An economic analysis of online streaming: How the music industry can generate revenues from cloud computing*

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Abstract

This paper investigates the upcoming business model of online streaming services allowing music consumers either to subscribe to a service which provides free-of-charge access to streaming music and which is funded by advertising, or to pay a monthly flat fee in order to get ad-free access to the content of the service accompanied with additional benefits. By imposing a two-sided market model on the one hand combined with a direct transaction between the streaming service and its flat-rate subscribers on the other hand, the investigation shows that it can be highly profitable to launch a business which is free-of-charge for subscribers if advertising imposes a weak nuisance to music consumers. If this is the case, and by imposing an endogenously determined level of advertising which is provided by homogeneous advertisers, we find that a monopolistic streaming service increases the price for its flat-rate subscribers in order to stimulate free-of-charge demand and to capture higher revenues from advertisers. An extension of the model by illegal file-sharing shows that an increase in copyright enforcement shifts rents from music consumers to the monopolist.

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1 Introduction

Music labels suffer great losses since music is amenable to digitalization and everyone who is able to handle a personal computer may download music free-of-charge from hundreds of P2P-file-sharing-platforms.[1] Although this proceeding is illegal, the mass of offered platforms and transactions on it makes it impossible for owners of the respective artwork to enforce their copyrights. According to the IFPI Digital Music Report 2010, global sales of record companies in the digital era have been declined by around 30 per cent from 2004 to 2009. At the same time, opening up new sources of income from digital markets gained in massive importance for the music industry. In 2003, the digital music store *iTunes* was launched and it was reported that in 2009 the share of global revenues of record companies which was generated through digital channels was about 27% and grew by about 12% as compared with 2008 (see IFPI 2010). In the US market, the volume of digital music sales grew from \$0.2 billion to \$3.1 billion from 2004 to 2009.[2] Thus, it is straightforward that the music industry experiences a tremendous transformation process.[3] Recently, many ideas have emerged to explore new sources of revenues for the beleaguered music industry from digital markets.[4] Technical progress and growing availability of online services let recently upcoming business models, which provide a service allowing for listening to streaming music on the internet, enormously gain in importance.[5] The underlying idea

[1] In the US market for example, total revenues of the music industry fell from \$ 14.6 billion to \$ 6.3 billion from 1999 (the year of birth of the file-sharing network *Napster*) to 2009 (See: "Music's lost decade: Sales cut in half" by David Goldman (http://money.cnn.com/2010/02/02/news/companies/napster_music_industry/, February 3, 2010; last consulted on March 25, 2011)).

[2] See: RIAA, American Music Business Brochure (http://76.74.24.142/2DB721AD-3A69-11D3-1FA4-E3E59BEC0CE6.pdf, last consulted on April 3, 2011).

[3] A detailed survey concerning the economic aspects of digitalized music is provided by Peitz and Waelbroeck (2006). Liebowitz (2006) specifically focusses on the impact of file-sharing on the music industry.

[4] Such ideas also include business models funded on payments on a voluntary basis. Regner and Barria (2009) investigated the voluntary willingness to pay for an album of customers of the online music label Magnatune, after they had the possibility to listen to free samples, and they find out that customers pay far more than the minimum price charged by the label.

[5] In 2009 Eric Daugan, Senior Vice President, Commercial Strategy, Warner Music International EMEA, enunciated the future of selling music as 'a vision that music is available everywhere, at any time and in any place, but the biggest question is how do we monetise it in an environment of widespread piracy?' (see: IFPI (2010)).

consists in inducing music consumers to listen on demand to music from the so called "cloud" on the internet instead of possessing this music physically in the form of digitalized files.[6] Such business models are basically financed through two sources. First, users can listen free-of-charge to music which will be interrupted by commercial breaks between the particular songs. Beside this free-of-charge access, music consumers is given the possibility to sign flat-rate contracts in order to gain unlimited access to the content of the streaming service as well as further benefits like offline listening and applications for smartphones or tablet computers which enable listening of streaming music on mobile devices. Furthermore, the music provided by the flat-rate business is of better technical quality. The importance of the enormously growing streaming business is supported by having a closer look on the development of the digital music database *Spotify* which was launched in Sweden in 2006. According to the *Financial Times*, the content of *Spotify* contains now more than ten million songs and is used by more than ten million people across Europe. But it is still a point of criticism that only fewer than one million subscribers actually use the advertising-free premium service and pay a monthly charge, and although it reported enormous growth rates in the acquisition of subscribers, *Spotify* made losses of £16.6m according to its 2009 annual report.[7] But one has to mention that 2 years are a quite long period regarding e-business models and actually, *Spotify* is into negotiations with the four big major labels to launch its business in the USA and it received large inputs from investors letting them achieve a billion-dollar valuation.[8] Moreover, a business cooperation with *Facebook* is planned allowing facebook users for sharing streaming music with their friends.[9]

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[6] Long way before the launch of streaming business models, namely in 2005, the Indicare project, which investigates consumer issues of Digital Rights Management, published a survey within 51 percent of the surveyed music consumers respond that listening of music is of higher importance for them than storing (see: Indicare (2005)).

[7] See: "Spotify to expand music service despite losses of £16m" (http://www.bbc.co.uk/news/business-11821021, November 23, 2010; last consulted on February 25, 2011).

[8] See: "Fundraising to lift Spotify to $1bn valuation" by Tim Bradshaw and Andrew Edgecliffe-Johnson (http://www.ft.com/cms/s/2/743bbb6e-3ded-11e0-99ac-00144feabdc0.html#axzz1Exps1eSj, February 21, 2011; last consulted on February 25, 2011).

[9] (See: "Facebook to partner with spotify" by Shane Richmond (http://www.telegraph.co.uk/technology/news/8538810/Facebook-to-partner-with-Spotify.html, May 26, 2011; last consulted on May 31, 2011).

In countries like Germany, where interests of artists are defended by strong copyright protection, the launch of streaming services is strongly regulated.[10] Despite the versatile criticism, it seems that major labels invest hope in streaming business models as a future source of income. In the case of *Spotify*, licensing agreements with all the four important major record labels (Universal, Sony BMG, EMI and Warner) were made. Moreover, all these four big record labels have been reportedly invested in *Spotify*.[11] Actually, the start-up business *Simfy*, which has about eight millions songs available in its content and which has made agreements with the most important record labels as well as with the GEMA, is the most significant provider of streaming music in the German market.[12] It seems that streaming music business models are gaining in importance for music labels which search for alternative forms of funding. Raising appropriability of online market places and the advantages of the web 2.0 paved the way for the implementation of services which intend to take revenues from streaming music. The innovative and revolutionary character of this business model is visible in accounts allowing for features for mobile devices and offline listening which lead to redundancy of the physical possession of digital music files. This paper intends to present a theoretical model which investigates strategic decisions of an online streaming service which launches a mixed funded business given a monopolistic market structure.[13] Our investigation focuses on the behavior of all agents in the model depending on the nuisance caused by commercial breaks.

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[10] The German property rights organization "Gesellschaft für musikalische Aufführungs- und Vervielfältigungsrechte" (GEMA) successfully prevents a launch of *Spotify*.

[11] See: "Behind the music: The real reason why the major labels love Spotify" by Helienne Lindvall (http://www.guardian.co.uk/music/musicblog/2009/aug/17/major-labels-spotify, August 17, 2009; last consulted on February 26, 2011).

[12] See: "Hören statt besitzen" (http://newsticker.sueddeutsche.de/list/id/1122076, March 9, 2011; last consulted on March 9, 2011). Moreover, with 1.777.867 visits in October 2010, *Simfy* is listed on number two on the top-five newcomer list of the German online magazine Meedia. (See: "Gewinner und Verlierer der Online-IVW" (http://meedia.de/nc/details-topstory/article/die-gewinner-und-verlierer-der-online-ivw_100031402.html; last consulted on March 30, 2011)).

[13] The assumption of a monopolistic market structure is supported by the Swedish market where *Spotify* registers enormous growth rates and is actually assumed to be used by almost 15 percent of the population (See: "Spotify defends business model" by Tim Bradshaw (http://cachef.ft.com/cms/s/0/edb427aa-262f-11df-aff3-00144feabdc0.html#axzz1Ey5Zom00, March 2, 2010; last consulted on February 25, 2011).

2 Related Literature

Despite the promising launch of *Spotify* or *Simfy* it is currently inevitable that one has to assess that streaming services still do not earn enough revenues to compensate artists sufficiently.[14] The trend is towards that musicians are increasingly forced to gain revenues from complementary products and services to their music. This includes ticket sales for live performances[15] as well as merchandising. Gayer and Shy (2006) develop such a model and show that free-of-charge provision of music increases popularity of an artist and therefore increases the demand for complementary products and services to the artist's music. In addition to an increase in the sales of complementary goods, free-of-charge music listening can lead to a positive effect on revenues due to sampling. The idea behind is that music consumers can use free-of-charge music in order to prescreen the variety of music they are interested in, and then they are assumed to be willing to pay for the original material if they find a perfect match between the music and their preferences. Listening of advertising financed free-of-charge streaming music can be seen as an approach to promote sampling trusting to engender a benefit for the music industry. Peitz and Waelbroeck (2005) develop such a theoretical model which predicts that free-of-charge listening may let the profits of music labels increase. Duchêne and Waelbroeck (2006) called such a strategy of the music industry where music listeners expend effort to acquire information about music which can lead to a purchase decision an 'information-pull technology'. A further approach, emphasizing the benefits of free-of-charge music listening, is provided by Gayer and Shy (2005) who argue that network effects can be related to the music market. They suppose that an increasing number of free-of-charge listeners enhance the value of music for legal buyers and, given a sufficiently large network effect, also increases profits of a monopolistic provider. Of course, within these models free-of-charge listening is identical to illegal downloading. A survey concerning the therory of digital piracy is

[14] One million plays of Lady Gaga's song "Poker Face" earned the artist $ 167 (See: "Spotify rejects claims that it 'rips off artists'" by Emma Barnett (http://www.telegraph.co.uk/technology/7590782/Spotify-rejects-claims-that-it-rips-off-artists.html), April 14, 2010; last consulted on March 9, 2011) .

[15] This development is supported by Krueger (2005) who surveys that from 2000 to 2003 ticket prices for live performances sharply increased compared to the growth of inflation.

provided by Belleflamme and Peitz (2010). Our model contributes to the theory of digital file-sharing by an extension allowing music consumers for downloading music illegally under consideration of the costs due to legal enforcement. The success of the launch of a flat-rate business can be supported by Shiller and Waldfogel (2009) who investigate survey based data on 465 students' valuation for 50 songs. They find that bundling of the 50 songs not only increases the revenue of a monopolistic provider by more than 9 percent relative to uniform pricing, but also increases consumer surplus.

Beyond the literature which investigates the difficulties for the music industry to generate revenues due to digitalization of music, this paper attends to contribute to the theory of "two-sided markets".[16] The paper is closely related to the investigation of informative advertising on media platforms by Armstrong (2006). In recent years advertising on media platforms attracted a lot of attention. Most of the literature deals with media platforms competing for customers in advertising as well as in content. Such models which investigate spatial competition with respect to the programming of media platforms and which assume advertisers competing strategically by placing their ads on the respective media platforms, are for example developed by Gal-Or and Dukes (2003) or by Gantman and Shy (2007). Jay Pil Choi (2006) investigates the reverse effects of the number of market entrants and the level of advertising on social welfare in a free-entry model. Armstrong and Weeds (2007) show (in a duopoly with vertical quality differentiation) that programme quality is higher under charged access than under advertising funding. Two noteworthy articles investigating social optimality of advertising with respect to the nuisance of advertising are those by Anderson and Coate (2005) and by Peitz and Valleti (2008). Both articles show that the level of advertising is either too high or too low depending on the one side on the nuisance customers have to bear from advertising and on the other side on the willingness to pay of advertisers to contact customers.

In contrast to the aforementioned articles, our model endogenizes the demand of the advertisers for advertising space by implementing a free-entry equilibrium which drives advertisers' profits down to zero and which eliminates strategical behavior of advertisers.

[16] The notion "two-sided market" was inroduced by Rochet and Tirole (2003) and Armstrong (2006).

Furthermore, we consider a monopolistic firm which contemporaneously imposes a free-of-charge and advertising funded business and a business which is financed by charging subscribers who want to listen to streaming music.

The paper is organized as follows. The model is described in section 3. Section 4 solves for the equilibrium containing three types of agents, namely the streaming service, advertisers and subscribers and addresses welfare-theoretical aspects concerning advertising. Section 5 extends the model by considering illegal file-sharing. The monopolist maximizes its profit under a given prosecution for piracy. We then investigate music consumers' aggregate surplus and social welfare under intensification of legal enforcement. Section 6 concludes.

3 The Model

This paper investigates a monopolistic streaming service which offers two differently funded business models to its potential subscribers. The assumption of a monopoly simplifies our analysis insofar as we can focus on the strategic choice of the subscribers between the respective businesses without having to consider differentiated providers of streaming music. The advertising funded free-of-charge business is modeled as a two-sided market model with cross-group externalities[17], where the streaming service acts as a platform which tries to acquire subscribers on the one side of the market and advertising firms on the other side. Those firms placing ads on the online platform intend to reach free-of-charge subscribers in order to win them over customers. In contrast, the flat-rate business is a direct transaction between the streaming service and those subscribers who decide to sign a flat-rate contract and are willing to pay a monthly flat fee. *Figure 1* pictures all interactions between the agents in the model.

The model is organized as a three-stage game. In the first stage, the streaming service announces the flat fee p^b as well as the charge r which has to be payed by one advertising firm in order to place its commercial. In the second stage, advertising firms observe the

[17] Firms placing an ad benefit from an increasing size of subscribers, while subscribers suffer from an increase in commercials. We assume subscribers not to gain from information provided by advertising firms. Following the notation in Anderson and Coate (2005), commercial breaks are a nuisance for subscribers.

flat-rate price p^b and the charge r and simultaneously decide if they demand for advertising space or not. The number of advertisers a, who are willing to place their commercial, is determined by a free-entry equilibrium, meaning that advertisers only face the charge r as costs, and thus demand for advertising space until their profits are driven down to zero. For simplicity we make the assumption that advertising firms place only one commercial which is perceived by each free-of-charge subscriber. Thus, a is indeed the number of advertising spots which has to be tolerated by free-of-charge subscribers. By means of the flat price and the charge they have to pay, advertising firms are able to anticipate the number of commercials broadcasted on the free-of-charge business and therefore the number of free-of-charge subscribers who are also their potential customers. Finally, in the third stage, all potential subscribers decide independently whether to subscribe to the flat-rate business, to the free-of-charge business or not to become a user of the streaming service at all. We solve this model by backward induction and start with the determination of the demand of potential subscribers.

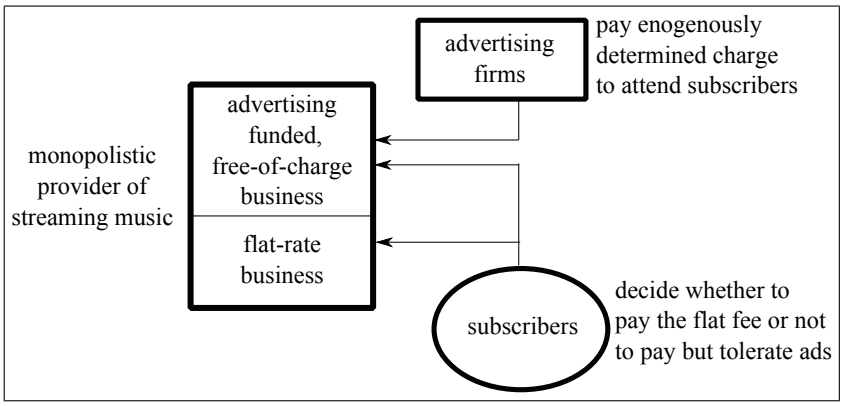

Figure 1: Online streaming business of a monopolistic provider

3.1 Subscribers

We consider a market which is served by a monopolistic online streaming service intending to attract a very large number of m potential users. Those users strategically decide according to a given level of commercials a and a given flat price p^b whether to sign a flat-rate contract, to subscribe to the free-of-charge business or to refuse to subscribe to the streaming service at all. We assume all users to be homogeneous except for their preference type θ ranging them according to an ascending 'addiction to music'. Suppose

that θ is uniformly distributed on the interval $[0,1]$.[18] Subscribers who are characterized by a low θ are more likely to listen to advertising or to refuse to subscribe than to pay the flat-rate price. Those subscribers are assumed to value unlimited listening to music and a higher streaming rate or further benefits, like mobile applications and offline listening, not that much. Subscribers who decide to use the free-of-charge business have to tolerate a certain level of advertising a creating a decrease in the gross value of music, measured by the nuisance costs parameter δ which is identical for all potential users. In contrast, customers of a flat-rate contract are supposed to prefer ad-free unlimited listening, a high streaming rate and additional features. In order to get premium access they have to pay a flat-rate price p^b. Finally, v_A and v_B can be described as intrinsic values of the streaming music provided by the respective businesses of the streaming service. The terms θv_B and θv_A therefore describe a respective subscriber's gross valuation for a given music content, fitting best to the preferences of this subscriber. We assume that the net valuation of a given subscriber of a subscription depending on her or his addiction to music can be measured by her or his utility which can be written as

$$U_\theta = \begin{cases} 0 & \text{if } \theta \in [0, \underline{\theta}] \\ \theta v_A - \delta a & \text{if } \theta \in [\underline{\theta}, \widehat{\theta}] \\ \theta v_B - p^b & \text{if } \theta \in [1-\widehat{\theta}, 1]. \end{cases} \tag{1}$$

The variable $\widehat{\theta}$ defines the indifference level between the free-of-charge and the flat-rate business, whereas $\underline{\theta}$ determines the indifference level between the free-of-charge business and not subscribing at all. As already mentioned, some streaming services provide a higher streaming rate, unlimited access or mobile applications to their paying subscribers. Therefore we assume that v_A is relatively small compared to v_B and because otherwise no subscriber would voluntarily pay the flat fee. To define an equilibrium where subscribers strategically choose according to their addiction to one has to impose *Assumption 1.*

[18] This allows us to define percentages allocating all potential subscribers to either the flat-rate business, the free-of-charge business or to refusal of streaming music.

Assumption 1. $v_B > v_A$.

Equating the utility functions of a potential user with $\theta \in [\underline{\theta}, \widehat{\theta}]$ and a potential user with $\theta \in [\widehat{\theta}, 1]$ let us derive the indifferent subscriber $\widehat{\theta}$, who will differentiate potential users into those preferring a free-of-charge membership with advertising and those choosing a membership liable to pay the flat price. Solving for θ one obtains

$$\widehat{\theta} = \frac{p^b - \delta a}{\eta}. \tag{2}$$

The term $\eta = v_B - v_A$ determines the difference between the intrinsic values provided by the two businesses. The indifferent subscriber is characterized by the relation between the difference of the individual costs of the two businesses and η. The higher the nuisance from advertising δa and η are, the higher will be the share of subscribers to the flat-rate business, while an increase in the flat fee naturally reduces flat-rate subscriptions. One has to consider that it is not profitable for all of the m potential users to become a subscriber to the streaming service. Potential users who are characterized by a very low θ may incur losses from using the service because interruption from advertising exceeds their gross valuation of a free-of-charge subscription. Equating the utility function of a subscriber for the free-of-charge business with zero and solving for θ yields

$$\underline{\theta} = \frac{\delta a}{v_A}. \tag{3}$$

All potential users to the left of $\underline{\theta}$ decide not to subscribe and all potential users to the right of $\underline{\theta}$ subscribe to the free-of-charge business. It is straightforward that the share of potential users who will not subscribe to the streaming service increases with an increasing nuisance from advertising and a decreasing intrinsic value from a free-of-charge subscription.

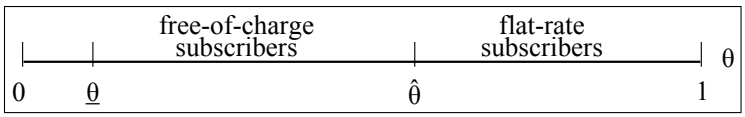

Figure 2: Segmentation of potential users with respect to θ.

Figure 2 plots the segmentation of the potential users according to their 'addiction to music'. Note that the monopolistic streaming provider will choose the level of advertising

as well as the flat-rate price according to the exogenous parameters δ and σ in order to maximize its profit. Depending on the exogenous parameters, this implies a measure of utilities and a resulting demand for the respective businesses which is bigger or equal to zero. Thus, $a \leq \frac{\delta}{v_A}$ and $p^b \leq v_B$ will be ensured.

The demand function for the respective businesses is derived by the multiplication of the interval length which determines the segmental share of the respective business with the number of potential users in the market. Having m potential users in the market, the number of subscribers to the free-of-charge and the flat-rate business can be written as

$$D_A(p^b, a) = m(\widehat{\theta} - \underline{\theta}) = \frac{m(p^b v_A - \delta a v_B)}{v_A \eta} \tag{4}$$

and

$$D_B(p^b, a) = m(1 - \widehat{\theta}) = \frac{m(\eta + \delta a - p^b)}{\eta}. \tag{5}$$

Not surprisingly, the demand for the free-of-charge business is negatively dependent on the nuisance created by advertising and positively dependent on the flat-rate price. One can observe that an increasing difference in the intrinsic values η causes a negative impact on the demand for the free-of-charge business. The demand for the flat-rate business is reversely dependent on these variables. Note that an increasing nuisance created by advertising causes a subscriber drift 'on both sides' of the interval which determines the demand for the free-of-charge business. On the one hand, subscribers with a low θ tend to quit their free-of-charge subscription, as the nuisance from advertising exceeds their gross valuation θv_A. On the other hand, it is getting more attractive for subscribers characterized by a high θ, who have been listened to commercials so far, to shift from the free-of-charge business to the flat-rate business.

3.2 Advertisers

In the second stage of the model, firms which specify their demand for advertising space and which therefore fund the free-of-charge business should be investigated.[19] Suppose that there are many advertisers who have to decide either to place exactly one commercial or not. If this commercial is placed, it is assumed to reach all subscribers to the free-of-charge business. As is customary in two-sided market models dealing with advertising, cross-externalities occur. The demand for advertising space is positively dependent on the demand of subscribers for the free-of-charge business, while the demand for the ad funded business decreases with an increasing level of commercials.

Advertisers know the flat-rate price as well as the charge they have to pay, allowing them for anticipating the equilibrium level of commercials on the free-of-charge business. This level is equal to the number of advertisers who decide to place a commercial in equilibrium, meaning that the streaming service broadcasts each commercial which is supplied to it. Each advertiser faces costs which solely consist of the charge r which is of identical extent for all advertisers and which is asked by the streaming service in order to place a commercial. Thus, the streaming service may strategically regulate the equilibrium level of commercials by varying r. By anticipating the level of commercials as well as by knowing the flat-rate price, advertisers may estimate the equilibrium demand for the free-of-charge business.

To reach one free-of-charge subscriber with its commercial is of a certain value for an advertiser. The parameter σ represents this value. All advertisers are supposed to provide homogeneous services or products and can therefore be assumed to be homogeneous with respect to σ.[20] Without loss of generality one can suppose that the marginal costs of the production of a commercial can be set equal to zero.

It follows that a representative advertiser generates a profit of $\pi^a = D_A\sigma - r$ from placing

[19] For a better understanding, if we refer to these firms, we define them as advertisers.

[20] This is in contrast to the nascent literature concerning the provision of advertising on media platforms (See e.g. Anderson and Gabszewicz (2006), Anderson and Coate (2005) and Peitz and Valletti (2005) who describe advertisers as producers of products being differentiated by characteristics what makes them more or less interesting for customers.)

a commercial on the free-of-charge business which can be written as

$$\pi^a = \left[\frac{m(p^b v_A - \delta a v_B)}{v_A \eta}\right]\sigma - r. \tag{6}$$

Because of their homogeneity, strategic interaction between advertisers is absent. To determine the level of commercials, free-entry in equilibrium is assumed meaning that advertisers demand for advertising space until their profits vanish.[21] According to the charge r, advertisers place commercials until the demand of the free-of-charge business decreased in a way that all advertisers who have been placed a commercial in equilibrium make zero profit. The endogenously determined level of commercials which will be broadcasted by the streaming service can be computed by equating the profit function of one representative advertiser with zero and by solving for a and thus, one obtains

$$\widetilde{a}(p^b, r) = \frac{v_A(\sigma m p^b - r\eta)}{\delta \sigma m v_B}. \tag{7}$$

Of course, an increasing charge r as well as an increasing η let $\widetilde{a}(p^b, r)$ decrease while an increasing flat-rate price induces advertisers to raise their demand for advertising space. The charge r will be determined in a way that the revenues of the monopolistic streaming service from advertising are maximized. By using $\widetilde{a}(p^b, r)$ one can derive the demand functions for the ad funded business $D_A(\widetilde{a}(p^b, r))$ as well as for the flat-rate business $D_B(\widetilde{a}(p^b, r))$ depending on the level of commercials which will be placed according to the equilibrium advertising charge and the equilibrium flat fee. Those demand functions are given by

$$D_A(\widetilde{a}(p^b, r)) = \frac{r}{\sigma} \tag{8}$$

and

$$D_B(\widetilde{a}(p^b, r)) = \frac{m\sigma(v_B - p^b) - r v_A}{\sigma v_B}. \tag{9}$$

[21] The assumption of a free-entry equilibrium is in the spirit of the concept of Chamberlinian monopolistic competition, of course with the distinction that advertisers do not compete in a differentiated product space. This let us neglect strategic aspects of advertising, but rather focus on the level of commercials.

Equation (9) determines the demand for the flat-rate business if one considers potential users being informed about the equilibrium flat fee as well as about the level of commercials they have to tolerate in equilibrium if they subscribe to the free-of-charge business. The demand for the free-of-charge business with respect to the equilibrium advertising level and the equilibrium flat-rate price (8) is simply the relation between the equilibrium advertising charge and the valuation of one advertiser to reach one free-of-charge subscriber.

3.3 Streaming Service

In equilibrium, the streaming service provides an endogenously determined level of advertising a^* at the equilibrium charge r^* and sets the flat-rate price p^* to derive a profit maximizing allocation of subscribers to its businesses. Without loss of generality, we assume that the monopolistic streaming service faces neither marginal costs nor fixed costs in the provision of its businesses and therefore revenues are tantamount to profits.[22] If one has a closer look on ad funding, the revenue per subscriber which can be generated by the streaming service is given by

$$\beta(p^b, r) = \frac{\widetilde{a}(p^b, r)r}{D_A(p^b, r)} \tag{10}$$

which is just the relation between the revenues from advertisers and the demand for the free-of-charge business. Inserting (8) yields

$$\beta(\widetilde{a}(p^b, r)) = \widetilde{a}(p^b, r)\sigma \tag{11}$$

which can be computed as the number of advertisers who decide to place a commercial multiplied by their valuation to reach one free-of-charge subscriber. Multiplying $\beta(\widetilde{a}(p^b, r))$ by the demand for the free-of-charge business $D_A(\widetilde{a}(p^b, r))$ leads us back to the total revenue from advertising which is given by $\widetilde{a}(r, p^b)r$. The total profit of the monopolistic

[22] An interesting extension of our model would be the consideration of musicians or music labels who decide whether to make their artwork available for such a business model or not. By doing this, royalties for artists and labels could be considered as the costs of the streaming service.

streaming service can therefore be written in a two-way manner and is given by

$$\pi^M = \beta(\widetilde{a}(p^b, r))D_A(\widetilde{a}(p^b, r)) + D_B(\widetilde{a}(p^b, r))p^b \tag{12}$$

which is equivalent to

$$\pi^M = \widetilde{a}(r, p^b)r + D_B(\widetilde{a}(p^b, r))p^b. \tag{13}$$

In what follows, equation (13) will be used. Inserting (7) as well as the deduced demand for the flat-rate business (9) in (13) yields

$$\pi^M = \left(\frac{mp^b\sigma - r(v_B - v_A)}{\delta m v_B \sigma}\right) r + \left(\frac{m\sigma(v_B - p^b) - rv_A}{\sigma v_B}\right) p^b. \tag{14}$$

The streaming service uses the two strategic variables r and p^b to maximize its profit. The maximization problem therefore results in the two first-order conditions $\partial\pi^M/\partial r = 0$ and $\partial\pi^M/\partial p^b = 0$. To ensure the existence of an equilibrium, the following restriction on parameter values must hold.

Assumption 2. $4\delta\sigma v_B > v_A(\delta + \sigma)^2$.

Solving for δ, one derives the lower equilibrium threshold of the nuisance parameter

$$\delta^{min} = \frac{2v_B\sigma - v_A\sigma - 2\sqrt{\sigma^2 v_B(v_B - v_A)}}{v_A}. \tag{15}$$

For all values $\delta > \delta^{min}$, the profit function of the streaming service is concave in r and p^b. A detailed derivation is provided in the Appendix.
First order conditions from profit maximization can be written as

$$r^* = \frac{mp^b(\sigma - \delta)}{2(v_B - v_A)} \tag{16}$$

and

$$p^{b*} = \frac{v_B}{2} + \frac{rv_A(\sigma - \delta)}{2m\sigma\delta}. \tag{17}$$

As long as σ exceeds δ and *Assumption 1* holds, the equilibrium flat-rate price and ad-

vertising charge are strictly positive and linearly interdependent. The investigation is restricted to cases in which no subsidization of advertisers or flat-rate subscribers is allowed, implying that the charge as well as the flat-rate price have to be positive. The fact that the equilibrium charge is positively dependent on the flat-rate price seems to be counterintuitive at the first sight. Imagine that the monopolist asks for a high flat fee in equilibrium. Two resulting effects on the free-of-charge equilibrium demand can be observed. First, a high flat-rate price induces potential users with a relatively high addiction to music to choose a free-of-charge subscription instead of a flat-rate subscription. Second, advertisers expect the resulting high free-of-charge demand and are therefore willing to place a high level of commercials. Potential users characterized by a low addiction to music would therefore be better off with a refusal of a subscription. Hence, the monopolist will choose a high equilibrium charge which allows for capturing a bigger surplus from less advertisers and which offsets the loss of free-of-charge subscribers due to an increase in ads. Commercials in equilibrium will thus be relatively expensive and will persist on a rather moderate level. Vice versa, the equilibrium flat-rate price is positively dependent on the advertising charge. An explanation of this strategic behavior of the monopolistic streaming service (by taking account of the nuisance from advertising) will be provided in the following section. Solving the linear system (16) - (17) yields

$$r^{**} = \frac{m v_B \delta\sigma(\sigma - \delta)}{4 v_B \delta\sigma - v_A(\delta + \sigma)^2} \tag{18}$$

and

$$p^{b**} = \frac{2 v_B \delta\sigma(v_B - v_A)}{4 v_B \delta\sigma - v_A(\delta + \sigma)^2}. \tag{19}$$

From *Assumption 1* and *Assumption 2* one can perceive that p^{b**} is strictly positive. The same is true for r^{**} as long as σ exceeds the nuisance cost parameter δ.

Inserting (18) and (19) in (7) yields the equilibrium level of advertising which is given by

$$a^*(p^{b**}, r^{**}) = \frac{v_A(v_B - v_A)(\delta + \sigma)}{4 v_B \delta\sigma - v_A(\delta + \sigma)^2}. \tag{20}$$

One can see that this equilibrium level of commercials is strictly positive as long as *Assumption 1* and *Assumption 2* hold.

3.4 Equilibrium Analysis

The flat-rate price and the charge for advertisers in the static equilibrium arise as a result of the level of nuisance created by advertising. This section provides a detailed analysis of the equilibrium outcome according to the parameter δ which measures this annoyance created by commercials on the free-of-charge version.

Proposition 1. *A monopolistic streaming service only imposes a mixed funded business model consisting of a coexisting advertising funded and flat-rate business within the nuisance bounds* $\delta \in [\frac{v_A\sigma}{2v_B - v_A}, \sigma]$. *For* $\delta \in [\delta^{min}, \frac{v_A\sigma}{2v_B - v_A}]$, *profits will solely be generated from launching an ad funded business, while for* $\delta \geq \sigma$, *nuisance from commercials reaches a level such that the monopolist maximizes its profit by solely selling flat-rate contracts.*

Proof. By using (2) and (3) as well as (19) and (20) one can derive the position of those indifferent subscribers $\underline{\theta}^*(p^{b**}, r^{**})$ and $\widehat{\theta}^*(p^{b**}, r^{**})$ who determine the equilibrium demand for the free-of-charge as well as for the flat-rate business and who are given by

$$\underline{\theta}^*(p^{b**}, r^{**}) = \frac{\delta(v_B - v_A)(\delta + \sigma)}{4v_B\delta\sigma - v_A(\delta + \sigma)^2} \tag{21}$$

and

$$\widehat{\theta}^*(p^{b**}, r^{**}) = \frac{\delta(2v_B\sigma - v_A(\delta + \sigma))}{4v_B\delta\sigma - v_A(\delta + \sigma)^2}. \tag{22}$$

To derive a segmental allocation of potential users in equilibrium which ensures the existence of a mixed funded business, $\underline{\theta}^*(p^{b**}, r^{**})$ must fall below of $\widehat{\theta}^*(p^{b**}, r^{**})$. Solving $\underline{\theta}^*(p^{b**}, r^{**}) = \widehat{\theta}^*(p^{b**}, r^{**})$ for δ yields $\delta^{max} = \sigma$ which determines the upper nuisance parameter bound for which this constraint is fulfilled. It follows that all values of the nuisance parameter which fall short of the advertisers' valuation to reach one single subscriber fulfill the constraint $\underline{\theta}^* \leq \widehat{\theta}^*$. A second constraint which will be necessary to enable a mixed funded business in equilibrium is fulfilled if $\widehat{\theta}^*(p^{b**}, r^{**}) \leq 1$. Solving this equation for δ let us derive the lower bound of the nuisance parameter $\widehat{\delta} = \frac{v_A\sigma}{2v_B - v_A}$, for which a monopolistic streaming service launches a mixed funded business in equilibrium. Note that $\widehat{\delta} > \delta^{min}$ which imposes a stronger restriction than *Assumption 2.* □

Proposition 2. *Within the interval* $[\widehat{\delta}, \delta^{max}]$*, a monopolistic streaming service sets its highest equilibrium flat-rate price as well as its highest equilibrium advertising charge at* $\delta = \widehat{\delta}$*, where the demand for the free-of-charge business as well as the level of commercials will reach their maximum. The same is true for the profit of the monopolistic streaming service which is maximal at the lower nuisance threshold* $\widehat{\delta}$*.*

Proof. We first maximize and minimize $p^{b**}(\delta)$ as well as $r^{**}(\delta)$ with respect to δ within the nuisance parameter thresholds imposed by *Proposition 1.* By setting up this maximization problem we have to consider the constraints of *Assumption 1*, namely that v_B must exceed v_A and we assume non-negativity of all variables and parameters. The results are presented in *Table 1.* In the left column, the arguments of the maximum and the minimum of p^{b**} and r^{**} with respect to δ are listed. The right column lists the associated maximal and minimal values of p^{b**} and r^{**} within $[\widehat{\delta}, \delta^{max}]$.

	δ	$p^{b**}(\delta), r^{**}(\delta)$
$\arg\max_{\delta\in[\widehat{\delta},\delta^{max}]} p^{b**}(\delta)$	$\widehat{\delta} = \frac{v_A\sigma}{2v_B - v_A}$	$v_B - \frac{v_A}{2}$
$\arg\min_{\delta\in[\widehat{\delta},\delta^{max}]} p^{b**}(\delta)$	$\delta^{max} = \sigma$	$\frac{v_B}{2}$
$\arg\max_{\delta\in[\widehat{\delta},\delta^{max}]} r^{**}(\delta)$	$\widehat{\delta} = \frac{v_A\sigma}{2v_B - v_A}$	$\frac{m\sigma}{2}$
$\arg\min_{\delta\in[\widehat{\delta},\delta^{max}]} r^{**}(\delta)$	$\delta^{max} = \sigma$	0

Table 1: Equilibrium flat-rate prices and advertising charges depending on δ within the interval $[\widehat{\delta}, \delta^{max}]$.

An interesting observation is that $p^{b**}(\widehat{\delta})$ exceeds $p^{b**}(\delta^{max})$. This is always true as long as *Assumption 1* is binding. Moreover, the monopolist asks for its maximal equilibrium flat-rate price when the aversion against advertising is at the lower bound $\widehat{\delta}$ for which a mixed funded business will be launched. The equilibrium flat-rate price is a decreasing function in within the interval $[\widehat{\delta}, \delta^{max}]$. This result is due to the interdependency of the equilibrium flat-rate price and the equilibrium charge which determine the equilibrium level of commercials and the resulting demand functions. When δ converges to $\widehat{\delta}$ the equilibrium charge for advertisers sharply increases. The economic intuition behind is that for values of δ around $\widehat{\delta}$, the demand for the free-of-charge business and therefore the supply of commercials is of such an extent, that the streaming service maximizes its profits not only by increasing its advertising charge, but also by increasing its flat-rate

price in order to induce more subscribers to quit their flat-rate contracts and to become free-of-charge users. Thus, when δ is on a low level, it turns out to be a greater benefit for the monopolist to charge a high flat-rate price in equilibrium in order to induce users to shift from the flat-rate to the free-of-charge business, than to keep these users as flat-rate subscribers by imposing a moderate flat fee. In contrast, when δ is on a high level around σ, the demand for the free-of-charge business will turn to zero. Hence, the monopolist relies on its flat-rate business and intends to acquire regular users by charging a moderate flat fee. Further inside can be gained from the equilibrium demand functions for the respective businesses. Therefore we insert the equilibrium flat-rate price and the resulting level of commercials in (4) and (5) and the equilibrium demand functions for the respective businesses can be written as

$$D_A^*(p^{b**}, r^{**}, \delta) = \frac{m v_B \delta(\sigma - \delta)}{4 v_B \delta \sigma - v_A(\delta + \sigma)^2} \tag{23}$$

and

$$D_B^*(p^{b**}, r^{**}, \delta) = \frac{\sigma m(2 v_B \delta - v_A(\delta + \sigma))}{4 v_B \delta \sigma - v_A(\delta + \sigma)^2}. \tag{24}$$

The equilibrium demand for the flat-rate business reaches its maximum at δ^{max}, while the the demand for the free-of-charge offer will be maximal at $\widehat{\delta}$ within the bounds introduced by *Proposition 1.* Again, one has to consider the familiar restraints and computing the argument of the maximum of D_B^* and D_A^* with respect to δ on the interval $[\widehat{\delta}, \delta^{max}]$ yields

$$\arg\max_{\delta \in [\widehat{\delta}, \delta^{max}]} D_B^* = \sigma = \delta^{max}$$

and

$$\arg\max_{\delta \in [\widehat{\delta}, \delta^{max}]} D_A^* = \frac{v_A \sigma}{2 v_B - v_A} = \widehat{\delta}.$$

$D_A^*(\widehat{\delta})$ as well as $D_B^*(\delta^{max})$ are computed as $m/2$ and one can conclude that at most half of all m potential subscribers subscribe to either one of the two businesses. Unsurprisingly, one can observe the argument of the minimum of D_B^* with respect to δ at $\widehat{\delta}$, while the argument of the minimum of D_A^* with respect to δ can be observed at δ^{max}. Inserting δ^{max} in (23) yields $D_A^*(\delta^{max}) = 0$, as well as inserting $\widehat{\delta}$ in (24) yields $D_B^*(\widehat{\delta}) = 0$.

At δ^{max}, the monopolist sets its flat-rate price and advertising charge in order that no

free-of-charge business will be launched. In contrast, a low nuisance from advertising accompanied by a high flat fee implies an increase in the equilibrium demand for the free-of-charge business, while the demand for the flat-rate business turns to zero at $\widehat{\delta}$. The increasing equilibrium demand for the free-of-charge business is only dampened by the equilibrium level of commercials which will be maximal at $\widehat{\delta}$.[23]

Turning next to a closer look at equilibrium revenues generated by the monopolistic streaming service from launching the respective businesses. Multiplying (20) by (18) yields the equilibrium revenue function from launching a free-of-charge business

$$R_A^*(\delta) = a^* r^{**} = \frac{m v_A v_B \delta\sigma(v_A - v_B)(\delta - \sigma)(\delta + \sigma)}{\left(v_A(\delta + \sigma)^2 - 4 v_B \delta\sigma\right)^2}. \tag{25}$$

Not surprisingly (and in line with the behavior of D_A^*, a^* and r^{**} with respect to δ) equilibrium revenues from the funds of the advertisers will be lower, the higher δ is and they will be zero when δ converges to σ. The higher the degree, users feel interrupted by advertising, the less will subscribe to the free-of-charge business and consequently, the less advertisers decide to place commercials in equilibrium forcing the streaming service to ask for a lower advertising charge. In contrast, one can observe the maximum equilibrium revenue from advertising within $[\widehat{\delta}, \delta^{max}]$ at

$$\arg\max_{\delta \in [\widehat{\delta}, \delta^{max}]} R_A^*(\delta) = \frac{v_A \sigma}{2 v_B - v_A} = \widehat{\delta}$$

and the resulting maximum equilibrium revenue from ad funding is

$$\max_{\delta \in [\widehat{\delta}, \delta^{max}]} R_A^*(\delta) = R_A^*(\widehat{\delta}) = \frac{m}{4}(2 v_B - v_A).$$

Focusing on the flat-rate business, equilibrium revenues are

$$R_B^*(\delta) = D_B^* p^{b**} = \frac{2 m v_B \delta \sigma^2 (v_B - v_A)(2\delta v_B - v_A(\delta + \sigma))}{\left(v_A(\delta + \sigma)^2 - 4 v_B \delta\sigma\right)^2}. \tag{26}$$

[23] The maximum equilibrium level of advertising which is given by $\max_{\delta \in [\widehat{\delta}, \delta^{max}]} a^*(\delta) = \frac{2 v_B - v_A}{2\sigma}$, can be observed at $\widehat{\delta}$.

Of course, equilibrium revenues from launching a flat-rate business behave conversely to the revenues from advertising within $[\widehat{\delta}, \delta^{max}]$. Searching for the argument of the maximum yields

$$\arg\max_{\delta \in [\widehat{\delta}, \delta^{max}]} R_B^*(\delta) = \sigma = \delta^{max}.$$

Just like the equilibrium demand for the flat-rate business, equilibrium revenues from launching a flat-rate business turn to zero when δ converges to $\widehat{\delta}$. The respective maximally attainable equilibrium revenue from implementing a flat-rate business within $[\widehat{\delta}, \delta^{max}]$ at δ^{max} is

$$\max_{\delta \in [\widehat{\delta}, \delta^{max}]} R_B^*(\delta) = R_B^*(\delta^{max}) = \frac{m v_B}{4}.$$

It is remarkable that $R_B^*(\delta^{max})$ always falls below of $R_A^*(\widehat{\delta})$ as long as $v_B > v_A$ is binding, meaning that the streaming service benefits from a low equilibrium nuisance level created by advertising and the resulting gain in importance of the free-of-charge business.
Equilibrium revenue functions must intersect within the interval $[\widehat{\delta}, \delta^{max}]$. Equating (25) and (26) and solving for δ yields the point of intersection

$$\widetilde{\delta} = \frac{\sigma(v_A - 2v_B) + 2\sqrt{\sigma^2(v_A^2 + v_B^2 - v_A v_B)}}{v_A}. \tag{27}$$

Finally, the monopolistic streaming service achieves a total equilibrium profit which can be determined as the summation of the equilibrium revenue functions from the respective businesses and which can therefore be written as

$$\pi^{M*}(p^{b**}, r^{**}, \delta) = \frac{m\delta\sigma v_B(v_B - v_A)}{4v_B\delta\sigma - v_A(\delta + \sigma)^2}. \tag{28}$$

Again, focusing attention on the investigation of π^{M*} according to its argument of the maximum as well as its argument of the minimum with respect to δ within $[\widehat{\delta}, \delta^{max}]$ yields

$$\arg\max_{\delta \in [\widehat{\delta}, \delta^{max}]} \pi^{M*}(\delta) = \frac{v_A\sigma}{2v_B - v_A} = \widehat{\delta}$$

and

$$\arg\min_{\delta \in [\widehat{\delta}, \delta^{max}]} \pi^{M*}(\delta) = \sigma = \delta^{max}.$$

The maximum equilibrium profit within $[\widehat{\delta}, \delta^{max}]$ can be observed at $\widehat{\delta}$, that is with a profit

fully captured from ad-funds while the minimum equilibrium profit can be reached at δ^{max}, where the monopolist solely acquires flat-rate subscribers. However, the equilibrium profit of the streaming service is a decreasing function as advertising becomes more of a nuisance, and the associated maximal and minimal equilibrium profits can therefore be computed as

$$\max_{\delta\in[\widehat{\delta},\delta^{max}]}\pi^{M*}(\delta) = \pi^{M*}(\widehat{\delta}) = \frac{m}{4}(2v_B - v_A) \tag{29}$$

and

$$\min_{\delta\in[\widehat{\delta},\delta^{max}]}\pi^{M*}(\delta) = \pi^{M*}(\delta^{max}) = \frac{mv_B}{4}. \tag{30}$$

Thus, the monopolist is better off with a low aversion against commercial breaks in equilibrium and the resulting focus on the ad-funded business. □

But it still remains to have a closer look at the revenues of the streaming service for values of δ which do not allow for launching a mixed funded business in equilibrium.

Proposition 3. *For values of δ outside of $[\widehat{\delta}, \delta^{max}]$, the monopolist generates the following equilibrium revenues depending on δ:*

- *For $\delta \in [\delta^{min}, \widehat{\delta}]$, the streaming service is solely financed through advertising. Nuisance created by commercials breaks is small in such a way that it is profit maximizing for the monopolist only to launch a free-of-charge business which yields a equilibrium profit of*

$$\pi^{M*}(\delta \leq \widehat{\delta}) = R_A^*(\delta \leq \widehat{\delta}) = \frac{\sigma m v_A}{4\delta}. \tag{31}$$

- *In contrast, for $\delta \in [\delta^{max}, \infty]$, potential users are disturbed from advertising in a way that it is profit maximizing for the monopolist to abandon ad-funding. The respective equilibrium profits which are solely generated by selling flat-rate contracts are thus given by*

$$\pi^{M*}(\delta \geq \delta^{max}) = R_B^*(\delta \geq \delta^{max}) = \frac{mv_B}{4}. \tag{32}$$

Proof. See Appendix.

These results can be clarified by employing a numerical plot. *Figure 3* illustrates the

equilibrium revenue functions of the monopolist from the free-of-charge as well as from the flat-rate business depending on δ. In doing so, the black and downward shaped curve illustrates total equilibrium profits $\pi^{M*}(\delta)$. For values of $\delta \leq \widehat{\delta}$ nuisance created by commercials is on such a low level that equilibrium profits will solely consist of revenues from ad-funding and no flat-rate contracts will be sold.

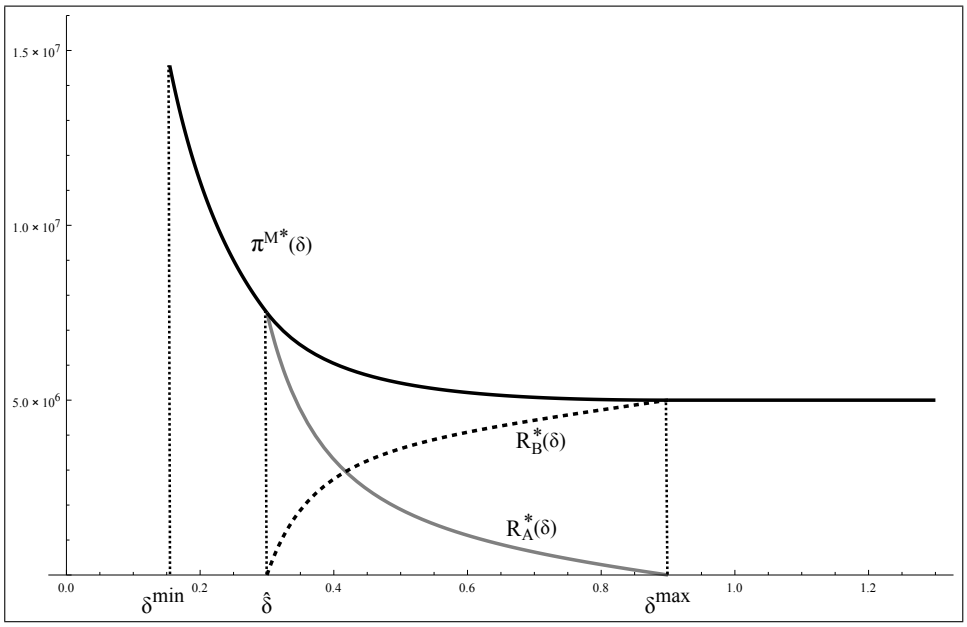

Figure 3: Equilibrium revenues of the monopolistic streaming service with respect to δ. Note: $\sigma = 0.9$, $v_A = 1000$, $v_B = 2000$ and $m = 10000$.

It is remarkable that $\pi^{M*}(\delta \leq \widehat{\delta})$ sharply increases with a decreasing nuisance from advertising. This is due to the structure of the model and can be explained by having a closer look on the timing of the model. Consider a decreasing δ in equilibrium. Thus, advertising becomes less of a nuisance for the users of streaming music. Before users decide to subscribe, advertisers expect an increasing free-of-charge demand and therefore they increase advertising space. Albeit potential users are now rather willing to listen to advertising, they would have to tolerate a larger equilibrium level of commercials if they decide to become a subscriber. Those effects completely offset each other and the equilibrium number of subscribers is therefore independent on δ and constant for values of $\delta \leq \widehat{\delta}$. The entire revenue from advertisers will be captured by the monopolist and if it does not implement a flat-rate business, the profit maximizing advertising charge is just the equilibrium demand multiplied by σ. This combination of a constant equilibrium

advertising charge accompanied by an increasing advertising space which does not create changes in the equilibrium free-of-charge demand due to a decreasing δ, allows for generating huge profits from launching an ad-funded business.

For values of $\delta \in [\widehat{\delta}, \delta^{max}]$, the streaming service maximizes its equilibrium profits by imposing a mixed funded business. The gray downward shaped curve illustrates the equilibrium revenues from the free-of-charge business while the dashed and upward sloped curve illustrates equilibrium revenues from launching a flat-rate business. Total equilibrium profits slightly decrease within this interval. For values of $\delta \geq \delta^{max}$, equilibrium demand for the free-of-charge business and thus, ad-funding turns to zero. The equilibrium profit, which no longer depends on δ, is now solely generated from launching a flat-rate business.

If one considers the revenue possibilities for the music industry, it seems as if it can be highly beneficial for an online-streaming provider to launch either a mixed funded business or a business model which does not charge its subscribers and which is financed by commercials. Depending on the degree of aversion to commercials, a monopolistic streaming provider either earns more from the ad-funded business or from the flat-rate business, but total equilibrium profits are always greater than or equal to revenues from a pure flat-rate funding. But if the aversion to commercial breaks exceeds a certain level, launching a flat-rate business remains the only revenue source from streaming music in an equilibrium which is determined by a monopolistic provider. However, the precondition for the assessment of profitability of such a business model is to estimate the aversion to commercial breaks of the potential subscribers, but detailed empirical research is needed to determine the disutility from advertising.

Proposition 4. *Within the interval $[\widehat{\delta}, \delta^{max}]$, equilibrium revenues of the streaming service and the aggregated equilibrium subscriber surplus drift apart with a decreasing nuisance from advertising. The higher the marginal social benefit of one additional commercial depending on δ is, the more excessive will be the equilibrium use of advertising of the streaming service in order to siphon all benefits from advertisers at subscribers' cost.*

Proof. We found that a monopolistic streaming service only launches an ad-funded business for values of $\delta < \sigma$ in equilibrium. From a perspective regarding social benefits from advertising and under the assumptions that *i*) δ and σ are exogenously given and *ii*) free-entry for advertisers is still binding, this implies that a monopolistic streaming service is only willing to launch an advertising funded business if social benefits exceed social costs from advertising, and if this social surplus from advertising can entirely be captured by the monopolist. The higher the margin between δ and σ (and therefore social surplus from advertising) is in equilibrium, the more excessive will be the level of commercial breaks and thus, the focusing on the free-of-charge business.

In a model investigating a media platform which funds its business through simultaneously charging its subscribers and broadcasting advertising, Anderson and Coate (2005) found that the number of advertisers takes an optimal level if marginal the social benefit and the marginal social cost of one additional advertiser are equated. Here, the social cost of one more commercial aired on the free-of-charge business in equilibrium should be investigated compared to the social benefit. Therefore, the streaming service reduces its charge to attract one additional advertiser, but we do not suppose the monopolist to react with an adjustment in the flat-rate price[24]. Thus, the marginal social cost consists on the one hand of the increased aggregated nuisance from advertising for free-of-charge subscribers and on the other hand of the flat fee which has to be payed by those users, who have been induced to switch from the free-of-charge to the flat-rate business. If one additional commercial is aired in equilibrium, one can observe a shift of free-of-charge subscribers in both directions: users with a low gross valuation for music will quit their free-of-charge subscription, while those who are characterized by a high gross valuation for music will become flat-rate users. From $\partial D_A/\partial a < 0$, it follows that a new number of free-of-charge users is now bothered by one additional commercial, parameterized by δ.

[24] Flat fees are often given by catchy amounts, like £9.99 for the *Spotify* premium account, familiarizing customers with the service and therefore are reluctantly modified by the service.

The marginal social cost of one more advertising spot in equilibrium is therefore given by

$$MSC(a) = \delta \underbrace{\left(\frac{\partial D_A}{\partial a} \frac{\partial a}{\partial r} \mathrm{d}r \right)}_{\mathrm{d}D_A} + \underbrace{\left(\frac{\partial D_B}{\partial a} \frac{\partial a}{\partial r} \mathrm{d}r \right)}_{\mathrm{d}D_B} p^b. \tag{33}$$

In contrast, the social benefit from one additional commercial in equilibrium is equal to the marginal revenue of the monopolist. The streaming service gains the charge from the new advertiser on its free-of-charge business which corresponds to the new reduced number of free-of-charge users multiplied by σ. As is a cost for subscribers, the increase in the number of subscribers to the flat-rate business multiplied by the price for a contract is a gain for the monopolist. The marginal social benefit from one additional advertiser is therefore

$$MSB(a) = \sigma \underbrace{\left(\frac{\partial D_A}{\partial a} \frac{\partial a}{\partial r} \mathrm{d}r \right)}_{\mathrm{d}D_A} + \underbrace{\left(\frac{\partial D_B}{\partial a} \frac{\partial a}{\partial r} \mathrm{d}r \right)}_{\mathrm{d}D_B} p^b. \tag{34}$$

Thus, the difference between the marginal social benefit and the marginal social loss of one more commercial is given by the margin between σ and δ and will be fully internalized by the streaming service. Hence, the higher this margin is, the stronger will be the focus of the streaming service on the free-of-charge business in equilibrium. The result can be clarified by employing a numerical example. Aggregated short-term subscriber surplus

$$SS(\delta) = m \int_{\frac{\delta a}{v_A}}^{\frac{p-\delta a}{\eta}} (\theta v_A - \delta a) \mathrm{d}\theta + m \int_{\frac{p-\delta a}{\eta}}^{1} (\theta v_B - p^b) \mathrm{d}\theta \tag{35}$$

is the summation of the net valuations of all subscribers to the free-of-charge and the flat-rate business. Inserting the equilibrium flat-rate price (19) and the equilibrium level of commercials (20) yields

$$SS^*(\delta) = \frac{m v_B (4\delta^2 \sigma^2 v_B^2 + \delta v_A v_B (\delta^3 - 6\delta^2 \sigma + \delta \sigma^2 - 4\sigma^3) + \sigma v_A^2 (2\delta^3 + \delta^2 \sigma + \sigma^3))}{2(v_A(\delta + \sigma)^2 - 4\delta \sigma v_B)^2}. \tag{36}$$

Figure 4 plots the aggregated equilibrium subscriber surplus compared to the equilibrium profits of the streaming service according to the nuisance cost parameter δ. Again, the black curve illustrates equilibrium profits of the streaming service, while the gray curve is the aggregated equilibrium surplus of subscribers depending on the nuisance cost param-

eter δ. It is easy to see that with a decreasing nuisance from advertising, the aggregated equilibrium subscriber surplus and the equilibrium profits of the streaming service visibly drift apart for values of $\delta < \delta^{max}$ for which an ad-funded business will be launched.

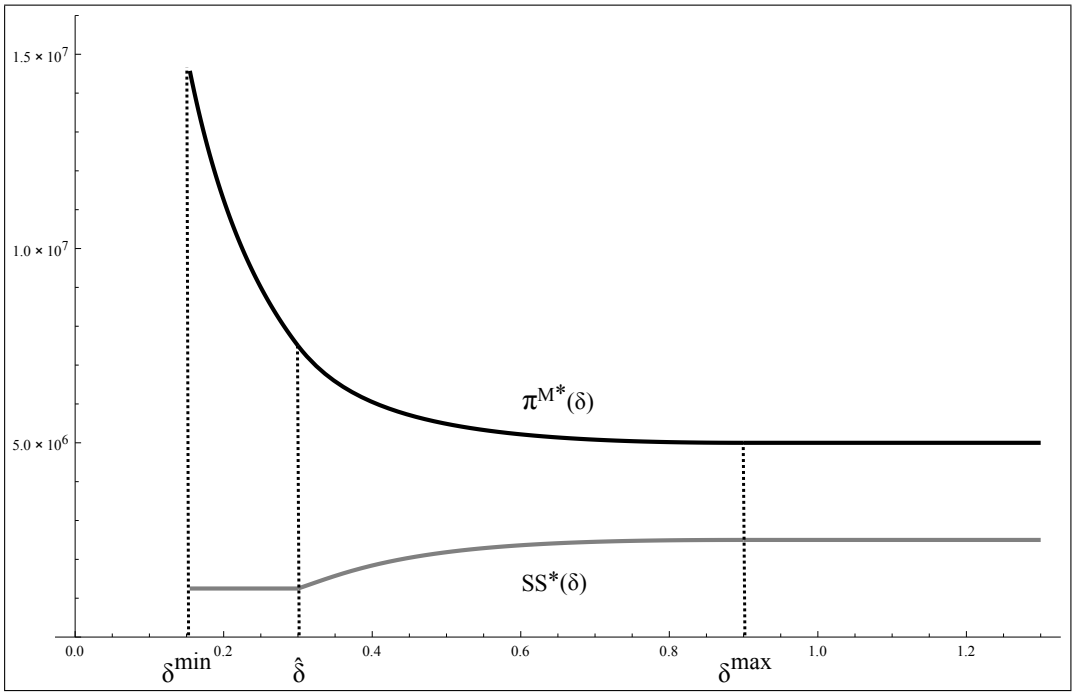

Figure 4: Equilibrium profits of the monopolistic streaming service compared to the aggregated equilibrium subscriber surplus with respect to δ. Note: $\sigma = 0.9$, $v_A = 1000$, $v_B = 2000$ and $m = 10000$.

For values of $\delta \leq \widehat{\delta}$, the aggregated equilibrium subscriber surplus is independent on δ and of a constant level. The same is true for values of $\delta \geq \delta^{max}$. We found in the proof of *Proposition 3* that a variation of δ for $\delta \leq \widehat{\delta}$, associated with the resulting shift in commercials, is neutral in terms of the net valuation of the free-of-charge subscribers. Thus, the equilibrium demand for the free-of-charge business as well as the aggregated equilibrium surplus of the free-of-charge users remain constant. Said constant levels of aggregated equilibrium subscriber surplus can be written as

$$SS^*(\delta \leq \widehat{\delta}) = m \int_{\frac{\delta a^*(\delta \leq \widehat{\delta})}{v_A}}^{1} (\theta v_A - \delta a^*(\delta \leq \widehat{\delta}))\mathrm{d}\theta = \frac{m v_A}{8} \tag{37}$$

and

$$SS^*(\delta \geq \delta^{max}) = m \int_{\frac{p^{b**}(\delta \geq \delta^{max})}{v_B}}^{1} (\theta v_B - p^{b**}(\delta \geq \delta^{max}))\mathrm{d}\theta = \frac{m v_B}{8}. \tag{38}$$

The loss in aggregated equilibrium subscriber surplus within the interval $[\widehat{\delta}, \delta^{max}]$ is created by those subscribers who are forced to switch from the flat-rate to the free-of-charge business by the price, which will be increased by the monopolist due to a decrease in δ. For those subscribers who are characterized by a relatively high gross valuation for music, it would be rather beneficial to sign a flat-rate contract than to listen to advertising. □

The following section intends to shed light on a situation where potential users have an alternative to the streaming service in terms of illegal file-sharing.

4 Piracy

Advertising funded streaming services intend to generate profits for the music industry from those music consumers who are not willing to pay for music. P2P file-sharing platforms allow for access to music which is not charged and as we mentioned in the introduction, are assumed to be responsible for enormously swooning revenues of the music industry. Consider that piracy, which appears in the form of illegally downloading of mp3-files, is now considered as alternative for those subscribers who are not willing to pay the flat-rate price in order to listen to music free of advertising.[25]

Suppose that the illegal download of music provides an intrinsic value of v_P which is assumed to exceed v_A but to fall below v_B. This strict assumption is due to the fact that illegally downloading allows for the physical ownership of the respective song, what is assumed to be of greater value than streaming with a low streaming rate and further quality constraints. In contrast, the intrinsic value provided by the flat-rate business is assumed to be higher because its additional features like mobile application or its linkage to social network sites.[26] Furthermore one can argue that music consumers do not have

[25] We assume that a flat-rate streaming account is preferred by those music consumers who listen to a lot of music and who tend to file-sharing compared to the essential more expensive alternative of buying mp3-files (Note that the digital music store *iTunes* prices a song between 69 cents and $ 1.29).

[26] In terms of quality one can also invoke that the compression of mp3-files reduces sonic quality compared to high quality streaming music. Additionally, streaming providers allow their customers for creating tracklists which can be listened in offline mode constituting equivalence between the physical ownership and the stream.

costs in order to search and eventually find a P2P-platform which offers the songs they like to download. Note that illegal downloads are associated with a certain level of risk for the music consumer due to enforcement of copyright law.[27] To measure this risk we introduce the parameter α which could be understand as a direct cost due to the expected punishment in case of being sued by the music industry.[28] Following *Assumption 1* of our model without piracy we impose the following restriction, namely $v_B > v_P > v_A$. Again, the streaming service sets the advertising charge and the flat-rate price in order to maximize revenues from the free-of-charge and the flat-rate business. Thus, according to the exogenously given parameters δ and σ, nuisance created by commercials will fall below v_A and the flat fee will fall below v_B. Again, the net valuation of a potential user according to her or his addiction to music can be measured by her or his utility which can now be written as

$$U_\theta = \begin{cases} 0 & \text{if } \theta \in [0, \underline{\theta}] \\ \theta v_A - \delta a & \text{if } \theta \in [\underline{\theta}, \theta^P] \\ \theta v_P - \alpha & \text{if } \theta \in [\theta^P, \widehat{\theta}] \\ \theta v_B - p^b & \text{if } \theta \in [1 - \widehat{\theta}, 1]. \end{cases} \tag{39}$$

The potential user characterized by θ^P is exactly indifferent between a free-of-charge subscription and P2P-file-sharing, while all potential users to the right of $\widehat{\theta}$ subscribe to the flat-rate business. All respective indifferent potential users can be determined as it was done in the previous model without piracy by equating utility functions. Thus,

$$\theta^P = \frac{\alpha - \delta a}{v_P - v_A} \tag{40}$$

and

$$\widehat{\theta} = \frac{p^b - \alpha}{v_B - v_P}. \tag{41}$$

[27] Since the successful suit of the music industry against *Napster*, the industry continued to sue individual users instead of taking legal action against P2P file-sharing platforms with the intension to discourage illegal downloads by threaten drastic punishment (See e.g. Bhattacharjee et al. (2006)).

[28] A similar parameter which measures the costs of illegal file-sharing was introduced by Duchêne and Waelbroeck (2006).

Note that the potential user who is indifferent between the free-of-charge business and not subscribing at all remains unchanged. *Figure 5* illustrates the respective intervals which determine the segmental allocation of potential users under consideration of piracy.

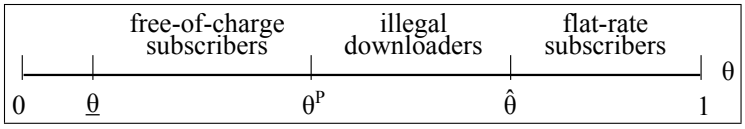

Figure 5: Segmentation of potential users under piracy with respect to θ.

The respective number of potential users who decide either to subscribe to the free-of-charge business or to the flat-rate business can be computed as

$$D_A(a,\alpha) = m(\theta^P - \underline{\theta}) = \frac{m(\delta a v_P - \alpha v_A)}{v_A(v_A - v_P)} \tag{42}$$

and

$$D_B(p^b,\alpha) = m(1 - \widehat{\theta}) = \frac{m(v_B - v_P + \alpha - p^b)}{v_B - v_P}. \tag{43}$$

Finally, the number of potential users who illegally download is

$$P(p^b,a,\alpha) = m(\widehat{\theta} - \theta^P) = m\left(\frac{p^b - \alpha}{v_B - v_P} - \frac{\alpha - \delta a}{v_P - v_A}\right). \tag{44}$$

The demand for advertising space of the advertisers is now depending on $D_A(a,\alpha)$ and we still assume advertisers to enter the free-of-charge business until their profits vanish. The level of commercials which will be provided by advertisers can thus be written as

$$\widetilde{a}(r,\alpha) = \frac{m\sigma\alpha + v_A(r(v_A - v_P))}{m\delta\sigma v_P}. \tag{45}$$

Of course one can observe a positive impact of an increase in α on $\widetilde{a}(r,\alpha)$ due to an increasing demand for the free-of-charge business caused by an increasing expected penalty through legal action. Thus, under consideration of piracy, the monopolist faces the following profit function

$$\pi^M(p^b,r,\alpha) = \left(\frac{v_A(r(v_A - v_P) + m\alpha\sigma)}{m v_P \delta\sigma}\right) r + \left(\frac{m(v_B - v_P + \alpha - p^b)}{v_B - v_P}\right) p^b. \tag{46}$$

It is assumed that the streaming service maximizes its profits under an exogenously given

degree of copyright enforcement. Again, the maximization problem is reduced to the two first-order conditions $\partial\pi^M/\partial r = 0$ and $\partial\pi^M/\partial p^b = 0$. FOC can thus be written as

$$r^*(\alpha) = \frac{m\alpha\sigma}{2(v_P - v_A)} \tag{47}$$

and

$$p^{b*}(\alpha) = \frac{v_B - v_P + \alpha}{2}. \tag{48}$$

In contrast to the previous model without piracy, the equilibrium advertising charge and flat-rate price are no longer linearly interdependent, but both increase due to an increase in α in equilibrium. Inserting (47) in (45) yields the equilibrium level of commercials

$$a^*(r^*, \alpha) = \frac{v_A\alpha}{2v_P\delta}. \tag{49}$$

Using this equilibrium level of commercials allows for the determination of the position of the subscriber who is indifferent between a free-of-charge subscription and no subscription at all

$$\underline{\theta}^*(r^*, \alpha) = \frac{\alpha}{2v_P}. \tag{50}$$

By means of $r^*(\alpha)$ and $p^{b*}(\alpha)$, one can derive the position of those subscribers $\theta^{P*}(r^*, \alpha)$ and $\widehat{\theta}^*(p^{b*}, \alpha)$ who determine the equilibrium allocation of potential users to the P2P-file-sharing platforms and the respective businesses of the streaming service. We have that

$$\theta^{P*}(r^*, \alpha) = \frac{\alpha(2v_P - v_A)}{2v_P(v_P - v_A)} \tag{51}$$

and

$$\widehat{\theta}^*(p^{b*}, \alpha) = \frac{v_B - v_P - \alpha}{2(v_B - v_P)}. \tag{52}$$

It is straightforward that the upper bound of the segmental allocation which determines the number of illegal file-sharers $\widehat{\theta}^*(p^{b*}, \alpha)$ depends negatively on α, while the lower bound $\theta^{P*}(r^*, \alpha)$ depends positively on α in equilibrium. Equating (51) and (52) and solving for α yields the maximal necessary effort of legal action in order to strengthen the expected punishment due to copyright enforcement in a way that induces all potential users who have

illegally downloaded music to become subscribers to the streaming service in equilibrium.[29] This level is given by

$$\alpha^{max}(p^{b*}, r^*) = \frac{v_P(v_B - v_P)(v_P - v_A)}{v_P(2v_B - v_P) - v_A v_B}. \tag{53}$$

Furthermore, it can be shown that $\theta^{P*}(r^*, \alpha)$ always exceeds $\underline{\theta}^*(r^*, \alpha)$ for values of $\alpha > 0$, meaning that the monopolist always launches an ad-funded business as long as there is a threat of being sued due to illegal file-sharing. Thus, one can determine the equilibrium range of the level of the legal threats parameter which is given by

$$0 < \alpha \leq \frac{v_P(v_B - v_P)(v_P - v_A)}{v_P(2v_B - v_P) - v_A v_B}.$$

Proposition 5. *Under piracy and the presence of a profit maximizing monopolist, all potential users suffer losses from an increase in legal threats, while overall welfare will be maximized at α^{max}. The level of α can therefore be understand as a rent allocation mechanism between the monopolist and potential users.*

Proof. The short-term aggregated surplus of all potential users in equilibrium can again be written as the summation of the net valuations of all the subscribers to the free-of-charge and the flat-rate business as well as of those music listeners who illegally download music. Thus, aggregated equilibrium "music consumer surplus" is

$$\begin{aligned} CS^{P*}(\alpha) = m \int_{\frac{\alpha}{2v_P}}^{\frac{\alpha(2v_P - v_A)}{2v_P(v_P - v_A)}} \left(\theta v_A - \delta\left(\frac{v_A \alpha}{2v_P \delta}\right)\right) \mathrm{d}\theta \\ + m \int_{\frac{\alpha(2v_P - v_A)}{2v_P(v_P - v_A)}}^{\frac{v_B - v_P - \alpha}{2(v_B - v_P)}} (\theta v_P - \alpha)\mathrm{d}\theta + m \int_{\frac{v_B - v_P - \alpha}{2(v_B - v_P)}}^{1} \left(\theta v_B - \frac{v_B - v_P + \alpha}{2}\right) \mathrm{d}\theta. \end{aligned} \tag{54}$$

By taking the argument of the maximum as well as the argument of the minimum of

[29] We determine α^{max} as the upper threshold for the legal threats parameter, because a further increase would lead to economic unintuitive results.

$CS^{P*}(\alpha)$ with respect to α within the legal threats interval $[0, \alpha^{max}]$, one can show that $CS^{P*}(\alpha)$ is a decreasing function in the strength of legal threats. By considering the restrictions $0 < v_A < v_P < v_B$ as well as $\delta > 0$ one derives

$$\arg\max_{\alpha \in [0,\alpha^{max}]} CS^{P*}(\alpha) = 0$$

and

$$\arg\min_{\alpha \in [0,\alpha^{max}]} CS^{P*}(\alpha) = \frac{v_P(v_B - v_P)(v_P - v_A)}{v_P(2v_B - v_P) - v_A v_B} = \alpha^{max}.$$

Thus, aggregated equilibrium music consumer surplus will be maximal at $\alpha = 0$ and its minimum at α^{max}, within the interval of legal threats. An increase in legal threats in equilibrium induces more potential users not to listen to streaming music at all. A social planner who is acting in music consumers' interest, would therefore reduce prosecution for illegal file-sharing to allow as many music consumers as possible for listening music regardless of whether this will be done by subscribing to the streaming service or by illegally downloading. The overall short-term equilibrium welfare function under piracy is given by the summation of the aggregated equilibrium surplus of music consumers and the equilibrium profit of the streaming service and can therefore be written as

$$\begin{aligned} W^{P*}(\alpha) = CS^{P*}(\alpha) &+ \left(\frac{\alpha v_A}{2\delta v_P}\right)\left(\frac{m\alpha\sigma}{2(v_P - v_A)}\right) \\ &+ m\left(1 - \frac{v_B - v_P - \alpha}{2(v_B - v_P)}\right)\left(\frac{v_B - v_P + \alpha}{2}\right). \end{aligned} \tag{55}$$

Note that under piracy σ may not urgently exceed δ in order to induce the streaming service to launch a free-of-charge business. The equilibrium revenue of the streaming service from launching a free-of-charge business are strictly positive being independent on the relation between σ and δ. Again, we search for the argument of the maximum of $W^{P*}(\alpha)$ with respect to α in the interval $[0, \alpha^{max}]$ under the familiar constraints and we find that

$$\arg\max_{\alpha \in [0,\alpha^{max}]} W^{P*}(\alpha) = \frac{v_P(v_B - v_P)(v_P - v_A)}{v_P(2v_B - v_P) - v_A v_B} = \alpha^{max}.$$

One can show that $W^{P*}(\alpha)$ is convex in α within the interval $[0, \alpha^{max}]$. Note that $W^{P*}(\alpha)$ slightly starts to decrease if one increases α in equilibrium starting from $\alpha = 0$. But the higher α will be in equilibrium, the stronger will be the increase of

the equilibrium profit of the streaming service. The positive effect of the increasing marginal profit of the monopolist completely offsets the negative effect of the marginal loss in the number of music listeners (and thus the negative effect of the marginal loss in aggregated equilibrium surplus of music consumers). Hence, overall short-term welfare in equilibrium is always maximal at the upper threshold of the interval for legal threats. □

Furthermore, $W^{P*}(\alpha^{max})$ increases with an increasing difference between the social benefit and the social cost of a marginal increase in α in equilibrium. To show this, one has to set up the equilibrium demand functions for the free-of-charge and the flat-rate business under piracy which can be written as

$$D_A^*(r^*, \alpha) = m(\theta^{P*}(r^*, \alpha) - \underline{\theta}^*(r^*, \alpha)) = \frac{\alpha m}{2(v_P - v_A)} \tag{56}$$

and

$$D_B^*(p^{b*}, \alpha) = m(1 - \widehat{\theta}^*(p^{b*}, \alpha)) = \frac{m(v_B - v_P + \alpha)}{2(v_B - v_P)}. \tag{57}$$

Let us first have closer look at the effect of a marginal increase of α in equilibrium on the ad-funded business which consists of two contrary impacts. At first, a more rigorous prosecution of illegal file-sharing induces potential users who previously have been illegal file-sharers to turn into free-of-charge subscribers. One can observe an increase in $\theta^{P*}(r^*, \alpha)$ and therefore an increase in the demand for the free-of-charge business. This incremental demand causes an increase in the number of commercials on the free-of-charge business[30], which is accountable for a loss of subscribers who decide to quit the free-of-charge business and not to subscribe at all. Therefore we have that $\partial\underline{\theta}^*(r^*, \alpha)/\partial\alpha > 0$. But this effect will be outweighed by the first one and taken together, one can observe that $\partial D_A^*(r^*, \alpha)/\partial\alpha > 0$. The social cost of a marginal increase in α consists of the nuisance from the increased level of commercials for the increased number of free-of-charge subscribers and of the flat-rate price, which has to be payed by those potential users who will be induced to sign a flat contract.

[30] We have $\partial a^*(\alpha)/\partial\alpha > 0$, although this causes an increasing equilibrium advertising charge.

The social cost of a marginal increase of α is therefore given by

$$MSC(\alpha) = \delta \left(\frac{\partial D_A}{\partial \alpha} \mathrm{d}\alpha \right) \left(\frac{\partial a}{\partial \alpha} \mathrm{d}\alpha \right) + \left(\frac{\partial D_B}{\partial \alpha} \mathrm{d}\alpha \right) p^b.$$

In contrast, the marginal social benefit is the increase in the profit of the monopolist due to a marginal increase in α and is thus given by

$$MSB(\alpha) = \sigma \left(\frac{\partial D_A}{\partial \alpha} \mathrm{d}\alpha \right) \left(\frac{\partial a}{\partial \alpha} \mathrm{d}\alpha \right) + \left(\frac{\partial D_B}{\partial \alpha} \mathrm{d}\alpha \right) p^b.$$

Consider the demand for the flat-rate business. An increase in α induces former illegal file-sharers to become subscribers to the flat-rate business and the loss they have to bear, namely the price p^b, which is again supposed to remain constant, is identical to the additional benefit for the streaming service. It follows that a marginal increase of α is neutral in terms of welfare with respect to the flat-rate business. Note that the marginal social cost and the marginal social benefit strictly increase when α increases in equilibrium and it is easy to see that as long as the willingness to pay of an advertiser to reach one free-of-charge subscriber exceeds the nuisance cost parameter in equilibrium, the social benefit of a marginal increase in α exceeds the social cost. The social benefit of a marginal increase in α will completely be captured by the monopolist. Thus, the higher the margin between the social benefit and the social cost of a marginal increase in α is, the higher will be the increase in the profit of the monopolist compared to the decrease in the aggregated equilibrium surplus of music consumers due to this marginal increase in α. Hence, an increasing margin between σ and δ let $\max_{\alpha \in [0, \alpha^{max}]} W^{P*}(\alpha)$ increase.

5 Concluding Remarks

This paper has investigated the performance of a monopolistic provider of streaming music which offers a free-of-charge advertising funded business on the one hand and a flat-rate business on the other hand to its subscribers, who strategically decide whether to subscribe to one of these businesses or to refuse to streaming music at all. The model was extended by illegal file-sharing, meaning that music consumers may be able to illegally download and to possess digitalized music files if they are willing to face the costs of being sued.

The investigation focused on an analysis of the equilibrium with respect to the nuisance created by commercials. By doing so, threshold values with respect to the aversion to advertising for which either imposing an ad-funded business or a flat-rate business is not gainful anymore for a monopolistic streaming service were determined. A monopolistic streaming provider only launches an advertising funded free-of-charge business if it can fully internalize all revenues which are generated from advertisers. Advertising funding can be highly profitable if nuisance caused by commercial breaks is relatively weak. When a low aversion to advertising can be observed, a monopolistic provider would increase its flat-rate price in order to induce more paying subscribers to turn to free-of-charge users which leads to losses in the welfare of subscribers. If music consumers posses the alternative of illegally downloading music by using P2P file-sharing platforms, the streaming service always launches a free-of-charge business in equilibrium as long as there is a risk of being sued by the music industry. In equilibrium, aggregated surplus of music consumers is reduced due to an increase in the enforcement of the copyright law. Thus, strengthening of legal threats allows a social planner for allocating rents from music consumers to the monopolistic provider.

An interesting question would be: how and to what extent music labels and their affiliated artists benefit or suffer from streaming businesses in the long run. As it was mentioned in the introduction of this paper, a major point of criticism concerning streaming businesses are insufficient royalty payments for musicians compared to suppliers which sell digital music files like *iTunes*. If this criticism comes true, it would be interesting to investigate whether such streaming business models were deteriorating long term welfare due to impaired revenues for artists which may be of a restrictive effect on the variety of published music. An approach concerning long term welfare losses due to a decrease in production is provided by Novos and Waldman (1984) who show in the case of illegal copying of a nonexcludable good that due to an increase in copyright protection, social welfare loss caused by underproduction decreases.

A Appendix

Proof of Assumption 2.

To derive the optimal strategy of the monopolistic streaming provider with respect to its two revenue sources in equilibrium second order conditions must hold. The corresponding Hessian matrix of the streaming service's profit function is given by

$$H(\pi^M) = \begin{bmatrix} -\frac{2m}{v_B} & \frac{v_A}{\delta v_B} - \frac{v_A}{\sigma v_B} \\ \frac{v_A}{\delta v_B} - \frac{v_A}{\sigma v_B} & \frac{2v_A(v_A - v_B)}{m v_B \delta \sigma} \end{bmatrix}.$$

To fulfill the second-order conditions the Hessian must be negative semi-definite for all (r, p^b). This is fulfilled if the first leading principal minor will be negative and the determinant will be positive. The first principal minor $-\frac{2m}{v_B}$ is always negative and the determinant is given by $\det H = \frac{4v_A}{\delta\sigma v_B} - \frac{v_A^2}{v_B^2}\left(\frac{1}{\delta^2} + \frac{1}{\sigma^2} + \frac{2}{\delta\sigma}\right)$. The determinant is positive if

$$4\delta\sigma v_B > v_A(\sigma + \delta)^2$$

is fulfilled. □

Proof of Proposition 3.

For values of $\delta \leq \widehat{\delta}$, the streaming service maximizes its profit by solely launching an ad-funded business. The respective demand is therefore reduced to

$$D_A(\delta \leq \widehat{\delta}) = m\left(1 - \frac{\delta a}{v_A}\right).$$

The endogenously determined level of commercials can thus be computed as

$$\widetilde{a}(\delta \leq \widehat{\delta}) = \frac{v_A(\sigma m - r)}{\delta\sigma m}.$$

Without launching a flat-rate business, the profit function of the streaming service is now reduced to $\pi^M(\delta \leq \widehat{\delta}) = \widetilde{a}(\delta \leq \widehat{\delta})r$. Maximizing with respect to r yields

$$r^*(\delta \leq \widehat{\delta}) = \frac{m\sigma}{2}.$$

Note that the streaming service asks for an equilibrium charge $r^*(\delta \leq \widehat{\delta})$ which is now independent on δ. The equilibrium level of commercials is

$$a^*(\delta \leq \widehat{\delta}) = \frac{v_A}{2\delta}$$

which yields a constant equilibrium free-of-charge demand of

$$D_A^*(a^*(\delta \leq \widehat{\delta})) = \frac{m}{2}.$$

Finally, equilibrium profits of the streaming service, which are reduced to revenues from launching a free-of-charge business, are given by

$$\pi^{M*}(\delta \leq \widehat{\delta}) = R_A^*(\delta \leq \widehat{\delta}) = \frac{\sigma m v_A}{4\delta}.$$

For $\delta \geq \delta^{max}$ advertising becomes of such a nuisance that it is not profitable for the monopolis to launch an ad-funded business. The demand for the flat-rate business is thus given by

$$D_b(\delta \geq \delta^{max}) = m\left(1 - \frac{p^b}{v_B}\right).$$

The profit of the streaming service is now reduced to $\pi^M(\delta \geq \delta^{max}) = D_b(\delta \geq \delta^{max})p^b$. Maximizing with respect to p^b let us derive the equilibrium flat-rate price

$$p^{b*}(\delta \geq \delta^{max}) = \frac{v_B}{2}.$$

The associated demand is now independent on δ and can be written as

$$D_b^*(p^{b*}(\delta \geq \delta^{max})) = \frac{m}{2}.$$

Hence, equilibrium profits of the streaming service can are

$$\pi^{M*}(\delta \geq \delta^{max}) = R_B^*(\delta \geq \delta^{max}) = \frac{m v_B}{4}.$$

□

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