

When Does a Platform Create Value by Limiting Choice?*

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When Does a Platform Create Value by Limiting Choice?

Abstract

We present a theory for why it might be rational for a platform to limit the number of applications available on it. Our model is based on the observation that even if users prefer application variety, applications often also exhibit direct network effects. When there are direct network effects, users prefer to consume the same applications to benefit from consumption complementarities. We show that the combination of preference for variety and consumption complementarities gives rise to (i) a commons problem (to better satisfy their individual preference for variety, users have an incentive to consume more applications than the number that maximizes joint utility); (ii) an equilibrium selection problem (consumption complementarities often lead to multiple equilibria, which result in different utility levels for the users); and (iii) a coordination problem (lacking perfect foresight, it is unlikely that users will end up buying the same set of applications). The analysis shows that the platform can resolve these problems by limiting the number of applications available. By limiting choice, the platform may create new equilibria (including the allocation that maximizes users' utility); eliminate equilibria that give lower utility to the users; and reduce the severity of the coordination problem faced by users.

Classification-JEL: D21, D42, L12, L82, L86

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1 Introduction

Platforms such as computer operating systems (Windows), video game systems (Nintendo), betting exchanges (Betfair), stock exchanges (NYSE), or online gaming sites (Kaixin001) are institutions that facilitate users' access to applications (defined as opportunities to fulfill users' particular purposes—such as writing documents, playing games, betting money, or investing capital).¹ Among the many governance choices that platform providers make, they determine the number of applications to provide access to (e.g., how many games to offer by a given online gaming platform, how many firms to list by a given stock exchange, and so on). In this paper, we study the relationship between the number of applications available on a platform and users' equilibrium utility. We find that narrow choice often increases utility and thus creates value.

Platforms are characterized by the presence of *indirect* network effects: the larger the number of users, the more firms are willing to join thus increasing the diversity of applications available, which, in turn, raises users' valuation of the platform. For example, firms' desire to list their shares in the New York Stock Exchange grows with the number of investors who are expected to trade there; likewise, the larger the number of firms expected to be listed in the NYSE, the more willing the investors are to invest there (Cantillon and Yin 2011). Naturally, indirect network effects induced by users' *preference for application variety* have played a prominent role in models of platforms, beginning, at least, from the pioneering work of Church and Gandal (1992) and Chou and Shy (1996) and spanning to recent contributions such as Hagiu (2009) or Weyl (2010).

When the value of a platform increases with the number of applications offered, common wisdom dictates that platforms should provide as many applications as possible. Indeed, suboptimal exploitation of indirect network effects may have dreadful consequences: superior platforms (better technology, better capitalized, early movers...) may perish in their competition against second-rate alternatives. Arthur (1990), for example, describes how Sony lost its battle against JVC in the 1980s whose VHS standard was inferior to Betamax, due largely to lesser movie availability on Sony's standard. Likewise, it is widely believed that Apple lost its battle against the PC in the late 1980s because of a dearth of applications. While Microsoft aggressively evangelized independent software vendors and provided them with tools and support, Apple based its approach on in-house development of a small

¹Examples of applications include: word processors or spreadsheet programs (in the case of computer operating systems), games (in video game systems or online gaming sites), sports events (in betting exchanges), and listed companies (in stock exchanges).

number of applications. By the early 1990s, the number of applications available for the Mac was a small fraction to that for the PC.

Given the wealth of evidence suggesting that maximizing application variety is a good idea, it is puzzling that successful platform providers such as Betfair, Nintendo, or Kaixin001 appear to have actively limited the number of applications available on their platforms. Betfair provides an electronic platform that allows its customers to back teams to win in sports such as soccer or horse races, but also to lay odds for others to bet on. The company began operations in the U.K. in 2000 as a second mover after Flutter.com. Although Flutter was the first mover and had better access to capital (its initial funding was \$43.7 million vs. £1 million for Betfair), Betfair won over the market.² A key difference between the two betting exchanges was that while Flutter would allow users to bet on any event they wished to create (such as next week’s weather), Betfair adamantly restricted the number of events (applications) on which users could bet. Interestingly, the platform that offered fewer applications ended up faring better.

Similarly, in the late 1980s Nintendo restricted the number of games that developers were allowed to release each year for the Nintendo Entertainment System (NES) to five. The company also restricted the number of developers who could sell games for the NES. Nintendo went on to become the dominant player (market share and profit) for the 8-bit generation.³ Likewise, the leading online social networking site, Kaixin001, provides a limited number of games for users to engage in (e.g., Parking Cars and Stealing Crops) when many more could be offered. The site offers the smallest number of social games among the top social networking sites in China and lags behind its competitors in making its platform open to third party application developers,⁴ but the site has the most highly active users among them.⁵ These examples run counter the conventional wisdom that when considering application variety in platforms “more is always better.”

In this paper, we ask: Why might it be rational for a platform to limit the number of applications when indirect network effects are at play? Our answer is that by limiting the number of applications the platform may resolve three problems faced by users: a *commons* problem, an *equilibrium selection* problem, and a *coordination* problem.

²Betfair acquired Flutter in December 2001 and is currently the dominant betting exchange in Europe. See Casadesus-Masanell and Campbell (2008).

³The NES was the leading second-generation (8-bit) game console. Nintendo’s global market share for 8-bit consoles in 1990 was greater than 90%. See Brandenburger (1995).

⁴<http://www.nth-wave.com/wordpress/?p=32985>

⁵<http://www.nytimes.com/external/venturebeat/2010/04/07/07venturebeat-chinas-top-four-social-networks-renren-kaixi-55248.html>

Our theory is based on the observation that even when platforms enjoy indirect network effects, applications often exhibit *direct* network effects, i.e., users are better off using the same applications as other users due to *consumption complementarities*. For example, Cantillon and Yin (2010) demonstrate that there are important direct network effects in derivatives' trading. Specifically, as the number of traders for a particular derivative increases, so does liquidity. Similarly, the richness of gameplay in massively multiplayer online games (MMOG), such as World of Warcraft, is based on the number of interactions between players; MMOGs are not fun if played alone. When users have limited resources (such as finite time to enjoy applications or an income constraint) and there are many applications available, they must pick and choose which ones to use. If direct network effects are at play, users are better off by purchasing and consuming the same limited set of applications.

We show that when users prefer application variety but also benefit from consumption complementarities, three issues may arise. First, the number of applications that maximizes users' utility may not be part of an equilibrium as each user may find it optimal to unilaterally deviate to consume more applications so as to better satisfy her craving for variety. Second, multiple equilibria often arise. With the usual assumption that users have perfect foresight, any one of those equilibria could, in principle, be selected. While some equilibria lead to higher user utility than others, nothing guarantees that the equilibrium yielding the highest utility will be selected. Third, if users lack perfect foresight on each others' choices in equilibrium, it is unlikely that they will end up purchasing and consuming the exact same set of applications, but such coordination is necessary to fully exploit consumption complementarities.

Our analysis demonstrates that by limiting the choice of applications, the platform can accomplish three tasks. First, it can create equilibria that did not exist when application choice was broad. In particular, the allocation that maximizes users' utility can be guaranteed to be an equilibrium thus relieving the commons problem. Second, it can eliminate socially inferior equilibria, effectively resolving the equilibrium selection problem. Third, it can reduce the severity of the coordination problem faced by users when they do not know other users' choices in equilibrium. With a smaller choice set, it is more likely that users will end up purchasing and consuming the same applications and thus more likely that they will enjoy consumption complementarities. We conclude that when direct and indirect network effects are at play, platforms may create value by limiting choice.

1.1 Literature

Our paper contributes to the literature on platforms and two-sided markets, a literature that has flourished on the basis of industry-specific models. Rochet and Tirole (2003), for example, is inspired by the credit card industry, Armstrong (2006) captures well the economics of newspapers, and Hagiu (2009) is about competition between video game systems. While most of the literature on platforms has studied questions related to pricing, our focus is on one aspect of platform governance that has received little attention thus far: the effect of limiting the choice of applications on user behavior and, ultimately, on the value created by the platform.

The only two papers we are aware of that are directly related to the question that we address here are Zhao (2010) and Hałaburda and Piskorski (2010). Zhao (2010) studies hardware/software platforms and explores the effects of quantity constraints on product quality and variety on a monopolistic two-sided platform where quality is uncontractible. He finds that when users cannot perfectly observe application quality, developers underinvest in quality and that the platform can then use quantity restraints to help mitigate free-riding and increase overall application quality. While Zhao (2010) studies the effects of quantity limitations on the behavior of developers, we study the effects on the behavior of users. A second point of differentiation is that while he provides an explanation for why it may make sense for the platform to limit the number of applications per developer, in his theory the platform gains nothing from limiting the number of developers. Therefore, contrary to ours, his theory is silent about the benefits of limiting the overall number of applications offered by the platform.

Hałaburda and Piskorski (2010) study dating platforms, an environment with indirect network effects: men prefer a market with a larger number of women, and women prefer a market with more men. Nonetheless, they show that users may benefit when dating platforms limit the number of candidates among which to find a match. This is because dating platforms limit the number of candidates on both sides. Thus by limiting choice, platforms also limit competition between agents in the same side. Some agents prefer a platform with less choice, because it increases the probability that they will find a match. The current paper differs from Hałaburda and Piskorski (2010) in two ways. First, Hałaburda and Piskorski (2010) is the best suited for markets with one-to-one matching, like dating or housing markets. The current paper focuses on markets where users can consume a large number of applications. Moreover, applications are infinitely duplicable: When one user consumes an application, it does not limit the availability of the same application to other

users. Second, our setting lacks the competitive effect that drives the result in Halaburda and Piskorski (2010). To the contrary: as a result of consumption complementarity, the direct network effect is positive. Users gain if more users (on the same side of the market) consume the same applications. Thus users benefit when the platform restricts choice because it helps them take advantage of consumption complementarities to a fuller extent, rather than avoiding competition.

The paper is organized as follows. In Section 2 we present the game with perfect foresight, solve for equilibria under direct and indirect network effects, and discuss the utility implications of the platform limiting choice. In Section 3 we recast the model as one where users have no foresight about other users' choices in equilibrium. In Section 4 we discuss our main modeling choices as well as some extensions to the analysis. Section 5 concludes. All proofs are in an appendix.

2 Game with perfect foresight

We consider a platform which brings together developers and users of applications. There is a set \mathbb{A} of available applications and N users. We denote the cardinality of \mathbb{A} by A . We treat N and \mathbb{A} as exogenous.

Let x_a^k denote user k 's consumption of application a . The consumption utility that user k derives from consuming $\mathbf{x}^k = (x_1^k, x_2^k, \dots, x_A^k)$ applications is given by

$$u(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) = \underbrace{\left(\sum_{a \in \mathbb{A}} (x_a^k)^{1/R} \right)^R}_{\text{preference for variety}} + \underbrace{\alpha \sum_{a \in \mathbb{A}} (x_a^k \sum_{l \neq k} x_a^l)}_{\text{consumption complementarity}},$$

where $\alpha \geq 0$ captures the strength of consumption complementarity, and $1 \leq R < 2$ captures the intensity of the user's preference for variety.⁶ The larger is α , the more the users benefit from consuming the same applications. Likewise, the larger is R , the more the users prefer application variety, i.e., consuming a larger number of applications.

Consumption utility u captures both, direct and indirect network effects. *Indirect* net-

⁶Note that when $\alpha = 0$, preferences are as in Dixit and Stiglitz (1977). Moreover, the analysis is only interesting for $R < 2$. For such R the marginal benefit from consuming another application is positive but decreasing. Conversely, for $R \geq 2$, the marginal benefit from consuming another application is increasing or constant. Thus, since for $p < X$ a user finds it beneficial to consume one application, then for $R \geq 2$ she will always find beneficial to consume *all* applications. To keep the analysis interesting, we restrict $R < 2$.

work effects originate from users' preference for variety: users prefer platforms with more users because it is more likely that more applications will be developed for that platform. Therefore, when R is larger, the source of indirect network effects is stronger. When $R = 1$, however, users have no preference for variety and, therefore, there are no indirect network effects.

Direct network effects are present when a user's utility from consuming an application increases with other users' consumption levels of the same application. For example, users of video games enjoy a given game more if their friends also consume the same game, as they can discuss strategies to beat the game. Direct network effects are captured by the term $\alpha \cdot x_a^k \cdot \sum_{l \neq k} x_a^l$: user k 's enjoyment of her consumption of application a is larger the more the other users ($l \neq k$) consume application a . We let $\alpha \geq 0$. When $\alpha = 0$, there are no direct network effects and as α increases, direct network effects become stronger. In summary, user preferences may exhibit direct or indirect network effects, or both, depending on the value of parameters α and R .

We assume that users have a budget of X units of time to consume applications and interpret $x_a^k \geq 0$ as the amount of time that user k spends consuming application a . Thus, if user k consumes a set $\mathbb{Q}^k \subseteq \mathbb{A}$ of applications, she must satisfy the time budget constraint: $X \geq \sum_{a \in \mathbb{Q}^k} x_a^k$. Each application is sold at exogenous monetary price $p > 0$, regardless of how much time users spend consuming it.⁷ Since the monetary dimension is different from the time dimension, spending p does not detract from the time budget X . We assume that p is sufficiently low for users to find it desirable to purchase and consume at least one application, i.e., we let $p < X$. Therefore, it follows that users consume at least one application, i.e. $Q^k \geq 1$, where Q^k denotes the cardinality of \mathbb{Q}^k .

User k 's net utility from consuming \mathbf{x}^k when price is p is given by

$$U(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) = u(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) - p \cdot \sum_{a \in \mathbb{A}} \mathbf{1}(x_a^k), \quad (1)$$

where $\mathbf{1}(\cdot)$ is an indicator function taking value 1 when its argument is different from zero.

Since the focus of our analysis is on the value of limiting choice, we also assume that absent action by the platform to constrain the set of available applications, the cardinality of \mathbb{A} is large. Specifically, we assume that $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$. We will show that this guarantees that there are sufficiently many different applications available for users to satisfy their preference for variety.

⁷We present the results for $p = 0$ in Appendix B.

We consider the following two-stage game: In the first stage, all users decide simultaneously which applications to purchase at price p . In the second stage, users decide simultaneously how to allocate their time budget X across the applications they have purchased. We solve for the subgame-perfect Nash equilibria in pure strategies and follow Katz and Shapiro (1985) in assuming that expectations are fulfilled in equilibrium.

Formally, given that user k has already purchased set of applications \mathbb{Q}^k , in the second stage she chooses consumption \mathbf{x}^k to maximize her own consumption utility u given the expected consumption of all other $N - 1$ users, \mathbf{x}^l for $l \neq k$:

$$\max_{x_a^k, a \in \mathbb{Q}^k} u(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) \quad \text{subject to} \quad X \geq \sum_{a \in \mathbb{Q}^k} x_a^k. \quad (2)$$

In the first stage, users choose the set of applications to purchase, $\mathbb{Q}^k \subseteq \mathbb{A}$, anticipating their own consumption and that of all other users in the second stage. User k 's objective is to maximize her own net utility U .

We end the description of the model by presenting two definitions that are helpful for the discussion of equilibria.

Definition 1 (balanced strategy) *Let $\mathbb{Q}^k = \{a | x_a^k > 0\}$ be the set of applications consumed by user k . And let Q^k be the cardinality of \mathbb{Q}^k . We say that user k 's strategy is balanced if $x_a^k = \frac{X}{Q^k}$ for all $a \in \mathbb{Q}^k$.*

Thus, a balanced strategy is one where the user allocates her time budget equally across all the applications she consumes. Note that balanced strategies are pure strategies and that for any \mathbb{Q}^k there is a unique balanced strategy.

Definition 2 (balanced equilibrium) *An equilibrium is balanced if all users play balanced strategies.*

In this section, we solve the game under the assumption that users have perfect foresight about other users' choices in equilibrium. This is a classic assumption of rational beliefs, a part of Nash equilibrium. Later, in Section 3 we relax the perfect-foresight assumption.

In the remainder of this section, we investigate each type of network effect separately before considering the interplay of both types together. We first study the model with direct network effects and find that users consume one single application so as to take full advantage of consumption complementarities (Section 2.1). Then, we move on to studying the model with pure indirect network effects and find that users choose to consume a large number

of applications, driven by their preference for variety (Section 2.2). Next, we study the interplay between the two types of network effects and find that there is a tradeoff between harnessing consumption complementarities and the utility gains from product variety. In the equilibrium that yields the highest utility to users, they always consume a smaller number of applications than under pure indirect network effects (Section 2.3). Finally, we show that the platform can create value by limiting the number of applications available even if users have perfect foresight about each others' purchase and consumption decisions (Section 2.4).

2.1 Direct network effects

There are pure direct network effects when users derive utility from consuming the same applications as other users but not from product variety. Therefore, consumption utility u exhibits pure direct network effects when $R = 1$ and $\alpha > 0$. In this case, user k 's net utility (1) takes the form

$$U_D(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) = \sum_{a \in \mathbb{A}} x_a^k + \alpha \sum_{a \in \mathbb{A}} (x_a^k \sum_{l \neq k} x_a^l) - p \cdot \sum_{a \in \mathbb{A}} \mathbf{1}(x_a^k).$$

User k 's consumption of application a in an equilibrium is denoted by \hat{x}_a^k . Let $\mathbb{Q}_D^k \subseteq \mathbb{A}$ be a set of applications that user k consumes in equilibrium in an environment with pure direct network effects. Then, the cardinality of \mathbb{Q}_D^k is $Q_D^k = \sum_{a \in \mathbb{A}} \mathbf{1}(\hat{x}_a^k)$. Remark 1 characterizes the equilibria in this case.

Remark 1 *When $R = 1$ and $\alpha > 0$, in every equilibrium $\mathbb{Q}_D^k = \mathbb{Q}_D$ for all k and the number of applications consumed is $Q_D^k = Q_D = 1$ for all k . There are A equilibria. All equilibria are balanced and yield the highest possible utility to the users.*

Proof. See Appendix A, page 30.

Because $R = 1$, users derive no utility from product variety. However, because $\alpha > 0$ they derive utility from other users consuming the same applications for longer periods of time. Indeed, user k 's marginal utility of consuming application a is increasing in other users' aggregate consumption of a ,

$$\frac{\partial u_D(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k})}{\partial x_a^k} = 1 + \alpha \cdot \sum_{l \neq k} x_a^l.$$

Therefore, the more other users consume application a , the more user k desires to consume a . Since the same applies to all users, in equilibrium all users consume the same application. Users could coordinate on any one of the A applications available, since all users and all applications are homogeneous.

2.2 Indirect network effects

There are pure indirect network effects when users derive utility from product variety but not from consuming the same applications as other users. Therefore, consumption utility u exhibits pure indirect network effects when $1 < R < 2$ and $\alpha = 0$. In such a case, user k 's net utility (1) takes the form

$$U_I(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) = \left(\sum_{a \in \mathbb{A}} (x_a^k)^{1/R} \right)^R - p \cdot \sum_{a \in \mathbb{A}} \mathbf{1}(x_a^k). \quad (3)$$

Note that (3) is essentially the same as the setup in Dixit and Stiglitz (1977), with two exceptions. First, the cost of time spent using application a is set in our model to 1 for all $a \in \mathbb{A}$. Second, we impose a price $p > 0$ that users must pay to use an application.⁸ Remark 2 characterizes the equilibria under pure indirect network effects.

Remark 2 *Assume $1 < R < 2$ and $\alpha = 0$. In every equilibrium the number of applications consumed is $Q_I^k = Q_I$ for each user k , where $Q_I = \max\{1, (\frac{(R-1)X}{p})^{\frac{1}{2-R}}\}$. All equilibria are balanced and yield the highest possible utility to the users.⁹*

Proof. See Appendix A, page 31.

To understand this result, notice that Dixit and Stiglitz (1977) implies that when $\alpha = 0$ and $p \rightarrow 0$, the solution to optimization problem (2) is $Q_I \rightarrow \infty$ and $\hat{x}_a^k = \frac{X}{Q_I} \rightarrow 0$. Users derive utility from product variety and find it optimal to consume as many applications as possible in equal proportions. The result is driven by the fact that, as long as $R > 1$,

⁸More precisely, our cost of time (which we normalize to 1) corresponds to the application prices in the original Dixit-Stiglitz's formulation. In contrast to Dixit-Stiglitz, we assume that users must pay a fixed price for access to each application she consumes, $p > 0$. This price is independent of the usage. For example, when users buy a particular videogame title, they pay for it once regardless of the usage, and then they allocate scarce time to playing the game. In our model, the price of the game is p and the opportunity cost of time allocated to playing the game is 1.

⁹There are $\frac{N \cdot A!}{Q_I!(A-Q_I)!}$ pure-strategy subgame-perfect Nash equilibria and continuum mixed strategy equilibria.

applications have infinite marginal consumption utility around zero:

$$\lim_{x_a^k \rightarrow 0} \frac{\partial u_I(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k})}{\partial x_a^k} = \infty$$

and that this marginal utility decreases as consumption increases. Therefore, spreading the time budget evenly across $Q + 1$ applications yields more utility than spreading the same time budget across Q applications.

To determine how many applications to purchase, users must compare the additional benefit from consuming an additional application and the price p that they must pay for that application. Specifically, if Q applications are consumed by a user in optimal consumption schedule, her utility is $(Q(\frac{X}{Q})^{\frac{1}{R}})^R - pQ = Q^{R-1}X - pQ$. Therefore, the marginal benefit from increasing Q is $(R - 1)Q^{R-2}X$. The marginal cost of an additional application is p . The number of applications at which the marginal benefit and marginal cost are equal is $(\frac{(R-1)X}{p})^{\frac{1}{2-R}}$.

As customary in the platforms literature (e.g., Ellison and Fudenberg 2003), we ignore the integer problem and treat the number of applications Q as a continuous variable. As indicated on page 6, users consume at least one application. Thus, if $(\frac{(R-1)X}{p})^{\frac{1}{2-R}} < 1$, the user consumes one application. That is, the optimal consumption is characterized by $Q_I = \max\{1, (\frac{(R-1)X}{p})^{\frac{1}{2-R}}\}$. As we show later, the number of applications consumed under direct and indirect network effects is never larger than Q_I . Therefore, to focus on non-trivial analysis, from now on we assume $Q_I > 1$,¹⁰ which implies $Q_I = (\frac{(R-1)X}{p})^{\frac{1}{2-R}}$.

Let $\mathbb{Q}_I^k \subseteq \mathbb{A}$ be the set of applications that user k consumes in equilibrium in an environment with pure indirect network effects. Remark 2 states that all users consume the same number of applications in equilibrium, i.e., $Q_I^k = Q_I$ for all k . However, it does not need to be that users consume the same applications, i.e., it may be that $\mathbb{Q}_I^k \neq \mathbb{Q}_I^l$ for k and $l \neq k$. This is because users gain no utility from consuming the same applications as others. Thus, any N subsets of \mathbb{A} with cardinality Q_I constitutes an equilibrium.

2.3 Interplay between direct and indirect network effects

Now we investigate what happens when users in the platform experience both direct and indirect network effects, so that they derive utility from product variety and from consuming the same applications as other users. In such a case, $1 < R < 2$ and $\alpha > 0$. Let $\mathbb{Q}_{DI}^k \subseteq \mathbb{A}$

¹⁰The working paper version (Casadesus-Masanell and Halaburda 2010) also considers the case where $Q_I = 1$.

be a set of applications that user k consumes in equilibrium in an environment with direct and indirect network effects, and let Q_{DI}^k be the cardinality of \mathbb{Q}_{DI}^k . Note that in the cases of pure direct and of pure indirect network effects, in all equilibria we could uniquely characterize the number of applications user k consumed in an equilibrium, i.e., $Q_D^k = 1$ and $Q_I^k = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}} > 1$. However, as we show below, when both direct and indirect network effects are present, multiple values of Q_{DI}^k are possible.

The study of this hybrid specification is substantially more complex than the cases of pure direct and pure indirect network effects. We will show that there is always a set of equilibria close to Q_I , the number of applications that users would choose if only indirect network effects were at play. And if consumption complementarity is sufficiently strong relative to preference for variety, another set of equilibria emerges around consuming $Q_D = 1$, the equilibrium number of applications consumed under pure direct network effects. Specifically, under the hybrid network effects equilibria emerge which are not equilibria under pure network effects of either type. Moreover, possible cardinalities of the consumption set in an equilibrium depend on the strength of consumption complementarity relative to that of preference for variety. We present the analysis in parts, beginning with two helpful lemmas.

Lemma 1 *Assume that $1 < R < 2$ and $\alpha > 0$. In every balanced equilibrium $\mathbb{Q}_{DI}^k = \mathbb{Q}_{DI}$ for all k .*

Proof. See Appendix A, page 33.

Lemma 2 *Assume that $1 < R < 2$ and $\alpha > 0$. If Q_{DI} is the cardinality of the consumption set in a balanced equilibrium, then any set of applications $\mathbb{Q}_{DI} \subseteq \mathbb{A}$ of cardinality Q_{DI} constitutes a balanced equilibrium.*

Proof. See Appendix A, page 35.

Lemma 1 says that in every balanced equilibrium all users consume the same applications. It is driven by presence of direct network effects. Lemma 2 says that if Q_{DI} is the number of applications consumed in a particular balanced equilibrium, then there are $C_{Q_{DI}}^A$ equilibria with the same number of applications consumed. For example, if $A = \{1, 2, 3, 4\}$, $Q_{DI} = 2$ characterizes six balanced equilibria: $\mathbb{Q}_{DI_1} = \{1, 2\}$; $\mathbb{Q}_{DI_2} = \{1, 3\}$; $\mathbb{Q}_{DI_3} = \{1, 4\}$; $\mathbb{Q}_{DI_4} = \{2, 3\}$; $\mathbb{Q}_{DI_5} = \{2, 4\}$; and $\mathbb{Q}_{DI_6} = \{3, 4\}$. It is easy to see that users derive the same utility in all of these equilibria and, thus, we think of them as equivalent. The lemmas imply that

in the case of $1 < R < 2$ and $\alpha > 0$ we may completely characterize balanced equilibria by simply stating equilibrium cardinalities Q_{DI} . For clarity of exposition, we refer to balanced equilibria by just indicating their cardinality, Q_{DI} .

Suppose that all users play balanced strategies and consume the same set of applications of cardinality Q . Then, each user's net utility (1) is given by $V(Q)$:

$$V(Q) = Q^{R-1}X + \alpha \frac{X^2}{Q}(N-1) - pQ. \quad (4)$$

Function $V(Q)$ is helpful in studying balanced equilibria. Not every Q constitutes an equilibrium. But Lemma 1 implies that the net utility in every balanced equilibrium must be given by $V(Q)$.

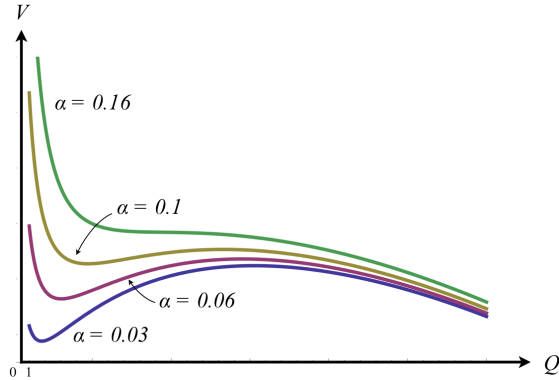


Figure 1: Shape of V for different values of α .
($R = 1.7135$, $A = 30$, $X = 2$, $N = 16$, $p = 0.646$.)

Figure 1 illustrates the shape of V for different values of α . The shape of V is driven by the weight of consumption complementarity relative to that of preference for variety. As shown by Remark 1, consumption complementarity and the resulting direct network effects induce users to consume one application only. Remark 2, however, shows that preference for variety and the resulting indirect network effects induce users to consume more applications. The graph in Figure 1 shows that when users have strong preference for variety compared to consumption complementarity (low α relative to R), indirect network effects outweigh direct network effects and the Q that maximizes V is interior. When preference for variety is weak relative to consumption complementarity direct network effects outweigh indirect network effects and $Q = 1$ maximizes V .

Let

$$\widehat{Q} = \max \left\{ 1, Q \text{ such that } \frac{dV}{dQ} = 0 \right\}.$$

If V has interior maxima, then \widehat{Q} is the unique interior maximum. Otherwise, V reaches its maximum at $\widehat{Q} = 1$. As we can see in Figure 1, when α is large, $\widehat{Q} = 1$ (cf. $\alpha = 0.16$ in the figure). Otherwise, $\widehat{Q} > 1$ (other values of α in the figure). The value \widehat{Q} is important for the shape of V . Specifically, for $Q > \widehat{Q}$, V is always decreasing. However, for $\widehat{Q} > 1$, when $Q < \widehat{Q}$, V first decreases and then increases. It is possible for some $Q < \widehat{Q}$ that $V(Q) > V(\widehat{Q})$. Let Q_* be $Q < \widehat{Q}$ such that $V(Q_*) = V(\widehat{Q})$, when $\widehat{Q} > 1$. (See Figure 2, below, for an example.)

The following remark states that \widehat{Q} is lower than Q_I , the equilibrium number of applications consumed when there are no direct network effects (as defined in Remark 2).

Remark 3 *Assume that $1 < R < 2$, $\alpha > 0$ and $Q_I > 1$. Then $\widehat{Q} < Q_I$.*

Proof. See Appendix A, page 35.

Intuitively, the presence of direct network effects prompts users to allocate their limited time budget to fewer applications. Consumption complementarity, due to other users consuming the same applications, compensates for the loss of application variety. The fact that we consider $Q_I > 1$ guarantees that the comparison between \widehat{Q} and Q_I is nontrivial.¹¹

As noted above, $\widehat{Q} > 1$ is the unique interior maximum of V . The following proposition shows that when $\widehat{Q} > 1$, users face a commons problem. Specifically, when all users consume $\widehat{Q} > 1$ applications, every user finds it profitable to unilaterally deviate upwards. However, when all of them deviate, they receive a lower utility. Notice that $\widehat{Q} > 1$ when preference for variety, R , is large relative to consumption complementarity, α .

Proposition 1 (commons problem) *Assume that $1 < R < 2$ and $\alpha > 0$. If $\widehat{Q} > 1$, then \widehat{Q} is not a balanced equilibrium. Specifically, for any user k , $U(Q^k = \widehat{Q} + \varepsilon, \{Q^l = \widehat{Q}\}_{l \neq k}) > V(\widehat{Q}) > V(\widehat{Q} + \varepsilon)$.*

Proof. See Appendix A, page 36.

The proposition states that when \widehat{Q} is interior, it cannot be a balanced equilibrium. This is because there is a profitable upward deviation, i.e., each user has incentive to consume more

¹¹For $Q_I = 1$, $\widehat{Q} = Q_I = 1$.

applications. By the definition of \hat{Q} , if $\hat{Q} > 1$, it must be that $\frac{\partial V(Q)}{\partial Q}|_{Q=\hat{Q}} = 0$. Therefore, an incremental balanced deviation upward to $\hat{Q} + \varepsilon$ has no effect on the utility of the deviator. However, the optimal unilateral deviation upward is *not* balanced.¹² The optimal upward deviation is strictly more beneficial to the user than the balanced deviation. Therefore, an optimal upward deviation is strictly profitable. Since all users have the same incentives, every user will deviate upwards to $\hat{Q} + \varepsilon$. As a consequence, every user will receive payoff $V(\hat{Q} + \varepsilon)$ which is lower than $V(\hat{Q})$. Therefore, users face a commons problem.

The following lemma shows that there is a large set of Q s that cannot characterize balanced equilibria. The result is helpful because it significantly constrains the set of Q s that may characterize equilibria.

Lemma 3 *Assume that $1 < R < 2$ and $\alpha > 0$. Then for any Q such that $\max\{1, Q_\star\} \leq Q < \hat{Q}$ or $Q > Q_I$, Q cannot characterize a balanced equilibrium.*

Proof. See Appendix A, page 37.

Figure 2 illustrates Lemma 3.

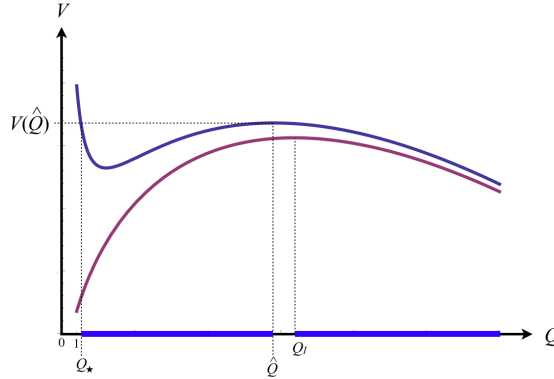


Figure 2: Intervals of Q that cannot be an equilibrium as described in Lemma 3.

To understand this result, consider first $Q > Q_I$. Given that all other users consume Q applications, any user has incentive to deviate downward to Q_I . The utility for user k from

¹²In the case of deviation upward, the deviator consumes some applications that no other user consumes. Due to consumption complementarity, an optimal consumption schedule then calls for more consumption of those applications that other users consume, and less (but positive) consumption of applications that only the deviator consumes.

deviating to $Q^k < Q$ is

$$U_{DI}(Q^k \leq Q) = (Q^k)^{R-1} X + \underbrace{\alpha Q^k \frac{X}{Q^k} (N-1) \frac{X}{Q}}_{\text{consumption complementarity}} - p Q^k. \quad (5)$$

Note that the consumption complementarity term is independent of Q^k . Since she consumes $Q^k < Q$, the deviator consumes only applications also consumed by the other users.¹³ Each of those applications is consumed by all other users at the level of $(N-1) \frac{X}{Q}$. The deviator divides her time budget X amongst the Q^k applications that she consumes, $Q^k \cdot \frac{X}{Q^k}$. Therefore, the benefit of the direct network effect is constant, no matter what $Q^k < Q$ the deviator chooses. However, the net benefit of variety $(Q^k)^{R-1} X - p Q^k$ is maximized at Q_I which is lower than Q . As a consequence, the deviator would want to deviate to Q_I . We conclude that $Q > Q_I$ may not be an equilibrium. Intuitively, consuming more than Q_I applications leads to “too much” application variety for the price. Moreover, if it had an effect, consumption complementarity would push users to consume fewer applications also.

For $Q \in [\max\{1, Q_\star\}, \widehat{Q})$, however, there is a profitable deviation upwards. In what follows we impose that the deviator balances her time budget across all the applications that she consumes. Even though this is not the optimal deviation, we show that it is a profitable deviation (and therefore, the optimal deviation is also profitable). Given that all other users consume Q applications in a balanced way, the utility of the deviator from a balanced consumption of Q^k applications is:

$$U_{DI}(Q^k \geq Q) = (Q^k)^{R-1} X + \underbrace{\alpha Q \frac{X}{Q^k} (N-1) \frac{X}{Q}}_{\text{consumption complementarity}} - p Q^k. \quad (6)$$

Note that $U_{DI}(Q^k \geq Q)$ is the same function of Q^k as V in equation (4) which has a local maximum at $\widehat{Q} > Q$. Moreover, for all $Q \in [\max\{1, Q_\star\}, \widehat{Q})$, $U_{DI}(\widehat{Q} > Q) > U_{DI}(Q)$. Thus, for all those values of Q , there is a profitable upward deviation. We conclude that $Q \in [\max\{1, Q_\star\}, \widehat{Q})$ may not be an equilibrium.

Intuitively, consuming more applications satisfies the deviator’s preference for variety to a

¹³Consider user k and suppose that all other users play balanced strategies consuming the same set of applications \mathbb{Q} . Directly, we can see that if user k decides to also consume Q applications, she consumes exactly the applications in \mathbb{Q} and no other. Moreover, it is optimal for her to consume them in equal amounts, i.e., she consumes them according to a balanced strategy. However, user k may also consider deviations that involve consuming a different number of applications.

greater extent. However, consuming less of each application consumed by other users means that the utility from consumption complementarity is lower. When $Q \in [Q_*, \widehat{Q}]$ the tradeoff is resolved in favor of consuming more applications.

Note that for $Q \in [1, Q_*]$ and $Q \in [\widehat{Q}, Q_I]$ the same tradeoff is at play. However, it is possible that the tradeoff is resolved in favor of consumption complementarity which means that it is not worth it for users to deviate upwards. In combination with Lemma 4 this observation implies that equilibria are possible in the intervals $Q \in [1, Q_*]$ and $Q \in [\widehat{Q}, Q_I]$. Lemmas 5 and 6 show that multiple equilibria exist in these intervals. We show that there are two aspects to this multiplicity. First, as described in Lemma 2, for any given cardinality Q_{DI} there may exist multiple sets \mathbb{Q}_{DI} —each constituting a separate equilibrium. Second, there may exist many different values of Q_{DI} that characterize equilibria. The former type of multiplicity is of no consequence to user utility while the latter has important utility implications. Thus, we focus only on the second type of multiplicity in our analysis.

The following lemma assures that so long as $Q \leq Q_I$, it is never beneficial for user k to deviate to a strategy with a lower number of applications. Thus, in searching for balanced equilibria, we need to focus only on deviations to a larger number of applications.

Lemma 4 *Assume that $1 < R < 2$ and $\alpha > 0$. If all users play balanced strategy \mathbb{Q} with cardinality $Q \leq Q_I$, then any unilateral deviation by user k to any other strategy with $Q^k < Q$ leads to lower utility for player k .*

Proof. See Appendix A, page 38.

To understand this result, suppose that all users are consuming $Q \leq Q_I$ and consider a deviation to $Q^k < Q$. We do not restrict the user to deviate to a balanced strategy with Q^k . However, from among all possible deviations to $Q^k < Q$, a balanced consumption of Q^k applications from the set \mathbb{Q} is the most profitable. Thus, the utility from the most profitable deviation to Q^k is given by formula (5):

$$U_{DI}(Q^k \leq Q) = (Q^k)^{R-1} X + \underbrace{\alpha Q^k \frac{X}{Q^k} (N-1) \frac{X}{Q}}_{\text{consumption complementarity}} - p Q^k.$$

Note that $U_{DI}(Q^k \leq Q)$ is increasing for all $Q^k \leq Q_I$ and therefore it is maximized at $Q^k = Q$. Thus, if $Q \leq Q_I$, there is no incentive to deviate downward.

Intuitively, consuming fewer applications satisfies user k 's preference for variety to a lesser extent. At the same time, there is no benefit from consumption complementarity. The reason is that each of the applications used by the deviator are consumed by all other users at the level of $(N - 1)\frac{X}{Q}$. Therefore, it is optimal to the deviator to divide her time budget X equally among the Q^k applications that she consumes, whereby she consumes $\frac{X}{Q^k}$ of each. Since $Q^k \cdot \frac{X}{Q^k} = X$, the benefit of the direct network effect is constant, no matter what Q^k the deviator chooses.

We use Lemma 4 to prove the result in Lemma 5. Lemma 5 states that there always exists a balanced equilibrium where all users consume Q_I applications and that Q s close but lower than Q_I also characterize equilibria. Together with Lemma 3, Lemma 5 indicates that Q_I is the equilibrium with the largest number of applications consumed.

Lemma 5 *When $1 < R < 2$ and $\alpha > 0$, there always exist balanced equilibria with $Q_{DI} = Q_I$, where $Q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}} > 1$. Furthermore, there exists $Q^o < Q_I$ such that any $Q \in [Q^o, Q_I]$ characterizes balanced equilibria, i.e., $Q = Q_{DI}$.*

Proof. See Appendix A, page 39.

Figure 3a illustrates the result in Lemma 5. This result means that so long as users exhibit preference for variety, no matter how small, there are balanced equilibria with the same number of applications, Q_I , that users would choose to consume if there were no direct network effects.

To understand why Q_I is an equilibrium, by Lemma 4 we need only consider deviations upward. By the same argument to that following equation (6), a deviation upward (balanced or unbalanced) cannot improve the utility from consumption complementarity. Moreover, Q_I maximizes utility from preference for variety. Therefore, there are no incentives to deviate and Q_I is an equilibrium.

A deviation upward always decreases utility from consumption complementarity. Notwithstanding, for $Q < Q_I$ there is some benefit from increased variety. For Q less than but close to Q_I , however, this benefit is infinitesimally small (the FOC is satisfied at Q_I) and it is outweighed by the utility loss from consumption complementarity. Therefore, Q s less than but close to Q_I also characterize equilibria.

Lemma 6 shows that for some parameters there may also exist equilibria with $Q_{DI} = 1$.

Lemma 6 *Assume that $1 < R < 2$ and $\alpha > 0$. There exist parameter values such that $Q_{DI} = 1$ while $Q_I > 1$.*

Proof. See Appendix A, page 41.

Notice that if $\hat{Q} > 1$, it is necessary that $V(Q=1) > V(\hat{Q})$ for $Q=1$ to be an equilibrium. It follows from Lemma 3. However, there is nothing in the proof that connects $Q_{DI} = 1$ to Q° . So the equilibrium at $Q_{DI} = 1$ may be disconnected from the set of equilibria around Q_I .

The result of Lemma 6 is illustrated in Figure 3b. There we can see that equilibria exist in two disconnected intervals: one interval around $Q = 1$ (recall that following Remark 1 $Q_D = 1$ is the equilibrium under pure direct network effects) and the other one around Q_I . In the interval around $Q_D = 1$, the strong consumption complementarities (users consume the same few applications intensely) guarantee that users do not want to deviate to consume more applications. In the interval around Q_I , the weak consumption complementarities (users consume little of many applications) guarantee that users do not want to deviate to consume fewer applications.

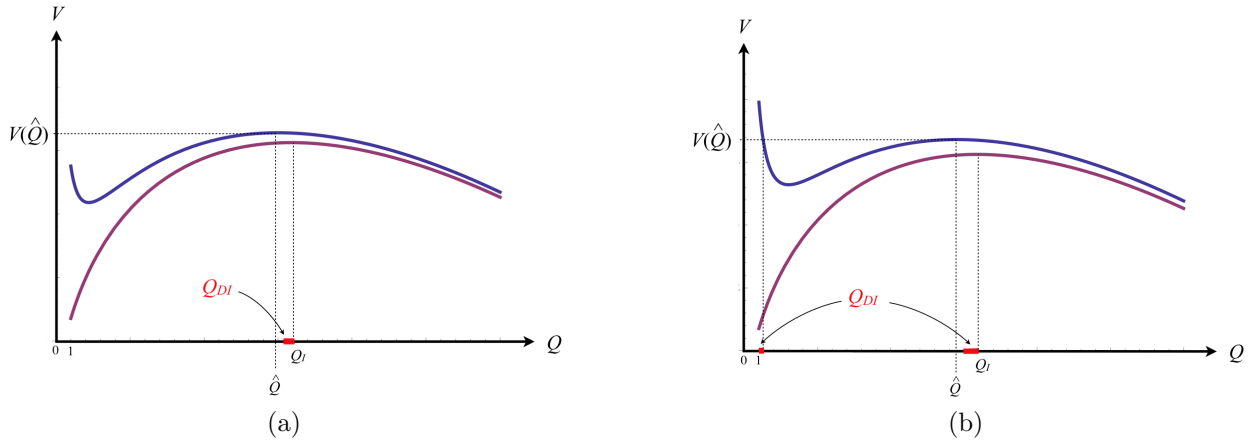


Figure 3: Intervals of Q_{DI} .

Lemmas 5 and 6 show that there are always multiple equilibria. We now show that the equilibria can be ranked according to users' utility. In particular, equilibria with fewer applications consumed yield higher utility than equilibria with more applications. However, even when consumption complementarity is large relative to preference for variety and the allocation that maximizes users' utility is an equilibrium, the model does not predict which of the many equilibria will be played or the utility that users will achieve. Therefore, users face an equilibrium selection problem.

Proposition 2 (equilibrium selection problem) *When $1 < R < 2$ and $\alpha > 0$ there exist multiple balanced equilibria with different values of Q_{DI} . Equilibria with smaller Q_{DI} yield higher utility than equilibria with larger Q_{DI} .*

Proof. See Appendix A, page 42.

To understand the intuition, recall that function $V(Q)$ is user utility in a situation where every user consumes Q applications in a balanced way. Therefore, for values of Q_{DI} that constitute a balanced equilibrium, $V(Q_{DI})$ is the utility that users obtain in equilibrium. As follows from Lemma 3 and illustrated by Figure 2, equilibria only occur for values of Q such that $V(Q)$ is decreasing. Therefore, equilibrium utility must be decreasing in Q_{DI} .

In conclusion, Propositions 1 and 2 identify two undesirable properties of equilibria in environments where both direct and indirect network effects are present. When preference for variety is large relative to consumption complementarity, users face a common problem because the allocation that maximizes users' utility is not an equilibrium. Moreover, regardless of the values of α and R there are always multiple equilibria which yield different levels of utility. Thus users also face an equilibrium selection problem, even when consumption complementarity is large relative to preference for variety and the allocation that maximizes users' utility is an equilibrium. In the following subsection we show how the platform can alleviate these problems by limiting the number of applications available.

2.4 On the role of the platform: creating value by limiting choice

We conclude Section 2 by showing that users may benefit when the platform limits the number of applications available; but only when both direct and indirect network effects are present. To examine the platform's choice of the number of applications available, A , we relax the assumption that $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$.

Notice first that when pure direct network effects are present (i.e., $R = 1$), users achieve the same net utility in an equilibrium, for any $A \geq 1$. The platform cannot improve on this.

Corollary 1 *Assume that $R = 1$ and $\alpha > 0$. Then the platform cannot change the net utility that users achieve in an equilibrium, for any $A \geq 1$.*

Likewise, the platform cannot improve the equilibrium under pure indirect network effects (i.e., $\alpha = 0$). From Section 2.2 we know that when $A \geq Q_I$ in an equilibrium under pure indirect network effects, every user consumes Q_I applications. As Corollary 2 states, when

the platform sets $1 < A < Q_I$, users consume all available applications, but this yields lower utility than consuming Q_I applications.

Corollary 2 *Assume that $1 < R < 2$ and $\alpha = 0$. If $1 < A < Q_I$, then there exists a unique equilibrium. This is a balanced equilibrium where all users consume all A applications. The net utility of users in equilibrium strictly increases with A for $A < Q_I$.*

Therefore, in the case of pure indirect network effects the platform can only decrease users' utility when limiting the number of available applications. When $A < Q_I$ users strictly gain from access to a larger number of applications. And when $A \geq Q_I$, the users do not gain or lose by having more applications available.

We now turn to the case where both direct and indirect network effects are present (i.e., $1 < R < 2$ and $\alpha > 0$). The following definition is helpful for the arguments that follow. Let

$$Q^{**} = \arg \max V(Q). \quad (7)$$

From the shape of V follows that Q^{**} may be either 1 or \widehat{Q} . In both cases $Q^{**} \leq \widehat{Q} < Q_I$.¹⁴ When $Q^{**} = \widehat{Q} > 1$, then by Proposition 1, Q^{**} never characterizes a balanced equilibrium. When $Q^{**} = 1$, it may characterize a balanced equilibrium (as Lemma 6 shows), but it not always does. We show that by limiting choice when Q^{**} is not an equilibrium, the platform helps users solve the commons problem shown in Proposition 1. I.e., the platform creates an equilibrium at Q^{**} . And when Q^{**} is an equilibrium, it will typically be one of many equilibria which yield lower utility than Q^{**} . Thus in this case, by limiting choice the platform helps users solve the selection problem in Proposition 2.

Proposition 3 shows that regardless of whether Q^{**} is in the equilibrium set of the original game, the platform can ensure that Q^{**} becomes the only equilibrium of the game by restricting A to Q^{**} .

Proposition 3 *Assume that $1 < R < 2$ and $\alpha > 0$. If the platform sets $A = Q^{**}$, then there exists a unique balanced equilibrium where all users consume Q^{**} applications.*

Proof. See Appendix A, page 43.

¹⁴Notice that whether $Q^{**} = 1$ or $Q^{**} = \widehat{Q} > 1$ depends on the value of α relative to R . For small α (as $\alpha = 0.03$ in Figure 1) $Q^{**} = \widehat{Q} > 1$. For larger α (as $\alpha = 0.06$ and $\alpha = 0.1$ in the figure), $\widehat{Q} > 1$, but $Q^{**} = 1$. For even larger α (as $\alpha = 0.16$ in the figure), $Q^{**} = \widehat{Q} = 1$.

The proposition implies that the equilibrium set may change with changes in A . In particular, when the platform sets $A = Q^{**}$, Q^{**} becomes the unique equilibrium. Therefore, when Q^{**} is in the original equilibrium set, if the platform constrains A to be equal to Q^{**} , it eliminates all equilibria that yield lower utility for the users and, thus, eliminates the possibility that users select an inferior equilibrium. Hence,

Corollary 3 *Assume that $1 < R < 2$ and $\alpha > 0$. When Q^{**} is in the equilibrium set, users may benefit when platform restricts the number of available applications to Q^{**} .*

On the other hand, if Q^{**} is not in the original equilibrium set, when the platform constrains A to be equal to Q^{**} , it creates a new equilibrium that makes users better off than all the original equilibria. Thus,

Corollary 4 *Assume that $1 < R < 2$ and $\alpha > 0$. When Q^{**} is not in the equilibrium set, users strictly benefit when platform restricts the number of available applications to Q^{**} .*

In summary, when consumption complementarity is large relative to preference for variety, then Q^{**} is in the equilibrium set and the platform can eliminate other equilibria (which yield lower utility) by limiting the number of applications available. When preference for variety is large relative to consumption complementarity, then Q^{**} is not in the equilibrium set and by limiting the number of applications the platform creates a new, unique, equilibrium that yields highest possible utility.

3 Game with no foresight

Whenever direct network effects are present, the equilibria studied in Section 2 require users to know exactly which applications are consumed by all other users. That is, our assumption has been one of *perfect foresight* about other users' choices in equilibrium. In many environments, such perfect foresight may be difficult to achieve.

The literature has pointed out that in the presence of network effects, equilibria are influenced by the way users form their expectations.¹⁵ Therefore, it is important to analyze how the equilibrium set in our model changes when we step away from perfect foresight in beliefs formation. We focus here on a specific alternative way in which users form their beliefs. Specifically, we assume that users have no way of knowing which applications other users will consume when they make their own choices.

¹⁵See, for example, Hurkens and Lopez (2010).

Perfect foresight assumes that in equilibrium player k knows the cardinality and the identity of the applications that all other users will consume. Moreover, she also knows how much of each application other users consume. As an alternative, now we assume *no foresight* by which we mean that users initially assign equal probability to any feasible strategy of other users. However, they refine their beliefs by Bayesian updating and eventually reach equilibrium beliefs.¹⁶

In the game with no foresight, users face a coordination problem. Since users do not know which applications are consumed by other users, some of the benefit to the direct network effects is lost. The utility that users can achieve in this environment is lower than in the environment with perfect foresight, because users cannot exploit consumption complementarities as well due to lack of coordination. In such a situation, the platform can create value by limiting the number of available applications. By providing fewer applications, the platform alleviates this coordination problem.

Absence of perfect foresight may be interpreted as a situation where it is costly for users to obtain information about other users' consumption. The no-foresight game that we analyze below assume the extreme case where acquiring such information is prohibitively costly and thus users make their decisions without knowing other users' consumption.

The economic literature on platforms has focused on the case of perfect foresight which is embedded in the notion of Nash equilibrium. We are aware that neither of the two extreme cases (perfect foresight and no-foresight) are likely to hold in reality. However, understanding no-foresight equilibria provides insight into the in-between cases.¹⁷

3.1 Setup

We now describe the game with no foresight. The only difference with the game in Section 2 is what we assume about users' beliefs. Recall that $\mathbf{x}^k = \{x_1^k, x_2^k, \dots, x_A^k\}$ such that $\sum_{a \in \mathbb{A}} x_a^k = X$ denotes a feasible consumption vector. We use \mathbf{x}^k to also denote a pure strategy. Because all users are identical, they all have access to the same set of pure strategies. Let \mathbb{X} denote the set of all pure strategies for any given user. Let $\phi_l^k \sim U[\mathbb{X}]$ denote user k 's beliefs on user l 's choice of pure strategy. Let $\boldsymbol{\phi}^k = \{\phi_l^k\}_{l \neq k}$ be a vector that denotes user k 's beliefs on all other users' choices of pure strategy.

¹⁶We consider such updating to be realistic. In the experiment of El-Gamal and Grether (1995) overwhelming majority of subjects used Bayes updating rule.

¹⁷A detailed analysis of the in-between case of imperfect foresight is complex and deserves separate study in another paper.

With this, user k 's utility from consuming vector \mathbf{x}^k is:

$$\mathbf{E}_{\phi^k} u(\mathbf{x}^k) = \left(\sum_{a \in \mathbb{A}} (x_a^k)^{1/R} \right)^R + \alpha \sum_{a \in \mathbb{A}} (x_a^k \mathbf{E}_{\phi^k} \sum_{l \neq k} x_a^l),$$

and the optimization problem (2) becomes

$$\max_{x_a^k, a \in \mathbb{A}} \left\{ \mathbf{E}_{\phi^k} u(\mathbf{x}^k) - p \cdot \sum_{a \in \mathbb{A}} \mathbf{1}(x_a^k) \right\} \quad \text{subject to} \quad X \geq \sum_{a \in \mathbb{A}} x_a^k. \quad (8)$$

As Lemma 7 shows, under no foresight the expectation over consumption of any application a by any other user $l \neq k$ significantly simplifies.

Lemma 7 *For every l and k and a ,*

$$\mathbf{E}_{\phi_l^k} x_a^l = \frac{X}{A}.$$

Proof. See Appendix A, page 43.

Therefore,

$$\mathbf{E}_{\phi^k} \sum_{l \neq k} x_a^l = \sum_{l \neq k} \mathbf{E}_{\phi_l^k} x_a^l = \sum_{l \neq k} \frac{X}{A} = (N-1) \frac{X}{A}. \quad (9)$$

Note that this expectation does not depend on how many applications or which applications all other users consume, therefore there is no interdependence between users' choices. Given this, we now can find the optimal choice by user k (which in our setting is independent of what all other users do). Whatever is the number of applications G^k that user k wishes to consume, her optimal consumption pattern is balanced consumption, i.e., dividing the time budget equally among the applications consumed. Once user k decides that G^* is the optimal number of applications for her to consume, it does not matter which subset of \mathbb{A} she chooses, as all yield the same utility. Therefore, G^* fully describes k 's set of best responses.

Moreover, G^* identifies what are the dominated strategies for user k : any strategy with cardinality different from G^* and any strategy with cardinality equal to G^* but with non-balanced consumption pattern. User k knows that all other users are the same and she knows that all other users are rational. Therefore, user k cannot believe that other users will play dominated strategies. Because users are homogeneous, user k can infer dominated strategies of other users. When user k finds an optimal number of applications for her to

consume, G^* , she knows that that number is the same for all other users and also that all users are going to consume the G^* applications in a balanced way.

User k updates her beliefs using Bayes' Rule. Therefore, she assigns zero probability to dominated strategies and equal probability to undominated strategies. Since with no foresight she does not know which applications they consume, she believes that every subset of \mathbb{A} with cardinality G^* is equally likely to be consumed by user $l \neq k$. That is, the updating does not tell her which precise applications other users will consume.

Finally, user k recalculates her best response under the updated beliefs. This recalculated best response is exactly the same as the original best response. This is because under the new beliefs the expected consumption of any application a by agent $l \neq k$ is still $\frac{X}{A}$, as in Lemma 7. If every user behaves this way, beliefs are consistent with strategies and this constitutes a *no-foresight equilibrium*.

3.2 No-foresight equilibrium

We consider the case where both direct and indirect network effects are present (i.e., $1 < R < 2$ and $\alpha > 0$).¹⁸ Let G_{DI}^k be the number of applications consumed by user k in a no-foresight equilibrium.

The next proposition shows that in a game with no foresight users will typically consume different applications. Moreover, the more applications are available, the less likely it is that users will end up consuming the same applications. Due to this lack of coordination, users achieve lower expected utility the larger the number of applications available.

Proposition 4 (Coordination Problem) *Suppose $1 < R < 2$ and $\alpha > 0$. In every no-foresight equilibrium, every user k consumes $G_{DI}^k = G_{DI} = Q_I$ applications in equal amount, where $Q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}} > 1$. The expected equilibrium net utility is*

$$\mathbf{E}U_{DI}(G_{DI}) = G_{DI}^{R-1}X + \alpha X(N-1)\frac{X}{A} - pG_{DI}. \quad (10)$$

Proof. See Appendix A, page 44.

Notice that the equilibrium net utility decreases with A for $A \geq G_{DI}$. Clearly, the platform may increase users' utility by limiting A to G_{DI} . In Section 3.3 we show that the

¹⁸The analysis of pure direct and pure indirect network effects is straightforward and can be found in the working paper Casadesus-Masanell and Halaburda (2010).

platform may further increase users' utility by setting A below G_{DI} .

Equation (9) shows that $\mathbf{E}_{\phi^k} \sum_{l \neq k} x_a^l = (N - 1) \frac{X}{A}$. This says that the expected consumption of any application by other users does not depend on the number of applications consumed. Therefore, the direct network effect does not influence the number of applications consumed in equilibrium, i.e., $G_{DI} = Q_I$. Still, the direct network effects affect the expected utility achieved in equilibrium (see (10)).

Combining this result with Lemmas 3 and 5, we see that the number of applications consumed in equilibria with perfect foresight is weakly lower than in the no-foresight equilibrium, i.e., $Q_{DI} \leq G_{DI}$. Moreover, as shown in Proposition 2, the worst of the equilibria with perfect foresight in terms of users' utility occurs when $Q_{DI} = Q_I$. Since $G_{DI} = Q_I$, the platform can create value by limiting the number of applications available. We turn to this issue in the next subsection.

3.3 On the role of the platform: creating value by limiting choice

In Section 2 we have shown that with perfect foresight and in the presence of direct and indirect network effects, users may benefit when the platform limits the set of applications available. Specifically, users always face an *equilibrium selection* problem and may also face a *commons problem*. Both of these issues can be resolved by the platform limiting the number of applications available.

We now show that under no foresight there is a different reason why users benefit from limited choice: resolving the *coordination problem* that users face when direct network effects are at play. We note that under no foresight, users do not face neither equilibrium selection nor commons problems.

Proposition 5 *Suppose that $\alpha > 0$, $1 \leq R < 2$. Under no foresight, the platform maximizes users' net utility by setting the number of available applications to $A = Q^{**}$, where Q^{**} is given by (7).*

Proof. See Appendix A, page 44.

Note that the result applies whenever direct network effects are at play, regardless of the preference for variety. Intuitively, by reducing A the platform alleviates the coordination problem as it is more likely that users consume the same applications and gain utility from direct network effects.

So long as $A > Q^{**}$, the equilibrium is inefficient, especially when A is large. Only when $A = Q^{**}$ the efficient outcome is an equilibrium. The platform creates value by creating a new equilibrium.

4 Discussion

In this section we discuss several aspects of our approach. First, although for expositional simplicity we have presented the model with reference to network effects, all that we need for the results to go through is the presence of consumption complementarities and preference for variety. Consumption complementarities always imply direct network effects. Preference for variety, however, not always implies indirect network effects. To illustrate this point, note a key difference between hardware-software platforms (e.g., Nintendo) and betting platforms (e.g., Betfair). In the case of Nintendo, users benefit from game variety as provided by a large number of independent developers, and developers benefit from a large number of users to sell games. Thus preference for variety and indirect network effects go hand-in-hand in this case. This contrasts with Betfair where punters (back and lay sides) benefit from a large variety of sporting events to bet on, but where there are no *independent* event providers that benefit from there being more punters (as Betfair is the only provider of events on its platform). Although there are no indirect network effects in this case, our analysis and results apply.

Second, we have analyzed the user side only and assumed that the price of accessing an application p is exogenous. While to better understand the interactions between users and developers it would be interesting to extend the model to endogenous p , to do so would require imposing substantial assumptions on industry structure on the developer side (entry conditions, production cost, number of games sold by each developer, and so on). Of course, the equilibrium p would not be innocuous to such assumptions. However, a critical implication of our analysis is that the platform cannot induce users to consume the optimal number of applications by manipulating p .¹⁹ Put differently, we have shown that *regardless of the value of p* , the commons, equilibrium selection, and coordination problems will arise when, in addition to preference for variety, there are direct network effects. Therefore, our conclusion that it is valuable for platforms to manage the number of applications available

¹⁹The exception is a situation where the platform drives p so high that $Q_I = 1$, which we assumed away on page 10. In this case, users consume one application which is the optimal number. For details, see the working paper version (Casadesus-Masanell and Hałaburda 2010).

holds regardless of whether p is endogenous or exogenous.²⁰

Third, while the method we have considered in Sections 2.4 and 3.3 for correcting the distortions which can exist in consumer decisions on platforms—outright restriction on the number of applications available—might seem brutally direct, there are indirect ways to implement it. One such way is through manipulation of the access fees and/or royalties charged to developers. High prices to developers will lead to less entry and a smaller set of applications available and, thus, to possibly more value for users. An interesting and counterintuitive implication of our results is that to the extent that higher access prices to developers result in net utility gains to users, the platform will be able to charge higher prices to the user side. Of course, this runs counter to the conventional wisdom that to earn more from one side of the market, the platform must set lower prices to the other side.

Another indirect way to narrow down the set of applications available is by tinkering with user search. For example, in the late 1980s and 1990s, Nintendo used *Nintendo Power*—an in-house magazine priced to break even and carried no advertising—to promote particular games. Two years after it had launched, it had become the highest-circulation publication targeted to children in the United States. Games not featured on *Nintendo Power* were much less likely to become commercial successes.²¹ Likewise, the current search capabilities on Apple’s App Store are notoriously deficient.²² Applications appear ranked by number of downloads which, of course, reinforces direct network effects for applications—such as word processors (*Pages*), spreadsheet programs (*Numbers*), or presentation software (*Keynote*)—that exhibit consumption complementarities.

Fourth, our results suggest a possible resolution to an issue in the video game market that has traditionally been seen as a puzzle. Console makers typically charge royalties to game developers. This seems like a bad idea due to double marginalization. One explanation is that royalties are an instrument to compel developers to raise the quality of games. Because games are more expensive in the presence of royalties, fewer but better games are developed. Thus, royalties are often seen as resolving a tradeoff between quality and quantity. Our analysis shows that there may be no tradeoff because the platform may prefer both, better-

²⁰Although p is often endogenous, it is not hard to think of cases where it is exogenous, particularly when technological reasons constrain p to be zero. For example, open television channels are free to users as it is impossible to exclude access to them. In this case, p may not be used as a coordinating device. (In Appendix B we present the complete analysis of the model with $p = 0$.)

²¹See Brandenburger (1995).

²²See, for example, <http://accidentaltechnologist.com/apple/apple-please-fix-the-app-store-search/> or <http://www.eweek.com/c/a/Mobile-and-Wireless/10-Apple-App-Store-Problems-That-Need-Fixing-Now-412975/>

quality games but also fewer games. Moreover, because they ultimately limit the number of applications available, the motive for royalties in the video game industry could be exactly the same as the rationale for Betfair to limit the number of betting events on its platform. To the best of our knowledge, the idea that *royalties* and *restrictions on application variety* might be derived from the same underlying force is new to the literature.

Last, while analysis of competing platforms is well beyond the scope of this paper, it is easy to see an interesting tradeoff that is likely to emerge when direct and indirect network effects are at play. Consider a situation with two platforms competing for a given set of users. If one of the platforms limits the number of applications when there is preference for variety, users will likely expend some of their budget on applications from the second platform. Thus, by limiting choice, the platform may potentially create additional value, but users are more likely to multi-home and crowd out some of their limited resources to the other platform. As a result, competition for users is likely to have a mitigating effect on the platform's desire to limit choice.

5 Conclusion

We have shown that when users enjoy application variety but also benefit from consumption complementarities, three problems may arise: the socially optimal number of applications may not be part of an equilibrium; multiple equilibria ensue; and, users will likely find it hard to coordinate consumption. The analysis has demonstrated that by limiting the number of applications, the platform can provide a fix to these problems. Specifically, by limiting choice the platform may create new equilibria that do not exist when application choice is broad. In addition, it can eliminate equilibria that yield lower utility. Moreover, it can reduce the severity of the coordination problem faced by users.

The overall conclusion is that when direct and indirect network effects are at play, an important governance decision that platforms face is the choice of the number of applications that should be allowed to run on them. To implement such a choice, the platform may directly suppress access to developers and impose quantity constraints, or it may limit the number of applications indirectly through setting high access prices to developers.

While we have shown that the platform may create value by limiting choice, the recommendation to practitioners is obviously not “provide as few applications as possible.” Rather, it is that even in settings where users have a strong preference for variety, the platform provider must be cognizant that there may be a number beyond which offering more

applications will decrease users' utility and, thus, overall platform value. This recommendation is in stark contrast to the conventional wisdom that platforms should encourage the development of complements to the maximal possible extent.

The obvious next step in this research is the endogenization of access prices in a setting with competing platforms and direct and indirect network effects. Given the complexity of the analysis when users are the only strategic players, we expect these extensions to be challenging. It is our hope to have provided a solid first step on which to build general theories of platform competition that will shed further light on the value that platforms may create by acting as gatekeepers.

APPENDIX

A Proofs

Proof of Remark 1 (page 8)

Proof. Let $R = 1$ and $\alpha > 0$. Suppose that users $l \neq k$ play pure strategies \mathbf{x}^l . The proof proceeds in following steps: First, we find the optimal consumption patterns, given that user k has access to some set $\mathbb{Q} \subseteq \mathbb{A}$ of applications, where the cardinality of \mathbb{Q} is $Q \geq 1$. Second, given the consumption pattern, we find the optimal set of applications consumed, \mathbb{Q}_D^k . We characterize subgame-perfect Nash equilibria where all users decide which applications to consume and at which level.

Suppose that user k has access to a set $\mathbb{Q} \subseteq \mathbb{A}$ applications. Given \mathbb{Q} , user k 's objective is to allocate the consumption in order to maximize her utility, i.e.,

$$\max_{x_a^k, a \in \mathbb{Q}} \left\{ \sum_{a \in \mathbb{Q}} x_a^k + \alpha \sum_{a \in \mathbb{Q}} \left(x_a^k \sum_{l \neq k} x_a^l \right) \right\} \quad \text{s.t. } X \geq \sum x_a^k.$$

The Lagrangian of this maximization problem, including the constraint is

$$\mathbb{L} = \sum_{a \in \mathbb{Q}} x_a^k + \alpha \sum_{a \in \mathbb{Q}} \left(x_a^k \sum_{l \neq k} x_a^l \right) + \lambda (X - \sum x_a^k).$$

The derivative of the Lagrangian with respect to x_a^k is

$$\frac{\partial \mathbb{L}}{\partial x_a^k} = 1 + \alpha \sum_{l \neq k} x_a^l - \lambda.$$

This derivative does not depend on x_a^k . Let a' be an application such that $\sum_{l \neq k} x_{a'}^l \geq \sum_{l \neq k} x_a^l$ for all $a \in \mathbb{Q}$. There may be one or more such applications. Those applications yield the largest derivative $\frac{\partial \mathbb{L}}{\partial x_{a'}^k}$, i.e., additional consumption of those applications brings more additional consumption utility than other applications. In equilibrium, user k does not consume other applications than a' . If there is only one a' , the best response of user k is to consume only this one application, i.e., $x_{a'}^k = X$ and $x_a^k = 0$ for $a \neq a'$. If there is more than one a' , any allocation of time budget X across all those applications yields exactly the same consumption utility. A special case of such allocation is allocating whole X to one application a' .

Given this consumption pattern, user k needs to decide on the set of applications that she consumes, \mathbb{Q} , in order to maximize her net utility. If there exists unique $a' \in \mathbb{A}$ such that $\sum_{l \neq k} x_{a'}^l \geq \sum_{l \neq k} x_a^l$ for all $a \in \mathbb{A}$, then the optimal set of applications consumed by user k is a singleton $\mathbb{Q}_D^k = \{a'\}$. Notice that it leads to an equilibrium, where all users allocate their whole time budget to the same application, i.e. $\mathbb{Q}_D^k = \mathbb{Q}_D = \{a'\}$ and $x_{a'}^k = X$ for all k . Therefore, it is a balanced equilibrium. Since any $a' \in \mathbb{A}$ would constitute such an equilibrium, there are A equilibria of this form. Every user's net utility in such an equilibrium is $X + \alpha X^2(N - 1) - p$. The following paragraph shows that no other equilibrium exists. In particular, there is no equilibrium that makes users better off than an equilibrium with \mathbb{Q}_D . Therefore, all those equilibria yield the highest possible utility to the users.

Suppose now that there is more than one a' such that $\sum_{l \neq k} x_{a'}^l \geq \sum_{l \neq k} x_a^l$. Since the price $p > 0$, user k 's best response is to consume only one of a' applications. This is because consuming more of those applications yields exactly the same consumption utility, but user k needs to pay additional price p for each additional application. The net utility is lower when more applications are consumed. Therefore, there cannot be an equilibrium with $Q_D \geq 2$. Moreover, since $p < X$, it is always better for any user to consume one application to none. We conclude that in each equilibrium exactly one application is consumed by all users.

This completes the proof of Remark 1. ■

Proof of Remark 2 (page 9)

Proof. Suppose that $1 < R < 2$ and $\alpha = 0$. User k 's consumption utility (and net utility) does not depend on other users' consumption, due to $\alpha = 0$. Thus, the equilibrium consumption decision of user k does not depend on the decisions of other users (i.e., the equilibrium strategy is simply the optimization result of each user).

The proof proceeds in two steps: First, we find the optimal consumption pattern, given that user k has access to some set \mathbb{Q} of applications, where cardinality of \mathbb{Q} is $Q \geq 1$. Second, given the consumption pattern, we find the optimal set of applications consumed, \mathbb{Q}_I^k .

Suppose that user k has access to set $\mathbb{Q} \subseteq \mathbb{A}$ of applications. Given \mathbb{Q} , user k 's objective is to allocate the consumption in order to maximize her utility, i.e.,

$$\max_{x_a^k, a \in \mathbb{Q}} \left(\sum_{a \in \mathbb{Q}} (x_a^k)^{\frac{1}{R}} \right)^R \quad \text{s.t.} \quad X \geq \sum_{a \in \mathbb{Q}} x_a^k.$$

The Lagrangian associated with this problem, including the constraint is

$$\mathbb{L} = \left(\sum_{a \in \mathbb{Q}} (x_a^k)^{\frac{1}{R}} \right)^R + \lambda (X - \sum_{a \in \mathbb{Q}} x_a^k).$$

The first order condition for a particular application $a' \in \mathbb{Q}$, $\frac{\partial \mathbb{L}}{\partial x_{a'}^k} = 0$ yields

$$\left(\sum_{a \in \mathbb{Q}} (x_a^k)^{\frac{1}{R}} \right)^{R-1} \cdot (x_{a'}^k)^{\frac{1}{R}-1} = \lambda \iff x_{a'}^k = \frac{\left(\sum_{a \in \mathbb{Q}} (x_a^k)^{\frac{1}{R}} \right)^R}{\lambda^{\frac{R}{R-1}}}, \quad \forall a' \in \mathbb{Q}.$$

Thus, in the consumption schedule that maximizes the consumption utility, every application is consumed in the same amount, i.e., $\hat{x}_a^k = \hat{x}$ for all $a \in \mathbb{Q}$. To reach the maximum the constraint $X \geq \sum_{a \in \mathbb{Q}} x_a^k$ needs to bind. Therefore $Q \cdot \hat{x} = X$ and $\hat{x} = \frac{X}{Q}$. That implies that every equilibrium must be a balanced equilibrium.

With $\hat{x} = \frac{X}{Q}$ the maximal consumption utility given \mathbb{Q} is

$$u_I(\hat{x}; \mathbb{Q}) = \left(\sum_{a \in \mathbb{Q}} \left(\frac{X}{Q} \right)^{\frac{1}{R}} \right)^R = \left(Q \left(\frac{X}{Q} \right)^{\frac{1}{R}} \right)^R = Q^{R-1} X.$$

This consumption utility is the same for any set \mathbb{Q} of cardinality Q . The net utility also depends solely on the cardinality of set \mathbb{Q} . For any set \mathbb{Q} with cardinality Q , the user k 's maximal net utility is

$$U_I(\hat{x}; Q) = Q^{R-1} X - pQ.$$

For $p > 0$, the optimal number of applications consumed by user k is characterized by the first order condition

$$(R-1) Q^{R-2} X = p \iff Q = \left(\frac{X(R-1)}{p} \right)^{\frac{1}{2-R}}. \quad (11)$$

$$\text{Let } q_I = \left(\frac{X(R-1)}{p} \right)^{\frac{1}{2-R}}.$$

The number of applications consumed cannot be greater than A or smaller than 1. We have assumed that the number of applications is large enough.²³ Specifically, we have assumed that $A \geq q_I$. Therefore, we need to assure that the number of applications consumed

²³If we had allowed for $A < q_I$, it would be optimal for a user to consume all A applications. This is because the derivative of $U_I(\hat{x}; Q)$ is strictly positive for all $Q < q_I$. So it would be positive on the whole domain $[1, A]$ for $A < q_I$.

is not lower than 1. Therefore, the optimal number of applications consumed by any user k is $Q_I^k = \max\{1, q_I\}$.

Since the optimal number of applications consumed is the same for all users, let Q_I denote Q_I^k for any k . Each user is indifferent between consumption of any subset with cardinality Q_I . Any collection of sets $\{\mathbb{Q}_I^1, \dots, \mathbb{Q}_I^N\}$ constitutes an equilibrium, as long as for any k , cardinality of \mathbb{Q}_I^k is Q_I . There are $\frac{A!}{Q_I!(A-Q_I)!} \cdot N$ such collections of sets. Therefore there is that many pure strategy Nash equilibria. There is also a continuum of mixed strategy equilibria: any probability distribution over all the pure strategies described above constitutes an equilibrium strategy for user k (given that all the subsets have the same cardinality, any of such mixed strategies yields the same utility as a pure strategy).

If there existed any other equilibrium, it would involve some users consuming other number of applications than Q_I with a positive probability. That is suboptimal strategy for those users. Therefore, there are no other equilibria.

Since in all subgame perfect Nash equilibria every user consumes Q_I applications, each in equal amount, all the equilibria yield the same net utility to all users. Moreover, there does not exist an equilibrium where some users could achieve a higher net utility. Thus, all equilibria yield the highest possible utility to the users.

This completes the proof of Remark 2. ■

Proof of Lemma 1 (page 11)

Proof. Assume that $1 < R < 2$ and $\alpha > 0$. Suppose, to the contrary, that in some equilibrium $\mathbb{Q}^k \neq \mathbb{Q}^l$ for some l and k (we drop the subscript DI in this proof for clarity of exposition). We show that this cannot be an equilibrium.

First, consider the case where $Q^k = Q^l$, i.e., user k and user l consume the same amount of applications, but different ones. This cannot be an equilibrium. Take an application a' that k consumes, but l does not, and application a'' that l consumes but k does not. Suppose, without loss of generality that $\sum_{j \neq l, k} x_{a'}^j \leq \sum_{j \neq l, k} x_{a''}^j$ (otherwise, we switch k and l). User k 's net utility in such a candidate equilibrium is

$$\left(\sum_{a \in \mathbb{Q}^k} (x_a^k)^{\frac{1}{R}} \right)^R + \alpha \sum_{a \in \mathbb{Q}^k \setminus \{a'\}} \left(x_a^k \sum_{j \neq k} x_a^j \right) + \alpha x_{a''}^k \sum_{j \neq k, l} x_{a''}^j - p Q^k.$$

If user k spends $x_{a''}^k$ consuming application a'' instead of a' (without changing anything else),

she increases her utility to

$$\left(\sum_{a \in \mathbb{Q}^k} (x_a^k)^{\frac{1}{R}} \right)^R + \alpha \sum_{a \in \mathbb{Q}^k \setminus \{a'\}} \left(x_a^k \sum_{j \neq k} x_a^j \right) + \alpha x_{a'}^k \underbrace{\left(\sum_{j \neq k, l} x_{a'}^j + x_{a'}^l \right)}_{> \sum_{j \neq k, l} x_{a'}^j} - p Q^k.$$

Therefore, it is not an equilibrium for users to consume different application, since $\alpha > 0$.

For the same reason, if $Q^k < Q^l$, user k consumes only applications that l consumes, i.e., $\mathbb{Q}^k \subset \mathbb{Q}^l$. However, $Q^k < Q^l$ cannot be an equilibrium.

Suppose, to the contrary, that in a balanced equilibrium $Q^l > Q^k$ and $\mathbb{Q}^k \subset \mathbb{Q}^l$. Since they place balanced strategies, $x_a^l = \frac{X}{Q^l}$ for $a \in \mathbb{Q}^l$, and $x_a^k = \frac{X}{Q^k}$ for $a \in \mathbb{Q}^k$ and $x_a^k = 0$ for all other applications, especially for $a \in \mathbb{Q}^k \setminus \mathbb{Q}^k$. The consumption of all other users is $\sum_{j \neq k, l} x_a^j$ for all $a \in \mathbb{A}$. For k , it is optimal to consume Q^k . Such consumption yields the net utility

$$\left(\sum_{a \in \mathbb{Q}^k} (x_a^k)^{\frac{1}{R}} \right)^R + \sum_{a \in \mathbb{Q}^k} x_a^k \left(\sum_{j \neq k, l} x_a^j + x_a^l \right) - p Q^k.$$

And after substituting $x_a^k = \frac{X}{Q^k}$ and $x_a^l = \frac{X}{Q^l}$

$$(Q^k)^{R-1} X + \alpha \frac{X}{Q^k} \sum_{a \in \mathbb{Q}^k} \left(\sum_{j \neq k, l} x_a^j \right) + \alpha \frac{X}{Q^k} Q^k \frac{X}{Q^l} - p Q^k.$$

In particular, consuming Q^k applications yields higher utility for user k than consuming the same Q^l applications as user l , i.e.²⁴

$$\begin{aligned} (Q^l)^{R-1} X + \alpha \left(\frac{X}{Q^l} \right)^2 Q^k + \alpha \frac{X}{Q^l} \sum_{a \in \mathbb{Q}^l} \left(\sum_{j \neq k, l} x_a^j \right) - p Q^l &\leq (Q^k)^{R-1} X + \alpha \frac{X^2}{Q^l} + \alpha \frac{X}{Q^k} \sum_{a \in \mathbb{Q}^k} \left(\sum_{j \neq k, l} x_a^j \right) - p Q^k \implies \\ \implies X \left((Q^l)^{R-1} - (Q^k)^{R-1} \right) + \alpha X \left(\frac{1}{Q^l} \sum_{a \in \mathbb{Q}^l} \left(\sum_{j \neq k, l} x_a^j \right) - \frac{1}{Q^k} \sum_{a \in \mathbb{Q}^k} \left(\sum_{j \neq k, l} x_a^j \right) \right) - p (Q^l - Q^k) &\leq \alpha \frac{X^2}{Q^l} \left(1 - \frac{Q^k}{Q^l} \right). \end{aligned} \tag{12}$$

²⁴The utility if user k would consume Q^l applications in a balanced strategy is calculated with the following formula:

$$(Q^l)^{R-1} X + \alpha \frac{X}{Q^l} \left(Q_{DI}^k \frac{X}{Q^k} + \sum_{a \in \mathbb{Q}_{DI}^k} \left(\sum_{j \neq k, l} x_a^j \right) \right) + \alpha \frac{X}{Q^l} \sum_{a \in \mathbb{Q}^k \setminus \mathbb{Q}^l} \left(\sum_{j \neq k, l} x_a^j \right) - p Q^l.$$

For l it is optimal to consume

$$\begin{aligned} \left(\sum_{a \in \mathbb{Q}^l} (x_a^l)^{\frac{1}{R}} \right)^R + \sum_{a \in \mathbb{Q}^k} x_a^l \left(\sum_{j \neq k, l} x_a^j + x_a^k \right) + \sum_{a \in \mathbb{Q}^l \setminus \mathbb{Q}^k} x_a^l \left(\sum_{j \neq k, l} x_a^j \right) - p Q^l &= \\ &= (Q^l)^{R-1} + \alpha \frac{X}{Q^l} X + \alpha \frac{X}{Q^l} \sum_{a \in \mathbb{Q}^l} \left(\sum_{j \neq k, l} x_a^j \right) - p Q^l. \end{aligned}$$

In particular, consuming Q^l applications yields higher utility for user l than consuming only Q^k applications, i.e.,

$$X \left((Q^l)^{R-1} - (Q^k)^{R-1} \right) + \alpha X \left(\frac{1}{Q^l} \sum_{a \in \mathbb{Q}^l} \left(\sum_{j \neq k, l} x_a^j \right) - \frac{1}{Q^k} \sum_{a \in \mathbb{Q}^k} \left(\sum_{j \neq k, l} x_a^j \right) \right) - p(Q^l - Q^k) \geq \alpha X^2 \left(\frac{1}{Q^k} - \frac{1}{Q^l} \right). \quad (13)$$

However, for $Q^l > Q^k \geq 1$, $\alpha \frac{X^2}{Q^l} \left(1 - \frac{Q^k}{Q^l} \right) < \alpha X^2 \left(\frac{1}{Q^k} - \frac{1}{Q^l} \right)$. Therefore, both inequalities (12) and (13) cannot be satisfied at the same time. Thus, it cannot be that there is a balanced equilibrium where $Q^l > Q^k$.

This completes the proof of Lemma 1. ■

Proof of Lemma 2 (page 11)

Proof. Suppose that in a balanced equilibrium user k consumes $Q_{DI}^k = Q_{DI}$ applications. By Lemma 1, we know that all users consume the same Q_{DI} applications, where \mathbb{Q}_{DI} denotes the consumption set. Notice that the net utility of users does not depend on the identity of the applications. The net utility is the same as long as all users consume the same Q_{DI} applications, whichever they are. Therefore, any subset of applications \mathbb{Q}_{DI} of cardinality Q_{DI} constitutes an equilibrium. ■

Proof of Remark 3 (page 13)

The remark directly follows from Lemma 8.

Lemma 8 *For all parameters $\alpha \geq 0$ and $1 \leq R < 2$, $Q_I \geq \widehat{Q}$. Moreover when $Q_I > 1$, then $\widehat{Q} < Q_I$, and when $Q_I = 1$ then $\widehat{Q} = Q_I$.*

Proof. Recall that Q_I is defined based on the solution (q_I) to the following first order condition

$$\underbrace{(R-1)Q^{R-2}X - p}_{D_I(Q)} = 0. \quad (14)$$

For $Q \rightarrow 0^+$ the derivative $D_I \rightarrow \infty$. Moreover, the derivative is always decreasing. For any $p > 0$, the first order condition $D_I(Q) = 0$ has exactly one solution, at q_I .

Similarly, \widehat{Q} is defined based on the solution to another first order condition

$$\widehat{D}(Q) = \underbrace{(R-1)Q^{R-2}X - p}_{D_I(Q)} - \underbrace{\alpha X(N-1)\frac{X}{Q^2}}_{+} = 0. \quad (15)$$

For any Q the derivative \widehat{D} is smaller than the derivative D_I . Therefore, whenever $\widehat{D} = 0$ for some \widetilde{Q} , then $D_I > 0$ for this \widetilde{Q} . Moreover, since the derivative D_I is decreasing, $D_I = 0$ for a larger Q than \widetilde{Q} . Therefore, the solution (q_I) to the first order condition (14) is always larger than any solution to the first order condition (15), if the solution to the latter exists.

We focus on the non-trivial case when $Q_I > 1$.²⁵ This happens when $q_I > 1$. The value of \widehat{Q} is either a solution to (15) or 1. In either case $\widehat{Q} < q_I = Q_I$.

This completes the proof of Lemma 8. ■

Proof of Proposition 1 (page 13)

Proof. The optimal upward deviation involves non-balanced consumption. It yields strictly higher utility than an upward deviation with balanced consumption. Suppose that $\widehat{Q} > 1$. Then $V'(\widehat{Q}) = 0$. Note that the payoff from an upward deviation to Q^k under balanced consumption is

$$U_{DI}(Q^k \geq \widehat{Q} | \text{balanced}) = (Q^k)^{R-1} X + \widetilde{Q} \alpha \frac{X}{Q^k} (N-1) \frac{X}{\widetilde{Q}} - p Q^k,$$

which is the same as $V(Q^k)$. And since $V'(\widehat{Q}) = 0$, an incremental upward deviation with balanced consumption from \widehat{Q} yields 0 benefit. But that means that the optimal upward deviation (which is better than balanced deviation) from \widehat{Q} yields strictly positive benefit. Thus, \widehat{Q} is not a balanced equilibrium.

This completes the proof of Proposition 1.²⁶ ■

²⁵For $Q_I = 1$, which happens when $q_I < 1$. Therefore, any solution to (15) must also be smaller than 1. Then $\widehat{Q} = 1 = Q_I$.

²⁶Notice the implication of this result for the incentives in the market: Suppose that the platform limits the number of applications to \widehat{Q} , and \widehat{Q} is optimal. Thus, the platform guarantees users the best equilibrium outcome. Nonetheless, the users are not happy with this restriction. They may believe (because they look at their profitable deviation upward) that if one more application would be available, they would be better off. But, of course, in an equilibrium they wouldn't.

Proof of Lemma 3 (page 14)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that all users play a balanced strategy, where they consume a set of applications \mathbb{Q} with cardinality Q .

The proof proceeds in two steps: In the first step, we show that for $Q > Q_I$ any user has incentive to deviate from this strategy and consume fewer applications. In the second step, we show that for Q such that $\max\{1, Q_\star\} \leq Q < \widehat{Q}$ any user has incentive to deviate and consume more applications. Therefore, no Q in those two intervals can characterize a balanced equilibrium.

Suppose that $Q > Q_I$. If user k consumes Q or fewer applications, i.e., $Q^k \leq Q$, she consumes the same applications as other users, i.e., $\mathbb{Q}^k \subseteq \mathbb{Q}$. This is because, due to direct network effects ($\alpha > 0$), user k 's consumption utility would be lower if she consumed other applications instead. Moreover, if user k consumes $Q^k \leq Q$ applications, it is optimal for her to consume them according to a balanced consumption schedule: $\frac{X}{Q^k}$ of each. This is because each application presents the same benefit through consumption complementarity. Therefore, the net utility when user k consumes $Q^k \leq Q$ applications is

$$U_{DI}(Q^k \leq Q) = (Q^k)^{R-1} X + Q^k \alpha \frac{X}{Q^k} (N-1) \frac{X}{Q} - p Q^k.$$

Since $p > 0$, the optimal number of applications that user k would like to consume is characterized by the first order condition

$$\frac{\partial U_{DI}(Q^k \leq Q)}{\partial Q^k} = (R-1) (Q^k)^{R-2} X - p = 0. \quad (16)$$

Note that this is the same condition as (11) in the proof of Remark 2. So $Q^k = q_I$ is the only positive value satisfying this condition. Moreover, for any $Q^k > q_I$, the derivative in (16) is negative. Therefore, for any $Q > Q_I$, user k can profitably deviate to consuming Q_I applications.

In the second step of the proof, we turn to Q such that $\max\{1, Q_\star\} \leq Q < \widehat{Q}$, and we show that any user can profitably deviate by consuming more applications. When user k consumes more applications than Q , she consumes all applications in \mathbb{Q} , and $Q^k - Q$ applications that no other user consumes. The optimal consumption schedule in such a deviation is *not* a balanced consumption schedule. If we impose the balanced consumption schedule on the upward deviation, it yields lower utility than the optimal deviation. Even though it is not the optimal deviation, we show that an upward deviation with a balanced consumption

schedule is profitable for any user. The net utility from user k 's balanced consumption of $Q^k \geq Q$ applications is

$$U_{DI}(Q^k \geq Q | \text{balanced}) = (Q^k)^{R-1} X + \alpha \frac{X}{Q^k} (N-1) \frac{X}{Q} - p Q^k.$$

Note that $U_{DI}(Q^k \geq Q | \text{balanced})$ is the same as $V(Q)$ in equation (4) which has a local maximum at $\hat{Q} > Q$. Moreover, if there does not exist $Q_* \leq 1$, then for any $Q \in [1, \hat{Q})$, and when $Q_* \leq 1$ exists, then for any $Q \in (Q_*, \hat{Q})$, $U_{DI}(\hat{Q} > Q) > U_{DI}(Q)$. That is, it is strictly profitable for a user to deviate upwards (to \hat{Q} from those Q s). It reminds to show that there exists a profitable deviation away from $Q_* \leq 1$. By the definition of Q_* , $U_{DI}(\hat{Q} > Q) = U_{DI}(Q_*)$. The most profitable deviation, however involves a non-balanced consumption schedule, and yields strictly higher utility than $U_{DI}(Q^k > Q)$. Therefore, the optimal deviation away from Q_* is profitable.

This completes the proof of Lemma 3. ■

Proof of Lemma 4 (page 16)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that all other users $l \neq k$ play a balanced strategy where they consume a set of applications \mathbb{Q} with cardinality Q . If user k consumes Q or fewer applications, i.e. $Q^k \leq Q$, she consumes the same applications as other users, i.e. $\mathbb{Q}^k \subseteq \mathbb{Q}$. This is because, due to direct network effects ($\alpha > 0$), user k 's net utility would be lower if she consumed other applications instead.

User k 's consumption utility from consuming $Q^k \leq Q$ applications is

$$u(\mathbf{x}^k, Q^k; Q) = \left(\sum_{a \in \mathbb{Q}^k} (x_a^k)^{\frac{1}{R}} \right)^R + \alpha \underbrace{\sum_{a \in \mathbb{Q}^k} x_a^k}_{=X} (N-1) \frac{X}{Q}.$$

By usual arguments we find that the consumption schedule maximizing the consumption utility, under the constraint $\sum_{a \in \mathbb{Q}^k} x_a^k \leq X$ is balanced strategy, i.e., $x_a^k = \frac{X}{Q^k}$ for all $a \in \mathbb{Q}^k$.

Therefore, the net utility of user k from consuming Q^k applications is

$$U_{DI}(Q^k; Q) = (Q^k)^{R-1} X + \alpha X (N-1) \frac{X}{Q} - p Q^k. \quad (17)$$

Notice that this utility is strictly increasing for $Q^k < q_I$ and strictly decreasing for $Q^k > q_I$.

Under $Q_I > 1$, suppose that $Q \leq Q_I$.²⁷ Then if $Q^k < Q \leq Q_I$, then the utility in (17) increases with Q^k . That is, the user achieves a lower utility if she deviates from Q to $Q^k < Q$.

This completes the proof of Lemma 4. ■

Proof of Lemma 5 (page 17)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that all other users play balanced strategies and consume the same set of applications \mathbb{Q}_I with cardinality Q_I . Since Lemma 4 shows that there is no profitable deviation downward, it is enough to show that there is no profitable deviation upward to prove that Q_I is a balanced equilibrium.

Consider user k who consumes $Q^k > Q_I$ applications. When user k diverts part of her time y away from the Q_I applications that all other users consume, it is optimal for her to consume the same amount of each application in \mathbb{Q}_I , $\frac{X-y}{Q_I}$. Moreover, it is also optimal to consume the same amount of each application that user k consumes outside \mathbb{Q}_I , $\frac{y}{Q^k - Q_I}$. Then, the net utility of user k is

$$U_{DI}(Q^k > Q_I | y) = \left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R + \alpha \frac{X(X-y)}{Q_I} (N-1) - p Q^k.$$

Consider first only the part of the net utility without the direct network effects:

$$\left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R - p Q^k.$$

This is the same as the utility under pure indirect network effects. We know from the proof of Remark 2 that for any Q^k , the utility maximizing consumption schedule is balanced. However, since $\alpha > 0$, in this case the optimal deviation upward must involve un-balanced consumption (in an optimal deviation user consumes more of each application that other users consume and less of each applications that she alone consumes), i.e., $y < \frac{X}{Q^k}(Q^k - Q_I)$. Therefore, if $Q^k > Q_I$, then

$$\left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R - p Q^k < \left(Q^k \left(\frac{X}{Q^k} \right)^{\frac{1}{R}} \right)^R - p Q^k.$$

²⁷If $q_I < 1$, then $Q_I = 1$ and it is not possible that $Q^k < Q_I$.

Recall that Q_I maximizes the net utility under pure indirect network effects. Therefore, for $Q^k > Q_I$,

$$\left(Q^k \left(\frac{X}{Q^k} \right)^{\frac{1}{R}} \right)^R - p Q^k < \left(Q_I \left(\frac{X}{Q_I} \right)^{\frac{1}{R}} \right)^R - p Q_I.$$

Putting those two inequalities together yields

$$\left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R - p Q^k < \left(Q_I \left(\frac{X}{Q_I} \right)^{\frac{1}{R}} \right)^R - p Q_I.$$

Moreover, for any $y > 0$,

$$\alpha \frac{X(X-y)}{Q_I} (N-1) < \alpha \frac{X^2}{Q_I} (N-1).$$

Therefore, any positive deviation, $y > 0$, toward consuming more applications, $Q^k > Q_I$, yields strictly worse net utility for user k ,

$$\begin{aligned} U_{DI}(Q^k > Q_I | y) &= \left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R + \alpha \frac{X(X-y)}{Q_I} (N-1) - p Q^k < \\ &< \left(Q_I \left(\frac{X}{Q_I} \right)^{\frac{1}{R}} \right)^R + \alpha \frac{X(X-y)}{Q_I} (N-1) - p Q_I < \\ &< \left(Q_I \left(\frac{X}{Q_I} \right)^{\frac{1}{R}} \right)^R + \alpha \frac{X^2}{Q_I} (N-1) - p Q_I = U_{DI}(Q_I). \end{aligned}$$

Therefore, any set of applications Q_I with cardinality Q_I constitutes a balanced equilibrium.

Notice that the optimal deviation y^* that maximizes the consumption utility is always positive.²⁸ That is, if y^* satisfies the first order condition $\left. \frac{\partial U_{DI}(Q^k > Q_I | y)}{\partial y} \right|_{y=y^*} = 0$, it must be that $y^* > 0$. However, because the user needs to pay a positive price $p > 0$ for diverting even small y , it is not optimal to do so at Q_I . (As shown by declining net utility.)

Below we show that since $Q_I > 1$, then also for Q s slightly smaller than Q_I , users have no incentive to deviate upward. And so those Q s constitute balanced equilibria.

Suppose that $\widehat{Q} > 1$. By Lemma 4, for any Q such that $\widehat{Q} < Q < Q_I$, no user has a profitable deviation downward. Any such Q constitutes a balanced equilibrium if there is also no profitable deviation upward.

²⁸For formal proof of this property, see the proof of Proposition 6, for $p = 0$.

For Q such that $\widehat{Q} < Q < Q_I$, suppose that all other users consume Q applications, while user k consider diverting y of her time toward more applications, $Q^k > Q$. Utility at this deviation is

$$U_{DI}(Q^k > Q | y > 0) = \left(Q \left(\frac{X-y}{Q} \right)^{\frac{1}{R}} + (Q^k - Q) \left(\frac{y}{Q^k - Q} \right)^{\frac{1}{R}} \right)^R + \alpha \frac{X(X-y)}{Q} (N-1) - p Q^k.$$

User k chooses a deviation $y > 0$ and $Q^k > Q$ to maximize this utility, $\max_{y>0, Q^k>Q} U_{DI}(Q^k > Q | y > 0)$. The profitability of the deviation

$$DevProf(Q) = \max_{y>0, Q^k>Q} U_{DI}(Q^k > Q | y > 0) - V(Q).$$

From Proposition 1, we know that $DevProf(\widehat{Q}) > 0$. We can also show that $DevProf(Q_I) < 0$. This follows from the fact that an infinitesimal upward deviation from Q_I under pure indirect yields 0 profit. Due to the loss of the consumption complementarity, under both indirect and direct network effects the optimal deviation yields smaller utility. Therefore, the deviation is not profitable. Function $DevProf(Q)$ is continuous in Q . Therefore, there must exist Q^0 , $\widehat{Q} < Q^0 < Q_I$ such that $DevProf(Q^0) = 0$. If there are multiple Q satisfying this condition, let Q^0 be the largest. Then, for all $Q \in [Q^0, Q_I]$, $DevProf(Q) \leq 0$, i.e., there is no profitable deviation from Q . Hence, all $Q \in [Q^0, Q_I]$ constitute balanced equilibria.

This completes the proof of Lemma 5. ■

Proof of Lemma 6 (page 17)

Proof. Suppose that $Q_I > 1$, which implies that $q_I > 1$ or $(R-1)X > p$. When all users consume one application only, their consumption utility is:

$$u(Q=1) = X + \alpha X^2(N-1).$$

Now, if a user deviates to consume y of second application, her consumption utility is:

$$u(Q=2) = \left((X-y)^{\frac{1}{R}} + y^{\frac{1}{R}} \right)^R + \alpha X(N-1)(X-y).$$

The optimal level of deviation y^* is characterized by the first order condition:

$$\frac{\partial u(Q=2)}{\partial y} = \left((X - y^*)^{\frac{1}{R}} + y^{*\frac{1}{R}} \right)^{R-1} \left(\left(\frac{1}{y^*} \right)^{1-\frac{1}{R}} - \left(\frac{1}{X - y^*} \right)^{1-\frac{1}{R}} \right) - \alpha X(N - 1) = 0.$$

Notice that y^* decreases with N and $y^* \rightarrow 0$ as $N \rightarrow \infty$. Therefore, as N increases, y^* decreases but Q_I is not affected.

To find out if the value of the optimal deviation is larger than the price of the second application, we compute:

$$\begin{aligned} u(Q=2|y=y^*) - u(Q=1) &= \\ &= \left((X - y^*)^{\frac{1}{R}} + y^{*\frac{1}{R}} \right)^R + \alpha X(N - 1)(X - y^*) - (X + \alpha X^2(N - 1)) < \\ &< \left((X - y^*)^{\frac{1}{R}} + y^{*\frac{1}{R}} \right)^R - X. \end{aligned}$$

Note that $\left((X - y^*)^{\frac{1}{R}} + y^{*\frac{1}{R}} \right)^R - X$ is continuous, takes value zero at $y^* = 0$ and it is strictly increasing in y^* . Therefore for any price p , we can find N large enough so that y^* is low enough so that

$$u(Q=2|y=y^*) - u(Q=1) < p,$$

and the deviation is not profitable.

This completes the proof of Lemma 6. ■

Proof of Proposition 2 (page 18)

Proof. Directly from Lemma 5 we obtain the existence of multiple equilibria with different values of Q_{DI} .

The result that the equilibria with a smaller Q_{DI} yield higher utility follows directly from the shape of $V(Q)$ and Lemma 3: All possible equilibria need to be included in the interval $[1, Q_\star) \cup (\widehat{Q}, Q_I]$. (The set of equilibria is a strict subset of this interval). The utility obtained by every user in each equilibrium Q is $V(Q)$. Since $V(Q)$ is strictly increasing on the interval $[1, Q_\star) \cup (\widehat{Q}, Q_I]$, a lower equilibrium Q yields higher utility for every user than a higher equilibrium Q . ■

Proof of Proposition 3 (page 20)

Proof. The shape of V implies that either $Q^{**} = 1$ or $Q^{**} = \widehat{Q}$. The proof first considers $Q^{**} = \widehat{Q} > 1$, and then $Q^{**} = 1$.

Suppose that $Q^{**} = \widehat{Q} > 1$. Then, Q_* (as defined for Lemma 3) does not exist. Therefore, by Lemma 3, no $Q < \widehat{Q}$ may constitute a balanced equilibrium. As in the proof of Lemma 3, users are better off deviating upward to consuming \widehat{Q} applications. When $A > \widehat{Q}$, then \widehat{Q} is not a balanced equilibrium, by Proposition 1. This is because there exists profitable deviation upward, toward consuming larger number of applications. However, when $A = Q^{**} = \widehat{Q}$, such deviation is not possible. Therefore, consuming all \widehat{Q} constitutes the only equilibrium.

Now, suppose that $Q^{**} = 1$. When platform sets $A = Q^{**} = 1$ then trivially, in the only equilibrium all users consume the only application in the market.

This completes the proof of Proposition 3. ■

Proof of Lemma 7 (page 23)

Proof. Suppose that l consumes G applications for some G . Given G , any application a is consumed by l with probability $\frac{G}{A}$. Since when every strategy is equally likely, any subset of cardinality G is equally likely to be consumed. The probability that particular application a is in a consumption set is

$$\begin{aligned} & \frac{\text{how many subsets with } G \text{ can you choose from } \mathbb{A} \text{ that will include } a}{\text{how many subsets with } G \text{ can you choose from } \mathbb{A} \text{ overall}} = \\ & = \frac{\text{choose } G-1 \text{ out of } A-1}{\text{choose } G \text{ out of } A} = \frac{(A-1)!}{(G-1)!(A-G)!} \bigg/ \frac{A!}{G!(A-G)!} = \frac{G}{A}. \end{aligned}$$

Now, we calculate the expected level of consumption $\mathbf{E}(x_a^l | x_a^l \in \mathbb{G})$ conditionally on a being in a given consumption set \mathbb{G} of l . Since we know that all the consumption schedules over the set \mathbb{G} satisfy $\sum_{a \in \mathbb{G}} x_a^l = X$, therefore

$$\mathbf{E}\left(\sum x\right) = X \implies \sum \mathbf{E}(x) = X,$$

due to the linearity of the sum.

But the applications are not distinct (they are interchangeable). If every consumption schedule is equally likely, the expected consumption of every a in the consumption set is the same.

Suppose to the contrary, that the expected consumption of some $a' \in \mathbb{G}$ is higher than

some other application, a'' , $\mathbf{E}(x_{a'}^l) > \mathbf{E}(x_{a''}^l)$. Then, in the set of all possible consumption schedules, switch a' and a'' in every schedule. The set of all possible consumption schedules remains unchanged, but now $\mathbf{E}(x_{a''}^l) > \mathbf{E}(x_{a'}^l)$. Hence, contradiction. Therefore, the expected consumption of every $a \in \mathbb{G}$ is the same $\mathbf{E}(x_a^l) = \frac{X}{G}$.

Then, the overall expected level of consumption of any application a is

$$\mathbf{E}_{\phi_i^k} x_a^l = \frac{G}{A} \cdot \frac{X}{G} = \frac{X}{A}.$$

(Technically, it is for a given G . But since the expectation for any G is the same, any probability distribution over G 's gives the same expected value.)

This completes the proof of Lemma 7. ■

Proof of Proposition 4 (page 24)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that user k consumes G^k applications in a no-foresight environment. For any given number of applications, G^k , the optimal consumption schedule is a balanced consumption. This is because for any application, the expected level of consumption by other users is the same (Lemma 7). User k 's expected net utility (using Lemma 7) is then

$$\mathbf{E} U_{DI}(G^k) = (G^k)^{R-1} X + \alpha(N-1) \frac{X^2}{A} - p G^k. \quad (18)$$

Note that the benefit from the direct network effect does not depend on G^k . This leads to a result similar to the one in Remark 2: The above function U_{DI} is maximized by $G^k = q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, for any k .

This completes the proof of Proposition 4. ■

Proof of Proposition 5 (page 25)

Proof. Let's consider separately the case for $R = 1$ and for $1 < R < 2$.

Suppose first that $\alpha > 0$ and $R = 1$. It is easy to show that for any $A \geq 1$, every user optimally consumes one application. The expected net utility for any user in an equilibrium for $A \geq 1$ is

$$\mathbf{E} U_D^* = X + \alpha(N-1) \frac{X^2}{A} - p.$$

Clearly, $\mathbf{E} U_D^*$ is maximized by $A = 1$. Moreover, for $\alpha > 0$ and $R = 1$, the unique maximum

of $V(Q)$ is always $Q^{**} = 1$. Therefore, the platform maximizes users' net utility when it sets the number of available applications to $A = Q^{**} = 1$.

Suppose now that $\alpha > 0$ and $1 < R < 2$. By Proposition 4, we know that when $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the optimal number of applications consumed by any user k is $G_{DI}^k = Q_I$, as this number maximizes $\mathbf{E}U_{DI}(G^k)$ in (18).

Since $Q_I > 1$,²⁹ when $A \geq Q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the expected net utility of a user in equilibrium is

$$\mathbf{E}U_{DI}^*(Q_I|A \geq Q_I) = (Q_I)^{R-1} X + \alpha(N-1)\frac{X^2}{A} - pQ_I.$$

On the possible range of $A \geq Q_I$, this utility is maximized for $A = Q_I$.

Since $\mathbf{E}U_{DI}(G^k)$ strictly increases in G^k for $G^k < Q_I$, every user consumes all applications, if there is fewer applications available than Q_I . Thus, for $A \leq Q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the expected net utility of a user in equilibrium is

$$\mathbf{E}U_{DI}^*(A|A \leq Q_I) = (A)^{R-1} X + \alpha(N-1)\frac{X^2}{A} - pA.$$

Note that this function of A is the same as V (with the exception that V is a function of Q). Moreover, since $Q_I > 1$, it must be that $Q^{**} < Q_I$. Because Q^{**} is the value that maximizes V , then $A = Q^{**} < Q_I$ also maximizes the expected net utility $\mathbf{E}U_{DI}^*(A|A \leq Q_I)$. Moreover, notice that $\mathbf{E}U_{DI}^*(Q_I|A \geq Q_I)$ is maximized at $A = Q_I$, but $\mathbf{E}U_{DI}^*(Q^{**}) > \mathbf{E}U_{DI}^*(Q_I)$. So, $A = Q^{**}$ maximizes the expected utility $\mathbf{E}U_{DI}^*$ on the whole range $A \geq 1$. Therefore, the platform maximizes users' net utility when it sets the number of available applications to $A = Q^{**} < Q_I$.

This completes the proof of Proposition 5. ■

²⁹When $Q_I = 1$, the expected net utility of any user in equilibrium for $A \geq 1$ is

$$\mathbf{E}U_{DI}^*(Q_I=1) = X + \alpha(N-1)\frac{X^2}{A} - p,$$

which is maximized by $A = 1$. By Lemma 8, $\widehat{Q} = 1$ when $Q_I = 1$. Thus, the unique maximum of $V(Q)$ is always $Q^{**} = 1$. In result, the platform maximizes user's net utility when it sets the number of available applications to $A = Q^{**} = 1$.

B Results for $p = 0$

Suppose that the access price for any application is $p = 0$. Then, the net utility is the same as the consumption utility, and every user chooses the set of consumed applications and the consumption schedule to maximize her consumption utility.

Note that under $p = 0$ the assumption on A does not make sense any more. It is not possible to make A “large enough.” In this appendix, we allow for arbitrary A .

B.1 Game with perfect foresight

In this section, we assume that every agent knows (or correctly predicts) the number and identity of applications consumed by all other users (i.e., the user knows the consumption sets and consumption schedules of all other users) in equilibrium.

B.1.1 Game with perfect foresight: direct network effects

Remark 4 *Assume $R = 1$, $\alpha > 0$ and $p = 0$. For any $\mathbb{Q} \subseteq \mathbb{A}$, there exists a balanced equilibrium where all users consume the set of applications \mathbb{Q} . There is no other balanced equilibrium.*

Proof. Let $R = 1$, $\alpha > 0$ and $p = 0$. Suppose that all other users play a balanced strategy where they consume a set $\mathbb{Q} \subseteq \mathbb{A}$ applications. If user k consumes fewer applications than other users, she consumes the same applications as other users (due to consumption complementarity it would make sense otherwise). The optimal consumption schedule then is balanced. User k 's net utility of consuming $Q^k \leq Q$ applications is

$$U_D(Q^k \leq Q) = \cancel{(Q^k)^{R-1}} X + \cancel{Q^k} \alpha \frac{X}{\cancel{Q^k}} (N-1) \frac{X}{Q}.$$

(The term $(Q^k)^{R-1}$ cancels because $R = 1$.) This utility does not depend on Q^k . Thus, user k does not have incentive to deviate downward, i.e., consume fewer applications than other users.

Now suppose that user k consumes more applications than other users. She diverts $y \leq X$ of her time to applications that no other users consume. It is optimal for her to still consume all \mathbb{Q} applications that other users consume (hence $y < X$), and among those applications, each application is consumed at the same lever, $\frac{X-y}{Q}$. Because $R = 1$, independently of how

many more applications user k consume, her net utility is

$$U_D(Q^k \geq Q) = X + \alpha X(N - 1) \frac{X - y}{Q}.$$

This utility is strictly decreasing in y , and it reaches its maximum for $y = 0$, i.e., when user k does not divert *any* time to applications other than those consumed by other users. So, user k has no incentive to deviate upward, i.e., consume more applications.

Therefore, if all users consume \mathbb{Q} applications in a balanced strategy, it constitutes a balanced equilibrium, for any $\mathbb{Q} \subseteq \mathbb{A}$.

To show that there is no other balanced equilibrium, notice that if other balanced equilibrium existed, users would need to consume different applications or different number of applications in an equilibrium. Suppose that $\mathbb{Q} \subseteq \mathbb{A}$ is a set of applications that is consumed at strictly positive level by at least one other user,

$$\mathbb{Q} = \{a, \exists l \neq k \text{ s.t. } x_a^l > 0\}.$$

If the aggregate consumption of all other users is not the same for all applications in \mathbb{Q} , the best response of user k is an un-balanced strategy, where she consumes larger levels of applications that have higher aggregate consumption levels by other users. Therefore, it cannot be a balanced equilibrium.

Suppose then that aggregate consumption by all other users is the same for all applications in \mathbb{Q} , $\sum_{l \neq k} x_a^l = \sum_{l \neq k} x_{a'}^l$ for all $a, a' \in \mathbb{Q}$. Then user k 's best response is a balanced strategy where she consumes all \mathbb{Q} applications. But if $x_a^l = x_{a'}^l$ for all l and all $a, a' \in \mathbb{Q}$, then it is a balanced equilibrium where all users consume \mathbb{Q} . To show that it cannot be any other balanced equilibrium, suppose that for some l and l' and some $a, x_a^l \neq x_a^{l'}$. But then for user l it is not true that $\sum_{j \neq l} x_a^j = \sum_{j \neq l} x_{a'}^j$. Therefore, it cannot be that user l 's best response is a balanced strategy. Therefore, it cannot be a balanced equilibrium. The only balanced equilibrium is when all users consume all applications in \mathbb{Q} .

This completes the proof of Remark 4. ■

Corollary 5 *Assume $R = 1$, $\alpha > 0$ and $p = 0$. Suppose that in an equilibrium users consume \mathbb{Q} applications, where Q is cardinality of \mathbb{Q} . The equilibria with $Q = 1$ yield the highest utility to the users.*

Proof. Let $R = 1$, $\alpha > 0$ and $p = 0$, and suppose that in an equilibrium users consume \mathbb{Q} applications, where Q is cardinality of \mathbb{Q} . The net utility of each user in an equilibrium with

$Q \geq 1$ is

$$U_D^*(Q) = X + \alpha \sum_{a=1}^Q \left(\frac{X}{Q} (N-1) \frac{X}{Q} \right) = X + \alpha(N-1) \frac{X^2}{Q}.$$

This utility strictly decreases with Q , and is maximized when $Q = 1$. Whenever more than one application is consumed in equilibrium, the equilibrium yields lower utility to the users than an equilibrium where one application is consumed.

Therefore, the equilibria where $\mathbb{Q}_D^k = \mathbb{Q}_D = \{a\}$ for some $a \in \mathbb{A}$ and all k —i.e., equilibria where all users consume one and the same application—yield the highest utility. ■

B.1.2 Game with perfect foresight: indirect network effects

Remark 5 *Assume $1 < R < 2$, $\alpha = 0$ and $p = 0$. There exists a unique equilibrium, where $\mathbb{Q}_I^k = \mathbb{A}$ each user k , i.e., every user k consumes all available applications. Moreover, the unique equilibrium is balanced.*

Proof. Let $1 < R < 2$, $\alpha = 0$ and $p = 0$.

As in the proof for $p > 0$, we find that the optimal consumption schedule, given that user k has access to some set \mathbb{Q} applications is to consume each of them in the amount of $\hat{x} = \frac{X}{Q}$. Therefore, every equilibrium is balanced equilibrium.

To find the equilibrium consumption set for user k , recall that her consumption utility given Q is

$$u_I(\hat{x}; Q) = \left(Q \left(\frac{X}{Q} \right)^{\frac{1}{R}} \right)^R = Q^{R-1} X.$$

This utility is always strictly increasing in Q , i.e., user k always prefers to consume as many applications as possible. In such a case, user k 's consumption set is only limited by \mathbb{A} , i.e., she optimally consumes *all* applications available, each at the level $\hat{x} = \frac{X}{A}$.

This completes the proof of Remark 5. ■

Equilibria may differ in the utility that the users achieve. However, sometimes there exist allocations that are not equilibria, but that yield higher utility to the users than any equilibrium. The following corollary shows that this is not the case when only indirect network effects are present.

Corollary 6 *In the case for $1 < R < 2$, $\alpha = 0$ and $p = 0$, the unique balanced equilibrium also yields the highest possible utility to the users.*

Proof. For $\alpha = 0$, the equilibrium utility does not depend on other users' consumption, and no other number of consumed application yields higher utility for user k than the equilibrium number, A . Therefore, this equilibrium yields the highest possible utility to the users. ■

B.1.3 Game with perfect foresight: interplay between direct and indirect network effects

Proposition 6 *Assume $1 < R < 2$, $\alpha > 0$ and $p = 0$. There exists a unique balanced equilibrium, where $Q_{DI}^k = A$ each user k , i.e., every user k consumes all available applications.*

Proof. Let $1 < R < 2$, $\alpha > 0$ and $p = 0$. Suppose that all other users consume Q applications according to a balanced strategy. If user k consumes fewer applications than other users, $Q^k \leq Q$, it is optimal for her to consume them according to a balanced consumptions schedule. Then, the net utility of user k is

$$U_{DI}(Q^k \leq Q) = (Q^k)^{R-1} X + Q^k \frac{X}{Q^k} \alpha \frac{X}{Q} (N-1).$$

This utility strictly increases with Q^k , and yields the highest value for $Q^k = Q$. Therefore, user k has no incentive to deviate downward, and consume fewer applications than other users.

Now, we show that if $Q < A$, then user k always has incentive to consume more applications than other users. Suppose that user k consumes one more application than other users. She diverts some y of her time to the new application, while it is optimal for her to consume at the same level all the applications that other users consume, $\frac{X-y}{Q}$. Then, user k 's utility is

$$\left(Q \left(\frac{X-y}{Q} \right)^{\frac{1}{R}} + y^{\frac{1}{R}} \right)^R + Q \frac{X}{Q} (X-y) \alpha (N-1).$$

Using additional application brings user k benefit due to preference for variety. However, diverting time from applications that are consumed by other users decreases user k payoff due to consumption complementarity (the direct network effect). The marginal "cost" of diverting consumption due to direct network effect is $\frac{\partial X(X-y)\alpha(N-1)}{\partial y} = \alpha X(N-1)$. The marginal benefit due to preference for variety is

$$\frac{\partial \left(Q^{1-\frac{1}{R}}(X-y)^{\frac{1}{R}} + y^{\frac{1}{R}} \right)^R}{\partial y} = \underbrace{\left(Q^{1-\frac{1}{R}}(X-y)^{\frac{1}{R}} + y^{\frac{1}{R}} \right)^{R-1}}_{f(y)} \left(y^{1-\frac{1}{R}} - Q^{1-\frac{1}{R}}(X-y)^{\frac{1}{R}-1} \right).$$

Function $f(y)$ is strictly decreasing in y , and as $y \rightarrow 0^+$, $f(y) \rightarrow \infty$. Therefore, for any value of $\alpha X(N - 1)$, there exists small enough y for which $f(y) > \alpha X(N - 1)$. That means that there always exists a consumption schedule (characterized by y for which it is beneficial for user k to deviate from Q and consume one more application.³⁰

Since for all $Q < A$ user k has incentive to deviate toward consuming more applications, such Q cannot characterize an equilibrium. When $Q = A$ a deviation upward is not feasible, and no user finds it profitable to deviate downward. Therefore, in a balanced equilibrium all applications are consumed by all users. There is only one such equilibrium.

This completes the proof of Proposition 6. ■

Corollary 7 *In the case for $1 < R < 2$, $\alpha > 0$ and $p = 0$, the unique balanced equilibrium may or may not yield the highest possible utility to the users. When $V(A) \geq V(1)$, then the unique balanced equilibrium, $Q_{DI} = A$, yields the highest possible utility to the users. But when $V(A) < V(1)$, the maximal utility is achieved at $Q = 1$.*

Proof. Let $1 < R < 2$, $\alpha > 0$ and $p = 0$. In the unique balanced equilibrium all users consume all A available applications, which yields utility

$$U_{DI}^*(Q_{DI}=A) = A^{R-1}X + \alpha \frac{X^2}{A}(N - 1).$$

Now, suppose that all users would play a balanced strategy where they all consume a set of applications \mathbb{Q} of cardinality Q . Then, user k 's payoff is

$$\underbrace{Q^{R-1}X + \alpha \frac{X^2}{Q}(N - 1)}_{V(Q)},$$

³⁰Notice that here it is fine to compare derivatives, while in the case of $p > 0$ it is not. Here for every little bit of y , we lose $y \cdot \alpha X(N - 1)$, and we benefit more than $y \cdot f(y)$ (this is because it is an underestimation, for lower y 's, $f(y)$ is higher). In the case of $p > 0$, user k needs to pay the whole p , even if using infinitesimally small amount of y .

which is the same function as V for $p = 0$. This function $V(Q)$ has only one optimum,³¹ at $Q = \left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}}$. This is a minimum.³² So, if $\left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}} \leq 1$, $V(Q)$ is increasing for all $A \geq 1$. When $\left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}} > 1$, $V(Q)$ is first decreasing and then increasing. In such a case, $V(Q)$ has two local maxima: at $Q = 1$ and at $Q = A$. It is possible that $V(1) > V(A)$, even though $Q = 1$ is not an equilibrium. When $V(A) \geq V(1)$, then the unique balanced equilibrium yields the highest possible utility to the users. But when $V(A) < V(1)$, the equilibrium yields lower utility than the allocation where $Q = 1$.

This completes the proof of Corollary 7. ■

B.1.4 Game with perfect foresight: on the role of the platform

With pure direct network effects there exist many possible equilibria. However, equilibria where exactly one application is consumed yield higher utility to the users than other equilibria. Equilibria where exactly one application is consumed always exist. But if more than one application is available, $A > 1$, there also exist other equilibria, which yield lower utility. The platform eliminates the equilibria that yield lower utility by setting $A = 1$.

With pure indirect network effects there exists a unique equilibrium. This equilibrium yields the highest possible utility to the users for a given A , i.e., in a given environment, users could not be better off if they consumed any other number of applications. However, the equilibrium net utility of each user: $A^{R-1}X$ increases with A . Therefore, the larger the number of applications the platform provides, the larger is users' net utility in the equilibrium.

³¹The first order condition

$$\frac{\partial V(Q)}{\partial Q} = \frac{X}{Q^2} [(R-1)Q^R - \alpha X(N-1)] = 0$$

is satisfied only for $Q = \left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}}$.

³²This result may be obtained in two ways: First, it is enough to show that for Q lower than this threshold, the derivative is negative; and for Q higher than the threshold, the derivative is positive. In the second approach, we show that the second derivative of $V(Q)$ is negative for $Q = \left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}}$.

$$V''(Q) = (R-1) \underbrace{(R-2)}_{-} Q^{R-3} X + 2\alpha \frac{X^2}{Q^3} (N-1) = 0 \iff Q^R = \frac{2\alpha X(N-1)}{(R-1)(2-R)}.$$

For $Q < \left(\frac{2\alpha X(N-1)}{(R-1)(2-R)}\right)^{\frac{1}{R}}$, $V''(Q) < 0$. And $\left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}} < \left(\frac{2\alpha X(N-1)}{(R-1)(2-R)}\right)^{\frac{1}{R}}$, so the second derivative is negative where the first order condition is satisfied.

In the presence of both direct and indirect network effect there exists a unique equilibrium. But for a given A it may or may not yield the highest possible utility to the users (Corollary 7). If the platform is bounded in setting A and cannot provide more applications than \bar{A} , then it needs to consider whether $V(1) \leq V(\bar{A})$ or not. When $V(1) \leq V(\bar{A})$, then setting $A = \bar{A}$ would maximize users' net utility. But when $V(1) > V(\bar{A})$, then $A = 1$ maximizes users' utility.

If the platform is not bounded in setting A . It should set as large A as possible. This is because $U_{DI}^*(A) \rightarrow \infty$ when $A \rightarrow \infty$. Therefore, for large enough A , $U_{DI}^*(A) = V(A) > V(1)$, and then $U_{DI}^*(A)$ is only decreasing in A .

B.2 Game with no foresight

B.2.1 No-foresight equilibrium

Proposition 7 *Assume that $1 < R < 2$ and $\alpha > 0$. There exists a unique no-foresight equilibrium, where $\mathbb{G}_{DI}^k = \mathbb{A}$ each user k , i.e., every user k consumes all available applications. Moreover, all users play balanced strategies in this equilibrium.*

Proof. Suppose that that $1 < R < 2$ and $\alpha > 0$. Under the assumption of no-foresight, and $p = 0$, user k 's net utility from consuming a set of applications \mathbb{G}^k is

$$U_{DI}(\{x_a^k\} | x_a^k \in \mathbb{G}^k) = \left(\sum_{x_a^k \in \mathbb{G}^k} (x_a^k)^{\frac{1}{R}} \right)^R + \alpha(N-1) \frac{X^2}{A}.$$

Maximizing $U_{DI}(\{x_a^k\} | x_a^k \in \mathbb{G}^k)$ under the constraint that $\sum_{x_a^k \in \mathbb{G}^k} x_a^k \leq X$, yields the same first order condition for every x_a^k . Therefore, the optimal consumption schedule is a balanced consumption.

Under balanced consumption, the utility of user k 's from consuming G^k applications is $U_{DI}(G^k) = (G^k)^{R-1} X + \alpha(N-1) \frac{X^2}{A}$. This utility is strictly increasing in G^k . Therefore, every user finds it optimal to consume all A available applications, and in equilibrium all users play a balanced strategy and consume $\mathbb{G}_{DI}^k = \mathbb{A}$.

This completes the proof of Proposition 7. ■

B.2.2 On the role of the platform

Under pure direct network effects, when the platform provides $A \geq 1$ applications, each user's utility in any equilibrium is

$$U_D^* = X + \alpha(N - 1)\frac{X^2}{A}.$$

This utility strictly decreases with A . The platform with the objective to maximize users' net utility should set $A = 1$.

Under pure indirect network effects, foresight plays no role. The cases of no-foresight and perfect foresight are the same: The users' utility strictly increases in A and, therefore, the platform should provide as many applications as possible.

When both network effects are present, the unique equilibrium under no-foresight equilibrium is the same as the unique balanced equilibrium under perfect foresight: All users consume all A available applications according to the balanced consumption schedule, and achieve the equilibrium utility of

$$U_{DI}^*(G_{DI}^k = A) = A^{R-1}X + \alpha\frac{X^2}{A}(N - 1).$$

Therefore, the same analysis as in the case of perfect foresight leads us to conclusion that if the platform is unbounded while setting A , it should set as large A as possible. When it is bounded by \bar{A} , it needs to consider whether $V(\bar{A}) \geq V(1)$ or not.

References

- Armstrong, Mark. 2006. Competition in two-sided markets. *Rand Journal of Economics* **37**(3) 668–691.
- Arthur, Brian W. 1990. Positive feedbacks in the economy. *Scientific American* **262** 92–99.
- Brandenburger, A. 1995. Power Play (A): Nintendo in 8-bit Video Games. *Harvard Business School Case and Teaching Note* **795-102, 795-167**.
- Cantillon, Estelle, Pai-Ling Yin. 2010. Competition between exchanges: Lessons from the battle of the bund. *MIT Working Paper* .
- Cantillon, Estelle, Pai-Ling Yin. 2011. Competition between exchanges: A research agenda. *International Journal of Industrial Organization* (forthcoming).
- Casadesus-Masanell, R., N. Campbell. 2008. Betfair vs. the UK Bookmakers. *Harvard Business School Case and Teaching Note* **709-417, 709-418**.
- Casadesus-Masanell, Ramon, Hanna Hałaburda. 2010. When does a platform create value by limiting choice? NET Institute Working Paper No. 10-04.
- Chou, Chien-fu, Oz Shy. 1996. Do consumers gain or lose when more people buy the same brand. *European Journal of Political Economy* **12**(1) 309–330.
- Church, Jeffrey, Neil Gandal. 1992. Network effects, software provision and standardization. *Journal of Industrial Economics* **40**(1) 85–104.
- Dixit, Avinash K., Joseph E. Stiglitz. 1977. Monopolistic competition and optimum product diversity. *American Economic Review* **67**(3) 297–308.
- El-Gamal, Mahmoud A., David M. Grether. 1995. Are people bayesian? uncovering behavioral strategies. *Journal of the American Statistical Association* **90** 1137–1145.
- Ellison, Glenn, Drew Fudenberg. 2003. Knife-edge or plateau: When do market models tip? *Quarterly Journal of Economics* **118**(4) 1249–1278.
- Hagiu, Andrei. 2009. Two-sided platforms: Pricing and social efficiency. *Journal of Economics & Management Strategy*, **18**(4).
- Hałaburda, Hanna, Mikołaj Piskorski. 2010. Platforms and limits to network effects. Harvard Business School Working Paper.
- Hurkens, Sjaak, Angel L. Lopez. 2010. Mobile termination, network externalities, and consumer expectations. IESE Working Paper, D/850-E.
- Katz, Michael L., Carl Shapiro. 1985. Network externalities, competition, and compatibility. *American Economic Review* **75**(3) 424–440.
- Rochet, Jean-Charles, Jean Tirole. 2003. Platform competition in two-sided markets. *Journal of European Economic Association* **1**(4) 990–1029.

Weyl, Glen. 2010. A price theory of multi-sided platforms. *American Economic Review* **100**(4) 1642–72.

Zhao, Luyi. 2010. Quantity constraints on two-sided platforms. *Harvard College Senior Thesis*.