An Adverse Selection Model of Optimal Unemployment Insurance - Technical Appendix

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Abstract

We ask whether offering a menu of unemployment insurance contracts is welfare-improving in a heterogeneous population. We adopt a repeated moral hazard framework as in Shavell/Weiss (1979), supplemented by unobserved heterogeneity about agents' job opportunities. Our main theoretical contribution is a quasi-recursive formulation of our adverse selection problem, including a geometric characterization of the state space. Our main economic result is that optimal contracts for "bad" searchers tend to be upward-sloping due to an adverse selection effect. This is in contrast to the well-known optimal decreasing time profile of benefits in pure moral hazard environments that continue to be optimal for "good" searchers in our model.

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Appendix A.

Proof of Lemma 2.3

Proof. First, we prove that $p_B(a_t^B) > p_G(a_t^G)$ ensues from the Spence-Mirrlees property. We calculate

$$\frac{\partial V_t^i}{\partial z_{t+1}} = \beta p_i(a_t^i)\frac{\partial V_{t+1}^i}{\partial z_{t+1}} = \beta p_i(a_t^i),$$

where we use the Envelope Theorem. The assertion now follows immediately.

Second, assume that $p_B(a_t^B) > p_G(a_t^G)$ holds. In the case where $s = 1$, the Spence-Mirrlees property follows from what we have shown above. So let $s > 1$. Then

$$\frac{\partial V_t^B}{\partial z_{t+s}} - \frac{\partial V_t^G}{\partial z_{t+s}} = \beta p_B \frac{\partial V_{t+1}^B}{\partial z_{t+s}} - \beta p_G \frac{\partial V_{t+1}^G}{\partial z_{t+s}},$$

where we have used the Envelope Theorem once more. The assertion follows by induction. ■

Proof of Proposition 3.2

Proof. Let assumption 3.1 hold. We prove the second assertion by contradiction: Assume that there is a pooling instead of a separating solution to the principal's problem, i.e. the problem is solved by one contract p, $\{z_1^p, ..., z_{T-1}^p, z_T^p\}$, for both agents that generates a total expected utility of V_t^B and V_t^G in period t for agents B and G respectively.

To construct the contradiction, we first have a look at the "pure moral hazard" solutions for the last two periods that generate the same utilities V_{T-1}^B and V_{T-1}^G as p. Hereby, we refer to the solution of the problem of guaranteeing agent i a utility of V_{T-1}^i at the lowest cost, where the choice of effort by each agent is hidden action, but where the type of agent is known (i.e. the problem considered by Shavell and Weiss [22] with full information on the agents' search technology). We want to show that the optimal Shavell-Weiss (SW) contract for agent B , (z_{T-1}^b, z_T^b), is 'flatter' than that of agent G (z_{T-1}^g, z_T^g) (see the formal definition for 'flat' below).

Formally, the problem is stated as follows:

$$\min_{z_{T-1}^i, z_T^i} c(z_{T-1}^i) + \beta p_i(a^i) c(z_T^i)$$
$$s.t. \quad V_{T-1}^i = z_{T-1}^i - a^i + \beta[p(a^i) z_T^i + (1 - p(a^i)) u(w)]$$
$$1 = \beta p_i'(a^i)[z_T^i - u(w)]$$

This is the two-period cost minimization problem (the principal's problem in this framework) in the case of agent i, subject to the promise-keeping constraint and the first order condition of the agent's problem, determining the choice of effort a^i. Plugging the entitlement constraint into the objective function and making use of the Envelope Theorem, we derive the following first order conditions for the principal with respect to z_T (we abbreviate $p_i = p_i(a^i)$):

$$
\begin{aligned}
c'(z^b_{T-1}) &= -\frac{(p'_B)^3}{p_B p''_B} c(z^b_T) + c'(z^b_T) && \text{(A.1)} \\
c'(z^g_{T-1}) &= -\frac{(p'_G)^3}{p_G p''_G} c(z^b_T) + c'(z^b_T). && \text{(A.2)}
\end{aligned}
$$

The factor in front of the cost function $c(.)$ on the right-hand side (RHS) of equations (A.1) and (A.2) is:

$$
-\frac{(p'_i)^3}{p_i p''_i} = \frac{1}{\pi_i(z_T)} \frac{\partial \pi_i(z_T)}{\partial z_T},
$$

and so we see that the RHS is identical to the relative expected marginal cost. By Condition 3.1, part 1, we know that the factor of the cost function is higher for agent G than for agent B for a given z_T. By its second part we know that this has to hold in equilibrium, too, and so the RHS is greater for agent G in (A.2) than for agent B in (A.1).[1]

We may therefore deduce that the SW contract of agent B is flatter than its counterpart for agent G, where we define "flatter" in the following sense:

$$
\frac{z^G_{T-1}}{z^G_T} > \frac{z^B_{T-1}}{z^B_T}.
$$

In the following we will discuss the last two periods of the pooling contract only and show that it cannot be optimal to offer it to both agents.

First, suppose that the pooling contract p is flatter than the SW contract g of agent G, denoted by g. Then the principal can offer p and a second contract g' that is identical to contract p except

[1] Note that we could weaken Condition 3.1: To ensure that the RHS of G is higher than the RHS of B it is sufficient to assume that the relative marginal probability of remaining unemployed $\frac{1}{\pi_i(z)} \frac{\partial \pi_i(z)}{\partial z}$ is higher for agent G than for agent B.

for the last two periods, where z^p_{T-1} and z^p_T are substituted by z^g_{T-1} and z^g_{T-1} from the SW contract g. We show that offering these two contracts would be both feasible and less costly for the principal, leading to a contradiction. Offering p and g' is incentive-compatible: First, agent G is indifferent between p and g' by construction of g. Second, suppose that agent B (weakly) preferred g' over p. Then for period 1 to $T-2$, he can exert the same effort a^g_1 to a^g_{T-2} (i.e. that he chooses in the case of contract g') when facing contract p, and thus the stochastically discounted utility from the benefits z_1 to z_{T-2} is identical for both contracts. In the last two periods, in contrast, agent B - exerting effort optimally - gains a higher utility from the flatter contract p than from contract g' because of the Spence-Mirrlees property. So agent B cannot prefer g' over p. Offering the two contracts p and g' is also cheaper for the principal, because g' is the (unique) cost-optimizing contract for agent G during the last two periods. So this contradicts the optimality of the pooling contract p.

Second, suppose that the pooling contract p is identical to or steeper than the SW contract g of agent G. The principal then offers p and a second contract b' that is identical to contract p with z^p_{T-1} and z^p_T substituted by z^b_{T-1} and z^b_T from the SW contract. Since the SW contract b of agent B is flatter than the SW contract g of agent G, as we have seen, we can infer the contradiction in the same way as in the first case. ∎

Appendix B.

Proof of Proposition 3.3

Before proving the different parts of Proposition 3.3, we restate the problem of offering unemployment insurance contracts to unemployed agents (ASUI):

$$\min_{\{z^b_1,\dots,z^b_T\},\{z^g_1,\dots,z^g_T\}} q[c(z^b_1) + \beta p_B(\hat{a}^B_1)[c(z^b_2) + \beta p_B(\hat{a}^B_2)[c(z^b_3) + \dots]\dots]] +$$
$$(1-q)[c(z^g_1) + \beta p_G(\hat{a}^G_1)[c(z^g_2) + \beta p_G(\hat{a}^G_2)[c(z^g_3) + \dots]\dots]]$$

subject to the *entitlement constraints* (EC)

$$V_1^{b,B} \geq \underline{V}, \tag{B.1}$$

$$V_1^{g,G} \geq \underline{V}, \tag{B.2}$$

the *adverse selection incentive constraints* (AS-IC)

$$V_1^{b,B} \geq V_1^{g,B}, \tag{B.3}$$

$$V_1^{g,G} \geq V_1^{b,G}. \tag{B.4}$$

and subject to the choice of effort by the agents. We can now begin the proof.

Proof. Proposition 3.3 gives a quasi-recursive - not a recursive - formulation of (ASUI) [2]. By this we mean that (ASUI) is decomposed into a minimization problem in two cost functions C_1^b and C_1^g, one for each contract, that are themselves solutions to recursive minimization problems. Whereas the choice variables of the (ASUI) problem are the benefits of the contract for agent B and agent G respectively, the quasi-recursive problem maximizes its objective function with respect to the state variables in the two cost functions C_1^B and C_1^G, subject to the same entitlement and incentive constraints as in problem (ASUI). The cost functions give the costs of contract b and g (in period 1) respectively. The state variables are the lifetime utilities $V^{i,B} = V_1^{i,B}$ and $V^{i,G} = V_1^{i,G}$ that contract i guarantees to agent B and G, for both contract $i = b$ and $i = g$, which are entitlements from the principal's viewpoint. As $V^{i,B}$ and $V^{i,G}$ are generated by the same contract i, their values are not independent and it is clear that the pairs $(V^{i,B}, V^{i,G})$ that actually correspond to a contract will form a subset of $\mathbf{R^2}$. This holds true also for pairs $(V_t^{i,B}, V_t^{i,G})$ that indicate the remaining lifetime utilities in period t, engendered by the residual of contract i, i.e. $(z_t^i, ..., z_T^i)$. The variables $(V_t^{i,B}, V_t^{i,G})$ are state variables in the recursive formulation of finding the cost minimizing residual contract $(z_t^i, ..., z_T^i)$, with cost function $C_t(V_t^{i,B}, V_t^{i,G})$. The choice variables are $(V_{t+1}^{i,B}, V_{t+1}^{i,G})$, together with z_t. Economically speaking the pair $(V_t^{i,B}, V_t^{i,G})$ indicates the entitlement utilities that the principal has to observe in period t, while $(V_{t+1}^{i,B}, V_{t+1}^{i,G})$ are the

[2] As explained in the text, we cannot obtain a recursive formulation, as the first period is distinguished from all other periods by the revelation of the agents' type.

new entitlements for the subsequent period that he chooses to promise. State variables and choice variables are related by a law of motion - in this case the agents' problem that sums up current utility and promised utility that have to add up to the entitlement utility. The first part of the proof formalizes the notion of state and choice variables, defining a correspondence $\Gamma_t(V_t^{i,B}, V_t^{i,G})$ that maps the pair of state variables $(V_t^{i,B}, V_t^{i,G})$ onto the set of corresponding choice variables. It thus establishes a -formal- characterization of those pairs $(V_t^{i,B}, V_t^{i,G})$ that correspond to a contract i. The geometric characterization of Γ_t, that serves as the basis for the numerical computation of the UI contracts, is topic of Proposition 3.4. The second part decomposes ASUI into a minimization problem with two minimization subproblems. The third part gives a recursive formulation of the subproblems that define the cost functions C_t^i.

Formalizing correspondence Γ_t*:*. We give a formal definition of $\Gamma_t(V_t^{i,B}, V_t^{i,G})$. Recall that

$$V_t^{i,j} = z_t^i - \hat{a}_t^{i,j} + \beta(p_j(\hat{a}_t^{i,j})[z_{t+1}^i - \hat{a}_{t+1}^{i,j} + \beta(p_j(\hat{a}_{t+1}^{i,j})[...] + ...)] + (1 - p_j(\hat{a}_t^{i,j}))W_t), \tag{B.5}$$

where i denotes the type of contract, j the type of agent and $\hat{a}_t^{i,j}$ the choice of effort by agent j, given contract i at time t. Then the formal definition of Γ_t is straightforward:

$$\begin{aligned}
\Gamma_t(V_t^{i,B}, V_t^{i,G}) \quad = \quad & \{(z_t^i, V_{t+1}^{i,B}, V_{t+1}^{i,G}) \mid \exists(z_t^i, \hat{z}_{t+1}^i, ..., \hat{z}_T^i) s.t.\ j \in \{B, G\} \\
& V_t^{i,j} = z_t^i - \hat{a}_t^{i,j} + \beta(p_j(\hat{a}_t^{i,j})[\hat{z}_{t+1}...]...) \ \wedge \\
& V_t^{i,j} = z_t^i - \hat{a}_t^{i,j} + \beta(p_j(\hat{a}_t^{i,j})V_{t+1}^{i,j} + (1 - p_j(\hat{a}_t^{i,j})W_t)\}.
\end{aligned}$$

In other words: The correspondence Γ_t maps a pair of state variables $(V_t^{i,B}, V_t^{i,G})$ in a given period t onto all triples $(z_t, V_{t+1}^{i,B}, V_{t+1}^{i,G})$ (where z_t denotes current utility and $V_{t+1}^{i,j}$ promised utilities) to which a contract $(z_t, ..., \hat{z}_T)$ exists that generates the corresponding lifetime utilities for agents B and G. Note that $\Gamma_t(V_t^{i,B}, V_t^{i,G})$ can be empty, because for some values of $(V_t^{i,B}, V_t^{i,G})$ of pairs of lifetime utilities of B and of G, there might be no sustaining contract.

Note also that the support of Γ_t is the "largest" set of pairs of lifetime utilities - all possible contracts are represented. Thus, adding the constraint

$$\Gamma_1(V^{b,B}, V^{b,G}) \neq \emptyset \qquad \Gamma_1(V^{g,B}, V^{g,G}) \neq \emptyset,$$

to ASUI does not change the problem. It is, however, crucial for the subsequent decomposition in two recursive subproblems.

Decomposition of (ASUI):. Given the formal notion of state variables we can now reformulate (ASUI) as follows:

$$\min_{(V^{b,B},V^{b,G},V^{g,B},V^{g,G})} \Big(\min_{\{z_1^b,...,z_T^b\},\{z_1^g,...,z_T^g\}} q[c(z_1^b) + \beta p_B(\hat{a}_1^{b,B})[c(z_2^b) + \beta p_B(\hat{a}_2^{b,B})[c(z_3^b) + ...]...]] + \\ (1-q)[c(z_1^g) + \beta p_G(\hat{a}_1^{g,G})[c(z_2^g) + \beta p_G(\hat{a}_2^{g,G})[c(z_3^g) + ...]...]] \Big)$$

subject to

$$\Gamma_1(V^{b,B}, V^{b,G}) \neq \emptyset \qquad \Gamma_1(V^{g,B}, V^{g,G}) \neq \emptyset, \tag{B.6}$$

subject to (EC)

$$V_1^{b,B} \geq \underline{V}, \tag{B.7}$$

$$V_1^{g,G} \geq \underline{V}, \tag{B.8}$$

and subject to (AS-IC)

$$V_1^{b,B} \geq V_1^{g,B}, \tag{B.9}$$

$$V_1^{g,G} \geq V_1^{b,G}. \tag{B.10}$$

At this stage, the additional minimization over $(V^{b,B}, V^{b,G}, V^{g,B}, V^{g,G})$, together with the additional constraint (B.6) is empty since we allow for all pairs of utilities that correspond to a contract. As in the formulation of (ASUI) we have not explicitly stated the implicit constraint on the choice of $(z_1^b, ..., z_T^b)$ and $(z_1^g, ..., z_T^g)$ by definition (B.5) of $V_1^{b,B}$, $V_1^{b,G}$, $V_1^{g,B}$ and $V_1^{g,G}$. At this stage, this is an empty constraint because we minimize over all pairs $(V_1^{b,B}, V_1^{b,G})$ and $(V_1^{g,B}, V_1^{g,G})$, to which a corresponding contract b and g exists. In the subsequent recursive formulation $(V^{b,B}, V^{b,G})$

become state variables $(V^{g,B}, V^{g,G})$ and thus part of a (binding) constraint, the law of motion.

Next, we decompose the objective function in the inner minimization problem into the sum of two separate minimization problems:

$$\min_{\{z_1^b,\ldots,z_T^b\}} q[c(z_1^b) + \beta p_B(\hat{a}_1^B)[c(z_2^b) + \beta p_B(\hat{a}_2^B)[c(z_3^b) + \ldots]\ldots]] +$$
$$\min_{\{z_1^g,\ldots,z_T^g\}} (1-q)[c(z_1^g) + \beta p_G(\hat{a}_1^G)[c(z_2^g) + \beta p_G(\hat{a}_2^G)[c(z_3^g) + \ldots]\ldots]]$$

This decomposition is mathematically correct, because inside the brackets there is no interdependence of the two summands of the objective function or the implicit constraints (B.5).

The recursive problems defining C_1^i:. We are left to show that the recursive formulation of the contract problem, given by (16), solves the minimization problem

$$\min_{\{z_1^b,\ldots,z_T^b\}} c(z_1^b) + \beta p_B(\hat{a}_1^B)[c(z_2^b) + \beta p_B(\hat{a}_2^B)[c(z_3^b) + \ldots]\ldots] \tag{B.11}$$

subject to

$$\begin{aligned} V^{b,B} &= z_1^b - \hat{a}_1^{b,B} + \beta(p_B(\hat{a}_1^{b,B})[z_2^b - \hat{a}_2^{b,B} + \beta(p_B(\hat{a}_2^{b,B})[\ldots] + \ldots)] + (1 - p_B(\hat{a}_1^{b,B}))W_1) \\ V^{b,G} &= z_1^b - \hat{a}_1^{b,G} + \beta(p_G(\hat{a}_1^{b,G})[z_2^b - \hat{a}_2^{b,G} + \beta(p_G(\hat{a}_2^{b,G})[\ldots] + \ldots)] + (1 - p_G(\hat{a}_1^{b,G}))W_1) \end{aligned}$$

and the choice of effort by the agents CE (compare equation (2)).

We prove the claim by induction over the number of periods T.

For $T = 2$ we have to show that the following two formulations are equivalent:

$$
\begin{aligned}
\min_{\{z_1^b, z_2^b\}} & \quad c(z_1^b) + \beta p_B(\hat{a}_1^B) c(z_2^b) \\
s.t. & \\
V^{b,B} &= z_1^b - \hat{a}_1^{b,B} + \beta(p_B(\hat{a}_1^{b,B}) z_2^b + (1 - p_B(\hat{a}_1^{b,B})) u(w)) \\
V^{b,G} &= z_1^b - \hat{a}_1^{b,G} + \beta(p_G(\hat{a}_1^{b,G}) z_2^b + (1 - p_G(\hat{a}_1^{b,G})) u(w)) \\
CE &
\end{aligned}
$$

and

$$
\begin{aligned}
C_1^B(V^{b,B}, V^{b,G}) &= \min_{\{z_1, V_2^{b,B}, V_2^{b,G}\} \in \Gamma_1(V^{b,B}, V^{b,G})} c(z_t) + \beta p_B(a^B) C_2^B(V_2^{b,B}, V_2^{b,G}) \\
s.t. & \\
V^{b,B} &= z_1 - \hat{a}_1^{b,B} + \beta[p_B(\hat{a}_1^{b,B}) V_2^{b,B} + (1 - p_B(\hat{a}_1^{b,B})) u(w)] \\
V^{b,G} &= z_1 - \hat{a}_1^{b,G} + \beta[p_G(\hat{a}_1^{b,G}) V_2^{b,G} + (1 - p_G(\hat{a}_1^{b,G})) u(w)] \\
CE & \\
V_2^{b,B} &= V_2^{b,G} = z_2^b.
\end{aligned}
$$

Recall that the last constraint is due to the death of the agents at the end of period $T = 2$. Substituting z_2^b for $V_2^{b,B}$ and $V_2^{b,G}$ and $c(z_2^b)$ for $C_2^B(V_2^{b,B}, V_2^{b,G})$ delivers the equivalence.

Now assume the claim holds for $T-1$. Then the problem (B.11) for T periods can be rewritten as:

$$
\begin{aligned}
\min_{\{z_1^b\}} & \quad c(z_1^b) + \beta p_B(\hat{a}_1^B)\big(\min_{\{z_2^b, \ldots, z_T^b\}} c(z_2^b) + \beta p_B(\hat{a}_2^B)[c(z_3^b) + \ldots]\ldots\big) \\
s.t. & \\
V^{b,B} &= z_1^b - \hat{a}_1^{b,B} + \beta(p_B(\hat{a}_1^{b,B}) V_2^{b,B} + (1 - p_B(\hat{a}_1^{b,B})) W_1) \\
V^{b,G} &= z_1^b - \hat{a}_1^{b,G} + \beta(p_G(\hat{a}_1^{b,G}) V_2^{b,G} + (1 - p_G(\hat{a}_1^{b,G})) W_1) \\
CE &
\end{aligned}
$$

Note that all we have done is:

1. to split up the summation in the objective function
2. to separate the choice variable z_1^b from the other choice variables $z_2^b, ..., z_T^b$
3. to reformulate the constraints in terms of the lifetime utilities $V_t^{i,j}$ as defined by equation (B.5).

By using the induction hypothesis we may identify the variables $V_2^{b,B}$ and $V_2^{b,G}$ with the state variables and the term in the brackets with the recursive cost function $C_2(V_2^{b,B}, V_2^{b,G})$ of the $T-1$ version of the problem. Limiting the choice of $(z_1^b, V_2^{b,B}, V_2^{b,G})$ to elements of $\Gamma_1(V^{b,B}, V^{b,G})$ ensures by construction of Γ that the former correspond to feasible contracts $(z_1^b, z_2^b, ..., z_T^b)$. This proves the last claim of the Proposition. ■

Appendix C.

Proof of Proposition 3.4

Use of the Proposition. In Proposition 3.3 we have given a formal definition of the correspondence $\Gamma_t(V_t^B, V_t^G)$. It corresponds to the correspondence Γ in the formulation of the Bellman equation in Stokey et al. [24] (see the functional equation (FE) on page 66).[3] As its counterpart it characterizes both the space of state variables and of choice variables in the recursive formulation (16) of contract i: The support of Γ_t is the set of state variables (V_t^B, V_t^G) feasible in period t, i.e. pairs of utilities $\Gamma_t(V_t^B, V_t^G)$ for which there is a sustaining contract $(z_t, ..., z_T)$. Given such a pair (V_t^B, V_t^G), Γ_t maps onto the set of corresponding control variables in the recursive formulation, i.e. the triple $(z_t^i, V_{t+1}^{i,B}, V_{t+1}^{i,G})$, consisting of utility z_t^i from the UI benefit in period t, and entitlement utilities for the subsequent period $t-1$. The formal version of the definition of Γ_t is, however, insufficient for any computational implementation of the recursive formulation - an algorithm is needed to calculate the support and the image of Γ_t, so that the minimization problem in (16) can be computed subsequently. In giving a geometric characterization of Γ_t, proposition 3.4 provides the basis for the algorithm in the computational section 4.

[3]Although we deal with the recursive formulation of a stochastic model, we give the reference to the deterministic Bellman equation: Of our two (stochastic) states, employment and unemployment, employment is absorbing, so that effectively only one state remains and the problem becomes formally equivalent to a deterministic one.

The computation of the state space -and along with it, the space of control variables- is a general challenge in this type of model. As pointed out in the main text, our model builds on the strand of literature based on Spear and Srivastava [23], Thomas and Worrall [25], Abreu et al. [1], Atkeson and Lucas [2] and Chang [5]. The sets of jointly feasible entitlements (V_t^B, V_t^G), i.e. the support of Γ_t, are the finite-dimensional analogue of the set of sequential equilibrium payoffs (of the agents' game) in the infinite-dimensional framework of Abreu et al. [1] or the set of sustainable outcomes in the (again infinite-dimensional) framework of Chang [5]. Abreu et al. [1] and Chang [5] characterize these sets as the largest fixed point of a set operator. Moreover, they show that the fixed point can be obtained by a fixed-point iteration of sets. This is theoretically sound. However, it does not provide a geometrical description of the sets, as subsets of $\mathbf{R^n}$, nor a (good) algorithm to calculate them numerically, in particular if the state space is more than one-dimensional. The numerical determination of these sets is generally a tricky issue in simulations of models building on these methods. Judd et al. [12] develop a general algorithm to calculate the sets of sequential equilibrium payoffs in Abreu et al. [1] for the case where these sets are known to be convex. Essentially, it approximates them by inner and outer hyperplanes (the authors refer to this as a "ray method"). Our approach is different.

Proposition 3.4 gives - for our model - a geometric description of the sets of state variables, i.e. the support of Γ_t, as well as a description of the corresponding sets of control variables. First, the sets of state variables are limited by boundaries that are continuous functions. Thus in particular, the sets are compact, connected and contractible.[4] The characterization is crucial for the numerical implementation of our solution. Our algorithm exploits the fact that the boundary can be approximated by narrowing the distance between outer and inner points in a bracketing procedure. To apply this algorithm, there must be no "holes" in the set of state variables - the bracketing would then stop without result. This is how we make use of the Proposition first. Second, Proposition 3.4 states that the principal's choice problem in a given period is essentially one-dimensional (in the sense that the correspondence describes a smooth one-dimensional path in the three-dimensional real space, with this path being parameterized in a).[5] By characterizing the sets explicitly, the minimization problem in (16) is largely facilitated in the subsequent computation. This is the

[4]Here, "contractible" is a term from algebraic topology. Intuitively, a topological space is contractible if it contains no holes. Formally, all closed loops in the space are homotopic to a single point.

[5]Except for the next-to-last period, where there is only one choice left.

second use of the Proposition for the algorithm.

Strategy of the Proof. The basic structure of the proof is a backward induction over time periods. Given the structure of the state space of entitlement utilities (V_{t+1}^B, V_{t+1}^G) in period $t+1$, we check which entitlement utilities (V_t^B, V_t^G) in period t give rise to non-empty sets of control variables. To do so, we use the fact that state variables (V_{t+1}^B, V_{t+1}^G) in $t+1$ are control variables in period t - economically speaking, we ask which utility triples $(z_{t+1}, V_{t+1}^B, V_{t+1}^G)$ are possible realizations of the entitlement utilities (V_t^B, V_t^G) in period t, given the search efforts by the agents and the corresponding stochastic discounting. If there are such triples in the support of Γ_{t+1}, satisfying the law of motion in (16), then (V_t^B, V_t^G) are feasible state variables. The task is to show that those pairs (V_t^B, V_t^G) that map into non-empty sets of control variables take themselves the form of subsets of $\mathbf{R^2}$ bounded by continuous functions (point 1 of the Proposition). The next-to-last period is distinguished from the others in that the utility from the UI contract is the same for both types in the last period - the different stochastic discounting, present in the previous periods, has come to an end. The additional equality of utility $V_T^B = V_T^G$ reduces the dimension of the control space in period $T-1$ to zero, i.e. there is at maximum one possible choice of effort (point 2 of the Proposition).

From the geometric point of view, the set of control variables $(z_{t+1}, V_{t+1}^B, V_{t+1}^G)$ in period t is -for a given pair of state variables (V_t^B, V_t^G)- a subset (more precisely: a submanifold) of $\mathbf{R^3}$ spanned by the law of motion (LOM) and the equations determining the choice of effort (MH-IC). In the proof we show that these four equations reduce the subset to a one-dimensional path in $\mathbf{R^3}$, parameterized in one of the effort variables. Much of the calculations below are dedicated to this task. Each pair of state variables (V_t^B, V_t^G) gives rise to one such path. Whenever the projection of the path into the $\mathbf{R^2}$ plane defined by the promise variables (V_{t+1}^B, V_{t+1}^G) has an intersection with the support of Γ_{t+1}, we know that the corresponding promise variables (V_{t+1}^B, V_{t+1}^G) are sustained as state variables in period $t+1$ and thus are admissible. In that case (V_t^B, V_t^G) are admissible state variables in period t. This holds for the periods $t < T-1$, in $t = T-1$ the additional restriction $V_T^B = V_T^G$ reduces the path to a single point. The difficulty in the proof is to show that the pairs (V_t^B, V_t^G) of state variables that generate paths with an intersection in the continuously bounded set of state variables of period $t+1$, i.e. the support of Γ_{t+1}, form themselves a set bounded by continuous functions. What is shown in the proof is essentially that the paths are parameterized in the difference $V_t^G - V_t^B$ and that for small values of the difference the path lies above the support of Γ_{t+1} and for large values it lies below.

Proof.

In order to simplify the proof we introduce a normalization: The utility from consuming the wage w is set to zero. Thus, all W_t become zero too, and the entitlement utilities of the unemployed agents take non-positive values. Note that the lower bounds for the entitlements, stemming from the lower bound on the benefit utility $\underline{z}$, thus shift downward each period along the backward induction.

A further comment before starting the proof is that the upper bound W_t on V_t^G - stated in the Proposition - is artificial: Of course the principal can ensure lifetime utilities above the value of secure lifetime income from work. However, this reduces the search effort to zero. We therefore exclude lifetime utilities above W_t from our considerations.

First, we look at the agents' problem. Recall it takes the form

$$V_t^i = \max_a z_t - a + \beta[p_i(a)V_{t+1}^i + (1 - p_i(a))W_{t+1}].$$

Given our normalization, we obtain the following first order condition at an interior solution:

$$p_i'(a_t^i) = \frac{1}{\beta V_{t+1}^i} \tag{C.1}$$

By using the Inada condition in Condition 2.1 we ensure that the interior solution always applies.

The case of $\mathbf{t = T - 1}$*:*. We start with the case of $\Gamma_t(V_t^B, V_t^G)$ with $t = T - 1$ (assertion 2 of the Proposition). There are two points to show: First, for a given pair of (would-be) entitlements (V_{T-1}^B, V_{T-1}^G) there is either one or no corresponding value of the triple of control variables (z_T, V_T^B, V_T^G), parameterized in one of the effort variables a. Second, the set of those (V_{T-1}^B, V_{T-1}^G) that give rise to a control variable is bounded by continuous functions. Mathematically speaking, we analyze the space spanned by the law of motion (LOM) and choice of effort (MH-IC) in the recursive formulation 16, as well as boundary conditions (19) and (20), which take the form $V_T^B = V_T^G = z_T$.

To show the first point, let us look at the Law of Motion (LOM) for the state variables V_{T-1}^B

and V^G_{T-1}:

$$\begin{aligned} z_{T-1} - a^B_{T-1} + \beta p_B(a^B_{T-1}) V^B_T &= V^B_{T-1}, \\ z_{T-1} - a^G_{T-1} + \beta p_G(a^G_{T-1}) V^G_T &= V^G_{T-1}, \end{aligned}$$

where we will drop the time index from the effort variables a^i_{T-1}. In the following, we will denote the difference between the entitlements of the agents by:

$$\Delta_t := V^G_t - V^B_t. \tag{C.2}$$

With this new notation, and remembering both our normalization and $V^i_T = z_T$, we solve the LOM for z_{T-1}, equalize both equations and solve for Δ_{T-1}:

$$\Delta_{T-1} = a^B - a^G + \beta p_G(a^G) z_T - \beta p_B(a^B) z_T \tag{C.3}$$

We want to further simplify equation (C.3). In the next-to-last period, the first order condition of the agents' problem (C.1) takes the following form

$$p'_B(a^B) = p'_G(a^G) = \frac{1}{\beta z_T}. \tag{C.4}$$

Again by using Condition 2.1, the p'_i are strictly increasing functions

$$p'_i : (0, \infty) \longrightarrow (-\infty, 0).$$

Note that the normalization $W_t = 0$ implies $V^i_T = z_T < 0$. From this we deduce that the p'_i are one-to-one and onto. Therefore the following function $\gamma(a^G)$ is well-defined:

$$\gamma(a^G) := (p'_B)^{-1} \circ p'_G(a^G).$$

Now we have everything at hand to define Δ_{T-1} as a function of a^G:

$$\Delta_{T-1}(a^G) = \gamma(a^G) - a^G + \frac{p_G(a^G) - p_B(\gamma(a^G))}{p'_G(a^G)} \tag{C.5}$$

In order to show point 2 of Proposition 3.4, we have to demonstrate that $\Delta_{T-1}(.)$ is invertible. We do so by proving:

$$\Delta'_{T-1}(a^G) > 0. \tag{C.6}$$

Using the agents' first order condition (C.1) and

$$\gamma'(a^G) = \frac{p''_G(a^G)}{p''_B(\gamma(a^G))}$$

we calculate

$$\Delta'_{T-1}(a^G) = [p_B(\gamma(a^G)) - p_G(a^G)]\frac{p''_G(a^G)}{(p'_G(a^G))^2}. \tag{C.7}$$

By Condition 2.1 we know that $p''_G(.) > 0$, and since $p_B(\gamma(a^G)) > p_G(a^G)$ by Condition 2.2, assertion (C.6) follows.

Finally we observe that, again by Condition 2.1:

$$\lim_{a^G \to 0} \Delta_{T-1}(a^G) = 0. \tag{C.8}$$

Together with (C.6) we deduce that as agent B's entitlement V^B_{T-1} approaches agent G's one V^G_{T-1}, the effort of the agent G a^G (as well as the effort of agent B) goes to zero. Because of (C.1) this means that the benefit for the last period z_T has to converge to zero, i.e. the wage consumption utility.

Summarizing our results so far, we can state the following: Given entitlements V^B_{T-1} and V^G_{T-1} such that $\Delta_{T-1} = V^B_{T-1} - V^G_{T-1} \geq 0$, we can -at least up to a certain value of Δ_{T-1}- find a unique corresponding choice of effort by agent G a^G (for the time being, we neglect the lower bound $\underline{z}$ on the benefits z_t). From this we can calculate - uniquely - the choice of effort by agent B a^B and the benefit for the last period z_T from equation (C.1), and the benefit of the next to last period z_{T-1} from LOM. All these functions are differentiable. As Δ_{T-1} goes to zero, the benefit of the last period z_T goes to zero, i.e. the cost of the benefit converges to that of the wage. That proves the first point.

The second point to show is the geometric form of the set of feasible entitlements (V^B_{T-1}, V^G_{T-1}). This amounts essentially to determining its boundaries. If $\underline{z} = -\infty$, so z can take any value, we infer from (C.8) that the upper bound $\overline{V}_t^B(.)$ on V_t^B, given V_t^G, is

$$\overline{V}_t^B(V_t^G) = V_t^G.$$

As for the lower bound, we calculate

$$\underline{V}_t^B(V_t^G) = \lim_{a^G \to \infty} V_t^G - \Delta_{T-1}(a^G).$$

Now let $\underline{z} > -\infty$. Then there is a natural lower bound $\underline{V}_{T-1}^G$, namely the stochastically discounted sum of the bounds on z_{T-1} and z_T, discounted with $p_G(a^G)$. Given $V_{T-1}^G \in [\underline{V}_{T-1}^G, 0]$ we now have to prove that there is a lower and an upper bound $\underline{V}_{T-1}^B(V_{T-1}^G)$ and $\overline{V}_{T-1}^B(V_{T-1}^G)$ on the corresponding feasible V_{T-1}^B. Because of (C.1), the lower bound on z_T translates into an upper bound $\overline{a}^G$ on the corresponding choices of effort of agent G. It is attained with equality. By (C.2) and (C.6) we find the lower bound

$$\underline{V}_{T-1}^B(V_{T-1}^G) = V_{T-1}^G - \Delta_{T-1}(\overline{a}^G).$$

As for the upper bound $\overline{V}_{T-1}^B(V_{T-1}^G)$, one can see intuitively that V_{T-1}^B is bounded by V_{T-1}^G (for a rigorous argument, see point 1 in the proof of 3.5). However, V_{T-1}^B does not necessarily attain this bound because of an additional constraint: $z_{T-1} \geq \underline{z}$. From the LOM and (C.1) we know

$$z_{T-1} = V_{T-1}^G + a^G - \frac{p_i(a^G)}{p_i'(a^G)}$$

The RHS is increasing in a^G, so a lower bound on z_{T-1} implies a lower bound on the effort of the second type, $\underline{a}^G$ (note that because of our normalization, the reference points for each period have been shifted downwards). Because of (C.6), a lower bound on Δ_{T-1} ensues. Given V_{T-1}^G, we thus find the upper bound on V_{T-1}^B:

$$\overline{V}_{T-1}^B(V_{T-1}^G) = V_{T-1}^G - \Delta_{T-1}(\underline{a}^G).$$

We see that V_{T-1}^B attains V_{T-1}^G only if the lower bound $\underline{a}^G$ becomes zero (the smallest possible effort). Since $\Delta_{T-1}(.)$ is an increasing function, we see that all values $V_{T-1}^B \in [\underline{V}_{T-1}^B, \overline{V}_{t-1}^B]$

are attainable as long as $\overline{a}^G > \underline{a}^G$. This must be the case for $V_{T-1}^G \geq \underline{V}_{T-1}^G$, since then there are corresponding benefit values z_{T-1}, z_T such that $z_i \geq \underline{z}$. Finally, because of the Theorem of the Maximum, both $\underline{a}_{T-1}$ and $\overline{a}_{T-1}$ depend continuously on V_{T-1}^G, and since Δ_{T-1} is a smooth function, the lower and the upper bound $\underline{V}_{T-1}^B$ and $\overline{V}_{T-1}^B$ are continuous functions of V_{T-1}^G.

So for period $T-1$, we have shown that the set of feasible values takes the form stated in the theorem. Note in particular that this set is compact and connected.

The case of $\mathbf{t \leq T-2}$:. In this subsection, we prove assertion 1 of Proposition 3.4 by backward induction over t. The strategy is similar to the one of assertion 1: First, we have to show that -for a given pair of state variables (V_{T-1}^B, V_{T-1}^G)- the corresponding set of control variables takes the form stated in the Proposition, i.e. the one of a path of choice variables $(z_{t-1}(.), V_t^B(.), V_t^G(.))$ parameterized in a. It turns out that the proof is slightly more complicated than the corresponding one in assertion 1, as we must not only derive the functional form of the path, but have to show that it is connected as well. Second, we have to show that the the set of state variables in t, i.e. the support of Γ_t, takes the form stated in the proposition, i.e. the one of a set bounded by continuous functions. Again, we make use of the law-of-motion and the choice-of-effort by the agents to find the geometric characterization of the space of state and control variables. As said before, the crucial difference between the next-to-last period and the previous ones is the boundary condition of the last periods (19) and (20).

To prove the first point, we have a look at the LOM once more. With the help of the agents' first order condition we transform it into

$$\begin{aligned} z_t - a^B + \frac{p_B(a^B)}{p_B'(a^B)} &= V_t^B, \\ z_t - a^G + \frac{p_G(a^G)}{p_G'(a^G)} &= V_t^G, \end{aligned}$$

where again we have dropped the time index from a_t^i. This inspires the definition of the following functions $(i = 1, 2)$

$$f_i(a^i) = a^i - \frac{p_i(a^i)}{p_i'(a^i)}.$$

From the LOM we can now derive a necessary equation for the choice variables (as represented by the a^is, replacing the V_t^is) to hold:

$$\Delta_t + f_G(a^G) = f_B(a^B), \tag{C.9}$$

where we have used definition (C.2).

Let us have a closer look now at f_i. From

$$f_i' = \frac{p_i p_i''}{(p_i')^2} > 0 \tag{C.10}$$

we can see that it is a strictly increasing function (bearing in mind Condition 2.1). Moreover, we calculate

$$\lim_{a^i \to 0} f_i(a^i) = 0, \tag{C.11}$$

$$\lim_{a^i \to \infty} f_i(a^i) = \infty. \tag{C.12}$$

Now note that there is a natural lower bound $\underline{V}_t^G$ on each V_t^G, namely the stochastically discounted sum of the $z_{\hat{t}}$s (where $\hat{t} = t, ..., T$). In the case of $T-1$, we have shown that the set of jointly feasible values V_{T-1}^B, V_{T-1}^G takes the form stated in the theorem. So let $\Gamma_t(V_t^B, V_t^G)$ be non-empty and take the form of a path in the space $(z_t, V_{t+1}^B, V_{t+1}^G)$ for $V_t^B \in [\underline{V}_t^B(V_t^G), \overline{V}_t^B(V_t^G)]$ with $V_t^G \geq \underline{V}_t^G$. We have to show first that $\Gamma_{t-1}(V_{t-1}^B, V_{t-1}^G)$ is then non-empty for $V_{t-1}^B \in [\underline{V}_{t-1}^B(V_t^G), \overline{V}_{t-1}^B(V_t^G)]$ for some continuous functions $\underline{V}_{t-1}^B$, $\overline{V}_{t-1}^B$ when $V_{t-1}^G \geq \underline{V}_{t-1}^G$, and takes the form of a path in (z_{t-1}, V_t^B, V_t^G).

Put differently, we have to ask for which pairs (V_{t-1}^B, V_{t-1}^G) are there choice variables (z_{t-1}, V_t^B, V_t^G) that are jointly feasible. By the agents' first order condition (C.1) we can replace V_t^B and V_t^G by the corresponding choices of effort a_{t-1}^B and a_{t-1}^G (we will drop the time index in the sequel). The effort choices a^B and a^G have to satisfy equation (C.9). Since $\Delta_{t-1} \geq 0$ and by (C.10), (C.11) and (C.12), for all $a^G \geq 0$ we can find a corresponding $a^B \geq 0$ - this gives a the variable $a = a^G$ to parameterize the path of control variables. By the LOM, we can always determine z_{t-1} once a^G is given. Thus the control variables $(z_{t-1}(a^G), V_t^B(a^G), V_t^G(a^G))$ are each parameterized in the parameter a^G - we have found the path (we still have to show that it is connected). All functions are combinations of differentiable functions and thus differentiable - therefore the path is differentiable, too. The curve $\phi_{\Delta_{t-1}}$, which is parameterized in a^G, is defined as the projection of the triple of choice variables into the two-dimensional space $(V_t^B(a^G), V_t^G(a^G))$.

We have exploited the LOM and MH-IC to reduce the choice problem to a one dimensional problem, but for which a^G do the (z_{t-1}, V_t^B, V_t^G) correspond to *feasible* choice variables? Two further constraints are to be taken into account. First, we look at the constraint $z_{t-1} \geq \underline{z}$. As in the preceding subsection, by using the LOM

$$z_{t-1} = V_{t-1}^G + f_G(a^G)$$

it translates into a constraint[6]

$$a^G \geq \underline{a}^G = \begin{cases} f_G^{-1}(\underline{z} - V_{t-1}^G) & : \quad V_{t-1}^G \leq \underline{z} \\ 0 & : \quad V_{t-1}^G > \underline{z} \end{cases} \tag{C.13}$$

Second we have to ask: Which of the pairs of entitlements $(V_t^B(a^G), V_t^G(a^G))$ are feasible? The answer is, those for which $\Gamma_t(V_t^B, V_t^G)$ is non-empty. In other words: Given V_{t-1}^B and V_{t-1}^G, the set of feasible choices is the intersection of the curve $\phi_{\Delta_{t-1}}$ defined by (C.9), parameterized in a^G with $a^G \geq \underline{a}^G$, and the support of $\Gamma_t(.,.)$. Figure 7 depicts the intersection for the case of period 4 of 12 in an example from our simulation. The solid lines represent the bounds $\underline{V}_4^B(V_4^G)$ and $\overline{V}_4^B(V_4^G)$, while the dotted and the dashed line are curves ϕ_{Δ_3} with two different values for Δ_3.

Two things remain to be shown:

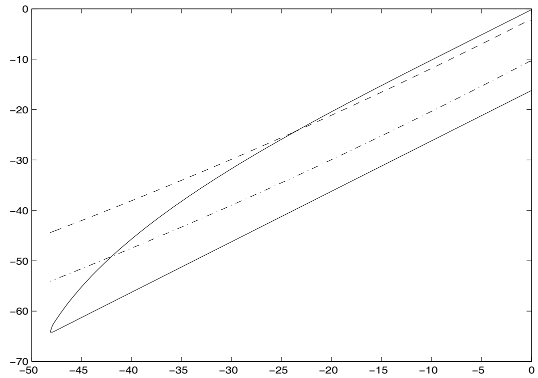
1. We have to show that the set of (V_{t-1}^B, V_{t-1}^G), for which the intersection is non-empty, itself takes the form of a set bounded by functions $\underline{V}_{t-1}^B$ and $\overline{V}_{t-1}^B$ (point 2).
2. We have to show that if the curve defined by (C.9) intersects the set of feasible values (V_t^B, V_t^G), it cuts the bounds at most twice, so that the set of feasible choices is connected.

To show the *first assertion*, we look more closely at the family of curves

$$\phi_{\Delta_{t-1}} : a^G \longrightarrow [\phi_B(a^G), \phi_G(a^G)]_{\Delta_{t-1}},$$

[6] If $\underline{z} = -\infty$, by our definition there is no limit on a^G.

Figure C.1: Set of Jointly Feasible Entitlements in Period 4 of 12



x-axis: Entitlements of type B, y-axis: Entitlements of type G
with projection of sets of agents' choice sets

where

$$\phi_B(a^G) = \frac{1}{\beta p_B'(f_B^{-1}(\Delta_{t-1} + f_G(a^G)))}, \tag{C.14}$$

$$\phi_G(a^G) = \frac{1}{\beta p_G'(a^G)}. \tag{C.15}$$

Since ϕ_G is one-to-one, the curves can also be understood as a function

$$V_t^B = \phi_{\Delta_{t-1}}(V_t^G).$$

We now want to prove the following. The curves are "decreasing" in Δ_{t-1}, i.e.

$$\Delta_{t-1} < \Delta_{t-1}^* \Rightarrow \phi_{\Delta_{t-1}}(V_t^G) > \phi_{\Delta_{t-1}^*}(V_t^G). \tag{C.16}$$

We do so by calculating the derivative

$$\partial_{\Delta_{t-1}}(\phi_{\Delta_{t-1}})(V^G) = -\frac{1}{(\beta p_1'(a^B))^2} \cdot \beta p_1''(a^B) \cdot \frac{1}{f_{\theta_1}'(\Delta_{t-1} + f_{\theta_2}(a^G))} < 0,$$

which is negative because of (C.10) and Condition 2.1. The property of $\phi_{\Delta_{t-1}}$ is reflected by its dotted line and the dashed representation in Figure 7.

By the induction hypothesis, the support of $\Gamma_t(V_t^B, V_t^G)$, is bounded by continuous functions, in particular, it is compact and connected. From this and (C.16) we deduce that there are $\overline{\Delta}_{t-1}$ and $\underline{\Delta}_{t-1}$ so that the curves $\phi_{\Delta_{t-1}}$ intersect the set for $\underline{\Delta}_{t-1} \leq \Delta_{t-1} \leq \overline{\Delta}_{t-1}$ and do not intersect for $\Delta_{t-1} < \underline{\Delta}_{t-1}$ and $\Delta_{t-1} > \overline{\Delta}_{t-1}$ (of course $\underline{\Delta}_{t-1}$ could be smaller than zero, the lower limit for Δ_{t-1}). From this ensues the existence of two bounds, $\underline{V}_{t-1}^B(V_{t-1}^G)$ and $\overline{V}_{t-1}^B(V_{t-1}^G)$, limiting the set of feasible pairs (V_{t-1}^B, V_{t-1}^G) from above and below.

To show the *second assertion*, we have to look more closely at the shape of the curve $\phi_{\Delta_{t-1}}$ as well as the limiting functions $\underline{V}_{t-1}^B(.)$ and $\overline{V}_{t-1}^B(.)$. First, we prove that the derivative of $\phi_{\Delta_{t-1}}$ is *smaller than one*. We do so by showing that

$$D(.) \circ \phi_G^{-1}(V_{t-1}^G) := (\phi_G(.) - \phi_B(.)) \circ \phi_G^{-1}(V_{t-1}^G)$$

is increasing in V_{t-1}^G, i.e. the derivative of $\phi_{\Delta_{t-1}}$ is below the one of the diagonal:

$$\partial_{V_{t-1}^G} D(\phi_G^{-1}(V_{t-1}^G)) = 1 - \partial_{V_{t-1}^G} \phi_B(\phi_G^{-1}(V_{t-1}^G)) > 0.$$

Since we know that

$$\partial_{V_{t-1}^G} (\phi_G^{-1})(V_{t-1}^G) < 0$$

from (C.1), it is sufficient to show that

$$D'(a^G) < 0.$$

Using $a^B := f_B^{-1}(\Delta_{t-1} + f_G(a^G))$ we calculate

$$\begin{aligned}
D'(a^G) &= -\frac{p_G''(a^G)}{\beta(p_G'(a^G))^2} + \frac{p_B''(a^B)}{\beta(p_B'(a^B))^2} \cdot \frac{f_G'(a^G)}{f_B'(a^B)} \\
&= -\frac{p_G''(a^G)}{\beta(p_G'(a^G))^2} + \frac{p_B''(a^B)}{\beta(p_B'(a^B))^2} \cdot \frac{p_G(a^G)p_G''(a^G)}{(p_G'(a^G))^2} \cdot \frac{(p_B'(a^B))^2}{p_B(a^B)p_B''(a^B)} \\
&= \left(\frac{p_G(a^G)}{p_B(a^B)} - 1\right) \cdot \frac{p_G''(a^G)}{\beta(p_G'(a^G))^2}.
\end{aligned}$$

The last expression is negative by Conditions 2.1 and 2.2. Now, the second assertion follows if we can show that the derivative of the boundary functions $\underline{V}^B(.)$ and $\overline{V}^B(.)$ is *greater than one*, for then $\phi_{\Delta_{t-1}}$ crosses them at most once. So by the induction hypothesis, assume that $\underline{V}_t^B(.)$ and $\overline{V}_t^B(.)$ have a derivative greater or equal than one (note that this is certainly true for the case of $t = T - 1$).

According to what we have shown above, $\overline{\Delta}_{t-1}$ and $\underline{\Delta}_{t-1}$ limit the set of values $\Delta_{t-1} = V_{t-1}^G - V_{t-1}^B$ for which $\phi_{\Delta_{t-1}}$ intersects the set of feasible (V_t^B, V_t^G). From this we might be tempted to deduce immediately both $\underline{V}_{t-1}^B$ and $\overline{V}_{t-1}^B$ must be linear functions with derivative one, for apparently the limits only depend on the difference $\Delta_{t-1} = V_{t-1}^G - V_{t-1}^B$. Note, however, that the starting point $\underline{a}^G$ (see equation (C.13)) for each curve $\phi_{\Delta_{t-1}}$ is shifting upwards as V_{t-1}^G is falling. Thus, since by induction hypothesis $\underline{V}_t^B$ and $\overline{V}_t^B$ are increasing more steeply than the $\phi_{\Delta_{t-1}}$, we may deduce that

1. $\underline{V}_{t-1}^B(.)$ is indeed linear with derivative one because the $\phi_{\Delta_{t-1}}$s cross the function $\underline{V}_t^B(.)$ at the lower bound $\underline{V}_t^G$ at a high value for a^G.
2. For lower values of V_{t-1}^G, the smallest Δ_{t-1} for which $\phi_{\Delta_{t-1}}$ intersects the set of feasible values (V_t^B, V_t^G) is below the one that would have been obtained with $\underline{a}^G$ fixed. Since the latter would have corresponded to a linear upper bound $\overline{V}_t^B(.)$ with derivative one, we conclude that $\overline{V}_t^B(.)$ has to rise more steeply than this, i.e. that its derivative is greater than one.

Thus by induction, we have shown that $\phi_{\Delta_{t-1}}$ and $\underline{V}_{t-1}^B(.)$ and $\overline{V}_{t-1}^B(.)$ cross only once and the second assertion about the form of the correspondence Γ_{t-1} ensues. This concludes the proof of Proposition 3.4. ■

A final remark on Proposition 3.4 concerns the sense of the lower bound $\underline{z}$ on utility from consuming the benefit.

Remark Appendix C.1. *The lower bound on z_t in Proposition 3.4 is introduced for technical reasons: Some utility functions map onto the real line $\mathbf{R}$, while some only onto the half-line $\mathbf{R}_+$ (or $\mathbf{R}_-$). Constant Absolute Risk Aversion (CARA) are an example of the former kind and CRRA utility functions of the latter. Without a lower bound we would allow*

Proof of Proposition 3.5

Proof. Beginning with point 1 we show that for all contracts, $V^G > V^B$. The assertion then follows by $V^{b,G} > V^{b,B}$ and agent B's entitlement constraint (14).

So we consider a feasible UI contract. Given any set of effort choices $(a_1^B, a_2^B, ..., a_{T-1}^B)$ of agent B, the same set of choices would yield a higher value of total expected lifetime utility for agent G than for agent B, $V^G(\vec{a}^B) > V^B(\vec{a}^B)$. This is the case because firstly (total) utility when employed is higher than (total) utility when unemployed, and secondly by condition 2.2, first part, $p_B(a^B) > p_G(a^B)$ for any $a^B > 0$. Thus, in particular, at the optimum $V^G > V^B$.

We now prove point 2 by contradiction. Suppose that for the solution contracts b, $(z_1^b, ..., z_T^b)$, and g, $(z_1^g, ..., z_T^g)$, the constraint (14) did not bind. For sufficiently high $\underline{V}$ we may assume that all $z_t^i > \underline{z}$ for all t, in particular for $t = 1$. But then create new contracts b' and g' by replacing z_1^i by $z_1^i - \epsilon$ $(i = b, g)$ for some $\epsilon > 0$ with $z_1^i - \epsilon > \underline{z}$. These contracts are certainly feasible. They are also incentive compatible, since the entitlements $V_1^{i,j}$ are reduced by the same amount. However, the new contracts b' and g' are less costly for the principal, since the cost function $c(.)$ is strictly increasing. This is a contradiction.

Point 3 is proved by an argument simular to the one in point 2. ■

Proof of Corollary 3.6

Proof. 1. Note that, given that $V^{g,B}$ is chosen optimally for each value of $V^{g,G}$, the cost function of contract g strictly increases in $V^{g,G}$. Moreover, in a full information optimum (i.e. the pure moral hazard case for both contracts), the optimal $V^{b,G}$ (optimal with respect to $V^{b,B} = \underline{V}$) can be characterized by a first order condition. We thus obtain a first order reduction of costs for contract g by lowering $V^{g,G} = V^{b,G}$ (constraint (13) is binding) below the value of $V^{b,G}$ in a full information optimum, whereas there is only a second order increase in costs for contract b.

2. In our framework, we can recover the SW contracts (i.e. the contracts from the pure moral hazard environment) at a given level of entitlement $V^{i,i}$ by solving $(i \neq j)$:

$$\begin{aligned} &\min_{V^{i,j}} C_1^i(V^{i,i}, V^{i,j}) \\ s.t. \quad &LOM, MH - IC \end{aligned}$$

and applying forward induction afterwards. This is because by minimizing the costs of contract i with respect to its value for agent j, we simply neglect the impact of this value for the optimal contract.

Now, if our objective function is optimized without further restriction, we recover the optimal contract from the pure moral hazard environment, because the value $V^{g,B}$ of contract g for type B does not appear in the cost function of contract b. ■

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