The Direction of Innovation in the Long-Run

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FRAME Research Training, Mannheim
October 18, 2018

This project has received funding from the European Union's Horizon 2020 Research and Innovation Programme under grant agreement No 727073

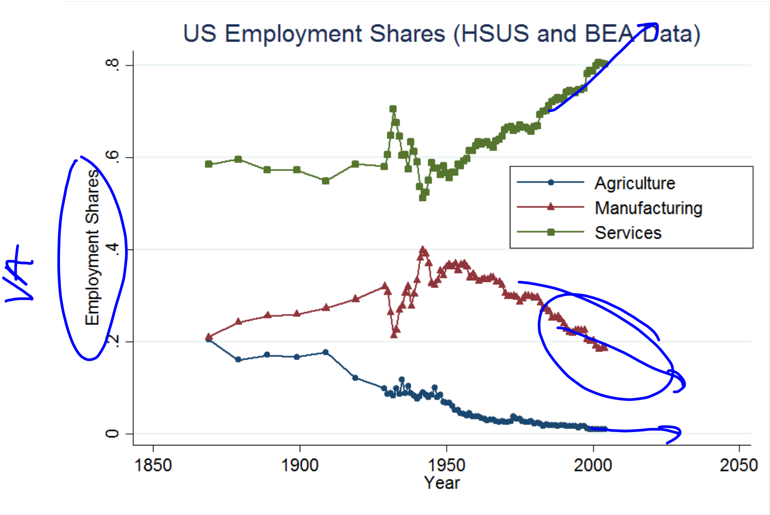
Introduction

- Technology is a key driver of modern economic growth (Klenow and Rodríguez-Clare 1997, Clark and Feenstra 2003)
- Typically,
  - Solow residual.
  - Zoom in to particular technologies.
  - Some exceptions, e.g., CHAT dataset.
- Related to sweeping changes in the sectoral composition of the economy.

Plan for the Lecture

- Review some new and old stylized facts on innovation.
- Present a framework of multi-sectoral growth model of structural change where the direction of innovation is **endogenous**.
- Discuss how to simulate the resulting economy.

Structural Change & Future of Growth



- Implications of these transformations for the future of growth?

Two Views on Structural Change

1. Nonohomtheticity in preferences.
2. Differential technological progress across sectors.

Two Views on Structural Change

1. Nonohomtheticity in preferences.
2. Differential technological progress across sectors.

- **Goal**: Combine two views endogenizing innovation process.
  - Endogenous direction of innovation across sectors/directed technical change.
  - Use non-homothetic demand system consistent with Engel curves not asymptoting to 1 (homotheticity).
- Result: Long-run trends proportional to income elasticity.

Core Mechanism

- Relative demand:

$$\frac{Y_s}{Y_m} = \tilde{\mathfrak{D}}\left(\frac{P_s}{P_m}; C_{tot}\right)$$

Agg Consumption

Rel Prices

- Relative prices/technology:

$$\boxed{\frac{P_s}{P_m} = \mathfrak{P}\left(\frac{N_s}{N_m}\right)}$$

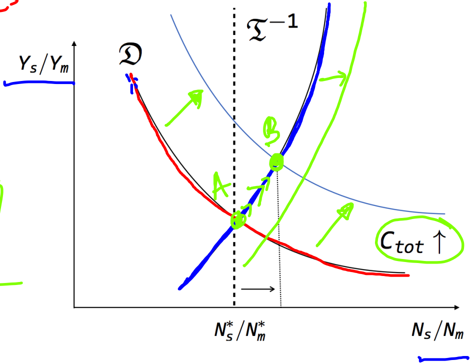
$$\Rightarrow \frac{Y_s}{Y_m} = \mathfrak{D}\left(\frac{N_s}{N_m}; C_{tot}\right)$$

- Technology supply:

$$\boxed{\frac{N_s}{N_m} = \mathfrak{T}\left(\frac{Y_s}{Y_m}\right)}$$

- Sector s more income elastic

C_{tot} ↑



Key Elements of the Lecture

facts

- Document structural transformation in innovation
  - Use long-run evidence from patents
- Document income elasticities of US industry outputs correlated with
  - Rates of growth of patenting
  - Rates of growth of R&D expenditure

- Construct multisector growth model with
  1. Nonhomothetic CES demand → Arbitrary number of sectors.
  2. Intersectoral knowledge spillovers
  3. Endogenous sectoral productivity growth

- Show the equilibria asymptotically predict correlation between income elasticity and innovation growth

Related Lit.: Structural Change vs Biased Technical Change

- Modern literature on biased technical change:
  Autor et al. (1998); Acemoglu (1998, 2002, 2007)
  - Assume aggregate production function in aggregate factor inputs
  - Study response of factor-augmenting technology to shock in relative factor inputs?

    However, in LR, factor supply endogenous?

- Our approach:
  - Assume empirically grounded heterogeneity in sectoral demand
  - Study LR heterogeneity in rates of innovation and growth

Related Literature II

- Endogeneous Structural Change
  - Homothetic Preferences: Herrendorf and Valentinyi (2016), Hori et al. (2016), ...
  - Non-homothetic Preferences: Boppart and Weiss (2015), Foellmi, Reto and Zweimuller (2014), Foellmi and Zweimuller (2008), Matsuyama (2002), 2018)
- Determinants of Sectoral R&D and Innovation
  - Macro: Ngai and Samaniego (2011), Klenow (1996),. . .
  - IO: Schmookler (1966), Cohen and Levin (1989),. . .

Outline

1. Reduced form Evidence on Income Elasticities.
    - Universe of U.S. patents.
        - Berkes Mestieri (2017): Universe US patents.
        - USPTO: 1976 - onwards
    - U.S. Census of manufacturers.

    70's
2. Model Set-up.
3. Long-run drivers of innovation, around CGP approximation.
4. Analytic Example.

Reduced form Evidence on Role of Income Elasticity

- Do more income elastic sectors have more innovation?
- Historical Data: Universe US Patents Berkes Mestieri 2017
- Last three decades: Two proxies for innovation
  - U.S. Patents $\longrightarrow$ Universe 1978 - 2014.
  - R&D expenditure $\longrightarrow$ U.S. Census Data + Compustat.
- For Income Elasticity
  - Structural Estimates using Nonhomothetic-CES.
  - Robust to Aguiar and Bils elasticities estimates from CEX.

Historical Evidence – Berkes Mestieri (2017)

- Digitize *all* US patents, 1836 to 2016, from three sources.
- Use algorithm to identify citations pre-1940 citations (as they are in text).
  - We also have each patent geo-localized (not today).
- We identify the leading technological classes in each year of the sample as the most represented class in the top 10% patents in terms of forward citations in that year.

Data Collection

- Digitalized and OCR'ed all patents issued by USPTO into text.
- Used redundant external sources as checks.
  - USPTO digitalized patents.
  - Google Patents (and Maps).
  - Local repositories (e.g. Wyoming Inventor's Database)
  - HistPat.

Patent Examples

UNITED STATES PATENT OFFICE.

WM. F. GOODWIN, OF NEW YORK, N. Y.

IMPROVEMENT IN MOUNTING HAND-MORTARS.

Specification forming part of Letters Patent No. **46,101**, dated January 31, 1865.

To all whom it may concern:

Be it known that I, WILLIAM F. GOODWIN, of the city, county, and State of New York, have invented a new and useful Improvement in Mortars; and I do hereby declare that the following is a full, clear, and exact description thereof, which will enable others skilled in the art to make and use the same, reference being had to the accompanying drawings, forming part of this specification, in which—

Figure 1 is a longitudinal axial section of a mortar constructed according to my invention. Fig. 2 is a like section on a larger scale, a part of the stake upon which the mortar is mounted being broken away. Fig. 3 is a side view from the opposite side to that presented in the other figures.

Similar letters of reference indicate like parts.

This invention consists, among other things, in mounting a mortar upon one end of a stake of wood or other suitable material, the other end of which is made pointed to enable one to insert it in the ground.

A is the mortar, formed with a powder-chamber, K, in its bottom. A cone, *d*, rising from the bottom of the powder-chamber, communicates through a channel, *e*, with the nipple.

B is a stout sleeve extending from the base of the mortar, to enable it to be attached to a stake, F, as shown in the drawings.

C is an elastic cushion, formed of rubber or equivalent material, placed at the bottom of the sleeve, so as to bear directly against the end of the stake. A slot, E, is made through the stake in that part covered by the sleeve, and it receives a pin, D, which passes through and is secured in the sides of the sleeve, so that when the mortar is fired it may slide longitudinally upon the stake, and yet be prevented from becoming displaced or being torn from the stake. The opposite end of the stake is shod with a pointed metallic ferrule or shoe, G, to enable it to be placed in the ground with facility.

H represents a lock, whose hammer-piece *f* is in outline the arc of a circle, and has a groove formed on its front side, which affords a path or channel for the passage upward of the gases which arise from the explosion of the cap, so that the lock shall not become injured thereby. The trigger *b* and sere *a* are operated by means of a cord or chain, (shown in red,) and the hammer is thrown down by the usual spring, J, inclosed in the lock-frame I, fast on one side of the sleeve B. A small hollow cylinder, O, closed at its rear end, is fastened on the right-hand side of the lock-frame, as seen in Fig. 3. A spiral spring in its bottom throws a small bolt, *c*, outward against the end of the trigger *b*, so as to restore the sere *a* to its place against the tumbler after every pull of the chain on the trigger, the latter resting against the end of the bolt when the hammer is cocked, as shown in that figure. The bolt will be retained in the cylinder between the spring and trigger by their mutual pressure against it.

The mortar may be made of bronze or any other suitable material. The stake or other support upon which it is mounted is to be portable, so that the mortar can be easily transported and fixed in the ground, or otherwise temporarily but firmly secured in a suitable position and inclination for the proper and efficient use of the weapon after the usual manner of using mortars. The axis of the stake upon which the mortar is mounted is coincident with that of the mortar; or, if not made coincident, they are always to be in parallel planes.

I am aware of the Letters Patent No. 43,881, granted August 16, 1864, to Ralph Graham, of Brooklyn, Kings county, New York, for a hand fire-arm adapted to projecting grenades or small bombs, and I do not claim the invention therein shown; but

What I do claim as new and of my invention, and for which I desire Letters Patent, is—

1. Constructing a mortar with a hollow sleeve projecting from its base, instead of trunnions or cheeks, substantially as above described, for the purpose of receiving the elastic cushion, or any equivalent spring, and the end of a stake, as above set forth.

2. The combination of the slot E and pin D with the aforesaid mortar A, sleeve B, and spring C, as and for the purposes specified.

WM. F. GOODWIN.

Witnesses:
M. M. LIVINGSTON,
THEO. TUSCH.

Patent Examples

Citations before 1947 and name.

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Patent Examples

Names, location, dates.

UNITED STATES PATENT OFFICE.

SPENCER LEE FRASER AND WILLIAM A. BRIGHAM, OF TOLEDO, OHIO.

OYSTER-REFRIGERATOR.

SPECIFICATION forming part of Letters Patent No. 300,061, dated June 10, 1884.

Application filed October 12, 1883. (No model.)

To all whom it may concern:

Be it known that we, SPENCER LEE FRASER and WILLIAM A. BRIGHAM, of Toledo, in the county of Lucas and State of Ohio, have invented certain new and useful Improvements

for the receptacle B. When access is desired to the receptacle for the removal of its contents, it is only necessary to remove the cover G, the ice in the box *a* being thus at all times covered and not exposed to the air at any

Patent Examples

Reference list after 1947.

ing a forward impact face on which the resilient material in the openings in said second block is exposed.

LAYTON R. FETTEROLF.

REFERENCES CITED

The following references are of record in the file of this patent:

UNITED STATES PATENTS

Number	Name	Date
723,258	Felton	Mar. 24, 1903
819,900	Martin	May 8, 1906
1,088,571	Heferman	Feb. 24, 1914
1,154,490	Davis	Sept. 21, 1915
1,504,326	Cullinan	Aug. 12, 1924
1,664,257	McCullough	Mar. 27, 1928
1,943,399	Smith	Jan. 16, 1934
1,968,626	Young	July 31, 1934
1,982,526	Lussky	Nov. 27, 1934
2,046,164	Herkner	June 30, 1936

FOREIGN PATENTS

Number	Country	Date
18,134	Great Britain	1902
431,884	Great Britain	July 17, 1935

Leading Technology Classes

- **Period 1830-1876**:
  1. Agriculture; Forestry; Animal Husbandry; Hunting; Trapping; Fishing
  2. Heating; Ranges; Ventilating
- **Period 1877-1958**:
  1. Engineering Elements or Units; General Measures for Producing and Maintaining Effective Functioning of Machines or Installations; Thermal Insulation in General
- **Period 1959-1969**:
  1. Conveying; Packing; Storing Handling Thin or Filamentary Material
  2. Organic Chemistry

Leading Technology Classes

- **Period 1976-1983**:
  1. Measuring; Testing
- **Period 1984-1995**:
  1. Medical or Veterinary Science; Hygiene
- **Period 1996-Present**:
  1. Computing; Calculating; Counting

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- Consistent with a picture where demand for different items was very different in 1830 than now.
  - Market size of agricultural products has declined, while it has increased for computing...

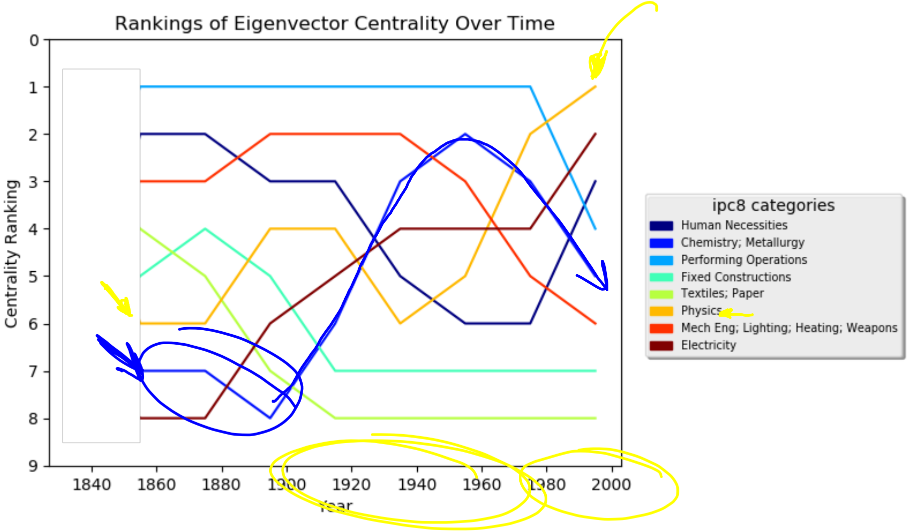
Leading Patent Categories

Most cited Category in Top 10% Patents

Years	IPC Class	Description
1836–1871	F24	Heating; Ranges; Ventilating
	A01	Agriculture; Forestry; Animal Husbandry; Hunting; Trapping; Fishing
1872-75	F16	Engineering Elements or Units; General Measures for Producing and Maintaining Effective Functioning of Machines or Installations; Thermal Insulation in General
1876	A01	Agriculture; Forestry; Animal Husbandry; Hunting; Trapping; Fishing
1877–1958	F16	Engineering Elements or Units; General Measures for Producing and Maintaining Effective Functioning of Machines or Installations; Thermal Insulation in General
1959–65	B65	Conveying; Packing; Storing Handling Thin or Filamentary Material
1966–67	C07	Organic Chemistry
1968–69	B65	Conveying; Packing; Storing Handling Thin or Filamentary Material
1970–75	C07	Organic Chemistry
1976–83	G01	Measuring; Testing
1984–95	A61	Medical or Veterinary Science; Hygiene
1996–present	G06	Computing; Calculating; Counting

Eigenvector Centrality

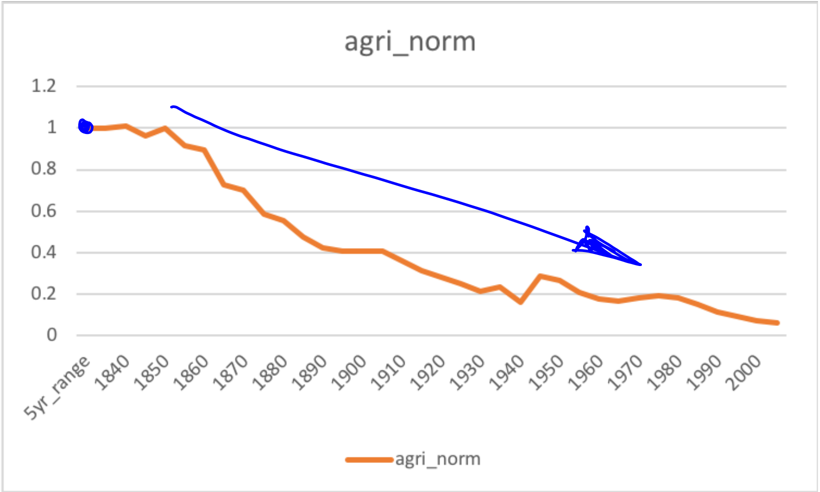
- Use citations across different patent classes.



Evolution by Three Broad Sectors (CLM2)

- Assign each patent category to
  1. Agriculture
  2. Manufacturing
  3. Services
- Plot evolution normalizing initial level to 1.

Evolution by Three Broad Sectors (CLM2)



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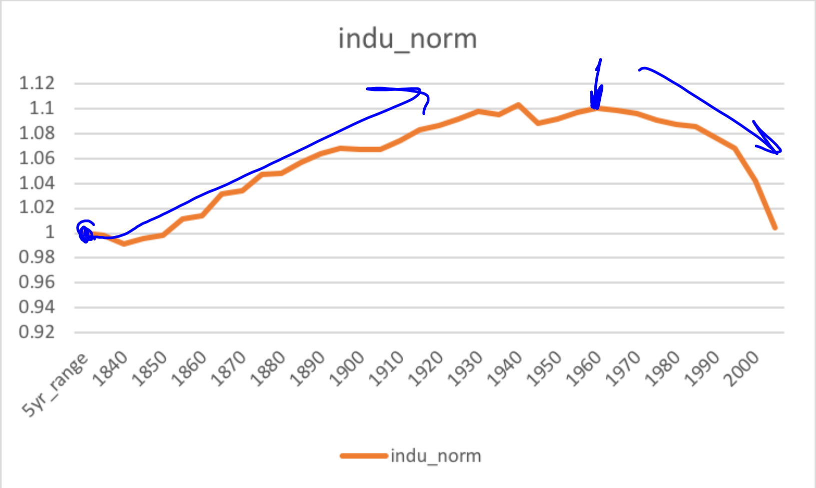

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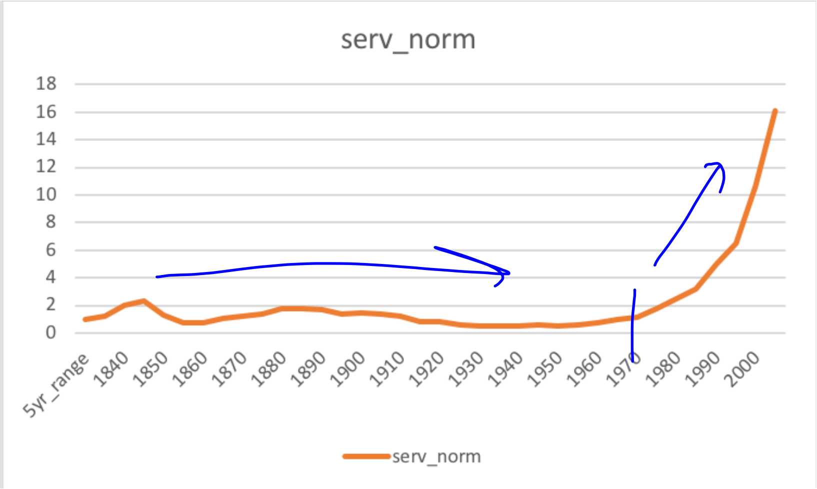

Evolution by Three Broad Sectors (CLM2)

Sectoral Share of Innovation (over the 5-years period)

	Min (Year)	Average	Max (Year)
Agriculture	0.7% (2015)	4.5%	11% (1835)
Industry	89% (1840, 2015)	94.1%	98% (1945)
Services	<0.1% (<1950)	1.4%	10% (2015)

Reduced form Evidence on Role of Income Elasticity II

- Run the following type of regression

$$y_{it} = \alpha + \beta\varepsilon_i + \delta_t + \delta_I + \nu_{it},$$

where

  - y_{it} is growth in R&D and patents in sector i,
  - δ_t is time fixed effect,
  - δ_I broad sector FE (SIC 1 for R&D), (NAICS 1 patents).

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  where
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- Median growth of patents in the sample: 0.024
- P90 growth 12%, P10 -13%
- Median income elasticity in the sample: 1.05
- P90 income elasticity 1.3, P10, 0.88

Results for Patents

1974 - 2010

$$\text{Yearly Patent Growth}_{it} = \alpha + \beta \varepsilon_i + \delta_t + \delta_{NAICS1} + \nu_{it},$$

	Raw Citations			Weighted Citations		
	(1)	(2)	(3)	(4)	(5)	(6)
Elasticity	0.024	.024***	.016***	.025	.025***	.016***
	(.020)	(.007)	(.007)	(.021)	(.006)	(.008)
Year FE	No	Yes	Yes	No	Yes	Yes
Ind. FE	No	No	Yes	No	No	Yes
R^2	.0004	.91	.91	.0005	.90	.90

Obs. 3002, s.e.: robust, clustered at year and NAICS 1, respectively.

Results for R&D Expenditure – Census of Manufacturers

$$\text{Yearly R\&D Growth}_{it} = \alpha + \beta\varepsilon_i + \delta_t + \delta_{SIC1} + \nu_{it},$$

$+\gamma_1 Sh.College_i$
$+\gamma_1 Capital\ Share_i$
$+ \dots$

	(1)	(2)	(3)
Elasticity	0.001	.136***	.496***
	(.069)	(.06)	(.127)
Year FE	No	Yes	Yes
Broad Ind. FE	No	No	Yes
R-squared	.004	.257	.349

Number obs. is 1120. Robust standard errors in parenthesis. Weighted regression by number of obs. by industry.

- Also holds for Compustat sample (actually looks better).

Theory

Static Core of the Theory

- Demand for i vs. j goods:

$$\boxed{\frac{Y_i}{Y_j}} = \boxed{\frac{C_i}{C_j}} = \mathcal{D}\left(\underbrace{\frac{P_i}{P_j}}; \underbrace{C_{tot}}\right).$$

Static Core of the Theory

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- Relative prices given by the state of technology

$$\boxed{\frac{P_i}{P_j}} = \mathcal{T}\left(\frac{N_i}{N_j}\right).$$

Static Core of the Theory

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- Relative prices given by the state of technology

$$\frac{P_i}{P_j} = \mathcal{T}\left(\frac{N_i}{N_j}\right).$$

- Equilibrium technology $\mathbf{N}\left(C_{tot}\right)$, determined by trade-off b/w
  - Relative sectoral profits $\frac{\Pi_i}{\Pi_j} \propto \frac{C_i}{C_j}$;
  - Relative costs of technological innovation $\frac{\eta_i}{\eta_j}$.

Nonhomothetic CES Preferences

income elast ≠ 1 ↑Price Elasticity.

Preferences

Define consumption aggregator $C(t)$ over goods $\{C_i\}_{i=1}^{I}$ as

$$C \cdot \sum_{i=1}^{I} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}} = 1; \cdot C$$

with $\sigma \geq 0$, and $\epsilon_i \geq 0$.

- Multilplying by C,

$$\boxed{C} = \sum_{i=1}^{I} \underbrace{C^{\boxed{1-\epsilon_i \frac{\sigma-1}{\sigma}}}}_{\text{Weight}(C,\epsilon_i)} C_i^{\frac{\sigma-1}{\sigma}},$$

j

- $\epsilon_i = 1$ recovers standard homothetic CES.

Nonhomothetic CES Preferences Demand

- Maximize C defined by nonhomothetic CES.
- Vector of sectoral prices $\mathbf{P} = \{P_i\}_{i=1}^{I}$, total expenditure E.

$$\min_{C_i} \sum_i P_i C_i \quad \text{s.t} \quad \begin{cases} \sum_i \left(\frac{C_i}{C^{\varepsilon_i}}\right)^{\frac{\sigma-1}{\sigma}} = 1 \leftarrow \\ \sum P_i C_i \leq E \leftarrow \end{cases}$$

Comin, Lashkari & Mestieri "Structural Change with Long-run ..."

Nonhomothetic CES Preferences Demand

- Maximize C defined by nonhomothetic CES.
- Vector of sectoral prices $\mathbf{P} = \{P_i\}_{i=1}^{I}$, total expenditure E.
- Demand given by

Homoth. CES

if $\epsilon_i \neq 1$

$$\boxed{\frac{C_i}{C}} = \boxed{\left(\frac{P_i}{P}\right)^{-\sigma}} \underbrace{C^{(1-\sigma)(\underbrace{\epsilon_i-1}_{\neq 0})}}_{\text{Non-Homoth.}},$$

where the price index is given by

$$P \equiv P\left(C; \mathbf{P}\right) = \left(\sum_i \left(C^{\epsilon_i-1}P_i\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

if $\epsilon_i = 1 \; \forall i \Rightarrow P^{1-\sigma} = \sum P_i^{1-\sigma}$

and $E = P \cdot C$.

Shares.

$\frac{P_i C_i}{PC} = \text{share}_i = \left(\frac{P_i}{P}\right)^{1-\sigma} C^{(1-\sigma)(\epsilon_i - 1)}$

$\epsilon_i = 1 \longrightarrow$ exp. $\text{share}_i = \left(\frac{P_i}{P}\right)^{1-\sigma}$

Model of Multi-sector Endogenous Growth

- Preferences: – Unitary Household
  - Intertemporal preferences:

$$\text{Total Utility} = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

  - $C(t)$: nonhomothetic CES aggregator with $\left(\sigma; \{\epsilon_i\}_{i=1}^I\right)$.

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- Romer-model with multi-sector production:
  - Production: competitive sectoral producers combine sectoral intermediate goods.
  - Innovation: sectoral R&D firms employing $Z_i(t)$ in sector i.

Model of Multi-sector Endogenous Growth

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  - Production: competitive sectoral producers combine sectoral intermediate goods.
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- Labor the single factor of production:

$$H = \underbrace{Z(t)}_{\text{R\&D workers}} + \underbrace{L(t)}_{\text{Production workers}}$$

Production

- Competitive producers of sectoral goods:

consumer

$$Y_i(t) = \left(\int_0^{N_i} X_{iv}(t)^{\frac{\zeta}{\zeta+1}} dv \right)^{\frac{\zeta+1}{\zeta}}$$

i = agric, manuf, etc.

- Monopolist producers of sectoral intermediate inputs:

$$X_{iv}(t) = \psi L_{iv}(t),$$

  - Constant marginal cost normalized $\psi = \frac{\zeta}{1+\zeta}$

Production

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Acemoglu's Modern Econ Growth (endogenous growth)

- Demand for intermediate goods:

$$X_{iv}(t) = \left(\frac{P_i(t)}{P_{iv}(t)} \right)^{1+\zeta} Y_i(t)$$

  - Monopolist pricing of intermediate goods $\boxed{P_{iv}(t) = \frac{1+\zeta}{\zeta}\psi = 1.}$

$\pi = P_{iv} X_{iv} - \psi L_{iv}$

Market Size and Profits of Monopolists

- Profits of a monopolist v is sector i:

$$\Pi_{iv}(t) = \frac{1}{\zeta} \underbrace{P_i(t)^{\zeta}}_{\text{price effect} \equiv \frac{1}{N_i(t)}} \times \underbrace{\left(\frac{P_i(t)\, Y_i(t)}{Y(t)}\right)}_{\text{market size effect} \equiv \Omega_i(t)} \times Y(t)$$

- Relative profits of monopolists in two sectors i and j:

$i \ \gamma$
$j \ \gamma' \rightarrow$

$$\frac{\Pi_{iv}(t)}{\Pi_{jv'}(t)} = \underbrace{\left(\frac{N_j(t)}{N_i(t)}\right)^{1+\frac{1-\sigma}{\zeta}}}_{\text{Technology effect}} \underbrace{C(t)^{(1-\sigma)\left(\epsilon_i-\epsilon_j\right)}}_{\text{Income effect}},$$

Innovation and Technology Push

- R&D firms in sector i hire $Z_i(t)$ workers to create new intermediate good varieties:

$$\frac{dN_i}{dt} = \dot{N}_i(t) = \frac{1}{\eta_i} S_i(t) \cdot \boxed{Z_i(t)}$$

- $S_i(t)$: Relevant knowledge sector i from past innovations

$$S_i(t) = \mathcal{S}_i\left(N_1(t), \cdots, N_I(t)\right)$$

  - Determines costs of innovation in sector i

Assumptions on Innovation Technology

$$\dot{N}_i(t) = \frac{1}{\eta_i} S_i(\mathbf{N}(t)) Z_i(t),$$

- $\partial S_i / \partial N_j \geq 0$ for all i and j.
- Each S_i is homogenous of degree 1 in its arguments
- The following limit exists and satisfies

$$\lim_{N_i \to \infty} \frac{S_i(\mathbf{N})}{N_i} > 0,$$

- The matrix $[\Sigma_{ij}] \equiv \left[\frac{\partial \log S_i}{\partial \log N_j}\right]_{ij}$ is positive definite.

Example of S_i

- Nested CES

$$
\begin{aligned}
S_i(\mathbf{N}) &\equiv \frac{1}{\eta_i}\left[\delta_i^{1-\psi_i} \underline{N_i^{\psi_i}} + (1-\delta_i)^{1-\psi_i}\, \underline{\tilde{S}_i(\mathbf{N})^{\psi_i}}\right]^{\frac{1}{\psi_i}}, \\
\tilde{S}_i(\mathbf{N}) &\equiv \left(\sum_{j\neq i} \vartheta_{ij}^{1-\varsigma_i} N_j^{\varsigma_i}\right)^{\frac{1}{\varsigma_i}},
\end{aligned}
$$

$\tilde{S}(t)$: an economy-wide, general purpose stock of knowledge

R&D Market Free Entry Condition

- $V_i(t)$: value of owning intermediate input firm in sector i:

$$R(t)\,V_i(t) - \dot{V}_i(t) = \Pi_i(t)$$

- Free entry condition:

$$\text{wage} = 1 = \underbrace{\frac{S_i(t)}{\eta_i}}_{\text{innovative productivity of labor}} \times V_i(t)$$

- Rewrite as:

$$\underbrace{N_i(t)\,V_i(t)}_{\text{total assets in } i} = \underbrace{\eta_i \frac{N_i(t)}{S_i(t)}}_{\text{cost of growth } i} = \frac{Z_i(t)}{\dot{N}_i(t)\,/N_i(t)}$$

R&D Market Free Entry Condition

- $V_i(t)$: value of owning intermediate input firm in sector i:

$$R(t)\,V_i(t) - \dot{V}_i(t) = \Pi_i(t)$$

- Free entry condition:

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- Define *Sectoral Share of Total Corporate Assets:*

$$\Lambda_i(t) \equiv \frac{N_i(t)\,V_i(t)}{\sum_j N_j(t)\,V_j(t)}$$

Market Equilibrium

Definition

An allocation $(C, \mathbf{\Omega}, \mathbf{N}, \mathbf{S}, \mathbf{Z}, \mathbf{L}, \mathbf{\Pi}, \mathbf{V})_{t\geq 0}$ is a market equilibrium if there exists time paths of prices $(R, P, \mathbf{P})_{t\geq 0}$ such that prices and technologies satisfy $P_i(t) = N_i(t)^{1/\zeta}$, aggregate price index and aggregate consumption satisfy eqs. on slide 10, (3) sectoral shares of consumption expenditure $\boxed{\Omega_i(t) \equiv P_i(t)\, C_i(t) / E(t)}$ satisfy $\Omega_i(t) = \Xi_i \left(\frac{P_i(t)}{P(t)}\right)^{1-\sigma} C(t)^{(1-\sigma)(\epsilon_i - 1)} = \frac{L_i(t)}{L(t)}$, (4) total consumption expenditure satisfies $E(t) = P(t)\, C(t) = (1+\zeta)\, L(t) / \zeta H$, (5) corporate profits and (6) value of company stocks satisfy evolution eqns in slide 17, (7) sectoral spillovers satisfy assumptions in slide 14 (8) and labor markets clear, $(H = L(t) + Z(t)$, etc.).

Dynamics of the Direction of Innovation*

- Given state of technology at time t:
  - $\Omega_i(t)$: Production (consumption) shares given from demand
  - $\Lambda_i(t)$: Corporate asset shares given from costs of innovation

- Technological growth

tech   demand shares

$$\begin{pmatrix} \dot{N}_1 \\ \vdots \\ \dot{N}_k \end{pmatrix} = \dot{\mathbf{n}} = \mathbf{\Sigma}^{-1}\left(\frac{L}{\zeta}\left(\frac{\mathbf{\Omega}}{\mathbf{\Lambda}}\right) - r\mathbf{1}\right).$$

entry of innovators

$\uparrow$

V

- Closed form solution in the example.

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- Closed form solution in the example.

$$\frac{\dot{N}_i(t)}{N_i(t)} = \gamma(t) + \frac{1+\zeta}{\zeta^2\delta}\left(\frac{\Omega_i(t)}{\Lambda_i(t)}\right)\frac{L(t)}{H},$$

where $\gamma(t)$ an economy-wide index of growth:

$$\gamma(t) \equiv \frac{Z(t) - \frac{1+\zeta}{\zeta^2\delta}L(t)}{H}$$

Equilibrium Dynamics*

- Allocations fully characterized through aggregate consumption $C(t)$ and state of technology $\mathbf{N}(t) = (N_1(t), \cdots, N_I(t))$
  - Economy starts from initial technological state $\mathbf{N}(0) = (N_1(0), \cdots, N_I(0))$
  - Equilibrium path fully characterized with the choice of initial level of consumption $C(0)$

- Evolution of the economy:

$$\boxed{\begin{aligned} \frac{\dot{C}(t)}{C(t)} &= \mathcal{F}(C(t), \mathbf{N}(t)), \\ \frac{\dot{N}_i(t)}{N_i(t)} &= \mathcal{G}_i(C(t), \mathbf{N}(t)), \end{aligned}} \quad \text{for } 1 \leq i \leq I,$$

Constant Growth Path and Structural Change

- Equilibrium such that aggregate consumption $C(t)$ and sectoral technologies asymptotically grow at constant rates.
- There exist values $\boxed{\left(g_C^*, g_{N_1}^*, \cdots, g_{N_I}^*\right)}$ such that:

$$
\left\{
\begin{aligned}
\lim_{t\to\infty} \frac{d}{dt} \log C(t) &= g^*, \\
\lim_{t\to\infty} \frac{d}{dt} \log N_i(t) &= \gamma_i g^*, \qquad \text{for } 1 \leq i \leq I.
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\end{aligned}$$

- Sectoral consumption expenditure (also employment) and total production employment converges to a constant,

$$\begin{aligned}
\lim_{t\to\infty} \Omega_i(t) &= \Omega_i^*, \\
\lim_{t\to\infty} L(t) &= L^* > 0.
\end{aligned}$$

- Value of assets converges to $\Lambda_i^* = \eta_i \delta_i^{\frac{\psi_i - 1}{\psi_i}}$ in the example.

Characterization for $0 < \sigma < 1$

Sectoral Innovation Growth

- The asymptotic growth rates in $\mathcal{I}^*$ is

$$\boxed{\gamma_i = \xi \epsilon_i.}$$

Sketch of Proof

- Consistent with the "reduced" form regressions.

Characterization for $0 < \sigma < 1$

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technology

markup

Sketch of Proof

  - Consistent with the "reduced" form regressions.

- Sectoral Innovation Growth near CGP with example spillovers

$$\gamma_i = \xi \epsilon_i \left(1 + \frac{\xi_i}{\xi_i + 1 - \sigma} \left(\frac{\epsilon_{max}}{\epsilon_i} - 1 \right) \right),$$

where $\xi_i \equiv \xi$ if $\psi_i > 0$ and $\xi_i \equiv \xi (1 - \delta_i)$ if $\psi_i \to 0$. Details

  - Vanishing sector has higher productivity growth (services vs. manufacturing).

Results for the Particular Specification of Spillovers*

- Recall Nested CES Structure

$$\left\{\begin{aligned} S_i(\mathbf{N}) &\equiv \frac{1}{\eta_i}\left[\delta_i^{1-\psi_i} N_i^{\psi_i} + (1-\delta_i)^{1-\psi_i} \tilde{S}_i(\mathbf{N})^{\psi_i}\right]^{\frac{1}{\psi_i}}, \\ \tilde{S}_i(\mathbf{N}) &\equiv \left(\sum_{j\neq i} \vartheta_{ij}^{1-\varsigma_i} N_j^{\varsigma_i}\right)^{\frac{1}{\varsigma_i}}, \end{aligned}\right.$$

Non-vanishing set of Sectors $\mathcal{I}^*$

Set of sectors that asymptotically constitute a nonvanishing share of economic activity $\mathcal{I}^*$ consists of

1. Any sector i with $\varsigma_i > 0$ and $\psi_i < 0$ or $\varsigma_i < 0$ and $\psi_i > 0$
2. Any sector i with $\varsigma_i < 0$ and $\psi_i < 0$ if $\epsilon_i \leq \epsilon_{i'}$ for all i' ,
3. Any sector i with $\varsigma_i > 0$ and $\psi_i \geq 0$ if $\epsilon_i \geq \epsilon_{i'}$ for all i'.

Characterization for $0 < \sigma < 1$

- The asymptotic growth rates of technologies in different sectors

$$\xi = \min_i \left\{ \frac{\gamma_i}{\epsilon_i} \right\} = \frac{\overline{\gamma}^*}{\overline{\epsilon}^*}, \tag{1}$$

$\overline{\gamma}^*$ and $\overline{\epsilon}^*$ denote average rates under distribution $\{\Omega_i^*\}_i$.

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$\overline{\gamma}^*$ and $\overline{\epsilon}^*$ denote average rates under distribution $\{\Omega_i^*\}_i$.

- Let $\mathcal{I}^*$ denote the set of industries that achieve minimum in (4). The production shares for $i \notin \mathcal{I}^*$ declines at

$$\boxed{\lim_{t\to\infty} \dot{\omega}_i(t) = (1-\sigma)\left(\epsilon_i - \frac{\gamma_i}{\xi}\right) g^* \leq 0.}$$

$\rightarrow$ disappearing sectors have $\xi_i > \xi$.

Characterization Innovation Side

- Economy converges to a stationary distribution $\{\Lambda_i^*\}$.
- Sector i's share of R&D emp. and Λ_i fall at a rate

$$\boxed{\lim_{t\to\infty} \dot{z}_i(t) = \lim_{t\to\infty} \dot{\lambda}_i(t) = \left(\gamma_i - \gamma_i^S\right) g^* \leq 0,}$$

where asymptotic rate of growth of innovation spillovers to sector i defined as $\gamma_i^S g^*$. Along CGP,

$$\gamma_i^S \equiv \lim_{t\to\infty} \frac{\dot{s}_i(t)}{g^*} = \sum_j \Sigma_{ij}^* \gamma_j \geq \gamma_i,$$

- Let $\mathcal{I}^\dagger$ the set of sectors that satisfy with equality.
- Technological growth for different sectors

$$\gamma = (1-\sigma)\left[\left(1 + \frac{1-\sigma}{\zeta}\right)\mathbf{I} - \mathbf{\Sigma}^*\right]^{-1} \epsilon.$$

Characterization (ct'd)

- Remark: $\mathcal{I}^* = \mathcal{I}^\dagger$.
- For the sectors *surviving asymptotically*,

$$\gamma_i = \xi \epsilon_i.$$

- Implication: More innovation in more income elastic goods within sectors in $\mathcal{I}^*$.

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Results for the Particular Specification of Spillovers* (ct'd)

- First Order Approx. Growth near CGP:

$$\gamma_i = \xi \epsilon_i \left(1 + \frac{\xi_i}{\xi_i + 1 - \sigma} \left(\frac{\epsilon_{max}}{\epsilon_i} - 1 \right) \right), \tag{2}$$

  where $\xi_i \equiv \xi$ if $\psi_i > 0$ and $\xi_i \equiv \xi \left(1 - \delta_i\right)$ if $\psi_i \to 0$.
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Results for the Particular Specification of Spillovers* (ct'd)

Equilibrium Characterization

Let $A^* \equiv \frac{1}{H}\sum_{i\in\mathcal{I}^*}\Lambda_i^*$ and denote by $\langle\epsilon_i\rangle^*$ and $Var\,\langle\epsilon_i\rangle^*$ under $\{\Lambda_i\}$. Suppose $\frac{1}{\zeta A^*}\left(1-\frac{\theta+\langle\epsilon_i\rangle^*-1+(\zeta+1)\langle\epsilon_i\rangle^*}{1+(\zeta+1)\langle\epsilon_i\rangle^*}\right)<\rho<\frac{1}{\zeta A^*}$. Then CGP exists and is unique. Determined by

$$\begin{aligned}
\bar{\epsilon}^* &= \langle\epsilon_i\rangle^* + \frac{Var\,\langle\epsilon_i\rangle^*}{r^*+\zeta\,\langle\epsilon_i\rangle^*},\\
r^* &= \frac{1}{\zeta A^*} - \frac{(1+\zeta)\,\langle\epsilon_i\rangle^*}{\bar{\epsilon}^*+\theta-1+(1+\zeta)\,\langle\epsilon_i\rangle^*}\left(\frac{1}{\zeta A^*}-\rho\right).\\
g^* &= \frac{1/\zeta A^*-\rho}{\theta+\bar{\epsilon}^*-1+(\zeta+1)\,\langle\epsilon_i\rangle^*},\\
\frac{L^*}{H} &= \frac{\rho\zeta A^*\,\langle\epsilon_i\rangle^*+\theta+\bar{\epsilon}^*-1+\zeta\,\langle\epsilon_i\rangle^*}{\theta+\bar{\epsilon}^*-1+(\zeta+1)\,\langle\epsilon_i\rangle^*}.\\
\Omega_i^* &= \frac{\zeta}{L^*}\left(r^*+\zeta g^*\epsilon_i\right)\Lambda_i^*, \qquad \text{for all } i\in\mathcal{I}^*.
\end{aligned}$$

Conclusion

FACTS
Sweeping in sectoral innovation
Growth rate innov ⊕ corr w/ inc. elast.

Theory

- Provided endogenous theory of directed technical change at the sectoral level.
- Investigated implications of long-run demand nonhomotheticity for the direction of profit-driven R&D.
- Long-run growth rate innovation proportional to income effect.
- Way ahead: quantitative version of the model to assess "demand pull" vs. "technology push."
  - Use Ethier/Bénassy formulation of gains from variety as HV.
  - Role of different types of labor?

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