APPENDICES

Appendix A: From equations (11a)-(11e) to equations (12a)-(12c).

(12a) and (12b) follow if we differentiate the definitions $z \equiv \frac{c}{k}$ and $y \equiv gk$ with respect to time and then use (11c)-(11e). To get (12c), we take logarithms on both sides of (11a), differentiate with respect to time and substitute (11b)-(11e).

Appendix B: Proof of Proposition 1.

Equation (15) in the text is an equation in $\tilde{q}$ only. Since the right-hand-side (RHS) is negative (because $(1-a-q)<0$ along the optimal path), the left-hand-side (LHS) must be negative too. Consider first the RHS. For $\tilde{q} \to (1-a)$, it tends to minus infinity. For $\tilde{q} \to 1$, it becomes a negative number; in particular, $-\frac{nr}{a}$. Also, the RHS is monotonically increasing in $\tilde{q}$. Consider now the LHS. For $\tilde{q} \to (1-a)$, it becomes a negative number. For $\tilde{q} \to 1$, it becomes another negative number; in particular, $A \equiv -\left(a^{\frac{1}{a}} b^{\frac{1-a}{a}}\right)\left[1+\frac{n}{N}\left(1+\frac{\hat{d}}{\tilde{G}_y}\right)\right]-\left(\frac{n\hat{d}}{1-b}\right)\left(1+\frac{\hat{d}}{\tilde{G}_y}\right)$. Also, if $\left(1+\frac{\hat{d}}{\tilde{G}_y}\right)>0$ (which is condition (16a) in the text), the LHS is decreasing in $\tilde{q}$ (this is a sufficient condition). Therefore, if $-\frac{nr}{a}>A$ (which is condition (16b) in the text), the LHS and the RHS intersect once as shown in Figure 1. This completes Proposition 1.

Appendix C: The Jacobian matrix in equation (18).

The elements of the Jacobian matrix evaluated in steady state are:

$$J_{zz} \equiv \frac{¶\dot{z}}{¶z} = r+\hat{d} > 0,\quad J_{zy} \equiv \frac{¶\dot{z}}{¶y} = 0,\quad J_{zq} \equiv \frac{¶\dot{z}}{¶q} = 0,$$

$$J_{yz} \equiv \frac{¶\dot{y}}{¶z} = -\tilde{y} < 0,\quad J_{yy} \equiv \frac{¶\dot{y}}{¶y} = 0,\quad J_{yq} \equiv \frac{¶\dot{y}}{¶q} = \frac{n\hat{d}}{a(1-b)\tilde{q}^{2}\Delta(\tilde{q})} > 0,$$

$$J_{qz} \equiv \frac{\P \dot{q}}{\P z} = 0\,, \; J_{qy} \equiv \frac{\P \dot{q}}{\P y} = \hat{d}\Phi(\tilde{q}) < 0\,,$$

$$J_{qq} \equiv \frac{\P \dot{q}}{\P q} = \Phi(\tilde{q})\left[\frac{nr\Delta(\tilde{q})[1-\tilde{q}-a(2-a)]}{a\tilde{q}[\Delta(\tilde{q})(1-a-\tilde{q})]^2} + \frac{n\hat{d}}{a(1-b)\tilde{q}^2\Delta(\tilde{q})}\right]$$

Appendix D: Transitional Dynamics.

If $J_{qq} > 0$, there are three sign alterations in (19), so that Descartes' Theorem (which states that the number of positive roots cannot be higher than the number of sign alterations) implies that there are at most three positive roots. Now define $b' \equiv -b$. In this case, there are no sign alterations in (19). Hence, we cannot have a positive root for b', so that we cannot have a negative root for b. Combining these results, it follows that when $J_{qq} > 0$, there are three positive roots. Hence, there is local determinacy.

If $J_{qq} < 0$ so that $[(r+\hat{d})J_{qq} + \frac{\det(J)}{(r+\hat{d})}] < 0$, there is one sign alteration in (19) and so at most one positive root. Now define $b' \equiv -b$. In this case, there are two sign alterations in (19), and so at most two positive roots for b', or equivalently at most two negative roots for b. Combining these results, it follows that when $[(r+\hat{d})J_{qq} + \frac{\det(J)}{(r+\hat{d})}] < 0$ (which is condition (20) in the text), there is one positive and two negative roots. Hence, there is local indeterminacy.

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