Firms' Optimal Digital Rights Management (DRM) Strategies: The Effects of Public Copy Protection and DRM Compatibility

By

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Abstract

The purpose of this paper is to investigate how different types of demand structure affect firms' optimal pricing and private copy protection (DRM) level of digital products. We construct a model where the equilibrium prices and protection levels are dependent upon the two parameters: public copy protection of digital products and degree of compatibility of DRM in terms of hacking technology. We show that the optimal levels of DRM and prices are determined by different types of demand structure depending on the strategic interaction between the firms. As a result, the effects of an increase in public copy protection and less compatible hacking technology on the optimal levels of DRM, prices, quantity demanded, and profits have direct and indirect channels of which the latter is induced by the change of DRM. It is shown that total effects are dependent upon the demand structure and the strategic nature of DRM. The paper also discusses results of social welfare analysis.

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1. Introduction

In the last decade or so, there has been a surge in consumers' usage of the Internet and digital products, such as CDs, DVDs, iPods, downloadable software, e-books, and the like. Since digitalization and the Internet make (illegal) copying easier, content providers have used protection technologies to protect their digital products from illegal copying. These protection technologies, such as encryption and copy controls, are collectively termed "digital rights management" (DRM) (Park and Scotchmer, 2005). Consumers, however, have financial incentives to acquire digital products through illegal copying, and recent advances in technologies have made this process easier. Facing this threat of piracy, content providers have ratcheted up their protection, which irk honest consumers because high level of DRM restricts usage of digital products that consumers acquire legally (Wingfield and Smith, 2007). As such, piracy and DRM have been controversial issues for content vendors of digital products and consumers alike.

Piracy and DRM have been topics of intense debate in the industries (e.g., music, movie, and software industries) that are affected by actual and potential loss due to piracy. The most intense debate has occurred in the music industry, which has been affected by piracy more than any other industries. In a recent online essay, Steve Jobs, CEO of Apple, contends that the major music companies should consider allowing content distributors to sell songs without DRM software (Jobs, 2007). He argues that the current DRM system does not prevent piracy effectively and abandoning DRM would spur growth in the overall music industry. While it is not clear whether the major music companies will follow his recommendation, his essay clearly shows how controversial and significant piracy and associated DRM have become in the music industry.

The movie industry has been affected greatly by piracy as well. According to a study conducted by LEK Consulting, in 2005 the worldwide movie industry lost $18.2 billion as a result of piracy and U.S. movie studios lost about $6.1 billion to piracy worldwide (McBride and Fowler, 2006). Sixty-two percent of the $6.1 billion losses in the U.S. resulted from piracy of hard goods, such as DVDs, and 38% from Internet piracy, which has increased rapidly in recent years. Major U.S. movie studios take measures to deal with losses due to piracy by embedding copy-protection software on their DVD products, and by lobbying the U.S. government to pressure governments in piracy-rampant countries to crack down on piracy more aggressively (King, 2007).

Piracy and DRM have also drawn substantial attention from academia. Early research on piracy focused on photocopying and addressed the issue of how publishers can appropriate some of their lost revenues from copied products (e.g., Liebowitz, 1985). Later research turned to copyright issues and examined how copyright protection affects the level of piracy, pricing, development incentives, and social welfare (e.g., Bae and Choi, 2006; Besen and Raskind, 1991). Most of this research, however, focuses on monopoly cases, and only a handful of studies address duopoly cases (Belleflamme and Picard, 2007; Johnson, 1985; Park and Scotchmer, 2005). Given the observation that only a small number of large companies dominate the industries with digital products, analysis of duopoly is more realistic than that of monopoly. Even, those studies which consider duopoly, do not adequately reflect the reality of the industries. Belleflamme and Picard (2007) and Park and Scotchmer (2005) model duopoly settings focusing only on pricing, rather than addressing both DRM and pricing for digital goods providers. Since DRM is one of the central issues in piracy, it makes more sense to treat DRM as a decision variable, rather than a parameter. They also assume that all consumers consume either one or two products uniformly, while, in reality, some consumers might consume only one product and other consumers might consume two products. And, Johnson (1985) analyzes information goods producers who are price-takers, while digital goods producers are price-setters in reality.

This paper addresses all of the issues mentioned above. It analyzes two competitors that offer similar, but not necessarily substitute, digital products. It also examines markets where some consumers prefer using only one product and other consumers prefer using two products, which is more in line with the reality. As a focal point, this paper discusses optimal DRM levels as well as optimal pricing schemes for companies faced with disparate demand structures. To the best of our knowledge, this paper is the first study that addresses both optimal DRM levels and pricing in a duopoly setting.

The purpose of this paper is to investigate how different types of demand structure affect firms' competitive behavior in terms of their choice of DRM level and pricing of digital products. Specifically, utilizing Hotelling's linear city model with two sellers[1] (or content providers) at the ends, this paper develops a model that identifies three different cases of demand structure,

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[1] In this paper, we use content providers, sellers, and firms interchangeably. We define these concepts broadly to include companies that produce digital products (e.g., music companies and movie studios), online and offline retailers, and rental stores. Also, we do not differentiate "producers" of digital products from "sellers" of digital products.

analyzes how the sellers compete in the market, and how they cope with pirated products preferred by some consumers. In the model, the equilibrium prices and protection levels are dependent upon the two parameters: public copy protection of the digital products and the degree of compatibility of DRM for the digital products. The results show that when some consumers buy only one legitimate digital product and the other consumers use unauthorized copies of both digital products, the sellers regard their individual protection levels as strategic complements. On the other hand, when some consumers buy only one legitimate digital product and other consumers buy one legitimate digital product and use one pirated product, and the other consumers use pirated copies of both digital products, the sellers regard their individual protection levels as strategic substitutes.[2] In equilibrium, as public copy protection increases, the directions of changes in the optimal DRM levels, prices, and the demands for the two legitimate products depend on particular demand structures and the strategic nature of DRM. For instance, when the content providers consider their DRM levels as strategic substitutes [complements], the effects of stronger public protection result in lower [higher] levels of private protection. This has the important implication that policymakers should be careful in evaluating the effectiveness of public copy protection. However, an increase in the degree of DRM compatibility results in increases in the optimal DRM levels, prices, and the demand for the two legitimate products, although the magnitudes of the increases depend on the strategic nature of DRM. Moreover, these effects are shown to have direct and indirect channels, of which the latter are induced by the fact that the changes in the DRM level, in turn, affect other equilibrium variables.

In addition, we perform a welfare analysis under each regime and show that the effects of strengthening copy protection and less compatible copy technology on social welfare depend on the various factors. They increase gross copy cost for consumer who continue to copy either one or both of the products, induce demand switches between legal and illegal copies, change total usage of the illegal product, and affect total production costs of DRM. As expected from the results of comparative statics in section 3, the total effects of these various factors on social welfare are ambiguous since the change of the optimal private protection level responding to the marginal changes in consumers' incentive to copy is dependent upon the strategic nature of DRM under different regimes and different policy measures as well.

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[2] For strategic complements and substitutes, see Bulow et al. (1985).

The remainder of this paper is organized as follows: In Section 2, we develop models with disparate demand structures and derive equilibrium solutions in each case. In the next section, we discuss comparative static results in detail based on the analysis conducted in the previous section. In Section 4, we perform a welfare analysis. The paper concludes with some remarks and number of future research.

2. Model

A. Overview of the Model

The model in this paper is based on Hotelling's linear city model. The number of consumers is normalized to 1, and they are uniformly distributed on the unit interval [0,1]. Firm 1 is located at point 0, and firm 2 is located at point 1. Firm 1 sells a digital product, product 1, and firm 2 sells another digital product, product 2 to consumers. These digital products are horizontally differentiated perfectly, so they are not substitutes for each other. The two products are assumed to contain quite different contents, so that if a consumer likes to consume one product, then the consumer does not like to consume the other product in general. In other words, the firms have local monopoly power over consumers nearby. Those consumers in the middle of the market do not value the two products enough to buy a legitimate copy of any of the products. Hence, the market is "not covered" with legitimate copies of the two products. Two example products might be rap music and classical music. In general, consumers who are fans of rap music do not enjoy classical music as much as they enjoy rap music, and vice versa. And, consumers who love country music would not buy rap or classical music because they do not value those types of music enough.

The two products are "piratable," i.e., they are imperfectly protected. Therefore, there are always some consumers who want to and are able to make illegal copies of the products, depending on their valuation (or their maximum willingness to pay) of the products, prices, and protection levels.

In the model, consumers are not required to purchase one or two products uniformly. Based on their valuation of the products, prices, transaction costs, and availability of illegal copies, consumers may decide to (1) buy one legitimate product and obtain one illegal copy of the other product, (2) purchase only one legitimate product, (3) obtain only one illegal copy of a product, or (4) obtain copies of the two products. Consumers receive the same amount of valua-

tion (i.e., $v > 0$) from consuming product 1 or 2. The valuation (v) is the utility from consuming either (legitimate) product 1 or 2 within the usage that the products' DRM permits and is assumed the same across consumers. Also, consumers suffer disutility (e.g., transportation cost) from choosing a variant that differs from their ideal. Depending upon their location (x_i) in the interval, consumers incur disutility of tx_i when they acquire a legitimate or illegal copy of product 1 and transaction costs of $t(1 - x_i)$ for product 2.

Consumers value a legitimate copy and an illegal copy of product 1 or 2 differently, and this is captured by α, where $0 < \alpha < 1$. When a consumer located at x_i purchases a legitimate copy of product 1, the consumer's net utility is $v - tx_i$. In the case of an illegal copy, however, the consumer's net utility becomes $(1 - \alpha)(v - tx_i) + \alpha \cdot 0$ because the consumer can be caught by the government (or the governing authority) with a probability of α and can avoid being caught with a probability of $(1 - \alpha)$. When the consumer is caught, her net utility is assumed to be zero for simplicity.[3] As such, α measures the intensity of public copy protection by the government. In addition to a possibility of being caught, consumers also face reproduction cost (e) when making an illegal reproduction (Yoon, 2002). The reproduction cost includes the physical cost (e.g., CDs to hold illegally copied songs) and the hacking cost to hack the DRM system of a digital product. Since the physical cost is currently close to negligible, the reproduction cost generally means the hacking cost. In our model, the reproduction cost is determined by the sellers of the digital goods. The rationale is that if a seller sets a high level of DRM for its product, the reproduction cost will also high because a hacker needs to make more efforts to hack the DRM system.[4] We assume that there is a synergy effect in hacking digital products. That is, if a hacker succeeds in hacking one digital product, then the hacker can hack the other product with a lower reproduction cost than that for the first product. This synergy effect is

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[3] Png and Chen (1999) use a similar interpretation of α as the content publisher's monitoring rate against potential copiers. On the other hand, other literature considers α to be a degradation rate which is a quality difference between an original content and an illegal copy (Bae and Choi, 2006; Hui and Png, 2003). Since making a copy of digital content does not generate quality degradation in most cases, we follow the interpretation in Png and Chen (1999).
[4] A typical DRM is an encryption program to control access to or usage of digital content. DRM used in downloaded songs for instance has different methods to restrict access or usage. They can limit the number of times a song can be played, limit the number of computers to store songs, decide to make it iPod compatible, and whether or not it can be burned to CD. We assume here that the more complex control in DRM, the more costly to hack into.

captured by β, where $\frac{1}{2} \leq \beta \leq 1$. More formally, β measures the compatibility of DRM systems between the digital products in terms of hacking technology.

The two firms compete in a two-stage game. In the first stage, each firm decides the optimal protection level for its digital product. Then in the second stage, each firm chooses the price that maximizes its profits, given the protection levels set by the two firms in the first stage. Like most sequential games, we derive equilibrium outcomes by solving backward.

Let $u(x_i;(A_1, A_2))$ denote the gross utility for consumer i who is located at x_i with choices of acquiring product 1 and 2. A_1 is consumer i's choice of acquiring product 1 with $A = \{B, C, O\}$ where $B, C,$ and O are abbreviations of buy, copy and no use, respectively. For example, $u(\frac{1}{2};(B, C))$ denotes the gross utility of the consumer located at $\frac{1}{2}$ who buys product 1 and makes an illegal copy of product 2. Let p_1 denote the price for a legitimate copy of product 1 and e_1 denote the reproduction cost of product 1. Then, we can derive the gross utilities for all combinations of consumption choices of product 1 and 2, which are shown in Table 1.

Insert Table 1 about here.

B. Demand Structures

Before we describe the demand structures, we first define notations witch will be used intensively throughout the paper. Define $\hat{x}(B,O)$ as the x intercept of $u(x_i;(B,O))$ and define $\hat{x}(C,O)$, $\hat{x}(O,B)$, and $\hat{x}(O,C)$ in the same way. Also, let $u\left(\tilde{x}_i\left((A_1, A_2),(A_1^{'}, A_2^{'})\right)\right)$ denote the gross utility of the consumer whose gross utility is the same between the product choices of (A_1, A_2) and $(A_1^{'}, A_2^{'})$. To have a more meaningful analysis, we eliminate cases, in which either (B,B) or (C,C) dominates all other choices. To serve this purpose, we have the following two assumptions:

Assumption 1: $p_1 + p_2 + t > 2v$

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Assumption 2: $\hat{x}(O,B) < \hat{x}(C,O)$ and $\hat{x}(O,C) < \hat{x}(B,O)$.

Assumption 1 describes the condition where the consumer who would be indifferent between product 1 and 2 does not buy both products and thus the market is not covered (Tirole, 1988). In other words, each firm has local monopoly power. Assumption 2 is used to ensure interactions between firm 1 and 2 in the regimes we consider. Cases where Assumption 2 is not met become either monopoly situations or one of the three regimes addressed in this paper (i.e., 'no strategic interaction regime,' as termed later in the paper). Since monopoly is not a focus in this paper, those cases with monopoly situations are eliminated (see Bae and Choi (2006) for detailed examination on monopoly and piracy). The rest of the cases where Assumption 2 is not met basically entail the same analysis and results as those for the no strategic interaction regime. Hence, such cases are not considered in the following analysis.

Our model consists of three regimes, each characterized by a different demand structure. In the first regime, which we term "strategic substitute regime," some consumers buy only one product (either product 1 or 2), other consumers buy one product and obtain an illegal copy of the other product, and the rest of the consumers obtain illegal copies of both products. The demand structure of this regime is shown in Figure 1 (a). The lines in the figure represent utilities from the product bundles (e.g., $(B,O),(B,C),(C,C)$, etc.) for all consumers. The consumer located at x_i chooses the product bundle that provides the highest utility. Therefore, the market in this regime is divided into five consumer segments that prefer the following product bundles: $(B,O),(B,C),(C,C),(C,B)$, and (O,B). In the second regime, which we term "strategic complement regime," there are two types of consumers, i.e., consumers who buy only one product (either product 1 or 2) and consumers who obtain pirated copies of both products. In the third regime, which we term "no strategic interaction regime," some consumers buy only one product, other consumers obtain only one pirated copy of either product 1 or 2, and the rest of the consumers obtain pirated copies of both products. The demand structures of the second and third regimes are illustrated in Figure 1 (b) and (c), respectively.

Insert Table 2 and 3 about here.

<u>Case Classification</u>

To calculate the demand for product 1 (focusing on the vicinity of firm 1), we need to know how many consumers would buy a legal copy of product 1 in each regime, which will determine the demand for product 1. For such consumers, their gross utility must satisfy the following condition:

$$\left(v - tx_i - p_1\right) + \max\left\{\left(1-\alpha\right)\left(v - t\left(1-x_i\right)\right) - e_2, 0\right\} \geq \max\left\{u\left(x_i;(C,C)\right), u\left(x_i;(C,O)\right)\right\} \quad (1)$$

When the price of product 1 is low enough, there are some consumers who prefer (B, C) to (C, C) even with the synergy effect of hacking technology. Then, the above inequality becomes

$$\left(v - tx_i - p_1\right) + \left(1-\alpha\right)\left(v - t\left(1-x_i\right)\right) - e_2 \geq u\left(x_i;(C,C)\right), \quad (2)$$

which is equivalent to Constraint 1A (C 1A, hereafter) in Table 2 for those consumers. If C 1A holds, this means that the utility of a consumer who is indifferent between *(B, O)* and *(B, C)* is greater than that from *(C, C)*. This constraint provides possibilities that some consumers prefer *(B, C)* to *(B, O)* and *(C, C)*.

On the other hand, as the price of product 1 increases, the choice, *(B, C)*, is dominated by *(C, C)*, which describes Constraint 1B (C 1B, hereafter). If C 1B holds, the demand structure depends on whether some consumers prefer *(C, O)* to *(C, C)*. When the utility of a consumer who is indifferent between *(B, O)* and *(C, O)* is less than, or equal to, that from *(C, C)*, which is referred as Constraint 2A (C 2A, hereafter), there are no consumers who prefer *(C, O)* to any other alternatives. In other words, C 2A eliminates the possibility of a market for *(C, O)*. Since consumers do not choose *(B, C)* and *(C, O)* in this case, they have two options, *(B, O)* or *(C, C)*, to choose for consumption. For the consumer who is indifferent between *(B, O)* and *(C, O)*, the following inequality holds:

$$\left(v - tx_i - p_1\right) \leq u\left(x_i;(C,C)\right). \quad (3)$$

If the opposite case of C 2A holds, which is referred to as Constraint 2B (C 2B, hereafter), some consumers prefer *(C, O)* to *(B, O)* and *(C, C)*. In this case, the following inequality holds for the consumer who is indifferent between *(B, O)* and *(C, O)*:

$$\left(v - tx_i - p_1\right) > u\left(x_i;(C,C)\right). \quad (4)$$

The three regimes in the model are determined depending on which constraints in Table 2, Table 3, and Figure 1 hold. Focusing on the vicinity of firm 1, C 1A and C 1B determine whether a regime is a strategic substitute regime or not. And, C 2A and C 2B indicate whether a regime falls into a strategic complement regime or a no strategic interaction regime. We can make the same, parallel arguments regarding the vicinity of firm 2, and the corresponding constraints are shown in Table 3.

Strategic Substitute Regime[5]

In this regime (based on Figure 1 (a)), those consumers who buy product 1 purchase one of the following bundles: *(B, O)* or *(B, C)*. Hence, we can derive the demand for product 1 by identifying the consumer who is indifferent between *(B, C)* and *(C, C)*. By setting the gross utilities of (B,C) and (C,C) equal and solving the expression with respect to x_i, we obtain the following demand for product 1 (q_1):

$$q_1 = \frac{1}{\alpha t}\left[\beta e_1 - (1-\beta)e_2 - p_1 + \alpha v\right]. \tag{5}$$

By using C 1A′ and the corresponding gross utilities, we can derive the following demand for product 2 (q_2):

$$q_2 = \frac{1}{\alpha t}\left[\beta e_2 - (1-\beta)e_1 - p_2 + \alpha v\right]. \tag{6}$$

When C 1A holds, some consumers prefer to make illegal copies of both products if they do not have a strong preference for either of the products. Specifically, under this strategic substitute regime (SS regime, hereafter), the market is divided into five segments: 1) consumers who buy a legitimate copy of product 1 only; 2) consumers who buy a legitimate copy of product 1 and obtain an illegal copy of product 2; 3) consumers who buy illegal copies of both product 1 and 2; 4) consumers who obtain an illegal copy of product 1 and buy a legitimate copy of product 2; and 5) consumers who buy a legitimate copy of product 2 only. Figure 1 (a) illustrates this regime.

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[5] In this regime, when $\beta = 1$, the analysis becomes the same as that in No Strategic Interaction Regime. Thus, the range for β is changed to $\frac{1}{2} \leq \beta < 1$ for this regime.

To derive a specific condition for C 1A, we can rewrite C 1A as follows in terms of prices and DRM levels:

C 1A: $\alpha(2v-t) + \beta(e_1+e_2) - \dfrac{1}{1-\alpha}e_2 > p_1 \Rightarrow \Gamma_1 > p_1$,

where $\Gamma_1 = \alpha(2v-t) + \beta(e_1+e_2) - \dfrac{1}{1-\alpha}e_2$. (7)

Strategic Complement Regime

If C 1B and C 2A are satisfied, a different demand structure emerges. In this strategic complement regime (SC regime, hereafter), we have a case where dual piracy becomes a more attractive option to acquire both products, and consumers' product choices are simpler than in the SS regime. Specifically, consumers in the entire market are segmented into the following three groups: 1) consumers who buy a legitimate copy of product 1 only; 2) consumers who obtain illegal copies of both product 1 and 2; and 3) consumers who buy a legitimate copy of product 2 only. This case is illustrated in Figure 1 (b).

In this regime, the conditions, C 1B and C 2A can be expressed as follows:

C 1B: $\alpha(2v-t) + \beta(e_1+e_2) - \dfrac{1}{1-\alpha}e_2 \leq p_1 \Rightarrow \Gamma_1 \leq p_1$, (8)

C 2A: $\alpha(2v-t) - \dfrac{\alpha\beta}{1-\alpha}(e_1+e_2) + \dfrac{1}{1-\alpha}e_1 \geq p_1 \Rightarrow \Gamma_2 \geq p_1$,

where $\Gamma_2 = \alpha(2v-t) - \dfrac{\alpha\beta}{1-\alpha}(e_1+e_2) + \dfrac{1}{1-\alpha}e_1$. (9)

Since $\dfrac{1}{2} \leq \beta \leq 1$, we know that $\Gamma_1 \leq \Gamma_2$. Thus, the binding constraint for p_1 is $\Gamma_1 \leq p_1 \leq \Gamma_2$.

Given this constraint, we can derive the demand for product 1 using the same procedure in the

SS regime. In this regime, the critical consumer for determining the demand for product 1 is the consumer who is indifferent between *(B, O)* and *(C, C)*. Therefore, by setting the gross utilities of (B,O) and (C,C) equal and solving it with respect to x_i, we can derive the specific demand for product 1 as follows:

$$q_1 = \frac{1}{t}\left[\beta(e_1 + e_2) - p_1 + t - v - \alpha t + 2\alpha v\right]. \tag{10}$$

By using C 1B′, C 2A′ and the corresponding gross utilities, we can derive the following demand for product 2:

$$q_2 = \frac{1}{t}\left[\beta(e_1 + e_2) - p_2 + t - v - \alpha t + 2\alpha v\right]. \tag{11}$$

No Strategic Interaction Regime

When C 1B and C 2B hold, another regime is formed. In this no strategic interaction regime (NSI regime, hereafter), the market is divided into five different segments: 1) consumers who buy a legitimate copy of product 1 only; 2) consumers who obtain an illegal copy of product 1 only; 3) consumers who obtain illegal copies of both product 1 and 2; 4) consumers who obtain an illegal copy of product 2 only; and 5) consumers who buy a legitimate copy of product 2 only. Figure 1 (c) shows the demand structure in this regime.

Insert Figure 1 (c) about here.

Although there are two constraints in this regime, C 2B is the only binding constraint. After rewriting C 2B in terms of prices and DRM levels, we have the following constraint:

$$\text{C 2B: } \alpha(2v - t) - \frac{\alpha\beta}{1-\alpha}(e_1 + e_2) + \frac{1}{1-\alpha}e_1 < p_1 \Rightarrow \Gamma_2 < p_1. \tag{12}$$

In this regime, the critical consumer is the consumer who is indifferent between *(B, O)* and *(C, O)*. As in the previous two regimes, we can derive the demand for product 1 by setting the gross utilities of (B,O) and (C,O) equal and solving with respect to x_i. The resulting demands for product 1 and product 2 are shown below:

$$q_1 = \frac{1}{\alpha t}\left[e_1 - p_1 + \alpha v\right], \tag{13}$$

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$$q_2 = \frac{1}{\alpha t}\left[e_2 - p_2 + \alpha v\right].$$ (14)

Aggregate Demand

Combining the demands for product 1 in the above three regimes, we have the following aggregate demand function (in the order of the SS regime, the SC regime, and the NSI regime):

$$q_1(p_1, e_1, e_2) = \begin{cases} \dfrac{1}{\alpha t}\left[\beta e_1 - (1-\beta)e_2 - p_1 + \alpha v\right] & \text{if } p_1 < \Gamma_1 \\[2mm] \dfrac{1}{t}\left[\beta(e_1 + e_2) - p_1 + t - v - \alpha t + 2\alpha v\right] & \text{if } \Gamma_1 \le p_1 \le \Gamma_2 \\[2mm] \dfrac{1}{\alpha t}\left[e_1 - p_1 + \alpha v\right] & \text{if } p_1 > \Gamma_2 \end{cases}$$ (15)

C. Equilibrium Prices, Protection Levels, and Profits

Firm 1 has the following profit function to maximize in each regime:

$$\pi_1^R = p_1^R q_1^R - \left[K + \frac{m}{2}\left\{e_1^R\right\}^2\right], \text{ where } R = \{SS, SC, NSI\}.$$ (16)

The terms in the brackets represent the cost structure of firm 1 to create its own copy protection (i.e., DRM) for its product. Specifically, K is the fixed cost of creating its copy protection, and m determines the marginal cost of copy protection.

The two firms in this model compete in a two-stage game, where they set their DRM levels in the first stage and then choose their prices in the second stage, given the DRM levels set in the first stage. As most sequential games are solved backward, we first examine the second-stage competition and then return to the first-stage DRM-setting competition.

The Second Stage

In the second stage, both firms simultaneously choose the optimal prices that maximize their profits, taking the DRM levels for product 1 and product 2 as given. Since the two firms employ the identical price-setting mechanism, we focus only on firm 1 in this stage. Firm 1 sets the price that maximizes equation (16). After substituting the aggregate demand in expression (15) into equation (16) in each regime, differentiating equation (16) with respect to p_1 and solving for p_1, we have the following equilibrium prices (in the order of the SS regime, the SC regime, and the NSI regime) for the second stage:

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$$p_1^*(e_1, e_2) = \begin{cases} \dfrac{1}{2}\left(\beta e_1 - (1-\beta)e_2 + \alpha v\right) & \text{if } \hat{e}_1^{SS} \leq e_1 \\[2mm] \dfrac{1}{2}\left[\beta(e_1+e_2)+t-v-\alpha t+2\alpha v\right] & \text{if } \hat{e}_1^{SC} \leq e_1 \leq \max\left\{\hat{e}_2^{SC}, \hat{e}_3^{SC}\right\} \\[2mm] \dfrac{1}{2}\left[e_1 + \alpha v\right] & \text{if } e_1 > \max\left\{\hat{e}_1^{NSI}, \hat{e}_2^{NSI}\right\} \end{cases} \qquad ^6 \qquad (17)$$

In the above equilibrium prices, we include additional profit constraints, denoted as $\hat{e}_3^{SC}$ and $\hat{e}_2^{NSI}$, under the SC and NSI regimes in order to eliminate any incentive for firm 1 to deviate from one regime to another.

One notable fact regarding the above equilibrium prices is that they are not functions of firm 2's prices in any of the regimes. Similarly, the equilibrium prices of firm 2 are not functions of firm 1's prices in all regimes. This means that the two firms do not compete directly with each other in prices in the second stage. Their actual competition takes place in the first stage.

The First Stage

In the first stage, firms determine their optimal DRM levels that maximize their profits simultaneously. As in the second stage, because both firms use the same DRM-setting mechanism, we focus only on the results for firm 1 in this stage. In order to derive the reaction function for the DRM level of firm 1 in each regime, we plug the equilibrium price in expression (17) into equation (16) and differentiate equation (16) with respect to its DRM level (i.e., e_1). Solving the FOC for e_1 yields the following reaction function for firm 1 (in the order of the SS regime, the SC regime, and the NSI regime):

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[6] $\hat{e}_1^{SS} \equiv \left(\dfrac{\alpha+1-(1-\alpha)\beta}{(1-\alpha)\beta}\right)e_2 - \dfrac{\alpha(3v-2t)}{\beta}$ (from $p_1 \leq \Gamma_1$)

$\hat{e}_1^{SC} = \dfrac{(1+\alpha)\beta e_2 + (1-\alpha)(t-v+\alpha t - 2\alpha v)}{2-\alpha\beta-\beta}$ (from $p_1 \leq \Gamma_2$)

$\hat{e}_2^{SC} = \left(\dfrac{2-(1-\alpha)\beta}{(1-\alpha)\beta}\right)e_2 + \dfrac{t-v+\alpha t - 2\alpha v}{\beta}$ (from $p_1 > \Gamma_1$)

$\hat{e}_3^{SC} = \hat{e}_2^{NSI} \equiv \dfrac{\sqrt{\alpha}\beta e_2 + \sqrt{\alpha}\Delta - \alpha v}{1-\sqrt{\alpha}\beta}$ (from $\pi_1^{SC} = \pi_1^{NSI}$)

$\hat{e}_1^{NSI} = \dfrac{2\alpha\beta e_2 + \sqrt{\alpha}\Delta - \alpha v}{1-\sqrt{\alpha}\beta}$ (from $p_1 > \Gamma_2$), and $\Delta = t-v-\alpha t + 2\alpha v$.

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$$
e_1^*(e_2) = \begin{cases} -\dfrac{\beta(1-\beta)}{2\alpha mt - \beta^2}e_2 + \dfrac{\alpha\beta v}{2\alpha mt - \beta^2} & \text{if } e_2 \le \hat{\beta}_1^{ss} \\[3mm] \dfrac{\beta^2}{2mt - \beta^2}e_2 + \dfrac{(t - v - \alpha t + 2\alpha v)\beta}{2mt - \beta^2} & \text{if } \max\left\{\hat{\beta}_2^{SC}, \hat{\beta}_3^{SC}\right\} \le e_2 \le \hat{\beta}_1^{SC} \quad {}^{7} \\[3mm] \dfrac{\alpha v}{2\alpha mt - 1} & \text{if } e_2 < \min\left\{\hat{\beta}_1^{NSI}, \hat{\beta}_2^{NSI}\right\} \end{cases}
\tag{18}
$$

We can derive the reaction function for firm 2 with the same procedure although it is not shown here. Since the reaction function for firm 1 is a function of firm 2's DRM (and vice versa), the optimal DRM levels set by the two firms are strategically related. In the SS regime, the slopes of the reaction functions for firm 1 and firm 2 are negative. Hence, with the following stability and convergence condition

$$
2\alpha mt - \beta > 0,
\tag{19}
$$

the optimal DRM levels set by the firms are strategic substitutes. In the SC regime, however, the slopes of the reaction functions for firm 1 and firm 2 are positive. Therefore, the optimal DRM levels for firm 1 and firm 2 are strategic complements, given the following two conditions for stability and convergence:

$$
mt > \beta^2 \text{ and } t(1-\alpha) + v(2\alpha - 1) > 0.
\tag{20}
$$

One intuitive explanation for the firms' strategic interaction in the SS regime is that when firm 2 increases its protection level (i.e., e_2), the demand for product 2 increases because some consumers who consume (C, C) switch to (C, B) due to an increase in reproduction cost for

---

[7] $\hat{\beta}_1^{ss} \equiv \dfrac{(1-\alpha)(\alpha mt(3v - 2t) + (t - v)\beta)}{(1 + \alpha - (1-\alpha)\beta)mt - \beta^2}$ (from $\hat{e}_1^{SS} \le e_1$)

$\hat{\beta}_1^{SC} \equiv \dfrac{\beta(\Delta - \alpha\beta v) - (1-\alpha)mt(t - v + \alpha t - 2\alpha v)}{((1+\alpha)mt - \beta)\beta}$ (from $\hat{e}_1^{SC} \le e$)

$\hat{\beta}_2^{SC} \equiv \dfrac{(1-\alpha)((t-v)\beta^2 - mt(t - v + \alpha t - 2\alpha v))}{(2 - \beta + \alpha\beta)mt - \beta^2}$ (from $e_1 \le \hat{e}_2^{SC}$ if $\hat{e}_2^{SC} \ge \hat{e}_3^{SC}$)

$\hat{\beta}_3^{SC} \equiv \dfrac{2mt(\alpha v - \sqrt{a}\Delta) + (\alpha\beta v - \Delta)\beta}{\beta(2\sqrt{a}mt - \beta)}$ (from $e_1 \le \hat{e}_3^{SC}$ if $\hat{e}_3^{SC} \ge \hat{e}_2^{SC}$)

$\hat{\beta}_1^{NSI} \equiv \dfrac{\Delta - \alpha(1-\alpha)mt(2t - 3v) - \alpha\beta v}{2\alpha mt - \beta}$ (from $e_1 > \hat{e}_1^{NSI}$ if $\hat{e}_1^{NSI} \ge \hat{e}_2^{NSI}$)

$\hat{\beta}_2^{NSI} = \dfrac{2\alpha mt(\sqrt{a}v - \Delta) + \Delta - \alpha\beta v}{(2\alpha mt - 1)\beta}$ (from $e_1 > \hat{e}_2^{NSI}$ if $\hat{e}_2^{NSI} \ge \hat{e}_1^{NSI}$), and $\Delta = t - v - \alpha t + 2\alpha v$.

product 2. But, the increase in firm 2's protection level has a negative impact on the demand for product 1 (negative demand-shift effect). The reason is that consumers who consume (B,C) don't value product 2 enough to buy a legitimate copy of product 2 facing the increased protection level for product 2. In order to reduce the overall cost of consuming both products and increase the gross utility, some of the consumers who consume (B,C) obtain an illegal copy of product 1, instead of buying a legal copy of product 1. Hence, some consumers in that segment switch to (C,C), which results in a decrease in the demand for product 1. To boost its demand, firm 1 needs to reduce its price, which gives consumers less incentive to pirate product 1. With this lowered piracy level, firm 1 can reduce its protection level to reduce its cost and thus increase its profit.[8]

A real-life example of the SS regime can be found in the music business. While most of the major music companies attempt to increase their DRM requests to online sellers, such as Apple and Amazon.com, to curb music piracy, EMI Group has recently decided to license its music to online sellers without copy protection (Smith and Vara, 2007). EMI Group's decision is the opposite of the DRM policy adopted by its major competitor, Universal Music Group, which is greatly concerned about online piracy. EMI Group's strategic behavior in this example can be explained by the above intuition. Another example is in the download movie business. Paramount, a major movie studio, has refused to make its movies available on Apple's iTunes due to piracy concerns, although iPod and iTunes are very popular among consumers. It is unlikely that Paramount would agree to license their movies to Apple, until Apple makes its sharing rules on downloaded movies stricter and put more protection for their movies (Grover, 2007). Contrary to this position, however, Walt Disney, another major movie studio, has agreed to license its movies to Apple and made its movies available on iTunes. Clearly, Disney has a different strategic mindset on Apple's DRM that is the opposite of the viewpoint of its major competitor. The above intuition provides a good rationale for Walt Disney's decision on offering its movies to Apple.

---

[8] Taking the partial derivative of equation (17) and (18) with respect to e_2 in the case of the SS regime yields
$$\frac{\partial p_1^*\left(e_1,e_2\right)}{\partial e_2} = -\frac{1}{2}\left(1-\beta\right) < 0 \text{ and } \frac{de_1^*\left(e_2\right)}{de_2} = -\frac{\beta\left(1-\beta\right)}{2\alpha mt - \beta^2} < 0 \text{ respectively.}$$

The intuition behind the firms' strategic interaction in the SC regime is as follows. If firm 2 increases its protection level (i.e., e_2), the demand for product 2 increases because some consumers who consume (C,C) and strongly prefer product 2 switch to (O,B) facing an increase in the reproduction cost for product 2. In this case, however, the increase in firm 2's protection level has a positive impact on the demand for product 1 (positive demand-shift effect). With the increased protection level for product 2, consumers who consume (C,C) and strongly prefer product 1 switch to (B,O) because the increase in reproduction cost for product 2 is too much for them. Instead, they purchase a legitimate copy of product 1 and opt not to consume product 2 (legitimate or illegal), which results in an increase in the demand for product 1. Since its demand is increased, firm 1 can increase its price. At the same time, firm 1 needs to increase its protection level to reduce the increased piracy incentive for its product due to the price increase.[9]

This strategic nature of the SC regime can be illustrated by decisions made by companies that sell music or movies through the Internet. In 2006, CinemaNow announced that it would allow consumers to burn copies of some movies onto a DVD that can be watched on a television, which means less restricted copy protection (McBride, 2006). Responding to CinemaNow's move, Movielink, a competing movie download service, attempted to strike a similar deal with movie studios. Another real-life example is a recent announcement by Amazon.com. that its music-download service would only sell music that comes without copy protection (Smith and Vara, 2007). This announcement was made as Apple was preparing to offer music without copy protection on its iTunes. Essentially, Amazon.com followed suit and reduced its private copy protection when its major rival, Apple, planned to reduce its private copy protection. These two examples illustrate the strategic nature of the SC regime.

Now, we derive optimal DRM levels, which is the ultimate objective in the first stage. We can calculate specific optimal DRM levels for both firms by solving the reaction functions of firm 1 and firm 2 for e_1 and e_2 simultaneously in each regime. The following proposition summarizes the optimal DRM levels determined by firm 1 in the three regimes:

---

[9] Taking the partial derivative of equation (17) and (18) with respect to e_2 in the case of the SC regime yields $\dfrac{\partial p_1^*(e_1,e_2)}{\partial e_2} = \dfrac{\beta}{2} > 0$ and $\dfrac{de_1^*(e_2)}{de_2} = \dfrac{\beta^2}{2mt-\beta^2} > 0$ respectively.

Proposition 1: *If condition (19) is satisfied in the SS regime and condition (20) in the SC regime, the optimal DRM levels chosen by firm 1 are (in the order of the SS regime, the SC regime, and the NSI regime)*

$$
e_1^* = \begin{cases} \dfrac{\alpha\beta v}{2\alpha mt + \beta - 2\beta^2} & \text{if } \hat{\alpha}_1^{ss} \leq \beta \\[3mm] \dfrac{\beta\Delta}{2\left(mt - \beta^2\right)} & \text{if } \hat{\alpha}_3^{SC} \leq \beta \leq (<)\min\left\{\hat{\alpha}_1^{SC}, \hat{\alpha}_2^{SC}\right\} \\[3mm] \dfrac{\alpha v}{2\alpha mt - 1} & \text{if } \beta > \hat{\alpha}_2^{\text{NSI}} \end{cases} \tag{21}
$$

$$
\text{with } \frac{1}{2} < \hat{\alpha}_1^{ss} < \hat{\alpha}_3^{SC} < \min\left\{\hat{\alpha}_1^{SC}, \hat{\alpha}_2^{SC}\right\} < \hat{\alpha}_2^{\text{NSI}} .^{10}
$$

3. Comparative Static Analysis

We now analyze the effects of a marginal increase in public copy protection, which is comparable to Intellectual Property Rights (IPR) protection. As with previous studies in the literature (e.g., Bae and Choi, 2006; Novos and Waldman, 1984), we model the increase in IPR protection as an increase in the cost of piracy, which makes the option of piracy less attractive. For example, Bae and Choi (2006) provide the generalized results of the effects associated with two different types of costs associated with piracy for the case of monopoly: constant reproduction cost and proportional degradation rate. Since the optimal level of DRM, which corresponds to the reproduction cost, is endogenously determined by content providers, we concentrate on the other measure of IPR protection, which is the public copy protection. Unlike previous literature, in this model we have composite effects which can be written as

$$
\frac{dp_1^{R*}}{d\alpha} = \frac{\partial p_1^{R*}}{\partial\alpha} + \frac{\partial p_1^{R*}}{\partial e_1}\frac{de_1^{R*}}{d\alpha} \text{ and } \frac{dq_1^{R*}}{d\alpha} = \frac{\partial q_1^{R*}}{\partial\alpha} + \frac{\partial q_1^{R*}}{\partial e_1}\frac{de_1^{R*}}{d\alpha} . \tag{22}
$$

---

[10] $\hat{\alpha}_1^{ss} \equiv \dfrac{\Delta + \sqrt{8\alpha(1-\alpha)^2 mt(2t-3v)(t-v)+\Delta^2}}{4(1-\alpha)(t-v)}$, $\hat{\alpha}_1^{SC} \equiv \dfrac{\Delta + \sqrt{\Delta^2 - 8\alpha(1-\alpha)mtv(t-v+\alpha t-2\alpha v)}}{4\alpha v}$,

$\hat{\alpha}_2^{SC} \equiv \dfrac{\Delta + \sqrt{\Delta^2 + 8(1-\alpha)^2 mt(t-v)(t-v+\alpha t-2\alpha v)}}{4(1-\alpha)(t-v)}$, $\hat{\alpha}_3^{SC} \equiv \dfrac{\Delta + \sqrt{16\alpha mtv(\alpha v - \sqrt{\alpha}\Delta)+\Delta^2}}{4\alpha v}$,

$\hat{\alpha}_2^{\text{NSI}} \equiv \dfrac{\Delta - 2\alpha mt(\sqrt{\alpha} v - \Delta)}{2\alpha v}$, and $\Delta = t - v - \alpha t + 2\alpha v$.

The first term shows the usual direct effect, the indirect effect in the second term comes from the fact that an increase in public copy protection changes the optimal level of DRM, thus affecting the optimal price and quantity in each regime. Therefore, the total effect of α on $\left(p^{R*}, q^{R*} \right)$ is the sum of these two effects.

The signs of $\partial p_1^{R*}/\partial \alpha$, $\partial p_1^{R*}/\partial e_1$, $\partial q_1^{R*}/\partial \alpha$, and $\partial q_1^{R*}/\partial e_1$ are determined by how these changes affect demand for legal products under different regimes. An increase in public copy protection under the SC regime and the level of DRM has the same directional impact in the copy cost across consumers, which is equivalent to an outward parallel shift in demand for the original goods. Facing higher demand, content providers respond with the price increase but this is not enough to offset the initial demand increase: $\dfrac{\partial p_1^{SC*}}{\partial a} = \dfrac{2v-t}{2} > 0$, $\dfrac{\partial q_1^{SC*}}{\partial \alpha} = \dfrac{2v-t}{2t} > 0$,

$\dfrac{\partial p_1^{R*}}{\partial e_1} > 0$ and $\dfrac{\partial q_1^{R*}}{\partial e_1} > 0$.[11] On the other hand, stricter public copy protection implies a pivot change in demand that affects the slope of the demand curve for legal copies under the SS and NSI regimes. Due to a proportional increase in the copy cost, higher valuation consumers are more adversely affected by an increase in public copy protection, which reduces it's the demand elasticity. The content providers are more interested in serving only the high valuation consumers, which means $\dfrac{\partial p_1^{NSI*}}{\partial \alpha} = \dfrac{v}{2} > 0$ and $\dfrac{\partial q_1^{NSI*}}{\partial \alpha} = -\dfrac{e_1^{NSI}}{2\alpha^2 t} < 0$.[12]

We now calculate the effect of an increase in public copy protection on the optimal DRM under each regime. It is shown that the effect can have different implications depending on which regime the content providers are operating under and the strategic nature of DRM. Let $e_1^{R*} = R_1(\alpha, e_2^{R*})$ and $e_2^{R*} = R_2(\alpha, e_1^{R*})$ be the optimal choice of DRM under regime R, where $R = \{SS, SC\}$. We differentiate the equilibrium condition $e_1^{R*} = R_1(\alpha, R_2(\alpha, e_1^{R*}))$ with respect to α to set

---

[11] Simple comparative statics exercise of taking the partial derivative of the optimal price and demand in the second period with respect to α and e respectively confirms the results.

[12] Another possible outcome under the SS regime if $e_2 - \beta(e_1 + e_2) > 0$ would be a rightward shift in demand with a steeper slope where the effect of higher α on price and quantity are similar to those under the SC regime.

$$\frac{de_1^{R*}}{d\alpha} = \frac{\partial e_1^{R*}}{\partial \alpha} + \frac{\partial R_1}{\partial e_2}\frac{\partial R_2}{\partial e_1}\frac{de_1^{R*}}{d\alpha} + \frac{\partial R_1}{\partial e_2}\frac{dR_2}{d\alpha}, \text{ which yields}$$

$$\frac{de_1^{R*}}{d\alpha} = \frac{1}{\left(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1\right)}\left[\frac{\partial e_1^{R*}}{\partial \alpha} + \frac{\partial R_1}{\partial e_2}\frac{dR_2}{d\alpha}\right]. \tag{23}$$

The denominator is always positive to ensure the stability of the equilibrium. Therefore, the effect of α on e_1^{R*} is shown to depend on three factors: 1) the direct effect of α on e_1^{R*} $\left[\partial e_1^{R*}/\partial \alpha\right]$, 2) the slope of the reaction function $\left[\partial R_1/\partial e_2\right]$ and 3) firm 2's response to an increase in α $\left[dR_2/d\alpha\right]$. Since the first and the third effects are symmetric, we only concentrate on the third effect for analytical convenience.

   To illustrate how a higher level of public copy protection affects the level of DRM in the equilibrium, we first look at the changes in reaction function curves. Under the SS regime, we have shown that $\partial R_1/\partial e_2 < 0$, and the sign of $dR_2/d\alpha$ depends on the level of e_1. If

$$e_1 > \frac{\beta^2 v}{2mt(1-\beta)},$$ we have $\frac{dR_2}{d\alpha} > 0$. Otherwise, $\frac{dR_2}{d\alpha} < 0$. These effects are shown as the counter-

clockwise rotation of firm 1's reaction curve around $e_1 = \frac{\beta^2 v}{2mt(1-\beta)}(> e_1^{SS*})$. The new reaction

curves are illustrated by the dotted lines in Figure 2 (a). The new equilibrium point moves to the "south-west", which implies that the higher public copy protection rate induces the optimal DRM to decrease. Under the SC regime, in contrast, we have a situation of strategic complements: an anticipated increase in firm 2's DRM level causes firm 1 to raise its protection [i.e., $\partial R_1/\partial e_2 > 0$]. As depicted in Figure 2 (b), a higher public copy protection rate makes both firms more "soft", leading them to choose a higher level of DRM given any choice of their rivals [i.e., $dR_2/d\alpha > 0$], which make the new reaction curves shift outward.[13] Therefore, with a higher public copy protection rate under the SC regime, we observe a higher level of DRM which moves to the "north-east" in the figure 2 (b). Under other regimes, there is no strategic interaction between

---

[13] The derivation of $\frac{dR_2}{d\alpha}$ comes from differentiating the reaction function of firm 1 with respect to α, which yields

$\frac{\beta}{(2\alpha mt - \beta^2)^2}\left[2(1-\beta)mte_1 - \beta^2 v\right]$ under the SS regime and $\frac{\beta}{2mt - \beta^2}\left[2v - t\right]$ under the SC regime.

content providers, which means $\partial e_1^{R*} / \partial e_2^{R*} = 0$. Proposition 2 and Table 4 summarizes the results of the comparative statics.

Insert Figure 2 about here.

Insert Table 4 about here.

Proposition 2: *The effects of an increase in the public copy protection rate on the optimal level of DRM, price, and the authorized usage crucially depend on the demand structure and the strategic nature of DRM under different regimes.*

    Proof. Calculation of the optimal level of DRM with respect to α under different regimes yields

$$\frac{de_1^{SS*}}{d\alpha} = \frac{1}{\left(1 - \partial R_1 / \partial e_2 \cdot \partial R_2 / \partial e_1\right)} \left[\underbrace{\frac{\partial e_1^{SS*}}{\partial \alpha}}_{(-)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(-)} \underbrace{\frac{dR_2}{d\alpha}}_{(-)} \right] = -\frac{(2\beta-1)\beta^2 v}{(2\alpha m t + \beta - 2\beta^2)^2} \le 0 \quad ,$$

$$\frac{de_1^{SC*}}{d\alpha} = \frac{1}{\left(1 - \partial R_1 / \partial e_2 \cdot \partial R_2 / \partial e_1\right)} \left[\underbrace{\frac{\partial e_1^{SC*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(+)} \underbrace{\frac{dR_2}{d\alpha}}_{(+)} \right] = \frac{m(2v-t)}{2(mt - \beta^2)} \ge 0, \text{ and}$$

$$\frac{de_1^{NSI*}}{d\alpha} = -\frac{v}{(2\alpha m t - 1)^2} \le 0.$$

We now are ready to determine the effect of α on the equilibrium price and quantity under different regimes.

$$\frac{\partial p_1^{SS*}}{\partial \alpha} = \underbrace{\frac{\partial p_1^{SS*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial p_1^{SS*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SS*}}{d\alpha}}_{(-)} = \frac{2\alpha m t v \left(\alpha m t + \beta - 2\beta^2\right)}{(2\alpha m t + \beta - 2\beta^2)^2} \overset{?}{\ge} 0,$$

$$\frac{\partial p_1^{SC*}}{\partial \alpha} = \underbrace{\frac{\partial p_1^{SC*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial p_1^{SC*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SC*}}{d\alpha}}_{(+)} = \frac{mt(2v-t)}{2(mt - \beta^2)} \ge 0,$$

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$$\frac{\partial p_1^{NSI*}}{\partial \alpha} = \underbrace{\frac{\partial p_1^{NSI*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial p_1^{NSI*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{NSI*}}{d\alpha}}_{(-)} = \frac{2\alpha mtv(\alpha mt - 1)}{(2\alpha mt - 1)^2} \overset{?}{\geq} 0,$$

$$\frac{\partial q_1^{NSI*}}{\partial \alpha} = \underbrace{\frac{\partial q_1^{NSI*}}{\partial \alpha}}_{(-)} + \underbrace{\frac{\partial q_1^{NSI*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{NSI*}}{d\alpha}}_{(-)} = -\frac{mv}{(2\alpha mt - 1)^2} \leq 0,$$

$$\frac{\partial q_1^{SS*}}{\partial \alpha} = \underbrace{\frac{\partial q_1^{SS*}}{\partial \alpha}}_{(-)} + \underbrace{\frac{\partial q_1^{SS*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SS*}}{d\alpha}}_{(-)} = \frac{(1-2\beta)\beta mv}{(2\alpha mt + \beta - 2\beta^2)^2} \leq 0, \text{ and}$$

$$\frac{\partial q_1^{SC*}}{\partial \alpha} = \underbrace{\frac{\partial q_1^{SC*}}{\partial \alpha}}_{(+)} + \underbrace{\frac{\partial q_1^{SC*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SC*}}{d\alpha}}_{(+)} = \frac{m(2v-t)}{2(mt - \beta^2)} \geq 0. \qquad\qquad \textbf{Q.E.D.}$$

The intuition underlying this result is the following. Facing less threat from dual piracy with an increase in the public copy protection rate, the direct effect $\left[\partial e_1 / \partial \alpha < 0\right]$ under the SS regime results in a reduction in firm 1's DRM because the content providers are only interested in serving high valuation consumers with pivot change in demand. This direct effect is, however, lessened by strategic interaction between firms due to the strategic nature of DRM. Using the terminology of Fudenberg and Tirole (1984) we can explain the strategic effect as follows: when firm 2 regards firm 1's DRM as a strategic substitute $\left[\partial R_2 / \partial e_1 < 0\right]$ and an increase in the public copy protection rate makes it "soft" $\left[dR_2 / d\alpha < 0\right]$, the appropriate strategy for firm 2 would be that of " stay lean and hungry" if firm 2 were able to control the level of public copy protection.[14] That is, firm 2 wants to have a lower level of α in order to commit to being more aggressive without considering the direct effects. Firm 1 then decides on a less aggressive level of DRM. As a result, an increase in public copy protection via a higher level of α exerts two opposite effects on the optimal level of DRM, in which the net effect is a lower level of DRM.

On the other hand, an outward-parallel shift of demand with an increase in the public copy protection rate under the SC regime makes the content providers respond with a higher level of DRM which expands the demand further. This effect is further augmented by the strategic effect with strategic complements. We can again explain the strategic effect with the

[14] To be consistent with the direct effect we examine firm 2's strategic incentive.

terminology of Fudenberg and Tirole (1984). When firm 2 regards firm 1's DRM as a strategic complement $\left[\partial R_2/\partial e_1 > 0\right]$ and an increase in public copy protection makes it "soft" $\left[dR_2/d\alpha > 0\right]$, the appropriate strategy for firm 2 would be that of "fat cat" if firm 2 were able to control the level of public copy protection. That is, firm 2 wants to have a higher level of α in order to commit to being less aggressive. Firm 1 then replies with a higher level of DRM. Under the SC regime therefore a change in the public copy protection rate leads to the same-direction direct and strategic effects on the optimal level of DRM.

We also determine the total effect of a higher level of α on firm 2's profits by taking the total derivative of π_2^R, where $R=\{SS,\ SC\}$, with respect to α, which yields

$$\frac{d\pi_2^R}{d\alpha} = \frac{\partial \pi_2^R}{\partial \alpha} + \frac{\partial \pi_2^R}{\partial e_1^R}\frac{de_1^R}{d\alpha}\ . \tag{24}$$

The first term in the right-hand side of equation (24) is the direct effect on firm 2's profit from a higher α; the second term is the strategic effect as the equilibrium response of firm 1 to the change in α. Firm 2 always prefers a higher level of public copy protection since we have both positive direct and strategic effects under both regimes while the nature of strategic interaction across regimes is different; $\dfrac{\partial \pi_2^{SS}}{\partial e_1^{SS}} < 0$, $\dfrac{\partial \pi_2^{SC}}{\partial e_1^{SC}} > 0$, $\dfrac{de_1^{SS}}{d\alpha} < 0$, and $\dfrac{de_1^{SC}}{d\alpha} > 0$.

Another interesting comparative statics exercise is how less compatible hacking technology affects the optimal level of DRM, price, and quantity. Since the derivation of these effects is the simple reiteration of that of an increase in public copy protection rate, we only show the results in Proposition 3 and Table 4.

Proposition 3: *An increase in the compatibility of DRM in term of hacking technology has positive effects on the optimal level of DRM, price and the authorized usage across regimes but its magnitude crucially depends on the strategic nature of DRM under different regimes.*

Proof. Calculation of the optimal level of DRM with respect to β under different regimes yields

$$\frac{de_1^{SS*}}{d\beta} = \frac{1}{\left(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1\right)}\left[\underbrace{\frac{\partial e_1^{SS*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(-)}\underbrace{\frac{dR_2}{d\beta}}_{(+)}\right] = \frac{\alpha m v\left(\beta^2 + \alpha m t\right)}{\left(2\alpha m t + \beta - 2\beta^2\right)^2} \geq 0,$$

$$\frac{de_1^{SC*}}{d\beta} = \frac{1}{\left(1 - \partial R_1/\partial e_2 \cdot \partial R_2/\partial e_1\right)} \left[\underbrace{\frac{\partial e_1^{SC*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial R_1}{\partial e_2}}_{(+)} \underbrace{\frac{dR_2}{d\beta}}_{(+)} \right] = \frac{\Delta(mt + \beta^2)}{2(mt - \beta^2)^2} \geq 0.$$

We now are ready to determine the effect of β on the equilibrium price and quantity under different regimes.

$$\frac{\partial p_1^{SS*}}{\partial \beta} = \underbrace{\frac{\partial p_1^{SS*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial p_1^{SS*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SS*}}{d\beta}}_{(+)} = \frac{\alpha^2 mtv(4\beta - 1)}{(2\alpha mt + \beta - 2\beta^2)^2} \geq 0,$$

$$\frac{\partial p_1^{SC*}}{\partial \beta} = \underbrace{\frac{\partial p_1^{SC*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial p_1^{SC*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SC*}}{d\beta}}_{(+)} = \frac{\beta mt\Delta}{(mt - \beta^2)^2} \geq 0,$$

$$\frac{\partial q_1^{SS*}}{\partial \beta} = \underbrace{\frac{\partial q_1^{SS*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial q_1^{SS*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SS*}}{d\beta}}_{(+)} = \frac{\alpha mv(4\beta - 1)}{(2\alpha mt + \beta - 2\beta^2)^2} \geq 0, \text{ and}$$

$$\frac{\partial q_1^{SC*}}{\partial \beta} = \underbrace{\frac{\partial q_1^{SC*}}{\partial \beta}}_{(+)} + \underbrace{\frac{\partial q_1^{SC*}}{\partial e}}_{(+)} \underbrace{\frac{de_1^{SC*}}{d\beta}}_{(+)} = \frac{\beta m\Delta}{(mt - \beta^2)^2} \geq 0. \qquad \textbf{Q.E.D.}$$

With a higher degree of the compatibility of DRM in term of hacking technology, all consumers face the same increase in dual copy cost, which is equivalent to an overall demand increase for firms. Hence the firms benefit from higher demand by increasing the optimal level of DRM inducing even higher price, yet increasing sales at the same time under both the SS and SC regimes. This direct effect is, however, either augmented or lessened depending on the nature of strategic interaction between firms, which is shown in Figure 3.

Insert Figure 3 about here.
----

4. Welfare Analysis

We are now in a position to examine the effects of an increase in the public copy protection and the degree of compatibility of hacking technology on social welfare. Social welfare depend on how an increase in public copy protection and decrease in DRM compatibility affect

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on consumers' incentive to make illegal copies as well as the firms' incentive to adjust their DRM level. In order to separate different effects on social welfare, we identify the following channels; *demand switch, single copy usage change, single copy cost, dual copy cost, DRM production cost*.

As can be seen from the equations (A1) to (A6) in the Appendix, we can separate five different channels through which an increase in public copy protection and decrease in DRM compatibility affects social welfare. The two terms indicated by 'dual copy cost increase' and 'single copy cost increase' in equations (A1) to (A5) are always negative and represent social welfare loss due to increase in gross copy cost for consumers who continue to copy either one or both of the products. The first term in equations (A1) to (A5) represents the demand switch effect between legal and illegal copies, which induces welfare gain or loss depending on the direction of demand switches. One distinctive feature is that the demand switch effect is a sum of the direct effect, which has been discussed in the previous literature (e.g., Bae and Choi, 2006; Belleflamme and Picard, 2007), and the indirect effect which shows how the marginal increase in protection affects the private incentive of firms to protect their own products through $\frac{\partial q_1^{R*}}{\partial e} \frac{de_1^{R*}}{d\alpha}$.

The total demand switch effect decreases social welfare in the case of an increase in public copy protection under the SS and the NSI regime, since the marginal consumer who was indifferent between *(B, C)* and *(C, C)* [*(B, O)* and *(C, O)*] under the SS [NSI] regime now switch to (C, C) [(C, O)] which are produced inefficiently and suffer from the increased gross copy cost. However, the demand switch effect resulting from an increase in public copy protection under the SC regime and the decreased compatibility in DRM under the SS and SC regimes is positive, since it induces the marginal consumer to switch to legal copies as demonstrated in comparative statics.

Another term denoted as 'single copy usage change' in equations (A1), (A3), (A4) and (A6) explains how these effects affect the marginal consumer who was indifferent between *(B, O)* and *(B, C)* [*(C, O)* and *(C, C)*] under the SS [NSI] regime. It has a negative impact on social welfare if the marginal consumer stops using the illegal product, which reduces the total usage. The last term 'DRM production cost' affects social welfare directly through changes in firms' production cost of DRM in response to the marginal change in IPR protection. Therefore, the overall effect on social welfare is ambiguous and depends on the relative magnitudes of the five effects. However, one implication that deserves attention here is that under the SC regime, the

effect of an increase in public copy protection and compatibility of DRM changes social welfare in the same direction since both of these effects induce parallel shifts in the demand of firms. Under the SS regime, in contrast, an increase in public copy protection results in a positive demand switch and production cost savings because of the lower DRM, but an increase in compatibility of DRM brings a negative demand switch and production costs increase due to higher DRM. Proposition 4 and Table 45 summarizes the results of the welfare analysis.

Proposition 4: *The effects of an increase in the public copy protection rate and the compatibility of DRM in term of hacking technology on social welfare depend on the relative magnitude of the following effects under different regimes; demand switch, single copy usage change, single copy cost, dual copy cost, DRM production cost.*

Proof*.* See Appendix

Insert Table 5 about here.

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5. Conclusion

This paper examines how disparate demand structures influence firms' competitive behavior and their optimal DRM levels and prices. For this purpose, we first develop a model that includes three different demand structures. Then, we derive the optimal DRM levels, prices, and quantity demanded for the legitimate products under each of the demand structures. We find that the impacts of public copy protection on the optimal DRM levels, prices, and the quantity demanded for the products is different from those of the degree of DRM compatibility. Specifically, the impacts of public copy protection vary depending on demand structures, while the impacts of the degree of DRM compatibility are the same (i.e., positive) across the demand structures, regardless of whether the content providers regard their optimal DRM levels as strategic substitutes or strategic complements. In terms of social welfare, the effects of changes in public copy protection and degree of compatibility of DRM systems on social welfare are determined by the relative magnitudes of the following five factors: demand switch, single copy usage change, single copy cost, dual copy cost, and DRM production cost.

This paper provides new, important insights to the literature on piracy. Most of the past studies on piracy examine monopoly situations, and only a small number of studies deal with duopoly cases (e.g., Johnson, 1985; Park and Scotchmer, 2005). Those studies examining duopoly cases mainly focus on prices of information goods, rather than addressing DRM of information goods. This paper incorporates DRM as well as prices of digital goods as key decision variables in a duopoly setting and shows that when a competitor changes its DRM level, a firm adjusts its DRM level in the opposite direction under the SS regime and in the same direction under the SC regime. This paper suggests that policymakers' decision on public copy protection generates opposite results in content providers' decisions on private copy protection levels, depending on whether the content providers consider their private protection levels as strategic substitutes or strategic complements. Eventually, the policymakers' decision will affect overall consumers' welfare in the industry because consumers' welfare is influenced by content providers' private copy protection levels. This paper also reveals that, in situations where firms are *independent* of each other in the price-setting game, they become *interdependent* with each other through DRM competition when a significant portion of consumers prefer pirated goods.

This paper provides opportunities for future research. One possible direction would be modeling a situation where the two firms can cooperate or collude with each other, which is in line with the work by Park and Scotchmer (2005). In the current paper, the two content providers set their optimal DRM levels competitively. But, how would the results from this paper change if the content providers develop and own the same DRM system *jointly*? Would this situation offer a better approach in dealing with piracy? Would consumers be better off in this situation? Would the overall industry be better off? These are important and interesting research questions that need to be addressed in the area of piracy and DRM. Another direction would be to include content producers and content sellers as separate players in a model. In the current paper, firms produce *and* sell digital goods to consumers. In reality, however, content producers are not usually content sellers. Music companies and movie studios produce contents, but usually are not engaged in selling their products directly to consumers. They make their products available to consumers through online and offline retailers. The following would be a key research question for this direction: How would this setup change the optimal levels of DRM and prices for digital goods? Research in this direction will be a meaningful extension of this paper.

Table 1 Gross Utilities of Consumer *i* with Various Options of Acquiring Product 1 and 2

2 \ 1	B	C	O
B	N.A.	$(1-\alpha)(v-tx_i)-e_1+v-t(1-x_i)-p_2$	$v-t(1-x_i)-p_2$
C	$v-tx_i-p_1+(1-\alpha)(v-t(1-x_i))-e_2$	$(1-\alpha)(2v-t)-\beta(e_1+e_2)$	$(1-\alpha)(v-t(1-x_i))-e_2$
O	$v-tx_i-p_1$	$(1-\alpha)(v-tx_i)-e_1$	0

Table 2 Constraints Regarding Firm 1

Constraint Number	Constraint name	Corresponding expression
1A	Strategic substitute constraint	$u\left(\tilde{x}_i\left((B,O),(B,C)\right)\right)>u\left(x_i;(C,C)\right)$
1B	No strategic substitute constraint	$u\left(\tilde{x}_i\left((B,O),(B,C)\right)\right)\leq u\left(x_i;(C,C)\right)$
2A	Strategic complement constraint	$u\left(\tilde{x}_i\left((B,O),(C,O)\right)\right)\leq u\left(x_i;(C,C)\right)$
2B	No strategic complement constraint	$u\left(\tilde{x}_i\left((B,O),(C,O)\right)\right)>u\left(x_i;(C,C)\right)$

Table 3 Constraints Regarding Firm 2

Constraint Number	Constraint name	Corresponding expression
1A′	Strategic substitute constraint	$u\left(\tilde{x}_i\left((O,B),(C,B)\right)\right)>u\left(x_i;(C,C)\right)$
1B′	No strategic substitute constraint	$u\left(\tilde{x}_i\left((O,B),(C,B)\right)\right)\leq u\left(x_i;(C,C)\right)$
2A′	Strategic complement constraint	$u\left(\tilde{x}_i\left((O,B),(O,C)\right)\right)\leq u\left(x_i;(C,C)\right)$
2B′	No strategic complement constraint	$u\left(\tilde{x}_i\left((O,B),(O,C)\right)\right)>u\left(x_i;(C,C)\right)$

Table 4 Summary of Comparative Statics

|	e^{R*}	p^{R*}	q^{R*}
SS	$\dfrac{de^{SS*}}{d\alpha} \le 0$, $\dfrac{de^{SS*}}{d\beta} \ge 0$	$\dfrac{dp^{SS*}}{d\alpha} \lesseqgtr 0$, $\dfrac{dp^{SS*}}{d\beta} \ge 0$	$\dfrac{dq^{SS*}}{d\alpha} \le 0$, $\dfrac{dq^{SS*}}{d\beta} \ge 0$
SC	$\dfrac{de^{SC*}}{d\alpha} \ge 0$, $\dfrac{de^{SC*}}{d\beta} \ge 0$	$\dfrac{dp^{SC*}}{d\alpha} \ge 0$, $\dfrac{dp^{SC*}}{d\beta} \ge 0$	$\dfrac{dq^{SC*}}{d\alpha} \ge 0$, $\dfrac{dq^{SC*}}{d\beta} \ge 0$
NSI	$\dfrac{de^{NSI*}}{d\alpha} \le 0$	$\dfrac{dp^{NSI*}}{d\alpha} \lesseqgtr 0$	$\dfrac{dq^{NSI*}}{d\alpha} \le 0$

Table 5 Summary of Welfare Analysis

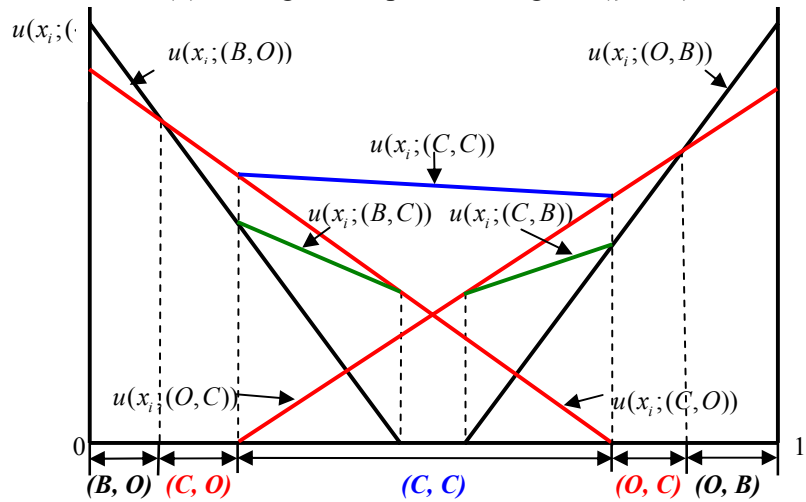| Regime	parameters	Demand switch	Single copy usage	Dual copy cost	Single copy cost	DRM
SS	$\dfrac{\partial SW^{SS}}{\partial \alpha}$	(−)	(−)	(−)	(−)	(+)
	$\dfrac{\partial SW^{SS}}{\partial \beta}$	(+)	(+)/(−)	(−)	(−)	(−)
SC	$\dfrac{\partial SW^{SC}}{\partial \alpha}$	(+)	N/A	(−)	N/A	(−)
	$\dfrac{\partial SW^{SC}}{\partial \beta}$	(+)	N/A	(−)	N/A	(−)
NSI	$\dfrac{\partial SW^{NSI}}{\partial \alpha}$	(−)	(+)/(−)	(−)	(−)	(+)
	$\dfrac{\partial SW^{NSI}}{\partial \beta}$	N/A	(−)	(−)	N/A	N/A

Figure 1 Demand Structures under different regimes
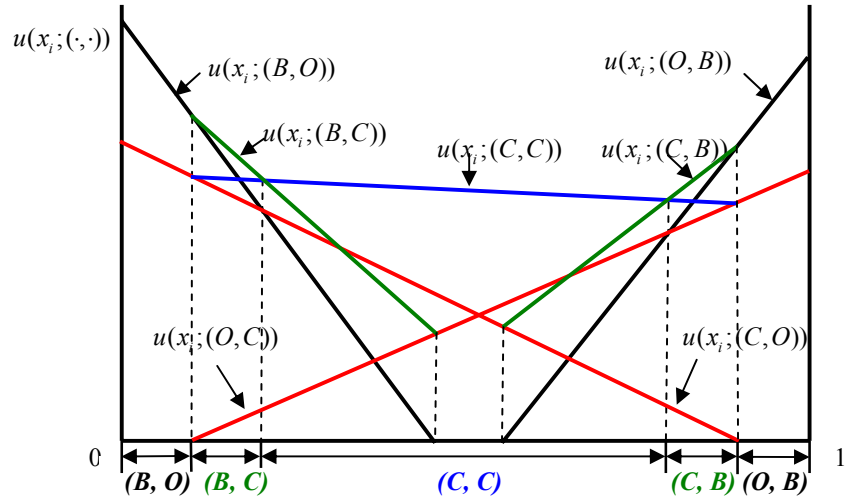
(a) Strategic Substitute Regime ($\beta \in [1/2, 1)$)
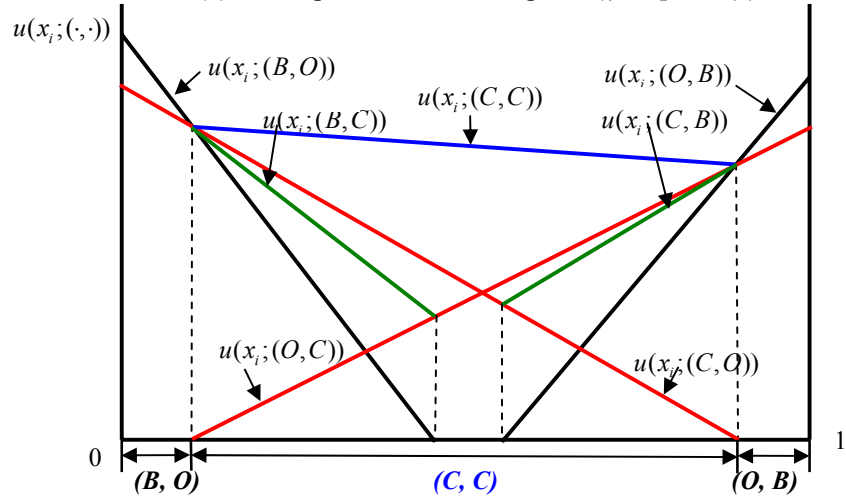
(b) Strategic Complement Regime ($\beta = 1$)

(c) No Strategic Interaction Regime ($\beta = 1$)

Figure 2 The Effect of the Public Copy Protection Rate Increase with $\alpha < \alpha'$
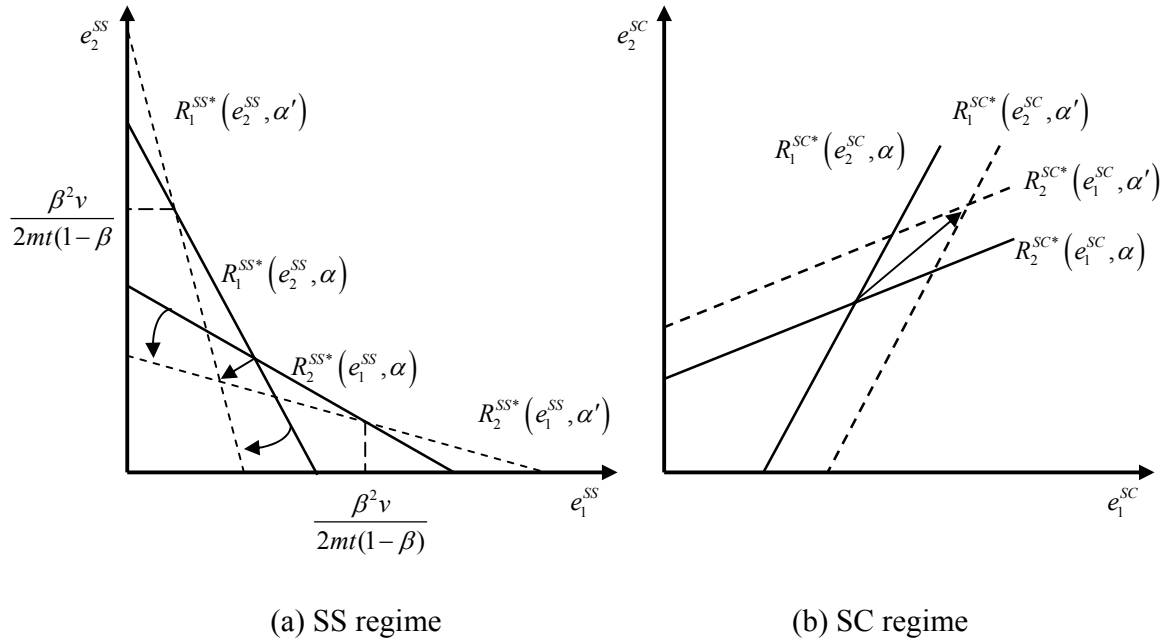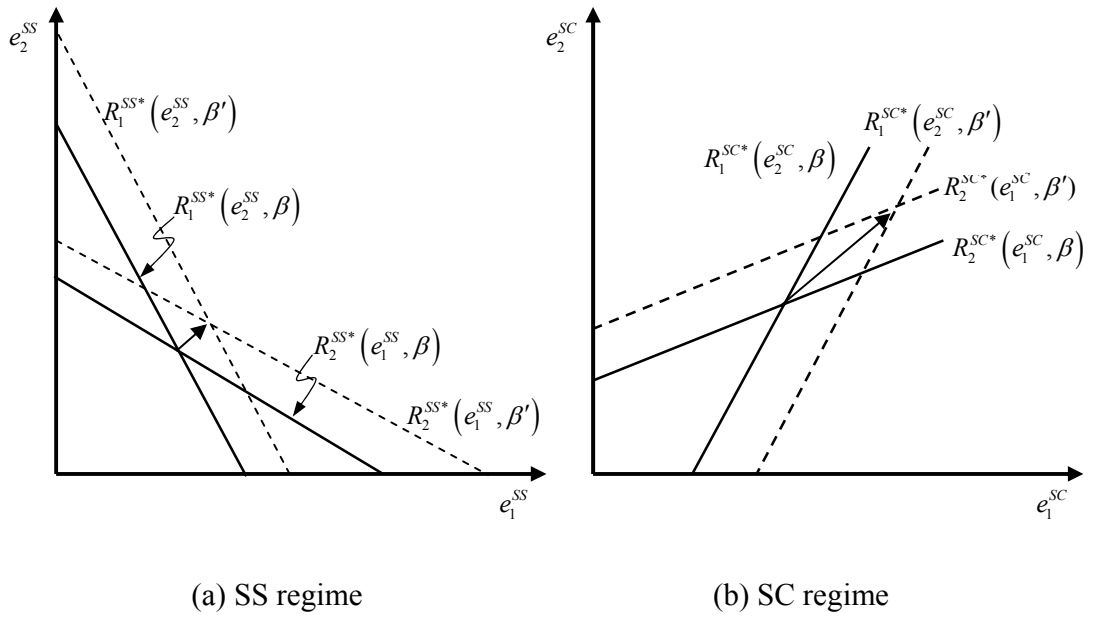

$$\frac{\beta^2 v}{2mt(1-\beta)}$$

$R_1^{SS*}\left(e_2^{SS}, \alpha'\right)$

$R_1^{SS*}\left(e_2^{SS}, \alpha\right)$

$R_2^{SS*}\left(e_1^{SS}, \alpha\right)$

$R_2^{SS*}\left(e_1^{SS}, \alpha'\right)$

$$\frac{\beta^2 v}{2mt(1-\beta)}$$

e_1^{SS}

e_2^{SS}

$R_1^{SC*}\left(e_2^{SC}, \alpha\right)$

$R_1^{SC*}\left(e_2^{SC}, \alpha'\right)$

$R_2^{SC*}\left(e_1^{SC}, \alpha'\right)$

$R_2^{SC*}\left(e_1^{SC}, \alpha\right)$

e_2^{SC}

e_1^{SC}

(a) SS regime   (b) SC regime

Figure 3 The Effect of the Compatibility of DRM with $\beta < \beta'$

$R_1^{SS*}\left(e_2^{SS}, \beta'\right)$

$R_1^{SS*}\left(e_2^{SS}, \beta\right)$

$R_2^{SS*}\left(e_1^{SS}, \beta\right)$

$R_2^{SS*}\left(e_1^{SS}, \beta'\right)$

e_2^{SS}

e_1^{SS}

$R_1^{SC*}\left(e_2^{SC}, \beta\right)$

$R_1^{SC*}\left(e_2^{SC}, \beta'\right)$

$R_2^{SC*}\left(e_1^{SC}, \beta'\right)$

$R_2^{SC*}\left(e_1^{SC}, \beta\right)$

e_2^{SC}

e_1^{SC}

(a) SS regime   (b) SC regime

Appendix. Proof of Proposition 4

Social welfare can be derived from the sum of the firms' profits and the consumer's surplus under different regime. However, we apply symmetry argument to each firm's profits and consumer surplus between $\left[0,\frac{1}{2}\right]$ and $\left[\frac{1}{2},1\right]$ so that we define the social welfare function as following for analytical convenience:

$SW^R = 2\left(\pi_1^R + CS^R\right)$ where

$$CS^R = \begin{cases} CS^{SS} = \int_0^{\tilde{x}(BC,CC)} u\left(x_i;(B,O)\right)dx + \int_{\tilde{x}(BO,BC)}^{\tilde{x}(BC,CC)} u\left(x_i;(O,C)\right)dx + \int_{\tilde{x}(BC,CC)}^{\frac{1}{2}} u\left(x_i;(C,C)\right)dx \\ CS^{SC} = \int_0^{\tilde{x}(BO,CC)} u\left(x_i;(B,O)\right)dx + \int_{\tilde{x}(BO,CC)}^{\frac{1}{2}} u\left(x_i;(C,C)\right)dx \\ CS^{NSI} = \int_0^{\tilde{x}(BO,CO)} u\left(x_i;(B,O)\right)dx + \int_{\tilde{x}(BO,CO)}^{\tilde{x}(CO,CC)} u\left(x_i;(C,O)\right)dx + \int_{\tilde{x}(CO,CC)}^{\frac{1}{2}} u\left(x_i;(C,C)\right)dx \end{cases}.$$

We examine the effect of an increase in the public protection on social welfare as

$\dfrac{\partial SW^{SS}}{\partial \alpha}$

$$= 2\left[\underbrace{\left[v - t\tilde{x}\left((B,C),(C,C)\right) + (1-\alpha)\left(v - t\left(1 - \tilde{x}\left((B,C),(C,C)\right)\right)\right) - e_2 - \left((1-\alpha)(2v-t) - \beta(e_1 + e_2)\right)\right]\frac{\partial \tilde{x}\left((B,C),(C,C)\right)}{\partial \alpha}}_{\text{demand switch (-)}} \\ \underbrace{-\left[(1-\alpha)\left(v - t\left(1 - \tilde{x}\left((B,O),(B,C)\right)\right) - e_2\right)\right]\frac{\partial \tilde{x}\left((B,C),(B,C)\right)}{\partial \alpha}}_{\text{single copy usage change (-)}} \underbrace{- \int_{\tilde{x}((B,C),(C,C))}^{\frac{1}{2}}\left(2v - t + \beta\left(\frac{\partial e_1}{\partial \alpha} + \frac{\partial e_2}{\partial \alpha}\right)\right)dx}_{\text{dual copy cost increase (-)}} \\ \underbrace{-\int_{\tilde{x}((B,O),(B,C))}^{\tilde{x}((B,C),(C,C))}\left(v - t(1-x) + \frac{\partial e_2}{\partial \alpha}\right)dx}_{\text{single copy cost increase (-)}} \quad \underbrace{-me_1\frac{\partial e_1}{\partial \alpha}}_{\substack{\text{DRM production cost}\\ \text{decrease (+)}}} \right] \quad \text{(A1)}$$

$$\frac{\partial SW^{SC}}{\partial \alpha} = 2\left[\underbrace{\left[v - t\tilde{x}\left((B,O),(C,C)\right) - \left((1-\alpha)(2v-t) - \beta(e_1 + e_2)\right)\right]\frac{\partial \tilde{x}\left((B,O),(C,C)\right)}{\partial \alpha}}_{\text{demand switch (+)}} \\ \underbrace{-\int_{\tilde{x}((B,O),(C,C))}^{\frac{1}{2}}\left(2v - t + \beta\left(\frac{\partial e_1}{\partial \alpha} + \frac{\partial e_2}{\partial \alpha}\right)\right)dx}_{\text{dual copy cost increase (-)}} \quad \underbrace{-me_1\frac{\partial e_1}{\partial \alpha}}_{\substack{\text{DRM production cost}\\ \text{increase (-)}}} \right] \quad \text{(A2)}$$

$$\frac{\partial SW^{NSI}}{\partial \alpha}$$

$$= 2 \left[\begin{array}{l} \underbrace{\left[v - t\tilde{x}\big((B,O),(C,O)\big) + (1-\alpha)\big(v - t\tilde{x}\big((B,O),(C,O)\big)\big) - e_1 \right] \dfrac{\partial \tilde{x}\big((B,O),(C,O)\big)}{\partial \alpha}}_{\text{demand switch (-)}} \\[2em] \underbrace{+ \left[(1-\alpha)\big(v - t\tilde{x}\big((C,O),(C,C)\big) - e_1\big) - \big((1-\alpha)(2v-t) - \beta(e_1+e_2)\big) \right] \dfrac{\partial \tilde{x}\big((C,O),(C,C)\big)}{\partial \alpha}}_{\text{single copy usage change (-)/(+)}} \\[2em] \underbrace{- \int_{\tilde{x}((C,O),(C,C))}^{\frac{1}{2}} \left(2v - t + \beta\left(\dfrac{\partial e_1}{\partial \alpha} + \dfrac{\partial e_2}{\partial \alpha}\right) \right) dx}_{\text{dual copy cost increase (-)}} \underbrace{- \int_{\tilde{x}((B,O),(C,O))}^{\tilde{x}((C,O),(C,C))} \left(v - tx + \dfrac{\partial e_1}{\partial \alpha} \right) dx}_{\text{single copy cost increase (-)}} \underbrace{- m e_1 \dfrac{\partial e_1}{\partial \alpha}}_{\substack{\text{DRM production cost} \\ \text{decrease (+)}}} \end{array} \right] \quad \text{(A4)}$$

Also, we examine the effect of an increase in the compatibility of DRM in term of hacking technology on social welfare as

$$\frac{\partial SW^{SS}}{\partial \beta}$$

$$= 2 \left[\begin{array}{l} \underbrace{\left[v - t\tilde{x}\big((B,C),(C,C)\big) + (1-\alpha)\big(v - t\big(1 - \tilde{x}\big((B,C),(C,C)\big)\big)\big) - e_2 - \big((1-\alpha)(2v-t) - \beta(e_1+e_2)\big) \right] \dfrac{\partial \tilde{x}\big((B,C),(C,C)\big)}{\partial \beta}}_{\text{demand switch (+)}} \\[2em] \underbrace{- \left[(1-\alpha)\big(v - t\big(1 - \tilde{x}\big((B,O),(B,C)\big)\big) - e_2\big) \right] \dfrac{\partial \tilde{x}\big((B,O),(B,C)\big)}{\partial \beta}}_{\text{single copy usage change (+)/(-)}} \underbrace{- \int_{\tilde{x}((B,C),(C,C))}^{\frac{1}{2}} \left((e_1+e_2) + \beta\left(\dfrac{\partial e_1}{\partial \beta} + \dfrac{\partial e_2}{\partial \beta}\right) \right) dx}_{\text{dual copy cost increase (-)}} \\[2em] \underbrace{- \int_{\tilde{x}((B,O),(B,C))}^{\tilde{x}((B,C),(C,C))} \left(\dfrac{\partial e_2}{\partial \beta} \right) dx}_{\text{single copy cost increase (-)}} \underbrace{- m e_1 \dfrac{\partial e_1}{\partial \beta}}_{\substack{\text{DRM production cost} \\ \text{increase (-)}}} \end{array} \right] \quad \text{(A5)}$$

$$\frac{\partial SW^{SC}}{\partial \beta} = 2 \left[\begin{array}{l} \underbrace{\left[v - t\tilde{x}\big((B,O),(C,C)\big) - \big((1-\alpha)(2v-t) - \beta(e_1+e_2)\big) \right] \dfrac{\partial \tilde{x}\big((B,O),(C,C)\big)}{\partial \beta}}_{\text{demand switch (+)}} \\[2em] \underbrace{- \int_{\tilde{x}((B,O),(C,C))}^{\frac{1}{2}} \left((e_1+e_2) + \beta\left(\dfrac{\partial e_1}{\partial \beta} + \dfrac{\partial e_2}{\partial \beta}\right) \right) dx}_{\text{dual copy cost increase (-)}} \underbrace{- m e_1 \dfrac{\partial e_1}{\partial \beta}}_{\substack{\text{DRM production cost} \\ \text{increase (-)}}} \end{array} \right] \quad \text{(A6)}$$

$$\frac{\partial SW^{NSI}}{\partial \beta} = 2 \left[\underbrace{\left[(1-\alpha)\left(v - t\tilde{x}\big((C,O),(C,C)\big) - e_1\right) - \left((1-\alpha)(2v-t) - \beta(e_1+e_2)\right) \right] \frac{\partial \tilde{x}\big((C,O),(C,C)\big)}{\partial \beta}}_{\text{single copy usage change (-)}} \\ \underbrace{- \int_{\tilde{x}((C,O),(C,C))}^{\frac{1}{2}} \left(e_1 + e_2\right) dx}_{\text{dual copy cost increase (-)}} \right] \quad (A7)$$

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