

On the Dynamic Consistency of the Regulation of Next Generation Networks*

Duarte Brito[†]

UNL

Pedro Pereira[‡]

AdC and IST

João Vareda[§]

AdC and UNL

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Abstract

This article analyzes the dynamic consistency of three regulatory instruments for next generation networks. We model the industry as a duopoly, where a vertically integrated incumbent and a downstream entrant, that requires access to the incumbent's network, compete on Hotelling's line. The incumbent can invest in the deployment of a next generation network that improves the quality of the retail services. First, we show that for linear access tariffs, the dynamic consistency problem is particularly severe. Second, we show that two-part access tariffs mitigate, but do not completely solve, the dynamic consistency problem. Third, it is unclear whether, by itself, the separation of the retail and wholesale businesses of the incumbent solves the dynamic consistency problem.

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[†]DCSA, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal. E-mail: dmb@fct.unl.pt.

[‡]Autoridade da Concorrência, Rua Laura Alves, n.º 4, 6.º, 1050-188 Lisboa, Portugal. E-mail: jpe.pereira@netcabo.pt.

[§]Autoridade da Concorrência, Rua Laura Alves, n.º 4, 4.º, 1050-188 Lisboa, Portugal. E-mail: joao.vareda@concorrencia.pt.

Introduction

The deployment of next generation networks, leading to a multi-service networks for audio, video, and data services, sets the telecommunications sector on the verge of a new era.¹ In order to give firms the right incentives to invest, and to promote an efficient use of these infrastructures, sectoral regulators must set an adequate regulatory framework for these new telecommunications networks.

There is at least one important difference between the regulation of current and the next generation networks. The former are already deployed, whereas the latter are not.² This implies that the regulatory policy should balance the conflicting goals of reducing the incumbent's market power, namely on the wholesale market, and giving the incumbent incentives to invest in a next generation network. In other words, the regulatory policy should trade-off static and dynamic efficiency.³ This trade-off creates a problem of dynamic consistency for the regulatory policy. Before the network is deployed, it is socially optimal for the regulator to announce a policy that allows the incumbent to keep the marginal profits from his investment. This ensures that the incumbent has incentives to invest, but might involve setting the access price above marginal cost. After the network is deployed, it is socially optimal to set the access price to the new network at marginal cost. If the incumbent anticipates that the regulator will not be able to overcome this dynamic consistency problem, then he will reduce the investment on the next generation network.

A way to overcome the dynamic consistency problem is for the regulator to credibly commit to a policy. It is possible, in principle, for either the regulator or the legislator, to adopt measures that constrain the regulator's future actions. This is feasible for short periods.⁴ However, the investment cycle for telecommunications networks is very long. Therefore, in practice, it is hard for the regulators to credibly commit to a regulatory policy

¹A *Next Generation Network* is a "(...) packet-based network able to provide telecommunication services and able to make use of multiple broadband, QoS-enabled transport technologies and in which service-related functions are independent from underlying transport related technologies." See ITU (2001).

²Besides, the investment in the current networks occurred while the industry was a legal monopoly.

³According to ERG (2007), "welfare gains can result from two main sources: Static efficiency gains (derived from the most efficient use of existing technologies. Static efficiency is maximised through intense competition and subsequent lower prices), and dynamic efficiency gains (gains related to the additional value generated by innovative new technologies and services that may be produced at lower cost and customers may attach a higher value to)".

⁴Guthrie (2006) discusses the constraints on the regulator's actions adopted in several countries to prevent the regulator from acting opportunistically. For instance, the regulator can announce that it will set the access price at a certain level until the next scheduled review.

towards next generation networks. Even if the commitment takes the form of a law, laws can be changed. Take for example the case of the US cable television industry. It was deregulated in 1984 by the Cable Communications Policy Act, re-regulated in 1992 by the Cable Television Consumer Protection and Competition Act, and again deregulated in 1996 by the Telecommunications Act.

If it is hard for the regulator to commit for a policy for the duration of the investment cycle of the next generation network, then an alternative is to choose regulatory instruments that overcome, or mitigate, the dynamic consistency problem. In this article, we analyze how three regulatory instruments perform with respect to the dynamic consistency problem. The regulatory instruments are: **(i)** linear access tariffs, **(ii)** two-part access tariffs, and **(iii)** the separation of the retail and wholesale businesses of the incumbent with the absence of wholesale regulation.

We model the industry as a differentiated products duopoly, where an incumbent and an entrant compete on Hotelling's line (Hotelling, 1929). The incumbent is a vertically integrated firm that owns a network, and operates on the retail market. The entrant operates on the retail market, and requires access to the incumbent's network. The incumbent can invest in the deployment of a next generation network that improves the quality of the retail services. The sectoral regulator sets the access tariffs to the incumbent's network.

First, we show that for linear access tariffs, the dynamic consistency problem is particularly severe. To induce investment, the access tariff to the next generation network should be set above marginal cost. However, once the network is deployed, it is socially optimal to set the access tariff at marginal cost, to eliminate the competition distortions in the retail market. The incumbent anticipates that he will be expropriated from the marginal profits of his investment, and therefore reduces the investment.

Second, we show that two-part access tariffs mitigate, but do not completely solve, the dynamic consistency problem. For some parameter values, the regulator can set the variable part of the tariff at marginal cost, and use the fixed part to give the incumbent incentives to invest. The social optimality of this scheme does not change once the network is deployed. However, two-part access tariffs might involve politically unacceptable high payments from the entrant to the incumbent.

Third, we show that the separation of the retail and wholesale businesses of the incumbent, associated with the deregulation of the wholesale market and linear wholesale prices, provides incentives for investment, and ensures that the incumbent and the entrant pay the same wholesale price. However, it involves a positive wholesale mark-up, and the loss of

coordination economies. It is unclear whether the separation, by itself, solves the dynamic consistency problem. In the presence of a positive wholesale mark-up, the regulator will be under pressure to intervene. If the wholesale market is regulated, unless the regulator can commit to a regulatory policy, the separation only leads to the loss of coordination economies.

The academic literature on regulation only recently started to address the relation between access pricing and investment. Guthrie (2006) surveys the recent literature on the relationship between infrastructure investment and the different regulatory regimes. He concludes that much remains to be done. Valletti (2003) argues that one of the main issues that must be taken into account on the delineation of regulatory policies is the fact that regulators should be able to commit to rules over a reasonable time period. Regulators should try to stabilize their policies in order to signal to the firms that they can commit to their decisions. ERG (2007) states that transparency in the regulation of wholesale products is fundamental, since the predictability of the regulatory intervention is a key factor for firms's investment decisions.

Vareda (2007) studies the incumbent's incentives to invest in quality upgrades and cost reduction when the regulator forces him to unbundle his network. He shows that the regulator should commit to set a lower unbundling price when cost reduction is relatively less expensive than quality upgrades, and *vice-versa*. Vareda and Hoernig (2007) study the investment of two operators in a new infrastructure, which allows them to offer new services, and show that access holidays may be a necessary tool to give the leader the correct incentives to invest, at the same time that allows to charge a lower access price later on in order to delay the follower's investment. Foros (2004) shows that under some conditions the investment by an incumbent in the quality of his network is lower with price regulation since the access price is set equal to marginal cost. Kotakorpi (2006) considers a similar model with vertical differentiation, and obtains similar results. Brito et al. (2008) analyze the case where access to next generation networks is not regulated, and both firms can invest.

The remainder of the article is organized as follows. We describe the model in Section 2. In Section 3, we analyze linear access tariffs. In Section 4, we analyze two-part access tariffs. In Section 5, we analyze the separation of the retail and wholesale businesses of the incumbent. Finally, in Section 6, we conclude. All proofs are in the Appendix.

1 Model

1.1 Environment

Consider a telecommunications industry where two firms, the incumbent and the entrant, sell horizontally differentiated products. The *incumbent*, firm i , is a vertically integrated firm that owns a bottleneck input, to which we refer to as the old network. The *old network*, network o , is a telephone network with a local access network based on the twisted pair of copper wire. The incumbent can make an investment to deploy a next generation network. The next generation network is also a bottleneck input that allows the supply of retail products of a higher quality than those supplied through the old network. We refer to the next generation network as the *new network*, or network n . The *entrant*, firm e , only operates in the retail market, and has to buy access to the network of the incumbent. We index the firms with subscript $j = i, e$, and the networks with subscript $v = o, n$.

There is a third party in the industry, the *sectoral regulator*.

Costs and demand are common knowledge.

1.2 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. Consumers are uniformly distributed along a Hotelling line segment (Hotelling, 1929) of length 1, facing transportation costs tx to travel the distance x , with t on $[0, +\infty)$. Consumers are otherwise homogeneous. As in Biglaiser and DeGraba (2001), we assume each consumer has a demand function for telecommunications services given by $y_j = (z + \Delta_v) - p_j$, where y_j on $(0, z + \Delta_v)$ is the number of minutes of telecommunication purchased from firm j , p_j on $(0, z + \Delta_v)$ is the price per minute of firm j , z is a parameter on $(\frac{4}{3}\sqrt{6t}, +\infty)$, and Δ_v is a parameter that takes value 0 for products supplied through the old network and takes value Δ_d on $(0, +\infty)$ for products supplied through the new network. This means that consumers are willing to pay a premium for services delivered over the new network. The lower limit on z implies that all consumers have a positive surplus under the different market structures, and ensures that the incumbent's profits are increasing in the access price.

Denote by $S_v(p_j) := \frac{(z + \Delta_v - p_j)^2}{2}$, the gross surplus of a consumer for buying from firm j operating with network v at price p_j . This is a gross consumer surplus because it does not account for transportation costs or fixed fees.

1.3 Sectoral Regulator

The regulator sets the wholesale tariff for which the entrant can have access to network $v = o, n$, the *access tariff*, denoted by $A_v(y_e) = K_v + \alpha_v y_e$, where α_v on $[0, +\infty)$ is the price per minute of telecommunications services, and K_v on $[0, +\infty)$ is the fixed part, independent of the number of minutes and of the number of consumers.⁵ In Section 2, we assume that the access tariff is composed only of a variable part, i.e., $K_v = 0$. In Section 3, we allow the access tariff to be composed of a variable and a fixed part, i.e., $K_v \geq 0$.

The regulator maximizes social welfare, i.e., the sum of the firms' profits and the consumer surplus, denoted by W .

1.4 Firms

The incumbent produces an input that: (i) uses in the production of a retail product, and (ii) sells to the entrant. All of the incumbent's marginal costs are constant and equal to zero. The entrant has marginal costs α_v on $\{\alpha_o, \alpha_n\}$.

The incumbent is located at point 0 and the entrant at point 1 of the line segment where consumers are distributed.

Firms charge consumers two-part tariffs, denoted by $T_j(y_j) = F_j + p_j y_j$, $j = i, e$, where F_j on $[0, +\infty)$ is the fixed part of firm j .

At a cost C , the incumbent can deploy a new network.⁶ We assume that C belongs to $(0, \frac{1}{2} [(z + \Delta_d)^2 - 3t])$. This ensures that the incumbent would invest if this allowed him to move from a duopoly on the old network with $K_o = \alpha_o = 0$, to a monopoly on the new network. To simplify the exposition, we assume that the old network is phased out when the new network is deployed.

Denote by D_j , the demand, in terms of consumers, for firm $j = i, e$. The profits of firm $j = i, e$ for the whole game are:

$$\pi_i = [p_i (z + \Delta_v - p_i) + F_i] D_i + K_v + \alpha_v (z + \Delta_v - p_e) D_e - \frac{\Delta_v}{\Delta_d} C \quad (1)$$

$$\pi_e = [(p_e - \alpha_v) (z + \Delta_v - p_e) + F_e] D_e - K_v. \quad (2)$$

⁵Regulating telecommunications markets by intervening at the wholesale level, namely by setting access prices, corresponds to the current EU and US practice.

⁶We assume that the cost of the entrant investing on a new network is larger than the cost of the incumbent, and too high for the investment to be profitable, i.e., belongs to $(\frac{1}{2} (z + \Delta_d)^2 - t, +\infty)$. This happens because the entrant has to build a network from scratch, while the incumbent just needs to upgrade his old network. Alternatively, the entrant has a higher cost of capital.

1.5 Timing of the Game

We consider two games. In the *no-commitment game*, the sectoral regulator cannot commit to a regulation policy towards the new network before the investment is made. In the *commitment game*, the sectoral regulator can commit to a regulation policy towards the new network before the investment is made.

The *no-commitment game* has five stages, which unfold as follows. In stage 1, the regulator sets the access tariff to the old network. In stage 2, the incumbent decides whether to invest. In stage 3, the sectoral regulator sets the access tariff to the new network. In stage 4, the entrant decides if he stays in the market or exits. In stage 5, the incumbent and the entrant compete on retail tariffs.

The *commitment game* has four stages which unfold as follows. In stage 1, the sectoral regulator sets the access tariffs to the old and the new networks. In stage 2, the incumbent decides whether to invest. In stage 3, the entrant decides if he stays in the market or exits. In stage 4, the incumbent and entrant compete on retail tariffs.

These games represent two polar cases. In practice, the regulator has some ability to commit to a policy, particularly for a short period, but cannot commit completely to a policy, particularly for a long period. Thus, the critical issue is whether the regulator can commit for a regulatory policy for a period as long as the investment cycle of the new network.

1.6 Equilibrium Concept

The sub-game perfect Nash equilibrium for the no-commitment game is: **(i)** a pair of access tariffs, **(ii)** an investment decision, **(iii)** a decision of to stay or exit the market, **(iv)** a set of retail tariffs, such that:

- (E1)** the retail tariffs maximize the firms' profits, given the access prices and the market structure;
- (E2)** the decision to stay or exit the market maximizes the entrant's profits, given the access tariffs and the incumbent's investment decision;
- (E3)** the access tariffs for the new network maximizes social welfare, given the access tariff for the old network and the incumbent's decision to invest;
- (E4)** the investment decision maximizes the incumbent's profits, given the access tariff for the old network;
- (E5)** the access tariffs for the old network maximizes social welfare.

Similarly for the commitment game.

2 Linear Tariffs

In this Section, we characterize the equilibrium of the no-commitment game with linear access tariffs, which we construct by working backwards.

2.1 Retail

Next we characterize the equilibria of the retail price game for four cases: **(i)** the incumbent does not invest in the new network, and the entrant exits the industry, **(ii)** the incumbent invests in the new network, and the entrant exits the industry, **(iii)** the incumbent does not invest in the new network, and the entrant stays in the market, **(iv)** the incumbent invests in the new network, and the entrant stays in the market.⁷ In cases (i)-(ii) the retail market is a monopoly. In cases (iii)-(iv) the retail market is a duopoly. We use superscripts m_o , m_n , d_o , d_n to denote variables or functions associated with cases (i)-(iv), respectively.

We start with the following Lemma.

Lemma 1: *In equilibrium, firms set the marginal price of the two-part tariff at marginal cost, i.e., $p_i = 0$ and $p_e = \alpha$.* ■

As usual with two-part tariffs, firms set the variable part of the retail tariff at marginal cost to maximize the gross consumer surplus, and then try to extract this surplus through the fixed part.

Given Lemma 1, from now on we only discuss the determination of the fixed fees.

2.1.1 Monopoly

Next, we characterize the equilibria of the retail price game for the two cases where the retail market is a monopoly, which are given by the next Lemma.

Lemma 2: *If the retail market is a monopoly, in equilibrium, the incumbent charges the fixed fee, for $v = n, o$:*

$$F_i^{m_v}(\Delta_v) = \frac{(z + \Delta_v)^2}{2} - t. \quad (3)$$

■

⁷A duopoly where the incumbent uses the new network and the entrant uses the old network is impossible. By assumption, the old network is phased out once the new network is deployed.

The profits of the incumbent for $v = n, o$, are:

$$\pi_i^{m_v}(\Delta_v, C) = \frac{1}{2}(z + \Delta_v)^2 - t - \frac{\Delta_v}{\Delta_d}C, \quad (4)$$

The total welfare for $v = n, o$, is:

$$W^{m_v}(\Delta_v, C) = \frac{1}{2}[(z + \Delta_v)^2 - t] - \frac{\Delta_v}{\Delta_d}C. \quad (5)$$

2.1.2 Duopolies

Next, we characterize the equilibria of the retail price game for the two cases where the retail market is a duopoly, which are given by the next Lemma.

Lemma 3: *If the retail market is a duopoly, in equilibrium, the incumbent and the entrant charge the fixed fees, for $v = n, o$:*

$$\begin{aligned} F_i^{d_v}(\Delta_v, \alpha_v) &= \begin{cases} t + \frac{1}{6}\alpha_v[6(z + \Delta_v) - 5\alpha_v] & \text{for } \alpha_v \text{ on } [0, \sqrt{6t}) \\ \alpha_v(z + \Delta_v) - \frac{1}{2}\alpha_v^2 - t & \text{for } \alpha_v \text{ on } [\sqrt{6t}, z + \Delta_v] \end{cases} \\ F_e^{d_v}(\Delta_v, \alpha_v) &= \begin{cases} t - \frac{1}{6}\alpha_v^2 & \text{for } \alpha_v \text{ on } [0, \sqrt{6t}) \\ 0 & \text{for } \alpha_v \text{ on } [\sqrt{6t}, z + \Delta_v]. \end{cases} \end{aligned}$$

■

The profits of the incumbent and the entrant for $v = n, o$ are, respectively:⁸

$$\pi_i^{d_v}(\Delta_v, \alpha_v; C) = \begin{cases} \frac{(36t^2 + \alpha_v^4 - 60t\alpha_v^2) + 72\alpha_v t(z + \Delta_v)}{72t} - \frac{\Delta_v}{\Delta_d}C & \text{for } \alpha \text{ on } [0, \sqrt{6t}) \\ \alpha_v(z + \Delta_v) - \frac{1}{2}\alpha_v^2 - t - \frac{\Delta_v}{\Delta_d}C & \text{for } \alpha \text{ on } [\sqrt{6t}, z + \Delta_v], \end{cases}$$

and

$$\pi_e^{d_v}(\Delta_v, \alpha_v) = \begin{cases} \frac{[6t - \alpha_v^2]^2}{72t} & \text{for } \alpha_v \text{ on } [0, \sqrt{6t}) \\ 0 & \text{for } \alpha_v \text{ on } [\sqrt{6t}, z + \Delta_v]. \end{cases}$$

Social welfare is:

$$W^{d_v}(\Delta_v, \alpha_v; C) = \begin{cases} \frac{72t(z + \Delta_v)^2 + 5\alpha_v^4 - 36t(t + \alpha_v^2)}{144t} - \frac{\Delta_v}{\Delta_d}C & \text{for } \alpha_v \text{ on } [0, \sqrt{6t}) \\ \frac{(z + \Delta_v)^2}{2} - \frac{t}{2} - \frac{\Delta_v}{\Delta_d}C & \text{for } \alpha_v \text{ on } [\sqrt{6t}, z + \Delta_v]. \end{cases}$$

The next Corollary presents an useful auxiliary result.

Corollary 1: *In a duopoly on the old or new network, the profit of the incumbent is non-decreasing in the respective access price, while the profit of the entrant is non-increasing in the respective access price.* ■

⁸When $\alpha_v = z + \Delta_v$ the incumbent's profit equals the monopoly profits defined in the previous section.

When the access price increases, the marginal cost of the entrant increases relative to that of the incumbent. As a consequence, the market share, and thereby the profit of the incumbent increases, while the entrant's profit decreases.

2.2 Exit Decision

Next, we characterize the entrant's optimal decision of whether to stay or exit the market.

When indifferent between staying in the market or exiting, the entrant chooses the latter.

Lemma 4: *Suppose that the incumbent owns network v . Then, the entrant:*

$$\begin{cases} \text{stays in the market} & \text{for } \alpha_v \text{ on } [0, \sqrt{6t}) \\ \text{exits} & \text{for } \alpha_v \text{ on } [\sqrt{6t}, z + \Delta_v). \end{cases}$$

■

2.3 Regulation of the New Network

Next, we characterize the socially optimal access price to the new network, assuming that it has already been deployed, i.e., assuming that the investment costs have already been sunk.

Formally, the regulator chooses α_n to maximize $W^{dn}(\Delta_d, \alpha_n; C)$.

The next Lemma presents the socially optimal access price to the new network.

Lemma 5: *Suppose that the regulator sets the access price to the new network after the incumbent invested in the new network. The socially optimal access price to the new network is: $\alpha_n = 0$.*

■

For $\alpha_n = 0$ the entrant asks for access to the new network. In addition, given Lemmas 1 and 3, the entrant charges the lowest possible retail price per minute, and firms charge the same retail tariff. Thus they share the market equally, which minimizes the consumers' transportation costs.

2.4 Investment Decision

Next, we characterize the incumbent's optimal investment decision, which is given by the next Lemma.

Lemma 6: *Suppose that the regulator sets the access price to the new network after the incumbent makes his investment decision. In equilibrium, the incumbent does not invest in the new network.* ■

The incumbent foresees that the regulator will set the access price to the new network at marginal cost, eliminating the marginal profits from the investment. Thus, he does not invest in the new network.⁹

In Hotelling's model, if the firms have the same marginal costs, i.e., if $\alpha_v = 0$, the firms' profits depend only on the difference in the quality of the products, and not on the absolute value of the products' quality. Thus, if both firms offer products through the new network, their profit levels are the same as when using the old network, and consumers get all the benefits of the investment.

2.5 Regulation of the Old Network

Next, we analyze the socially optimal access price to the old network. Formally, the regulator chooses α_o to maximize $W^{do}(\alpha_o)$, taking into account that the firms will only supply services through the old network.

The next Lemma presents the socially optimal access price to the old network.

Lemma 7: *Suppose that the regulator sets the access price to the new network after the incumbent makes his investment decision. The socially optimal access price to the old network is: $\alpha_o = 0$.* ■

The regulator foresees that there will be no investment in a new network. Thus, when he sets the socially optimal access price to the old network he ignores the incentives for investment. Therefore, and given that in $W^{do}(\alpha_o)$ the access price and the level of quality of the services do not interact, the socially optimal access price equals marginal cost.

2.6 Equilibrium of the Whole Game

Having solved all the five stages of the no-commitment game, we can now summarize the equilibrium of the whole game, which we present in the next Proposition for further

⁹This result is similar to the result of Vareda (2007), where the incumbent also does not invest, neither in quality upgrades nor in cost reduction when the regulator is not able to commit to an unbundling price.

reference.

Proposition 1: *For the no-commitment game, in equilibrium: (i) the regulator sets $\alpha_o^* = 0$, (ii) the incumbent does not invest in the new technology, (iii) the regulator sets $\alpha_n^* = 0$, (iv) the entrant stays in the market, and (v) the incumbent and the entrant set $F_i^* = F_e^* = t$, and $p_i^* = p_e^* = 0$. ■*

If the regulator is unable to commit to a policy towards the new network, the incumbent foresees that he will be expropriated from the incremental profit of the investment and does not invest. This highlights the main theme of this article. When choosing the policy towards next generation networks, regulators have to take into account the impact on the retail market, but also the impact of the firms' incentives to invest in the deployment of the new network, i.e., regulators have to trade-off static and dynamic efficiency.

2.7 Equilibrium of the Game with Commitment

In this Section, we summarize the equilibria of the commitment game with linear access tariffs. The purpose of this exercise is to clarify the role of the regulator's inability to commit to a policy on the results of the previous Section.

Depending on the access prices (α_o, α_n) and C , the investment may or may not occur. If the investment cost is too high, there is no-investment whatever the levels of (α_o, α_n) . However, if the investment cost is not too high, by choosing (α_o, α_n) , the regulator can influence the outcome of the investment decision.

Denote the incremental profit of the investment, net of the investment cost by:

$$\begin{aligned} \Delta\Pi(\Delta_d, \alpha_n, \alpha_o; C) = \\ = \begin{cases} \pi_i^{d_n}(\Delta_d, \alpha_n; C) - \pi_i^{d_o}(0, \alpha_o; C) & \text{for } (\alpha_o, \alpha_n) \text{ on } [0, \sqrt{6t}) \times [0, \sqrt{6t}) \\ \pi_i^{d_n}(\Delta_d, \alpha_n; C) - \pi_i^{m_o}(0, C) & \text{for } (\alpha_o, \alpha_n) \text{ on } [\sqrt{6t}, z) \times [0, \sqrt{6t}) \\ \pi_i^{m_n}(\Delta_d, C) - \pi_i^{d_o}(0, \alpha_o; C) & \text{for } (\alpha_o, \alpha_n) \text{ on } [0, \sqrt{6t}] \times [\sqrt{6t}, z + \Delta_d) \\ \pi_i^{m_n}(\Delta_d, C) - \pi_i^{m_o}(0, C) & \text{for } (\alpha_o, \alpha_n) \text{ on } [\sqrt{6t}, z) \times [\sqrt{6t}, z + \Delta_d) . \end{cases} \end{aligned}$$

If the regulator does not want to induce investment, he should set $(\alpha_n, \alpha_o) = (0, 0)$. If the regulator wants to induce investment he should set (α_n, α_o) such that the net incremental profit of the investment is non-negative. This means setting $\alpha_o = 0$, so that the pre-investment profit is the lowest possible, thereby increasing the range of values of C for

which there is investment.¹⁰

Denote by $A_n(C)$, the set of access prices for the new network that induce the incumbent to invest, given C , i.e.,

$$A_n(C) := \begin{cases} \{\alpha_n : \alpha_n \geq a_n\} & \text{for } C \text{ on } [0, (z + \Delta_d) \sqrt{6t} - \frac{9}{2}t) \\ \{\alpha_n : \alpha_n \geq \sqrt{6t}\} & \text{for } C \text{ on } [(z + \Delta_d) \sqrt{6t} - \frac{9}{2}t, \frac{(z + \Delta_d)^2 - 3t}{2}] \end{cases},$$

with $a_n(C)$ implicitly defined by $\Delta\Pi(\Delta_d, a_n, 0; C) \equiv 0$. To induce investment, the regulator picks the welfare maximizing α_n from $A_n(C)$.

The welfare function under duopoly is quasi-convex in α_n . It is decreasing between $\alpha_n = 0$ and $\alpha_n = \frac{3}{5}\sqrt{10t}$, and then increases until $\alpha_n = \sqrt{6t}$. Thus, to induce investment the regulator should set:

$$\alpha_n^*(C) = \begin{cases} \min A_n(C) & \text{if } \min A_n(C) \leq \sqrt{\frac{6t}{5}} \\ \max A_n(C) & \text{if } \min A_n(C) > \sqrt{\frac{6t}{5}}. \end{cases}$$

Denote by \tilde{C} , the investment cost level for which the incremental welfare benefit of the investment, net of the investment cost, is 0, under duopoly, i.e., $W^{d_n}(\Delta_d, \alpha_n; \tilde{C}) - W^{d_o}(\alpha_o) \equiv 0$. The next Proposition characterizes the equilibria of the commitment game with linear access tariffs.

Proposition 2: *The commitment game has the following equilibria:*

(I) Let $\min \left\{ \tilde{C}, \sqrt{\frac{6}{5}t} (z + \Delta_d) - \frac{49}{50}t \right\} = \tilde{C}$.

(a) If C is on $(0, \tilde{C}]$: (i) the regulator sets access prices $(\alpha_o, \alpha_n) = (0, \alpha_n^*(C))$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = F_i^{d_n}(\alpha_n^*(C))$, $F_e^* = F_e^{d_n}(\alpha_n^*(C))$, and $p_i^* = 0$, $p_e^* = \alpha_n^*(C)$.

(b) If C is on $(\tilde{C}, \frac{(z + \Delta_d)^2 - 3t}{2})$: (i) the regulator sets $(\alpha_o, \alpha_n) = (0, 0)$, (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

(II) Let $\min \left\{ \tilde{C}, \sqrt{\frac{6}{5}t} (z + \Delta_d) - \frac{49}{50}t \right\} = \sqrt{\frac{6}{5}t} (z + \Delta_d) - \frac{49}{50}t$.

(a) If C is on $(0, \sqrt{\frac{6}{5}t} (z + \Delta_d) - \frac{49}{50}t)$: (i) the regulator sets access prices $(\alpha_o, \alpha_n) = (0, \alpha_n^*(C))$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = F_i^{d_n}(\alpha_n^*(C))$, $F_e^* = F_e^{d_n}(\alpha_n^*(C))$, and $p_i^* = 0$, $p_e^* = \alpha_n^*(C)$.

¹⁰If investment takes place, welfare is independent of α_o . Hence, the regulator should always set $\alpha_o = 0$.

(b) If C is on $\left(\sqrt{\frac{6}{5}}t(z + \Delta_d) - \frac{49}{50}t, \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t\right)$: (i) the regulator sets access prices $(\alpha_o, \alpha_n) = (0, a_n^*(C))$, (ii) the incumbent invests in the new technology, (iii) the entrant exits the market, and (iv) the incumbent sets retail tariffs $F_i^* = F_i^{m_n}(\Delta_n) = \frac{(z+\Delta_n)^2}{2} - t$, and $p_i^* = 0$.

(c) If C is on $\left(\frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t, \frac{(z+\Delta_d)^2-3t}{2}\right)$: (i) the regulator sets $(\alpha_o, \alpha_n) = (0, 0)$, (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$. ■

If the investment cost is low, the regulator should set the access prices such that the incumbent invests in a new network, and the entrant stays in the market. This implies a positive markup over marginal cost for the access price. As a consequence, the market share of the incumbent becomes larger than half of the market, since he has a lower marginal cost than the entrant.

If the investment cost takes intermediate values, the regulator should set the access prices such that the incumbent invests in the new network, and the entrant exits the industry. Inducing the exit of the entrant ensures that all consumers buy the optimal amount of minutes.

If the investment cost is high, the regulator should set the access prices to discourage the investment in the new network. The social incremental benefit, in terms of increased product quality, is lower than the cost of the investment and the losses introduced by the access price distortions.

The comparison of Propositions 1 and 2 leads to the next Corollary.

Corollary 3: *The incumbent's incentives to invest are higher if the regulator can commit to a regulatory policy than if the regulator cannot commit to a regulatory policy.* ■

With linear access tariffs, to induce the incumbent to invest in the deployment of the new network, the access price to the new network has to be set above marginal cost. However, once the new network is deployed, it is socially optimal to set the access price at marginal cost. Thus, if the regulator is unable to commit to a regulatory policy, the incumbent anticipates that he will be expropriated from the rents of his investment. This reduces his incentive to invest, and the socially beneficial opportunity of deploying a new network is lost.

In our model, the regulator's inability to commit to a policy completely eliminates the incumbents' incentives to invest. This extreme result is a consequence of our model being very simple. The point is that the regulator's inability to commit to a policy reduces the incumbent's incentives to invest, not that it completely eliminates the incumbent's incentives to invest. Next, we explain how the model could be modified so that the regulator's inability to commit reduces, but does not eliminate completely the incumbent's incentives to invest.

First, suppose that the product of the incumbent on the new network is of higher quality than the product of the entrant.¹¹ In these circumstances, the incumbent appropriates some of the rents from the investment. Thus, the regulator's inability to commit reduces the incumbent's incentives to invest, but does not eliminate them completely.¹²

Second, suppose that the market is not covered when the firms use only the old network. In these circumstances, the increase in the quality of the products brings new consumers to the market, increasing the firms' profits. Hence, even though the incumbent would not gain any rents with the entrant's consumers for a zero access price, he would earn rents with his new consumers.¹³

Third, suppose that the regulator maximizes welfare subject to the constraint that the incumbent earns a "fair" return on his investment. Then, the access price would equal the marginal cost plus some markup. This would give the incumbent some incentives to invest.

These modifications complicate the model, and particularly the exposition, but do not change qualitatively our results.

3 Two-Part Tariffs

In this Section, we characterize the equilibrium of the no-commitment game with two-part access tariffs, which is presented in the next Proposition.

Proposition 3: *The no-commitment game has two equilibria:*

(*I*) If C is on $(0, \min \{ \frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d) \})$ (*i*) the regulator sets access prices $(K_o^*, \alpha_o^*) = (0, 0)$, (*ii*) the incumbent invests in the new technology, (*iii*) the regulator sets $(K_n^*, \alpha_n^*) = (\pi_e^{d_n}(\Delta_d, 0; C), 0)$, (*iv*) the entrant stays in the market, and (*v*) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

¹¹This could happen if the incumbent had a relatively higher ability to convert the infrastructure investment into new services valued by consumers.

¹²This is the assumption on Foros (2004) and Kotakorpi (2006).

¹³This is similar to the result of DeBijl and Peitz (2004).

(II) If C is on $\left(\min\left\{\frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d)\right\}, \frac{(z+\Delta_d)^2-3t}{2}\right)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*) = (0, 0)$, (ii) the incumbent does not invest in the new technology, (iii) the regulator sets $(K_n^*, \alpha_n^*) = (0, 0)$, (iv) the entrant stays in the market, and (v) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

The following Corollary compares the equilibrium of the commitment and no-commitment games. In the appendix, in Proposition 3a, we present the equilibrium of the commitment game with two-part tariffs.

Corollary 4: Suppose that the regulator sets two-part access tariffs. If C is on $\left(0, \min\left\{\frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d)\right\}\right)$, then the commitment and the no-commitment games have the same the equilibrium. ■

If the investment cost is not too high, i.e., if C is on $\left(0, \min\left\{\frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d)\right\}\right)$, the socially optimal access tariff to the new network set before investment is also socially optimal after the investment. With two-part tariffs, the regulator can use the fixed component of the access tariff to give the incumbent incentives to invest. Since the fixed component only involves a redistribution from the entrant to the incumbent, it has no net impact on welfare. Hence, the regulator has no reason to revise it after the new network is deployed. This result depends, therefore, on the regulator giving the same weight to the profit of the incumbent and the profit of the entrant.

Corollary 5: Consider the no-commitment game. If C is on $\left(0, \min\left\{\frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d)\right\}\right)$, the amount paid by the entrant to the incumbent to have access to the new network is no smaller than the cost of the investment. ■

With two-part access tariffs, the regulator gains an additional instrument. This enables him to give the incumbent incentives to invest, even when he cannot commit to a regulatory policy. However, two-part access tariffs do not solve completely the dynamic consistency problem. First, without commitment, two-part access tariffs only enable the regulator to give the incumbent incentives to invest for some parameter values. Second, without commitment, to induce investment may involve setting the fixed component of the access tariff at a politically unacceptable high level. In our model, inducing investment involves the entrant making a fixed payment to the incumbent no smaller than the cost of the investment,

which goes beyond the policies of sharing the investment cost adopted by some regulatory authorities. With the changes to the model discussed in Section 2.7, the fixed payment of the entrant to the incumbent would not be so large. However, the situation remains qualitatively the same in the sense that the fixed payment would still be very large, and it is unclear whether it would be politically possible to implement such a solution.

4 Separation

In this Section, we characterize the equilibrium of the case where the retail and wholesale businesses of the incumbent are separated and there is no wholesale regulation.

Consider the model of Section 1, except that the regulator imposes the structural separation of the wholesale and retail activities of the incumbent, but abstains from the regulation of the wholesale tariffs. There are *coordination economies* in the vertical integration of the retail and wholesale businesses of the incumbent. Coordination economies stem from the wholesale and retail businesses sharing resources if they are vertically integrated. The separation of the retail and the wholesale activities increases marginal cost by c , due to the loss of *coordination economies*. Parameter c takes values on $[0, (z + \Delta_v) - \sqrt{12t}]$ to ensure that, in equilibrium, the market is fully covered. The timing of this game is as follows. First, the regulator separates the retail and the wholesale businesses of the incumbent, after which he ceases to intervene in the market. Second, the wholesaler decides whether to invest in the new network. Third, the wholesaler sets the wholesale tariff, common to both retailers. Finally, the incumbent and the entrant compete on retail tariffs. The following Proposition presents the equilibrium of this game.

Proposition 4: *Consider the case of structural separation with commitment not to regulate the access price. There are two equilibria:*

(I) *If C is on $(0, \frac{1}{4}\Delta_d[2(z - c) + \Delta_d])$: (i) the wholesaler invests in the new network, (ii) the wholesaler sets $\alpha_n^* = \frac{1}{2}(z + c + \Delta_d)$, (iii) the incumbent and the entrant set $F_i^* = F_e^* = t$, and $p_i^* = p_e^* = \alpha_n^*$.*

(II) *If C is on $[\frac{1}{4}\Delta_d[2(z - c) + \Delta_d], \frac{(z + \Delta_d)^2 - 3t}{2})$: (i) the wholesaler does not invest in the new network, (ii) the wholesaler sets $\alpha_o^* = \frac{1}{2}(z + c)$, (iii) the incumbent and the entrant set $F_i^* = F_e^* = t$, and $p_i^* = p_e^* = \alpha_o^*$. ■*

Both retailers pay the same wholesale price. However, the wholesale marginal cost is

higher than in the case of vertical integration and there is a markup. Thus, commitment is still an issue as it is not clear whether the regulator could credibly commit not to intervene in the market after imposing the separation. The type of commitment problem considered here is different from the one present in the two previous sections. Arguably, one could claim that committing not to intervene in a market is easier than committing not to change a specified value of a regulatory instrument. If the regulator cannot commit not to intervene in the market, the equilibrium is the one presented in the following remark.

Remark 1: *If the sectoral regulator is unable to commit not to regulate the wholesale price then, if the incumbent invests in the new network, the regulator sets $\alpha_n = c$. Anticipating this move, the wholesaler decides not to invest.*

Thus, separation imposed by a regulator unable to commit with respect to regulation of the wholesale prices would only lead to the loss of coordination economies.¹⁴

5 Conclusion

In this article, we analyzed the dynamic consistency of three regulatory instruments for next generation networks.

First, we showed that with linear access tariffs, the dynamic consistency problem is particularly severe. To induce investment, the access tariff to the next generation network should be set above marginal cost. However, once the network is deployed, it is socially optimal to set the access tariff at marginal cost, to eliminate the competition distortions in the retail market. The incumbent anticipates that he will be expropriated from the marginal profits of his investment, and therefore reduces the investment.

Second, we showed that if the regulator can set a two-part tariff for the access to the next generation network this problem is mitigated, but not completely solved. Moreover, to induce investment might involve setting the fixed component of the access tariff at a politically unacceptable high level.

Third, we showed that the separation of the retail and wholesale businesses of the incumbent, associated with the deregulation of the wholesale market, provides incentives for

¹⁴Introducing two-part wholesale tariffs does not change the results above because both retailers' profits are independent of the access price. The two-part access tariff would merely transfer the retailers' profits to the wholesaler with no impact on welfare, prices or on the incentives to invest.

investment, and ensures that the incumbent and the entrant pay the same wholesale price. However, it involves a positive wholesale mark-up, and the loss of coordination economies. It is unclear whether the separation, by itself, solves the dynamic consistency problem since, in the presence of a positive wholesale mark-up, the regulator will be under pressure to intervene.

Appendix

Lemma 1: See Biglaiser and DeGraba (2001). ■

Lemma 2: We first analyze the case where the entrant is a monopolist in the retail market using network $v = o, n$. Consumers purchase if and only if

$$\frac{(z + \Delta_v)^2}{2} - tx - F_i > 0 \Leftrightarrow x < \frac{1}{t} \left[\frac{1}{2} (z + \Delta_v)^2 - F_i \right].$$

Assuming an interior solution, the profit maximizing price and respective profits (excluding investment costs) are

$$\begin{aligned} F_i^{m_v}(\Delta_v) &= \frac{(z + \Delta_v)^2}{4} \\ \pi_i^{m_v}(\Delta_v) &= \frac{(z + \Delta_v)^4}{16t}. \end{aligned}$$

However, we do not have an interior solution since, given our assumption on z ,

$$x^{m_v} = \frac{(z + \Delta_v)^2}{4t} > 1.$$

In this case, the optimal fixed charge and profits are:

$$F_i^{m_v}(\Delta_v) = \pi_i^{m_v}(\Delta_v) = \frac{(z + \Delta_v)^2}{2} - t$$

Welfare is equal to

$$W^{m_v}(\Delta_v) = \frac{(z + \Delta_v)^2}{2} - \frac{1}{2}t. \quad \blacksquare$$

Lemma 3: We start by finding the consumer who is indifferent between buying from the incumbent or from the entrant when both firms use network $v = o, n$:

$$\begin{aligned} \frac{(z + \Delta_v)^2}{2} - tx - F_i &= \frac{(z + \Delta_v - \alpha_v)^2}{2} - t(1 - x) - F_e \Leftrightarrow \\ x(F_i, F_e, \Delta_v, \alpha_v) &= \left(\frac{1}{2} - \frac{F_i - F_e}{2t} - \frac{(z - \alpha_v + \Delta_v)^2 - (z + \Delta_v)^2}{4t} \right). \end{aligned}$$

with $\alpha_v < z + \Delta_v$.

Given this indifferent consumer, and the fact that $p_i = 0$ and $p_e = \alpha_v$, profit functions, excluding investment costs, become:

$$\begin{aligned} \pi_i &= F_i x(F_i, F_e, \Delta_v, \alpha_v) + \alpha_v (z + \Delta_v - \alpha_v) (1 - x(F_i, F_e, \Delta_v, \alpha_v)) \\ \pi_e &= F_e (1 - x(F_i, F_e, \Delta_v, \alpha_v)). \end{aligned}$$

Maximizing each profit function with respect to the fixed fee, we find:

$$\begin{aligned} F_i^{d_v}(\Delta_v, \alpha_v) &= t + \frac{1}{6}\alpha_v [6(z + \Delta_v) - 5\alpha_v] \\ F_e^{d_v}(\Delta_v, \alpha_v) &= t - \frac{1}{6}\alpha_v^2 \end{aligned}$$

The indifferent consumer is given by

$$x^{d_v} = \frac{1}{2} + \frac{\alpha_v^2}{12t},$$

with $\alpha_v \leq \sqrt{6t}$.

Equilibrium profits are then:

$$\begin{aligned} \pi_i^{d_v}(\Delta_v, \alpha_v) &= \frac{(36t^2 + \alpha_v^4 - 60t\alpha_v^2) + 72\alpha_v t(z + \Delta_v)}{72t} \\ \pi_e^{d_v}(\Delta_v, \alpha_v) &= \frac{[6t - \alpha_v^2]^2}{72t}. \end{aligned}$$

Regarding consumers, we have to ensure that all consumers have a positive surplus, independently of the network in use.

$$\begin{aligned} \frac{(z + \Delta_v)^2}{2} - tx^{d_v} - F_i^{d_v} &> 0 \Leftrightarrow \\ ((-2)\Delta_v(2\alpha_v - 2z - \Delta_v) - 4z\alpha_v - 6t + 2z^2 + 3\alpha_v^2) &> 0. \end{aligned}$$

This expression is minimized when $\Delta_v = 0$ at $(-4z\alpha_v - 6t + 2z^2 + 3\alpha_v^2) > 0$. Given that $z > \max\{\alpha_v, \frac{4}{3}\sqrt{6t}\}$ this is always verified.

Finally, total welfare is:

$$W^{d_v}(\Delta_v, \alpha) = \frac{72t(z + \Delta_v)^2 + 5\alpha_v^4 - 36t(t + \alpha_v^2)}{144t}$$

For $\alpha_v > \sqrt{6t}$, the indifferent consumer is at $x^{d_v} > 1$, and therefore we do not have an interior solution. In this case, the optimal fixed fees and profits are:

$$\begin{aligned} \pi_i^{d_v}(\Delta_v, \alpha_v) &= F_i^{d_v} = \alpha_v(z + \Delta_v) - \frac{1}{2}\alpha_v^2 - t \\ \pi_e^{d_v}(\Delta_v, \alpha_v) &= F_e^{d_v} = 0, \end{aligned}$$

and welfare is

$$W^{d_v}(\Delta_{\Delta_v}, \alpha_v) = \frac{(z + \Delta_v)^2}{2} - \frac{t}{2}.$$

■

Corollary 1: The second part is immediate from the observation of the entrant's profit function. With respect to the incumbent's profit, we need to analyze it carefully. Taking

the first and second derivatives we find that, for $\alpha_v < \sqrt{6t}$,

$$\begin{aligned}\frac{\partial \pi_i^{dv}(\Delta_v, \alpha_v, C)}{\partial \alpha_v} &= \frac{18t(z + \Delta_v) - 30t\alpha_v + \alpha_v^3}{18t} \\ \frac{\partial^2 \pi_i^{dv}(\Delta_v, \alpha_v, C)}{\partial \alpha_v^2} &= \frac{3\alpha_v^2 - 30t}{18t} < 0.\end{aligned}$$

Moreover, we find that $\left. \frac{\partial \pi_i^{dv}(\Delta_v, \alpha_v, C)}{\partial \alpha_v} \right|_{\alpha_v = \sqrt{6t}} = (z + \Delta_v) - \frac{4}{3}\sqrt{6t}$, which is positive given our assumption on z . Thus, $\pi_i^{dv}(\Delta_v, \alpha_v, C)$ is increasing in α_v . For $\alpha_v \in [\sqrt{6t}, z + \Delta_v]$, we have:

$$\frac{\partial \pi_i^{dv}(\Delta_v, \alpha_v, C)}{\partial \alpha_v} = z + \Delta_v - \alpha_v > 0$$

■

Lemma 4: This follows directly from the entrant's profit function. ■

Lemma 5: Taking the first derivative of $W^{dn}(\alpha_n, C)$, we obtain as candidates to extrema $\alpha_n = -\frac{3}{5}\sqrt{10t}$, $\alpha_n = 0$ and $\alpha_n = \frac{3}{5}\sqrt{10t}$. Taking the second derivative, we find that it is equal to 1 for $\alpha_n = \left| \frac{3}{5}\sqrt{10t} \right|$ and it is negative for $\alpha_n = 0$. Therefore, the candidate to maximizer is $\alpha_n = 0$, at which welfare is equal to $W^{dn}(0, C) = \frac{1}{2}(z + \Delta_d)^2 - \frac{1}{4}t - C$. We also need to check if it is not better to have a monopoly instead. For $\alpha_n \geq \sqrt{6t}$, welfare is given by $W^{mn}(C) = \frac{1}{2}(z + \Delta_d)^2 - \frac{1}{2}t - C < \frac{1}{2}(z + \Delta_d)^2 - \frac{1}{4}t - C$, and therefore welfare is maximized at $\alpha_n = 0$. ■

Lemma 6: If the incumbent invests, and given Lemma 5, his ex-post profit will be $\pi_i^{dn}(0, C) = \frac{1}{2}t - C$. If he does not invest and $\alpha_o < \sqrt{6t}$, his profit is $\pi_i^{do}(\alpha_o) = \frac{1}{2}t + \frac{\alpha_o^4 + 72t\alpha_o - 60t\alpha_o^2}{72t} > \pi_i^{dn}(0, C)$, while if $\alpha_o \geq \sqrt{6t}$ it is $\pi_i^{mo} = \frac{1}{2}z^2 - t > \pi_i^{dn}(0, C)$. ■

Lemma 7: See Lemma 5, with $\Delta_d = 0$ and α_o taking the place of α_n . ■

Proposition 1: Follows Lemmas 1 to 7. ■

Proposition 2: The incumbent invests in the new network if and only if the incremental profit is positive, i.e., if and only if $\Delta\Pi(\Delta_d, \alpha_n, \alpha_o; C) > 0$, with

$$\begin{aligned}\Delta\Pi(\Delta_d, \alpha_n, \alpha_o; C) &= \\ &= \begin{cases} \pi_i^{dn}(\Delta_d, \alpha_n; C) - \pi_i^{do}(0, \alpha_o; C) & (\alpha_o, \alpha_n) \in [0, \sqrt{6t}) \times [0, \sqrt{6t}) \\ \pi_i^{dn}(\Delta_d, \alpha_n; C) - \pi_i^{mo}(0, C) & (\alpha_o, \alpha_n) \in [\sqrt{6t}, z) \times [0, \sqrt{6t}) \\ \pi_i^{mn}(\Delta_d, C) - \pi_i^{do}(0, \alpha_o; C) & (\alpha_o, \alpha_n) \in [0, \sqrt{6t}) \times [\sqrt{6t}, z + \Delta_d) \\ \pi_i^{mn}(\Delta_d, C) - \pi_i^{mo}(0, C) & (\alpha_o, \alpha_n) \in [\sqrt{6t}, z) \times [\sqrt{6t}, z + \Delta_d) \end{cases} \text{ for } \end{aligned}$$

If the objective of the regulator is not to induce investment the solution is simple: he should set $\alpha_o = \alpha_n = 0$.

Assume now that the regulator wants to induce investment. Start by considering the case of (α_o, α_n) on $[0, \sqrt{6t}] \times [0, \sqrt{6t}]$. Then, in order to induce investment, $\pi_i^{d_n}(\Delta_d, \alpha_n; C) > \pi_i^{d_o}(0, \alpha_o; C)$ or $\pi_i^{m_n}(\Delta_d, C) > \pi_i^{d_o}(0, \alpha_o; C)$ would have to hold. This means that the regulator should also set $\alpha_o = 0$ to increase the range of values for C that result in investment. Hence, the regulator should always set α_o to the minimum when α_o is on $[0, \sqrt{6t}]$.

Assuming that $\alpha_o = 0$ the incremental profit is:

$$\Delta\Pi(\Delta_d, \alpha_n, \alpha_o; C) = \begin{cases} \pi_i^{d_n}(\Delta_d, \alpha_n; C) - \pi_i^{d_o}(0, 0; C) & \alpha_o = 0 \text{ and } \alpha_n \in [0, \sqrt{6t}] \\ \pi_i^{d_n}(\Delta_d, \alpha_n; C) - \pi_i^{m_o}(0, C) & \text{for } (\alpha_o, \alpha_n) \in [\sqrt{6t}, z] \times [0, \sqrt{6t}] \\ \pi_i^{m_n}(\Delta_d, C) - \pi_i^{d_o}(0, 0; C) & \alpha_o = 0 \text{ and } \alpha_n \in [\sqrt{6t}, z + \Delta_d] \\ \pi_i^{m_n}(\Delta_d, C) - \pi_i^{m_o}(0, C) & (\alpha_o, \alpha_n) \in [\sqrt{6t}, z] \times [\sqrt{6t}, z + \Delta_d] \end{cases}$$

If the regulator's objective is to induce investment, setting $\alpha_o = 0$ is also favorable, when compared to setting any value on $[\sqrt{6t}, z]$. Hence, the regulator should set $\alpha_o = 0$, regardless of its objective with respect to investment.

Hence, investment occurs if and only if $C \leq \bar{C}(\alpha_n)$ with

$$\bar{C}(\alpha_n) = \begin{cases} \pi_i^{d_n}(\Delta_d, \alpha_n; 0) - \pi_i^{d_o}(0, 0; C) = \frac{(72t(z+\Delta_d)-60t\alpha_n+\alpha_n^3)\alpha_n}{72t} & \alpha_n \text{ on } [0, \sqrt{6t}] \\ \pi_i^{m_n}(\Delta_d, 0) - \pi_i^{d_o}(0, 0; C) = \frac{(z+\Delta_d)^2-3t}{2} & \alpha_n \text{ on } [\sqrt{6t}, z + \Delta_d] \end{cases}$$

Clearly, $\bar{C}(\alpha_n)$ is increasing in α_n until $\sqrt{6t}$ and then is constant.¹⁵

For a given C , let $A_n(C)$ denote the set of access prices that induce the incumbent to invest. Then,

$$A_n(C) = \begin{cases} \{\alpha_n : \alpha_n \geq a_n\} & C \in [0, (z + \Delta_d)\sqrt{6t} - \frac{9}{2}t] \\ \{\alpha_n : \alpha_n \geq \sqrt{6t}\} & \text{for } C \in [(z + \Delta_d)\sqrt{6t} - \frac{9}{2}t, \frac{(z+\Delta_d)^2-3t}{2}] \end{cases}$$

with $a_n(C)$ implicitly defined by $\frac{(72t(z+\Delta_d)-60ta_n+a_n^3)a_n}{72t} = C$ with $\frac{\partial a_n(C)}{\partial C} = \frac{1}{z+\Delta_d+\frac{(a_n^2-30t)a_n}{18t}} > 0$.

When inducing investment, the regulator will set the welfare maximizing α_n in $A_n(C)$.

The welfare function is quasi-convex in α_n . It was shown in Lemma 5 that it is decreasing between $\alpha_n = 0$ and $\alpha_n = \frac{3}{5}\sqrt{10t}$ and then increases until $\alpha_n = \sqrt{6t}$. Additionally, $W^{d_n}(\Delta_d, \sqrt{\frac{6t}{5}}; C) = W^{d_n}(\Delta_d, \sqrt{6t}; C)$.

¹⁵ As $\frac{(72(z+\Delta_d)\sqrt{6t}-324t)}{72} - \frac{(z+\Delta_d)^2-3t}{2} = \frac{2\sqrt{6t}(z+\Delta_d)-(z+\Delta_d)^2-6t}{2} = -\frac{((z+\Delta_d)-\sqrt{6t})^2}{2} < 0$ the function shifts upwards at $\sqrt{6t}$.

Therefore, if the regulator wants to induce investment, he should set:

$$\alpha_n^*(C) = \begin{cases} \min A_n(C) & \text{if } \min A_n(C) \leq \sqrt{\frac{6t}{5}} \\ \max A_n(C) & \text{if } \min A_n(C) > \sqrt{\frac{6t}{5}}, \end{cases}$$

Note that $a_n(C) = \sqrt{\frac{6t}{5}}$ if and only if $C = \sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t > 0$. For higher values of C we have $a_n(C) > \sqrt{\frac{6t}{5}}$.

We now describe under which conditions the regulator wants investment to occur.

If there is no investment, we will have $\alpha_o = \alpha_n = 0$ and welfare will be $W^{d_o}(0, 0; C) = \frac{z^2}{2} - \frac{t}{4}$. If the regulator induces investment, he will set $\alpha_o = 0$ and $\alpha_n = \alpha_n^*(C)$ and welfare will be $W^{d_n}(\Delta_d, a_n(C); C)$ if $C \in \left[0, \sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t\right)$, or $W^{m_n}(\Delta_d; C)$ if $C \in \left[\sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t, \frac{(z + \Delta_d)^2 - 3t}{2}\right]$.

Therefore, the regulator prefers to induce investment if and only if $W^{d_n}(\Delta_d, a_n(C); C) > W^{d_o}(0, 0; C)$ when $C \in \left[0, \sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t\right)$ or $W^{m_n}(\Delta_d; C) > W^{d_o}(0, 0; C)$ when $C \in \left[\sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t, \frac{(z + \Delta_d)^2 - 3t}{2}\right]$. This is equivalent to:

$$\begin{aligned} C &< \frac{1}{2}\Delta_d(2z + \Delta_d) + \frac{(5(a_n(C))^2 - 36t)}{144t} (a_n(C))^2 & \text{for } C \in \left[0, \sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t\right) \\ C &< \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t & \text{for } C \in \left[\sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t, \frac{(z + \Delta_d)^2 - 3t}{2}\right] \end{aligned}$$

Let \tilde{C} be such that $f(\tilde{C}) := \tilde{C} - \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{(5(a_n(\tilde{C}))^2 - 36t)}{144t} (a_n(\tilde{C}))^2 = 0$. As $\frac{\partial f(C)}{\partial C} > 0$, we have that $C < \frac{1}{2}\Delta_d(2z + \Delta_d) + \frac{(5(a_n(C))^2 - 36t)}{144t} (a_n(C))^2$ is equivalent to $C < \tilde{C}$.¹⁶

Assume that $\tilde{C} < \sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t$. Then, for $C = \sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t$ we have $W^{d_n}(\Delta_d, a_n(C); C) = W^{m_n}(\Delta_d; C)$ and $W^{d_n}(\Delta_d, a_n(C); C) < W^{d_o}(0, 0; C)$. This implies that $W^{m_n}(\Delta_d; C) < W^{d_o}(0, 0; C)$ which means that $\sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t > \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t$. Thus, it is impossible to have $C < \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t$ and $C \in \left[\sqrt{\frac{6t}{5}}(z + \Delta_d) - \frac{49}{50}t, \frac{(z + \Delta_d)^2 - 3t}{2}\right]$.

Finally, note that $\frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t < \frac{(z + \Delta_d)^2 - 3t}{2} \Leftrightarrow z > \sqrt{\frac{5}{2}}t$, which is always true. ■

Corollary 3: From Propositions 1 and 2. ■

Proposition 3: With no commitment, the regulator will set $\alpha_o = \alpha_n = 0$. To encourage investment he will set $K_o = 0$ and $K_n = \pi_e^{d_n}(\Delta_d, 0) - \varepsilon$. There will be investment if and only if $\pi_i^{d_n}(\Delta_d, 0; 0) + \pi_e^{d_n}(\Delta_d, 0) - \pi_i^{d_o}(0, 0; C) = \frac{1}{2}t - C > 0$. The regulator wants to induce investment if $W^{d_n}(\Delta_d, 0; C) - W^{d_o}(0, 0; C) = \frac{1}{2}\Delta_d(2z + \Delta_d) - C > 0$. ■

¹⁶Note that $\frac{\partial f(C)}{\partial C} = \frac{3}{2} \frac{(12tz - 14ta_n + 12t\Delta_d - a_n^3)}{(18tz + 18t\Delta_d - 30ta_n + a_n^3)}$. The denominator is always positive. As for the denominator we have $12t(z + \Delta_d) - 14ta_n - a_n^3 > 12t(z + \Delta_d) - \frac{76}{5} \left(\sqrt{\frac{6t}{5}}\right)t > 0$.

Proposition 3a: *The commitment game has the following equilibria:*

(I) If $\min \left\{ \frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d) \right\} = \frac{1}{2}\Delta_d(2z + \Delta_d)$:

(a) If C is on $(0, \frac{1}{2}\Delta_d(2z + \Delta_d))$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*(C)) = (0, 0, \pi_e^{dn}(\Delta_d, 0; C), \alpha_n^{**}(C))$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

(b) If C is on $\left(\frac{1}{2}\Delta_d(2z + \Delta_d), \frac{(z + \Delta_d)^2 - 3t}{2} \right)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, 0, 0)$, (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

(II) If $\min \left\{ \frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d) \right\} = \frac{1}{2}t$ and $\tilde{C} < \sqrt{\frac{6}{5}}t(z + \Delta_d) - \frac{33}{50}t$:

(a) If C is on $(0, \frac{1}{2}t)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, \pi_e^{dn}(\Delta_d, 0; C), \alpha_n^{**}(C) = 0)$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

(b) If C is on $\left(\frac{1}{2}t, \tilde{C} \right)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, \pi_e^{dn}(\Delta_d, 0; C), \alpha_n^{**}(C))$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = \alpha_n^{**}(C)$.

(c) If C is on $\left(\tilde{C}, \frac{(z + \Delta_d)^2 - 3t}{2} \right)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, 0, 0)$, (ii) the incumbent does not invest in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

(III) If $\min \left\{ \frac{1}{2}t, \frac{1}{2}\Delta_d(2z + \Delta_d) \right\} = \frac{1}{2}t$ and $\tilde{C} > \sqrt{\frac{6}{5}}t(z + \Delta_d) - \frac{33}{50}t$:

(a) If C is on $(0, \frac{1}{2}t)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, b_n(C), \pi_e^{dn}(\Delta_d, 0; C), \alpha_n^{**}(C) = 0)$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$.

(b) If C is on $\left(\frac{1}{2}t, \sqrt{\frac{6}{5}}t(z + \Delta_d) - \frac{33}{50}t \right)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, \pi_e^{dn}(\Delta_d, 0; C), \alpha_n^{**}(C) > 0)$, (ii) the incumbent invests in the new technology, (iii) the entrant stays in the market, and (iv) the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = \alpha_n^{**}(C)$.

(c) If C is on $\left(\sqrt{\frac{6}{5}}t(z + \Delta_d) - \frac{33}{50}t, \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t \right)$: (i) the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, \pi_e^{dn}(\Delta_d, 0; C), \alpha_n^{**}(C) > \sqrt{6}t)$, (ii) the incumbent in-

vests in the new technology, **(iii)** the entrant exits the market, and **(iv)** the incumbent sets retail tariffs $F_i^* = F_i^{m_n}(\Delta_n) = \frac{(z+\Delta_n)^2}{2} - t$, and $p_i^* = 0$.

(d) If C is on $\left(\frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t, \frac{(z+\Delta_d)^2-3t}{2}\right)$: **(i)** the regulator sets access prices $(K_o^*, \alpha_o^*, K_n^*, \alpha_n^*) = (0, 0, 0, 0)$, **(ii)** the incumbent does not invest in the new technology, **(iii)** the entrant stays in the market, and **(iv)** the incumbent and the entrant set retail tariffs $F_i^* = t$, $F_e^* = t$, and $p_i^* = 0$, $p_e^* = 0$. ■

Proof: The incumbent invests in the new network if and only if the incremental profit is positive, i.e., if and only if $\Delta\Pi((\Delta_d, \alpha_n, \alpha_o; C) > 0$, with

$$\begin{aligned} \Delta\Pi((\Delta_d, \alpha_n, \alpha_o; C) &= \\ &= \begin{cases} \pi_i^{d_n}(\Delta_d, \alpha_n; C) + K_n - \pi_i^{d_o}(0, \alpha_o; C) - K_o & (\alpha_o, \alpha_n) \in [0, \sqrt{6t}) \times [0, \sqrt{6t}) \\ \pi_i^{d_n}(\Delta_d, \alpha_n; C) + K_n - \pi_i^{m_o}(0, C) & \text{for } (\alpha_o, \alpha_n) \in [\sqrt{6t}, z) \times [0, \sqrt{6t}) \\ \pi_i^{m_n}(\Delta_d, C) + K_n - \pi_i^{d_o}(0, \alpha_o; C) - K_o & (\alpha_o, \alpha_n) \in [0, \sqrt{6t}] \times [\sqrt{6t}, z + \Delta_d) \\ \pi_i^{m_n}(\Delta_d, C) + K_n - \pi_i^{m_o}(0, C) & (\alpha_o, \alpha_n) \in [\sqrt{6t}, z) \times [\sqrt{6t}, z + \Delta_d) \end{cases} \end{aligned}$$

If the regulator does not want to induce investment he will set $\alpha_n = K_n = 0$ and $\alpha_o = 0$ and a high K_o . If he wants to induce investment $\alpha_o = 0$ is also favorable as well as $K_o = 0$ (see Proposition 2) and the regulator could set $K_n = \pi_e^{d_n}(\Delta_d, \alpha_n; C) - \varepsilon$.

Note that

$$\pi_i^{d_v}(\Delta_v, \alpha_v; C) + \pi_e^{d_v}(\Delta_v, \alpha_v) = \frac{\alpha_v(36t(z + \Delta_v - \alpha_v) + \alpha_v^3)}{36t} + t - C$$

with

$$\begin{aligned} \frac{\partial(\pi_i^{d_v}(\Delta_v, \alpha_v; C) + \pi_e^{d_v}(\Delta_v, \alpha_v))}{\partial\alpha_v} &= z + \Delta_v + \frac{1}{9}t^{-1}(\alpha_v^2 - 18t)\alpha_v \\ \frac{\partial^2(\pi_i^{d_v}(\Delta_v, \alpha_v; C) + \pi_e^{d_v}(\Delta_v, \alpha_v))}{\partial\alpha_v^2} &= \frac{1}{3}t^{-1}(\alpha_v^2 - 6t) < 0 \end{aligned}$$

and

$$\left. \frac{\partial(\pi_i^{d_v}(\Delta_v, \alpha_v; C) + \pi_e^{d_v}(\Delta_v, \alpha_v))}{\partial\alpha_v} \right|_{\alpha_v=\sqrt{6t}} = z + \Delta_v - \frac{4}{3}\sqrt{6t} > 0$$

Hence, $\frac{\partial(\pi_i^{d_v}(\Delta_v, \alpha_v; C) + \pi_e^{d_v}(\Delta_v, \alpha_v))}{\partial\alpha_v} > 0$ for all α_n on $[0, \sqrt{6t})$.

Investment then occurs if and only if $C \leq \bar{\bar{C}}(\alpha_n)$ with:

$$\begin{aligned} \overline{\overline{C}}(\alpha_n) = & \begin{cases} \pi_i^{d_n}(\Delta_d, \alpha_n; 0) + \pi_e^{d_n}(\Delta_d, \alpha_n) - \pi_i^{d_o}(0, 0; C) & \alpha_n \text{ on } [0, \sqrt{6t}) \\ \pi_i^{m_n}(\Delta_d, 0) + 0 - \pi_i^{d_o}(0, 0; C) & \alpha_n \text{ on } [\sqrt{6t}, z + \Delta_d) \end{cases} = \\ & \begin{cases} \frac{1}{2}t + \frac{\alpha_n(36t(z+\Delta_d-\alpha_n)+\alpha_n^3)}{36t} & \alpha_n \text{ on } [0, \sqrt{6t}) \\ \frac{(z+\Delta_d)^2-3t}{2} & \alpha_n \text{ on } [\sqrt{6t}, z + \Delta_d) \end{cases}. \end{aligned}$$

Clearly, $\overline{\overline{C}}(\alpha_n)$ is increasing in α_n until $\sqrt{6t}$ and then is constant.¹⁷

For a given C , let $B_n(C)$ denote the set of access prices that induce the incumbent to invest. Then,

$$B_n(C) = \begin{cases} \{\alpha_n : \alpha_n \geq 0\} & C \in [0, \frac{1}{2}t) \\ \{\alpha_n : \alpha_n \geq b_n(C)\} & \text{for } C \in [\frac{1}{2}t, (z + \Delta_d)\sqrt{6t} - \frac{9}{2}t) \\ \{\alpha_n : \alpha_n \geq \sqrt{6t}\} & C \in [(z + \Delta_d)\sqrt{6t} - \frac{9}{2}t, \frac{(z+\Delta_d)^2-3t}{2}] \end{cases},$$

with $b_n(C)$ implicitly defined by $\frac{1}{2}t + \frac{b_n(36t(z+\Delta_d-b_n)+b_n^3)}{36t} = C$, with $\frac{\partial b_n(C)}{\partial C} = \frac{1}{z+\Delta_d + \frac{(b_n^2-18t)b_n}{9t}} > 0$.

Therefore, if the regulator wants to induce investment, he should set:

$$\alpha_n^{**}(C) = \begin{cases} \min B_n(C) & \text{if } \min B_n(C) \leq \sqrt{\frac{6t}{5}} \\ \max B_n(C) & \text{if } \min B_n(C) > \sqrt{\frac{6t}{5}}, \end{cases}$$

Note that $b_n(C) = \sqrt{\frac{6t}{5}}$ if and only if $C = \sqrt{\frac{6t}{5}}t(z + \Delta_d) - \frac{33}{50}t > 0$. For higher values of C we have $b_n(C) > \sqrt{\frac{6t}{5}}$.

We now describe under which conditions the regulator wants investment to occur.

If there is no investment we will have $\alpha_o = \alpha_n = 0$ and welfare will be $W^{d_o}(0, 0; C)$. If the regulator induces investment, he will set $\alpha_o = 0$ and $\alpha_n = \alpha_n^{**}(C)$ and welfare will be, $W^{d_n}(\Delta_d, 0; C)$ if $C \in [0, \frac{1}{2}t]$ or $W^{d_n}(\Delta_d, b_n(C); C)$ if $C \in (\frac{1}{2}t, \sqrt{\frac{6t}{5}}t(z + \Delta_d) - \frac{33}{50}t)$ or $W^{m_n}(\Delta_d; C)$ if $C \in [\sqrt{\frac{6t}{5}}t(z + \Delta_d) - \frac{33}{50}t, \frac{(z+\Delta_d)^2-3t}{2}]$.

Thus, the regulator induces investment if

¹⁷As $(\sqrt{6t}(z + \Delta_v) - \frac{9}{2}t) - \frac{(z+\Delta_d)^2-3t}{2} = -\frac{1}{2}((z + \Delta_d)^2 - 2(z + \Delta_v)\sqrt{6t} + 6t) < 0$ the function shifts upwards at $\sqrt{6t}$.

$$\begin{aligned}
C &< \frac{1}{2}\Delta_d(2z + \Delta_d) & C &\in [0, \frac{1}{2}t] \\
C &< \frac{1}{2}\Delta_d(2z + \Delta_d) + \frac{(b_n(C))^2 - 36t}{144t} (b_n(C))^2 & \text{for } C &\in \left(\frac{1}{2}t, \sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t\right) \\
C &< \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t & C &\in \left[\sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t, \frac{(z + \Delta_d)^2 - 3t}{2}\right],
\end{aligned}$$

Let $\tilde{\tilde{C}}$ be such that $g(\tilde{\tilde{C}}) := \tilde{\tilde{C}} - \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{(5b_n(\tilde{\tilde{C}})^2 - 36t)}{144t} b_n(\tilde{\tilde{C}})^2 = 0$. As $\frac{\partial g(C)}{\partial C} > 0$, we have that $C < \frac{1}{2}\Delta_d(2z + \Delta_d) + \frac{(5(b_n(C))^2 - 36t)}{144t} (b_n(C))^2$ is equivalent to $C < \tilde{\tilde{C}}$.¹⁸

Note that if $\frac{1}{2}t < \frac{1}{2}\Delta_d(2z + \Delta_d)$ then $\tilde{\tilde{C}} > \frac{1}{2}t$.

Assume that $\frac{1}{2}\Delta_d(2z + \Delta_d) < \frac{1}{2}t$. Then, for $\frac{1}{2}\Delta_d(2z + \Delta_d) < C < \frac{1}{2}t$ the regulator prefers not to induce investment although he could do so with $\alpha_n = 0$. This means that for larger values of C the regulator will prefer not to induce investment.

If $\frac{1}{2}\Delta_d(2z + \Delta_d) > \frac{1}{2}t$ the regulator induces investment whenever it can do so with $\alpha_n = 0$. Assume that $\tilde{\tilde{C}} < \sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t$. Then, for $C = \sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t$ we have $W^{d_n}(\Delta_d, a_n(C); C) = W^{m_n}(\Delta_d; C)$ and $W^{d_n}(\Delta_d, a_n(C); C) < W^{d_o}(0, 0; C)$. This implies that $W^{m_n}(\Delta_d; C) < W^{d_o}(0, 0; C)$ which means that $\sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t > \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t$. Thus, it is impossible to have $C < \frac{1}{2}\Delta_d(2z + \Delta_d) - \frac{1}{4}t$ and $C \in \left[\sqrt{\frac{6}{5}t}(z + \Delta_d) - \frac{33}{50}t, \frac{(z + \Delta_d)^2 - 3t}{2}\right]$.

Corollary 4: From Propositions 3 and 3a. ■

Corollary 5: The restriction which guarantees the incumbent's incentives to invest is $\pi_i^{d_n}(\Delta_d, 0; 0) + K_n - \pi_i^{d_o}(0, 0; C) > 0$ which is equivalent to $K_n - C \geq 0$. ■

Proposition 4: We start by finding the consumer indifferent between both firms

$$\frac{(z + \Delta_i - \alpha)^2}{2} - tx - F_i = \frac{(z + \Delta_i - \alpha)^2}{2} - t(1 - x) - F_e \Leftrightarrow x = \frac{t + F_e - F_i}{2t}$$

with $\alpha < z$, $\Delta_i = \Delta_d$ in the case of investment and $\Delta_i = 0$ in the case of no investment.

The demand function, in terms of consumers, facing the incumbent is

$$D_i(F_i, F_e, \Delta_i, \Delta_e, \alpha; z) = \begin{cases} 0 & F_i > F_e + t \\ 1 & F_i < F_e - t \\ x(F_i, F_e, \Delta_i, \alpha; z) & \text{else} \end{cases}$$

Given this indifferent consumer, and the fact that $p_i = p_e = \alpha$, profit functions become:

$$\begin{aligned}
\pi_i(F_i, F_e, \Delta_i, \Delta_e, \alpha; z) &= F_i x(F_i, F_e, \Delta_i, \alpha; z) \\
\pi_e(F_i, F_e, \Delta_i, \Delta_e, \alpha; z) &= F_e (1 - x(\Delta_i, \Delta_e, \alpha; z))
\end{aligned}$$

¹⁸Note that $\frac{\partial g(C)}{\partial C} = \frac{3}{4} \frac{(12t(z + \Delta_d) - 18tb_n + b_n^3)}{(9t(z + \Delta_d) - 18tb_n + b_n^3)} > 0$.

Interior best response functions are:

$$\begin{aligned}\frac{\partial \left(F_i \left(-\frac{1}{2t} (-t - F_e + F_i) \right) \right)}{\partial F_i} &= 0 \\ \frac{\partial \left(F_e \left(1 + \frac{1}{2t} (-t - F_e + F_i) \right) \right)}{\partial F_e} &= 0,\end{aligned}$$

from where we obtain

$$F_e = F_i = t.$$

The indifferent consumer is given by $x^* = \frac{1}{2}$ and the incumbent and entrant's profit is $t/2$.

The wholesaler's profit is

$$\pi_i = (\alpha - c) \left[\frac{1}{2}(z + \Delta_i - \alpha) + \frac{1}{2}(z + \Delta_i - \alpha) \right],$$

which is maximized at $\alpha^* = \frac{c+z+\Delta_i}{2}$. The corresponding wholesale profits are $\pi_i^*(z, \Delta_i, c) = \frac{1}{4}(z + \Delta_i - c)^2$.

The wholesaler finds it profitable to invest if and only if $\pi_i^*(z, \Delta_d, c) - C > \pi_i^*(z, 0, c)$, which is equivalent to

$$C < \frac{1}{4}\Delta_d(2z - 2c + \Delta_d).$$

■

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