To correct for this bias we use the same exclusion restriction to identify the treatment effect in regression (2) above. The result illustrates bundling has a causal effect on prices which lead to higher component prices for the bundled software and is consistent with our theoretical prediction. It is important to discuss that this result is not being identified from the fact that in high demand periods (or in low demand periods for that matter) software is subsidized when in a bundle causing our empirical model to interpret a higher price for standalone products. Since again the data used to investigate this prediction is *only* individual sales data from the three bundled games (e.g. data of software when sold individually) our dependent variable is the standalone price of the software and not a mixture of effective game price and standalone price. Crudely, identification is originating from variation in standalone bundle game prices and the presence of bundles overtime after controlling for selection. Lastly, we control for the concern that numerous high price sensitivity shoppers enter the market during the holiday months leading to lower prices and find the effect present.

Our third prediction determines whether the royalty rate levied by Nintendo decreases when mixed bundles are offered. Its important to point out that royalty rates are not exogenous and fixed; they do fall in video game industry as is evident by a 2003 Reuters story that announces a Nintendo royalty adjustment (Paul, 2003). Unfortunately, royalty rates are unobserved and we are unable to directly regress the royalty rates on a set of covariates. We, nonetheless, are able to determine if the royalty rates decrease indirectly with a simple assumption regarding the marginal cost of independent games. We assume the marginal cost of an independent piece of software in a given period t takes the form $mc_{jt} = \overline{mc}_j + r_t$, where $\overline{mc}$ is a constant plus the time varying royalty rate, r_t. With this assumption, we can infer the royalty rate declines if independent software prices decrease under the existence of bundles. We are aware that such an assumption is quite strong, but one can think of the following analysis as a first approximation of the causal effect. We analyze the last theoretical prediction with data which is restricted to include all available third party software (over 14,000 observations). If our theoretical model is consistent with the data then we expect the sign of the bundle measure to be negative, indicating the royalty rate declines as bundles enter.

There are, however, several alternative explanations that also would lead to a decline in software prices in the data and thus, if are not accounted for, will confound the bundle indicator estimate. The first of two alternative explanations is the entrance of lower quality independent software products. As lower quality games enter over time, these games will be priced lower, leading to a negative parameter estimate for the bundle indicator since bundles only occur in the middle of the data range. We correct for any such confoundedness from the entry of lower quality video games with the bundle indicator variable with the inclusion of video game fixed effects. The second explanation is one we touch upon above: dynamic evolution of consumer price sensitivity overtime (e.g. more price sensitive consumer enter during the holiday period). We control for this explanation with several different approaches which are contingent upon which type of methodology is employed. One approach is to employ a DID methodology which explicitly controls for such concerns by assuming the trends in both the control and treated groups are statistically identical.[20] With identical trends across groups any impact from price sensitive consumers entering the market will be controlled for with the use of the control group. To address this concern in the other methodologies, we deviate from the above analysis by replacing a seasonal indicator variable with month of year indicator variables given a larger data set to identify each month of year effect.

[20] Again, we test this assumption and find that the trends are staticially identical for all models

Table 6: Independent Software Price Regression

Price	1-OLS	2-IV	3-DID	4-Dynamic
I(Bundle)	-2.502**	-2.664**	-3.815**	-2.435**
	(0.111)	(0.155)	(0.3232)	(0.536)
Age	-1.033**	-0.956**	-0.561**	-0.006
	(0.011)	(0.014)	(0.0339)	(0.092)
Age^2	0.012**	0.011**	0.012**	0.002**
	(0.0003)	(0.0003)	(0.0005)	(0.0003)
$Price_{t-1}$				0.751**
				(0.014)
Game FE's	Yes	Yes	Yes	–
Month FE's	Yes	Yes	–	Yes
Number of Obs.	14,435	13,098	107,330	13,098

All models include an unreported constant and standard errors are clustered around software id.

**significant at 95% Test Statistic of Month FE of Control=FE of Treated: F(20, 72509) = 1.21 Prob > F = 0.2322

Hausman Test Stat: 376.66 (Prob>Chi2=0.00) A&B Test for 0 autocorr in FD of error: Z=1.5111 (Prob>Z=0.1308)

The results are in Table 5. There is clear evidence to support our theoretical prediction—–in each of the models and particularly in the IV and DID regression we empirically observe the predicted relationship between the presence of a bundle and price.

With each of the three theoretical predictions being present in the data, we conclude that our theoretical monopoly model of mixed bundling in the two-sided market setting is consistent with the portable video game market. Nonetheless, we by no means conclude that the above theoretical model is the only correct model that generate these results. Rather, there may be alternative theoretical models that produce predictions consistent with the empirical data.

6 Conclusion

This paper establishes a moderator between the existing bundling theory and two-sided markets literature. In addition to filling the theoretical gap, it provides a possible explanation for a few peculiar data trends which run contrary to the pricing structure of bundling in one-sided market——some standalone component prices fall with the introduction of bundling. We further extend the traditional literature on bundling and the burgeoning literature on two-sided markets by presenting a theoretical monopoly model of mixed bundling, in the context of the portable video game console market, a prototypical two-sided market.

Deviating from both the traditional bundling literature and standard two-sided markets literature, we find that under mixed bundling both the standalone platform price on the consumer side and the royalty rate on the content developer side are lower than their counterparts under independent pricing equilibrium. In our setting, mixed bundling acts as a price discrimination tool segmenting the market more efficiently as well as functions as a additional coordination device helping solve "the chicken or the egg" problem in two-sided markets. We further provide clear empirical evidence for the model predictions with new data from the portable video game console market.

Despite the model being somewhat stylized, there are several general insights we can draw from for business and public policy.

First, our model confirms that consumer heterogeneity is the primary reason for the firm to adopt a bundling strategy. More importantly, in the context of two-sided markets and network effect, bundling as a price discrimination tool can further help in restructuring the platform's pricing structure and increase its profit. Due to the cross-side network effect, business managers can set prices at low levels without suffering any loss. Therefore, failing to take this indirect network effect into consideration may severely underestimate the impact of promotion and penetration, especially in a two-sided markets where the demand is more elastic thanks to the network effect.

Turning to public policy, bundling has been a heated antitrust issue. When lower prices are associated with bundling, antitrust concerns on predatory pricing and exclusion may arise. Although we cannot draw a general policy conclusion in favor of bundling in our analysis, our model does point to a possibility of welfare-enhancing bundling in two-sided markets. Through bundling, the platform can internalize the cross-market externality by better segmenting consumers and coordinating two sides leading to consumers gaining too when the platform's profit increases.

With this paper being the first to link mixed bundling and two-sided markets there are some important limitations to our study that may serve as potential directions for further research in the future. First, we adopt a static model as almost all previous bundling or two-sided markets studies do; while the market is inherently dynamic. Theoretically, introducing dynamics into bundling decision in the two-sided markets contexts will considerably complicate the model and will likely lose tractability for analytical solutions. Empirically, Derdenger and Kumar (2011) present a dynamic structural model to examine how bundling affect sales of video game consoles and games.

Second, we don't model the independent content developer side in detail. This is because modelling independent content developer's dynamic pricing and entry decision would significantly complicate our model without providing too much insight into the main question——the platform's incentive to bundle. Consequently, we simply assume their entry decision is negatively affected by a royalty rate and suppress their pricing decision. It would be interesting to extend our model to look into some interesting questions such as how bundling affects independent content providers' decisions on when to enter, when to release their games, how to price and how to compete against other content.

Third, and probably the most fruitful area of research moving forward is to introduce competition into the platform market so one may understand how platform competition impacts pricing decisions. Nonetheless, our monopoly model does provide important insight into the optimal pricing of mixed bundles in two-sided markets, which is quite useful for marketing managers given the actual existence of such monopoly markets and the prevalence of asymmetric structure of most of two-sided markets.

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Appendix

Table A.1 provides the glossary of notations.

Table A.1: **Glossary of Notations**

p_c, P_c	Price for accessing the platform under independent pricing and under bundling, respectively
p_g, P_g	Integrated content price under independent pricing and under bundling, respectively
P_B	Bundle price under bundling
P_g^e	Effective price of the integrated content under bundling
r, R	Royalty rate under independent pricing and under bundling, respectively
t	Transportation cost per unit of length
α	Fraction of installed base
β	Marginal utility of the content
d, D	Quality of content provided by the independent content developer under independent pricing and under bundling, respectively
x	Consumer's distance from the origin
$u_{installed}$	Installed base consumer's gross utility from the integrated content
v_g	New consumer's intrinsic value for the integrated content
q_{new}, Q_{new}	Number of new consumers who purchase the access to the platform under independent pricing and under bundling, respectively
$q_{platform-only}, Q_{platform-only}$	Number of new consumers who purchase only the access to the platform under independent pricing and under bundling, respectively
q_{both}, Q_{both}	Number of new consumers who puchase both the access to the platform and the integrated content under independent pricing and under bundling, respectively
π^{IP}, π^B	Platform's profit under independent pricing and under bundling, respectively

Theoretical Proofs[21]

Proof of Lemma 1. We show that π^{IP} under (i) must be lower than that under (ii).[22]

[21] The Mathematica codes for computation are available on authors' Webpages.
[22] The proof for Π^B is parallel, with only modification on replacing p_g here by P_g^e then.

In (i), $2 - p_g > p_c - \beta d \geq 1$. And $q_{new} = q_{platform-only} = \frac{(1-p_c+\beta d+1-p_g)^2}{2t}$.

$$\pi^{IP}_{(i)} = \alpha \cdot p_g + (1-\alpha) \cdot (p_c + p_g) \cdot q_{new} + r \cdot (1-r) \cdot [\alpha + (1-\alpha) \cdot q_{new}].$$

Then we have

$$\begin{aligned}
\frac{\partial \pi^{IP}_{(i)}}{\partial p_c} &= (1-\alpha) \cdot [q_{new} + (p_c + p_g) \cdot \frac{\partial q_{new}}{\partial p_c} + r \cdot (1-r) \cdot \frac{\partial q_{new}}{\partial p_c}] \\
&= (1-\alpha) \cdot \frac{1 - p_c + \beta d + 1 - p_g}{2t} \cdot [2(1 - p_c + \beta d) - p_c - 3p_g - \beta d - 2r \cdot (1-r)] \\
&< 0,
\end{aligned}$$

where the term in the bracket is negative follows from the fact that in (i) $p_c - \beta d \geq 1$. Therefore, it is always profitable to reduce p_c as long as $p_c - \beta d \geq 1$. In other words, case (i) is impossible and we can thus focus on case (ii). ■

Proof of Lemma 2. Any IP menu (p_c, p_g) can be perfectly mimicked by (P_c, P_g, P_B) with $P_c = p_c$, $P_g = p_g$ and $P_B = p_c + p_g$. Due to the presence of installed base, offering mixed bundling gives the platform more freedom in extracting surplus. Thus, mixed bundling is strictly better than IP. Under pure bundling, neither the installed base nor the new consumers with low value for the integrated content would be served. Hence, pure bundling will be strictly dominated, too. ■

Proof of Proposition 3. When $\overline{R}$ is exogenously given,

$(p_c^{*\overline{R}}, p_g^{*\overline{R}})$ for IP are given by:	$(P_c^{*\overline{R}}, P_g^{e*\overline{R}})$ for bundling are given by:
$\frac{\partial B(p_c,p_g,\overline{R})}{\partial p_c} = 0$ $\frac{\partial B(p_c,p_g,\overline{R})}{\partial p_g} = s$, where $s = -\frac{\alpha}{1-\alpha} < 0$.	$\frac{\partial B(P_c,P_g^e,\overline{R})}{\partial P_c} = 0$ $\frac{\partial B(P_c,P_g^e,\overline{R})}{\partial P_g^e} = 0$.

Consequently, we can focus on the properties of $B(x, y, \overline{z})$ for comparative statics on s. Denote its Hessian matrix as $H \equiv D_i D_j[B(x, y, \overline{z})] = [h_{ij}]$.

Standard comparative statics gives that

- (change in p_g) : $\frac{\partial y}{\partial s} = \frac{h_{11}}{|H|} \because h_{11} = -\frac{1-\alpha}{t} \cdot 2 < 0 \therefore \frac{\partial y}{\partial s} < 0$
- (change in $(p_c + p_g)$) : $\frac{\partial x}{\partial s} + \frac{\partial y}{\partial s} = \frac{h_{11}-h_{12}}{|H|} \because h_{11} - h_{12} = -\frac{1-\alpha}{t} \cdot 3y < 0 \therefore \frac{\partial x}{\partial s} + \frac{\partial y}{\partial s} < 0$
- (change in p_c) : $\frac{\partial x}{\partial s} = \frac{-h_{12}}{|H|}$. It will depend on the sign of $-h_{12} = \frac{1-\alpha}{t} \cdot (2-3y)$. Thus, if $\frac{2}{3} > P_g^{e*\overline{R}}$, then $\frac{\partial x}{\partial s} > 0$. $\because \frac{\partial B}{\partial y}\Big|_{y=\frac{1}{2}} = \frac{1-\alpha}{2t} \cdot [-\frac{1}{4} - x - \overline{z}(1-\overline{z})] < 0 \therefore P_g^{e*\overline{R}} < \frac{1}{2} < \frac{2}{3}$.

For $p_g \geq \frac{2}{3}$, $P_g^e < p_g$ automatically satisfied. Hence, $\frac{\partial x}{\partial s} > 0$.

For the effective price index discussed in footnote 11, $\frac{\partial PI}{\partial s} = \frac{1}{t} \cdot [\frac{\partial x}{\partial s} + (1-y) \cdot \frac{\partial y}{s}] = -\frac{1-\alpha}{t^t \cdot |H|} \cdot y < 0$. Thus, $PI_{IP} > PI_B$. ■

Proof of Proposition 4. First, the change in standalone price of the integrated content for the installed base is a direct transfer to the platform resulting in total surplus to remain unchanged. Consequently, total surplus is dependent upon the change in surplus of new consumers and game developers.

Second, we show that mixed bundling increases consumer participation. Note that the number of new consumers is $q_{new} = \frac{1-p_c+\frac{(1-p_g)^2}{2}}{t} + \frac{\beta}{t} \cdot d(\overline{R})$. So the determinant is the change in $\Gamma \equiv 1 - p_c + \frac{(1-p_g)^2}{2}$ under two regimes. $\frac{\partial \Gamma}{\partial s} = -[\frac{\partial p_c}{\partial s} + (1-p_g) \cdot \frac{\partial p_g}{\partial s}] = -[\frac{-h_{12}}{|H|} + (1-y) \cdot \frac{h_{11}}{|H|}] = \frac{\frac{1-\alpha}{t} \cdot y}{|H|} > 0. (\because h_{11} = -\frac{1-\alpha}{t} \cdot 2, h_{12} = \frac{1-\alpha}{t} \cdot (2-3y))$ So q_{new} is higher under mixed bundling.

Third, we show the distribution change is welfare-enhancing. From Proposition 3, we know that when $\overline{R}$ is exogenously given, $P_B < p_c + p_g$, $P_g^e < p_g$, $P_c > p_c$. Therefore, the distribution change in new consumers is as shown in Figure A.

Although the total participation from the new consumers q_{new} increases as shown above, there is a caveat in the distribution change——area 2 is lost while area 1 is gained. Note the total surplus gains (area 1) are all above P_B, while the total surplus losses (area 2) are all below P_B. Combined with the fact that the overall participation from new consumers are higher, we can conclude that the gains dominate loss. The proposition follows. ■

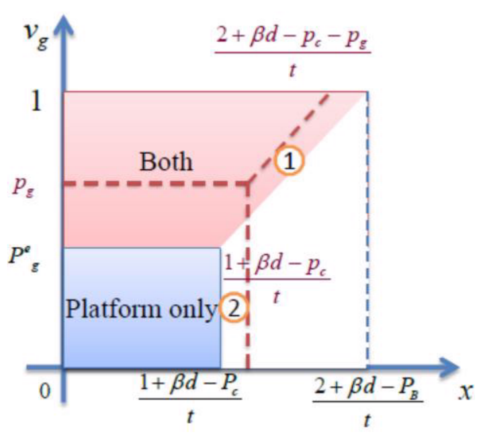

Figure A: The Change of New Consumers' Participation after Mixed Bundling

Proof of Proposition 4. When R is endogenously determined,

(p_c^*, p_g^*, r^*) for IP are given by:	(P_c^*, P_g^{e*}, R^*) for bundling are given by:
$\frac{\partial B(p_c,p_g,r)}{\partial p_c} = 0$ $\frac{\partial B(p_c,p_g,r)}{\partial p_g} = s$ $\frac{\partial B(p_c,p_g,r)}{\partial r} = 0$, where $s = -\frac{\alpha}{1-\alpha} < 0$.	$\frac{\partial B(P_c,P_g^e,R)}{\partial P_c} = 0$ $\frac{\partial B(P_c,P_g^e,R)}{\partial P_g^e} = 0$ $\frac{\partial B(P_c,P_g^e,R)}{\partial R} = 0$.

Similarly, we can focus on the properties of $B(x, y, z)$ for comparative statics on s. Denote its Hessian matrix as $K \equiv D_i D_j [B(x, y, z)] = [k_{ij}]$.

Standard comparative statics gives that

- (change in p_g) : $\frac{\partial y}{\partial s} = \frac{\begin{vmatrix} k_{11} & k_{13} \\ k_{31} & k_{33} \end{vmatrix}}{|K|} \because \begin{vmatrix} k_{11} & k_{13} \\ k_{31} & k_{33} \end{vmatrix} > 0, |K| < 0$ from negative definitiveness of $K \therefore \frac{\partial y}{\partial s} < 0$

- (change in r) : $\frac{\partial z}{\partial s} = -\frac{\begin{vmatrix} k_{11} & k_{12} \\ k_{31} & k_{32} \end{vmatrix}}{|K|} \because \begin{vmatrix} k_{11} & k_{12} \\ k_{31} & k_{32} \end{vmatrix} < 0 \therefore \frac{\partial z}{\partial s} < 0$

- (change in p_c) : $\frac{\partial x}{\partial s} = -\frac{\begin{vmatrix} k_{12} & k_{13} \\ k_{32} & k_{33} \end{vmatrix}}{|K|} \because \begin{vmatrix} k_{12} & k_{13} \\ k_{32} & k_{33} \end{vmatrix} < 0 \therefore \frac{\partial x}{\partial s} < 0$

- (change in $(p_c + p_g)$) : $\frac{\partial x}{\partial s} + \frac{\partial y}{\partial s} < 0$

■

Profitability of Bundling when Preferences are Positively Correlated

One may wonder about the correlation of preferences between goods and its role on the profitability of bundling. We assume new consumers' valuations for the platform and the integrated content are independent, which may appear peculiar for some empirical applications when one considers positively correlated preferences. However, as shown in our analysis below, the main results of this paper still holds when new consumers' valuations for the platform and the integrated content are positively correlated.

Let's consider a set up exactly the same as the one in Section 2 except for that new consumers' valuations for the platform and the integrated content are perfectly correlated.

To be more specific, the gross utility associated with a new consumer situated at point x who elects to purchase access to only platform is $(v - tx) \cdot 1\{\beta d > 0\} + \beta d$, while $2v - tx + \beta d$ if he purchases both the platform access and the integrated content, where $1\{\cdot\}$ is the indicator function. Here we assume consumers' valuations for the platform and for the integrated content are perfectly correlated, that is $v_c = v_g = v$. This is an extreme case for the positively correlated case, but serves analytically sufficient to illustrate our point. Similarly, we assume that v is drawn from the uniform distribution $U(\cdot)$ on $[0, 1]$.

Parallel analysis gives the new consumers' demand as shown in the figure.

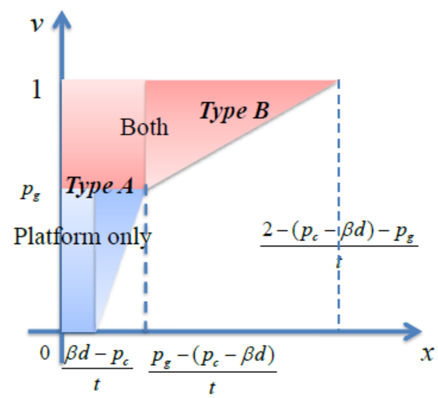

Figure B: New consumer demand with perfectly correlated preferences

$$
\begin{aligned}
q_{platform} &= \frac{p_g(\frac{p_g}{2} + \beta d - p_c)}{t} \\
q_{both} &= \frac{(1 - p_g)(1 + \beta d - p_c)}{t} \\
q_{new}^{(b2)} &= q_{platform} + q_{both} \\
&= \frac{\frac{1}{2} + \beta d - p_c + \frac{(1-p_g)^2}{2}}{t}.
\end{aligned}
$$

Comparing with the independent preferences case, the two demand systems look similar. This is because, for new consumers, the access to platform is essential. Thus, if new consumers decide to purchase there are only two options for them: 1) buy both the access to platform and the integrated content from the platform, or 2) buy only the access to the platform. Moreover, the borderline case for these two options is if $v - p_g \geq 0$ or if it is not, which is exactly the same as when consumer preferences are independent. This result

Table A.2: Number of Software Titles Regression

Price	1-OLS	2-IV	3-OLS	4-IV
I(Bundle)	17.057^{**}	32.878^{**}	24.576^{**}	34.756^{**}
	(6.426)	(10.049)	(5.064)	(7.056)
Installed Base	$.0000254^{**}$	$.0000258^{**}$	$.0000258^{**}$	$.0000262^{**}$
	(4.00e-07)	(6.41e-06)	(3.09e-07)	(3.95e-07)
Seasonal	11.572	9.524	–	–
	(7.627)	(8.276)		
Oct			39.878^{**}	45.196^{**}
			(11.136)	(12.161)
Nov			42.080^{**}	44.447^{**}
			(10.893)	(11.711)
Dec			2.041	3.66
			(10.890)	(11.680)
Number of Obs.	43	43	43	43

**significant at 95% *significant at 90%

Jan-Sept Month of Year Fixed Effects not reported in models 3 & 4 Constants not reported

is true not only for the perfectly correlated case, but also for any positively correlated case. Furthermore, if we write down the profit functions of two regimes for any positively correlated case, they will be in the same pattern as those for the independent case——the structure of Π^B will be identical to π^{IP} except for an extra term representing the surplus gain from the installed base. All these similar functional forms tells us that the main results will hold when preferences are positively correlated.

Moreover, they also confirm that the key reason for bundling adoption in our paper is the heterogeneity of valuations on the integrated content between installed base and new consumers. And the heterogeneity of valuations on the integrated content within new consumers is not critical. Consequently, positively correlated preferences within new consumers will not change our main results. We choose the independent preferences, because our objective is to use a simple model to illustrate the interesting and surprising pricing pattern shown in the paper.

Empirical Support-Hypothesis 4

We analyze a fourth prediction from the theoretical model above, which states that the number of video games available when a bundle is present should be larger than when it is not. The data we employ to empirically analyze this prediction is monthly time series data consisting of 45 months. We run four regression, an OLS and an instrumental variable regression to control for the endogeneity of the entry decision of the bundle. In each regression we include a constant, an indicator for the presence of a bundle, a measure of the installed base and a seasonal indicator or month of year fixed effects. The seasonal indictor variable (for the months of November and December) and month fixed effects are included to help control for the influx of games during the holiday period. We determine that all regressions illustrate that the presence of a bundle does lead to an increase in the number of available video games. Lastly, we see a sizable increase in the number of games being release around the holiday periods, specifically around October and November.