Beef Up Your Competitor: A Model of Advertising Cooperation between Internet Search Engines*

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Abstract

We propose a duopoly model of competition between internet search engines endowed with different technologies and study the effects of an agreement where the more advanced firm shares its technology with the inferior one. We show that the superior firm enters the agreement only if it results in a large enough increase in demand for advertising space at the competing firm and a relatively small improvement of the competitor's search quality. Although the superior firm gains market share, the agreement is beneficial for the inferior firm, as the later firm's additional revenues from a higher advertising demand outweigh its losses due to a smaller user pool. If the cooperation takes place it is in line with the advertisers' interests but is detrimental to users' welfare.

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1 Introduction

Few other markets have experienced such a dramatic growth during the last decade as internet search. The first search engines appeared in the early nineties as small enterprises and were only used by a narrow circle of users. By contrast, the search engine *Google* alone handled around 20 million searches per day in 2000.[1] In 2009 this figure amounted to near 200 million in the U.S. alone.[2] Within a few years, *Google*, the company that emerged to be the leader in online search and advertising reached a market capital of $102 billion in 2009.[3] The rise of internet search went hand-in-hand with a boom in online advertising. While in 1998 internet advertising revenues were around $2 billion in the United States, in 2008 they totalled over $22 billion.[4] A large share of this type of advertising is provided by search engines: between 2004 and 2008 search based advertising revenues accounted for around 40 percent of internet advertising revenues.[5]

Search engines deliver search results to keywords given by users. At the same time, they display advertisements next to the search results. These are paid links for a particular keyword bought by advertisers who wish to direct users from the search engine's results page to their own web pages. Search engines typically provide search service free of charge for users while charging advertisers for displaying their links next to the search results. Payments from the advertisers constitute the main source of revenues for the search engines.

Search engines face a twofold task: First, they need to maintain access to the available content in the internet. Second, they must be able to match the most relevant content to the users' search queries. Every search engine uses a particular ranking algorithm which determines the relevance of a certain web page to a user's query. These algorithms calculate the overall relevance of a retrieved web page based on several factors, such as the number and importance of links pointing to the page and the frequency as well as the environment of the keywords' occurrence at the page. The search results generated by such an algorithm are called *organic*

[1] Google Press Release, October 23, 2000, (http://www.google.com/press/pressrel/pressrelease39.html)

[2] Own calculation based on the monthly number of searches conducted in the United States, as reported by Nielsen Online. (http://www.nielsen-online.com/pr/pr_090310.pdf)

[3] Yahoo! Finance (http://finance.yahoo.com/q/ks?s=Goog), retrieved on March 13, 2009

[4] IAB Internet Advertising Revenue Report 2008 Q2 (http://www.iab.net/media/file/IAB_PWC_2008_6m.pdf)

[5] IAB Internet Advertising Revenue Report 2008 Q2 (http://www.iab.net/media/file/IAB_PWC_2008_6m.pdf)

results. Since the ranking algorithms are optimized so as to retrieve the most relevant web pages for a keyword combination, we regard organic results as the best match to the user's query. They are typically displayed on the left side of the search results page. Advertisements (so called *sponsored links)* are usually displayed next to the organic results on the top and on the right side of the page.

Search engines are two-sided businesses. They operate as platforms connecting users with advertisers. Advertisers aim to reach a possibly large audience with their advertisements and value a search engine with a larger user base more. On the other hand, users prefer to see either better matching advertisements or as few of them as possible as such sponsored links may bring distortion into the organic search results and reduce the overall quality of search. At the same time users value the design of a search engine's homepage, and the additional services that can be quickly reached from it, such as email, maps, etc. The task search engines face is to choose the number of sponsored links and the level of differentiation in a way that attracts users and advertisers in a profit-maximizing manner.

The last years have seen a strong concentration trend in the search engine market. *Google* emerged as the leading firm with a share of searches above 60 percent in the U.S. and even higher in Europe.[6] In the recent years *Google* has been continuously in the focus of antitrust authorities in the U.S. and Europe due to its participation in a number of transactions, such as its acquisition of *YouTube* and *DoubleClick*. The most recent arrangement raising the attention of competition agencies was *Google*'s planned cooperation with its closest competitor *Yahoo* in 2008.

In June 2008 *Google* and *Yahoo* announced plans for an agreement to cooperate in advertising. The planned partnership would have let *Yahoo* use *Google*'s technology to match advertisements with search keywords in Canada and the U.S. The proposed cooperation attracted antitrust scrutiny in several jurisdictions. The U.S. Department of Justice (DoJ) opened a formal investigation of the deal in July, and the Canadian Competition Bureau launched a review of the plans in August. Although the scope of the planned agreement was formally limited to Canada and the U.S., the European Commission also started a preliminary investigation into the potential effects of the deal. In November the DoJ announced that it would file suit to block

[6] Advertising Age Search Marketing Fact Pack 2008

the arrangement between *Yahoo* and *Google*. On the same day, the parties officially abandoned the agreement. The DoJ justified its approach by claiming that the deal would have virtually eliminated *Yahoo* as a competitor in the advertising market, and outsourcing advertisements to *Google* would have significantly reduced *Yahoo*'s incentives to invest in search advertising technology.[7]

It is widely accepted that *Google* possesses a superior technology for matching advertisements with search keywords and is able to place more relevant advertisements to queries. Sharing this technology with its competitor, *Google* would improve the quality of services provided by *Yahoo*. We distinguish between two effects this may have. First, making a superior technology available to *Yahoo* could have an effect on the advertisers' willingness to pay for advertisement space through the increased probability of a successful sale of the advertised good. Second, the users could benefit from increased search quality as *Yahoo* displayed more relevant advertisements on the search results page.

In the present paper we propose a model that takes into account the most important characteristics of the internet search market. In our model two horizontally and vertically differentiated search engines operators compete with each other. They provide search results to users and sell advertising space to advertisers. Advertisers aim to reach a possibly large audience while users prefer more relevant search results. Advertising has a dual role: on the one hand it is informative, making users value better matched advertisements more. On the other hand, advertising is a nuisance to users since more advertisements bring distortion into the organic search results. We take one firm to be endowed with a more advanced technology to match search queries with advertisements. This allows it to provide more relevant search results for users and to charge higher prices for its advertisement slots. We examine two research questions. First, we analyze what motivates the operator of a superior search engine to strengthen its competitor through providing it with a more advanced technology. Second, we address the welfare implications of such a cooperation agreement for advertisers and users. We conclude that the superior search engine will enter the agreement only if it results in a relatively large increase in demand from advertisers and a relatively small improvement of the competitor's search quality. Although the superior search engine gains market share the deal is beneficial for the inferior firm. If the deal

[7] DoJ (2008)

takes place, it is in line with the advertisers' interest but it is detrimental to users' welfare.

Our work can be placed into the broader context of three distinct strands of literature. First, it contributes to the emerging literature on two-sided markets, focusing on platform businesses that facilitate transaction between distinct groups of consumers in the presence of indirect network effects. Second, it fits into the literature related to advertising-supported media, which tends to present advertising as a nuisance for the consumers. Third, it adds to the literature on the economics of internet search engines.

In two-sided markets, platforms intermediate between two sets of agents, such that the participation of one group influences the value of participating for the other group. The seminal papers of Armstrong (2006) and Rochet and Tirole (2003) provide overviews on the economics of multi-sided platforms, and come to similar results. Armstrong (2006) focuses on the case of positive externalities arising on both sides of the market. He shows that a platform sets prices on each sides of the market in a way that the group generating larger network externalities or the one with more elastic demand will be subsidized by the other group. Rochet and Tirole (2003) provide a somewhat more conservative definition for what constitutes a two-side market, and show a similar pattern of cross-subsidization of the two sides. Reisinger (2004) models competing platforms in a setting where one side exerts a negative externality on the other side. Evans and Schmalensee (2005) address competition policy issues arising in a two-sided environment.

Our work is closely related to the literature on advertising-supported media (see Bagwell (2007) for a survey). In this strand, Anderson and Coate (2005) provide a seminal contribution to the literature on platform competition for advertisers. In their model of competition between two TV channels, they analyze how these platforms differ in terms of the levels of advertising and program choice. They find, that both equilibrium advertising levels as well as program quality can be either too high or too low. Gal-Or and Dukes (2006) analyze the conditions, under which a non-consolidating merger between two TV or radio stations can be profitable. In their model, consumers are averse to advertisement but may draw a positive utility from it, as it informs them about prices. A merger between stations increases the level of advertising which decreases the advertisers' prices and profits. This again decreases their willingness to pay for advertising, which may render the stations' merger unprofitable.

Our work contributes to the relatively new strand of research focusing on the economics of internet search engines. Evans (2008) provides an extensive overview of the history and characteristics of the online advertising industry. A lot of the research dealing with the economics of search engines focuses on the auction mechanisms search engines use to sell advertisement space on their web sites. Work in this area includes e.g. Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007). Telang, Rajan and Mukhopaday (2004) focus on the organization of internet search engines and Pollock (2008) analyzes factors facilitating concentration in the search engine market and discusses possible regulatory interventions. White (2008) examines the search engine's incentives to distort search quality by directing some users to paid links in order to extract revenues. Beschorner (2008) uses a model to review the latest series of deals involving *Google* including the planned cooperation agreement with *Yahoo*. He highlights the increased ability to provide customized services and content as a benefit to be weighed off against the increase in concentration which may result from these transactions.

The paper proceeds as follows. In the next section we present the set-up of the model. In Section 3 we derive and characterize the equilibrium. In Section 4 we apply our framework to the analysis of the advertising cooperation between the search engines. Finally, Section 5 concludes.

2 The Model

We analyze a two-sided market in which two horizontally and vertically differentiated search engine operators $i = \{1, 2\}$ provide search results to users and sell advertising space to advertisers. We assume that firm 1 possesses a superior technology to match advertisements to search queries and provides services of better quality to both users as well as advertisers. We will refer to firm 1 as the *superior* and to firm 2 as the *inferior* firm. The search results are free for users, while advertisers pay price p_i for an advertising slot at search engine i. Each search engine operator decides on the number of advertising slots it places on its web page, denoted by a_i. Search engines provide their services at zero marginal cost and realize profits

$$\pi_i = p_i a_i. \tag{1}$$

We assume that users single-home, i.e. every user conducts search at only one search engine (see Evans, 2008). We assume furthermore that the users' demand for the advertisers' goods is homogenous. It follows from these assumptions that the value of an advertisement placed at one search engine is independent of the value of an advertisement placed at the other, hence we have separate demand functions for advertisement slots at the two engines.

If an advertiser k places an advertisement of his product at search engine i, its expected profit $E\left(\pi_i^k\right)$ takes the form

$$E\left(\pi_i^k\right) = \Pr\{Sale\}\, n_i p^k - c^k - p_i,$$

where n_i, $p^k$and c^k denote the number of users at search engine i, the price of firm k's product and advertising costs respectively. We assume that every user buys exactly one unit of the good and normalize its price to unity ($p^k = 1$). The advertising costs c^k capture the firms' fixed costs associated with placing an advertisement other than the price paid for advertising space, such as the costs for designing the advertisement, acquiring the necessary computer literacy, etc. The advertisers are heterogeneous with respect to their costs which are uniformly distributed on the interval $c^k \in [0, \infty)$. $\Pr\{Sale\}$ denotes the probability that a user buys the product of advertiser k after having seen its advertisement on search engine i's result page. We assume that $\Pr\{Sale\} = 1 - \rho_i$, where the parameter ρ_i captures search engine i's ability to match search queries with advertisements. The maximum willingness to pay for an advertisement slot at search engine i is than given by

$$p_i = (1 - \rho_i) n_i - a_i. \tag{2}$$

With its superior technology, firm 1 can display more relevant advertisements which increases the probability of a successful sale by the advertiser. This translates into a higher willingness to pay for an advertising slot by the advertisers. We assume that for the superior search engine $\rho_1 = 0$, while $\rho_2 \in (0, 1)$ reflects the inferior firm's "handicap" in matching advertisements with keywords.

Users derive utility $\zeta_i q$ from conducting search at engine i with quality index $q > 0$. The term $\zeta_i q$ is higher if search results are matched better to queries. Search results include both organic results as well as advertisements. We allow users to derive utility from better matched advertisements since ads can be informative for users. We assume that the quality of the search results is higher at engine 1 and normalize ζ_1 to 1 while $\zeta_2 \in (0, 1)$. With $\zeta_2 < \zeta_1$ the engines are vertically differentiated. For notational simplicity in the following we will write ζ

and ρ instead of ζ_2 and ρ_2 respectively. Let Δ denote the quality advantage if firm 1, with $\Delta := (\zeta_1 - \zeta_2)q = (1-\zeta)q$. If $\Delta = 0$, firm 1 has no quality advantage compared to firm 2, while $\Delta > 0$ means that firm 1 has some quality advantage. We assume that the superior firm's quality advantage lies in the range $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\right)$ with $\underline{\Delta} = \mu\rho/[4(1+\mu)(1+\mu(1-\rho))]$ and $\overline{\Delta} = (1+3\mu)/[4(1+\mu)]$. We will analyze this restriction in more detail in the next section. The assumption that one firm is able to provide better matched search results for users and a higher probability of sale of the advertised products is realistic. It is widely accepted that *Google* has a more advanced technology than its closest competitor *Yahoo*. The New York Times notes that "*Google makes 60 percent more revenue for each search than Yahoo because of its superior technology and larger market share, which attracts more advertisers to bid in Google's ad auctions.*"[8]

Search engines differ in terms of the design of their home pages and the variety of extra services (such as email, maps, applications, etc.) they provide to users. It is reasonable to treat search engines as horizontally differentiated. Specific features of a search engine's home page seem to play an important role in determining users choice which search engine to use.[9]

The engines are placed on a unit circle such that the address of firm 1 is normalized to $s_1 = 0$, the address of the other firm is given by $s_2 \in (0,1)$. Users are uniformly distributed on the circle, with each having an address $t \in [0,1]$ reflecting their preferences for the optimal search engine. Searching at search engine i involves quadratic transportation costs which are positive if a user conducts search on an engine that is not located in his ideal position.

In our model advertisements (sponsored links) play a dual role. On the one hand they inform users of products they might be interested in, hence users benefit from more relevant advertisements. On the other hand advertisements distort search results. Organic search results are determined by the search engine's ranking algorithm, which takes into account a broad range of factors, including the clicking behavior of all users having conducted search before to predict which pages are the most relevant for a query. The order of organic results is the best predictor

[8] See "Google Says It Will Defend Competitive Rationale of a Yahoo Deal," New York Times online, May 22, 2008 (http://www.nytimes.com/2008/05/22/technology/22google.html).

[9] According to a survey carried out by Keynote Systems in 2008 "home page appeal" and "special features" are among the most important factors influencing customer satisfaction with a search engine. See "Search Portals Study," Keynote Systems, 2008 (http://www.keynote.com/docs/kcr/KCR_Search_2008.pdf).

of a page's relevance according to the engine's ranking algorithm, hence sponsored links distort the optimal ranking as they allow advertisers to "buy their place" at the top of the search result list. The users' disutility of advertisements depends on the number of ads and is given by a linear function μa_i with $\mu > 0$.

The utility function of a user searching at engine i than takes the form

$$U_i(t) = \zeta_i q - \delta_i^2 - \mu a_i \tag{3}$$

$$\text{with } \delta_1 = \min\{(t - s_1), (1 - t - s_1)\} \text{ and } \delta_2 = (t - s_2),$$

where δ_i^2 captures transportation costs.

In the following we restrict our attention to the case where the superior search engine serves more users than its competitor ($n_1 > n_2$). This assumption is realistic: Google is widely regarded to possess a more advanced technology to match queries with organic and sponsored links and has a significantly larger market share than its competitors in most jurisdictions.[10]

The timing of the game is as follows: First, search engine operator 2 decides on its location and both operators determine the number of advertisement slots at their web pages simultaneously. Second, users choose their preferred search engine and advertisers buy advertisement slots. We seek the subgame-perfect Nash equilibrium and solve the game backwards.

3 Equilibrium Analysis and Main Results

Every user chooses the search engine providing him the highest utility. We can find two marginal users with addresses t_1 and t_2 who are indifferent between searching at the two engines:

$$t_1(a_1, a_2, s_2) = \frac{\mu(a_2 - a_1) + s_2^2 + \Delta}{2s_2}, \tag{4}$$

$$t_2(a_1, a_2, s_2) = \frac{\mu(a_1 - a_2) + 1 - s_2^2 - \Delta}{2(1 - s_2)},$$

[10] In the United States around 70 percent of all searches are conducted on Google while Yahoo has a share of 17 percent. Other players balance on the verge of invisibility. See "Top 20 Sites & Engines," Hitwise, March 14, 2009 (http://www.hitwise.com/resources/data-center.php).

with $t_1 < t_2$. The market shares of the firms are then $n_1 = 1 - t_1 + t_2$ and $n_2 = t_2 - t_1$. This yields the following user demand at the search engines :

$$\begin{aligned} n_1(a_1, a_2, s_2) &= \frac{1}{2} + \frac{\Delta - \mu(a_1 - a_2)}{2s_2(1 - s_2)}, \\ n_2(a_1, a_2, s_2) &= \frac{1}{2} - \frac{\Delta - \mu(a_1 - a_2)}{2s_2(1 - s_2)}, \end{aligned} \tag{5}$$

with $\partial n_i / \partial a_i < 0$ and $\partial n_i / \partial a_j > 0$ for $i \neq j$. In the following we restrict our attention to the case where the superior search engine has a larger market share, that is $n_1 > n_2$. Plugging (5) into (2) we get the demand for advertisement slots as a function of a_1, a_2 and s_2. The search engine operators maximize their profits

$$\pi_i(a_1, a_2, s_2) = [(1 - \rho_i) n_i(a_1, a_2, s_2) - a_i]\, a_i$$

by choosing the number of slots and firm 2 chooses its position on the unit circle. In the following proposition we characterize the equilibrium in which both firms serve users and firm 1 has a larger user pool.

Proposition 1. *The market situation in which both firms serve users and firm* 1 *has a larger market share, is an equilibrium only if* $\underline{\Delta} < \Delta < \overline{\Delta}$ *with* $\underline{\Delta} = \mu\rho / [4(1 + \mu)(1 + \mu(1 - \rho))]$ *and* $\overline{\Delta} = (1+3\mu)/[4(1+\mu)]$. *Moreover, in this equilibrium the search engine operators choose maximal differentiation with* $s_2^* = 1/2$, *both search engines place positive number of advertisement slots on their web pages, firm* 1 *displays more advertisements and charges a higher price for its advertisement slots.*

Proof. See Appendix.

In equilibrium it is optimal for the firms to choose maximal differentiation in order to reduce competition for users from each other. Proposition 1 shows that a situation in which two firms are in the market with the superior firm having a larger market share is an equilibrium only if the quality advantage of the superior firm is not too large, but not too small either. If the quality advantage exceeds a certain upper threshold, search engine 1 would attract all the users even if firm 2 placed no advertisements. To understand the intuition behind this result it is instructive to consider the firms' reaction functions $a_1(a_2)$ and $a_2(a_1)$, which give the optimal amount of advertisement placed by firm i in response to the advertisement placed by firm j.

The reaction functions of firms 1 and 2 for $s_2 = 1/2$ are

$$a_1(a_2) = \frac{1 + 4(\Delta + \mu a_2)}{4(1 + 2\mu)} \tag{6}$$
$$a_2(a_1) = \max\left\{0, \frac{(1-\rho)(1 + 4(\mu a_1 - \Delta))}{4(1 + 2\mu(1-\rho))}\right\}.$$

Note, that the firms' decisions about the amount of advertisements placed are strategic complements as $\partial a_1(a_2)/\partial a_2 = \mu/(1 + 2\mu) > 0$ and $\partial a_2(a_1)/\partial a_1 = \mu/(1 + 2\mu(1-\rho) > 0$ hold. Since the superior firm has a quality advantage, it can place advertisements, even if its competitor does not do so. This is not the case for the inferior firm: the larger the superior firm's quality advantage, the more likely it is that the inferior firm does not advertise at all in equilibrium. Figure (1) represents two equilibria for the cases $\Delta < \overline{\Delta}$ and $\Delta = \overline{\Delta}$. In one equilibrium given by point A when the quality advantage of the superior firm is not too high (i.e. $\Delta < \overline{\Delta}$ holds), both firms display advertisements. In equilibrium B where the superior firm has a relatively large quality advantage (i.e. $\Delta = \overline{\Delta}$ holds), the inferior firm has no users and consequently does not display advertisements.

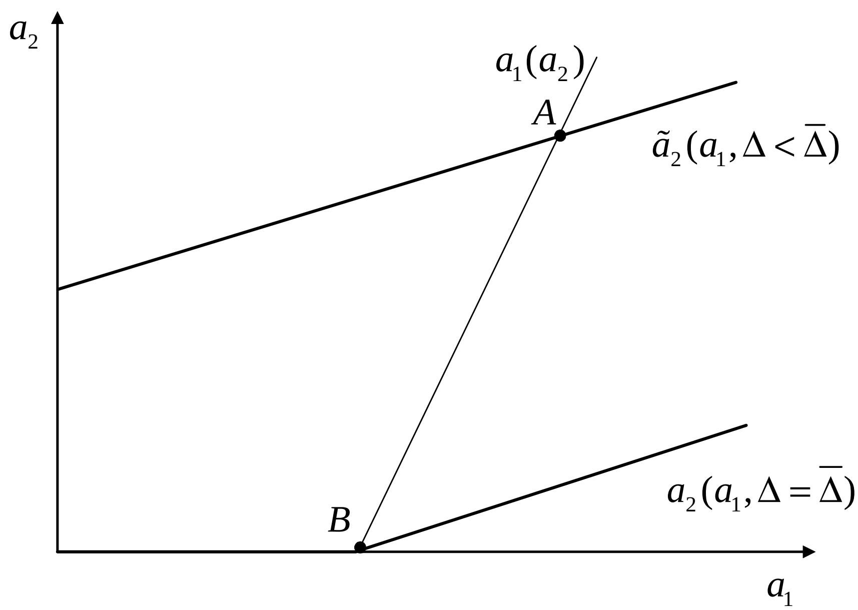

Figure 1: Reaction functions for different levels of quality advantage for firm 1

At the same time the quality advantage has a lower bound: it must be high enough for the superior search engine to have a larger share of users. The superior firm exploits its quality advantage and displays in equilibrium more advertisements than the inferior firm. In order to keep its dominant position, the quality advantage must be high enough to compensate for the disutility from showing more advertisements. Since the superior firm has a larger user pool, it faces higher demand for advertisement slots in equilibrium, and can not only place more advertisements, but also charge higher price per slot, with

$$
\begin{aligned}
p_1^* &= \left(1 - \frac{\partial n_1^*}{\partial a_1}\right) a_1^* = (1 + 2\mu)\, a_1^*, \\
p_2^* &= \left(1 - (1-\rho)\frac{\partial n_2^*}{\partial a_2}\right) a_2^* = (1 + (1-\rho) 2\mu)\, a_2^*,
\end{aligned}
\tag{7}
$$

where $p_1^* > p_2^*$.

In the next section we analyze the effects of a cooperation agreement (*ad-sharing agreement*) between the search engine operators.

4 Advertising Cooperation

We assume that the cooperation increases the quality of the services provided by the inferior search engine in two ways: *First*, it is able to provide more relevant search results to its users. *Second*, it can match the advertisements with the search queries better, which increases the probability of a successful sale by an advertiser. We capture this formally by assuming that the ad-sharing agreement results in a simultaneous increase in parameter ζ and a decrease in parameter ρ.

We analyze the incentives of the search engines to enter into an ad-sharing agreement and examine the effects of such an agreement on the advertisers' and consumers' surpluses. We start with analyzing the effect of an increase in the demand for advertisement slots resulting from a decrease in parameter ρ.

Proposition 2. *Suppose $\underline{\Delta} < \Delta < \overline{\Delta}$. As the demand for advertisement slots of the inferior firm rises (i.e. ρ decreases), the following holds:*

i) both search engines provide more advertisement slots,

ii) the superior (inferior) search engine gains (loses) market share of users,

iii) both search engines charge a higher price for the advertisement slots, *iv) both search engines make larger profits.*

Proof. See Appendix.

According to Proposition 2 both firms increase the number of advertisements with the rise in the demand for advertisement slots at the inferior search engine. With a decrease in parameter ρ the superior firm's reaction function remains unchanged while that of the inferior firm shifts outward for $a_2 > 0$.

The inferior firm displays more advertisements as it is directly affected by the increase in demand. The superior firm is affected indirectly, and can increase the number of advertisement slots if the competitor does so. It is instructive to inspect the reaction functions as given by Expression (6). Figure (2) illustrates the change in the equilibrium for two situations. In the first situation the reaction function of the inferior firm $a_2(a_1)$ is affected by the decrease in parameter ρ in two ways: its slope increases and it shifts upwards $(a_2^{\rho}(a_1))$. The equilibrium point moves from F to G. In the second case, the reaction function of the inferior firm $\widetilde{a}_2(a_1)$ rotates around point C.[11] The equilibrium point shifts from D to E. Both cases result in a higher number of advertisements at both search engines.

It is the sum of two effects that determines how the equilibrium number of advertising changes following a decrease in parameter ρ. The *direct effect* originates from a change in demand for advertising slots at the inferior firm and only the advertising decision of the inferior firm is affected by it. The *strategic effect* results from the observation that the firms' decisions on the number of advertisement slots are strategically related. If one firm displays more advertisements, the other can do so as well. The two effects can be disentangled using the firms' reaction functions:

$$\frac{\partial a_i^*}{\partial \rho} = \underbrace{\frac{\partial a_i(a_j = 0)}{\partial \rho}}_{\text{direct effect}} + \underbrace{\frac{\partial a_i(a_j)}{\partial a_j}\frac{\partial a_j^*}{\partial \rho}}_{\text{strategic effect}}.$$

The inferior firm is affected directly by the increase in demand for its advertisement slots. As advertising space becomes more valuable on its homepage, it displays more advertisements. The advertisers' demand at the superior firm remains unchanged, thus it is affected only indirectly by the change in parameter ρ, through the strategic effect. The strategic effect is at work at

[11] The number of advertisements the superior firm needs to place in order to drive the inferior firm out of the market does not depend on ρ.

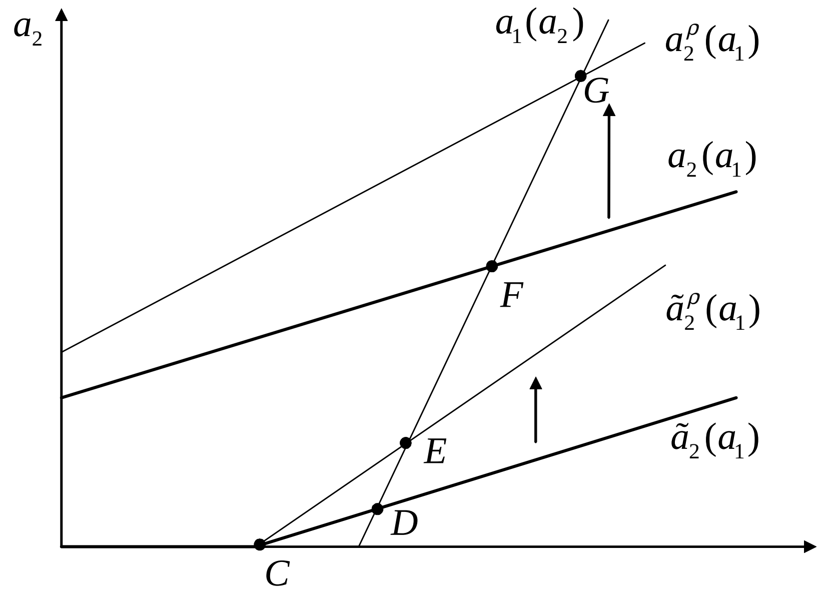

Figure 2: The effect of a change in parameter ρ on the equilibrium

the inferior search engine too and amplifies the positive direct effect. As a result, in the new equilibrium both search engines display more advertisements. Table 1 summarizes these effects.

	direct effect	strategic effect	total effect
a_1	0	+	+
a_2	+	+	+

Table 1: effects of a decrease in parameter ρ on the number of advertising slots

Both search engines benefit from the increased demand for advertising slots at the inferior firm since they can both place more advertisements and charge a higher price per slot.

We now analyze the effects of an increase in the users' utility from searching at the inferior firm due to the ad-sharing agreement. Both firms use a technology which matches advertisements with search queries. The superior firm displays more relevant advertisements next to the search results. This technology now becomes available for the other firm through the ad-sharing agreement resulting in an increase of parameter ζ (i.e. Δ decreases).

Proposition 3. *As the inferior firm provides higher utility from its services to users due to the increase of parameter ζ:*

i) the superior (inferior) search engine displays less (more) advertisements,

ii) the superior (inferior) search engine loses (gains) market share of users,

iii) the superior (inferior) firm chargers a lower (higher) price for the advertisement slots,

iv) the superior (inferior) firm makes lower (higher) profits,

v) the effect on the joint profits is negative.

Proof. See Appendix.

Again, it is helpful to distinguish between a *direct effect* and a *strategic effect* of the change in the market situation on the advertising decisions. These effects can be represented as

$$\frac{\partial a_i^*}{\partial \zeta} = \underbrace{\frac{\partial a_i(a_j = 0)}{\partial \zeta}}_{\text{direct effect}} + \underbrace{\frac{\partial a_i(a_j)}{\partial a_j}\frac{\partial a_j^*}{\partial \zeta}}_{\text{strategic effect}}.$$

The direct effect is driven by the change in the quality advantage of the superior firm. The strategic effect is related to the complementarity of the firms' advertising choices and captures by how much one firm changes its advertising level in response to a change by the other firm. In the case of an increase in parameter ζ the direct and strategic effects point into opposite directions at the two search engines. As the quality gap between the two firms narrows the direct effect is positive for the inferior firm and negative for the superior one. The search quality at the inferior firm increases with the cooperation, hence it can place more advertisements in equilibrium without losing users. At the same time the superior firm's quality advantage erodes and it has to reduce the number of advertisements to keep users from switching. The strategic effect is negative for the inferior firm: in equilibrium the superior firm decreases its advertising level, hence the strategic response of the inferior firm is to show less advertisements too. For the superior firm it is the other way around: since the inferior firm shows more advertisements in the new equilibrium, it displays more advertisements. The direct effect is stronger than the strategic effect, and the inferior firm increases while the superior firm decreases the number of advertisement slots in the new equilibrium. Table 2 summarizes the effects on the search engines' advertising decisions.

	direct effect	strategic effect	total effect
a_1	$-$	$+$	$-$
a_2	$+$	$-$	$+$

Table 2: effects of a decrease in parameter ζ on the number of advertising slots

As the utility from search increases at firm 2, it is able to attract users from firm 1 despite increasing the number of advertisements. At the same time the inferior firm charges a higher price per slot than before and makes higher profits. The reason for this is that equilibrium prices move in the same direction as advertising levels when ρ remains unchanged, as can be seen from Expression (7). For the superior firm it means that showing less advertisements results in lower per slot prices, hence its profit decreases.

Having derived the effects of the cooperation on the firms' profits in Propositions 2 and 3, we can address the question of the superior search engine operator's incentive to improve its inferior competitor by providing access to its higher quality search technology. While the inferior search engine surely benefits from the cooperation, it is not necessarily the case for the superior search engine. On the one hand, the superior firm benefits as demand for advertisement slots at its competitor increases. With advertising decisions being strategic complements, more advertisements displayed by the competitor means more advertisements and higher profits for the superior firm. On the other hand, the superior firm loses with the inferior firm providing higher search quality to users. As the superior firm's quality advantage erodes, some users switch to the competitor, making the superior search engine less attractive for advertisers. Whether the superior search engine has an incentive to share its technology with the competitor hence depends on which of the two effects is stronger. If the cooperation results in a relatively high increase in demand for advertising space for the competing firm and a relatively low improvement of the competitor's search quality, the superior firm is interested in the cooperation. If the opposite is the case it will refrain from sharing its technology with the other firm. We illustrate this condition for a numerical specification.

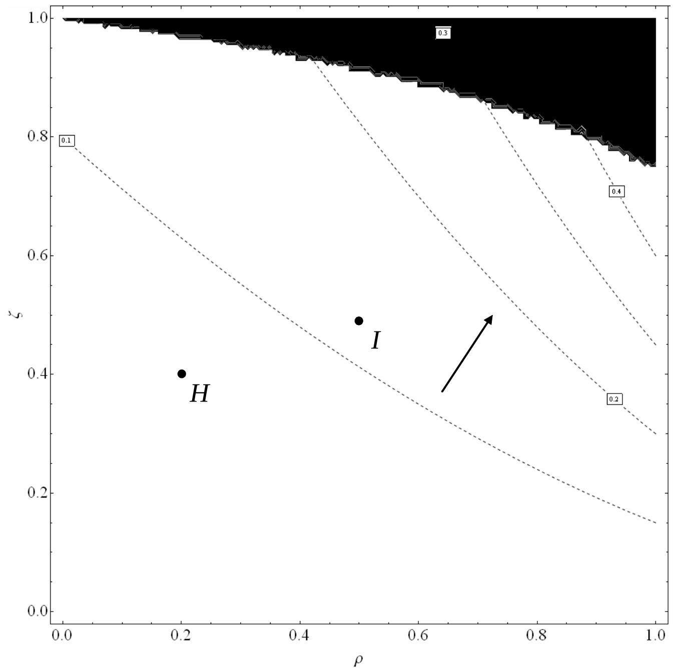

Figure 3: Critical values of $|\Delta\zeta/\Delta\rho|$ for firm 1 to break even with the cooperation.

5 Numerical Example

Firm 1 enters the agreement if it leads to a higher profit. The change in its profit can be approximated in the following way: $\Delta\pi_1^* \simeq (\partial\pi_1^*/\partial\rho)\Delta\rho + (\partial\pi_1^*/\partial\zeta)\Delta\zeta$, where $\Delta\pi_1^*$, $\Delta\rho$ and $\Delta\zeta$ denote the changes in the superior firm's equilibrium profit, the inferior firm's sale probability and the quality of its search result, respectively. We introduce a parameter λ with $\lambda = |\Delta\zeta/\Delta\rho|$, which measures the ratio of the change in ζ to the change in ρ as a result of the cooperation. Parameter λ is higher if the agreement results in a relatively high improvement of the inferior firm's search quality and a relatively low increase in demand for its advertising space. The lower the value of parameter λ, the more attractive is the cooperation for the superior firm. Figure 3 illustrates the critical initial values of ρ and ζ for firm 1 to enter the agreement for different levels of λ. We use $\mu = 1$ and $q = 1/2$ for the example.

Parameter combinations in the dark area are not feasible due to the restriction $\underline{\Delta} < \Delta < \overline{\Delta}$. Consider first initial levels of ζ and ρ given by point H. In this case the superior firm would have an incentive to enter the agreement if the resulting magnitude of decrease in ρ was more than ten times higher than the increase in parameter ζ, that is if $\lambda < 0.1$. If the initial parameters were given by point I, the superior firm would not enter the agreement if it led to the same change in ζ and ρ. We next analyze the welfare effects of the ad-sharing agreement and consider the effects on the advertisers' and users' surpluses.

6 Welfare Analysis

We start with the advertisers' surplus. For the linear demand for advertisement slots this is defined as

$$AS(a_1, a_2) = \sum_i \frac{[(1-\rho_i)n_i - p_i]\, a_i}{2} = \sum_i \frac{a_i^2}{2}. \tag{8}$$

We now turn to the users' surplus. From Expression (4) we get that $t_1(a_1, a_2, s_2^*) = 1 - t_2(a_1, a_2, s_2^*)$, hence in equilibrium the marginal users are located symmetrically on the circle. From Expression (3) we get the users' surplus as

$$US = 2\int_0^{t_1} (q - \mu a_1 - \delta_1^2(t))dt + 2\int_{t_1}^{1/2} (\zeta q - \mu a_2 - \delta_2^2(t))dt. \tag{9}$$

The effects of the cooperation on the users' and advertisers' surpluses are summarized in Proposition 4.

Proposition 4. *The agreement allowing the superior firm to share its technology with its competitor has two contrary effects on the users' and advertisers' surpluses:*

(i) as the demand for the advertisement slots of the inferior firm rises with the decrease in parameter ρ users' (advertisers') surplus decreases (increases),

(ii) as the inferior firm provides higher utility from its services to users due to the increase of parameter ζ users' (advertisers') surplus increases (decreases).

Proof. See Appendix.

As the inferior search engine becomes able to match more relevant advertisements to search keywords, the advertisers' demand for advertisement slots increases. Both search engines show more advertisements in the new equilibrium, which affects the utility of every user negatively. At the same time the advertisers' surplus increases due to the higher demand. With the better matching technology, the inferior search engine provides higher utility to users. Since the improved quality attracts some users to the inferior search engine it can increase the number of advertisements without losing market share. This puts pressure on the superior search engine to display less advertisements in order to keep its users. Overall, the disutility of the users conducting search at the inferior search engine due to being exposed to more advertisements is outweighed by the positive effects of the quality improvement: they benefit from the increased relevance of the advertisements and the users searching at the superior search engine gain higher utility as they are shown less advertisements.

7 Conclusion

In this paper we analyzed cooperation in advertising between internet search engines and considered an agreement where one firm shares its more advanced technology with its competitor. The first question we addressed was whether the firms have incentives to participate in such an arrangement. We identified the conditions under which a superior firm is interested in improving the quality of the services provided by its inferior competitor. We have shown, that the inferior firm always benefits from the cooperation. Whether the superior firm has an incentive to enter into such an arrangement depends on the relative strength of two different effects. The first effect results in an increase in the demand for advertisement slots at the inferior firm which leads to an increase of profit for the superior firm. Since advertising decisions are strategic complements the superior search engine benefits from the increased number of advertisement slots placed at the inferior search engine. The second effect results in an increase in the users' utility from searching at the inferior firm. As the inferior search engine becomes more attractive for users the profit of the superior firm decreases. Whether the superior search engine has an incentive to enter into such an agreement depends on which of the two effects will be stronger as a result of the cooperation. Although the superior firm gains market share the inferior firm benefits from the agreement: the increase in the value of the later's advertising slots due to the better technology offsets the profit loss due to the decreased market share.

We also analyzed the welfare implications of the agreement and found that these two effects work in opposite directions for the advertisers' and users' surpluses. However, the superior firm's decision to enter the agreement is always in line with the advertisers' and is contrary to the users' interests.

Appendix

Proof of Proposition 1. We start with the location choice of firm 2. The first order condition of firm 2 with respect to s_2 is

$$\frac{\partial \pi_2(s_2, a_1, a_2)}{\partial s_2} = \frac{a_2\,(1-\rho)\,(1-2s_2)\,[\Delta - \mu(a_1 - a_2)]}{2\,(1-s_2)^2\, s_2^2},$$

which is zero if either $s_2 = 1/2$ or $\Delta = \mu(a_1 - a_2)$ holds. It follows from Expression (5) that with $\Delta = (a_1 - a_2)\mu$ the firms have equal market shares, which we rule out per assumption. In the following we consider $s_2^* = 1/2$. Solving the first order conditions of firms 1 and 2 with respect to a_1 and a_2 simultaneously yields $a_1(s_2)$ and $a_2(s_2)$. By plugging in $s_2 = 1/2$ we get

$$\begin{aligned} a_1^* &= a_1\left(\frac{1}{2}\right) = \frac{1 + 3\mu(1-\rho) + 4\Delta(1 + \mu(1-\rho))}{4[3\mu^2(1-\rho) + 2\mu(2-\rho) + 1]} \\ a_2^* &= a_2\left(\frac{1}{2}\right) = \frac{(1-\rho)(1+\mu)(\overline{\Delta} - \Delta)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1}. \end{aligned} \tag{10}$$

The values $s_2^* = 1/2$, a_1^* and a_2^* constitute an equilibrium in which $n_1 > n_2 > 0$ if the corresponding second order conditions are fulfilled, all equilibrium values are positive and $n_1(s_2^*, a_1^*, a_2^*) > n_2(s_2^*, a_1^*, a_2^*) > 0$ holds. The second order conditions with respect to a_1 and a_2 are fulfilled as

$$\begin{aligned} \frac{\partial^2 \pi_1(s_2, a_1^*, a_2^*)}{\partial (a_1)^2} &= -2 - \frac{\mu}{(1-s_2)s_2} < 0, \\ \frac{\partial^2 \pi_2(s_2, a_1^*, a_2^*)}{\partial (a_2)^2} &= -2 - \frac{\mu(1-\rho)}{(1-s_2)s_2} < 0 \end{aligned}$$

is always true. The second order condition with respect to s_2 at the point s_2^*, a_1^* and a_2^* is given by

$$\frac{\partial^2 \pi_2(s_2^*, a_1^*, a_2^*)}{\partial (s_2)^2} = -\frac{16(1-\rho)^2(\mu+1)^2\,[1 + \mu(1-\rho)]\,(\Delta - \underline{\Delta})(\overline{\Delta} - \Delta)}{[3\mu^2(1-\rho) + 2\mu(2-\rho) + 1]^2}.$$

We introduce a function $\phi(\Delta) = (\Delta - \underline{\Delta})(\overline{\Delta} - \Delta)$ such that $sign\left[\partial^2 \pi_2(s_2^*, a_1^*, a_2^*)/\partial(s_2)^2\right] = sign\,[\phi(\Delta)]$. The function $\phi(\Delta)$ has two roots $\Delta_1 = \underline{\Delta}$ and $\Delta_2 = \overline{\Delta}$ and is strictly positive for $\underline{\Delta} < \Delta < \overline{\Delta}$. Hence, for $\underline{\Delta} < \Delta < \overline{\Delta}$ it holds that $\partial^2 \pi_2(s_2^*, a_1^*, a_2^*)/\partial(s_2)^2 < 0$. Using the equilibrium values s_2^*, a_1^* and a_2^* we calculate $p_i^*(s_2^*, a_1^*, a_2^*)$ and $n_i^*(s_2^*, a_1^*, a_2^*)$.

Claim 1. *Under the condition $\underline{\Delta} < \Delta$ it holds that $a_1^* > a_2^*$, $p_1^* > p_2^*$ and $n_1^* > n_2^*$.*

Proof. To compare the values we build the following differences:

$$a_1^* - a_2^* = \frac{\rho + 4\Delta(2\mu(1-\rho)+2-\rho)}{4(3\mu^2(1-\rho)+2\mu(2-\rho)+1)},$$

$$p_1^* - p_2^* = \frac{\rho[1+2\mu(1+(3\mu+1)(1-\rho))] + 4\Delta[2-\rho+2\mu(1+(1+\mu)(1-\rho)(2-\rho))]}{4(3\mu^2(1-\rho)+2\mu(2-\rho)+1)}$$

and

$$n_1^* - n_2^* = \frac{4(1+\mu)(1+\mu(1-\rho))(\Delta - \underline{\Delta})}{3\mu^2(1-\rho)2\mu(2-\rho)+1},$$

which are positive for $\Delta > \underline{\Delta}$.

Claim 2. *Under the condition $\Delta < \overline{\Delta}$ it holds that a_i^*, p_i^*,$n_i^* > 0$.*

Proof. With $\Delta < \overline{\Delta}$ it follows from Expression (10) that $a_2^* > 0$. The equilibrium price of the second firm p_2^* is

$$p_2^* = \frac{(1+2\mu(1-\rho))(1-\rho)(1+\mu)(\overline{\Delta} - \Delta)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1},$$

which is positive for $\Delta < \overline{\Delta}$. From Expression (2) we get that $n_2^* = (p_2^* + a_2^*)/(1-\rho) > 0$ if $p_2^*, a_2^* > 0$. In Claim 1 we showed that $a_1^* > a_2^*$, $p_1^* > p_2^*$ and $n_1^* > n_2^*$, hence, it also holds that $a_1^*, p_1^*, n_1^* > 0$ if $\Delta < \overline{\Delta}$.

We showed that if $\underline{\Delta} < \Delta < \overline{\Delta}$, then a_i^*, p_i^*, n_i^* and s^* constitute an equilibrium in which $n_1^* > n_2^* > 0$. *Q.E.D.*

Proof of Proposition 2. Under the maintained assumption $\underline{\Delta} < \Delta < \overline{\Delta}$ we know from Proposition 1 that in equilibrium firms choose maximal differentiation from each other. In the following we proceed with $s_2^* = 1/2$.

i) We start with the effect of a change in ρ on the number of advertisements displayed, a_i^*. Taking the derivative of Expression (10) with respect to ρ we get

$$\frac{\partial a_1^*}{\partial \rho} = -\frac{\mu(1+\mu)(\overline{\Delta} - \Delta)}{(3\mu^2(1-\rho)+2\mu(2-\rho)+1)^2}, \tag{11a}$$

$$\frac{\partial a_2^*}{\partial \rho} = -\frac{(1+2\mu)(1+\mu)(\overline{\Delta} - \Delta)}{(3\mu^2(1-\rho)+2\mu(2-\rho)+1)^2}. \tag{11b}$$

Both derivatives are negative for $\Delta < \overline{\Delta}$. Thus, if the demand for advertisement slots of the inferior firm rises due to a decrease in ρ both firms show more advertisements.

ii) We proceed with the effect of ρ on the market shares $n_i(a_i^*(\rho), a_j^*(\rho))$ by inspecting the expression $\partial n_i(a_i^*(\rho), a_j^*(\rho))/\partial \rho$ for $i \neq j$:

$$\frac{\partial n_i(a_i^*(\rho), a_i^*(\rho))}{\partial \rho} = \frac{\partial n_i^*}{\partial a_i^*}\frac{\partial a_i^*}{\partial \rho} + \frac{\partial n_i^*}{\partial a_j^*}\frac{\partial a_j^*}{\partial \rho}. \tag{12}$$

It follows from (5) that $\partial n_i^*/\partial a_j > 0$ and $\partial n_i^*/\partial a_i = -\partial n_i^*/\partial a_j$ for $i \neq j$. Hence, by rearranging Expression (12) we get

$$\frac{\partial n_i(a_i^*(\rho), a_j^*(\rho))}{\partial \rho} = \frac{\partial n_i^*}{\partial a_j^*}\left(\frac{\partial a_j^*}{\partial \rho} - \frac{\partial a_i^*}{\partial \rho}\right). \tag{13}$$

Note that $\partial n_1^*/\partial \rho = -\partial n_2^*/\partial \rho$. We evaluate the sign of the derivative $\partial(a_2^* - a_1^*)/\partial \rho$ by subtracting Expression (11a) from Expression (11b), to get

$$\frac{\partial a_2^*}{\partial \rho} - \frac{\partial a_1^*}{\partial \rho} = -\frac{(1+\mu)^2(\overline{\Delta} - \Delta)}{(3\mu^2(1-\rho) + 2\mu(2-\rho) + 1)^2}, \tag{14}$$

which is negative for $\Delta < \overline{\Delta}$. Hence, $\partial n_1(a_1^*(\rho), a_2^*(\rho))/\partial \rho < 0$ and $\partial n_2(a_1^*(\rho), a_2^*(\rho))/\partial \rho > 0$.

iii) Next, we turn to the effect of change in ρ on the prices of advertisement slots. We inspect $\partial p_1^*(n_1^*(a_1^*(\rho), a_2^*(\rho)), a_1^*(\rho))/\partial \rho$ and $\partial p_2^*(n_2^*(a_1^*(\rho), a_2^*(\rho)), a_2^*(\rho), \rho)/\partial \rho$. We use Expression (13) to rearrange these derivatives:

$$\frac{\partial p_1^*(n_1^*\left(a_1^*(\rho), a_2^*(\rho)\right), a_1^*(\rho))}{\partial \rho} = \frac{\partial n_1^*}{\partial a_2^*}\left(\frac{\partial a_2^*}{\partial \rho} - \frac{\partial a_1^*}{\partial \rho}\right) - \frac{\partial a_1^*}{\partial \rho} \tag{15a}$$

$$\frac{\partial p_2^*(n_2^*(a_1^*(\rho), a_2^*(\rho)), a_2^*(\rho), \rho)}{\partial \rho} = (1-\rho)\frac{\partial n_2^*}{\partial a_1^*}\left(\frac{\partial a_1^*}{\partial \rho} - \frac{\partial a_2^*}{\partial \rho}\right) - \frac{\partial a_2^*}{\partial \rho} - n_2^*. \tag{15b}$$

Taking the derivatives of Expressions (5) with respect to a_1 and a_2 yields $\partial n_i/\partial a_j = \mu/(2s_2(1-s_2))$. With $s_2^* = 1/2$ we arrive at

$$\frac{\partial n_i^*}{\partial a_j^*} = 2\mu. \tag{16}$$

We can now plug Expressions (16), (14) and (11a) into Expression (15a) to get

$$\frac{\partial p_1^*}{\partial \rho} = -\frac{\mu(1+2\mu)(1+\mu)(\overline{\Delta} - \Delta)}{(3\mu^2(1-\rho) + 2\mu(2-\rho) + 1)^2},$$

which is negative for $\Delta < \overline{\Delta}$. We next plug Expressions (16), (14), (11b) and n_2^* into Expression (15b) to get

$$\frac{\partial p_2^*}{\partial \rho} = -\frac{\left[1 + 2\mu\left(1 + \mu\left(1-\rho\right)\right)\left(1 + \left(3\mu + 2\right)\left(1-\rho\right)\right)\right](1+\mu)(\overline{\Delta} - \Delta)}{(3\mu^2(1-\rho) + 2\mu(2-\rho) + 1)^2}.$$

This expression is negative for $\Delta < \overline{\Delta}$.

iv) Finally, to analyze the influence of a change in ρ on the firms' profits we inspect the derivative $\partial \pi_i(a_i^*(\rho), a_j^*(\rho))/\partial \rho$ for $i \neq j$

$$\frac{\partial \pi_i(a_i^*(\rho), a_j^*(\rho))}{\partial \rho} = \frac{\partial p_i^*}{\partial \rho} a_i^* + p_i^* \frac{\partial a_i^*}{\partial \rho}.$$

We know from *i)* and *iii)* that the derivatives $\partial p_i^*/\partial \rho$ and $\partial a_i^*/\partial \rho$ are negative, hence, $\partial \pi_i/\partial \rho < 0$. With a decrease of parameter ρ, the profits of both firms increase. *Q.E.D.*

Proof of Proposition 3. Under the maintained assumption $\underline{\Delta} < \Delta < \overline{\Delta}$ we know from Proposition 1 that in equilibrium firms choose maximal differentiation from each other. In the following we proceed with $s_2^* = 1/2$.

i) We start with the effect of a change in ζ on the number of advertisements displayed by taking the derivatives of a_1^* and a_2^* with respect to ζ. The signs of the expressions $\partial a_1^*/\partial \zeta$ and $\partial a_2^*/\partial \zeta$ are straightforward:

$$\begin{aligned} \frac{\partial a_1^*}{\partial \zeta} &= -\frac{1 + \mu(1-\rho)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1} q < 0, \\ \frac{\partial a_2^*}{\partial \zeta} &= \frac{(1-\rho)(1+\mu)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1} q > 0. \end{aligned} \tag{17}$$

Hence, the superior (inferior) firm displays less (more) advertisements with an increase in ζ.

ii) Turning to the effect of ζ on the market shares n_i^*, we inspect the derivative $dn_i^*(a_1^*(\zeta), a_2^*(\zeta), \zeta)/d\zeta$. Using that $\partial n_i^*/\partial a_i = -\partial n_i^*/\partial a_j$ for $i \neq j$ we get

$$\begin{aligned} \frac{dn_i^*(a_i^*(\zeta), a_j^*(\zeta), \zeta)}{d\zeta} &= \frac{\partial n_i^*}{\partial a_i^*} \frac{da_i^*}{d\zeta} + \frac{\partial n_i^*}{\partial a_j^*} \frac{da_j^*}{d\zeta} + \frac{\partial n_i^*}{\partial \zeta} \\ &= \frac{\partial n_i^*}{\partial a_j^*} \left(\frac{da_j^*}{d\zeta} - \frac{da_i^*}{d\zeta} \right) + \frac{\partial n_i^*}{\partial \zeta} \end{aligned} \tag{18}$$

Since the market is always covered, it holds that $dn_1^*/d\zeta = -dn_2^*/d\zeta$. Hence, we will only analyze the sign of the derivative $dn_1^*/d\zeta$. From Expression (17) we compute the difference

$$\frac{da_2^*}{d\zeta} - \frac{da_1^*}{d\zeta} = \frac{1 + (1-\rho)(1+2\mu)}{3\mu^2(1-\rho) + 2\mu(2-\rho) + 1} q. \tag{19}$$

We now turn to the derivative $\partial n_1^*/\partial \zeta$. Taking the derivative of Expression (5) with respect to ζ we get $\partial n_1/\partial \zeta = -q/2s_2(1-s_2)$. Given the equilibrium location $s_2^* = 1/2$ we arrive at

$$\frac{\partial n_1^*}{\partial \zeta} = -2q. \tag{20}$$

Plugging Expressions (16), (19) and (20) into Expression (18) we get

$$\frac{dn_1^*(a_1^*(\zeta), a_2^*(\zeta), \zeta)}{d\zeta} = -\frac{2\,(\mu+1)\,(\mu(1-\rho)+1)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}q. \tag{21}$$

Hence, $\partial n_1^*/\partial\zeta < 0$ and $\partial n_2^*/\partial\zeta > 0$.

iii) We now turn to the effect of an increase in ζ on the firms' prices. The derivatives of the equilibrium prices with respect to ζ are given by

$$\begin{aligned}
\frac{dp_1^*(n_1^*(a_1^*(\zeta), a_2^*(\zeta))}{d\zeta} &= \frac{dn_1^*(a_1^*(\zeta), a_2^*(\zeta)}{d\zeta} + \frac{da_1^*(\zeta)}{d\zeta} \\
\frac{dp_2^*(n_2^*(a_1^*(\zeta), a_2^*(\zeta))}{d\zeta} &= (1-\rho)\frac{dn_2^*(a_1^*(\zeta), a_2^*(\zeta)}{d\zeta} + \frac{da_2^*(\zeta)}{d\zeta}.
\end{aligned}$$

Remember that $dn_1^*/d\zeta = -dn_2^*/d\zeta$. Using Expressions (17) and (21) we get

$$\begin{aligned}
\frac{dp_1^*(\zeta)}{d\zeta} &= -\frac{(2\mu+3)(\mu(1-\rho)+1)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}q < 0 \\
\frac{dp_2^*(\zeta)}{d\zeta} &= \frac{(1-\rho)\,(\mu+1)\,(2\mu(1-\rho)+3)}{3\mu^2(1-\rho)+2\mu(2-\rho)+1}q > 0.
\end{aligned} \tag{22}$$

With an increase in parameter ζ the superior (inferior) firm chargers a lower (higher) price for the advertisement slots.

iv) Finally, to analyze the influence of a change in ζ on the firms' profits we inspect the derivative $\partial\pi_i(a_i^*(\zeta), a_j^*(\zeta))/\partial\zeta$ for $i \neq j$

$$\frac{\partial\pi_i(a_i^*(\zeta), a_j^*(\zeta))}{\partial\zeta} = \frac{\partial p_i^*}{\partial\zeta}a_i^* + p_i^*\frac{\partial a_i^*}{\partial\zeta}.$$

Using Inequalities (17) and (22) we get that $\partial\pi_1^*/\partial\zeta < 0$ and $\partial\pi_2^*/\partial\zeta > 0$. With an increase in parameter ζ the superior (inferior) firm makes lower (higher) profits.

v) The total effect of a change in ζ on the firms' joint profit is negative if $|\partial\pi_1^*/\partial\zeta| > |\partial\pi_2^*/\partial\zeta|$, which is equivalent to

$$\left|\frac{\partial p_1^*}{\partial\zeta}\right|a_1^* + p_1^*\left|\frac{\partial a_1^*}{\partial\zeta}\right| > \left|\frac{\partial p_2^*}{\partial\zeta}\right|a_2^* + p_2^*\left|\frac{\partial a_2^*}{\partial\zeta}\right|. \tag{23}$$

Since $a_1^* > a_2^*$ and $p_1^* > p_2^*$, inequality (23) holds if $|\partial p_1^*/\partial\zeta| > |\partial p_2^*/\partial\zeta|$ and $|\partial a_1^*/\partial\zeta| > |\partial a_2^*/\partial\zeta|$. Comparing the expressions we get that

$$\left|\frac{\partial p_1^*}{\partial\zeta}\right| - \left|\frac{\partial p_2^*}{\partial\zeta}\right| = \frac{[2\mu((1+\mu)(1-\rho)+1)+3]\,\rho}{3\mu^2(1-\rho)+2\mu(2-\rho)+1} q > 0 \text{ and}$$

$$\left|\frac{\partial a_1^*}{\partial\zeta}\right| - \left|\frac{\partial a_2^*}{\partial\zeta}\right| = \frac{\rho q}{3\mu^2(1-\rho)+2\mu(2-\rho)+1} > 0, \tag{24}$$

which implies that $|\partial\pi_1^*/\partial\zeta| > |\partial\pi_2^*/\partial\zeta|$. The effect of an increase in ρ on the firms' joint profits is negative. *Q.E.D.*

Proof of Proposition 4. *i)* We first analyze the influence of the change in parameters on the advertisers' surplus. We see from Expression (8) that $AS(a_1, a_2)$ increases in both a_1 and a_2. In Proposition 2 we showed that with a decrease in ρ both a_1 and a_2 increase. It follows that $AS(a_1, a_2)$ increases as the demand for the advertisement slots of the inferior firm rises. To show the effect of parameter ζ on the advertisers' surplus, we first take the derivative of Expression (8) with respect to ζ:

$$\frac{\partial AS(a_1^*(\zeta), a_2^*(\zeta))}{\partial\zeta} = a_1^* \frac{\partial a_1^*}{\partial\zeta} + a_2^* \frac{\partial a_2^*}{\partial\zeta}.$$

We know from Proposition 3 that $\partial a_1^*/\partial\zeta < 0$ and $\partial a_2^*/\partial\zeta > 0$. Moreover, from Expression (24) we have that $|\partial a_1^*/\partial\zeta| > |\partial a_2^*/\partial\zeta|$. As stated in Proposition 1, $a_1(s_2^*) > a_2(s_2^*)$. It follows, that AS decreases as parameter ζ increases.

ii) We now turn to the analysis of the users' surplus. It is useful to distinguish between two groups of users: those who do not switch from the original engine in response to a change in parameters ρ or ζ, and those who do. We will refer to the former group of users as *switchers* and to the latter as *non-switchers*. We start with the effect of a change in ρ on the switchers' utility. Let t_1^ρ and t_2^ρ denote the locations of the marginal users and U_i^ρ the user's utility after a change in parameter ρ. We showed in Proposition 2 that n_1^* increases in response to a reduction in ρ, hence $t_1^\rho > t_1$ and $t_2^\rho < t_2$. Since marginal users are symmetric, we can restrict our analysis to the switchers with locations $t \in [t_1, t_1^\rho]$. Before the change in parameter ρ the switchers preferred search engine 2, hence for $t \in [t_1, t_1^\rho]$ it holds that $U_1(t) < U_2(t)$. We also know from Proposition 2 that a_1^* increases with a decrease in ρ, hence $U_1^\rho(t) < U_1(t)$ for any $t \in [t_1, t_1^\rho]$. Combining the two inequalities we get $U_1^\rho(t) < U_1(t) < U_2(t)$, hence the utility of the switchers decreases due to a decrease in ρ.

We now turn to the effect of a change in ρ on the non-switchers' utility. From Equation (3) we can distinguish three components of a user's utility: search quality ($\zeta_i q$), disutility from advertisements (μa_i) and transportation costs (δ_i^2). For non-switchers only the disutility from advertisements is affected by a change in ρ. We showed in Proposition 2 that both a_1^* and a_2^* increase with a decrease in ρ which results in a reduction in utility of non-switchers. We can conclude then that both switchers and non-switchers are worse-off due to the decrease in parameter ρ.

We now consider the effect of an increase in parameter ζ on the switchers' utility. Let t_1^ζ and t_2^ζ denote the locations of the marginal users and U_i^ζ the utility of a user after a change in parameter ζ. We showed in Proposition 3 that n_1^* decreases in response to an increase in parameter ζ, hence $t_1^\zeta < t_1$ and $t_2^\zeta > t_2$. Since marginal users are symmetric, we can restrict our analysis to the switchers with the locations $t \in [t_1^\zeta, t_1]$. After the change in parameter ζ, switchers prefer search engine 2, hence for $t \in [t_1^\zeta, t_1]$ it holds that $U_2^\zeta(t) > U_1^\zeta(t)$. We also know from Proposition 3 that a_1^* decreases with an increase in ζ, hence $U_1^\zeta(t) > U_1(t)$ must hold for any $t \in [t_1^\zeta, t_1]$. Combining the two inequalities we get $U_2^\zeta(t) > U_1^\zeta(t) > U_1(t)$, hence the utility of the switchers increases due to an increase in parameter ζ.

We finally turn to the effect of a change in ζ on the non-switchers' utility. As the marginal users are symmetric, we restrict our analysis to the users with the locations $t \in [0, 1/2]$. We know from Proposition 3 that a_1^* decreases and a_2^* increases in response to an increase in ζ. Moreover, we know that $|\partial a_1^*/\partial\zeta| > |\partial a_2^*/\partial\zeta|$, hence every non-switcher who searches at engine 1 (with location $t \in [0, t_1^\zeta]$) benefits more from an increase in ζ than any non-switcher who searches at engine 2 (with location $t \in [t_1, 1/2]$) loses. We also know that $n_1 > n_2$ for any ζ, hence using the symmetry of the marginal consumers and the fact that $t_1^\zeta < t_1$ we can conclude that $t_1^\zeta > 1/2 - t_1^\zeta > 1/2 - t_1$. It follows that the number of non-switchers who search at the engine 1 given by $2t_1^\zeta$ is larger than the number of non-switchers who search at the engine 2 given by $2(1/2 - t_1)$. The positive effect of the decrease in the number of advertisements placed at engine 1 is stronger than the negative effect due to the increase in the number of advertisements placed at engine 2 and non-switchers benefit from the increase in parameter ζ. Combining this result with the influence on the switchers' utility we conclude that the users' surplus increases in response to an increase in parameter ζ. *Q.E.D.*

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