III. OPTIMAL ECONOMIC POLICY

We now endogenize tax policy, q . To do so, we consider a benevolent government which maximizes the sum of all households' well-being.[16] The government plays Stackelberg *vis-a-vis* the private economy. In other words, the government takes into account the DCE characterized above. We assume commitment technologies so that the government chooses its policy once-and-for-all.

The government maximizes the sum of all i 's utility functions [see equations (4a)-(4b)] subject to each individual's i optimal decision rules in a DCE [this is summarized by equations (5), (6), (9) and (10d)] and its own budget constraint [this is summarized by equations (10a) and (10b)]. The current-value Hamiltonian, H , of this problem is:

$$H \equiv \sum_{i=1}^{N} \log c^i + \sum_{i=1}^{N} n \log\left((1-b)q\Delta(q)K - \sum_{i=1}^{N} d(\bar{k} - k^i) \right) +$$
$$+ \sum_{i=1}^{N} g^i[(1-q)\Delta(q)k^i + d(\bar{k} - k^i) - c^i] + \sum_{i=1}^{N} l^i c^i [(1-q)\Delta(q) - r - d(1-\frac{1}{N})]$$

where g^i and l^i are the multipliers associated with (5) and (6) respectively for each individual i . That is, g^i is the social marginal value of capital for individual i , and l^i is the social marginal value of the private marginal utility of assets for individual i . Note that in the problem above, we have used (10d), i.e. the sum of profit shares across individuals is zero.

The first-order conditions for q, c^i, l^i, g^i, k^i are respectively:

$$\sum_{i=1}^{N} \left(\frac{n\left((1-b)K[\Delta(q) + q\Delta_q(q)]\right)}{(1-b)q\Delta(q)K - \sum_{i=1}^{N} d(\bar{k} - k^i)} \right) + [(1-q)\Delta_q(q) - \Delta(q)] \sum_{i=1}^{N} [l^i c^i + g^i k^i] = 0 \quad \text{(11a)}$$

[16] Alternatively, we could use the median-voter approach. Our approach (i.e. benevolent government) is more general (see below). Note that we assume that all individuals are given the same weight by the government. Alternatively, we could assume that the weight given to poor people is higher than the one given to rich people, by appropriately using the relevant multipliers (see Bewley [1982] and Kehoe et al [1990]).

$$\dot{l}^i = r\,l^i - \frac{1}{c^i} - l^i[(1-q)\Delta(q) - r - d(1-\frac{1}{N})] + g^i \tag{11b}$$

$$\dot{c}^i = c^i[(1-q)\Delta(q) - r - d(1-\frac{1}{N})] \tag{11c}$$

$$\dot{k}^i = (1-q)\Delta(q)k^i + d(\bar{k} - k^i) - c^i \tag{11d}$$

$$\dot{g}^i = rg^i - \left(\frac{n[(1-b)q\Delta(q) + d(N-1)]}{(1-b)q\Delta(q)K - \sum_{i=1}^{N} d(\bar{k} - k^i)} \right) - g^i[(1-q)\Delta(q) - d(1-\frac{1}{N})] \tag{11e}$$

where $\Delta(q) > 0$ has been defined in (9) above, and $\Delta_q(q) = \frac{(1-a)\Delta(q)}{aq} > 0$.

These necessary conditions are completed with the addition of the transversality condition:

$$\left[(1-q)\Delta(q)\right] - r - d(1-\frac{1}{N}) < r \tag{11f}$$

which follows from (11c) and ensures utility is bounded.[17]

It can be easily shown that the utility function and the constraints are strictly concave in q, if $(1-a-q) < 0$ (this is a sufficient condition for concavity). Then, since the utility function and the constraints are continuous and bounded, and since the utility function is strictly concave in the controls (c^i, q) and the constraints are linear in c^i and strictly concave in q, existence is assured. Further, since the utility function and the constraints are both jointly concave in the controls (c^i, q) and the state variable (k^i), the necessary conditions in (11a)-(11f) are also sufficient for optimality. This establishes existence of a solution of the optimal control problem.

We point out four features of the model. First, the second-order conditions imply $(1-a-q) < 0$. That is, the optimal tax rate is within the subset $0 < 1-a < q < 1$. The optimal tax rate is higher than $1-a$ (which is the productivity of public production

[17] Capital cannot grow faster than consumption in steady state. Hence, the utility from public consumption services is also bounded if (11f) is satisfied.

services), because the government provides - in addition to public production services - public consumption services and transfer payments. This implies that when policy is endogenous, we can only be on the downward-sloping part of the growth-tax rate relation (compare it with the case in which policy is exogenous at the end of previous section). Therefore, along the optimal path, tax increases always reduce growth. This is an intuitive result: tax policy is not optimal when a higher tax rate can increase growth (which happens when $0 < q < 1 - a$).

Second, when public consumption services are absent (i.e. $n = 0$), equation (11a) implies $q = 1 - a$ in all time periods.[18] That is, the optimal tax rate is constant over time, and equal to the productivity of public production services. This special case gives Barro's [1990] flat tax rate. In this case there are no transitional dynamics. This is also the case in Alesina and Rodrik [1994].[19]

Third, when both public consumption and production services are absent (i.e. $n = 0$ and $a = 1$), $q = 0$ in all time periods. That is, the optimal tax rate is zero all the time. This implies zero tax revenues, and hence zero transfer payments. It also implies that the equilibrium return to capital is not bounded below (i.e. $\Delta(q) = 0$), and hence long-run growth is not optimal (see (11c)). Therefore, the government finds it optimal to redistribute income from the rich to the poor only when it can also provide public production services; it is the latter that generates long-run growth.

Fourth, for *given aggregate values* of consumption, capital and their shadow prices, total differentiation of (11a) implies that the tax rate, q, increases with $(\bar{k} - k^i)$. Thus, individuals with relative low (resp. high) capital stock prefer high (resp. low) tax rates. This is because those who are less capital-endowed than the average prefer higher redistribution and so higher tax rates. In turn, since the growth rate is negatively affected by the tax rate along the optimal path, it follows that more unequal societies grow faster. These results are similar to those in the median-voter literature (see e.g. Persson and Tabellini [1994a], Alesina and Rodrik [1994] and Benabou [1996]).

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[18] This is because $[(1-q)\Delta_q(q) - \Delta(q)] = \dfrac{(1-a-q)\Delta(q)}{aq}$. See (11a).

[19] Therefore, we extend Alesina and Rodrik [1994] in the following ways: (a) In our model, the government provides public (production and consumption) goods and explicitly redistributive transfers; (b) We have a moral hazard problem; (c) We have transitional dynamics.

IV. LONG-RUN EQUILIBRIUM PROPERTIES

This section studies the steady state of (11a)-(11e). We will focus on a steady state in which all individuals own ex post the same amount of capital. In other words, all individuals are alike *ex post*. This choice of the steady state follows naturally from the assumption that all individuals have the same rate of time preference. By contrast, heterogeneous rates of time preference would lead to a long-run equilibrium in which only patient agents hold capital.[20]

This implies that no actual transfers take place in long-run equilibrium. This is not very restrictive. We have already shown how inequality affects the tax rate, and in turn the growth rate, along the optimal path. The critical feature of redistribution is the *anticipation* of transfers of wealth as opposed to *actual* transfers of wealth. And this has been already captured in our model by moral hazard behavior. As Benabou [1996] points out, "the fight over the pie does not necessarily lead to higher transfers, just to higher distortions".

Therefore, we invoke the steady state conditions $c^i \equiv c$, $l^i \equiv l$, $g^i \equiv g$ and $k^i = k$ into the optimality conditions (11a)-(11e). Then, a careful observation of (11a)-(11e) reveals that if we use the transformations $z \equiv \frac{c}{k}$ and $y \equiv gk$, we can reduce the dimensionality of the system from five to three. In particular, Appendix A shows that the dynamics of (11a)-(11e) are equivalent to the dynamics of (12a)-(12c) below, which constitute a three-dimensional dynamic system in z, y, q. Thus,

$$\dot{z} = z^2 - (r + \hat{d})z \tag{12a}$$

$$\dot{y} = ry - \frac{n}{N} - \frac{n\hat{d}}{(1-b)q\Delta(q)} + \hat{d}y - zy \tag{12b}$$

[20] See also Yano [1981] and Bewley [1982]. If we use endogenous discount rates, they become identical to all agents in long-run, and thereby agents own the same amount of capital (see Epstein [1987] and Benhabib et al [1988]). Note that most of politico-economy models also use ex post homogeneity. By contrast, see Fernandez and Rogerson [1995] and Greenwood and Jovanovic [1990] for ex post heterogeneity.

$$\dot{q} = [\Phi(q)]\left[-1 - \frac{n}{N} - \frac{nr}{(1-a-q)\Delta(q)} - \frac{n\hat{d}}{(1-b)q\Delta(q)} + \hat{d}\,y\right] \tag{12c}$$

where $\hat{d} \equiv d(1-\frac{1}{N}) > 0$ is the effective redistributive parameter, and $\Phi(q) \equiv \frac{aq\Delta(q)(1-a-q)^2}{n[1-q-a(2-a)]} < 0$. The negative sign of $\Phi(q)$ follows from the negative sign of $[1-q-a(2-a)]$, which in turn follows from the negative sign of $(1-q-a)$ along the optimal path.

We can now look for Balanced Growth Paths (*BGPs*), i.e. steady states in which consumption (c) and capital (k) grow at constant positive rates. When this happens, (11c) and (11d) imply that c and k must grow at the same rate, denoted by $\tilde{G}_c > 0$ (throughout the paper, tildes over variables denote optimal steady state values). Hence, $\dot{z} = 0$ in (12a). We also look for a tax policy (q) that does not change. Hence, $\dot{q} = 0$ in (12c). Finally, we assume that y (where $y \equiv gk$) grows at a constant rate, denoted by $\tilde{G}_y$. For a steady state solution to exist, it must be that $\tilde{G}_y$ is negative (see also equation (14) below).[21] In other words, in long-run equilibrium, the social value of the capital stock ($y \equiv gk$) grows at a constant negative rate. The intuition behind a negative $\tilde{G}_y$ is as follows. In models of long-run growth in general, equilibrium returns to capital should be decreasing in order to maintain capital accumulation over time. Since our model includes public consumption, which requires higher capital accumulation, we need a stronger condition. Hence, $\tilde{G}_y < 0$.[22]

The rest of this section solves for $\tilde{z}, \tilde{y}$ and $\tilde{q}$. We start with $\tilde{z}$. Setting (12a) equal to zero, we have:

$$\tilde{z} = r + \hat{d} > 0 \tag{13}$$

[21] Setting long-run variables exogenously (e.g. to be zero) is standard practice in growth models.

[22] By contrast, when there is no redistribution $(\hat{d} = 0)$, the solution for the "actual" variables (z, q) is independent of the value of shadow prices (y). See the system (12a)-(12c) above.

so that the consumption-to-capital ratio, $\tilde{z}$, is unique and equals the discount factor, $r > 0$, plus the effective redistributive parameter, $\hat{d} > 0$.

We continue with $\tilde{y}$. Using $\tilde{G}_y \equiv \frac{\dot{y}}{y}$ and (13), we get from (12b):

$$\tilde{y} = -\frac{n}{\tilde{G}_y}\left(\frac{1}{N} + \frac{a^{\frac{-1}{a}} b^{\frac{-1+a}{a}} \hat{d} q^{\frac{-1}{a}}}{(1-b)}\right) > 0 \tag{14}$$

which makes clear that for $y \equiv gk$ to be positive, we need $\tilde{G}_y < 0$. Note that $y > 0$, because both $g > 0$ and $k > 0$.

Finally, by setting (12c) equal to zero, and using (13) and (14), we get:

$$-\left(a^{\frac{1}{a}} b^{\frac{1-a}{a}}\right)\left[1+\frac{n}{N}\left(1+\frac{\hat{d}}{\tilde{G}_y}\right)\right]\tilde{q}^{\frac{1}{a}} - \left(\frac{n\hat{d}}{1-b}\right)\left(1+\frac{\hat{d}}{\tilde{G}_y}\right) = \frac{nr\tilde{q}}{(1-a-\tilde{q})} \tag{15}$$

Equation (15) is an equation in the long-run tax rate, $\tilde{q}$, only. Since $(1-a-\tilde{q}) < 0$, the right-hand side of (15) is negative, so that the left-hand side must be also negative. Then, Appendix B shows:

Proposition 1: If the parameters satisfy the following conditions:

$$0 < \hat{d} < -\tilde{G}_y \tag{16a}$$

$$0 < r < \frac{a^{\frac{1+a}{a}} b^{\frac{1-a}{a}}}{n} + \left(1+\frac{\hat{d}}{\tilde{G}_y}\right)\left(\frac{a^{\frac{1+a}{a}} b^{\frac{1-a}{a}}}{N} + \frac{a\hat{d}}{1-b}\right) \tag{16b}$$

then equation (15) implies that there exists a unique long-run tax rate, $0 < 1 - a < \tilde{q} < 1$. *This tax rate supports a unique Balanced Growth Path* (BGP) *for consumption and capital.*[23]

Although equation (15) cannot be solved analytically for $\tilde{q}$, it implies that $\tilde{q}$ is a function of parameters $a, b, n, r, \hat{d}$ and the exogenous variable $\tilde{G}_y$. Denote this by $\tilde{q} = q(a, b, n, r, \hat{d}, \tilde{G}_y)$. It is of particular interest to study the effect of the redistributive parameter, d, on the optimal tax rate, $\tilde{q}$, where $\hat{d} \equiv d(1 - \frac{1}{N})$. Recalling that $0 < 1 - a < \tilde{q} < 1$, simple comparative static exercises in (15) can show that:

Lemma 1: If the conditions of Proposition 1 hold, then:

$$If \left(\frac{[a(1-a)]^{\frac{1}{a}} b^{\frac{1-a}{a}}}{N} + \frac{2\hat{d} + \tilde{G}_y}{1-b} \right) > 0, then \ \frac{¶\tilde{q}}{¶d} > 0 \tag{17a}$$

$$If \left(\frac{a^{\frac{1}{a}} b^{\frac{1-a}{a}}}{N} + \frac{2\hat{d} + \tilde{G}_y}{1-b} \right) < 0, \ then \ \frac{¶\tilde{q}}{¶d} < 0 \tag{17b}$$

To understand (17a)-(17b), we consider (without great loss of generality) the special case in which $N \to \infty$. Then, (17a) is reduced to $d > -\frac{\tilde{G}_y}{2} > 0$, and (17b) is reduced to $0 < d < -\frac{\tilde{G}_y}{2}$. In other words, if the redistributive parameter (d) is high enough relatively to the (absolute value of the) rate of change in the social value of capital

[23] If $\left[1 + \frac{n}{N}\left(1 + \frac{\hat{d}}{\tilde{g}_y}\right)\right] < 0$, multiple steady states are possible. Note that if $N \to \infty$, this cannot happen. That is, when there is a large number of individuals, multiplicity cannot arise.

$(\tilde{G}_y)$, the long-run optimal tax rate increases with d. On the other hand, if d is small enough relative to $\tilde{G}_y$, the long-run optimal tax rate decreases with d. Thus, the relation between anticipated redistributive policy and the long-run optimal tax rate is not monotonic.

As far as the Balanced Growth Path $(\tilde{G}_c)$ is concerned, recall that $\frac{¶\tilde{G}_c}{¶\tilde{q}} < 0$ (this follows from (11c) which implies $\tilde{G}_c = [(1-\tilde{q})\Delta(\tilde{q}) - r - d(1-\frac{1}{N})]$). Then, the relation between d and $\tilde{G}_c$ is the inverse of the relation between d and $\tilde{q}$, i.e. it is an inverted U-curve. Namely, for low d relative to $\tilde{G}_y$, $\tilde{G}_c$ increases with d, while for high d relative to $\tilde{G}_y$, $\tilde{G}_c$ decreases with d. All this can be summarized by the following Lemma:

Lemma 2: When the rate of redistribution is smaller than the fall in the social value of capital, growth increases with redistribution, while when the rate of redistribution is larger than the fall in the social value of capital, growth decreases with redistribution.

Note that the value of $\tilde{G}_y$ is critical for this non-monotonicity. These results are similar to those in Benabou [1996].

V. TRANSITIONAL DYNAMICS

We now move on to study local stability properties around the steady state. Ideally, we would like to linearize (11a)-(11e) around (13)-(15). However, due to heterogeneity along the optimal path, this is not a tractable problem. Since we cannot explicitly study the transitional dynamics of the economy under both *ex ante* and *ex post* heterogeneity, we will study the transitional dynamics in the special case there is a representative individual *ex post*, and then give an informal argument for the general case in which individuals differ both *ex ante* and *ex post*.

To study the case in which individuals are homogenous *ex post*, we linearize (12a)-(12c) around (13)-(15). Then, the dynamics are approximated by the linear system:

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} J_{zz} & J_{zy} & J_{zq} \\ J_{yz} & J_{yy} & J_{yq} \\ J_{qz} & J_{qy} & J_{qq} \end{bmatrix} \begin{bmatrix} z \\ y \\ q \end{bmatrix} \tag{18}$$

where the elements of the Jacobian evaluated at the steady state are in Appendix C.

The determinant of the Jacobian, $\det(J) = -\dfrac{n\hat{d}^2(r+\hat{d})\Phi(\tilde{q})}{a(1-b)\tilde{q}^2\Delta(\tilde{q})} > 0$, is positive (because $\Phi(\tilde{q}) < 0$). Given the three-dimensional model, this sign of the determinant indicates that there are two possibilities: either three positive eigenvalues, or two negative eigenvalues and one positive. Recall that here all three variables (z, y, q) are jump variables. The first possibility, i.e. three positive roots, implies that the BGP is locally determinate (i.e. saddle-path stable). The second possibility, i.e. two negative roots and one positive, implies that the BGP is locally indeterminate (i.e. there exists a continuum of equilibrium trajectories associated with a given initial condition and a unique BGP).

The characteristic equation of the Jacobian in (18) is:

$$b^3 - (r + \hat{d} + J_{qq})b^2 + [(r+\hat{d})J_{qq} + \frac{\det(J)}{(r+\hat{d})}]b - \det(J) = 0 \tag{19}$$

where b denotes the eigenvalues of the Jacobian evaluated at steady state. Observe that what is crucial is the sign of J_{qq}, which is ambiguous (see Appendix C).

If $J_{qq} > 0$ (which happens when the effective redistributive parameter $\hat{d}$ is small enough), Descartes' Theorem implies that there are three positive roots (see Appendix D). In this case, there is local determinacy.

If $J_{qq} < 0$ so that $[(r+\hat{d})J_{qq} + \dfrac{\det(J)}{(r+\hat{d})}] < 0$ (which happens when the effective redistributive parameter $\hat{d}$ is large enough), Descartes' Theorem implies that there is only one positive root (see Appendix D). In this case, there is local indeterminacy. Specifically, this happens when:

$$0 < -\frac{(1-b)[1-\tilde{q} - a(2-a)]\tilde{q}}{(1-a-\tilde{q})^2} < \frac{\hat{d}}{r+\hat{d}} \tag{20}$$

The above results are summarized by the following proposition:

Proposition 2: Under the conditions in Proposition 1, the unique BGP is locally indeterminate (resp. determinate), when condition (20) does hold (resp. does not hold), i.e. when the effective redistributive parameter $\hat{d}$ is high (resp. small) enough.

Observe that although the transitional dynamics are affected by various factors (see the parameters in (20)), they depend critically on $\hat{d}$. Thus, a sufficiently large $\hat{d}$ causes the BGP to be locally indeterminate. Also observe that since $\hat{d} \equiv d(1-\frac{1}{N})$, as N increases, $\hat{d}$ also increases, so that the possibility of indeterminacy increases. That is, when the number of individuals increases so that free-riding incentives become stronger, the possibility of indeterminacy increases.

What does Proposition 2 mean? The anticipation of sufficiently large redistributive transfers opens the door for multiplicity. That is, there are many possible equilibrium paths for tax policy, consumption and capital accumulation, each of which is consistent with a given initial condition and with convergence to a unique steady state. This result implies that, in the presence of moral hazard behavior, individuals who start with similar endowments may consume and save at different rates over time. In other words, the anticipation of redistribution can itself generate income inequality over time.

We finally discuss the general case in which individuals are heterogeneous not only *ex ante* but also *ex post*. As we said above, although we cannot explicitly study the transitional dynamics of this general case, we can use Proposition 2 above, to give an informal argument. It is obvious that when there is *ex post* heterogeneity, the dimensionality of the dynamic system (11a)-(11e) increases. Hence, stability cannot become easier when we move from the special case in which individuals are heterogenous *ex ante* to the general case in which individuals are heterogenous both *ex ante* and *ex post*. Then, Proposition 2 implies that the possibility of indeterminacy under a high enough $\hat{d}$ increases once individuals differ both *ex ante* and *ex post*. In turn, Lemma 1 implies that this also increases the possibility of long-run growth decreasing with redistribution.

VI. CONCLUSIONS AND EXTENSIONS

This paper has investigated the effects of redistributive transfers on desirable fiscal policies and economic growth. Although we remained within the context of the conventional wisdom (i.e. redistribution is basically bad for growth), we gave a more complex and realistic view of the effects of redistributive and allocative policies on economic growth.

We close with three possible extensions. First, we have not managed to study explicitly the properties of the model under *ex-post* heterogeneity. This was for technical reasons. Thus, the task to study equilibria in which individuals differ both *ex-ante* and *ex-post* still remains. A second limitation is that we have not taken a stand as to why redistribution occurs. Recently, there has been a literature on how to rationalize redistribution (see footnote 3 above). Here, we took redistribution as given. Third, it would be interesting to add human capital as an engine of growth, and assume that households have different human capital endowments (see Glomm and Ravikumar [1992], Perotti [1993] and Fernandez and Rogerson [1995, 1996]). We leave these extensions for future work.