Induced Technical Change with Congestion of a Limited Natural Resource

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Abstract

I study a simple Neoclassical Growth Model with endogenous technical change including a production externality from a fixed input, called 'land', which represents the carrying capacity of the earth's atmosphere. Land is assumed to be congested by the use of labor and capital in production. Innovation is induced, or cost-driven, and may be directed towards each of the inputs to production. Optimal pricing of the fixed input will set in motion the induced innovation engine and fostering land-augmenting technological progress which will reduce environmental stress. The unique equilibrium of this economy is found to be locally asymptotically stable in the numerical analysis for substitution elasticities smaller than 1. The corresponding direction of technical change is characterized by a positive growth rate of land-augmenting technologies and a Harrod-neutral growth path with respect to capital and labor, and involves constant shares of all inputs. The share of land in output can be interpreted as a mitigation function. A competitive economy where land is free, instead, will fail to reach a steady state, and is therefore prone to either environmental or industrial regress.

Keywords: Climate Change, Induced Innovation, Congestion, Capital Accumulation, Mitigation.
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1 Introduction

In recent years, the global level of attention on climate change has risen considerably, spreading from the scientific community to policy-making and the general public. Quoting from the 4^{th} IPCC Synthesis Report 2007, which summarizes the agreement reached by the nations of the world on the key findings and the uncertainties about the issues at stakes:[1]

> Warming of the climate system is unequivocal, as is now evident from observations of increases in global average air and ocean temperatures, widespread melting of snow and ice and rising global average sea level (p. 30).

There is also a widespread consensus that greenhouse gases (GHGs) are to be ascribed among the causes of global warming: the concentration of CO_2 (carbon dioxide), CH_4 (methane) and N_2O (nitrous oxide) in the atmosphere increased as a result of human activities, mostly because of fossil fuel use and agriculture. GHGs accumulate in the atmosphere for very long time, and concentration of such gases lead to warming of land and oceans, as data on global average surface temperature and sea level display without any doubt for the more recent periods. In the words of the 4^{th} IPCC Synthesis Report,

> There is *very high confidence* that the global average net effect of human activities since 1750 has been one of warming (*ibid.*, p. 37).

Other than melting of permanent ice packs and consequent sea rise, coastal erosions, floods, climate change has important economic effects, too, as the nations participating to the 1997 Kyoto Conference recognized in agreeing to put forward a system of economic incentives to limit the emissions of GHGs.

Global warming is likely to display its economic consequences in the long-run, due to geophysical time constants such as the half-life of atmospheric carbon dioxide, so that climate change is naturally incorporated in growth models. On the other hand, it is universally recognized that technological change is a key driving force of economic growth in the world's largest economies, which are also responsible . In a world facing fundamental challenges from global warming, two important questions for economic theory are: i) what can be the role of economic incentives on patterns of technical change directed at reducing GHG emission or environmental stress in general? ii) in particular, given that at each point in time the amount of resources to allocate to R&D is limited, what *should* be a composition of technical progress that will ensure economic growth and environmental preservation at once?

A framework in which the direction of technological change responds to economic incentives appears to be appropriate to address these questions. The notion along which this paper is centered is that of *induced* technical progress, that is technological improvements developed in response to increases in production costs. Theoretical models incorporating induced technical progress considerations have been developed formally after the 1960s, starting with the thought-provoking contribution by Kennedy [24], followed by a number of articles by some of the most prominent scholars of the past century (Drandakis and Phelps [12], Samuelson [37], Nordhaus [33] are only a few examples). The distinctive feature of the induced innovation framework is Kennedy's [24] concept of 'Innovation Possibility Frontier' (IPF henceforth), a reduced-form function describing the trade-off between different types of factor-augmentation for given growth possibilities of the economy. A

[1] Available for download at http://www.ipcc.ch/ipccreports/index.htm.

recent wave of interest on the bias of technical progress, stemming from the important contributions by Acemoglu [1], [2], explicitly considers the microeconomic foundations behind the notion of an IPF. A microfoundation closer to the literature on induced (as opposed to directed) technical change can be found in Funk [14].

This paper develops a Neoclassical model of optimal growth incorporating induced innovation to provide an answer to the questions above. The production function of the stylized economy I study features, together with capital and labor, a fixed input of production which, following Foley [13], I will call 'land' and which represents the carrying capacity of the earth's atmosphere. Consistently with the agreement reached by the nations participating to the IPCC, atmosphere capacity is assumed to be congested by 'human activities', namely by the use of labor and capital in production. Another fundamental assumption is that the trade-off between factor-augmenting technological change represented by Kennedy's [24] IPF includes land-augmenting technologies.

Optimal growth requires (shadow-) pricing of every input including land, thus setting in motion the induced innovation engine of technical change on it. The equilibrium path of technical change is characterized by constant input shares, a positive growth rate of both labor and land augmentation and a zero growth rate of capital-augmenting technical progress. In a market economy where land is free, instead, the induced technical change mechanism is prevented from operating by failure to price the fixed resource. In the unpriced land case, if any, land augmentation will be too small, and the economy will fail to reach its steady state progressively reducing its production possibilities. This process can be characterized by either one of three scenarios, according to the actual shape of the IPF: (i) never-ending capital deepening, which in turn will produce increasing congestion on land; (ii) steady capital accumulation but decreasing land-efficiency; (iii) industrial regress taking place through progressive capital decumulation. Therefore, in all the three contexts failure to price land is a cause of concern for a market economy.

2 The Model

2.1 Basic Assumptions and Definitions

In a simple one-sector economy, production of output requires labor, L, capital, K and a natural resource, which we call 'land', representing the carrying capacity of the earth's atmosphere. The production possibilities of this economy are bounded by the following function:

$$Y = h(\theta)F(AL, BK) \tag{1}$$

where $\theta \equiv T/F(AL, BK)$, T being a productivity parameter which summarizes the current state of knowledge on land. The total amount of land available in the economy is normalized to 1, and we assume $h' > 0, h'' < 0$. The term $h(\theta)$ in the production function is meant to capture the 'effects of human activities' on climate change, and therefore is an instantaneous counterpart to a 'damage function' (see Nordhaus [35]). The idea is that the way in which capital and labor are utilized for production purposes (described in a stylized way by F) congest atmosphere capacity. On the other hand, F is the typical linearly homogeneous neoclassical production function, with A, B being positive parameters denoting respectively the current state of knowledge on labor and capital. It is easy to check that the production function (1) displays constant returns to scale in inputs measured in their respective efficiency units. Defining $y \equiv Y/L, x \equiv \frac{BK}{AL}$, the intensive form of (1) is:

$$y = h(\theta)Af(x) = h\left(\frac{\phi}{Af(x)}\right)Af(x) \tag{2}$$

where $\phi \equiv T/L$. Also, if $k \equiv K/L$, we have $x = Bk/A$. The standard regularity (Inada) conditions on f are assumed to be satisfied. Population grows exponentially at the exogenous rate n. There is no public sector: aggregate demand will equal consumption plus investment. Denoting by δ the 'radioactive', exogenous depreciation rate, the accumulation equation is $\dot{K} = sY - \delta K$, where s is the propensity to save. From what above, accumulation of capital per worker follows the law of motion:

$$\dot{k} = sh(\theta)Af(x) - (\delta + n)k \tag{3}$$

Productive factors are paid their marginal product. Following Foley [13], let us distinguish the case of a market economy not pricing land from the case of a planned economy in which the land externality is accounted for. If land is not priced, the land share in output will be zero (although the elasticity of output with respect to land will not), and the production share in output will equal 1. The capital share in the market economy is $\frac{f'(x)}{f(x)}x$, and the market labor share is given by $\frac{f(x)-f'(x)}{f(x)}x \equiv \omega(x)$. Also, the elasticity of substitution between capital and labor in f is defined as:

$$\sigma \equiv -\frac{f'(x)[f(x) - xf'(x)]}{xf(x)f''(x)} \tag{4}$$

Conversely, denote the land share in the planned economy as $\frac{\partial Y/\partial T}{Y}T = \frac{h'(\theta)}{h(\theta)}\theta \equiv \lambda(\theta)$. In this case, the capital share will be $[xf'(x)/f(x)][h(\theta) - \theta h'(\theta)]/h(\theta) = (1 - \omega(x))(1 - \lambda(\theta))$, and the labor share will equal $\omega(x)(1 - \lambda(\theta))$. Hence, the production share in the planned economy will be $1 - \lambda(\theta)$. It is worth observing that what I call 'land share' is comparable to what is traditionally referred to as 'mitigation' in the climate change literature, as for instance in Nordhaus [35]. Symmetrically, we define:

$$\eta \equiv -\frac{h'(\theta)[h(\theta) - \theta h'(\theta)]}{\theta h(\theta)h''(\theta)} \tag{5}$$

Finally, we extend the traditional framework of induced innovation by assuming that at each moment in time the growth rates of labor-, land- and capital-augmenting technical change are related by a three-dimensional version of Kennedy's [1964] IPF. Denoting $\frac{\dot{A}}{A} \equiv \alpha, \frac{\dot{T}}{T} \equiv \tau, \frac{\dot{B}}{B} \equiv \beta, (\alpha, \tau, \beta) \in \Upsilon \subset \mathbb{R}^3$, the IPF written in explicit form is :

$$\beta = g(\alpha, \tau), \quad \text{with } \nabla g < 0, \quad D^2 g \text{ negative definite} \tag{6}$$

and we assume, following Drandakis and Phelps [12], that there exist $0 < \bar{\alpha} < \infty, 0 < \bar{\tau} < \infty$ such that $g(\bar{\alpha}, \bar{\tau}) = -\infty$, so that the frontier is allowed to cross the axes and take values below zero for finite values of its arguments.

The induced innovation theorists of the '60s and '70s utilized a functional specification of the kind $\beta = g(\alpha)$, thus imposing that land-augmenting technical change doesn't enter the trade-off between factor-augmenting improvements. The inclusion of τ in the domain of the IPF, together with the form of the production function (1), are the building blocks of the analysis carried in this paper.

2.2 Optimal Direction of Technical Change

Let us start with the problem of allocating the *given* growth possibilities of the economy into different factor-augmenting technologies, similarly to the models by Kennedy [24], Drandakis and Phelps [12], Samuelson [37]. The analysis, which extends the framework by Nordhaus [33] is notationally simpler than that including the choice of optimal rate of technical change, and thus leads to an easier understanding of the implications arising from the assumptions made on technology for the patterns of technical change and income distribution arising when the congestion effects of human activities on atmosphere are optimally accounted for.

Consider a representative agent willing to maximize the present discounted value of consumption per capita over an infinite horizon taking into account the externality from the fixed input. It is convenient for our purposes to use the savings rate instead of consumption per capita as a control variable. On the other hand, the frontier (6) describes the trade-off in allocating the given growth rate of technological progress in different factor-augmenting improvements, so that the other control variable in the problem are α and τ. Hence, the planning authority solves:

$$\begin{array}{lll} \text{Choose } s,\alpha,\tau & \text{to maximize} & V(0)=\displaystyle\int_0^\infty e^{-\rho t}\left[(1-s)h(\theta)Af\left(\frac{Bk}{A}\right)\right]dt \\ & \text{subject to} & \dot{k}=sh(\theta)Af(x)-(\delta+n)k \\ & & \dot{B}=g(\alpha,\tau)B \\ & & \dot{A}=\alpha A \\ & & \dot{T}=\tau T \end{array} \tag{7}$$

Also, the initial conditions

$$A(0)=A_0,\quad B(0)=B_0,\quad T(0)=T_0,\quad k(0)=k_0\quad \text{given} \tag{8}$$

must be fulfilled, together with non-negativity of the shadow-prices $p_i(t)\geq 0\,\forall t,\forall i=1,\ldots,4$, and the transversality conditions, which we state in terms of the shadow-prices given that our problem has a free-end point:[2]

$$\lim_{t\to\infty} e^{-\rho t}p_1(t)=\lim_{t\to\infty} e^{-(\rho-\alpha_{ss})t}p_2(t)=\lim_{t\to\infty} e^{-\rho t}p_3(t)=\lim_{t\to\infty} e^{-\rho t}p_4(t)=0 \tag{9}$$

from which $\rho>\alpha_{ss}$ must hold for any positive value of $p_2(t)$. The associated Hamiltonian is:

$$\begin{aligned} \mathcal{H} &= e^{-\rho t}\left\{(1-s)h\left(\tfrac{\phi}{Af(\frac{Bk}{A})}\right)Af\left(\tfrac{Bk}{A}\right)+p_1\left[sh\left(\tfrac{\phi}{Af(\frac{Bk}{A})}\right)Af\left(\tfrac{Bk}{A}\right)-(\delta+n)k\right]\right\} \\ &\quad +e^{-\rho t}\left\{p_2e^{\alpha_{ss}t}g(\alpha,\tau)B+p_3\alpha A+p_4\tau T\right\} \end{aligned} \tag{10}$$

where α_{ss} denotes the steady state value for the growth rate of labor augmentation.[3] The first order necessary conditions for an ordinary maximum of the Hamiltonian are:

$$\frac{\partial\mathcal{H}}{\partial s}=(p_1-1)Ah(\theta)f(x)=0 \tag{11}$$

$$\frac{\partial\mathcal{H}}{\partial \alpha}=p_2e^{\alpha_{ss}t}Bg_\alpha+p_3A=0 \tag{12}$$

$$\frac{\partial\mathcal{H}}{\partial \tau}=p_2e^{\alpha_{ss}t}Bg_\tau+p_4T=0 \tag{13}$$

[2] See Sethi and Thompson [38], p.75 for a taxonomy of terminal conditions for a broad class of models.

[3] Observe that the adjoint variable for $\dot{B}$ is assumed to be $p_2e^{\alpha_{ss}t}$, so that the paper compares directly with the analysis in Nordhaus [33].

and are also sufficient because of strict concavity of g with respect of both α, τ and the fact that $\partial^2\mathcal{H}/\partial s^2 = 0$. Also, recall that even if in principle B is allowed to grow exponentially, we are not maximizing over B but on what makes it grow, and our assumptions on the production function and the IPF are enough to ensure concavity. On the other hand, the necessary conditions for optimality are existence of continuous function $p_i(t), i = 1, \dots, 4$ such that, denoting $\gamma \equiv 1 - s(1 - p_1)$:

$$\rho p_1 - \dot{p}_1 = \gamma B f'(x)[h(\theta) - \theta h'(\theta)] - (\delta + n)p_1 \tag{14}$$

$$(\rho - \alpha_{ss})p_2 - \dot{p}_2 = \gamma e^{-\alpha_{ss}t} k f'(x)[h(\theta) - \theta h'(\theta)] + g(\alpha, \tau)p_2 \tag{15}$$

$$\rho p_3 - \dot{p}_3 = \gamma[h(\theta) - \theta h'(\theta)][f(x) - f'(x)x] + \alpha p_3 \tag{16}$$

$$\rho p_4 - \dot{p}_4 = \gamma \frac{h'(\theta)}{L} + \tau p_4 \tag{17}$$

Since the constraint set is convex, for f, h, g being strictly concave and the other state variables being described by linear functions, the above equations together with the transversality conditions are also sufficient to characterize the optimal path.

2.2.1 Steady State

Equations (8)-(17) describe a system of necessary and sufficient conditions for optimality of the program (7) whose long-run solution we are interested in. One way to find such solution, followed by Nordhaus [33], is to note that at a steady state all shadow-prices must be constant. Using the resulting equilibrium values of the adjoint variables, we are able to solve for the long-run quantities we are interested in, that is the effective capital-labor ratio x, the effective land θ, and the growth rates of labor-augmenting and land-augmenting technical change. Setting all shadow-prices constant, and noting that at an equilibrium $p_1 = 1 = \gamma$ from (11), we obtain:

$$Bf'(x) = \frac{(\rho + \delta + n)}{h(\theta) - \theta h'(\theta)} = \frac{(\rho + \delta + n)}{[1 - \lambda(\theta)]h(\theta)} \tag{18}$$

$$p_2 = \frac{A[h(\theta) - \theta h'(\theta)]x f'(x)}{B[\rho - \alpha_{ss} - g(\alpha, \tau)]} e^{-\alpha_{ss}t} = \frac{A[1 - \lambda(\theta)]h(\theta)[1 - \omega(x)]f(x)}{B[\rho - \alpha_{ss} - g(\alpha, \tau)]} e^{-\alpha_{ss}t} \tag{19}$$

$$p_3 = \frac{[h(\theta) - \theta h'(\theta)][f(x) - f'(x)x]}{\rho - \alpha} = \frac{[1 - \lambda(\theta)]h(\theta)\omega(x)f(x)}{\rho - \alpha} \tag{20}$$

$$p_4 = \frac{h'(\theta)}{(\rho - \tau)L} \tag{21}$$

It is first useful to derive an equation of motion for θ. Logarithmic differentiation of $\frac{\phi}{Af(x)}$ yields:

$$\begin{aligned} \frac{\dot{\theta}}{\theta} &= \left(\frac{\dot{\phi}}{\phi} - \frac{\dot{A}}{A} - \frac{xf'(x)}{f(x)}\frac{\dot{x}}{x}\right) \\ &= \left\{(\tau - \alpha - n) - [1 - \omega(x)]\frac{\dot{x}}{x}\right\} \end{aligned} \tag{22}$$

This equation is important in the present model. It tells that the only force preventing the stock of effective land from deteriorating because of human activities is land-augmenting technical progress.

The next step is to find a dynamic equation for x. Because $p_1 = 1$ always along an optimal control, we can differentiate totally with respect to time (18) to derive:

$$
\begin{aligned}
\dot{B}f'(x) + Bf''(x)\dot{x} &= \tfrac{\rho+\delta+n}{(1-\lambda)^2 h^2}\left[\lambda' h - h'(1-\lambda)\right]\dot{\theta} \\
&= Bf'(x)\left[\tfrac{\lambda'}{1-\lambda} - \tfrac{h'}{h}\right]\dot{\theta} \\
&= Bf'(x)\left[\tfrac{\eta-1}{\eta}\tfrac{\lambda}{\theta} - \tfrac{\lambda}{\theta}\right]\dot{\theta} \\
&= -Bf'(x)\frac{\lambda}{\eta}\frac{\dot{\theta}}{\theta}
\end{aligned}
$$

Divide both sides by $Bf'(x)$, multiply and divide the second addendum in the LHS of the last equation by $\frac{xf(x)}{f(x)-xf'(x)}$, and use (4) to get:

$$
\frac{\dot{B}}{B} - \frac{\omega}{\sigma}\frac{\dot{x}}{x} = -\frac{\lambda}{\eta}\frac{\dot{\theta}}{\theta}
$$

Substituting (22) and solving for $\dot{x}/x$, we obtain the law of motion for the effective capital-labor ratio:

$$
\frac{\dot{x}}{x} = \frac{\eta\sigma}{\eta\omega(x) + \sigma[1-\omega(x)]\lambda(\theta)}\left[g(\alpha,\tau) - \frac{\lambda(\theta)}{\eta}(\alpha+n-\tau)\right] \tag{23}
$$

where it is understood that η, λ are functions of θ and σ, ω depend on x.

Evaluation of (15) and (16) at constant shadow-prices and substitution in (12) yields the optimal direction of labor-augmenting technical change:

$$
-g_\alpha = \frac{\omega(x)}{1-\omega(x)}\left[\frac{\rho - \alpha_{ss} - g(\alpha_{ss}, \tau_{ss})}{\rho - \alpha_{ss}}\right] \tag{24}
$$

Whereas, inserting the equilibrium values (19) and (21) into (13) yields the optimal direction of land augmentation:

$$
-g_\tau = \frac{\lambda(\theta)}{1-\lambda(\theta)}\frac{1}{1-\omega(x)}\left[\frac{\rho - \alpha_{ss} - g(\alpha_{ss}, \tau_{ss})}{\rho - \tau_{ss}}\right] \tag{25}
$$

From which it is apparent that when land is not priced $g_\tau = 0$ as in the model without land.

We are now able to characterize the steady state of this model. Since in steady state $\dot{x}/x = 0$, $\tau_{ss} = \alpha_{ss} + n$ in (22). Hence, $g(\alpha_{ss}, \tau_{ss}) = 0$. Summarizing, a long-run equilibrium of our system is:

$$
\tau_{ss} = \alpha_{ss} + n \tag{26}
$$

$$
g(\alpha_{ss}, \tau_{ss}) = \beta_{ss} = 0 \tag{27}
$$

$$
-g_{\alpha,ss} = \frac{\omega(x_{ss})}{1-\omega(x_{ss})} \tag{28}
$$

$$
\frac{\lambda(\theta_{ss})}{1-\lambda(\theta_{ss})} = -g_{\tau,ss}[1-\omega(x_{ss})]\left(\frac{\rho - \alpha_{ss} - n}{\rho - \alpha_{ss}}\right) \tag{29}
$$

where the last equation is solved for the ratio $\lambda/(1-\lambda)$ for comparative statics purposes, and requires as an additional condition that $\rho > \alpha_{ss} - n = \tau_{ss}$. As the battery of equations above shows, at the long-run equilibrium, there is no growth in capital-augmenting technologies, and a constant growth rate of labor-augmenting technological progress, meaning that technical change is Harrod-neutral with respect to capital and labor. Also, the long-run equilibrium involves the constancy of factor shares, once θ_{ss} and x_{ss} are pinned down by equations (28) and (29).

The long-run equilibrium described by equations (26)-(29) exists and is unique for $\sigma \neq 1 \cap \eta \neq 1$, paralleling what shown by the cited authors. In fact, when either of the substitution elasticities equals one, the innovation possibility frontier is not able to pin down the ratio of factor shares, for it is only the form of the (Cobb-Douglas, in this case) production function determining the factor distribution of income. When $\sigma \neq 1$, instead, $\omega(x) \in [0,1]$ and $\omega(x)/(1-\omega(x)) \in [0,\infty)$. Similarly, for $\eta \neq 1$, $\lambda(\theta) \in [0,1]$ and $\lambda(\theta)/(1-\lambda(\theta)) \in [0,\infty)$. Observe also that, since (26) is sufficient to determine the equilibrium rate of land augmentation given (28), the role of (29) is to pin down the equilibrium value of θ.

The optimal direction of land-augmenting technical progress resulting from this model compares interestingly to the results in Foley [13]. In his paper, he expands on Kennedy's result assuming that the growth rate of land-augmenting technologies is a function of the land share only. If we increase the dimensionality of IPF, instead, we see that (i) land augmentation depends negatively on the capital share and thus positively on the labor share, and (ii) the direction of land augmentation relates positively on both the rate of labor-augmenting technical progress and on population growth rate. Hence, being the direction of land-augmenting technical change derived in an optimizing framework, it will display important feedback effects from the other endogenous and exogenous variables of the model, which were ruled out by assumption in the previous treatments of the subject.

We can also compute the optimal long-run savings rate for the planned economy. Since $\frac{\dot{x}}{x} = \left(\frac{\dot{B}}{B} + \frac{\dot{k}}{k} - \frac{\dot{A}}{A}\right) = 0$, we have:

$$g(\alpha_{ss}, \tau_{ss}) + \frac{sBh(\theta)f(x_{ss})}{x_{ss}} - (\delta + n) - \alpha_{ss} = 0$$

so that, using (18):

$$s_{ss} = [1 - \omega(x_{ss})][1 - \lambda(\theta_{ss})]\left(\frac{\alpha_{ss} + \delta + n}{\rho + \delta + n}\right) \qquad (30)$$

The optimal savings rate is always less than 1, for $\rho > \alpha_{ss}$ from the transversality conditions. The savings rate of the benchmark model with no land is easily obtained setting $\lambda = 0$.

2.3 The Dynamical System

The solution approach we adopted above amounts to find a long-run equilibrium of the system as values for the variables of interest that ensure constant shadow-prices of all the state variables. This method has the disadvantage of being silent of what happens out of equilibrium.

On the other hand, a closer look at the sufficient conditions for a maximum of (10) reveals that we can exploit (11) on the one hand, and make use of the IPF and of its relative shadow-price on the other, to end up in a fully determined 4-dimensional system in two state variables, θ, x, and two control variables, α, τ. In order to do so, we proceed as it is usually done in standard courses on growth theory in 'eliminating' the adjoint variables of the Hamiltonian by repeated substitutions, so that we can focus on the behavior of the control variables and state variables only.

Let us start in standard fashion by totally differentiating (12) with respect to time:

$$\dot{p}_2 e^{\alpha_{ss}t} B g_\alpha + \alpha p_2 e^{\alpha_{ss}t} B g_\alpha + p_2 e^{\alpha_{ss}t} \dot{B} g_\alpha + p_2 e^{\alpha_{ss}t} B g_{\alpha\alpha}\dot{\alpha} = -\dot{p}_3 A - p_3 \dot{A}$$

Using (12) and (15), we have, rearranging:

$$e^{\alpha_{ss}t}Bg_\alpha\left[(\rho-\alpha_{ss})p_2-\frac{A}{B}e^{-\alpha_{ss}t}(1-\omega(x)f(x)(1-\lambda(\theta)h(\theta)\right]+p_2e^{\alpha_{ss}t}Bg_{\alpha\alpha}\dot{\alpha}=-\dot{p}_3A$$

Making use of (16), and then of (12) again, we obtain:

$$e^{\alpha_{ss}t}Bg_\alpha\left[(\rho-\alpha_{ss})p_2-\frac{A}{B}e^{-\alpha_{ss}t}[1-\omega(x)]f(x)[1-\lambda(\theta)]h(\theta)\right]+p_2e^{\alpha_{ss}t}Bg_{\alpha\alpha}\dot{\alpha}$$
$$=A(1-\lambda(\theta)h(\theta)\omega(x)f(x)+(\rho-\alpha)p_2e^{\alpha_{ss}t}Bg_\alpha$$

We can now harmlessly substitute (19) in the previous equation. Simplifying, we obtain:

$$\frac{1}{\rho-\alpha_{ss}-g(\alpha,\tau)}g_{\alpha\alpha}\dot{\alpha}[1-\omega(x)]=\omega(x)+\frac{\rho-\alpha}{\rho-\alpha_{ss}-g(\alpha,\tau)}g_\alpha[1-\omega(x)]$$

from which, finally:

$$\dot{\alpha}=\frac{1}{g_{\alpha\alpha}}\left\{g_\alpha(\rho-\alpha)+\frac{\omega(x)}{1-\omega(x)}[\rho-\alpha_{ss}-g(\alpha,\tau)]\right\} \tag{31}$$

Similar calculations lead to:

$$\dot{\tau}=\frac{1}{g_{\tau\tau}}\left\{g_\tau(\rho-\tau)+\frac{\lambda(\theta)}{1-\lambda(\theta)}\frac{1}{1-\omega(x)}[\rho-\alpha_{ss}-g(\alpha,\tau)]\right\} \tag{32}$$

It is obvious that equations (31) and (32) alone have the same equilibrium values as (24) and (25), and yield the same long-run equilibrium we found above when considered together with (22) and (23).

Summing up, we have derived a dynamical system formed by (22), (23), (31) and (32). We can now study the local stability properties of this system.

2.4 Stability Analysis

I now study the behavior of the dynamical system above in a neighborhood of its steady state. I will consider the following three cases: (i) competitive economy without land; (ii) competitive economy with unpriced land, and (iii) competitive economy pricing land. Although the first case is not immediately relevant for the purposes of this paper, it is of interest in itself because Nordhaus [33], who first studied the model without land, did not provide an analysis of the solution paths outside the equilibrium. This special case is also interesting because it is simple enough to be studied analytically.

2.4.1 Competitive Economy without Land Constraint on Production

Suppose for a moment that land poses no limits to production anymore. That is, suppose to scrap the externality altogether in order to study the simplest scenario arising in the special case where $h(\theta)=1,\dot{\theta}/\theta=0$ always. A reason for studying such special case might be a strong prior on atmosphere carrying capacity as a non-scarce factor, or disagreement on the conclusions about the relation between human activities and environmental stress reached by the IPCC.[4] In this simple

[4]The expression 'very high confidence' in the quotation in the introduction means that 9/10 of the IPCC members agreed on the sentence. We may think about the remaining 10% of the panel engaging in solving the special case exercise.

case, the dynamics of our system take place in the two-dimensional plane (x, α). The nontrivial rest point of this economy is $-g_\alpha = \frac{\omega(x_{ss})}{1-\omega(x_{ss})}, g(\alpha_{ss}) = 0$ which ensures constancy of the effective capital-labor ratio. The Jacobian matrix evaluated at the steady state is:

$$J_{Nordhaus,ss} = \begin{pmatrix} 0 & -\frac{\sigma(x_{ss})}{1-\omega(x_{ss})}x_{ss} \\ \frac{1}{g_{\alpha\alpha}}\left(\frac{1-\sigma}{\sigma}\right)\left[\frac{\omega(x_{ss})}{(1-\omega(x_{ss}))x_{ss}}\right] & \rho - \alpha_{ss} + \frac{1}{g_{\alpha\alpha}}\left(\frac{\omega(x_{ss})}{1-\omega(x_{ss})}\right)^2 \end{pmatrix}$$

The determinant is finite and negative if and only if $\sigma \in (0, 1)$ and positive if $\sigma > 1$, for $g_{\alpha\alpha} < 0$. Thus, if $0 < \sigma < 1$ the two eigenvalues are of opposite sign, and the so-called Harrod equilibrium[5] is saddle-path stable. Conversely, when $\sigma > 1$, we need to look at the trace of the matrix, too. A necessary and sufficient condition for the trace to be negative is:

$$-g_{\alpha\alpha} > \frac{1}{\rho - \alpha_{ss}}\left(\frac{\omega(x_{ss})}{1-\omega(x_{ss})}\right)^2 \tag{33}$$

If this is the case, under $\sigma > 1$ both eigenvalues are of equal sign and sum up to a negative number: the long-run equilibrium is stable. Conversely, if the inequality has the wrong sign, both eigenvalues are positive and the system is unstable. Therefore, the term in the RHS of the inequality (33) acts as a bifurcation parameter of the two-dimensional version of our system, provided that labor and capital are gross-substitutes in the production function.[6] The inequality (33) says that, even when $\sigma > 1$, the system may be stable or unstable according to how concave is Kennedy's IPF. It is important to point out, however, that there is very little empirical evidence supporting a substitution elasticity between capital and labor greater than 1. I will briefly survey this evidence in describing the calibration and simulation exercise I carry in the general four-dimensional case. In light of such evidence, the analysis of the case $\sigma > 1$ is included only for completeness.

These new findings on the model pair interestingly with the earlier result that, if $\sigma < 1$, an economy that remains in the Harrod equilibrium maximizes the preference functional (10), whereas the Harrod equilibrium is not the optimal path if $\sigma > 1$.[7] Thus, our simple economy either has an optimal saddle-path stable equilibrium as in the typical Neoclassical Growth Model, or a potentially unstable equilibrium that, however, does not maximize the preference functional (10), and it is not grounded in the available evidence on capital-labor substitution.[8] Of course, when $\sigma = 1$ an equilibrium with a role for the IPF doesn't exist in this model, so that it is needless to study its properties.

[Figure 1 about here]

Figure 1 displays the results of a simulation round over 200 periods of the model without land under the following calibration: $g(\alpha) = q - \frac{a}{\nu}\alpha^\nu$, $q = .02, \nu = 2$, and a being calibrated internally so as to solve $g(\alpha) = 0$. The discount rate is set

[5] See Nordhaus [33].

[6] In defining productive inputs as gross-substitutes if the elasticity of substitution is greater than 1, I follow Acemoglu [1].

[7] See Nordhaus [33] for a proof.

[8] Drandakis and Phelps [12] studied a dynamical system without land in the plane $(K, 1-\omega)$, and found that the system is stable or unstable according to σ being less than or greater than one respectively. The same result holds true in the Classical scenario without a land constraint studied as a special case in Foley [13].

exogenously at $\rho = .05$, equal to the depreciation rate, the population growth rate $n = .02$ and the elasticity of substitution equals 1/2. The endogenous variable x is set as to solve $\omega(x_{ss}) = 2/3$, which is roughly the observed labor share in total output in advanced market economies.[9]

2.4.2 Competitive Economy where Land is Unpriced

Consider a market economy (whose relative variables are denoted by the subscript M) where land is not priced. In this case, $-g_{\alpha\, M} = \omega/(1-\omega)$, but $g_{\tau M} = 0$. This is easily seen from the fact that the land share is zero, although the (negative of the) elasticity of output with respect to land is not. Equivalently, $p_4 = 0$ in the market economy not pricing land, so that equation 13 implies $g_{\tau\, M} = 0$. Because of our assumptions on the IPF,

$$-g_{\tau M}^{-1}(0) < -g_{\tau}^{-1}\left[\frac{\lambda(\theta_{ss})}{1-\lambda(\theta_{ss})}\frac{1}{1-\omega(x_{ss})}\left(\frac{\rho-\alpha_{ss}}{\rho-\alpha_{ss}-n}\right)\right]$$

Hence, $g_{\tau M}^{-1}(0) = \tau_M < \alpha_{ss} + n$, so that $g(\alpha, \tau_M) > 0$. Depending on the actual shape of the IPF, and provided that $\sigma, \eta > 0$, three cases may arise, but a steady state is never reached in any of them. Hence, there is no need for the study of the Jacobian matrix. The three possible scenarios are:

1. $\dot{x}/x > 0$ (*Overaccumulation Catastrophe*). From (23), we see that this case occurs when $\frac{g(\alpha,\tau_M)}{\alpha+n-\tau_M} > \frac{\lambda(\theta)}{\eta}$. The effective capital-labor ratio fails to reach its steady state value, and grows forever at a strictly positive rate, as long as $\sigma, \eta > 0$ (recall that although land is not priced the elasticity of output with respect to land is never zero in this model). Failure to price land completely overthrows the ability of an elasticity of substitution smaller than one to hold back capital deepening, contrary to the typical feature of the early models of induced innovation without externalities.[10] As a consequence, $\dot{\theta}/\theta$ doesn't reach its steady state either and keeps growing at a negative rate, so that land becomes increasingly congested reducing the production possibilities of the economy. Output will inevitably tend to zero, due to the destructive interplay of diminishing productivity of x in f and increasingly negative impact of θ on h (recall that $h'' < 0$).
   Hence, the unpriced land scenario in this model is as hopeless as the correspondent case in the Classical model by Foley [13]. The same forces of endlessly increasing capital deepening that determine an always rising effective capital-labor ratio are at work here, and their effect is enhanced by the congestion externality on land. As a result, the market capital share rises depressing the labor share. But the higher the capital share, the worse the impact of a rising x on output per worker, as it is easily seen by differentiating y with respect to x when land congestion is taken into account:

$$\frac{\partial y}{\partial x} = Ah(\theta)f'(x) - Ah'(\theta)(1-\omega)f(x)$$

[9]Given that we are dealing with a two-point boundary value problem with a saddle-path stable equilibrium, in order to compute the solution I specified an initial condition for the state variable x and a terminal condition for the control variable α. Standard duality arguments guarantee that the terminal condtion on α is equivalent to a terminal condition on p_2. As expected, the eigenvalues of the Jacobian matrix, evaluated numerically at several points along the optimal trajectories, are of opposite sign. The software used for simulations is *Mathematica* 6, and the code is available from the author upon request.

[10]This is just another way of saying that an equilibrium with induced innovation involves a constant capital-labor ratio, and this condition will not be met in this case.

If the effective capital-effective labor ratio keeps growing without reaching its steady state value, the inability of a market economy to price land triggers all forces driving technical progress to work in the wrong direction.

2. $\dot{x}/x = 0$ (*Environmental Decline with Steady Capital Accumulation*). The effective capital-labor ratio is constant, but θ fails to reach a steady state, shrinking forever toward zero. Induced technical change succeeds in holding back capital deepening, but land augmentation is not enough to overcome the congestion effect of production on land. The eventual state of zero production, however, is reached at a slower pace than in the previous case, given that there is no overaccumulation.

3. $\dot{x}/x < 0$ (*Capital Decumulation*). The forces at work (and the inequality discussed) in case 1 are reversed: capital decumulates forever, and land becomes less and less congested without any need of land augmentation.[11] At lower levels of output, the effective capital-labor ratio becomes more productive, but decreasing returns to effective land are also at work in lowering the impact of diminishing factor intensity on production. Capital accumulation, and output as a consequence, keeps tending to but never reaches zero because of the Inada condition $f'(0) = \infty$ which rules out the option of not undertaking production on the basis of economic convenience.[12]

We conclude that failure to price land is never harmless on a market economy. In particular, case 2 must not be underestimated, in what it leads to a state of the world similar to case 1 without sharing with the latter its evident catastrophic signs.

2.4.3 Competitive Economy where Land is Priced: Numerical Analysis

In this section, I analyze the stability properties of the full model numerically. From a calibration standpoint, the moments of the endogenous variables to match are: i) a long-run growth rate of labor productivity roughly equal to 2%; ii) a roughly constant capital productivity; iii) a labor share of about 2/3 of output. Parameters exogenously given to the model will be the discount rate ρ, the depreciation rate δ, and the population growth rate n. The depreciation rate is assumed to be 5% per period, and the population growth rate to equal 2% per period, both in standard fashion in the macro literature. As for the discount rate, we need to assume ρ to be big enough to ensure a positive and finite share of land in output.[13] In particular, the calibration of the discount rate must satisfy $\rho > \alpha_{ss} + n$. Hence, for these simulation rounds, I set $\rho = .05$.

The next step is to assume a functional form and calibrate the IPF. In what above, I only required the function to be decreasing and concave in both α, τ. The easiest specification one can think of is an exponential one:

$$g(\alpha, \beta) = q - \frac{a}{\nu}(\alpha + \beta)^{\nu}$$

Under a quadratic exponent ν, the parameters a, q can be easily calibrated internally. In fact, observation of (28) reveals that the LHS of the equation doesn't

[11] An issue not considered here is that, if idle machines cannot be destroyed completely, disposal of capital in the environment may be a cause of land congestion, or more generally an environmental threat too, as are for instance idle nuclear plants.

[12] Such feature refers to an elasticity of substitution smaller than 1.

[13] This follows from the assumed risk-neutrality in the utility function.

depend on the intercept q. Evaluating the equation at $(\alpha_{ss}, \tau_{ss}) = (.02, .04)$[14] and solving for a yields $a = 33.333$. Using this value, we can then set q so as to solve $g(\alpha_{ss}, \tau_{ss}) = 0$, which returns $q = .06$.

Finally, we need to assign values to the substitution elasticities σ, η. The vast literature on estimation of the elasticity of substitution between capital and labor is far from conclusive, but most of the articles on the subject suggest that the aggregate elasticity of substitution is significantly less than 1.[15] Nadiri [31], Nerlove [32] and Hamermesh [20] survey a range of early estimates of the elasticity of substitution, which are generally between 0.3 and 0.7. David and Van de Klundert [11] provide an estimate of σ to be in the neighborhood of 0.3. Using the translog production function, Griffin and Gregory [16] estimate elasticities of substitution for nine OECD economies between 0.06 and 0.52. Berndt [4], on the other hand, finds an estimate of the elasticity of substitution equal to 1, but does not control for a time trend, creating a strong bias towards 1. Using more recent data, and various different specifications, Krusell, Ohanian, Rios-Rull, and Violante [19] and Antras [3] also find estimates of the elasticity significantly less than 1. Estimates implied by the response of investment to the user cost of capital also typically yield an elasticity of substitution between capital and labor significantly less than 1 (see, e.g., Chirinko [8], Chirinko, Fazzari and Mayer [9] and [10], or Mairesse, Hall and Mulkay [27]).

Assigning a value of .5 to the elasticity of substitution σ seems a fairly appropriate average of the estimation results surveyed above. On the other hand, an estimate of the substitution elasticity between effective land and f is not only not available but also in principle problematic to obtain. To break ties, I follow Foley [13] in assuming $\eta = \sigma$, and as a robustness exercise I evaluate solutions for these elasticities equal to 0.3, 0.5, 0.7, which imply steady a sequence of steady-state values $\{(x_{ss,i}, \theta_{ss,i})\}_{i=1}^{3}$ equal to $\{(1.35, 1.9), (2, 4.5), (5.04, 33.43)\}$ respectively.[16]

[Figure 2 about here]

The Jacobian matrix, evaluated numerically, has four distinct eigenvalues inside the unit circle, and therefore the equilibrium is locally stable. The simulation round displayed in Figure 2 show that the model converges fairly quickly to its balanced growth path. Given our calibration, labor augmentation settles onto a 2% growth rate, land augmentation grows at 4%, and the labor share converges to a long-run value of 2/3 of F. The land share stabilizes around 18% of total output, which is fairly high compared to the results on mitigation appearing, for instance, in the DICE model by Nordhaus [35]. The reason of this discrepancy has to be found in the requirement on the discount rate. Typically, models of climate change assume exogenous technical change, and for simulation purposes calibrate the discount rate very low compared to macroeconomic growth models. Here, the discount rate has to be set fairly high for consistency reasons: one the one hand, the use of the Maximum

[14] The calibration of τ_{ss} follows from (26).

[15] The following survey closely follows Acemoglu [2].

[16] The simulation round for the full model depicted in Figure 2 is obtained by taking a discrete-time approximation of the system. As it is clear from a close look at the dynamical system, this approximation is harmless. The system is solved in *Mathematica* using the function 'FindRoot'. Due to the two point-boundary nature of the problem, initial conditions are given for the state variables x, θ and terminal conditions for the variables α, τ. Standard duality arguments guarantee that a terminal condition on the control variables is equivalent to a terminal condition on the adjoint variables. On the other hand, as a consistency check on the optimality of the solutions, I computed equilibrium paths assigning different values to the terminal t, up to 2000 periods, and find that the solution for the first 200 periods is not sensitive to the choice of the terminal horizon.

Principle to solve the planner's problem requires $\rho > \alpha_{ss}$. On the other hand, we want the land share/mitigation to be positive and finite, which in turns imposes $\rho > \alpha_{ss} + n$.

2.5 Comparison of Growth Paths

The growth rate of the economy pricing land at a balanced growth path will be given by:

$$\begin{aligned} \frac{\dot{y}}{y} &= \frac{h'(\theta)}{h(\theta)}\theta\left(\frac{\dot{\phi}}{\phi} - \frac{\dot{A}}{A} - \frac{xf'(x)}{f(x)}\frac{\dot{x}}{x}\right) + \frac{\dot{A}}{A} + \frac{xf'(x)}{f(x)}\frac{\dot{x}}{x} \\ &= \lambda(\theta_{ss})(\tau_{ss} - n - \alpha_{ss}) + \alpha_{ss} \\ &= \alpha_{ss} \end{aligned} \tag{34}$$

On the other hand, a competitive economy with no externality from land grows at the warranted rate $\alpha_{ss} + n$. Finally, a competitive economy in the unpriced land case will grow at the (unbalanced) rate:

$$\frac{\dot{y_M}}{y} = \lambda\left(\frac{\eta + (1-\lambda)}{\eta}\right)(\tau - \alpha - n) + \alpha + \frac{\eta\sigma}{\eta\omega(x) + \sigma[1-\omega(x)]\lambda(\theta)}(1-\omega)(1-\lambda)g(\alpha, \tau)$$

As it is intuitive, there is a one-to-one correspondence between the three scenarios discussed above and $\frac{\dot{y}_M}{y} \gtreqless \alpha_{ss}$.

2.6 Comparative Dynamics

In analyzing the dynamical properties of our solution path, we found that the long-run equilibrium of the system is unique and locally stable. It is then of interest to study the effect of changes in the exogenous variables of the model on its steady state. Consider first an increase in the population growth rate. It is clear from (26) that the long-run growth rate of land-augmenting technical change must increase. On the other hand, differentiating the right-hand side of (29) with respect to n we can see that the ratio $\lambda/(1-\lambda)$, and therefore the land share, will fall, as it is intuitive given that there is full employment and the whole population growth will be absorbed in production. Also, the savings rate responds positively, and in the same way, to both n and δ, given $\rho - \alpha_{ss} > 0$.

An increase in the discount rate ρ will reduce the optimal savings rate given the higher degree of time-impatience, as it is standard in growth theory. What is perhaps surprising is that a higher discount rate determines a higher share of land in total output. In fact,

$$\frac{\partial\lambda/(1-\lambda)}{\partial\rho} = -g_{\tau,ss}\frac{(1-\omega)n}{(\rho-\alpha_{ss})^2} > 0$$

This result is, however, much less counterintuitive than it seems. A look at (17) reveals that when the discount rate increases, the shadow-price of land augmentation increases. The central plan office compensates for a higher time-impatience of its citizens by increasing technical change directed at reducing congestion on land. To put it differently, since an amount in ρ determines an increase in present consumption, the social planner has to compensate for the higher consumption through an increase in mitigation on land. Figure 3 plots the solution paths for $\rho = .1$. The increase in time-impatience determines a dramatic increase in the equilibrium land share. All the other parameters are calibrated as above.

[Figure 3 about here]

On the other hand, an increase in population growth requires recalibration of the IPF, and this is due to the fact that $\beta = 0$ must hold in equilibrium. In Figure 4, I increase population growth to .4, and consequently τ_{ss} =.6, and reassign values to a, q so as to ensure $g(.2, .6) = 0$.

[Figure 4 about here]

2.7 Optimal Rate of Technical Change

In the above setup, we studied the determination of the optimal direction of technological change. For the framework to become an endogenous growth model we need to include also the planner's choice of intensity, or rate, of technical change. The conclusions about the long-run equilibrium of the system reached in the previous section are not sensitive to the inclusion of the optimal rate of technical change in the planner's problem.

Consider again a representative agent determining both the direction and the rate of technological change by allocating part of her labor time into the educational sector, as in Uzawa [41]. Imposing full employment of labor, the amount of labor in the educational sector will be a fraction of the total labor force: $L_e = (1-u)L$. To further simplify the analysis, normalize $L = 1$. Also, workers in L_e can be employed in any of the factor-augmenting technologies. To keep things simple, I make the assumption that land augmentation is subject to the same technology as other factor-augmentations, and that the planner chooses the portion ν of workers L_e employed on production of land-augmenting technologies. Therefore, we can rewrite equations (6) as:

$$\dot{A} = \alpha\xi(1-\nu u)A, \quad \dot{T} = \tau\xi[1-u(1-\nu)]T, \quad \dot{B} = g(\alpha,\tau)\xi(1-\nu u)B \tag{35}$$

where g is exactly as above. Following Uzawa [41], we assume that the function ξ is concave enough to ensure that the present discounted value of consumption per capita converges as $t \to \infty$, that is we assume: $\xi(1) < \rho < \xi(0) + \xi'(0)$, and $\xi' > 0, \xi'' < 0$. The planning problem is to choose s, α, τ, u, ν to maximize (7) under the constraints (3), (35), (8) and (9). The Hamiltonian of the problem is:

$$\begin{aligned}\mathcal{H} = e^{-\rho t}\left\{(1-s)Ah\left(\frac{\phi}{Auf(x/u)}\right)uf\left(\frac{x}{u}\right) + p_1\left[sAh\left(\frac{\phi}{Auf(x/u)}\right)uf\left(\frac{x}{u}\right) - (\delta+n)\frac{A}{B}x\right]\right\} \\ +e^{-\rho t}\left\{p_2 e^{\alpha_{ss}\phi(1-\nu_{ss}u_{ss})t}g(\alpha,\tau)\phi(1-\nu u)B + p_3\alpha\phi(1-\nu u)A + p_4\tau\phi[(1-u(1-\nu)]T\right\}\end{aligned} \tag{36}$$

where the conjugate variable for B is now $p_2 e^{\alpha_{ss}\xi(1-\nu_{ss}u_{ss})t}$ because of the specification of the technology for production of factor-augmentation. The first-order conditions for an ordinary maximum of (36) are:

$$(p_1 - 1)Ah(\theta)uf\left(\frac{x}{u}\right) = 0 \tag{37}$$

$$\frac{\partial \mathcal{H}}{\partial \alpha} = p_2 e^{a_{ss}\xi(1-\nu_{ss}u_{ss})t}g_\alpha\xi(1-\nu u)B + p_3\xi(1-\nu u)A = 0 \tag{38}$$

$$\frac{\partial \mathcal{H}}{\partial \tau} = p_2 e^{\alpha_{ss}\xi(1-\nu_{ss}u_{ss})t}g_\tau\xi(1-\nu u)B + p_4\xi[1-u(1-\nu)]T = 0 \tag{39}$$

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial u} = \gamma A\left[h(\theta) - \theta h'(\theta)\right]\left[f\left(\tfrac{x}{u}\right) - \tfrac{x}{u}f'\left(\tfrac{x}{u}\right)\right] \\ -\xi'\left\{\nu\left[p_2 e^{\alpha_{ss}\xi(1-\nu_{ss}u_{ss})t}g(\alpha,\tau)B + p_3\alpha A\right] + (1-\nu)p_4\tau T\right\} = 0\end{aligned} \tag{40}$$

$$\frac{\partial \mathcal{H}}{\partial \nu} = p_4\xi' u\tau T - \left[p_2 e^{\alpha_{ss}\xi(1-\nu_{ss}u_{ss})t}g(\alpha,\tau)B + p_3\alpha A\right]\xi' u = 0 \tag{41}$$

Equations (14)-(17) modify as follows:

$$\dot{p}_1 = (\rho + \delta + n)p_1 - \gamma \left[h(\theta) - \theta h'(\theta)\right] f'\left(\frac{x}{u}\right) B \tag{42}$$

$$\begin{aligned} \dot{p}_2 \quad &= \left[\rho - \alpha_{ss}\xi(1-\nu_{ss}u_{ss}) - g(\alpha,\tau)\xi(1-\nu u)\right] p_2 \\ &- \left[h(\theta) - \theta h'(\theta)\right] \gamma f'\left(\tfrac{x}{u}\right) e^{-\alpha_{ss}\xi(1-\nu_{ss}u_{ss})t} \tfrac{A}{B} x \end{aligned} \tag{43}$$

$$\dot{p}_3 = [\rho - \alpha\xi(1-\nu u)]p_3 - \gamma \left[h(\theta) - \theta h'(\theta)\right] u \left[f\left(\frac{x}{u}\right) - \frac{x}{u} f'\left(\frac{x}{u}\right)\right] \tag{44}$$

$$\dot{p}_4 = \{\rho - \tau\xi[1 - u(1-\nu)]\}p_4 - \gamma \frac{h'(\theta)}{L} u f\left(\frac{x}{u}\right) \tag{45}$$

while equations (18) and (28)-(30) become:

$$B_{ss} f'\left(\frac{x_{ss}}{u_{ss}}\right) = \frac{\rho + \delta + n}{[1 - \lambda(\theta)]h(\theta)} \tag{46}$$

$$g(\alpha_{ss}, \tau_{ss}) = \psi(\theta_{ss}) \left\{\tau_{ss}\xi[1 - u_{ss}(1-\nu_{ss}) - n - \alpha_{ss}\xi(1-\nu_{ss}u_{ss})]\right\} = 0 \tag{47}$$

$$-g_\alpha = \frac{\omega(x)}{1 - \omega(x)} \tag{48}$$

$$-g_\tau = \frac{\lambda(\theta_{ss})}{1 - \lambda(\theta_{ss})} \frac{1}{1 - \omega(x_{ss})} \left[\frac{\rho - \alpha_{ss}\xi(1-\nu_{ss}u_{ss})}{\rho - \tau_{ss}\xi[1 - u_{ss}(1-\nu_{ss})}\right] \frac{\xi[1 - u_{ss}(1-\nu_{ss})]}{\xi(1-\nu_{ss}u_{ss})} \tag{49}$$

$$s_{ss} = [1 - \omega(x_{ss})][1 - \lambda(\theta_{ss})] \left[\frac{\alpha_{ss}\xi(1-\nu_{ss}u_{ss}) + \delta + n}{\rho + \delta + n}\right] \tag{50}$$

The difference with the equilibrium conditions in the previous section being only the appearance of the intensity values multiplying the factor-augmentation rates. Also, the equation determining the optimal innovation intensity is, from (40):

$$\xi'(u_{ss}) = \frac{\omega(x)}{\nu\left[\frac{(1-\omega(x))\lambda(\theta)(n-\tau_{ss})}{\rho - [\alpha_{ss} + \lambda(\theta)(n-\tau_{ss})]\xi(1-\nu_{ss}u_{ss})} + \frac{\omega(x)\alpha_{ss}\xi(1-\nu_{ss}u_{ss})}{\rho - \alpha_{ss}\xi(1-\nu_{ss}u_{ss})}\right] + (1-\nu_{ss})\frac{\lambda(\theta)\tau_{ss}\xi[1-u_{ss}(1-\nu_{ss})]}{\rho - \tau\xi[1-u_{ss}(1-\nu_{ss})]}} \tag{51}$$

and from (41) we have:

$$\frac{(1-\omega(x))\lambda(\theta)(n-\tau_{ss})}{\rho - [\alpha_{ss} + \lambda(\theta)(n-\tau_{ss})]\xi(1-\nu_{ss}u_{ss})} + \frac{\alpha_{ss}\omega(x)}{\rho - \alpha_{ss}\xi(1-\nu_{ss}u_{ss})} = \frac{\lambda(\theta)\tau_{ss}}{\rho - \tau_{ss}\xi[1 - u_{ss}(1-\nu_{ss})]} \tag{52}$$

The long-run equilibrium of the model including the choice of intensity of technical change is exactly the same as the simpler case of choice of direction only, and involves positive land and labor augmentation, zero capital augmentation, and constant shares of all inputs. The conclusions we reached above extend to the market economy where land is not priced. The same properties of the resulting dynamical system can be derived by assuming u, ν to be constant over time at their equilibrium values.

3 Discussion

Economists have been dealing with environmental issues for a long time. Sophisticated models for environmental policy evaluation have been developed over the past three decades, a recent example being the DICE-2007 studied by Nordhaus [35]. In that model, the issue of direction of technical change in presence of externalities from the atmosphere capacity is not addressed, as technical change is assumed to be Hicks-neutral. Hicks-neutrality of technical change is a natural outcome of models of induced innovation when there is no accumulating factor,[17] but it is at odd with a basic stylized fact of capitalist development: increasing labor productivity coupled with rising capital/labor ratio. The present model produces a Harrod-neutral path of technological progress with respect to capital and labor, and a long-run endogenous rate of land-augmenting technical progress equal to the sum of population growth rate and the rate of growth of labor-augmentation.

The viewpoint I took in this paper is that induced technical change can be a powerful determinant of patterns of economic growth, and that the early contributions on induced innovation have proved to be successful in reproducing some of the long-run features of capitalist economies. Hence, I chose to generalize a fairly old model of optimal technical change with induced innovation, first studied by Nordhaus [33]. The model presented here can be seen as a Neoclassical counterpart of the Classical framework developed more recently by Foley [13], and shares with the latter the basic idea: assigning a (shadow-) price to land will induce cost-reducing technical change directed at economizing the use of land in production, thus reducing environmental strees. Differently from Nordhaus, I introduced an externality arising from a fixed natural resource, affecting production of output. Unlike Foley's model, in which the dependence of each factor-augmenting technical change on its own share in costs is assumed, I augmented the dimensionality of Kennedy's [24] IPF to include land-augmenting technical change. Such feature of the present framework creates important equilibrium feedbacks among different kinds of factor-augmentation, including land, that were ruled out by assumption in the previous analyses of the subject. Another difference is that Foley's model is closed by a Goodwin predator-prey cycle, whereas full employment in the planned economy is imposed here. This feature of the model calls for relaxing the assumption of a fully-employed labor force in order to address the possible presence of trade-offs or trade-ins between labor market institutions and environmental policies. Finally, the scenario depicted by Foley in his discussion of the unpriced land case is only one among the three possible cases arising in the present model. The reason behind this different result lays in the different assumption on technology I made in this paper, which explicitly considers land congestion through human activities in agreement with the consensus reached by the nations participated in the IPCC.

The induced innovation approach has been sharply criticized for its lack of microfoundations, especially in [34], and this weakness, among others, was responsible for the decline of growth models based on induced technical change. At the firm level, in fact, it is not clear how innovation can be financed and priced if there are constant returns to scale and competition. The problem of reconciling economic growth with competition, however, is common to the whole early growth literature, and involves removing the assumption of constant returns to scale in the production function. Years of literature on Endogenous Growth have addressed this issue, spanning from AK frameworks, to models of human capital accumulation, to R&D-based growth models. All these strands of literature feature increasing returns. Recent models with decreasing returns to scale reconciled growth and competition, using the fact that firms in regime of decreasing returns have inframarginal rents to finance R&D expenditure.[18] Models of economically directed technical change adopting a different approach than the one taken here (Acemoglu [2]) assume that production of

[17] See Samuelson [37] for an enlightening illustration of this point.

[18] A very recent example of a model with decreasing returns and induced innovation, also providing detailed references to the literature, is Zamparelli [43]. The price to pay for this reconciliation between growth and competition, however, is that the optimal dimension of the firm tends to zero.

the final good occurs competitively with constant returns to scale to effective inputs, but uses intermediate goods that are produced by technology monopolists. To this extent, the model presented here provides a different microeconomic structure underlying technical change.

Another matter of criticism is the assumed stationarity of the trade-off between factor augmentations represented by the IPF. Magat [28], and Skott [39] explored the implications of depletion of innovation possibilities on the dynamics of factor shares in a model based on that of Drandakis and Phelps [12]. Depletion of innovation possibilities can be seen as a way to capture environmental decline occurring even at a faster pace than what is implied by the aggregate production function 1. This extension is left for future research. On the other hand, in the market counterpart of the present model based on imperfect competition, there is no need for an explicit specification of the IPF (see Acemoglu [1]), because each factor-augmenting technical progress will be determined by its own technology. 'Innovation possibility frontiers' will be implicitly determined by the relative slopes of such technologies, and their slope will be time-varying out of equilibrium. It is worth reiterating that the purpose of this paper is to study the normative implications of the optimal direction of technical progress and this justifies my choice of not using the microfounded IPF appearing in Acemoglu [1].

A key implication of the congestion hypothesis in the unpriced market case is that labor and capital will each appropriate a portion of land's contribution to the productive process, so that the market will remunerate factors more than it is socially optimal. As a consequence, if labor is fully employed in the planned economy, it cannot be in the market economy not pricing land. This poses problems additional to the ones already outlined in the comparisons of the two economies regarding their innovative ability. On the other hand, it points toward extensions of the framework to open economies in order to study the interaction between environmental policies and movements of labor and capital across countries, with the difficulty that such interaction will occur when the country not pricing land is out of the steady state path.

4 Conclusions

In this paper, I generalized an early model of induced technical change first studied by Nordhaus [33]. The extension amounts to: i) include a production externality from a fixed resource representing atmosphere carrying capacity, which I called 'land' following Foley [13], and which is congested by the use of labor and capital in production; ii) to increase the dimensionality of Kennedy's [24] IPF by adding land-augmenting technical change. In standard fashion, I was also able to study both analytically and numerically the dynamical system that results out of the infinite horizon problem (7), this way providing an account of the equilibrium and out-of-equilibrium dynamics of the model, and addressing long-standing questions remained unanswered in the framework. I showed that:

1. The competitive equilibrium without land is either saddle-path stable if the elasticity of substitution between labor and capital is less than one, or stable if the substitution elasticity is greater than one *and* the IPF is sufficiently steep at the steady state, a precise meaning of the adjective 'steep' being given by the fulfillment of the inequality (33).

2. A market economy not pricing land always fails to reach a steady state, and may end up in either one of three worrying scenarios: (i) a catastrophe led by

overaccumulation of capital: (ii) a slower environmental decline where capital deepening is held back by Induced technical change but land congestion is not; (iii) a path of industrial regress where capital progressively decumulates.

3. A competitive economy where land is priced has an equilibrium, unique when $\sigma, \eta \neq 1$, in which the shares of all inputs are constant, the rates of labor and land augmentation are positive, and the rate of capital-augmenting technical change is zero. I showed that, under the calibration proposed, the equilibrium path is locally asymptotically stable.

These findings lose some of their importance if we consider that the postulated existence of an IPF is just a parable, as empirical tests of the link between rates of factor-augmenting technical progress and relative factor shares are hardly available. Also, a different type of exercise might be to study the role of land congestion in the natural market counterpart of the growth model analyzed in section 2.7, that of human capital due to Lucas [26]. I also don't address intergenerational equity issues (see for instance Greiner and Semmler [15]), nor the role played by uncertainty in climate change (Weitzman [42]). All the above directions in which the induced innovation framework can be extended to include externalities from the atmosphere capacity appear to be fruitful areas for further investigation.

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Figures

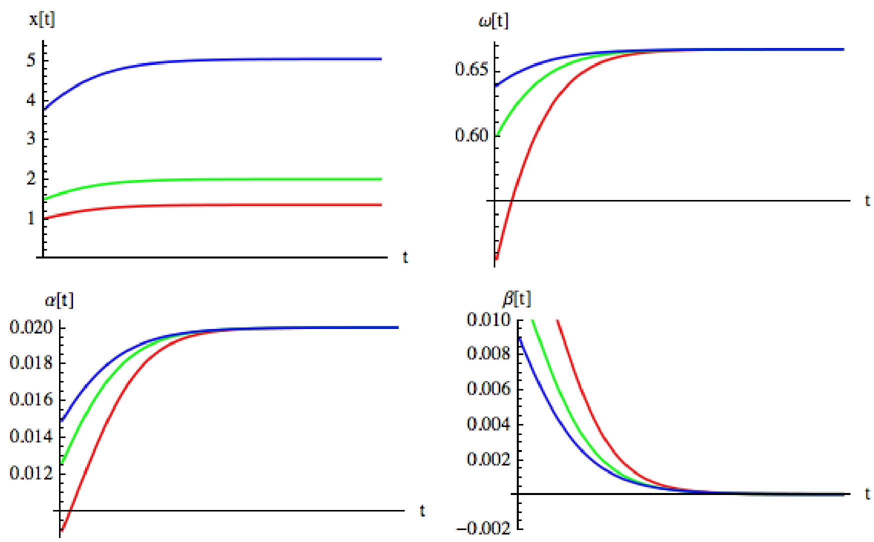

Figure 1: Simulation results for the planned economy without land over 200 periods. Elasticity of substitution: $\sigma = \{.3 \text{ (red)}, .5 \text{ (green)}, .7 \text{ blue}\}$.

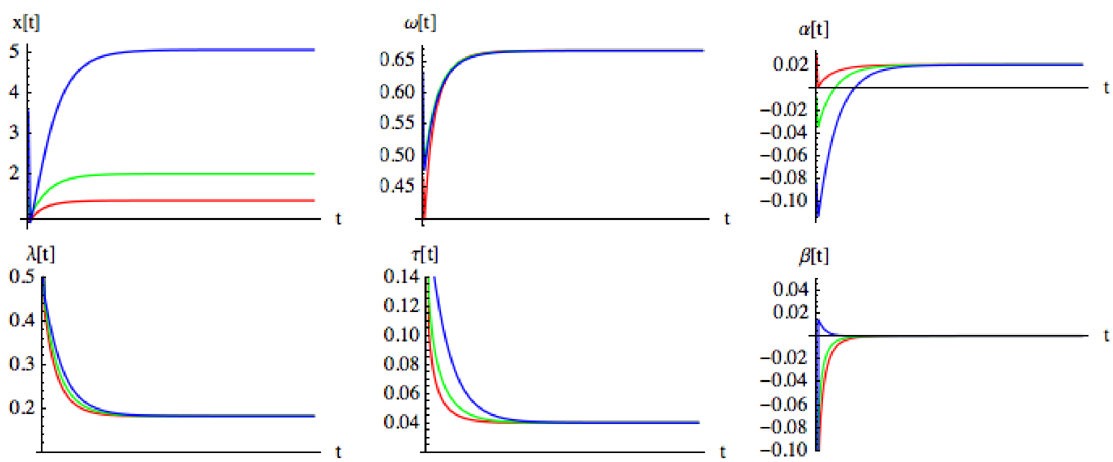

Figure 2: Simulation results for the full model: $\rho = .05$, $\delta = .05$, $n = .02$. Elasticities of substitution: $\sigma = \eta = \{0.3 \text{ (red)}, 0.5 \text{ (green)}, 0.7 \text{ (blue)}\}$.

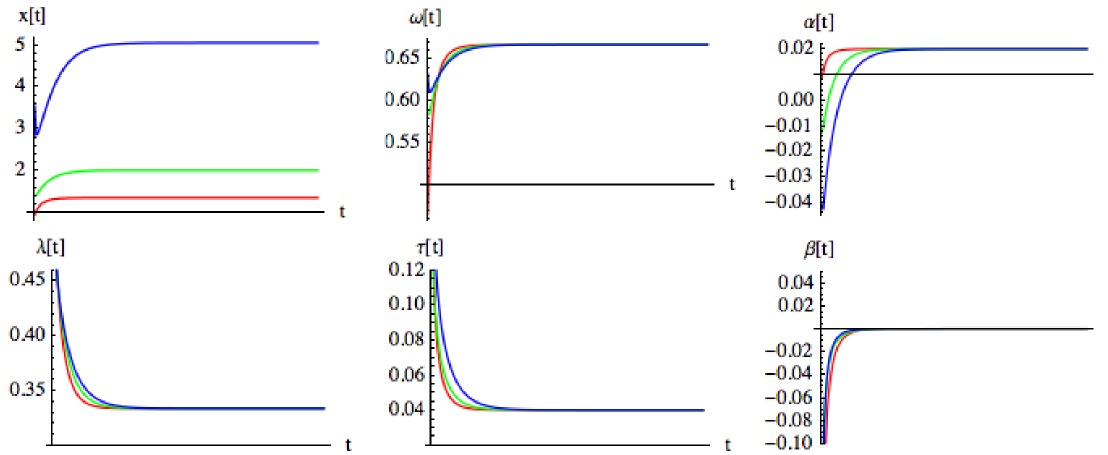

Figure 3: Simulation results for $\rho = .1$, all the other parameters as above.

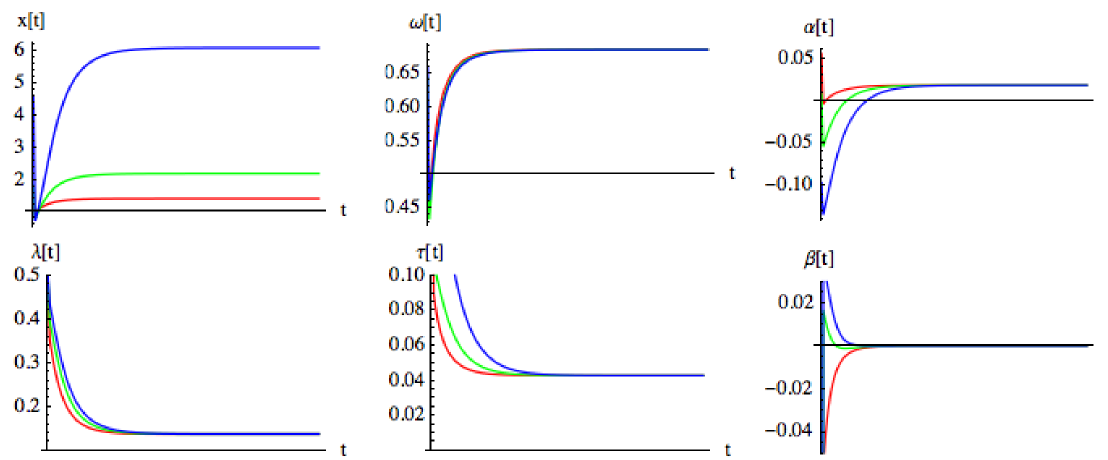

Figure 4: Simulation results for $n = .025$, all the other parameters as above.