Competition between multiple asymmetric networks: A toolkit and applications

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Abstract

This paper presents a model of competition between an arbitrary number of telecommunications networks, in the presence of tariff-mediated network externalities, call externalities, and cost and surplus asymmetries. We determine the Nash equilibria in linear and two-part tariffs, provide an appropriate stability criterium, and show how the model can calibrated to existing market outcomes. As an application, we reconsider the setting of mobile termination rates for calls from the fixed network, and between mobile networks, in the presence of many asymmetric networks.

Keywords: Telecommunications network competition, on/off-net pricing, asymmetry, call externality, multiple networks, linear tariffs, two-part tariffs

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1 Introduction

Two great obstacles of applying models of telecommunications competition to real-world markets are that most either assume symmetric firms and / or consider a duopoly. To our knowledge, there are few or no realistic cases that can be portrayed as a symmetric duopoly, since most telecommunications markets are characterized either by at least three firms which have entered at different points in time, as in mobile telephony, or by one large incumbent and several smaller rivals using different technologies, as is often the case in fixed telephony. One reason for the assumptions of symmetry and duopoly that is usually advanced is that models with several asymmetric networks are not tractable. Here we attempt to show otherwise.

While a series of recent papers has presented models of network competition with more than two networks, as listed below, all either have assumed symmetry or have not been able to give closed-form solutions for the equilibrium. In this paper we set out to develop and solve a rather general model of competition between interconnected telecommunications networks. As in Hoernig (2007) for two networks, there are tariff-mediated network externalities, i.e. networks price discriminate between on- and off-net calls, and call externalities, i.e. receiving calls conveys utility, and networks can be asymmetric in size. Still, we go beyond the scope of that paper by allowing for an arbitrary number of networks and asymmetries in network and per-customer fixed costs. While being at the centre of the ongoing debate about the regulation of mobile termination rates (MTRs) in the European Union, cost differences seem to have been largely ignored in the economic literature on network competition.

We show how to set up and solve network competition models with many asymmetric firms, both for competition in linear and two-part tariffs. The model and most of the calculations are rendered in matrix notation, exploiting maximally the underlying quadratic functional form of profits and the linearity of the demand structure. This vastly reduces the complexity of the derivations and leads to equilibrium conditions in the form of one-liners.

As a first step, we propose a generalization of the condition of stability in expectations introduced by Laffont, Rey and Tirole (1998b) to multiple networks. Effectively, it imposes an upper limit on the intensity of preferences, as a function of tariff-mediated network externalities. This stability condition assumes that networks compete in prices, but is independent of whether networks compete in linear or two-part tariffs.

We then derive the socially optimal prices and market shares in the presence of asymmetric cost and perceived consumer surplus. As expected, efficient prices reflect the true costs of origination and termination on the originating and terminating networks, respectively. Our main finding concerning market shares is that the socially optimal outcome can be implemented by setting fixed fees that reflect exactly the differences in fixed cost (net of fixed-to-mobile termination profits), if and only if network usage costs are symmetric. With cost differences, these fixed fees must be corrected for the effects of differing retail prices.

In the main part of the paper, we derive the Nash equilibria in the price competition games with linear and two-part tariffs. As concerns off-net prices, we allow networks to set a uniform off-net price to all other networks, or to set different prices to groups of other networks. With linear tariffs, we show that the condition in Hoernig (2007) which links the level of the off-net price to the level of the on-net price in the case of two networks, continues to hold "on average" in the case of many networks. If there is a uniform off-net price to all competing networks then this price is set based on average perceived off-net cost.

With two-part tariffs, we show that identical off-net prices to a group of competing networks are set based on average perceived off-net cost, and as if all competitors had the same average market share as the members of this group. Rather unsurprisingly, the on-net prices continue to be set at the efficient levels independently of cost asymmetries and the number of networks.

For the case of two-part tariffs we show how to calibrate the model to real-market cases by computing the fundamental differences in consumer surplus that give rise to the asymmetry in the first place, over and above any cost differences. This exercise is becoming ever more useful for academics and regulators as many countries in Europe, such as France and Portugal, decided recently to make available more spectrum for the entry of additional (fourth or fifth) mobile networks. In these cases it is essential to be able to model asymmetries in the presence of many networks.

As a final exercise, we explore the implications of our results for the setting of mobile termination rates. We first consider the "waterbed effect" in fixed-to-mobile interconnection, i.e. the phenomenon where profits from fixed-to-mobile termination are handed on to consumers, leading to lower retail prices. With linear tariffs, we predict this effect to exist, since both on- and off-net prices decrease with higher fixed-to-mobile termination profits, yet without being able to determine its strength. On the other hand, with two-part tariffs we show that even in the presence of many asymmetric networks the waterbed effect is full at the level of each individual network, as all of the termination profit is handed over to consumers through lower fixed fees. These findings imply that it continues to be true in the general case that higher fixed-to-mobile MTRs amount to a transfer of surplus from the customers of fixed networks to those of mobile networks (and in the case of linear tariffs, to mobile networks themselves).

Concerning mobile-to-mobile termination rates, we generalize the result of Gans and King (2001), with competition in two-part tariffs, to the case of many symmetric networks. Their finding was that networks maximize join profits by setting off-net prices below the efficient level and therefore MTRs below the true cost of termination. We show that as the number of networks increases, joint profit-maximizing off-net prices converge towards the efficient price. The corresponding MTRs only converge to termination cost in the absence of call externalities, otherwise they remain bounded further below cost.

Related literature: There is now a vast amount of work that has sprung from the seminal contributions of Armstrong (1998) and Laffont, Rey and Tirole (1998a,b). In the following we will mostly concentrate on the papers that consider price discrimination between on- and off-net prices, in the tradition of the third paper just mentioned. See the Laffont and Tirole (2000), Armstrong (2002) and Vogelsang (2003) for surveys about the literature on network competition.

Duopoly network competition in linear prices has been considered by Doganoglu and Tauman (2002), Berger (2004), de Bijl and Peitz (2004), DeGraba (2004), Hoernig (2007), and Geoffron and Wang (2008). Duopoly equilibrium results under two-part tariffs have been derived by Gans and King (2001), Peitz (2005), Berger (2005), and Hoernig (2007).

Call externalities have been considered in Jeon, Laffont and Tirole (2004), Berger (2004, 2005), Hoernig (2007), and Armstrong and Wright (2007). Our modeling of asymmetries based on differences in surplus that consumers derive directly from pertaining to one or the other network has been introduced by Carter and Wright (1999, 2003), and has been taken up in de Bijl and Peitz (2004), Peitz (2005) and Hoernig (2007).

Several papers have already considered network competition models with more than two networks, in different models where all firms compete with each other. Symmetric networks are assumed by: Calzada and Valletti (2008), and Armstrong and Wright (2007). Dewenter and Haucap (2005) also consider more than two asymmetric networks, but they take market shares as given and only solve for the resulting per-minute prices. Closest to our paper is Thompson, Renard and Wright (2007), in using a similar demand specification and considering an arbitrary number of networks which can differ in subscription surplus. Yet, networks in their model do not price-discriminate between on- and off-net calls, and no closed-form solution for the equilibrium can be derived.[1]

This paper has the following structure: Section 2 presents the model, discusses stability in consumer expectations and derives socially optimal prices and market shares. Section 3 presents the Nash equilibrium solutions in linear and nonlinear tariffs, while Section 4 considers the symmetric case. Finally, Sections 5 and 6 present results on fixed-to-mobile and mobile-to-mobile termination, while Section 7 concludes.

2 Model Setup

2.1 Demand, Market Shares and Consumer Surplus

The following model is a generalization of the network competition models of Laffont, Rey, Tirole (1998) and Carter and Wright (1999, 2002) to many asymmetric networks. It leads to a demand formulation that is similar to that of Armstrong and Wright (2007) and the "spokes model" by Chen and Riordan (2007), but allows explicitly for exogenous asymmetry between networks. All networks compete against each other, as in Calzada and Valletti (2008), which for more than three networks is different from the mostly used generalization of the Hotelling model to multiple firms, the circular city model of Salop. The equilibrium concept we employ is static Nash equilibrium of the pricing game between networks.

There are $n \geq 2$ networks, and consumers are located on $n(n-1)/2$ segments of length 1 which link all networks to each other. The total mass of consumers is 1, thus each segment has $2/n(n-1)$ consumers. Transport cost are linear, with unit cost $t = 1/2\sigma > 0$. Market shares are $\alpha_i > 0$ with $\sum_{i=1}^{n} \alpha_i = 1$. All networks are interconnected, thus consumers can make calls to any one of them.

A client of network i receives surplus $w_i + 2tA_i$, where A_i is the consumer's fixed surplus from being connected to network i (which may include brand value, trust etc.), and w_i is the surplus arising from making calls, defined below. We assume $A_1 \geq A_2 \geq ... \geq A_n = 0$, i.e. the lowest surplus level is normalized to zero since only the differences $A_i - A_j$ will matter. Network i

[1]Other models of competition between multiple symmetric networks under non-discriminatory pricing are Jeon and Hurkens (2008), Stennek and Tangerås (2008) and Tangerås (2009). On the other hand, Hurkens and Jeon (2008) only consider two networks under termination-based price discrimination.

charges a two-part tariff consisting of a fixed charge F_i, and prices per minute of p_{ii} for on-net calls and p_{ij} for off-net calls to network j.

Consumers' utility of calls is $u(q)$, with indirect utility $v(p) = \max_q u(q) - pq$ (with $v'(p) = q(p)$). Below we will denote the price elasticity of demand as $\eta = -pq'/q$, but note that we never assume it to be constant. Let v_{ij}, q_{ij}, u_{ij} be defined as $v(p_{ij})$, $q(p_{ij})$, $u(q_{ij})$. The utility of receiving calls is $\gamma u(q)$ where $\gamma \in [0,1)$. Assuming an *ex-ante* balanced calling pattern, w_i is given by

$$w_i = \sum_{j=1}^{n} \alpha_j (v_{ij} + \gamma u_{ji}) - F_i = \sum_{j=1}^{n} \alpha_j h_{ij} - F_i \tag{1}$$

Defining the $(n \times n)$ −matrix $h = (h_{ij})_{ij}$ and the $(n \times 1)$ −vectors $F = (F_i)_i$ and $\alpha = (\alpha_i)_i$, we can restate the above in matrix form as

$$w = h\alpha - F. \tag{2}$$

The matrix h is a function of prices, and will therefore be indirectly a function of costs, MTRs and market shares.

We assume throughout that on each segment both adjoining networks have clients, thus the indifferent consumer on segment ij is located at the distance x_j from network i, defined by

$$w_i + 2tA_i - tx_{ij} = w_j + 2tA_j - t\left(\frac{2}{n(n-1)} - x_{ij}\right). \tag{3}$$

Solving for x_{ij} yields network i's market share on segment ij as

$$x_{ij} = \frac{1}{n(n-1)} + A_i - A_j + \sigma(w_i - w_j).$$

Summing over segments yields network i's total market share:

$$\alpha_i = \sum_{j \neq i} x_{ij} = \frac{1}{n} + (n-1) A_i - \sum_{j \neq i} A_j + \sigma\left((n-1) w_i - \sum_{j \neq i} w_j\right) \tag{4}$$

or, in matrix notation,

$$\alpha = \alpha_0 + B(A + \sigma w), \tag{5}$$

where α_0 is the $(n \times 1)$ vector of symmetric market shares $1/n$ and B is an $(n \times n)$ matrix with the values $(n-1)$ on the diagonal and -1 elsewhere.

Market shares in a fully covered market must add up to 1, which is the case here: Let E be the $(n \times 1)$ vector of ones, then

$$\sum_{i=1}^{n} \alpha_i = E'\alpha = E'\alpha_0 + E'B\left(A + \sigma w\right) = n \times \frac{1}{n} = 1, \tag{6}$$

because $E'B = 0$.

Plugging (2) into (5) leads to

$$\left(I - \sigma Bh\right)\alpha = \alpha_0 + B\left(A - \sigma F\right), \tag{7}$$

and solving for α leads to

$$\begin{aligned} \alpha &= \left(I - \sigma Bh\right)^{-1}\left[\alpha_0 + B\left(A - \sigma F\right)\right] \\ &= G\alpha_0 + H\left(A - \sigma F\right), \end{aligned} \tag{8}$$

where I is the $(n \times n)$ identity matrix, $G = \left(I - \sigma Bh\right)^{-1}$ and $H = \left(I - \sigma Bh\right)^{-1} B$. Thus we have found a simple unique solution for market shares *given prices*.[2] The following Lemma states some properties of G and H which will be useful later on.

Lemma 1 *We have: $E'G = E'$, $E'H = 0$ and $HE = 0$. In particular, $\sum_{i=1}^{n} H_{ij} = 0$ for all j, and $\sum_{j=1}^{n} H_{ij} = 0$ for all i.*

Proof. First note that $E'\left(I - \sigma Bh\right) = E' - \sigma 0 h = E'$ since $E'B = 0$. Therefore

$$E'G = E'\left(I - \sigma Bh\right)\left(I - \sigma Bh\right)^{-1} = E'.$$

Note that $GE \neq E$ in general. Furthermore, $E'H = \left(E'G\right)B = E'B = 0$ and $HE = G\left(BE\right) = 0$ since $BE = 0$. ■

Consumer surplus: Total consumer surplus consists of the difference between the surplus from pertaining to networks and making calls, and "transport cost" which measures the welfare cost of a less than perfect fit with preferences:

$$\begin{aligned} S &= \sum_{i=1}^{n}\left[\alpha_i\left(w_i + 2tA_i\right) - \sum_{j \neq i}\int_0^{x_{ij}} tzdz\right] \\ &= \sum_{i=1}^{n}\left[\alpha_i\left(w_i + \frac{A_i}{\sigma}\right) - \frac{1}{4\sigma}\sum_{j \neq i} x_{ij}^2\right] \\ &= \alpha'\left(h\alpha - F + \frac{1}{\sigma}A\right) - \frac{1}{4\sigma}\sum_{i,j \neq i} x_{ij}^2. \end{aligned} \tag{9}$$

[2] This solution does not yield equilibrium market shares explicitly since both H_0 and H may depend on market shares indirectly through prices. We study price choice in the next section.

2.2 Stability

One important technical aspect, discussed at length in Laffont, Rey, Tirole (1998b) for the duopoly case, is the stability of equilibrium in consumer expectations. In this section we show how this stability condition can be generalized to the presence of an arbitrary number of firms.

Lemma 2 *The Nash equilibrium in the price competition game, no matter whether in linear or in two-part tariffs, is stable in consumer expectations if and only if $\alpha_i \geq 0$ for all $i = 1, ..., n$ and $\sigma \in (0, 1/\kappa)$, where κ is the largest eigenvalue of Bh.*

Proof. The condition that all α_i are non-negative is a pre-condition for a well-defined equilibrium candidate. Now consider, similar to Laffont, Rey, Tirole (1998b), a virtual tâtonnement process where consumers observe market shares α_{t-1} and then join networks based on the resulting welfare. This leads to market shares

$$\alpha_t = \alpha_0 + B\left(A + \sigma\left(h\alpha_{t-1} - F\right)\right) = \left[\alpha_0 + B\left(A - \sigma F\right)\right] + \sigma B h \alpha_{t-1}.$$

The effect of market shares at $t-1$ on market shares at time t is given by $d\alpha_t / d\alpha_{t-1} = \sigma B h$. For this tâtonnement process to converge, it is necessary that the largest eigenvalue of $\sigma B h$ be less than 1, which is equivalent to the condition stated in the Lemma. ■

Since B has rank $(n-1)$, one eigenvalue of Bh is zero. With symmetric prices, we have $h_{ii} \equiv h_{on}$, $h_{ij} \equiv h_{off}$, and the other $(n-1)$ eigenvalues of Bh are all equal to $n\left(h_{on} - h_{off}\right)$. Thus under symmetry equilibrium is stable if

$$\sigma < \bar{\sigma} = \frac{1}{n\left(h_{on} - h_{off}\right)}.$$

This leads to some straight-forward implications for market stability:

Proposition 1 *With symmetric networks competing in linear or two-part tariffs, the symmetric market equilibrium is less likely to be stable*

1. *for a higher number of firms, for given per-minute prices;*
2. *for a higher mobile termination rate a;*
3. *for a higher competitive intensity σ.*

Proof. 1. $\bar{\sigma}$ decreases in n for given $\left(h_{on} - h_{off}\right)$. 2. h_{off} decreases in a, and $\bar{\sigma}$ increases in h_{off}. 3. Higher σ more likely violates the stability condition. ■

2.3 Profits

Networks incur fixed cost per customer of f_i, and have on-net cost $c_{ii} = c_{oi} + c_{ti}$, where the indices o and t stand for origination and termination, respectively. The mobile termination charge on network i is a_i, so that costs of off-net calls from network i to network $j \neq i$ are $c_{ij} = c_{oi} + a_j$. The mobile termination margin is $m_i = a_i - c_{ti}$. Networks' profits are

$$\pi_i = \alpha_i \left(\sum\nolimits_{j=1}^{n} \alpha_j R_{ij} + F_i + Q_i - f_i \right), \tag{10}$$

where $R_{ij} = (p_{ij} - c_{oi} - a_j)\, q_{ij} + (a_i - c_{ti})\, q_{ji}$ are the profits from calls between networks i and j. Note that this simplifies to $R_{ii} = (p_{ii} - c_{ii})\, q_{ii}$, and $R_{ij} = (p_{ij} - c_{ij})\, q_{ij} + m_i q_{ji}$ for $j \neq i$. Furthermore, $Q_i = m_i q_{fi}$ are fixed-to-mobile termination profits.

Let J^{ij} be the matrix with entry 1 at position (i, j) and zero elsewhere, R be the $(n \times n)$ matrix with entries R_{ij}, and F, Q, f be the $(n \times 1)$-vectors with entries F_i, Q_i, and f_i, respectively. We can express network i's profits in matrix notation as

$$\pi_i = \alpha' J^{ii} \left(R\alpha + F + Q - f \right), \tag{11}$$

and, since $\sum_{i=1}^{n} J^{ii} = I$, joint profits of all networks as

$$\sum_{i=1}^{n} \pi_i = \alpha' \left(R\alpha + F + Q - f \right). \tag{12}$$

Total welfare in the market for mobile telephony is given by

$$\begin{aligned} W &= S + \sum_{i=1}^{n} \pi_i & (13) \\ &= \alpha' \left[(R + h)\, \alpha + \frac{1}{\sigma} A + Q - f \right] - \frac{1}{4\sigma} \sum_{i, j \neq i} x_{ij}^2 & (14) \end{aligned}$$

We can now describe first-best prices and market shares:

Proposition 2 *1. First-best per-minute prices are* $p_{ij} = \frac{c_{oi} + c_{tj}}{1+\gamma}$ *for all* $i, j = 1, ..., n.$

2. Let $M \equiv R + h$ *at first-best prices. Then socially optimal market shares in the mobile telephony market are*

$$\alpha^* = \left(I - \sigma B \left(M' + M \right) \right)^{-1} \left[\alpha_0 + B \left(A + \sigma \left(Q - f \right) \right) \right], \tag{15}$$

if asymmetries are small enough. With symmetric network cost, optimal market shares become

$$\alpha^* = \alpha_0 + B\left(A + \sigma\left(Q - f\right)\right). \tag{16}$$

Proof. In the expression for aggregate profits the terms corresponding to mobile-to-mobile termination costs and profits cancel, so that after some re-ordering of terms with indices ij and ji,

$$\alpha'\left(R + h\right)\alpha = \sum_{i,j}\alpha_i\alpha_j\left[\left(p_{ij} - c_{oi} - c_{tj}\right)q_{ij} + v_{ij} + \gamma u_{ij}\right].$$

Thus for each pair ij the same surplus maximization problem is posed, with first-order condition

$$q_{ij} + \left(p_{ij} - c_{oi} - c_{tj}\right)q'_{ij} - q_{ij} + \gamma u'_{ij}q'_{ij} = 0.$$

Since $u'_{ij} = p_{ij}$ at the consumer's optimal choice of call minutes the above result obtains.

Let $M \equiv R + h$ at the socially optimal prices. Then we need to maximize social surplus

$$W = \alpha' M\alpha + \alpha'\left(\frac{1}{\sigma}A + Q - f\right) - \frac{1}{4\sigma}\sum_{i,j\neq i}x_{ij}^2$$

subject to the conditions $x_{ji} = \frac{2}{n(n-1)} - x_{ij}$ and $x_{ij} \geq 0$ for all $j \neq i$, $i = 1, ..., n$. Omitting for the moment the non-negativity constraints, and substituting out x_{ji} in $\alpha_j = \sum_{k\neq j}x_{jk}$, we have $\frac{d\alpha}{dx_{ij}} = (e_i - e_j)$, where e_i and e_j are $(n \times 1)$ vectors with value 1 at position i and j, respectively, and zeros elsewhere. Thus, maintaining the substitution of x_{ji}, we have the first-order conditions, for all i and $j \neq i$,

$$\begin{aligned}\frac{dW}{dx_{ij}} &= \left(e_i - e_j\right)' M\alpha + \alpha' M\left(e_i - e_j\right) + \left(e_i - e_j\right)'\left(\frac{1}{\sigma}A + Q - f\right)\\ &- \frac{1}{2\sigma}x_{ij} + \frac{1}{2\sigma}\left(\frac{2}{n\left(n-1\right)} - x_{ij}\right) = 0.\end{aligned}$$

Taking into account that $\alpha' M\left(e_i - e_j\right) = \left(e_i - e_j\right)' M'\alpha$, and summing the conditions over $j \neq i$, we obtain

$$B_i\left(M' + M\right)\alpha + B_i\left(\frac{1}{\sigma}A + Q - f\right) - \frac{1}{\sigma}\alpha_i + \frac{1}{\sigma n} = 0,$$

where B_i is row i of the matrix B. Stacking these equations leads to

$$B\left(M'+M\right)\alpha+B\left(\frac{1}{\sigma}A+Q-f\right)-\frac{1}{\sigma}\alpha+\frac{1}{\sigma}\alpha_0=0$$

and the condition

$$\left(I-\sigma B\left(M'+M\right)\right)\alpha=\alpha_0+B\left(A+\sigma\left(Q-f\right)\right).$$

Note that $B\left(M'+M\right)=0$ with symmetric network cost. These results hold as long as all $x_{ij}\geq 0$, which holds if and only if the asymmetries in network cost and $A+\sigma\left(Q-f\right)$ are small enough. ■

< comment, interpret >

With symmetric network cost, since $Bh^*=0$, by (8) the socially optimal market shares α^* can be induced by introducing fixed fees equal to $F=f-Q+k$, where k has the same value in each component. That is, not the absolute value of fixed fees is relevant, only the differences between firms. What needs to be signalled to consumers is the difference in net fixed cost $(f-Q)$ per consumer. If on the other hand network costs are not symmetric, then there is no longer a simple correspondence between conditions (8) and (15), and fixed fees must be chosen such that

$$\sigma BF=\alpha_0+BA-\left(I-\sigma Bh\right)\left(I-\sigma B\left(M'+M\right)\right)^{-1}\left[\alpha_0+B\left(A+\sigma\left(Q-f\right)\right)\right].$$

Note, though, that our result on the first-best market share, and the resulting fixed fees, considers the transfer Q from the fixed telephony market as given. In particular, it does not take into account the welfare loss caused by this transfer. These optimal market shares also take the surplus asymmetry A, which distinguishes networks in the eyes of consumers, as given. Indeed, if consumers as a whole prefer some networks then at the social optimum these networks' market shares should be higher.

3 Pricing Equilibrium

In this section we will describe equilibrium prices and market shares under both linear and nonlinear pricing. As some results concerning the effects of mobile termination rates are known to differ between these two types of strategies (see e.g. Laffont, Rey and Tirole (1998a,b), it seems useful to consider the case of many firms for both of them.

3.1 Linear Tariffs

With linear tariffs, let $F = 0$. Each network chooses the prices p_{ii} and p_{ij} in order to maximize its profits

$$\pi_i = \alpha_i \left(\sum_{j=1}^{n} \alpha_j R_{ij} + Q_i - f_i \right) = \alpha' J^{ii} \left(R\alpha + Q - f \right).$$

We first state a central result about how per-minute prices affect market shares.

Lemma 3 *For any price p, we have* $\frac{d\alpha}{dp} = \sigma H \frac{dh}{dp} \alpha$*, with* $\sum_{i=1}^{n} \frac{d\alpha_i}{dp} = 0$*.*

Proof. From condition (7) we have $(I - \sigma B h)\alpha = \alpha_0 + BA$. Taking derivatives on both sides leads to

$$-\sigma B \frac{dh}{dp} \alpha + (I - \sigma B h) \frac{d\alpha}{dp} = 0,$$

from which the result follows. Furthermore, $E' \frac{d\alpha}{dp} = \sigma (E'H) \frac{dh}{dp} \alpha = 0$ since $E'H = 0$. ■

As is common in models network competition in linear prices, we cannot give explicit expressions for the equilibrium prices. Still, we can show how the equilibrium off-net prices relate to on-net prices. For the sake of generality, we consider the case where network i divides its competitors into separate groups K and charges a uniform off-price p_{iK} to each group. Extreme cases are where each group contains a single member (in which case there is price discrimination between all networks), or where all other networks are in the same group (the case of a uniform off-net price). We obtain the following results on equilibrium prices:

Proposition 3 *1. Network i's equilibrium on-net price satisfies the following condition:*

$$L_{ii} = \frac{p_{ii} - c_{ii}}{p_{ii}} = \frac{1}{\eta} - \frac{\sigma (1 + \gamma\eta) H_{ii}}{\eta} \left(\frac{\pi_i}{\alpha_i^2} + \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{17}$$

2. If network i sets uniform prices p_{iK} *to different groups K of competing networks, its average off-net Lerner index*

$$\bar{L}_{ij} = \frac{\sum_K \sum_{j \in K} \alpha_j (p_{iK} - c_{ij}) / p_{iK}}{1 - \alpha_i} \tag{18}$$

satisfies the condition

$$\bar{L}_{ij} = \frac{1}{\eta} + \frac{(1 + \gamma\eta)^{-1} - \alpha_i}{1 - \alpha_i} \left(L_{ii} - \frac{1}{\eta} \right). \tag{19}$$

3. Network i's profits are given by

$$\pi_i = \alpha_i^2 \left(\frac{1}{\sigma H_{ii}} \frac{1 - \eta L_{ii}}{1 + \gamma\eta} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{20}$$

Proof. For on-net prices we obtain

$$\frac{dh}{dp_{ii}} = \left(\gamma p_{ii} q_{ii}' - q_{ii}\right) J^{ii} = -\left(1 + \gamma\eta\right) q_{ii} J^{ii}.$$

Thus

$$\frac{d\alpha}{dp_{ii}} = -\sigma \left(1 + \gamma\eta\right) q_{ii} H J^{ii} \alpha = -\sigma \left(1 + \gamma\eta\right) q_{ii} \alpha_i H_{\cdot i},$$

where $H_{\cdot i}$ is the ith column of H. Furthermore, $\frac{dR}{dp_{ii}} = \left(1 - \eta L_{ii}\right) q_{ii} J^{ii}$, where $L_{ii} = \left(p_{ii} - c_{ii}\right)/p_{ii}$ is the Lerner index for on-net calls. The first-order condition for profit-maximization with respect to the on-net price is

$$\frac{d\alpha'}{dp_{ii}} J^{ii} \left(R\alpha + Q - f\right) + \alpha' J^{ii} \frac{dR}{dp_{ii}} \alpha + \alpha' J^{ii} R \frac{d\alpha}{dp_{ii}} = 0,$$

which simplifies to

$$-H_{ii} \left(R_{i\cdot}\alpha + Q_i - f_i\right) - \alpha_i R_{i\cdot} H_{\cdot i} + \frac{\alpha_i \left(1 - \eta L_{ii}\right)}{\sigma \left(1 + \gamma\eta\right)} = 0 \tag{21}$$

or

$$\pi_i = \alpha_i^2 \left(\frac{1}{\sigma H_{ii}} \frac{1 - \eta L_{ii}}{1 + \gamma\eta} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right). \tag{22}$$

Solving for L_{ii} leads to the condition on the on-net price.

2. Assume that network i sets a uniform off-net price p_{iK} to a set K of other networks. We have

$$\frac{dh}{dp_{iK}} = -q_{iK} J^{iK} + \gamma p_{iK} q_{iK}' J^{Ki} = -q_{iK} \left(J^{iK} + \gamma\eta J^{Ki}\right),$$

where J^{iK} and J^{Ki} are matrices with ones at locations ij and ji where $j \in K$, respectively, and zeros elsewhere. Thus

$$\frac{d\alpha}{dp_{iK}} = -\sigma q_{iK} H \left(J^{iK} + \gamma\eta J^{Ki}\right) \alpha = -\sigma q_{iK} \left(\sum_{j \in K} \alpha_j H_{\cdot i} + \gamma\eta\alpha_i \sum_{j \in K} H_{\cdot j} \right)$$

The first-order condition for a profit maximum becomes

$$\frac{d\alpha'}{dp_{iK}} J^{ii} (R\alpha + Q - f) + \alpha' J^{ii} \frac{dR}{dp_{iK}} \alpha + \alpha' J^{ii} R \frac{d\alpha}{dp_{iK}} = 0,$$

where $\frac{dR}{dp_{iK}}$ has elements $q_{iK} (1 - \eta L_{ij})$, where $L_{ij} = (p_{iK} - c_{ij}) / p_{iK}$ at locations ij, $j \in K$, and $m_j q'_{iK}$ at locations ji, $j \in K$. Note that off-net costs c_{ij} may differ between receiving networks j. This first-order condition can be rewritten as

$$\begin{aligned} 0 &= \left(\sum_{j \in K} \alpha_j H_{ii} + \gamma \eta \alpha_i \sum_{j \in K} H_{ij} \right) (R_{i \cdot} \alpha + Q_i - f_i) \\ & \quad + \alpha_i R_{i \cdot} \left(\sum_{j \in K} \alpha_j H_{\cdot i} + \gamma \eta \alpha_i \sum_{j \in K} H_{\cdot j} \right) - \frac{\alpha_i}{\sigma} \sum_{j \in K} \alpha_j (1 - \eta L_{ij}) . \end{aligned}$$

Summing over all sets K, and making use of $\sum_{j \neq i} H_{\cdot j} = -H_{\cdot i}$ from Lemma 1, leads to

$$-H_{ii} (R_{i \cdot} \alpha + Q_i - f_i) - \alpha_i R_{i \cdot} H_{\cdot i} + \frac{\alpha_i (1 - \alpha_i) \left(1 - \eta \bar{L}_{ij}\right)}{\sigma (1 - \alpha_i - \gamma \eta \alpha_i)} = 0, \quad (23)$$

where $\bar{L}_{ij} = \sum_{j \neq i} \alpha_j L_{ij} / (1 - \alpha_i)$ is the weighed average Lerner index of off-net prices, or

$$\pi_i = \alpha_i^2 \left(\frac{(1 - \alpha_i) \left(1 - \eta \bar{L}_{ij}\right)}{\sigma (1 - \alpha_i - \gamma \eta \alpha_i) H_{ii}} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right) . \quad (24)$$

Taking the difference between (24) and (22) we obtain

$$\frac{\alpha_i^2 (1 - \alpha_i) \left(1 - \eta \bar{L}_{ij}\right)}{\sigma (1 - \alpha_i - \gamma \eta \alpha_i) H_{ii}} = \frac{\alpha_i^2 (1 - \eta L_{ii})}{\sigma (1 + \gamma \eta) H_{ii}},$$

from which the above result follows. ■

Condition (17) is the generalization to n asymmetric networks of condition (12) in Laffont, Rey and Tirole (1998b). The result on off-net prices is the generalization to n networks with asymmetric costs, and up to n different off-net prices to groups of networks, of equations (6) in Laffont, Rey and Tirole (1998b) and (11) in Hoernig (2007) for two networks. It is remarkable that the relationship between the average level of off-net prices, as measured by $\bar{L}_{ij}$, and on-net prices remains the same even as the number of networks increases.

If network i charges a uniform off-net price p_{iu} to all other networks, then we can reformulate $\bar{L}_{ij}$ as follows:

$$\bar{L}_{ij} = \frac{\sum_{j \neq i} \alpha_j \left(p_{iu} - c_{ij}\right) / p_{iu}}{1 - \alpha_i} = \frac{p_{iu} - \bar{c}_{iof}}{p_{iu}}, \tag{25}$$

where $\bar{c}_{iof} = \left(\sum_{j \neq i} \alpha_j c_{ij}\right) / (1 - \alpha_i)$ is the weighted average off-net cost faced by network i. Thus for a uniform off-net price, $\bar{L}_{ij}$ simply becomes the Lerner index relative to weighted average off-net cost.

< interpretation for on/off-net differential >

As we will see below, the expression (20) uncovers a previously overlooked link between the equilibrium profits under competition in linear and two-part tariffs. This link may aid future research into the relationship between the two modes of competition.

3.2 Two-Part Tariffs

In this section, we determine the equilibrium prices, fixed fees and market shares for the case of competition in two-part tariffs. We find the following:

Proposition 4 *If networks compete in two-part tariffs,*

1. *On-net prices are set efficiently at* $p_{ii} = c_{ii} / (1 + \gamma)$.
2. *The uniform off-net price to a group* K *of competing networks is*

$$p_{iK} = \frac{\sum_{j \in K} \alpha_j c_{ij}}{\sum_{j \in K} \alpha_j - \frac{|K|}{n-1} \gamma \alpha_i}. \tag{26}$$

3. *Equilibrium fixed fees are given by*

$$F = f - Q - \left(\hat{R} + R\right) \alpha, \tag{27}$$

where $\hat{R}$ *is an* $(n \times n)$ *matrix with elements* $\hat{R}_{ii} = \sum_{j=1}^{n} \frac{H_{ji}}{H_{ii}} R_{ij} - \frac{1}{\sigma H_{ii}}$ *and* $\hat{R}_{ij} = 0$ *for* $j \neq i$.

Proof. 1. In order to determine equilibrium call prices, we follow the standard procedure of first keeping market shares α constant and solving (4) for F_i,

$$F_i = \sum_{j=1}^{n} \alpha_j v_{ij} + \alpha_i \gamma u_{ii} - \frac{\alpha_i}{n-1} \sum_{j \neq i} u_{ij} + const,$$

where "*const*" denotes terms that do not depend on network i's prices. Substituting this into profits leads to

$$\pi_i = \alpha_i \left(\sum_{j=1}^{n} \alpha_j \left(R_{ij} + v_{ij} \right) + \alpha_i \gamma u_{ii} - \frac{\alpha_i}{n-1} \sum_{j \neq i} u_{ij} \right) + const.$$

This expression can now be maximized over call prices. As concerns the on-net price, network i solves

$$\max_{p_{ii}} \left\{ R_{ii} + h_{ii} \right\} = \left\{ \left(p_{ii} - c_{ii} \right) q_{ii} + v_{ii} + \gamma u_{ii} \right\},$$

which has first-order condition

$$q_{ii} + \left(p_{ii} - c_{ii} \right) q'_{ii} - q_{ii} + \gamma u'_{ii} q'_{ii} = 0.$$

Since $u'_{ij} = p_{ij}$ at the consumer's optimal choice for all $i, j = 1, ..., n$, we obtain

$$p_{ii} = \frac{c_{ii}}{1+\gamma}, \tag{28}$$

i.e. on-net prices are set at the efficient level.

2. Assume now that network i wants to set a uniform off-net price p_{iK} towards a group K of other networks, solving

$$\max_{p_{iK}} \left\{ \sum_{j \in K} \left(\alpha_j \left(\left(p_{iK} - c_{ij} \right) q_{iK} + v_{iK} \right) - \frac{\alpha_i}{n-1} \gamma u_{iK} \right) \right\}.$$

Here $q_{iK} = q\left(p_{iK}\right)$, $v_{iK} = v\left(p_{iK}\right)$ and $u_{iK} = u\left(q_{iK}\right)$. Performing similar calculations as above leads to

$$p_{iK} = \frac{\sum_{j \in K} \alpha_j c_{ij}}{\sum_{j \in K} \alpha_j - \frac{|K|}{n-1} \gamma \alpha_i}. \tag{29}$$

3. Now we determine the equilibrium fixed fees. Take the call prices and fixed fees of networks $j \neq i$ as given, and consider the first-order condition of network i's profit maximum in (10) with respect to its fixed fee:

$$\frac{\partial \pi_i}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i} \left(\sum_{j=1}^{n} \alpha_j R_{ij} + F_i + Q_i - f_i \right) + \alpha_i \left(\sum_{j=1}^{n} \frac{\partial \alpha_j}{\partial F_i} R_{ij} + 1 \right) = 0.$$

From (8), for all $i, j = 1, ..., n$ we have $\frac{\partial \alpha_j}{\partial F_i} = -\sigma H_{ji}$, where H_{ji} is the ji-element of matrix H. The first-order condition can then be solved for F_i as

$$F_i = f_i - Q_i - \alpha_i \left(\sum_{j=1}^{n} \frac{H_{ji}}{H_{ii}} R_{ij} - \frac{1}{\sigma H_{ii}} \right) - \sum_{j=1}^{n} \alpha_j R_{ij}. \tag{30}$$

Letting $\hat{R}$ be an $(n \times n)$ matrix with $\hat{R}_{ii} = \sum_{j=1}^{n} \frac{H_{ji}}{H_{ii}} R_{ij} - \frac{1}{\sigma H_{ii}}$ and $\hat{R}_{ij} = 0$ if $j \neq i$, we can write

$$F = f - Q - \left(\hat{R} + R \right) \alpha. \;\blacksquare \tag{31}$$

Thus we confirm the standard result of the efficiency of on-net prices under two-part tariffs for the case of many asymmetric networks. If there are no call externalities ($\gamma = 0$) then $p_{ii} = c_{ii}$, while in the presence of the latter the efficient on-net price is below cost.

As concerns the off-net prices, in the absence of call externalities they are equal to weighted average off-net cost:

$$p_{iK} = \frac{\sum_{j \in K} \alpha_j c_{ij}}{\sum_{j \in K} \alpha_j}.$$

This is a natural generalization of the result for two firms. Furthermore, as in Jeon *et al.* (), Berger (2005) and Hoernig (2007), the off-net prices increase in γ and are above (weighted average) off-net cost if $\gamma > 0$. Expression (26) shows that network i sets its off-net price to a set K of networks as if it was setting a uniform off-net price to all networks, assuming they all have the same average market share as those in the set K.

Two special cases of off-net prices are a uniform off-net price

$$p_{iu} = \frac{\sum_{j \neq i} \alpha_j c_{ij}}{1 - \alpha_i - \gamma \alpha_i},$$

and price discrimination between all networks, with

$$p_{ij} = \frac{\alpha_j c_{ij}}{\alpha_j - \frac{1}{n-1} \gamma \alpha_i}.$$

We now consider equilibrium profits and market shares.

Proposition 5 *Equilibrium profits and market shares are, respectively,*

$$\pi_i^* = \alpha_i^2 \left(\frac{1}{\sigma H_{ii}} - \sum_{j=1}^{n} R_{ij} \frac{H_{ji}}{H_{ii}} \right), \tag{32}$$

$$\alpha^* = \left(I - \sigma B \left[h + \hat{R} + R \right] \right)^{-1} \left(\alpha_0 + B \left[A + \sigma \left(Q - f \right) \right] \right). \tag{33}$$

Proof. The expression for profits results from substituting equilibrium fixed fees into (10). Finally, substituting fixed fees into (7) yields the condition for the equilibrium market share. ■

One should take note that the expression for equilibrium profits (32) is every similar to the one in (20). Indeed, this similarity is no coincidence:

Corollary 1 *At efficient on-net prices, the expressions for equilibrium profits (20) under linear tariffs and (32) under two-part tariffs are formally identical.*

Proof. With $p_{ii} = \frac{c_{ii}}{1+\gamma}$, we have $\frac{1-\eta L_{ii}}{1+\gamma\eta} = 1$. Thus the additional term in (20) disappears. ■

The same argument holds for expression (24), by the way, since at the off-net prices (26) the average Lerner index has value $\bar{L}_{ij} = \gamma\frac{\alpha_i}{1-\alpha_i}$, which again makes the additional term disappear. These observations imply that the fundamental difference between competition in linear and two-part tariffs lies in how usage prices are set, rather than in the existence or not of a fixed fee. Maybe surprisingly, the expression for equilibrium profits under linear tariffs turns out to be more general than the one under two-part-tariffs, rather than less, as it applies to both cases (with different retail prices, sure enough). < explore relationship with DeGraba (2004) >

Note that an alternative expression for equilibrium profits under two-part tariffs is $\pi_i^* = -\alpha' J^{ii}\hat{R}\alpha$, which leads to the handy expression for joint equilibrium profits of

$$\sum_{i=1}^{n} \pi_i = -\alpha'\hat{R}\alpha. \tag{34}$$

The right-hand side of (33) in general depends indirectly on α through $h+\hat{R}+R$ and off-net prices. Contrary to the two-firm case, this is true even if there are no call externalities, since in this case the off-net prices are equal to off-net costs weighted by market shares. Only if off-net costs (including mobile termination rates) are symmetric will the dependence on α disappear. In the latter case (33) gives an explicit solution for market shares, but otherwise numerical methods need to be employed.

Calibration: If we want to calibrate the model using observed market shares and prices, the fixed surplus A can be calculated from (33), starting by the normalization $A_n = 0$:

$$BA = \left(I - \sigma B\left(\hat{R} + R + h\right)\right)\alpha - \alpha_0 - \sigma B\left(Q - f\right). \tag{35}$$

The matrix B cannot be inverted, but using $A_n = 0$ and solving the first $(n-1) \times (n-1)$ dimensions of this system yields the unique $A_1, ..., A_{n-1}$ that give rise to the observed market shares α.

4 The Special Case of Symmetric Networks

In this section we will shortly resume the outcomes of our models under symmetric networks, i.e. equal network and fixed costs, the same surplus A (normalized to zero) and the same mobile-to-mobile and fixed-to-mobile termination charges for all networks. This is the case considered in Armstrong and Wright (2007), and also in Calzada and Valletti (2008) for a different (logit) demand specification. Armstrong and Wright's model also contains call externalities, albeit of a different functional form. Thus our results in this section complement both papers just mentioned.

In a symmetric equilibrium market shares are $\alpha_i \equiv \frac{1}{n}$. Let network costs, including MTRs, be c_{on} and c_{off} for on- and off-net calls, respectively, the equilibrium surplus from on- and off-net calls be $h_{ii} \equiv h_{on}$ and $h_{ij} \equiv h_{off}$, and also $R_{ii} \equiv R_{on} = (p_{on} - c_{on}) q_{on}$ and $R_{ij} = R_{off} = (p_{off} - c_{on}) q_{off}$ for $j \neq i$.[3] Thus profits from (10) become[4]

$$\pi_i = \frac{1}{n} \left(\frac{1}{n} R_{on} + \frac{n-1}{n} R_{off} + F_i + Q_i - f_i \right). \tag{36}$$

Our first result is of technical nature:

Lemma 4 *With n symmetric networks,*

$$H_{ii} = \frac{n-1}{1 - n\sigma (h_{on} - h_{off})}, \quad H_{ij} = -\frac{1}{1 - n\sigma (h_{on} - h_{off})}. \tag{37}$$

With linear tariffs, we find from (19) that

$$\bar{L}_{ij} = \frac{1}{\eta} + \frac{n (1 + \gamma\eta)^{-1} - 1}{n-1} \left(L_{ii} - \frac{1}{\eta} \right). \tag{38}$$

Since the leading factor on the right-hand side increases in n, we find that in a market with more symmetric networks, the off-net Lerner will we relatively

[3]The occurrence of c_{on} in R_{off} is not a typo — it is due the cancelling-out of mobile-to-mobile interconnection profits.

[4]We keep the index i in order to not create confusion with the vectors F, Q and f, but assume that all elements are equal.

higher as compared to the on-net Lerner index, i.e. the on-/off-net differential will be larger.

< try to compare absolute levels of prices using (17) and (24) >

As concerns profits under linear tariffs, we have from (22),

$$\pi_i = \frac{1}{n^2}\left(\frac{1 - n\sigma\left(h_{on} - h_{off}\right)}{\sigma\left(n-1\right)}\frac{1 - \eta L_{ii}}{1 + \gamma\eta} - R_{on} + R_{off}\right). \quad (39)$$

With two-part tariffs and symmetric networks, off-net prices become

$$p_{iu} = p_{iK} = p_{ij} = \frac{n-1}{n-1-\gamma}c_{off}, \quad (40)$$

no matter whether *a priori* networks price discriminate off-net or not. Profits are

$$\pi_i^* = \frac{1}{n^2}\left(\frac{1 - n\sigma\left(h_{on} - h_{off}\right)}{\sigma\left(n-1\right)} - R_{on} + R_{off}\right), \quad (41)$$

where $R_{on} = -\frac{\gamma}{1+\gamma}c_{on}q_{on}$ and $R_{off} = \left(\frac{n-1}{n-1-\gamma}c_{off} - c_{on}\right)q_{off}$.

5 Fixed-To-Mobile Termination and the Waterbed effect

In this section we will state what our previous results imply for the fixed-to-mobile "waterbed effect", i.e. the phenomenon according to which termination profits accruing from interconnection to the fixed network lead to reductions in prices for mobile retail customers.

With linear tariffs, by (21) and (23), $\frac{\partial\pi_i}{\partial p_{ii}\partial Q_i}$ and $\frac{\partial\pi_i}{\partial p_{ij}\partial Q_i}$ both have the sign of $-H_{ii}$, which is negative at least if the market is close enough to the symmetric equilibrium. Thus a higher Q_i lowers both p_{ii} and p_{ij}, and network i's market share will increase. As long as prices are strategic complements, all equilibrium prices will fall. Therefore consumers of all networks will receive at least part of the rent due to higher fixed-to-mobile termination charges. Thus a fixed-to-mobile waterbed effect exists even with linear tariffs, but at this level of generality we cannot determine its extent. Furthermore, while it is clear that each single network prefers to have a high fixed-to-mobile MTR Q_i, the total effect on aggregate equilibrium profits is unclear.

With two-part tariffs the outcome is much easier to establish: Remember from (27) that equilibrium fixed fees are given by

$$F = f - Q - \left(\hat{R} + R\right)\alpha,$$

where Q is the vector of per-customer profits from fixed-to-mobile termination. Thus with two-part tariffs all termination profits are handed over to the consumer even in the case of a Nash equilibrium with many asymmetric networks, and the waterbed effect is full at the level of each single network.

As concerns the effect of different fixed-to-mobile termination charges on market shares, consider condition (33) defining equilibrium market shares under two-part tariffs:

$$\alpha^* = \left(I - \sigma B\left[h + \hat{R} + R\right]\right)^{-1}(\alpha_0 + B\left[A + \sigma(Q - f)\right]).$$

A higher Q_i has a similar effect as a higher perceived surplus A_i, and thus increases network i's market share. The same holds for lower fixed cost per customer f_i.

6 Mobile-to-mobile termination

As concerns two-part tariffs, for now we quickly consider the symmetric equilibrium and derive a generalization of the result of Gans and King (2001) to n networks. Joint profits are

$$n\pi_i^* = \frac{1}{n}\left(\frac{1 - n\sigma(h_{on} - h_{off})}{\sigma(n-1)} - R_{on} + R_{off}\right). \tag{42}$$

The effect of the mobile-to-mobile MTR on profits is indirect, through the effect of the off-net price p_{off} on h_{off} and R_{off}. As we have seen in the proof of point 1 of Proposition 4, if both h_{off} and R_{off} had the same relative weight in profits then p_{off} would be set efficiently. As it happens, though, with n networks h_{off} has weight $\frac{n}{n-1}$ relative to R_{off}, which implies that networks want to set an off-net price that is even better for consumers, i.e. too low from a socially optimal point of view. This is what Gans and King have shown. On the other hand, our result implies that this effect becomes less strong as n becomes large since $\frac{n}{n-1} \to 1$. Formally, when choosing their joint profit-maximizing off-net price networks maximize

$$\frac{n}{n-1}(v_{off} + \gamma u_{off}) + (p_{off} - c_{on})\, q_{off}.$$

The maximum is obtained at

$$p_{off} = \frac{\eta(n-1)\, c_{on}}{n\eta(1+\gamma) - \eta + 1} < \frac{c_{on}}{1+\gamma}, \tag{43}$$

which by force of $p_{off} = \frac{n-1}{n-1-\gamma}(c_{on} + a - c_t)$ from (40) implies

$$a = c_t - \frac{\gamma\eta + n\gamma\eta + 1}{n\gamma\eta + \eta n - \eta + 1} c_{on}. \tag{44}$$

As the number of networks is increased, the profit-maximizing MTR increases and converges towards $a = c_t - \frac{\gamma}{\gamma+1} c_{on}$. This MTR remains below cost because the joint profit-maximizing off-net price converges to the efficient price, while the Nash equilibrium price converges to off-net cost. Therefore only in the absence of call externalities will networks want to set MTRs close to the true cost of termination. < compare with existing results under multiple symmetric networks >

< linear tariffs: symmetry: similar to Berger (2004), from $\pi_i = \frac{1}{n^2}\left(\frac{1-n\sigma\left(h_{on}-h_{off}\right)}{\sigma(n-1)}\frac{1-\eta L_{ii}}{1+\gamma\eta} - R_{on} + R_{off}\right)$. Berger's result for two networks: for $\gamma \approx 0$ networks want MTR above cost, while for large γ the opposite holds. Similar to what we do above but here need to consider $(h_{on} - h_{off})$ vs $(R_{on} - R_{off})$), at $\gamma = 0$ and $a = c_t$ >

7 Conclusions

In this paper we have presented a tractable extension of network competition models with tariff-mediated externalities to an arbitrary number of asymmetric firms (surplus and cost asymmetry), and derived Nash equilibria under both linear and two-part tariffs. We derived a generalized stability condition and determined the first-best prices and market shares, and showed how to calibrate the model to markets with more than two networks under two-part tariffs. Finally, we uncovered an interesting new link between equilibrium profits under linear and two-part tariffs, and reconsidered the implications of multiple networks for the effect and choice of fixed-to-mobile and mobile-to-mobile termination rates.

Future versions of this paper will explore the effects of asymmetries for mobile termination, and present some actual calibrations for mobile telephony markets will at least four networks.

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