Conventional Markets versus Online Markets: Brand Effects and Entry Decision

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Abstract

Why did high-brand conventional retailers hesitate to enter online markets for a substantial period of time? This paper aims to explain the above phenomenon which did not derive much attention in the literature. I modeled entry decision of a conventional firm as a dilemma: Early entry by the firm to the online market will increase the popularity of the online market and as a result, cause the demand for the product to shift from the conventional market to the online market. On the other hand, the firm's failure to enter the online market early will allow the online firm to increase its brand value due to the relatively high demand for its online product. The results of the paper point out that given the take off probability -the probability that the popularity of the online market will increase- conventional firm will delay its entry to the online market whenever brand effects are not substantial, to protect its profits in the monopolistic conventional market. I have also shown that, if the difference in willingness to pay between agents is high for the online product then the conventional firm will enter early to be able to increase its profits by price discrimination. However, given the probability of take off, for high enough markup rates in the conventional market, the conventional firm will not risk its dominant position in the conventional market by early entry.

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1. Introduction:

With the development of e-commerce, online markets have been introduced for many products that were sold in conventional markets. Many leading firms of online markets gain a high popularity that may outrun their conventional counterparts, whereas many others failed to capture a significant portion of conventional markets. Examples for the former case includes Amazon in online book market, Ebay at online auction market, and Netflix at online DVD rental market, whereas example for the latter includes online electronics market where Best Buy, the retailer with the largest share in conventional market also dominates the online market. Amazon who started selling books online in 1994 has a high popularity that is comparable to Barnes and Noble, the book retailer with the highest market share in the conventional market. Another example is DVD rental market, where the online firm Netflix established in 1999 gained a popularity that can compete with long established Blockbuster's, the conventional firm with highest market share for DVD rentals.

These newly introduced online markets had a high potential to become popular if they could attain a substantial demand for their product. Yet, many existing dominant firms operating in conventional markets seemed to have been hesitant to use their established brand image to capture a large share of online markets, ending up becoming a second player in the market. Barnes and Noble entered to online market at 1997, 3 years after Amazon was established as the first online book retailer. Similarly, Barnes and Noble's online demand is only one fourth of Amazon, where the two networks supply 85 percent of the market share. In the case of DVD rentals, the lag of Blockbuster was 5 years as it entered online market at 2004. Competing through online markets with Netflix yielded only one thirds of market share of Netflix, as of 2005.

Why did Barnes and Noble and Blockbuster's hesitate to enter online market? Obviously, the probability of online markets to take off as a network market was an important determinant. However given the lucrative evolving market, it is somewhat puzzling why these incumbent firms in conventional market did not try to use their brand names in order to establish a dominant status in the online market. This paper tries to provide an answer to this puzzle.

To explain the behavior of a conventional firm with an established brand, a two stage game between a conventional firm operating at the conventional market and an online firm selling the same product in online market is constructed. The foremost assumption of this paper is that, a newly established online market behaves as a network market, that is, the utility derived from the online market depends on the size of installed base, which will be small if it is not used by a sufficient number of consumers. Therefore, a network market that is smaller than a critical mass cannot be sustained in the equilibrium, due to network effects.[1]

Due to his monopoly position in the conventional market, the conventional firm is assumed to have a high brand name, and therefore whenever he enters the online market, the utility derived from his high brand product will be high. Furthermore, it is assumed that, if the conventional firm enters the online market, the demand switching from conventional market to network market will be high enough such that the online network market will take off. However, online firm has a relatively low brand value in the beginning and therefore the willingness to pay for its product is low. Thus, if the conventional firm does not enter the network market, the demand for online market will be uncertain, which can be lower than critical mass with a positive probability. Whenever the online market takes off, the willingness to pay for the online market will also increase, whereas if it does not take off, then willingness to pay for the network market will not change. The willingness to pay for the conventional market is the same across all consumers, but consumers differ in their willingness to pay for the network good, which can be high or low.

The conventional firm decides whether to enter the newly established network market or not, and for the optimal time for entry if he decides to enter. It is assumed that conventional market is at least as profitable as network market, when network market succeeded. That is, willingness to pay for conventional good is higher than network good in the case of success, by a high type person who values the network higher than a low type. Therefore, the conventional firm will attain the highest profits in the case of failure of network. Thus the conventional firm will prefer failure of the online market to be able to enjoy monopoly profits from conventional market where the profit margin is relatively higher.

After conventional firm makes his entry decision about first period the performance of online market is realized and then firms set prices and consumers make the purchasing decisions. The conventional firm can decide to enter to online market either at the beginning of the first period without knowing the performance or at the beginning of the second period after success or failure of the online market is realized, or he may choose never to enter the online market at all.

The trade-off that the conventional firm faces is two-fold. If he does not enter at the first period and the market succeeds, the online firm would have

[1] See Economides and Himmelberg (1995) for an extensive discussion of critical mass and network size.

a high brand, and entering the online market at the second period will yield him a lower market share. Although, the conventional firm can enter the online market at the beginning of the first period securing a high market share over time, his entry to the online market will lead to the success of the online market which may have failed if didn't enter. By entering early the conventional firm can secure the highest profits for the second period if the market succeeds, however, failure of the online market will lead to the highest profits possible for the conventional firm.

It is shown that, whenever the increase in the value of online firm's brand is low and loss due to take off of online market by entering early is relatively high, then, not entering is the only equilibrium. But if the increase in brand value of online firm has a bigger effect on profits than not entering when the online market did not take off, conventional firm will find more profitable to enter early to secure a high market share in the second period whenever success probability and willingness to pay for conventional market are low whereas willingness to pay for network by high types, the increase in willingness to pay when the market take off, and the brand advantage of the network firm due to conventional markets late entry are high.

The organization of the paper is as follows. In section 2 literature is presented. In Section 3, model with consumers preferences over markets and brands and firm decisions is introduced. The set up is a two stage game between conventional and network firm. In section 4.1 equilibrium prices and profits are solved for the cases when conventional market set up a network market and not. In section 4.2 subgame perfect equilibrium and the conditions where the conventional firm will enter the market or wait for the second period are presented.

2.Literature:

Chevalier and Goolsbee (2003) empirically investigated the price sensitivity in the online markets for books. Using the data from leading online booksellers, Amazon and BN.com, they estimated the elasticities of demand for booksellers and created a price index for books. Their results point out that price sensitivity for both booksellers is high , where demand for BN.com is much more price elastic than demand for Amazon, both in his own price and Amazon's price. They found that BN.com's own price elasticity of demand is -3.5, whereas Amazon's own price elasticity of demand is -0.45. For cross price elasticities, they found that one percent increase in Amazon's price raises quantity demanded for BN.com by 3.5 percent. They have concluded that, given

Amazon sells 3 to 10 times as many books as BN.com, and this result implies every customer lost by Amazon instead buys the book at BN.com. They also stated that their results point to Amazon being the market leader in the online book business with BN.com serving as more of a price taking fringe. Their results are supporting the assumption of this study that the network market is shared between the two firms as the estimates for cross price elasticity are high.

Lee and Ward (1999) in their study examined the usefulness of brands as a substitute to consumer's own search activities in online purchases. They claim that for many users, especially the ones that are inexperienced to the internet, finding product information may be frustrating. Therefore, for such consumers brand names serve as a source of information. However, as consumers become more experienced with internet over time, they claim that the reliance on brand names should decrease. By using the usage and opinion survey data from Internet Community, they have reached the result that branding can facilitate consumers acceptance of electronic commerce. They conclude that there is a significant relationship between internet experience and search proficiency and brand reliance. Their results supports supports the assumption of this study that, the early entry of a high branded established firm to the online network market will increase sales in the network market and increase the popularity of the network market and cause positive network effects.

Brynjolfsson and Smith (2000) and Clay, Krishnan, Wolff and Fernandes (2002) empirically analyzed the pricing behavior for both online and conventional markets for books where the former study included and CD's . In their paper they examined if competition in the internet will lead to lower and more homogeneous prices. In the latter study, it has been found that, the average online prices are similar to physical stores and there is substantial price dispersion online. In the former study Brynjolfsson and Smith calculated the average prices for online markets to be lower than conventional markets. Also, contrary to their expectataion that online markets will lead to lower and more homogeneous prices they found substantial absolute price dispersion for internet retailers, an amount slightly higher than or the same as of conventional retailers.

On the other hand, their calculations when prices are weighted by market share shows that price dispersion for internet retailers is lower. They have pointed out that being the undisputed leader in online book sales, Amazon's prices are far from being lower than its competitors, which points out to a price premium. They have shown that internet retailers who also have conventional outlets are able to charge price premiums of about 9 percent for their products compared to retailers who only operate at internet market. They have stated the reasons for price dispersion among internet retailers as retailer heterogeneity with respect to the trust of consumers, which arise as a characteristic of the internet and associated value of branding where one way for Internet retailers to signal trust is by developing a reputation among customers for reliability, thus a high market share may be self perpetuating. Also there may be important network externalities to conveyance of trust through word of mouth. So they concluded that, contrary to their expectation that internet will equalize the retailers and eliminate branding, internet markets may have heightened the importance of differences in trust and branding among retailers.

In this study, parallel to the results of Brynjolfsson and Smith, it appears that the retailers who also have conventional markets will charge a price premium for their online market, regardless of his relatively higher valued brand, which will be higher for a conventional market with a high brand value. Their result of the existence of firm level network effects and the importance of firm level network effects in the brand value of a firm supports the assumption that a higher demand for an online firm will lead to a higher reputation and brand for this firm, which increases premium charged by the online firm for his brand.

Mason and Weeds (2000) examine the optimal timing of adoption of a technology whose returns are uncertain when there is an advantage to being first adopter and but also there is a network advantage from adopting when others are adopting. There are two possibilities in terms of the timing of adoption, either the leader preempts its rival or both firms adopt the technology simultaneously. They have investigated the optimal entry timing by Hotelling style model of entry to the horizontally differentiated market. They have found that whenever there is a preassigned leader, a simultaneous adoption can occur in equilibrium if and only if expected profits for either firm from adopting simultaneously is sufficiently high or expected profits for either firm from adopting sequentially is sufficiently low. However, in this study, given online firm is a preassigned leader in online market, early entry by conventional firm will occur whenever willingness to pay for conventional market is low, online firm's brand disadvantage is high or increase in willingness to pay for online market given the network takes off is low, which all effect profits of online firm negatively. The difference in the results occurs as, the network effects in online market effect the profits from conventional markets negatively, and thus conventional firm is not necessarily benefiting from network effects.

3. The Model:

3.1. Overview:

There are two markets, conventional market and online market, and two heterogeneous firms, namely Firm A and Firm B, who initially operate at different markets; the conventional market and the online market. Firm B is an established firm and has a monopoly position in the conventional market. Being an established monopoly firm, B has a high brand value that increases the utility derived from B's products. Firm A is a new firm who enters the online market at period 1. Being a new firm, the value of A's brand is low, and he has a brand disadvantage in the first period. B's relative brand advantage is normalized to 0, and A's brand disadvantage in the first period is represented by $z^H > 0$. Firm B's decision to enter the online market or not and the time of entry will effect firm A's brand value: if firm B enters early at the first period, first period demand for firm A's online product will be low. In that case firm A will not be able to make use of firm specific network effects and its brand value and popularity will stay low through the second period. On the other hand, if firm B chooses to enter the online market in the second period, or not to enter at all, due to its monopoly position in the first period in online market, A's brand value disadvantage will decrease to z^L from z^H.

If firm B enters to the online market in first period, the online market will take off. However, if firm B does not enter to the online market, the market will not take off with a positive probability denoted by $1-\eta$. If the conventional firm B enters online market at period 1, the popularity of the market will increase, and the online market is assumed to success with probability 1.

3.2. The Two Markets:

A single product is sold only at conventional market, before period 1. At period 1, an online market is introduced, where the same product is sold by an online firm A. The total demand for the product is fixed. The utility derived purchasing from conventional market and online market are different.

3.3. Consumer Preferences:

There is a continuum of consumers in the economy. Each consumer purchases one unit of commodity either from the conventional market or from the online market. Consumers differ over their tastes for brands A and B, and over two markets the conventional market and the online market. In the first period, willingness to pay for the online good is low, which is normalized to 1. In the second period the online market can take off and become very popular, then willingness to pay for online product will increase to $1 + \delta$, where δ is the increase in willingness to pay. Consumers differ in their tastes for the online product: A consumer's willingness to pay for the online product is θ^i where $\theta^i \in \left\{\theta^H, \theta^L\right\}$, in the first period, a θ^H type consumer's willingness to pay for the network good is high and θ^L type's willingness to pay is low with $\theta^H > \theta^L$. In the case of success of the network, his willingness to pay will increase to $\theta^i(1+\delta)$. Consumers also differ in their valuation of brands of the two firms. θ^i type consumers are distributed uniformly between $[0, 1]$ a la Hotelling, where 1 represents the location of firm B's brand and 0 represents location of firm A's brand. A consumer's utility from consuming a product decreases with his distance to the brand of a firm ; $1 - t$ is the distance of consumer t from firm B's brand and t is the distance from firm A's brand. Each consumer consumes one unit of product. The population of high and low type consumers are the same and is normalized to 1, and thus the total demand for the product is 2.

A consumer's utility from firm B's conventional product B_T, firm B's online product B_N and firm A's online product A_N with the corresponding prices P_{BT}, P_{BN} and P_A respectively for each product are given as:

$$U_{BT}(t, P_{BT}) = T - (1 - t) - P_{BT}$$
$$U_{BN}(t, P_{BN}) = N - (1 - t) - P_{BN}$$
$$U_A(t, P_A) = N - t - P_A - z^j$$

where:

$$N = \theta^i(1 + \delta)$$

The willingness to pay for conventional market is given by T and willingness to pay for the network is given by $\theta^i(1 + \delta)$ where $\theta^i \in \{\theta^L, \theta^H\}$ and $z^j \in \{z^L, z^H\}$. The increase in willingness to pay is denoted by δ where $\delta > 0$ if the market takes off and $\delta = 0$ if not.

3.4. Analysis:

Given this setup, consumers choose either between firm A's online market A_N, firm B's conventional market B_T or firm B's online market B_N. A consumer located at t will prefer firm A if and only if

$$U_A(t, P_A) > max\{U_{BN}(t, P_{BN}), U_{BT}(t, P_{BT})\}$$

which implies

$$(1+\delta)\theta^i - t - P_A > max\{T-(1-t)-P_{BT}, (1+\delta)\theta^i-(1-t)-P_{BN}\}$$

As a consumer's choice between firm B's online and conventional market does not depend on it's brand; for any t, a consumer will choose B's online market over conventional if

$$(1+\delta)\theta^i - P_{BN} > T - P_{BT}$$

and will choose conventional over B's online otherwise. If a type θ^i consumer prefers B's conventional market over B's online market, where $\theta^i \in \{\theta^L, \theta^H\}$, he chooses between B 's conventional market and A's online market depending on his location t. Conversely, if type θ^i consumer prefers B's online market over B's conventional market, he chooses between B's online market and A's online market depending on his location t. Hereby, for convenience, it is assumed that, the willingness to pay differences between high and low types for the online market are high enough such that whenever B sets up an online market, a high type consumer would prefer B's online product to B's conventional product and a low type consumer would prefer B's conventional product to B's online product. This condition is satisfied if

$$\theta^H(1+\delta) - T > P_{BN} - P_{BT} > \theta^H(1+\delta) - T$$

for $\delta \geq 0$. This condition represents firm B's commitment to price differentiate between high and low types through creating two different markets, online and conventional. Therefore, it is assumed ad hoc that price differentiation always yields higher profits for firm B.[2].

The total demand for firm m, $m \in \{A, B\}$ is D_T^m, which is the sum of demand from high type consumers, D_H^m and low type consumers D_L^m, $D_T^m = D_H^m + D_L^m$.

3.5 The Two Stage Game:

A two stage game of conventional firm's optimal entry decision is played between the conventional and the online firm, firms A and B. Firm B's strategies at each period is to decide whether to enter the online market if he didn't enter yet. For firm B, entering the online market yields higher profits in the first period. Yet if it enters in the first period, due to its high brand, the demand for online product will increase and consequently the online market will take off in the second period due to the network effects and lower firm B's profits.

[2]For incentive compatibility conditions of this assumption, see Appendix.

The trade off that the firm B faces is as follows: By entering online market in the first period, B can prevent firm A to increase its brand value in the second period, thus increase its profits for the second period given the market will take off. In other words, if the online market takes off, firm B will be able to have higher profits from online market, due to A's brand disadvantage. But if he chooses not to enter in the first period, although its profits in the first period will be lower, its second period profits will be higher whenever the online market does not take off due to the relatively high willingness to pay for conventional market or the absence of network effects. Yet, even though firm B did not enter the online market, the online market may take off with a positive probability. In that case, if the online market takes off, then the network effects will work in favor of A, given B did not enter in the first period. Through the high demand for its online product in the first period, A's brand value will increase. At each stage of the game, firm A and B set profit maximizing prices for their existing markets. Firms maximize sum of expected profits over periods one and two. The equilibrium concept of this two stage game is subgame perfect equilibrium.

4.1 Stage Equilibrium when the Online market is Monopolistic:

When firm B chooses not to enter the online market in the first period, both high and low type consumers would be choosing between firm B at conventional market and firm A at online market. A type θ^i consumer, where $\theta^i \in \{\theta^L, \theta^H\}$ will choose to purchase from firm B iff

$$U_T(B, t, P_{BT}) > U_N(A, t, P_A)$$

or

$$t > \frac{(1+\delta)\theta^i - T + 1 + P_{BT} - P_A - z}{2}$$

where $i \in \{H, L\}$ Thus the total demand for firm B will be the sum of demand of high types and low types for firm B:

$$D_T^B = T + 1 - \frac{(1+\delta)(\theta^L + \theta^H)}{2} - P_{BT} + P_A + z$$

where $z \in z^L, z^H$. The total profit $\Pi(B)$ of firm B is maximized where first order conditions are satisfied. Given P_A, the profit maximizing pricing rule for firm B is:

$$P_{BT}(P_A) = \frac{T + 1 + P_A + z}{2} - \frac{(1+\delta)(\theta^L + \theta^H)}{4}$$

therefore firm B's maximum profit at stage equilibrium when it chooses not to enter the online market is

$$\Pi_B^{ne}(P_A) = \left(\frac{T+1+z+P_A}{2} - \frac{(1+\delta)(\theta^L+\theta^H)}{4}\right)^2$$

Whenever firm B does not enter the online market, prices and profits of conventional firm are increasing with willingness to pay for conventional market; T, the brand disadvantage of firm A; z, and decreasing with willingness to pay for online market; θ^L and θ^H. If $\delta > 0$ that is if the online market takes off, firm B will enter and the online market will no longer be monopolistic. Therefore $\delta = 0$ whenever the online market is monopolistic. Note that, although θ^L, θ^H and T are said to be representing the willingness to pay for the online and conventional market. Yet, they may as well be representing the size of each market, for the two cases cases of interest: when the online market take off and does not take off, holding the willingness to pay for the two markets constant.

A type θ^i consumer, for $\theta^i \in \{\theta^L, \theta^H\}$ will choose A's online market if

$$U_N(A,t,P_A) > U_T(B,t,P_{BT})$$

that is when

$$t < \frac{(1+\delta)\theta^L - T + 1 + P_{BT} - P_A - z}{2}$$

So the demand of $i \in \{H, L\}$ type consumers for firm A is

$$D_i^A = \frac{(1+\delta)\theta^i - T + 1 + P_{BT} - P_A - z}{2}$$

Thus the total demand for firm A is

$$D_T^A = \sum_i D_i^A = 1 - T - P_A + P_{BT} - z + \frac{(1+\delta)(\theta^L+\theta^H)}{2}$$

Firm A maximizes its profit taking P_{BT} as given, with respect to his price P_A. Given P_{BT}, firm A's profit maximizing pricing rule is:

$$P_A(P_{BT}) = \frac{2 + (1+\delta)(\theta^L+\theta^H) - 2T + 2P_{BT} - 2z}{4}$$

therefore firm A's profits if B chooses not to enter is

$$\Pi_A(P_{BT}) = \frac{1}{2}\left(\frac{2 - 2z + (1+\delta)(\theta^L+\theta^H) - 2T + 2P_{BT}}{2}\right)^2$$

Whenever firm B does not enter online market, prices and profits of firm A are increasing with θ^H and θ^L, the willingness to pay for online market and decreasing with willingness to pay for conventional market T and brand disadvantage z. Cournot equilibrium prices for firms A and B are:

$$P_A^* = 1 - \frac{2T - \theta^L - \theta^H}{6} - \frac{z}{3}$$
$$P_T^* = 1 + \frac{2T - \theta^L - \theta^H}{6} + \frac{z}{3}$$

For $P_A^* > 0$ a sufficient condition is $6 - 2z > 2T - (1+\delta)\theta^L - (1+\delta)\theta^H$. Firm A and B's profits at Cournot equilibrium follows respectively as:

$$\Pi_T^* = (1 + \frac{z}{3} + \frac{(2T - \theta^L - \theta^H)}{6})^2$$
$$\Pi_A^* = (1 - \frac{z}{3} - \frac{(2T - \theta^L - \theta^H)}{6})^2$$

Whenever the online market is monopolistic, firm B's profits are increasing with the willingness to pay for conventional market and firm A's profits are increasing with the willingness to pay for the online market. If it is assumed that the willingness to pay for each market represents the size of each market and the total size of the book market -that is the sum of demands for each market- is fixed, a higher willingness to pay o size fo conventional market would mean higher profits for firm B, which is a monopoly in the conventional market and lower profits for A, which is a monopoly in the online market, with a smaller sized online market. On the contrary, high willingness to pay or size of online market will imply low willingness to pay or size for the conventional market and an increase in firm A's profits and a decrease in firm B's profits. When both firms are monopolies in the markets they have been operating, each one's profit depends on the relative popularity and therefore the size of its market that it is a monopoly.

4.2 Stage Equilibrium when online market is Duopoly:

When conventional firm enters the online market, low type consumer's preference will be between firm B at conventional market and A at online market and, the high type consumer's preference will be between firm B and firm A at online market due to the price discrimination assumption. In that case low types will choose firm B at the conventional market if

$$U_T(B, t, P_{BT}) > U_N(A, t, P_A)$$

or

$$t > \frac{(1+\delta)\theta^L - T + 1 + P_{BT} - P_A - z}{2}$$

So the demand for firm B at the conventional market will be

$$D_L^B = \frac{T - (1+\delta)\theta^L + 1 - P_{BT} + P_A + z}{2}$$

and high types will choose firm B at the online market if

$$U_N(B, t, P_{BN}) > U_N(A, t, P_A)$$

that is if

$$t > \frac{1 + P_{BN} - P_A - z}{2}$$

So the demand for firm B at the online market can be written as

$$D_H^B = \frac{1 - P_{BN} + P_A + z}{2}$$

Firm B maximizes its profit taking P_A as given, with respect to the prices at conventional and online markets; P_{BT} and P_{BN}. Its profit maximizing pricing rule will be:

$$P_{BT}^* = \frac{T + 1 - (1+\delta)\theta^L + P_A + z}{2}$$

$$P_{BN}^* = \frac{1 + P_A + z}{2}$$

therefore firm B's maximum profit if it chooses to enter is

$$\Pi_{B,e}^*(P_A) = \frac{1}{2}\left(\frac{T + 1 - (1+\delta)\theta^L + P_A + z}{2}\right)^2 + \frac{1}{2}\left(\frac{1 + P_A + z}{2}\right)^2$$

Whenever willingness to pay for conventional market is low, i.e. $T = \theta^L(1+\delta)$, the optimal price when not entered, P_B^{ne}, will be lower than P_{BT}^* and P_{BN}^* for a given P_A: B will charge more competitive prices for both markets if willingness to pay for conventional market is low. If willingness to pay for the online market is high, i.e. $T = \theta^H(1+\delta)$ then P_B^{ne} will be lower than price of conventional market and higher than price for online market when firm B enters the online market, given P_A. In this model, given the willingness to pay of the agents, entering the online market is more profitable for firm B: $\Pi_B^{e,*}(P_A) > \Pi_B^{ne,*}(P_A)$. The only case that not entering will yield higher profits is when willingness to pay for the online market for both high and low types is too low i.e.$\theta^H = \theta^L = 0$: then P_B^{ne} will be higher than price for online market and will be equal to the price for conventional market. In this case profits by not entering the online market will be higher for firm $\Pi_B^{ne,*}(P_A) > \Pi_B^{e,*}(P_A)$.

By setting up an online market, firm B will be able to charge a higher price for conventional market and can increase the profit from conventional market and also overall profit. Firm B will be able to compete for the two different types of consumers at two different markets, by both charging two different prices -that is price discrimination- and by providing an online product which is preferred by high type consumers.

Whenever $T < \theta^H(1+\delta)$, entering will yield higher profits to firm B.

Firm A will maximize his profits, given the prices of firm B at online market and conventional market as given. In this case, low types will choose A's online market if

$$U_N(A, t, P_A) > U_T(B, t, P_{BT})$$

that is when

$$t < \frac{(1+\delta)\theta^H - T + 1 + P_{BT} - P_A - z}{2}$$

So the demand of low type consumers for firm A is

$$D_L^A = \frac{(1+\delta)\theta^L - T + 1 + P_{BT} - P_A - z}{2}$$

and high types will choose firm A at the online market if

$$U_N(A, t, P_A) > U_N(B, t, P_{BN})$$

that is when

$$t < \frac{1 + P_{BN} - P_A - z}{2}$$

So the demand of high type consumer's demand for firm A's online product will be

$$D_H^A = \frac{1 + P_{BN} - P_A - z}{2}$$

Thus the total demand for firm A will be

$$D_T^A = 1 - P_A - z + \frac{(1+\delta)\theta^L - T + P_{BT} + P_{BN}}{2}$$

and the total profit $\Pi(A)$ of firm A is

$$\Pi_A = P_A(1 - z - P_A + \frac{(1+\delta)\theta^L - T + P_{BT} + P_{BN}}{2})$$

Firm A's profit maximizing pricing rule is a function of firm B's prices. Since firm A maximizes his profit taking P_{BN} and P_{BT} as given, with respect to his price P_A:

$$P_A^* = \frac{2 - 2z + (1+\delta)\theta^L - T + P_{BT} + P_{BN}}{4}$$

firm A's profits if B chooses to enter is

$$\Pi_A(P_A^*) = \frac{1}{2}\left(\frac{2 - 2z + (1+\delta)\theta^L - T + P_{BT} + P_{BN}}{2}\right)^2$$

By solving for firm A and B's prices from their optimal pricing rules, the stage equilibrium prices can be obtained:

$$P_A^* = 1 - \frac{(T - (1+\delta)\theta^L)}{6} - \frac{z}{3}$$

$$P_{BN}^* = 1 - \frac{(T - (1+\delta)\theta^L)}{12} + \frac{z}{3}$$

$$P_{BT}^* = 1 + \frac{5(T - (1+\delta)\theta^L)}{12} + \frac{z}{3}$$

For $P_A^* > 0$, the sufficient condition is $6 > 2z + T - (1+\delta)\theta^L$. Note that $P_A^{ne,*} = -(\frac{T-\theta^H(1+\delta)}{6}) + P_A^{e,*}$ Therefore if $T > \theta^H(1+\delta)$ then $P_A^{e,*} > P_A^{ne,*}$ and $P_A^{e,*} < P_A^{ne,*}$ otherwise. If θ^H is high enough or the online market succeed such that high types willingness to pay for online market is greater than their willingness to pay for conventional market T, then firm A's equilibrium prices will be higher when he is a monopoly, but if high types willingness to pay is lower than conventional market than A's equilibrium prices will be higher at duopoly. When willingness to pay for the online market is high for high types, A can charge a higher price for his online market as a monopoly, to extract its high willingness to pay advantage from high types, but if willingness to pay is low, willingness to pay advantage doesn't exist for A. If willingness to pay by high types is low, when duopoly, firm A will be overcoming this disadvantage due to difference in willingness to pay whenever B enters, as then, high types decision between firm A and B will only depend on brand, not on valuation of either network or conventional market.

If willingness to pay for conventional market T is very high relative to willingness to pay for online market, such that $3T > (\theta^L + 2\theta^H)$ then prices when firm B did not enter will be the highest in order to capture the high willingness to pay by both high and low types. When willingness to pay for online market is very low such that $5T < (\theta^L + 2\theta^H)$ then in order to be competitive even in low types market, price of B when he does not enter will be lower than P_{NB} and P_{TB} to capture more of both types whose willingness to pay for conventional market is very low.

By entering the online market, firm B would be price discriminating between high and low types. Only by entering the online market, even though it may be charging the same prices for both markets, firm B can increase its profits as the high type consumers demand for B online product will be higher compared to B's conventional product. Disregarding the effects of brand, z, B's online prices will be lower than A, as A will be losing revenues from low type consumers, due to his inability to price discriminate, unlike firm B.

Thus by setting up a online market, firm B will be able to compete more aggressively with firm A for high type consumers, through charging lower prices for its online product, without having to set a lower price for lower type consumers whose willingness to pay for conventional product is high but willingness to pay for online product is low. So firm B will have an advantage in both conventional and online markets as it can charge different prices for each market by maximizing its profits according to different values of willingness to pay by high and low type consumers.

Profits for firm B and A at Cournot duopoly follows respectively as:

$$\Pi_B^* = \frac{1}{2}\left(\frac{12 + 4z + 5(T - (1+\delta)\theta^L)}{12}\right)^2 + \frac{1}{2}\left(\frac{12 + 4z - (T - (1+\delta)\theta^L)}{12}\right)^2$$

$$\Pi_A^* = \frac{1}{36}(6 - 2z - (T - (1+\delta)\theta^L))^2$$

Firm B's prices for both markets and profits are increasing in A's brand disadvantage z; decreasing as the brand value of A increases, whose effect is the same for both markets. Therefore an increase in A's brand value from first to the second period will decrease equilibrium prices and quantity demanded for firm B and his profits. Conversely, as A's brand value increases , A's equilibrium demand and price increases, so a higher brand for A will imply larger profits for A and smaller for B, a profit transfer from firm B to A.

Let s represent the state of the online market, to indicate if the online market took off or not, where $s \in \{S, I\}$, I represents the state of the online market with low willingness to pay and S represent the case where the online market took off. Let z represent the value of A's brand with $z \in \left\{z^L, z^H\right\}$, where z^H represents the the case where A has a high valued brand and z^L represents the the case where A has a low valued brand. Let a represent the action of firm B where $a \in \{entry, non - entry\}$. For a given state of the online market s and brand value of A, z, for firm B, entry yields higher profits in the stage equilibrium, that is

$$\Pi_{B,z}^{*,s,entry} > \Pi_{B,z}^{*,s,non-entry}$$

for $s \in \{S, I\}$ and $z \in \left\{z^L, z^H\right\}$. Note that $\Pi_{B,z}^{*,s,entry} > \Pi_{B,z}^{*,s,non-entry}$ holds for a small z, and a high δ. Secondly, note that, for a given state of online market s and action of firm B, a, a low brand for A, -z^H- implies higher profits for firm B,

$$\Pi_{B,z^H}^{*,s,a} > \Pi_{B,z^L}^{*,s,a}$$

The trade off firm B faces occurs due to the effect of its high brand on take off probability of the online market. Due to its high brand value at the conventional market, its entry to the online market in the first period will create a popular online market and result in take off of the online market with probability 1. Take off of the online market hurts firm B as its profits without take off are higher as take off would mean relatively higher willingness to pay and demand for the more competitive online market and relatively lower willingness to pay and demand for the conventional market where it has a monopoly power and competitive advantage. Firm B can extract higher revenues from both types due to their relatively high willingness to pay for conventional market compared to online market whenever he does not enter. If it chooses to enter than he can charge low types higher due to their relatively high willingness to pay for conventional market and high types willingness to pay for the online market excluding the effects of brand will be the same. But in that case it can charge the profit maximizing price for the online market without worrying about its revenues from lower types (due to different prices). It is implicitly assumed here that the decrease in firm B's profits due to take off are more important compared to increase in profits due to entry in stage equilibrium such that even A's brand is higher, and B's profits with non-entry are higher when the online market did not take off compared to the case where A's brand is low and B enters when the online market take off , that is:

$$\Pi_{B,z^L}^{*,I,non-entry} > \Pi_{B,z^H}^{*,S,entry}$$

where $\Pi_{B,z^L}^{*,I,non-entry} > \Pi_{B,z^H}^{*,S,entry}$ holds for z^L high and z^H low, δ high, and $T > \theta^L(1 + \delta)$.

4.3. Conventional Firm's Entry Decision:

In the case of early entry, due to the high brand value of established conventional firm B, the online market takes off. If in stage 1, firm B does not enter, then,probability of online market to take off is η where $\eta \in [0, 1)$

Expected profits from entering in first period , given that it will lead to take off is:

$$E(\Pi_B^{early-entry}) = \Pi_{B,z^H}^{*,I,entry} + \Pi_{B,z^H}^{*,S,entry}$$

Where expected profits from not entering in the first period depends on takeoff probability and can be written as:

$$E(\Pi_B^{non-entry}) = \Pi_{B,z^H}^{*,I,non-entry} + (1-\eta)\Pi_{B,z^L}^{*,I,entry} + \eta\Pi_{B,z^L}^{*,S,entry}$$

The difference between profits with early entry and not entering in 1st period in the subgame perfect equilibrium can be written as

$$E(\Pi_B^{early-entry} - \Pi_B^{non-entry}) =$$

$$(\Pi_{B,z^H}^{*,I,entry} - \Pi_{B,z^H}^{*,I,non-entry}) + (\Pi_{B,z^H}^{*,S,entry} - \Pi_{B,z^L}^{*,I,entry}) + \eta(\Pi_{B,z^L}^{*,I,entry} - \Pi_{B,z^L}^{*,S,entry})$$

Note that $(\Pi_{B,z^H}^{*,I,entry} - \Pi_{B,z^H}^{*,I,non-entry}) > 0$, $(\Pi_{B,z^H}^{*,S,entry} - \Pi_{B,z^L}^{*,I,entry}) < 0$ and $(\Pi_{B,z^L}^{*,I,entry} - \Pi_{B,z^L}^{*,S,entry}) > 0$

Above equation can be rewritten as

$$E(\Pi_B^{early-entry} - \Pi_B^{non-entry}) = a + b\eta$$

where

$$a = (\Pi_{B,z^H}^{*,I,entry} - \Pi_{B,z^H}^{*,I,non-entry}) + (\Pi_{B,z^H}^{*,S,entry} - \Pi_{B,z^L}^{*,I,entry})$$

and

$$b = (\Pi_{B,z^L}^{*,I,entry} - \Pi_{B,z^L}^{*,S,entry})$$

.

When take off probability of the online market is 0, that is $\eta = 0$, then conventional firm will choose not to enter early if $a < 0$; that is if, for firm B, the loss due to take off substantially high compared to gain by entering, when the online market did not take off. When $\eta = 0$ and $a < 0$, $E(\Pi_B^{early-entry} - \Pi_B^{non-entry}) = a < 0$ and therefore not entering yields higher profits for conventional firm. Note that $a < 0$ holds for the parameter values $T = \theta^H = \theta^L(1+\delta)$ and $z^H = z^L$.

In the case where $\eta = 0$ network will take off if firm B enters in first period, and will not take off otherwise. In this case, not entering in the first period yields higher overall profits to firm B, and therefore subgame perfect Nash equilibrium for firm B is not to enter early. By not entering firm B will have lower profits in the first period but in the second period, his profits will be higher due to lower willingness to pay for the online market and relatively higher willingness to pay for conventional market.

When the take off probability of the online market is 1, that is $\eta = 1$, then firm B will choose to enter early if $a + b > 0$; that is if entry yields higher revenues whenever market does not take off, $\Pi^{*,s,entry}_{B,z} > \Pi^{*,s,non-entry}_{B,z}$. When $\eta = 1$ and $a + b > 0$, $E(\Pi^{early-entry}_{B} - \Pi^{non-entry}_{B}) = a + b > 0$ and therefore entering yields higher profits for conventional firm. Note that $a + b > 0$ holds for the parameter values $T = \theta^H = \theta^L(1 + \delta)$ and $z^H = z^L$.

When $\eta = 1$, that is probability of takeoff is 1, regardless of firm B's entry decision, then, given $\Pi^{*,s,entry}_{B,z^H} > \Pi^{*,s,non-entry}_{B,z^H}$ for $s \in \{S, I\}$, firm B's total profits will be higher by early entry, which will be the only subgame perfect Nash equilibrium.

Proposition 1: There is a critical probability η^*,

$$\eta^* = -\frac{(\Pi^{*,I,entry}_{B,z^H} - \Pi^{*,I,non-entry}_{B,z^H}) + (\Pi^{*,S,entry}_{B,z^H} - \Pi^{*,I,entry}_{B,z^L})}{(\Pi^{*,I,entry}_{B,z^L} - \Pi^{*,S,entry}_{B,z^L})}$$

such that
if $\eta < \eta^*$ then $E(\Pi^{early-entry}_{B}) < E(\Pi^{non-entry}_{B})$ and not entering early yields higher profits for the conventional firm.
if $\eta > \eta^*$ then $E(\Pi^{early-entry}_{B}) > E(\Pi^{non-entry}_{B})$ and entering early yields higher profits for the conventional firm.
So, for a high probability of take off for the online market, conventional firm will find it profitable to enter early, and for a low enough probability he will prefer not to enter early.

(1) For a given take off probability of the network, η^*, early entry will yield relatively higher profits for higher values of willingness to pay differences between high and low types; $\theta^H - \theta^L$ and the first period brand disadvantage of the online firm; z^H.

(2) For a given take off probability of the network, η^*, early entry will yield relatively higher profits for lower values of willingness to pay for the conventional market, T, which represents the profit margin, increase in willingness to pay for online market given take off occurred; δ and the brand advantage of firm A given firm B did not enter in the first period z^L.

Proposition 1 and the assumptions needed to attain Proposition 1 hold for the parameter values of $T = \theta^H = \theta^L(1+\delta)$, $z^H = z^L$.

Early entry is more profitable for a low willingness to pay for conventional market T, if the probability of takeoff η and willingness to pay for network θ^H and θ^L are low and increase in willingness to pay δ and A's brand value when it is high, z^L, is high, and less profitable otherwise. Firstly, when η is low, strategy of not entering becomes effective, as by not entering, firm B can prevent take off of the network with higher probability. Therefore, for a lower η, a higher willingness to pay for for conventional market T will imply higher foregone profits by take off. Similarly, for relatively lower willingness to pay's for network θ^H and θ^L, a higher T will imply relatively higher foregone profits by network's take off . And also a high δ will imply a high loss with take off of the market. Note that these values of parameters effect profits only if η is low. However if η is high, non entry is less effective and δ is low, early entry is less costly, therefore, for a higher T, first period profits due to price discrimination are higher, so that firm B can extract higher revenues due to high T in the first period, and thus early entry becomes more profitable for a higher T.

Early entry is more profitable for a high willingness to pay for online market by high types,θ^H for δ low, thus whenever willingness to pay for network by high types is high, the loss from not entering and in first period and thus not being able to price discriminate between high and low types will be high due to the high willingness to pay difference between types. As θ^H increases early entry becomes more profitable as by entry firm B can compete for high types without lowering his revenues from low types while his loss by not entering will be higher as the high types willingness to pay for network increases, whenever δ, the loss in second period due to increased willingness to pay is low.

Early entry is more profitable for a high brand disadvantage for firm A in the beginning of the first period, z^H. As firm B's profits are increasing with z, for a given z^L, the brand disadvantage (or advantage if $z^L < 0$) if firm B does not enter early, a higher z^H will imply a higher loss in the second period due to not entering early will be and higher as the increase in the brand value of A from period 1 to 2 will be relatively higher.

Not entering in first period is more profitable for a high brand disadvantage for firm A in the beginning of the first period, z^L, for the take off probability η low enough such that not entering strategy is effective. Whenever z^L is high, for a given initial brand disadvantage for firm A z^H , the loss of not entering due to an increase on A's brand value will be lower, so firm

B's revenue loss due to A's brand increase whenever he chooses not to enter in the first period will be lower and not entering early will be relatively more profitable.

Not entering in first period is more profitable for a high increase in willingness to pay for online market whenever in the case of a take off, δ, for the take off probability η low enough such that not entering strategy is effective. Firm B's profits are decreasing with the willingness to pay for the network, given that firm B's total profits are higher whenever the online market does not take off. But when η is low such that B can prevent take off, a higher δ will imply a higher loss from conventional market profits if the network take off, as willingness to pay for conventional market will be relatively low whenever the online market takes off. Thus for a higher δ not entering in the first period becomes more profitable.

5. An Extension to Infinite Horizon Game:

In this section, an extension of the two period game to an infinite horizon game is proposed. Firstly, it is assumed that, the probability of take off is constant throughout all periods of the infinite horizon game and is equal to η. It is assumed that the game begins at period $T \geq 0$. At period T, firm B, the conventional firm decides between entering and not entering the online market, given that it did not entered before. If it enters, it will capture part of the online market and its profits in the initial period, period T, will be higher compared to not entering. If firm B does not enter, the online market will stay as a monopoly. However, if firm B enters at period T, the online market will take off at the next period, period $T + 1$, where willingness to pay for the online market will be higher and willingness to pay for the conventional market will be relatively lower for periods $t \geq T + 1$. If firm B chooses not to enter to the online market at period T, then the online market will take off with probability η where $\eta > 0$. In that case willingness to pay for the online market will increase and be higher compared to willingness to pay for the conventional market for periods $t \geq T + 1$. With probability $(1 - \eta)$ the online market will not take off, and the game will extend to period $T + 1$ where firm B will have a chance to choose between entering and not entering the online market again. Firm A's brand disadvantage in period T is denoted by z_T.

In the infinite period game, if firm B decides to enter the online market at period T, firm A will lose its monopolistic advantage and will not be able to increase its brand value for periods $t \geq T + 1$. In that case firm A's brand disadvantage will be z_T for $t \geq T + 1$, and therefore the market shares of firm

A and B will be the same for $t \geq T+1$. But if firm B decides not to enter at period T, then firm A will be a monopoly in the online market at period T and will increase its brand value from period T to $T+1$, which will imply, $z_T > z_{T+1}$, that is a decrease in A's brand disadvantage. Firm A's market share in the online market at period $T+1$ will be higher if the market takes off at T. Or in the case where online market didn't take off and firm B decided to enter at period $T+1$. The discount ratio is set to be δ.

Payoffs to entry and non-entry strategies are examined in terms of stage profits of firm B. At period T, entering will yield to first period profit π_T^{eF} which is higher than not entering, π_T^{neF} that is, $\pi_T^{eF} > \pi_T^{neF}$. But if B enters at period T, its profits, π_T^{eS} will be lower for $t \geq T+1$, where compared to first period profits, π_T^{neF}, as the online market will take off at period $T+1$. If it does not enter in period T, and the online market takes off, it will enter in period $T+1$, and its profits, π_{T+1}^{eS}, will be lower compared to π_T^{eS}. What is more, I will assume that, for any $k > 0$, $\pi_{T+k+1}^{eS} - \pi_{T+k}^{eS} = x$ and $\pi_{T+k}^{eS} = \pi_T^{eS} - kx$ where x is constant. That is, the effect of the increase in firm A's brand value on firm B's profit is the same throughout all the periods. Similarly, it is assumed that , for any $k > 0$, $\pi_{T+k+1}^{eF} - \pi_{T+k}^{eF} = y$, due to the increase in A's brand as long as firm B does not enter.

Proposition 2: The only candidate strategies for subgame perfect Nash equilibrium for firm B are entering the online market at the present period, T and never entering.

Proof: The difference between expected payoffs from not entering for the first $k+1$ periods of the game and entering at period $T+k+2$ an expected payoffs from not entering for the first k periods of the game and entering at period $T+k+1$ can be written as:

$E(\text{period } T+k \text{ entry}) - E(\text{period } T+k+1 \text{ entry}) =$

$$\pi_T^{eF}\left[1-(1-\eta)\delta^2\right] + \pi_T^{eS}\left[\frac{\delta}{(1-\delta)}(1-\eta-(1-\eta)\delta^2)\right] - \pi_T^{neF}\left[1+(1-\eta)\delta\right]$$

$$+x\frac{\delta}{(1-\delta)}\left[(1-k)+nk+(1-\eta)\delta^2(k+1)\right] + y\left[(1-k)+(1-\eta)\delta^2(k+1)\right]$$

To find out when would entering one period earlier would be more profitable, sooner or later, the change of the difference between early and late entry can be calculated as:

$$\frac{\partial E_{T+k} - E_{T+k+1}}{\partial k} = x\frac{\delta}{(1-\delta)}\left[-1+n+(1-\eta)\delta^2\right] + y\left[(1-\eta)\delta^2 - 1\right] \leq 0$$

If at period T, expected payoff to enter is higher than delaying the entry, that is

$$E_T - E_{T+1} > 0$$

if $\delta = 0$ and $y = 0$ then $\frac{\partial E_{T+k} - E_{T+k+1}}{\partial k} = 0$ and entering in period T, the beginning period, would yield the highest expected payoffs. for $\delta > 0$, on the other hand, as $\frac{\partial^2 E_{T+k} - E_{T+k+1}}{\partial k^2} = 0$, entry payoffs would first decrease then increase such that, for some $m^* > 0$, and $m > m^*$

$$E_T < E_{T+m}$$

Note that for an m^* which is substantially large, entering early, at period T may be a preferred strategy. If onthe other hand, expected payoff to enter, is higher than delaying at period T, that is

$$E_T - E_{T+1} > 0$$

as $\frac{\partial^2 E_{T+k} - E_{T+k+1}}{\partial k^2} \leq 0$, not entering the online market and delaying entry would yield higher payoffs at each and every stage.$\|$

The proposition 2 shows that payoffs to entry are decreasing for firm B at period T, compared to not entering. As the time passes, entry will yield relatively lower profits, and entry strategy becomes less and less appealing. This result provides an insight to the conventional firms behavior; as the conventional firm delays its entry, firm A will be able to increase its brand value until the next period, and entry will become less profitable each period for firm B, due to firm A's relatively higher brand value, and thus its relatively smaller share in the online market.

Note that, if first period entry payoffs are substantially high and, the decrease in entry to online markets is relatively slow, firm B may prefer entry in the period T.

Another important result is that, early entry will yield relatively higher payoffs for higher values of π_T^{eS} and π_T^{eF}, that is $\frac{\partial E_{T+k} - E_{T+k+1}}{\partial \pi_T^{eS}} > 0$ and $\frac{\partial E_{T+k} - E_{T+k+1}}{\partial \pi_T^{eF}} > 0$. This result supports the previous result. A higher value of π_T^{eF} would imply that A's brand value is low at period T, and in general, for an earlier period compared to a later and therefore, entry would yield a higher market share and a higher payoff. Similarly, a higher value of π_T^{eF} would imply higher increase in willingness to pay for the online market and higher overall share of the market of firm A achieve with a higher brand value. In that case, delaying the entry would be more costly for firm B.

Additionally, early entry payoffs are decreasing with the amount of increase in brand value of firm A from one period to other, unless the discount rate is high, this result also supports the relative profitability of non-entry strategy.

6. Conclusion:

Model provides an insight to online markets, and propose an explanation to the behavior of conventional firms decision of not entering the online market until after the online markets take off. What is more, this model suggests explanations to the profitability of online firms strategies, the rapid development of their brand values and market shares, which seems to hinder the entry of conventional firm to the online market, once it missed the train.

7. References:

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8. Appendix:

At equilibrium, the utility of the consumer whose utility by purchasing the commodity is the lowest, who in this case is the type t consumer, should have a positive utility by purchasing the product:
$U_T(P_A^*, P_{BN}^*, P_{BT}^*) > 0$
$U_N(P_A^*, P_{BN}^*, P_{BT}^*) > 0$
$U_A(P_A^*, P_{BN}^*, P_{BT}^*) > 0$
Whenever firm B does not enter the online market, the types whose utility is the lowest utility by purchasing have a positive utility if, the following holds for high types:
$U_{BT}(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^H) = U_A(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^H) = \frac{T-3+(1+\delta)\theta^H - z}{2} > 0$
and for low types:
$U_{BT}(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^L) = U_A(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^L) = \frac{T-3+(1+\delta)\theta^L - z}{2} > 0$
Whenever firm B does not enter the online market, the types whose utility is the lowest utility by purchasing have a positive utility if, the following holds for high types:
$U_{BT}(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^H) = U_A(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^H) = \frac{T-12+8(1+\delta)\theta^H-(1+\delta)\theta^L-4z}{8} > 0$
and for low types:
$U_{BN}(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^L) = U_A(P_A^*, P_{BN}^*, P_{BT}^*)(\theta^L) = \frac{2T-9+4(1+\delta)\theta^L-3z}{6} > 0$

It is assumed that whenever firm B enters the online market, high types will choose between firms in online markets and low types will choose between firms in conventional markets. A sufficient condition for the incentive compatibility conditions for the assumption to hold are as follows:

$U_{BT}(\theta^L, t) \geq U_{BN}(\theta^L, t)$
$U_{BT}(\theta^H, t) \leq U_{BN}(\theta^H, t)$
that is
$T-(1+\delta)\theta^L \geq P_{BT} - P_{BN} \geq T-(1+\delta)\theta^L$
which is satisfied in equilibrium prices if
$2(T-\delta\theta^H) \leq T-\delta\theta^L$

Expected profits to each of the strategies, early entry and late entry are calculated as follows
$E(\Pi_B^{entry}) = \frac{1}{2}(1+\frac{z}{3}+\frac{5}{12}(T-(1+\delta)\theta^L))^2 + \frac{1}{2}(1+\frac{z}{3}-\frac{(T-(1+\delta)\theta^L)}{12})^2$
$E(\Pi_B^{non-entry}) = (1+\frac{z}{3}+\frac{2T-(1+\delta)(\theta^L+\theta^H)}{6})^2$

And the difference of expected profit between the two strategies can be written in terms of constants a and b and take off probability η where a and b are calculated as follows
$a = \frac{1}{2}(\frac{z^H - z^L}{3})(\frac{z^H + z^L}{3} + \frac{5(T-\theta^L)}{6}) + \frac{1}{2}(\frac{z^H - z^L}{3})(\frac{z^H + z^L}{3} - \frac{(T-\theta^L)}{6}) + \frac{1}{2}(1 + \frac{z^H}{3} + \frac{5(T-\theta^L(1+\delta))}{12})^2 + \frac{1}{2}(1 + \frac{z^H}{3} - \frac{(T-\theta^L(1+\delta))}{12})^2 - (1 + \frac{z^H}{3} + \frac{(2T-(\theta^L+\theta^H))}{6})^2$

$b = \frac{1}{2}(\frac{5\delta\theta^L}{12})(2 + \frac{2z^L}{3} + \frac{5(2T-(2+\delta)\theta^L)}{6}) - \frac{1}{2}(\frac{\delta\theta^L}{12})(2 + \frac{2z^L}{3} - \frac{(2T-(2+\delta)\theta^L)}{6})$

Therefore the difference between expected profits can be written as such:
$E(\Pi_B^{early-entry} - \Pi_B^{non-entry}) = \frac{1}{2}(\frac{z^H - z^L}{3})(\frac{z^H + z^L}{3} + \frac{5(T-\theta^L)}{6}) + \frac{1}{2}(\frac{z^H - z^L}{3})(\frac{z^H + z^L}{3} - \frac{(T-\theta^L)}{6}) + \frac{1}{2}(1 + \frac{z^H}{3} + \frac{5(T-\theta^L(1+\delta))}{12})^2 + \frac{1}{2}(1 + \frac{z^H}{3} - \frac{(T-\theta^L(1+\delta))}{12})^2 - (1 + \frac{z^H}{3} + \frac{(2T-(\theta^L+\theta^H))}{6})^2 + \eta(\frac{\delta\theta^L}{3}(1 + \frac{z^L}{3} + \frac{13}{24}(2T - (2+\delta)\theta^L))$.