Version compatibility and strategy against entry

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Abstract

The present paper studies the market of a durable good and its upgrade in the presence of network effects. We investigate the firm's strategic decision upon compatibility between the durable and the upgrade faced to a potential entrant. We consider backward, forward and two-way compatibility and characterize market equilibria under each regime. By making comparison between the equilibria, we elucidate the features of the regimes in terms of entry prevention and puts in light the incumbent's optimal compatibility decision.

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1   Introduction

With the advent of the information age, the computer has become predominant in our society. Nowadays, it is unimaginable to live our daily life without it. Along with the computer, the software business has grown to be one of the most important industries. Needless to say, computer software is a durable good and durable goods theory has an abundant literature(see for instance Coase (1972), Bulow (1982) and Gul, Sonnenschein, and Wilson (1986)).

Software, however, possesses the following three features which are not captured in the classic durable goods theory. Firstly, the software manufacturer sells one generation of a good and launches a new upgraded version later in time. Consumers who possess an old version very often purchase a new one even if the old one is still usable.

Secondly, software has network effects: the more consumers own a particular piece of software the larger utility it provides. The more people have a software product, the more easily a file created by it can be opened, accessed and processed.

Thirdly, software provides compatibility between the old and the new version. For instance, using computer software in actual life, we may have a bitter experience that a file we have made with an old version of the software some time ago cannot be opened or processed with a new version. The release 2 of Lotus 1-2-3 exactly lacked this compatibility. If we can process with a new version of software files created with the old version, this software provides backward compatibility. That an old version of software can process a file created with a new version is forward compatibility. If the above two compatibilities are present simultaneously, the product provides two-way compatibility.

Certainly, when there is a drastic technological change, assuring some of the compat-

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ibility regimes above mentioned might be technically difficult or too costly to the firm. However, compatibility provision is at least as much a strategic decision as a technical one and the firm can decide which compatibility to provide in terms of profit maximization. This article analyzes the market of computer software with the above three characteristics and investigates what effect each compatibility regime has on the manufacturer's customer capturing and how the manufacturer makes a strategic compatibility decision.

Theoretically, the firm has four options: no compatibility, backward compatibility, forward compatibility, two-way compatibility. In no compatibility, the users of either versions cannot enjoy network effects provided by the users of the other version. In backward compatibility, the buyers of the upgrade can profit from network effects from the buyers of the initial version whereas these users of the initial version cannot benefit from network externalities from the users of the initial version. In forward compatibility, the users of the initial version profit by network effects resulting from those of the new version whereas these users cannot enjoy network effects from the users of the old version. In two-way compatibility, the users of both versions can profit from network effects provided by the other. Accordingly, backward compatibility raises the value of the upgrade version with network effects and brings in more profit from the upgrade whereas forward compatibility brings in more profit from the initial version. It is not obvious which compatibility regime is more profitable to the firm in the dynamic context.

In this paper, however, we abstract from the aspect of the firm's dynamic decision. Instead, we investigate the role of the compatibility regimes as a counter-entry device. Needless to say, the firm's dynamic decision is of much interest in its own right but the static framework adopted in this paper already gives substantial insights into the feature of the compatibility regimes and the firm's strategic compatibility decision.

Initially, the incumbent has an initial version of a product on the market with heterogeneous customers. We assume that all the customers possess the incumbent's initial

version. At the next stage, the firm launches the upgrade version and decides which compatibility to provide. In our static framework, absent a competitor(entrant firm), the incumbent firm always chooses to provide backward compatibility since it increases the utility of the new version(with network effects) and promotes the sale whereas the initial version does not bring any additional profits. However, the entrance of a competitor drastically changes the picture. In the presence of a potential entrant, the incumbent firm may find it in its interest to render the initial version forward compatible. Increasing the utility of the version with larger network effects, this may persuade customers who choose not to buy the upgrade to stay with the old version instead of switching to the entrant's product. The present paper's most important message is that forward compatibility plays a role of a counter-entry device. This view has been lacking in academic literature, which sometimes claims that backward compatibility is best for the firm since it raises the value of the upgrade version and brings in more revenue.

Although there is an abundant literature on compatibility between competing firms' products, there are very few works on one firm's strategic compatibility decision between the versions of its own good. Choi (1994) studies a monopolistic firm's decision on whether to provide compatibility between the old and new good but he only considers two-way compatibility. He assumes that consumers are identical in their taste. Fudenberg and Tirole (1998) consider the monopolist's upgrade provision in a durable good market but leave out compatibility and network effects. Ellison and Fudenberg (2000) examine the monopolist's decision on supplying an upgrade but assume that the upgrade is backward compatible and thus rule out strategic compatibility decision. Fudenberg and Tirole (2000) study the pricing of a network good under entry threat. In contrast with the present paper, they assume that the entrant's good is of the same value to all consumers and they only consider two-way compatibility. The substantial difference with us, though, is that they do not consider consumers' upgrading and different compatibility regimes. Nahm (2003) is probably the only work which considers the firm's

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one-way compatibility decision. In contrast with the present paper, he considers inter-actions beween the hardware manufacturer's compatibility decisions(backward, forward, etc.) and the market of software products. He does not include network effects in the study.

In the next section, we present the model and in Section 3, we work out the equilibrium of each compatibility regime. In Section 4, we make comparison of the regimes, which includes the analysis of the optimal regime for the incumbent firm. The last section concludes the paper.

2   The model

There are three products, the old product and the new upgrade product which are provided by the incumbent manufacturer, and the product proffered by the entrant manufacturer. They are labeled respectively as L(low quality), H(high quality), E(the entrant's).

We assume that all the existing population have purchased in the first period and possess an old version of a product. By doing so, we focus upon the function of each compatibility. We assume that in the second period, the incumbent only sells the new but not the old version and that there is no second hand market. Consumers, therefore, can buy either the new or the entrant's product in the second period. If a consumer purchases one of the products in that period, the old version will have a scrap value. In other words, he does not enjoy accumulated utility from a newly bought and the old product.

The customers are different in the appreciation of the products called *type*, the range of which is a non empty interval of positive real numbers. Without the loss of generality, we normalize the interval to $\Theta = [0, 1]$, from which type θ takes a value. The type θ is distributed according to the uniform distribution.

We normalize the cost of production of all the goods to naught. This makes us free from consideration of cost and we will concentrate upon the incumbent's purely strategic decision on compatibility.

The product i, for $i = L, H, E$, offers basic utility u_i to the consumers. Having product i, type θ customer enjoys basic utility θu_i net of network effects. In our context, u_i for $i = L, H, E$ is known to all the parties.

We investigate the incumbent's strategic decision on compatibility between the old product and the upgrade. Theoretically, there are four possibilities of compatibility: no, backward, forward, two way compatibility regimes. Let N, B, F, T indicate respectively the regime when used as a superscript. By use of these notations, we denote the price of product i under regime j by p_i^j

Let M_i^j denote a number of consumers who purchase product i under compatibility regime j.[1] N_i^j is a network size, that is, a number of customers contributing to the network effect of product i. It varies according to the compatibility regime. We assume there is no network effects between the entrant's and the incumbent's product, whereby we have $M_E^j = N_E^j$. One justification for this is that the incumbent firm has a patent for its products and can protect them from competition.[2]

In no compatibility, the link between the old and the new product is completely severed and there is no network effect between them:

$$N_L^N = 1 - M_E^N - M_H^N, \qquad N_H^N = M_H^N.$$

The right hand in the first equation is the number of consumers who only have the old version, not buying any product in the second period.

In backward compatibility, the upgrade has compatibility with the old version but

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[1]On our assumption, all consumers possess the old version and thus $M_L^j = 1$.

[2]The licensing contract to a competitor can be the firm's strategic decision in terms of profit maximization. This issue is out of the scope of this article.

not vice versa. Therefore,

$$N_L^B = 1 - M_E^B - M_H^B, \qquad N_H^B = 1 - M_E^B.$$

The right hand in the second equation is the number of consumers who either possess the old or new version, in other words, who do not buy the entrant's product.

In forward compatibility, contrary to the backward case, the old version has compatibility with the new product but not the other way about.

$$N_L^F = 1 - M_E^F, \qquad N_H^F = M_H^F.$$

The first equation indicates that the network size of the old version consists of the consumers who possess the old or new product.

In two-way compatibility, the old and the new versions have compatibility with each other.

$$N_L^T = 1 - M_E^T, \qquad N_H^T = 1 - M_E^T.$$

The network size of both the products is identical and composed of the consumers who choose one of the incumbent's product.

Given the network size N_i^j, all consumers enjoy the network utility vN_i^j, where v is a positive number. The network utility is, by assumption, independent of the consumers' type.[3] Type θ consumer has the gross utility for product i, $\theta u_i + vN_i^j$.

Before going into the detail of the comsumers' behaviour, let us note that as is generally the case in analysis of network goods, there may be mutiple equilibria in our context. We adopt the standard convention that the consumers coordinate on an equilibrium selection which is largest sales of the goods (see Fudenberg and Tirole (2000)).

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[3]The network effect may take a more general form. However, the comparison of the compatibility regimes is rather complicated even in our simple setting. This is why we opt for our simplified assumption. For dependence of network utility on type, see Ellison and Fudenberg (2000).

Given one of the four compatibility regimes and also the price of each product, the consumers select one of the actions: clinging to the old version, buying the upgrade or buying the entrant's product. Type θ consumer respectively obtains the following utility under compatibility regime j:

$$U_H^j(\theta) = \theta u_H + v N_H^j - p_H^j, \quad U_E^j(\theta) = \theta u_E + v N_E^j - p_E^j, \quad U_L^j(\theta) = \theta u_L + v N_L^j.$$

Consumers purchase a good which brings them the highest utility. The measure of the types who purchase good $i = H, E$ under regime j is expressed as follows.

$$M_i^j = \mu\{\theta | U_i^j(\theta) \geq U_h^j(\theta),\ h = L,\ H,\ E\} \tag{1}$$

where μ is a measure deduced from the uniform distribution of θ, thus the Lebesgue measure.

Given compatibility regime j and the price of the new and the entrant's product[4], profit from product i to the manufacturers is written as

$$\Pi_i^j = p_i^j M_i^j \qquad \text{for } i = H,\ E. \tag{2}$$

3 Equilibria of each regime

Now we examine competition between the incumbent and the entrant. We study Bertrand competition, in which, given a compatibility regime, simultaneously, the incumbent decides upon the price of the upgrade and the entrant upon the price of its product. A few words might be in order here for a justification of the competition. The alternative possibility is naturally Stackelberg competition. We do not, however, have a clear an-

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[4]The old product has already been in the market.

swer to which manufacturer is a leader. Depending upon a situation, either party can be a leader. We opt, accordingly, for Bertrand as one of the equally plausible scenarios. It enables us to completely work out the prices and profits and make comparison across the different regimes.

In the sequel, we make an assumption on the basic utility of the products. Naturally, the upgrade offers higher basic utility than the initial version. In addition, we assume the following on the relative magnitude of the incumbent's and the entrant's goods:

$$u_L \leq u_H \leq u_E.$$

In other words, we limit ourselves to the case where the entrant's product is of the highest quality compared to the incumbent's. It is shown that the three products capture each an interval of customers regardless of the compatibility regime.

Proposition 1. *Given regime j, the entrant captures the highest portion of type $M_E^j = [\theta_E^j, 1]$, the upgrade the middle $M_H^j = [\theta_H^j, \theta_E^j]$, the old version the lowest $M_L^j = [0, \theta_H^j]$.*

Proof. The customers buying each product are completely determined by the two prices, p_H^j and p_E^j and so are the network size. Suppose that the prices are given. The consumers buying the entrant's product is

$$S = \{\theta | U_E^j(\theta) \geq U_i^j(\theta),\ i = L,\ H\}.$$

In our competition game, S is never empty since there is no production cost and the entrant always earns profit by setting such a price as to allow customers to buy. At the same time, if $\theta \in S$, then $\theta' \in S$ for any $\theta' \geq \theta$ owing to the assumption on the basic utility values. Therefore, there exists the cut-off type θ_E^j such that S= $[\theta_E^j, 1]$.

The consumers buying the upgrade is

$$T = \{\theta | U_H^j(\theta) \geq U_i^j(\theta),\ i = L,\ E\}.$$

As above, T is not empty because the upgrade incurs no cost and the incumbent is keen to earn money. If $U_H^j(\theta) \geq U_L^j(\theta)$, so is it for all $\theta' \geq \theta$ by the same argument as above. Therefore, there is the smallest type θ_H^j which buys the upgrade. We conclude that $T = [\theta_H^j, \theta_E^j]$.

$\square$

According to the proposition, the cut-off types or the lowest types of consumers who buy the upgrade and the entrant's product determine the choice of all consumers. We perform profit maximization in terms of the cut-off types instead of the prices.

From the proposition, the cut-off type of the new product is always indifferent between the new and the old[5],

$$U_H^j(\theta_H^j) = U_L^j(\theta_H^j).$$

This is rewritten in terms of the price:

$$p_H^j = \theta_H^j(u_H - u_L) + v(N_H^j - N_L^j). \tag{3}$$

In the similar manner, the incumbent offers the upgrade price in such a way that the lowest type choosing the entrant's product is indifferent between it and the upgrade:

$$U_E^j(\theta_E^j) = U_H^j(\theta_E^j).$$

This is equivalent to

$$p_E^j = \theta_E^j(u_E - u_H) + v(N_E^j - N_H^j) + p_H^j. \tag{4}$$

We have expressed the prices in terms of the cut-off types and we are ready to perform

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[5] As seen in the proof of Proposition 1, it will be obvious that the cut-off type of the new version is always indifferent between the new and the old version in our context of the profit maximizing incumbent. If not, the incumbent can raise the price of the upgrade by a little without losing any customer.

profit maximization.

3.1 Backward compatibility

It follows from Proposition 1 that

$$N_H^B = \theta_E^B, \quad N_E^B = 1 - \theta_E^B, \quad N_L^B = \theta_H^B.$$

Accordingly, from (2), the entrant's and the upgrade's profits are respectively,

$$\Pi_E^B = \left(\theta_E^B(u_E - u_H) - \theta_H^B(u_L - u_H) + v(1 - \theta_E^B - \theta_H^B)\right)\left(1 - \theta_E^B\right),$$

$$\Pi_H^B = \left(\theta_H^B(u_H - u_L) + v(\theta_E^B - \theta_H^B)\right)\left(\theta_E^B - \theta_H^B\right).$$

Proposition 2. *If $u_E - u_H - 2v \leq 0$,*

$$\theta_E^B = 0, \quad \theta_H^B = 0, \ p_E^B = v, \ p_H^B = 0, \ \Pi_E^B = v, \ \Pi_H^B = 0. \tag{5}$$

If $u_E - u_H - 2v > 0$,

- *if $u_H - u_L - 2v \leq 0$,*

$$\begin{cases} \theta_E^B = \frac{u_E - u_H - 2v}{2(u_E - u_H - v)}, \\ \theta_H^B = 0, \end{cases} \quad \begin{cases} p_E^B = \frac{u_E - u_H}{2}, \\ p_H^B = 0, \end{cases} \quad \begin{cases} \Pi_E^B = \frac{(u_E - u_H)^2}{4(u_E - u_H - v)}, \\ \Pi_H^B = 0. \end{cases} \tag{6}$$

- *if $u_H - u_L - 2v > 0$,*

$$\begin{cases} \theta_E^B = \frac{2(u_E - u_H - 2v)}{4u_E - 3u_H - u_L - 6v}, \\ \theta_H^B = \frac{(u_E - u_H - 2v)(u_H - u_L - 2v)}{(4u_E - 3u_H - u_L - 6v)(u_H - u_L - v)}, \end{cases} \tag{7}$$

in addition, we have

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$$\theta_E^B - \theta_H^B = \frac{(u_E - u_H - 2v)(u_H - u_L)}{(4u_E - 3u_H - u_L - 6v)(u_H - u_L - v)},$$

$$p_H^B = \frac{(u_E - u_H - 2v)(u_H - u_L)}{4u_E - 3u_H - u_L - 6v}, \quad \Pi_H^B = \frac{(u_H - u_L)^2(u_E - u_H - 2v)^2}{(4u_E - 3u_H - u_L - 6v)^2(u_H - u_L - v)}.$$

Proof. See the Appendix. □

The case in (5) shows that when the entrant's product quality is not so high compared to that of the upgrade, the entrant captures all types by charging a low price, which is just the network value.

In the case of $u_E - u_H - 2v > 0$, the entrant's product has a significant quality advantage relative to the competitor's upgrade. Then, by charging a high price, the entrant only captures high-end customers who appreciate the quality more and are ready to pay a higher price.

In the case of (6), the quality improvement of the upgrade is not so considerable compared to the old version, the incumbent sets a low price and gets all the remaining customers to upgrade to the new version.

In the final case (7), the quality of the upgrade is very high. Then, the incumbent sells the upgrade only to higher types of customers who more appreciate quality by charging a high price. As a result, the entrant captures the highest types and the upgrade the middle and the old the lowest.

3.2 Forward compatibility

$$N_H^T = \theta_E^F - \theta_H^F, \quad N_E^T = 1 - \theta_E^F, \quad N_L^T = \theta_E^F.$$

Similarly to the previous section, we have the profits:

$$\Pi_E^F = \left(\theta_E^F(u_E - u_H) - \theta_H^F(u_L - u_H) + v(1 - 2\theta_E^F)\right)\left(1 - \theta_E^F\right),$$

$$\Pi_H^F = \left(\theta_H^F(u_H - u_L) - v\theta_H^F\right)\left(\theta_E^F - \theta_H^F\right).$$

Proposition 3. *If $u_E - u_H - 3v \leq 0$,*

$$\theta_E^F = 0, \quad \theta_H^F = 0, \ p_E^F = v, \ p_H^F = 0, \ \Pi_E^F = v, \ \Pi_H^F = 0. \tag{8}$$

If $u_E - u_H - 3v > 0$,

- *if $u_H - u_L - v \leq 0$,*

$$\begin{cases} \theta_E^F = \frac{u_E - u_H - 3v}{2(u_E - u_H - 2v)}, \\ \theta_H^F = 0, \end{cases} \quad \begin{cases} p_E^F = \frac{u_E - u_H - v}{2}, \\ p_H^F = 0, \end{cases} \quad \begin{cases} \Pi_E^F = \frac{(u_E - u_H - v)^2}{4(u_E - u_H - 2v)}, \\ \Pi_H^F = 0; \end{cases} \tag{9}$$

- *if $u_H - u_L - v > 0$,*

$$\begin{cases} \theta_E^F = \frac{2(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v}, \\ \theta_H^F = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v}, \end{cases} \tag{10}$$

furthermore, we have

$$\theta_E^F - \theta_H^F = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v},$$

$$p_H^F = \frac{(u_E - u_H - 3v)(u_H - u_L - v)}{4u_E - 3u_H - u_L - 8v}, \ \Pi_H^F = \frac{(u_H - u_L - v)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2}.$$

Proof. See the Appendix.

$\square$

The customer capturing pattern in forward compatibility is exactly the same as in backward compatibility and all the explanations there apply here.

3.3   Two-way compatibility

$$N_H^T = \theta_E^T, \quad N_E^T = 1 - \theta_E^T, \quad N_L^T = \theta_E^T.$$

The entrant's profit is written as

$$\Pi_E^T = \left(\theta_E^T(u_E - u_H) - \theta_H^T(u_L - u_H) + v(1 - 2\theta_E^T)\right)\left(1 - \theta_E^T\right).$$

This is the same form as under forward compatibility. The profit of the upgrade is

$$\Pi_H^T = \theta_H^T(u_H - u_L)\left(\theta_E^T - \theta_H^T\right).$$

Proposition 4. *If $u_E - u_H - 3v \leq 0$,*

$$\theta_E^T = 0, \quad \theta_H^T = 0, \ p_E^T = v, \ p_H^T = 0, \ \Pi_E^T = v, \ \Pi_H^T = 0. \tag{11}$$

If $u_E - u_H - 3v > 0$,

$$\begin{cases} \theta_E^T = \frac{2(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v}, \\[2mm] \theta_H^T = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v}, \end{cases}$$

$$\theta_E^T - \theta_H^T = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v},$$

in addition, we have

$$p_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v}, \ \Pi_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2}.$$

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Proof. See the Appendix.

□

(11) and (4) are identical to (8) and (10) in forward compatibility. As in the other regimes, in the fist case where the entrant's product quality is not so superior to that of the upgrade, the entrant captures all customers by charging a low price. In the second case, the entrant's product quality is very high relative to the competitor's upgrade, the entrant sets a high price and only captures high-end customers who are sensitive to quality. The cut-off types in this case are the same as (10) under forward compatibility. However, the prices and profits are different. The reason is that the two compatibility regimes have distinct network effects.

4   Comparison of the regimes

In this section, we make comparison of the equilibria in the different regimes. Inspection of the propositions in the previous section immediately shows that when $u_E - u_H - 2v \leq 0$, the equilibria in each regime are identical as the ones of the first case of the propositions. In other words, when the entrant's product quality is relatively low to that of the upgrade, the equilibrium is the same across the compatibility regimes. In this section, we assume the following:

$$u_E - u_H - 2v > 0.$$

That is to say, we examine the case where the entrant's good has a large quality advantage, faced to the competitor.

Proposition 5.   • *The case of $u_E - u_H - 3v \leq 0$*

   − *If $u_H - u_L - 2v \leq 0$,*

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$$\theta_E^B - \theta_H^B = \frac{u_E - u_H - 2v}{2(u_E - u_H - v)} > \theta_E^F - \theta_H^F = \theta_E^T - \theta_H^T = 0;$$

$$1 - \theta_E^B = \frac{u_E - u_H}{2(u_E - u_H - v)} < 1 - \theta_E^F = 1 - \theta_E^T = 1. \qquad (12)$$

– If $u_H - u_L - 2v > 0$,

$$\theta_E^B - \theta_H^B = \frac{(u_E - u_H - 2v)(u_H - u_L)}{(4u_E - 3u_H - u_L - 6v)(u_H - u_L - v)} > \theta_E^F - \theta_H^F = \theta_E^T - \theta_H^T = 0;$$

$$1 - \theta_E^B = \frac{2u_E - u_H - u_L - 2v}{4u_E - 3u_H - u_L - 8v} < 1 - \theta_E^F = 1 - \theta_E^T = 1. \qquad (13)$$

• The case of $u_E - u_H - 3v > 0$

– If $u_H - u_L - 2v \le 0$ and $u_H - u_L - v \le 0$,

$$\theta_E^B - \theta_H^B = \frac{u_E - u_H - 2v}{2(u_E - u_H - v)} > \theta_E^F - \theta_H^F = \frac{u_E - u_H - 3v}{2(u_E - u_H - 2v)},$$

$$\theta_E^T - \theta_H^T = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v};$$

$$1 - \theta_E^B = \frac{u_E - u_H}{2(u_E - u_H - v)} < 1 - \theta_E^F = \frac{u_E - u_H - v}{2(u_E - u_H - 2v)},$$

$$1 - \theta_E^T = \frac{2u_E - u_H - u_L - 2v}{4u_E - 3u_H - u_L - 8v}. \qquad (14)$$

– If $u_H - u_L - 2v \le 0$ and $u_H - u_L - v > 0$,

$$\theta_E^B - \theta_H^B = \frac{u_E - u_H - 2v}{2(u_E - u_H - v)},$$

$$\theta_E^F - \theta_H^F = \theta_E^T - \theta_H^T = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v};$$

$$1 - \theta_E^B = \frac{u_E - u_H}{2(u_E - u_H - v)},$$

$$1 - \theta_E^F = 1 - \theta_E^T = \frac{2u_E - u_H - u_L - 2v}{4u_E - 3u_H - u_L - 8v}. \qquad (15)$$

- If $u_H - u_L - 2v > 0$,

$$\theta_E^B - \theta_H^B = \frac{(u_E - u_H - 2v)(u_H - u_L)}{(4u_E - 3u_H - u_L - 6v)(u_H - u_L - v)} >$$

$$\theta_E^F - \theta_H^F = \theta_E^T - \theta_H^T = \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v};$$

$$1 - \theta_E^B = \frac{2u_E - u_H - u_L - 2v}{4u_E - 3u_H - u_L - 6v} < 1 - \theta_E^F = 1 - \theta_E^T = \frac{2u_E - u_H - u_L - 2v}{4u_E - 3u_H - u_L - 8v}$$

$$(16)$$

Proof. From the propositions in the previous section, all follow easily except for $\theta_E^B - \theta_H^B > \theta_E^F - \theta_H^F$ in the case of $u_E - u_H - 3v > 0$ and $u_H - u_L - 2v > 0$. In this case, we have

$$\theta_E^B - \theta_H^B > \frac{u_E - u_H - 2v}{4u_E - 3u_H - u_L - 6v}.$$

It is easily shown that the right hand is larger than $\theta_E^F - \theta_H^F$.

$\square$

Backward compatibility adds to the utility of the new version with network effects. Customers are, therefore, more encouraged to switch to the new version from the old version. This is confirmed in the proposition except in (15).

On the other hand, forward compatibility increases the utility of the old version with network effects. As a result, forward compatibility may discourage consumers to

upgrade to the new good. Note, however, that it may be effective as an entry barrier. The customers who abstain from upgrading because utility provided by the upgrade is not large enough might be attracted by the entrant's product if this proffers higher utility. Forward compatibility dissuades these customers to switch to the entrant's by making the old version of more value. Consequently, the whole number of customers who stay with the incumbent whether with the old or the new may be larger than under backward compatibility. This is seen in the proposition as the value of $1 - \theta_i^j$. Also, when this dissuasion to buy the entrant's product is very large, forward compatibility might have a larger market share of the upgrade than backward compatibility(see (15)).

Two-way compatibility is a combination of the two above regimes. As such, in most cases, two-way compatibility has a smaller upgrade market share than backward compatibility since backward compatibility promotes the new version but forward compatibility makes it less attractive. The exceptions are the cases of (14) and (15). In these cases, forward compatibility is so effective as an entry barrier that two-way compatibility might have a larger market share of the upgrade than backward compatibility.

Proposition 6. • *The case of $u_E - u_H - 3v \leq 0$*

  − *If $u_H - u_L - 2v \leq 0$,*

$$p_H^B = p_H^F = p_H^T = 0.$$

  − *If $u_H - u_L - 2v > 0$,*

$$p_H^B = \frac{(u_E - u_H - 2v)(u_H - u_L)}{4u_E - 3u_H - u_L - 6v} > p_H^F = p_H^T = 0.$$

• *The case of $u_E - u_H - 3v > 0$*

  − *If $u_H - u_L - 2v \leq 0$ and $u_H - u_L - v \leq 0$,*

$$p_H^B = p_H^F = 0 < p_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v}.$$

– If $u_H - u_L - 2v \leq 0$ and $u_H - u_L - v > 0$,

$$p_H^B = 0 <$$
$$p_H^F = \frac{(u_H - u_L - v)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v} <$$
$$p_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v}.$$

– If $u_H - u_L - 2v > 0$,

$$p_H^B = \frac{(u_H - u_L)(u_E - u_H - 2v)}{4u_E - 3u_H - u_L - 6v} < p_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v};$$
$$p_H^F = \frac{(u_H - u_L - v)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v} < p_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v}.$$

Proof. From the value of $\theta_E - \theta_H$ in the previous proposition, (3) and (4), the results follow.

$\square$

Backward compatibility enhances utility of the new version. As a result, under this compatibility, it is expected that the incumbent can price the upgrade higher (see the case of $u_E - u_H - 3v \leq 0$). On the other hand, as stated before, forward compatibility might be effective as an entry prevention device. Likewise, two-way compatibility comprises a built-in counter entry device(forward compatibility). If the price under backward compatibility were as high as the two-way compatibility price, many of customers who abstain from buying the upgrade might go to purchase the entrant's good. Lacking the counter entry device, backward compatibility only can set a lower price to compete against entry–the case of $u_E - u_H - 3v > 0$.

Proposition 7. ● *The case of $u_E - u_H - 3v \leq 0$*

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– If $u_H - u_L - 2v \leq 0$,

$$\Pi_H^B = \Pi_H^F = \Pi_H^T = 0.$$

– If $u_H - u_L - 2v > 0$,

$$\Pi_H^B = \frac{(u_H - u_L)^2(u_E - u_H - 2v)^2}{(4u_E - 3u_H - u_L - 6v)^2(u_H - u_L - v)} > \Pi_H^F = \Pi_H^T = 0.$$

• *The case of $u_E - u_H - 3v > 0$*

  – If $u_H - u_L - 2v \leq 0$ and $u_H - u_L - v \leq 0$,

$$\Pi_H^B = \Pi_H^F = 0 < \Pi_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2}. \tag{17}$$

  – If $u_H - u_L - 2v \leq 0$ and $u_H - u_L - v > 0$,

$$\Pi_H^B = 0 < \Pi_H^F = \frac{(u_H - u_L - v)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2} <$$
$$\Pi_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2}. \tag{18}$$

  – If $u_H - u_L - 2v > 0$,

$$\Pi_H^F = \frac{(u_H - u_L - v)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2} < \Pi_H^B = \frac{(u_H - u_L)^2(u_E - u_H - 2v)^2}{(4u_E - 3u_H - u_L - 6v)^2(u_H - u_L - v)};$$
$$\Pi_H^F = \frac{(u_H - u_L - v)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2} < \Pi_H^T = \frac{(u_H - u_L)(u_E - u_H - 3v)^2}{(4u_E - 3u_H - u_L - 8v)^2}.$$

Proof. From the previous two propositions and (2), the results follow except for $\Pi_H^F < \Pi_H^B$ in the last case. For this, if we pose $u_E - u_H - 2v$, $4u_E - 3u_H - u_L - 6v$, $u_H - u_L$ as x, y, z, $\Pi_H^F < \Pi_H^B$ can be written as:

$$x(y - 2v)z > (x - v)y(z - v) \Longleftrightarrow 0 < zv(y - 2x) + yv(x - v).$$

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This is true since $(y - 2x) > 0$.

$\square$

It can be seen that the effect of the promotion of the upgrade under backward compatibility hinges upon the relative magnitude of the upgrade utility: $u_H - u_L - 2v$. If u_H is large enough, backward compatibility boosts the total utility of the upgrade with network effects and makes more profit than forward compatibility.

In the case of (18), the entry prevention device of forward compatibility is very effective and this keeps network effects even for the upgrade high. In consequence, the profit under forward compatibility is higher than under backward compatibility.

From the viewpoint of keeping customers from switching to the entrant, it does not matter whether customers stay with the old or the new version. In either case, they are the incumbent's customers. However, forward compatibility might discourage consumers from purchasing the upgrade by increasing the value of the old version. Two-way compatibility has a built-in entry prevention (forward compatibility) and upgrade promotion device (backward compatibility). In consequence, two-way compatibility does always better than forward compatibility.

Corollary 1. *If $u_E - u_H - 3v > 0$ and $u_H - u_L - 2v > 0$,*

$\Pi_H^B < \Pi_H^T$ *holds if and only if*

$$0 < 2(u_H - u_L)(u_E - u_H - 2v) - (u_E - u_L - 3v)(4u_E - 3u_H - u_L - 6v).$$

Proof. Pose $u_H - u_L$, $u_E - u_H - 2v$, $4u_E - 3u_H - u_L - 6v$ as x, y, z. Then, we have

$$\frac{(x - v)(y - v)^2}{(z - 2v)^2} < \Pi_H^T.$$

By putting the left hand as X, we obtain that $X - \Pi_H^B$ is equivalent to

$$0 < 2xy - (x + y - v)z.$$

$\square$

Backward compatibility promotes the sale of the upgrade whereas forward compatibility hampers the entrant but the upgrade as well. Forward compatibility has positive and negative effects for the incumbent. (17), (18) and the corollary show that the positive effects under two-way compatibility can be more significant than the negative ones. In most practical cases of common software such as the word processor, the firm offers two-way compatibility except for an occasion of a significant upgrade or fundamental technological change. By contrast, academic literature sometimes argues that backward compatibility does the firm better by promoting the upgrade and thus bringing more profits. Until now, this discrepancy seems to have been explained by the fact that the firm is afraid to suffer bad reputation by cutting suddenly compatibility between the two consecutive versions of a product. We have theoretically shown that two-way compatibility can be more profitable than backward compatibility. We have also shown that contrary to conventional belief, forward compatibility has an advantage for the firm compared to backward compatibility.

5   Conclusion

The present paper investigates the characteristics of each compatibility in the presence of the entrant. We have studied how a compatibility regime captures customers in competition with the entrant. In particular, we have elucidated how a compatibility regime uses its network effects to ward off the competitor. We have also taken a close look at when a particular regime manifests its advantage in competition and makes the largest profit among the other regimes.

To conclude, we list some issues deserving further study. Sometimes, the firm might

find it in its interest to sell the old version along with the new version in the second period. In practice, the withdrawal of the old product from the market is the firm's strategic decision. The sale of the old version may serve as an entry prevention device if it is sold at a low price along with the new version in the second period.

The paper abstracted from inter-temporal decision. In our model, the consumer population was fixed. In practice, however, it is sensible to think that new consumers arrive in the market over time.

In the dynamic situation, early pricing decision closely interacts with later pricing decision. In such a context, price discrimination between returning and new customers would be also the firm's strategic decision.

6 Appendix

6.1 Proof of Proposition 2

We maximize both the profits with respect to the cut off types. If we differentiate Π_E^B and Π_H^B in θ_E^B and θ_H^B respectively, we have

$$\frac{\partial \Pi_E^B}{\partial \theta_E^B} = -2\theta_E^B \left(u_E - u_H - v\right) + u_E - u_H - 2v + \theta_H^B \left(u_L - u_H + v\right),$$

$$\frac{\partial \Pi_H^B}{\partial \theta_H^B} = -2\theta_H^B \left(u_H - u_L - v\right) + \theta_E^B \left(u_H - u_L - 2v\right).$$

(i) The case of $u_H - u_L - v \leq 0$

Suppose that $u_H - u_L - v \leq 0$ and then from the second equation, Π_H^B is convex. We have

$$\frac{\partial \Pi_H^B}{\partial \theta_H^B}\big|_{\theta_H^B = \theta_E^B} = -\theta_E^B(u_H - u_L) \leq 0.$$

We obtain from this $\theta_H^B = 0$.

- $u_E - u_H - v \leq 0$

Then, from the second equation, it follows that Π_E^B is convex and that

$$\frac{\partial \Pi_E^B}{\partial \theta_E^B}\big|_{\theta_E^B=1} = -(u_E - u_H) < 0.$$

We conclude that $\theta_E^B = 0$.

- $u_E - u_H - v > 0$

  In this case, Π_E^B is concave and the second equation leads to

$$\theta_E^B = \begin{cases} 0 & \text{if } u_E - u_H - 2v \le 0, \\ \\ \frac{u_E - u_H - 2v}{2(u_E - u_H - v)} & \text{if } u_E - u_H - 2v > 0. \end{cases}$$

(ii) The case of $u_H - u_L - v > 0$

In this case, Π_H^B is concave and the second equation leads to

$$\theta_H^B = \begin{cases} 0 & \text{if } u_H - u_L - 2v \le 0, \\ \\ \frac{\theta_E^B(u_H - u_L - 2v)}{2(u_H - u_L - v)} & \text{if } u_H - u_L - 2v > 0. \end{cases} \tag{19}$$

Let us turn to the first equation.

(1) If $u_H - u_L - 2v \le 0$

- If $u_E - u_H - v \le 0$, Π_E^B is convex and

$$\theta_E^B = 0.$$

- If $u_E - u_H - v > 0$, Π_E^B is concave and

$$\theta_E^B = \begin{cases} 0 & \text{if } u_E - u_H - 2v \le 0, \\ \\ \frac{u_E - u_H - 2v}{2(u_E - u_H - v)} & \text{if } u_E - u_H - 2v > 0. \end{cases}$$

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(2) If $u_H - u_L - 2v > 0$

- If $u_E - u_H - v \leq 0$, Π_E^B is convex and

$$\theta_E^B = 0.$$

- If $u_E - u_H - v > 0$, Π_E^B is concave and

$$\theta_E^B = \begin{cases} 0 & \text{if } u_E - u_H - 2v \leq 0, \\ \frac{u_E - u_H - 2v + \theta_H^B(u_L - u_H + v)}{2(u_E - u_H - v)} & \text{if } u_E - u_H - 2v > 0. \end{cases}$$

We obtain, combining this with (19),

$$\theta_E^B = \begin{cases} 0 & \text{if } u_E - u_H - 2v \leq 0, \\ \frac{2(u_E - u_H - 2v)}{4u_E - 3u_H - u_L - 6v} & \text{if } u_E - u_H - 2v > 0, \end{cases}$$

and also

$$\theta_H^B = \begin{cases} 0 & \text{if } u_E - u_H - 2v \leq 0, \\ \frac{(u_E - u_H - 2v)(u_H - u_L - 2v)}{(4u_E - 3u_H - u_L - 6v)(u_H - u_L - v)} & \text{if } u_E - u_H - 2v > 0. \end{cases}$$

If we rearrange all the cases, the proposition follows.

6.2   Proof of Proposition 3

The first order conditions are

$$\frac{\partial \Pi_E^F}{\partial \theta_E^F} = -2\theta_E^F (u_E - u_H - 2v) + u_E - u_H - 3v + \theta_H^F (u_L - u_H),$$

$$\frac{\partial \Pi_H^F}{\partial \theta_H^F} = \left(-2\theta_H^F + \theta_E^F\right)(u_H - u_L - v).$$

(i) The case of $u_H - u_L - v \leq 0$

Then, Π_H^F is convex and it follows that

$$\frac{\partial \Pi_H^F}{\partial \theta_H^F}|_{\theta_H^F = \theta_E^F} = -\theta_E^F (u_H - u_L - v) \geq 0.$$

We conclude that $\theta_H^F = 0$.

(1) If $u_E - u_H - 2v \leq 0$

Then, Π_E^F is convex and

$$\frac{\partial \Pi_E^F}{\partial \theta_E^F}|_{\theta_E^F = 0} = u_E - u_H - 3v < 0.$$

We obtain that $\theta_E^F = 0$.

(2) If $u_E - u_H - 2v > 0$

Then, Π_E^F is concave and we obtain from $\frac{\partial \Pi_E^F}{\partial \theta_E^F}$

$$\theta_E^F = \begin{cases} 0 & \text{if } u_E - u_H - 3v \leq 0, \\ \\ \frac{u_E - u_H - 3v}{2(u_E - u_H - 2v)} & \text{if } u_E - u_H - 3v > 0. \end{cases}$$

(ii) The case of $u_H - u_L - v > 0$

Then, Π_H^F is concave and we have from $\frac{\partial \Pi_H^F}{\partial \theta_H^F}$

$$-2\theta_H^F + \theta_E^F = 0. \tag{20}$$

(1) If $u_E - u_H - 2v \leq 0$

Then, Π_E^F is convex and we obtain

$$\theta_E^F = 0.$$

From (20), we also have $\theta_H^F = 0$.

(2) If $u_E - u_H - 2v > 0$

Then, Π_E^F is concave and we obtain from $\frac{\partial \Pi_E^F}{\partial \theta_E^F}$

$$\theta_E^F = \begin{cases} 0 & \text{if } u_E - u_H - 3v \leq 0, \\[2ex] \frac{2(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v} & \text{if } u_E - u_H - 3v > 0, \end{cases}$$

and also from (20)

$$\theta_H^F = \begin{cases} 0 & \text{if } u_E - u_H - 3v \leq 0, \\[2ex] \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v} & \text{if } u_E - u_H - 3v > 0. \end{cases}$$

Rearrangement of all the cases lead to the proposition.

6.3 Proof of Proposition 4

The derivatives of the profits are the following:

$$\frac{\partial \Pi_E^T}{\partial \theta_E^T} = -2\theta_E^T (u_E - u_H - 2v) + u_E - u_H - 3v + \theta_H^T (u_L - u_H),$$
$$\frac{\partial \Pi_H^T}{\partial \theta_H^T} = (u_H - u_L)\left(\theta_E^T - 2\theta_H^T\right).$$

From the second equation it follows that Π_H^T is concave and that $\theta_E^T - 2\theta_H^T = 0$.

If $u_E - u_H - 2v \leq 0$, Π_E^T is convex and that $\theta_E^T = \theta_H^T = 0$.

If $u_E - u_H - 2v > 0$, Π_E^T is concave and from the first equation we have

$$\theta_E^T = \begin{cases} 0 & \text{if } u_E - u_H - 3v \leq 0, \\[2ex] \frac{2(u_E - u_H - 3v)}{4u_E - 3u_H - u_L - 8v} & \text{if } u_E - u_H - 3v > 0, \end{cases}$$

and also

$$\theta_H^T = \begin{cases} 0 & \text{if } u_E - u_H - 3v \leq 0, \\[2ex] \frac{u_E - u_H - 3v}{4u_E - 3u_H - u_L - 8v} & \text{if } u_E - u_H - 3v > 0. \end{cases}$$

The proposition obtains if we rearrange all the cases.

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