REGIONAL POLICIES AND INEQUALITIES:

ARE SUBSIDIES GOOD FOR YOU?

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Abstract: This paper analyses the effect of different types of regional subsidies to poor regions on industrial location, income inequality - between and inside regions - and welfare. Our main results are: 1) the impact on location of such subsidies is stronger when trade costs are low. 2) firms relocate to the poor region whether the subsidy is financed by a national tax or regional tax on income. 3) when firms are mobile, regional subsidies that take the form of tax breaks or subsidies to the fixed cost lead to higher profits for all firms because the equilibrium geography is such that profits are equalized across regions. Such subsidies given to firms in the poor region increase regional income inequality, as the rich region owns more capital: they actually lead to an income transfer from the poor to the rich region. It also leads to higher inequality within regions. However, regional subsidies to production may actually yield better results, as they could attract firms in the lagged region and reduce both regional income inequality and individual inequality.

JEL: R0, H2, H7

I. Introduction

A substantial share of the budget of regional policies consists of subsidies to private firms located in poor regions. The prime objective of these subsidies is to attract firms based on the assumption that this will increase employment, wages, and productivity in those regions. This paper has limited ambitions: we only look at certain types of subsidies and analyze the effects of such subsidies on industrial location, regional income inequality and inequalities between workers and capital owners. This analysis is done in a very simple general equilibrium model with agglomeration forces. Three main insights arise from the study of subsidies in the presence of agglomeration forces when firms are mobile.

Regional subsidies do attract firms – a result that would hold in almost any model, including models without agglomeration forces. Our first insight is that the impact of the regional subsidies becomes greater the higher is the level of goods-market integration. This is just a corollary of the home market magnification effect, but it has implications for regional integration. For example, if regional integration in the EU or among developing countries (MERCOSUR) continues to lower trade barriers without changing the level of permissible subsidies to firms in remote regions, the subsidies will lead to an increase distortion of the spatial allocation for industry.

The second insight is that when firms are mobile, regional subsidies to firms in one region lead to higher operating profits in *all* regions as the equilibrium geography is such that profits are equalized across regions. Thus, even firms that do not directly benefit from regional subsidies gain indirectly through the distortion on regional competition: as firms relocate to the region that provides the subsidies, competition becomes weaker in the other region so that profits increase there too. This raises interesting questions on the relation between spatial inequalities and

individual inequalities, especially between immobile workers and owners of mobile capital and the impact of regional policies on these types of inequalities. Implicitly or explicitly, regional policies are based on the belief that if spatial inequalities are decreased then inequality in general will be decreased. In a situation where capital is mobile but workers are not, this is not true. Not only do such subsidies increase inequalities between workers and capital owners. A subsidy given to firms that locate in the poor region can actually worsen nominal income inequality between the poor and the rich region: if profits increase due to the subsidy and most capital owners are in the rich region then these capital owners will disproportionately benefit from the subsidy intended to the poor region, and so will the rich region too. Hence, the subsidy to the poor region actually leads to a transfer from the poor to the rich region even when the subsidy is financed by a national proportional tax on income which falls therefore more heavily on the rich region. Another way to say this is that even though there is an official net transfer from the rich to the poor region, the net effective transfer of income is from the poor to the rich region.

The third insight concerns the effect of the way subsidies are financed. Financing the subsidy requires a tax that reduces market potential, and so profits of firms. Regional subsidies that are financed at the national level have the largest effect on relocation. When the subsidy is financed by the region itself, the local tax that satisfies the budget constraint alters local demand, and so reduces the initial impact of subsidy on relocation. An interesting result is that even when financed locally, regional subsidies lead to relocation in the region that gives the subsidy. Another way to put it is that the negative effect of taxing regional expenditure is more than compensated by the subsidies to regional production.

In what follows we will analyze the effects of a local subsidy to firms producing in poor regions in a general equilibrium model of endogenous location. Three types of subsidy are

successively studied: 1) a subsidy to profits of firms that locate in the poor region, that can be interpreted as a tax break on profits 2) a lump sum subsidy to firms that locate in the poor region. This can be interpreted as lump sum tax break or a subsidy to the fixed cost, such as when land is given (or sold below market price) to the firm to build the plant. 3) a subsidy to production of firms located in the poor region, such as subsidies proportional to the number of jobs created by the firms. They work as a decrease of the variable cost of production. This is for example the case in France with the main instrument of regional policy, the *Prime d'Aménagement au Territoire* (PAT) which gives to firms located in poor regions a subsidy of around 10000$ per job created. Because the subsidy is given once and for all and that it can also finance R&D of those firms, the PAT has also the characteristic of a lump sum subsidy. France however appears as an exception in Europe. Most countries subsidize investment rather than employment at the regional level and this translates into subsidies to capital rather than labor (see Yuill et al, 1994 and Fuest and Huber, 2000). An important example is the subsidy program provided to Eastern Germany. According to Fuest and Huber (2000), 90% of the subsidies to firms locating in Eastern Germany take the form of investment subsidies. At the European level, more than 400 types of subsidies exist that can help firms in poor regions. They take so many forms that it is difficult to put them in one of these three categories. It seems quite safe to characterize them as a complicated mix of the three.

We derive results of these subsidies on prices, profits, and wages, whether subsidies are financed nationally or by the region itself, and we analyze consequences on spatial inequality, regional income inequality and individual inequality.

The next section presents the framework, based on a model with capital mobility, labor immobility, increasing returns, the existence of operational profits and trade costs. To build up

intuition we then analyze in section III a partial equilibrium version of the model where the issue of financing is overlooked. The general equilibrium analysis is presented in section IV: both subsidies to profits and lump sum subsidies, which are similar in their effects, are studied. Section V shows that regional subsidies to production have quite different effects on location and income inequality. In section VI we analyze regional welfare and individual inequality issues.

II. Basic Framework

We will analyze the effect of regional subsidies in the "footloose capital" model developed by Martin and Rogers (1995) and further analyzed in Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud (BFMOR chapter 3) (2003).

This model features agglomeration, if we define agglomeration as the tendency of economic activity to generate forces that encourage further concentration of economic activity. Agglomeration stems from the home-market effect, which implies that a more than proportionate share of industry locates in the larger market. Agglomeration in the footloose capital model, however, is not self-reinforcing : there is no circular causality as in Core-Periphery models (Krugman 1991, Venables 1996, BMFOR chapter 2), as there is neither labor mobility nor vertical linkages. In the footloose capital model, location of mobile capital is determined by an arbitrage condition that equalizes profits across regions. We will only analyze the case of perfect capital mobility so that issues of stability and catastrophic agglomeration are ignored. It means that capital incomes must not be consumed in the region where capital is located : profits can be repatriated and firms can relocate without costs. In this case the stock of capital is exogenous, and the value of capital is given by profits.

II.1. Assumptions

There are two regions (North and South), two sectors, and two productive factors. Regions are symmetric in terms of tastes, technology, openness to trade, and labor endowments. The two sectors are referred to as manufacturing and traditional sector, and manufacturing is marked by increasing returns, monopolistic competition and iceberg trade costs. The traditional sector is assumed to produce a homogeneous good under Walrasian conditions (constant returns and perfect competition) and its output is traded costlessly.

The product factors are physical capital K and labor L, with K being the mobile factor and labor being the immobile factor. Physical capital can be employed in one region while its owner spends its reward in the other region – something that is clearly impossible when factors are associated with people as in the core-periphery model of Krugman (1991). Moreover, capital is only employed as a fixed cost of industrial firms; the variable cost only involves labor.[1] Importantly, the footloose capital model assumes that capital owners are completely immobile across regions. Thus, when pressures arise to concentrate production in one region, physical capital will move, but all of its reward will be repatriated to its country of origin. Total supplies of capital and labor are fixed, with the world's endowment denoted as K^w and $2L$.

Because physical capital can be separated from its owners, the region in which capital's income is spent may differ from the region in which it is employed. We must therefore distinguish the share of world capital <u>owned</u> by Northern residents (we denote this as $s_K \equiv K/K^w$) from the share of world capital <u>employed</u> in the North. Because we assume that each industrial

variety requires one unit of capital (see below), the share of the world capital stock employed in a region exactly equals the region's share of world industry. Consequently, we can use North's industry share, i.e. $s_n \equiv n/(n+n^*)$, to represent the share of capital employed in the North and the share of all varieties made in the North (n is the number of firms producing in the North, asterisk refers to the South).

The cost function of a typical industrial firm (the increasing-returns sector) is non-homothetic; the factor intensity of the fixed cost differs from the factor intensity of the variable cost. To keep things simple, we make the extreme assumption that the fixed cost involves *only* capital and the variable cost *only* involves labor. More specifically, each industrial firm requires one unit of K and a_m units of labor per unit of output. The implied cost function is:

(2.1) $$\pi + w_L a_m x$$

where π and w_L are the rewards to capital and labor, a_m is the variable unit input requirement, and x is firm-level output.

Technology in the traditional sector is kept as simple as possible. Producing traditional goods requires only labor, specifically, it takes one unit of labor to make one unit of this good. Note that this means that the increasing returns sector is intensive in the use of the mobile factor.

The representative consumer in each region has preferences given by:

(2.2) $$U = C; \qquad C \equiv C_M^{\mu} C_T^{1-\mu}, \quad C_M \equiv \left(\int_{i=0}^{n^w} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}, \quad 0 < \mu < 1 < \sigma$$

where C_M and C_T are, respectively, consumption of the composite of manufacturing sector

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[1] Viewing K as physical capital, we can think of the fixed cost in the M-sector as a factory. We can also interpret it as a patent required to start the production of a new variety.

varieties and consumption of the traditional sector. Also, n^w is the mass (roughly speaking, the number) of industrial varieties available worldwide, μ is the expenditure share on industrial varieties, and σ is the constant elasticity of substitution between any two varieties. The indirect utility function for the preferences in (2.2) is:

$$\textbf{(2.3)} \qquad V = \frac{E}{P}; \qquad P \equiv p_T^{1-\mu}\left(\int_{i=0}^{n^w} p_i^{1-\sigma}\, di\right)^{-\mu/(\sigma-1)}$$

where E is Northern expenditure (i.e. after-tax disposable income), P is 'perfect' price index, p_T is the price of the traditional good, p_i is the consumer price of industrial variety i (the variety subscript is dropped where clarity permits).[2] Analogous definitions hold for Southern variables, all of which are denoted by an asterisk.

II.2. Equilibrium without subsidies

Traditional Sector Results

Utility optimization implies that the demand function for the traditional good is $C_T=(1-\mu)E/p_T$. Perfect competition in this sector forces marginal cost pricing, i.e. $p_T=w_L$ and $p_T^*=w_L^*$. In addition, costless trade equalizes Northern and Southern prices and thus indirectly equalizes wage rates internationally: $w_L=w_L^*$ as long as some traditional good is produced in both regions. This condition – the so-called non-full-specialization (NFS) condition – requires that no region has enough labor to satisfy world demand for the traditional good. The exact condition is that total world spending on this good, namely $(1-\mu)E^w$, where E^w is world expenditure, is greater than the maximum value of production that is possible by either region, namely p_TL. This is assumed

to hold henceforth. Assuming workers are perfectly mobile between sectors, we get that wages are equalized in all sectors. Taking traditional good as numeraire, we get that wages are equal in both region to one : $p_T=w_L=p_T^*=w_L^*=1$.

Manufacturing Sector Results

Utility optimisation yields a constant division of expenditure between sectors, and CES demand functions for industrial varieties implies:

(2.4) $$c_j \equiv \frac{p_j^{-\sigma}\mu E}{\int_{i=0}^{n^w} p_i^{1-\sigma} di}; \quad E = \pi K + w_L L$$

where E is region-specific expenditure (and income when there is no taxation), π is the Northern rental rate of capital, K is the Northern stock ok capital, w_L is the Northern wage rate.

As usual, Dixit-Stiglitz monopolistic competition and (2.4) imply that 'mill pricing' is optimal for industrial firms, so the ratio of the price of a Northern variety in its local and export markets is just τ. Thus:

(2.5) $$p = \frac{w_L a_M}{1-1/\sigma}, \quad p^* = \frac{\tau w_L a_M}{1-1/\sigma}$$

Normalizing $a_M = \frac{\sigma - 1}{\sigma}$, and replacing w_L by one, we get the pricing rules of firms in the increasing-sector : domestic price is one, and export price is τ.

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[2] Using standard terminology, P is 'perfect' since real income defined with P measures utility.

The Mobile Factor's Reward

Since physical capital is used only in the fixed cost component of industrial production, the reward to capital is the Ricardian surplus of a typical variety, i.e. the operating profit of a typical variety. Since each unit of capital can be used to produce one industrial variety, the reward to capital would be bid up to the point where it equalled operating profit.

Under Dixit-Stiglitz competition, this operating profit is simply the value of sales divided by σ . In symbols, this means $\pi = x/\sigma$, where x is the scale of production, and an analogous expression holds for the Southern operating profit, π^*. Using the demand function and mill pricing, we can write these equilibrium expressions for π and π^* as:

(2.6)

$$\pi = b\frac{E^w}{K^w}\left[\frac{s_E}{s_n+\phi(1-s_n)}+\frac{\phi(1-s_E)}{s_n\phi+1-s_n}\right];\ b\equiv\frac{\mu}{\sigma}<1;\ \phi\equiv\tau^{1-\sigma}$$

$$\pi^* = b\frac{E^w}{K^w}\left[\frac{s_E\phi}{s_n+\phi(1-s_n)}+\frac{(1-s_E)}{s_n\phi+1-s_n}\right]$$

where E^w is world income, with s_E and $s_E^* \equiv 1-s_E$ being the North's and the South's share of expenditure ; s_n is the North's share of industry, and s_E is the North's share of expenditure. Finally ϕ is the usual transformation of transaction costs, reflecting the freeness of trade: ϕ equals 0 when transaction costs are infinite, and equals 1 when transaction costs are null.

Equilibrium of the Footloose Capital model:

The total level of income of the country is the sum of labor and capital income:

(2.7) $$E^w = 2L + \pi$$

where we assume that the two regions are of equal size and normalize the total stock of capital K^w to 1. In the long run, with free relocation, profits are equalized across regions so that $\pi = bE^W$ and

$$E^W = \frac{2L}{1-b}.$$

The spatial distribution of capital ownership is exogenous and given by s_K. With the assumption that regions are of equal size, we can define North's income as $E = L + s_K\pi$. The Northern share of total expenditure is therefore given by

(2.8) $$s_E = \frac{1}{2} + \frac{b(2s_K - 1)}{2}.$$

The North's share of industry s_n correspond to the geographical equilibrium, that is when profits are equalized across region. Solving $\pi = \pi^*$, we find

(2.9) $$s_n = \frac{1}{2} + \frac{(1+\phi)(2s_E - 1)}{2(1-\phi)}.$$

We assume that the differences in income shares and therefore capital ownership are not too large so that location is always at an interior equilibrium with $s_n < 1$.

III. The Location Effect of Regional Subsidies: partial equilibrium

III.1. Regional subsidies proportional to operating profits

We start the analysis of the location effect of these policies by overlooking the financing issue: hence this is only a partial equilibrium analysis. We first look at the effect of subsidies on the location of firms in the manufacturing sector given income shares. To simplify further we analyze in this section only a subsidy proportional to operating profits for firms located in the South. This can be interpreted as a tax break on profits for firms located in the South. Such a tax break in the traditional sector does not change anything, as this sector has zero operating profits. This particular form of subsidy has no impact on wages and therefore on labor income. In the

section where we analyze the effects of subsidies in general equilibrium, we also study the effects of a lump sum subsidy, and the case of a subsidy to production to firms locating in the South.

The condition that operating profits must be equalized for location of firms to be an equilibrium becomes in the case of subsidies to profits : $\pi = (1+z^*)\pi^*$ where z* denotes the subsidy proportional to operating profits for firms located in the South and π and π^* are the operating profits given by equation (2.6).

Using the equality of operating profits and for a given distribution of expenditure s_E (the share of expenditures in the North), which as we will see in the next section may be influenced by the subsidy itself, we get that the share of firms located in the North is:

(3.1) $$s_n = \frac{s_E(1-\phi^2)-\phi(1+z^*-\phi)}{(1-\phi)\left[1+z^*-\phi-z^* s_E(1+\phi)\right]} \qquad 0 < s_n < 1$$

Quite intuitively, for a given distribution of expenditure s_E, an increase in subsidies to firms located in the South decreases spatial concentration in the North as:

(3.2) $$\left.\frac{\partial s_n}{\partial z^*}\right|_{s_E \text{ constant}} = -\frac{(1+\phi)^2 s_E(1-s_E)}{\left[1+z^*-\phi-s_E z^*(1+\phi)\right]^2} < 0$$

Note that lower interregional trade costs (higher ϕ) magnify the relocation effect of the subsidy. The reason is that when those costs are low, firms are more willing to relocate to take advantage of a small differential in profits, as they can easily export to the region they leave. Profits in the North and in the South are given by (see appendix A):

(3.3) $$\pi = (1+z^*)\pi^* = \frac{bE^w}{K^w}\frac{\left[1+z^*-\phi-s_E z^*(1+\phi)\right](1+z^*)(1-\phi)}{\left[1-\phi(1+z^*)\right]\left[1+z^*-\phi\right]}$$

It can be checked that for a given level of total expenditure E^w, and expenditure shares s_E, firms' profits increase in both regions. In the South the reason is of course the subsidy. In the North, the reason is that as firms relocate in the South competition becomes less fierce and profits increase. Hence, the profit subsidy, in the case where capital is mobile, benefits firms in both regions.

III.2 Regional subsidies and regional income inequality

If profits increase then regional income inequality may be affected by a regional subsidy because of possible regional inequalities in capital stocks ownership. Suppose that the EU funds a regional subsidy to attract firms in the South of Italy. If firms are mobile across the country, then the subsidy will raise profits for all firms in Italy so that capital owners, wherever they are located, will be the main beneficiaries of the policy. If most of the capital owners are in the North, which is quite plausible, then the subsidy whose objective is to help the South may actually increase income inequality between the two regions. We derive this somewhat paradoxical result here in a partial equilibrium framework where the issue of the financing of the subsidy is not considered. The next section conducts the analysis in general equilibrium. The total level of income and expenditure of the country is the sum of labor and capital income: $E^w = 2L + \pi$. The share of income of the North is: $s_E = \left(L + \pi s_K\right)/E^w$ where s_K is the share of capital owned by Northerners which we assume to be more than 1/2, so that we take the North to be the rich region. We ask the following question: what does a small subsidy to firms located in the South do to regional income inequality which can be measured by s_E? It can first be shown that evaluated at z^*=0, a small increase in profits increases income inequality as long as where s_K>1/2, using the fact that $\pi = \frac{2Lb}{1-b}$ for z*=0 :

(3.4) $$\frac{\partial s_E}{\partial \pi}_{z^*=0} = \frac{(1-b)^2}{4L}(2s_K - 1) > 0$$

The subsidy is given to the South but it actually worsens nominal income inequality because Northerners benefit more from the subsidy than the Southerners. This is not due to the fact that Northerners own some firms located in the South but only to the assumption that the North owns more capital than the South and that capital is mobile. Introducing a subsidy distorts competition and increases capital owner's income, who are more numerous in the North. Hence, any increase in profits will raise regional income inequality because of unequal endowment in capital. This effect must be taken into account when analyzing the impact of regional subsidies on location.

IV. The impact of financing: general equilibrium effects

We now add the effect of the financing of regional subsidies on location.

IV.1. A local subsidy to profits financed by a national proportional income tax

We first look at the case in which the subsidy in the South is financed by a national proportional income tax. In this case, the share of expenditures in the North, s_E, is not affected directly as both incomes and expenditures in the two regions are proportionately affected:

(4.1) $$s_E = \frac{(1-t)(L+\pi s_K)}{(1-t)(2L+\pi)}$$

It remains true that $E^w = 2L + \pi$ if we interpret it as pre-tax income. The government budget constraint implies:

(4.2) $$t(2L+\pi) = z^*(1-s_n)\pi^* = \frac{z^*}{1+z^*}(1-s_n)\pi$$

which just says the revenues of national taxation on income equal the subsidy rate (z*) multiplied by the number of firms in the South ($1-s_n$) and the equilibrium level of profits (net of the subsidy) π^*.

The resource constraint at the national level (of the labor market in this case) implies:

(4.3) $$2L = (1-t)(2L+\pi)\left(1-b\right)$$

This equation says that labor supply (*2L*) is equal to labor demand which comes from the traditional and the manufacturing sectors, which themselves come from the equilibrium on the goods market. This relation already uses the fact that in equilibrium, geography must be such that profits are equalized across regions. This then implies that as wages are not affected by the subsidies, profits must rise in both regions. This is because taxes are effectively a transfer from workers to capital owners. Equation (4.3) indicates that total expenditure must remain constant. Also equation (3.4) tells us how a change in profits affects regional income inequality. Finally, the location of firms is still determined by equation (3.1).

We analyze the impact of a small subsidy given to profits of Southern firms. Hence, we differentiate the equilibrium with respect to z* and evaluate it at z*=0. We get the following relation from equation (4.3):

(4.4) $$\frac{\partial \pi}{\partial z^*}_{z^*=0} = \frac{2L}{1-b}\frac{\partial t}{\partial z^*}$$

which confirms that profit rise with subsidies. Then from equation (3.4):

(4.5) $$\frac{\partial s_E}{\partial z^*}_{z^*=0} = \frac{(2s_K-1)(1-b)}{2}\frac{\partial t}{\partial z^*}$$

which says that nominal inequalities rise when subsidies are given to firms because of the increase in profits and the initial inequality in capital endowments. In what follows, and for any nationally financed subsidy, we will use a general expression to express changes in expenditure shares and show the two effects of the subsidy on income inequality. On the one hand the effect via the change in profits and on the second hand the effect via the taxation of income :

(4.6) $$\frac{\partial s_E}{\partial z^*}_{z^*=0} = \frac{1-b}{2L} s_K \frac{\partial \pi}{\partial z^*}_{z^*=0} - \frac{1+b(2s_K-1)}{2} \frac{\partial t}{\partial z^*}_{z^*=0}$$

The government budget constraint implies:

(4.7) $$\frac{\partial t}{\partial z^*}_{z^*=0} = b(1-s_n) = b\frac{1-s_E(1+\phi)}{1-\phi}$$

which says that taxes increase as long as some firms are located in the South which we have assumed as we restrict ourselves to interior equilibrium where s_n is less than 1.

Differentiating $s_E = \left(L + \pi s_K\right)/E^w$, we get, putting (4.5) and (4.7) together:

(4.8) $$\frac{\partial s_E}{\partial z^*}_{z^*=0} = \frac{(2s_K-1)(1-b)b}{2} \frac{1-s_E(1+\phi)}{1-\phi}$$

which says that regional nominal income inequality increases with the subsidy as long as the North has more capital than the South ($s_K > 1/2$). This is because the increase in profits mostly benefits the Northern region, which has more capital than the South. Note again that for this to be true, it is not necessary for firms producing in the South and receiving the subsidy to be owned by Northerners. In general equilibrium, all that is required is that capital be mobile.

The policy implies an official transfer from the North to the South as the subsidy to Southern firms is financed by a national proportional income tax: tE is the share financed by the North and tE^* for the South. As the South is poorer than the North, more of the subsidy is

financed by the North than by the South. From this point of view, the regional policy looks redistributive towards the South. The general equilibrium analysis shows that the incidence of the subsidy is actually a net transfer of resources from the South to the North as the share of total income (which remains constant) that goes to the North increases. Again, capital mobility coupled with initial inequality of capital endowments (which presumably is the reason for the subsidy in the first place) is at the source of this result.

Differentiating equation (3.1), and using (3.2) and (4.8), we then get :

(4.9) $$\frac{ds_n}{dz^*}_{z^*=0} = \frac{\partial s_n}{\partial s_E}_{z^*=0} \frac{\partial s_E}{\partial z^*}_{z^*=0} + \frac{\partial s_n}{\partial z^*}_{\substack{z^*=0 \\ s_E \text{ constant}}}$$ which yields:

(4.10) $$\frac{ds_n}{dz^*}_{z^*=0} = \frac{1+\phi}{1-\phi} \frac{(2s_K-1)(1-b)b}{2} \frac{1-s_E(1+\phi)}{1-\phi} - \frac{(1+\phi)^2\left[1-b^2(2s_K-1)^2\right]}{4(1-\phi)^2}$$

There are two effects of the subsidy on the location of firms in the North as measured by s_n. The first term of equation (4.10) says that if regional nominal inequalities rise because of the subsidy, this will have a positive impact on spatial inequalities because of the Home Market Effect: firms relocate where income and expenditures rise. The second term is the direct effect of the subsidy on location choices of firms, which decreases spatial inequalities: firms relocate where profit subsidies exist.

The net effect is given by:

(4.11) $$\frac{ds_n}{dz^*}_{z^*=0} = \frac{(1+\phi)b(2s_K-1)}{4(1-\phi)^2}\left[(1-b)(1-\phi)+(1+\phi)b^2(2s_K-1)\right] - \frac{(1+\phi)^2}{4(1-\phi)^2}.$$

Using the restriction that $s_n < 1$ (the equilibrium is an interior one so that not all firms are located in the North), we can show that (4.11) is negative. Hence, the regional subsidy financed by a national proportional income tax leads to relocation of firms in the South.

Another implication is that such a subsidy also worsens income inequality between regions and between workers and capital owners in both regions. We analyze this question in section VI.

IV.2. A local subsidy to profits financed by a local proportional income tax

Regional subsidies are not always financed at the national level. Some are financed at the regional level. In this case, the effect of subsidies on location of firms is not so clear. On the one hand, firms will be attracted by such subsidies. On the other hand, the local tax on incomes will lead to a smaller market size which will deter profits of the manufacturing sector. Hence, we now assume that only Southerners are taxed, so that the share of expenditures in the North s_E increases automatically via a tax effect:

(4.12) $$s_E = \frac{E}{E^W - tE^*},$$

where E, E* and E^w still define respectively Northern income, Southern income and the world income before the tax, as given in the previous section.

The location of firms is still determined by equation (3.1). The Southern government budget constraint now implies:

(4.13) $$t(L + \pi(1 - s_K)) = z^*(1 - s_n)\pi^* = \frac{z^*}{1 + z^*}(1 - s_n)\pi$$

The resource constraint (of the labor market again) is:

(4.14) $$2L = (1 - b)\left[2L + \pi - t(L + \pi(1 - s_K))\right]$$

By construction (taxes and subsidies are pure transfers) we know that total expenditure (E^w-tE^*) must remain constant. Using equation (4.14) we get again that profits increase with the subsidy :

$$\frac{\partial \pi}{\partial z^*}_{z^*=0} = \frac{L\left[1-b\left(2s_K-1\right)\right]}{1-b}\frac{\partial t}{\partial z^*}_{z^*=0}$$ where we use the fact that $E^* = \frac{L\left[1-b\left(2s_K-1\right)\right]}{1-b}$.

The government budget constraint implies:

(4.15) $$\frac{\partial t}{\partial z^*}_{z^*=0} = \frac{\pi}{E^*}(1-s_n) = b\frac{1-\phi-b(1+\phi)\left(2s_K-1\right)}{\left(1-\phi\right)\left[1-b\left(2s_K-1\right)\right]} > 0$$

Finally from equation (4.12) and using (4.13) and (4.14) we get that regional nominal income inequality increases with the subsidy :

(4.16) $$\frac{\partial s_E}{\partial z^*}_{z^*=0} = \frac{1-\phi-b\left(1+\phi\right)\left(2s_K-1\right)}{2(1-\phi)}bs_K > 0$$

Again, even though the subsidy is given only to Southern firms, capital mobility implies that in general equilibrium the income inequality must rise in favor of the North. Quite intuitively, it can be checked that the increase in income inequality is larger in the case of local financing than in the case of a national proportional tax because Northerners do not pay the tax.

Differentiating equation (3.1), and using equation (4.16), we get the relocation effect of the local subsidy financed by a local proportional income tax :

(4.17) $$\frac{ds_n}{dz^*}_{z^*=0} = \frac{1+\phi}{4(1-\phi)^2}\left[2bs_K\left[1-\phi-b\left(1+\phi\right)\right]-\left(1+\phi\right)\left(1-b^2\right)\right] < 0$$

which can be proved to be negative[3]. Hence, even though the locally financed subsidy leads to a decrease in expenditure in the poor region (the tax effect) and to increased income inequality (the effect on profits), it attracts firms in the poor region. When subsidies are financed locally, only Southerners pay the tax, so that the equilibrium tax rate is higher than in the nationally financed case. The increase in profits is the same, whereas increase in regional income disparity s_E is greater, which yields fewer firms to relocate to the South.

IV.3. A lump sum subsidy financed by a national proportional income tax

Regional subsidies can take the form of a lump sum transfer to firms of the manufacturing sector which locate in the poor region. This may be the case for example if, as often happens, land is given to the firm to build the plant, if a lump sum tax break is given or if R&D is subsidized for firms located in the poor region. As the analysis and results are quite similar we only analyze the case of a subsidy financed at the national level and do not repeat all the steps.

The arbitrage equation now becomes

(4.18) $$\pi = \pi^* + F^*$$

where F^* is the lump sum subsidy given to firms located in the South.

The government budget constraint becomes:

(4.19) $$t(2L + \pi) = F^*(1 - s_n)$$

The resource constraint and the share of expenditures in the North are still given respectively by equations (4.3) and (4.1).

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[3] The expression in the parenthesis is maximum when $\phi=0$. Even in this case, the expression is always negative.

Differentiating this system around an equilibrium where F^* is zero, we get the following expression for changes in spatial inequality:

(4.20)

$$\frac{\partial s_n}{\partial F^*}_{F^*=0} = \frac{\partial s_n}{\partial F^*}_{\substack{F^*=0 \\ s_E \text{ constant}}} + \frac{\partial s_n}{\partial s_E}_{F^*=0} \frac{\partial s_E}{\partial \pi}_{F^*=0} \frac{\partial \pi}{\partial t}_{F^*=0} \frac{\partial t}{\partial F^*}_{F^*=0}$$

$$\frac{\partial s_n}{\partial F^*}_{F^*=0} = -\frac{(1+\phi)^2 s_E(1-s_E)}{b(1-t)E^W(1-\phi)^2} + \frac{1+\phi}{1-\phi}\frac{\partial s_E}{\partial F^*}_{F^*=0}$$

From equation (4.1), we get the variation of regional income disparity with respect to profits of manufacturing firms, which is the same expression as equation (3.4). From the resource constraint we know that total expenditure must remain constant. The resource constraint also gives us changes in profits expressed in terms of changes in tax rate :

(4.21)

$$\frac{\partial \pi}{\partial F^*}_{F^*=0} = \frac{2L}{1-b}\frac{\partial t}{\partial F^*}_{F^*=0}$$

Finally we get changes in tax rate using equation (4.19) :

(4.22)

$$\frac{\partial t}{\partial F^*}_{F^*=0} = (1-s_n)\frac{1-b}{2L}$$

To sum up, it is worth noting that all these results are proportional to the ones we obtain with a subsidy to profits financed by a national proportional income tax. Changes induced by a lump sum subsidy on our four endogenous variables (the share of firms producing in the North s_n, the share of expenditure in the North s_E, the equilibrium profits π and the tax rate on income t) are just proportional to the effects of a subsidy to profits. The multiplying factor is the equilibrium value of profits: $\pi = \frac{2Lb}{1-b}$. Hence, for example, $\frac{\partial s_n}{\partial F^*}_{F^*=0}\frac{2Lb}{1-b} = \frac{\partial s_n}{\partial z^*}_{z^*=0}$

We thus conclude that a lump sum subsidy leads to a decrease in spatial concentration of firms together with an increase in regional income disparity, as in the case of a subsidy to profits.

V. A local subsidy to firms production

Regional subsidies can also consist of a subsidy proportional to production or to the number of jobs created by firms. In this section we study the case of a nationally financed subsidy to both sectors productions in the South. As the traditional sector is perfectly competitive, marginal-cost pricing and zero profit condition imply: w* = (1+z*). Hence, such a subsidy acts as an increase in labor productivity and raises the equilibrium wage in the South, as the labor markets are perfectly competitive. Note that in a more general model with labor and capital as inputs in the production (remember here that capital is used only in the fixed cost), the production subsidy would also increase capital productivity so that the results would not be as beneficial to labor as in this example.

As for the manufacturing sector, profit maximization under monopolistic competition yields: $\pi^* = p^* x^* (1+z^*) - a_M (1+z^*) x^* \Rightarrow p^* = \frac{a_M \sigma}{\sigma - 1}$ and $\pi^* = (1+z^*)\frac{x^*}{\sigma}$. Hence, a subsidy to both sectors production lets prices unchanged, whereas profits and wages in the South are multiplied by *(1+z*).* There are four equations to solve the model. The arbitrage equation that determines the optimal location of firms is given by:

(5.1) $$\pi = (1+z^*)\pi^*$$

The definition of the Northern share of expenditure is:

(5.2) $$s_E = \frac{(1-t)(L + \pi s_K)}{(1-t)\left[L(2+z^*) + \pi\right]}$$

The resource constraint becomes:

(5.3) $$2L = (1-t)(1-b)\left[L(2+z^*) + \pi\right]$$

Finally the government budget constraint when subsidies are financed by a national proportional income tax can be shown to be (see appendix B):

(5.4) $$tE^W = z^*(1-s_n)\pi + z^*L$$

Because subsidies and taxes are pure transfers, we know that world expenditure must remain constant and equal to $\frac{2L}{1-b}$. We analyze again the effect of a small subsidy by differentiating the system around the equilibrium without subsidy. From equation (5.4), we obtain the increase in profits, which is lower than in the case of a subsidy to profits. This is so because the production subsidy leads to an increase in wages in the South and therefore to a smaller increase in profits in both regions:

(5.5) $$\frac{\partial \pi}{\partial z^*}_{z^*=0} = \frac{2L}{1-b}\frac{\partial t}{\partial z^*}_{z^*=0} - L \geq 0 \text{ for } s_n \leq 1$$

The subsidy has a positive effect on profits as long as we assume non-full specialization ($s_n<1$). From equation (5.2), using equation (4.6) and the fact that $E = L + \pi s_K = \frac{L}{1-b}\left[1 + b(2s_K - 1)\right]$, we get :

(5.6) $$\frac{\partial s_E}{\partial z^*}_{z^*=0} = \frac{(1-b)}{2L}\left[s_K \frac{\partial \pi}{\partial z^*}_{z^*=0} - \frac{L}{1-b}\left[1 + b\left(2s_K - 1\right)\right]\frac{\partial t}{\partial z^*}_{z^*=0} \right]$$

Again, the change in income inequality depends on the impact of the subsidy on profits and on the impact of proportional taxes on income. The first effect tends to increase income inequality if profits rise. The second effect decreases inequality when taxes increase, as they are proportional to income and therefore fall more heavily on the rich region.

From the government budget constraint (5.4) we get:

(5.7) $$\frac{\partial t}{\partial z^*}_{z^*=0} = (1-s_n)b + \frac{1-b}{2} = \frac{1-\phi-(1+\phi)b^2(2s_K-1)}{2(1-\phi)} > 0$$

so that

(5.8) $$\frac{\partial s_E}{\partial z^*}_{z^*=0} = -\frac{(1-b)}{4(1-\phi)}\left[(1-\phi)+b^2(1+\phi)(2s_K-1)^2\right] < 0$$

which says that expenditure (and after tax nominal income) inequality decreases with the subsidy to production. This results greatly differs from the other types of subsidies. Another implication is that relocation to the South is more important in the case of a production subsidy than in the case of a tax break (subsidy proportional to profits) for firms located in the South. Indeed, the effect of the subsidy on geography of production, using equation (3.2), is given by :

(5.9) $$\frac{\partial s_n}{\partial z^*}_{z^*=0} = -\frac{(1+\phi)}{4(1-\phi)^2}\left[(1+\phi)+(1-b)(1-\phi)-b^3(1+\phi)(2s_K-1)^2\right] < 0$$

which can be shown to be larger in absolute value than (4.11). Agglomeration is decreased to a larger extent with this kind of subsidy as the market size of the poor region is increased due to the positive effect on Southern wages.

VI. Welfare (to be completed)

The next issue to consider is the impact of the subsidy on real net income in both regions. The effect may be ambiguous as the price index in the South will decrease and nominal income inequality may rise. A related question is the impact on real net incomes for workers and capital owners.

VI.1. Regional welfare issue

Real net income in both regions reflects regional welfare level, and is obtained by dividing the after-tax and subsidy expenditure by the perfect price index. Both levels depend on the evolution of the distribution of expenditure between regions, given that total expenditure remains constant. Welfare levels also depend on changes in price indices, and therefore on the relocation of firms. Whatever the type of subsidy, changes in regional real net income are given by :

(6.1)
$$\frac{dV}{dz}_{Z=0} = P^{-1}\left[\frac{2L}{1-b}\frac{ds_E}{dz}_{Z=0} + \frac{2\alpha L(1-\phi)}{(1-b)(\sigma-1)(1+\phi)}\frac{ds_n}{dz}_{Z=0}\right]$$

(6.2)
$$\frac{dV^*}{dz}_{Z=0} = P^{*-1}\left[-\frac{2L}{1-b}\frac{ds_E}{dz}_{Z=0} - \frac{2\alpha L(1-\phi)}{(1-b)(\sigma-1)(1+\phi)}\frac{ds_n}{dz}_{Z=0}\right]$$

Then we use equation (4.9) in the case of subsidy to profits or production, and equation (4.23) in the case of a lump sum subsidy to express geographical changes. Note that the sign of the welfare change for the South is opposite to the one for the North. Changes in welfare are relatively symmetric.

Results in terms of regional welfare will be obvious in the case of a local subsidy to production. As both regional income disparity and agglomeration of firms decrease, the North loses and the South wins in terms of real income and welfare. However for other types of subsidies we studied, the North experiences an increase in both regional income and regional price index, whereas the opposite occurs in the South. We thus have to determine the net effect of subsidies to profit and lump sum subsidy on regional welfare.

We find that North welfare decreases (and South welfare increases) with a subsidy to profits if it is financed nationally; the same results hold for the lump sum subsidy. We however

find that if financed by the South, and even though it succeeds in attracting firms in the South and does not create distortions in terms of efficiency, such subsidy can increase welfare in the North and decrease it in the South. This will be the case if trade costs are high (so that few firms relocate with the subsidy) and if the initial distribution in capital ownership is large.

VI.2. Individual inequality issue

We show in previous sections that subsidies to private firms located in the poor region translate to a relocation of firms that changes price indexes. Indeed, the price index decreases in the South because a greater share of manufactured goods will be produced and buy locally with no trade costs, whereas the price index increases in the North. Moreover, the financing of the subsidy and the resources transfers it induces may yield different results for individuals characterized by different locations and endowments.

In this last section we analyze the relation between spatial inequality, regional income inequality and individual inequality. The latter reflects the unequal distribution of wealth across individuals. We thus imagine that all individuals are endowed with one unit of labor, from which they receive wage. But individual capital endowments reflect an initial unequal distribution of wealth between individuals. So that individual inequality evolves with the distribution and the remuneration of this factor. In the present work, capital ownership distribution is given, and we will concentrate on the effects of subsidies on factors remuneration. This kind of approach is similar to the studying of qualification premium in Macroeconomics of Inequality and Growth models (see Aghion, Caroli and Garcia-Penalosa (1999), or Atkinson and Bourguignon (2001) for surveys).

We assume that capital ownership is unevenly spread across individuals, and that individual's income is composed of labor income and capital ownership income. Note that the

latter source of income is at the basis of heterogeneity of individuals. We will define intraregional individual inequality as inequality between poor and rich individuals inside each region. We imagine the polar cases of a typical agent who is endowed with labor and capital, compared to an agent who only owns labor. We will refer to the former as a capitalist, and to the latter as a worker. We define individual pre-tax income as following, where L refers to a worker and Z to a capitalist, c_i is capitalist i share of total capital ownership, and asterisk refers to the South :

(6.3)

$$E^{L} = w;\ E^{Z,i} = w + c_i\pi$$
$$E^{L^*} = w^*;\ E^{Z^*,i} = w^* + c_i\pi$$

We then study changes in individual indirect utility induced by the different subsidies:

(6.4)

$$\frac{\partial V^i}{\partial z^*}_{z^*=0} = P^{-1}\left[\frac{\partial E^i}{\partial z^*}_{z^*=0} - E^i\frac{\partial t}{\partial z^*}_{z^*=0} + E^i\frac{2\alpha(1-\phi)}{(\sigma-1)(1+\phi)\left[1+b(2s_K-1)\right]}\frac{\partial s_n}{\partial z^*}_{z^*=0}\right]$$
$$\frac{\partial V^{i^*}}{\partial z^*}_{z^*=0} = P^{*-1}\left[\frac{\partial E^{i^*}}{\partial z^*}_{z^*=0} - E^{i^*}\frac{\partial t}{\partial z^*}_{z^*=0} - E^{i^*}\frac{2\alpha(1-\phi)}{(\sigma-1)(1+\phi)\left[1-b(2s_K-1)\right]}\frac{\partial s_n}{\partial z^*}_{z^*=0}\right]$$

where i=Z,L. When subsidies are financed by a local proportional income tax, the second term into brackets in the first line is null.

We measure intraregional individual inequality as the ratio of a capitalist to a worker utility level, which is equivalent to the ratio of nominal net income, as they face the same price index. We know from the previous analysis that regional subsidies do not affect labor income and capital income in the same way. Certain types of subsidies operate a factorial income transfer from labor to capital. Hence these subsidies and their financing will actually lead to a transfer from workers to capitalists because labor reward decreases and capital reward increases. The more an agent is capital rich, the more he gains from the subsidy, wherever he lives. On the contrary, the more an agent is labor rich, relative to its endowment in capital, the more he is hurt

by the subsidy. We shall then conclude to an increase in intraregional individual inequality. The point is that intraregional individual inequality will increase if capitalist nominal net income relative increase is superior to a worker nominal net income relative increase:

(6.5)
$$\frac{\frac{dE^Z}{dz^*}}{E^Z} > \frac{\frac{dE^L}{dz^*}}{E^L}$$

We thus compare the evolution of labor income to capital income, because capital income is at the basis of the difference of wealth between workers and capitalists. The idea is that if labor income and capital income face the same relative increase, inequality between workers and capitalists will remain unchanged. If capital income relative increase is greater than labor income relative increase, we will conclude to an increase of capitalist-worker inequality. We present in table 1 the relative evolution of nominal net reward of labor and capital.

[Insert table 1]

We can analyze how utility of agents is altered by the different subsidies. The locally financed subsidy is preferred by Northerners, not by Southerners, because only Southerners finance the subsidy. We can check that as the subsidy is financed locally, nominal net income of workers in the North remains unchanged by the subsidy, and capital nominal net income in the North is not altered by the financing of the subsidy, while individuals in the South face a decrease in their nominal income. Besides, it is no more sure that capital nominal net income in the South increases. Net nominal income of a pure capitalist living in the South increase if only if South regional income is superior to the total sum of profits. The case of a lump sum subsidy is strictly proportional to the nationally financed subsidy to profits.

The subsidy to production yields one more time some different results, compared to the other types of subsidies. As wages and profits increase in the South, and as everybody finance the

subsidy, the utility of workers is completely changed. On the one hand, workers in the South face now an increase in their nominal net income. On the other hand, Northerners heavily support the counterpart of the increase in wages and profits in the South. Hence nominal net income of workers living in the North collapses. For them, this is the worst case. Finally, another interesting result is that capitalists nominal net income decreases. This is due to the fact that they also have to finance the wage increase in the South, and also because the increase in profits is lower than with any other subsidy.

We can then conclude that intraregional individual inequality increase in both regions when subsidies to profits or lump sum subsidies are given to firms, whenever they are financed by a national or a local proportional income tax, because these subsidies actually change the factorial distribution of total income and operate a transfer of resources from workers to capital owners. However in the case of a subsidy to production, intraregional inequality decreases in both regions. In the South, labor income increases, while capital income decreases. In the North, capital income relative reduction is superior to labor income relative decrease, so that intraregional individual inequality decreases in the North too. Once again, subsidies to production yield completely different results, and seem to be a better economic policy to match the goal of a reduction of both regional inequality and individual inequality.

Conclusions

This paper has analyzed simple regional subsidies whose objective is to attract firms. The assumption that capital is mobile means that firms in all regions benefit from the regional subsidy and experience an increase in operating profits for subsidies that are proportional to profits or production and for lump sum subsidies. Firms in the region that does not get the subsidy benefit from it indirectly because as some firms leave the region the competitive pressure is weaker for

remaining ones. This raises interesting questions on the relation between spatial inequalities and individual inequalities, especially between immobile workers and owners of mobile capital and the impact of public policies on these types of inequalities. Implicitly or explicitly, regional policies in Europe are based on the belief that if spatial inequalities are decreased then inequalities in general will be decreased. In a situation where capital is mobile but workers are not, a fair assumption in the European case, this is not true. Another way to put it is that when subsidies are given to firms that locate in poor French regions, the main beneficiaries may be the capital owners (wherever their capital is located) who disproportionately leave in Paris. A somewhat paradoxical but logical result is that a subsidy to firms located in the poor region can increase income inequality between the poor and the rich region if the rich region owns more capital than the South. For this result to hold, it is not necessary that Northerners own the firms that benefit from the subsidy in the South but simply that firms be mobile.

We believe that the relation between spatial inequality, regional income inequality and individual inequality is an interesting subject that has received little attention (see Dupont, 2002). To clarify the relation between these different types of inequality and the way various public policies affect them should be important because the public debate on inequality often confuses them. In particular, the social and political motivation behind regional policies is that reducing the spatial inequality or spatial concentration of economic activities helps reduce inequalities in general. We have seen that this is not necessarily true : subsidies to profits and lump sum subsidies leads to a decrease in spatial inequality, but also to an increase in regional income disparity and an increase in intraregional inequality in both regions. In these cases, we shall conclude that in a sense poor worker individuals living in the rich region are financing a subsidy that mostly benefit the rich capitalist individuals living in the poor region. Subsidy to production

seems to be the best policy to attract firms, and reduce both regional income inequality and individual inequalities.

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Table 1 : Relative evolution of capital and labor rates of returns

Subsidy to profits financed by a national proportional income tax	Labor in both regions	$-b(1-s_n)<0$
	Capital in both regions	$(1-b)(1-s_n)>0$
Subsidy to profits financed by a local proportional income tax	Labor in the North	0
	Capital in the North	$1-s_n>0$
	Labor in the South	$\frac{-\pi}{E^*}(1-s_n)<0$
	Capital in the South	$\frac{E^*-\pi}{E^*}(1-s_n)$
Lump sum subsidy	Labor in both regions	$-b(1-sn)\frac{(1-b)}{2Lb}<0$
	Capital in both regions	$\frac{(1-b)^2}{2Lb}(1-sn)>0$
Subsidy to production	Labor in the North	$-\left[(1-s_n)b+\frac{1-b}{2}\right]<0$
	Labor in the South	$\frac{1-b}{2}+bs_n>0$
	Capitalist in both regions	$-\left[s_n-\frac{1}{2}\right](1-b)<0$

Appendix A: $\frac{\left[1+z^{*}-\phi-s_{E}z^{*}(1+\phi)\right]\left(1+z^{*}\right)(1-\phi)}{\left[1-\phi(1+z^{*})\right]\left[1+z^{*}-\phi\right]}$ is the bias induced by the subsidy to profits.

We can see that it is more than 1 if z*>0. The bias increases as z* and phi increase. This bias is positive as long as we restrict our attention to interior solutions where $0< s_n<1$. Introducing a subsidy proportional to operating profits of firms located in the South induces relocation and distorts competition, so that profits increase in both regions.

Appendix B: This equation comes from the assumption that both sectors are subsidized in the South: the manufacturing sector production at the level: $z^*(1-s_n)x^*$ and the production of the traditional sector. The latter is obtain by considering that there are $(1-s_n)$ monopolistic firms producing in the South, which produce $(1-s_n)$ x^* and employ $(1-s_n)a_Mx^*$ individuals among L workers living and working in the South (remember we assume $L=L^*$). As we assume full-employment, the traditional sector employs $L-(1-s_n)a_Mx^*$ workers and produces $L-(1-s_n)a_Mx^*$. So the production subsidy to traditional sector in the South is equal to : $[L-(1-s_n)a_Mx^*]z^*$. We get equation (5.4) by using the normalization of a_M to $\frac{\sigma-1}{\sigma}$.