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DISCUSSION PAPER

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Bidding for Subsidies With One's Patience

Bidding for subsidies with one's patience *

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Abstract

We study the problem of distributing subsidies in a market that includes both marginal individuals in need of assistance and infra-marginal individuals who would purchase the subsidized product without additional incentives. We propose the use of a wait time auction, where individuals bid the amount of time they are willing to wait in exchange for a specified subsidy amount. This design enables more direct targeting of marginal individuals, thereby enhancing the overall effectiveness of the subsidy program. Furthermore, screening is costless in equilibrium as no wait times are imposed, and practical robustness against deviations from equilibrium behavior can be ensured by implementing a maximum allowable bid.

Keywords: Subsidies, Market Design, Auctions

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1 Introduction

Subsidies are an important tool for governments to redirect investments or consumption towards more sustainable areas. Examples include support for energy-efficient building retrofits, as well as incentives for the purchase of electric vehicles, cargo bikes, balcony power plants, and heat pumps. However, many subsidy programs suffer from a targeting problem (see e.g., Gillingham et al. (2018)). Subsidies are often allocated indiscriminately, primarily benefiting those who would have purchased the targeted product even without additional incentives — referred to as infra-marginal individuals. As a result, these subsidies crowd out support for applicants who are either unable or unwilling to make the purchase at the current price — referred to as marginal individuals.

This paper adopts a market design perspective, proposing the use of wait time auctions to distribute subsidies more effectively. We begin by analyzing the status quo, where subsidies are distributed on a first come, first served basis. While this mechanism is incentive-compatible and ensures that subsidies are not given to individuals who would not utilize them, it fails to screen out infra-marginal individuals (Theorem 1). In contrast, we show that a wait time auction satisfies the same positive properties as the status quo mechanism, while at the same time prioritizing marginal individuals over infra-marginal individuals when assigning subsidies (Theorem 2). Specifically, as marginal individuals will not purchase the relevant products without receiving a subsidy, they are perfectly patient and will therefore always outbid all infra-marginal individuals.

Another important feature of the wait time auction is that the actual wait time imposed on individuals is determined endogenously by the lowest unsuccessful bidder. Consequently, if applying for subsidies entails a small cost, no wait time is imposed on successful applicants in equilibrium (see Theorem 3). In this scenario, screening is achieved without imposing any additional costs on society.

Finally, in practice, individuals may fail to coordinate on the relevant equilibrium, potentially resulting in wait times being imposed on successful applicants. To address this, we propose implementing an additional level of safety at the cost of reduced screening effectiveness. Specifically, a wait time auction can be designed with a maximum allowable wait time bid, which trivially limits the maximum wait time that can be imposed on individuals. However, the effectiveness of screening decreases as the maximum allowable wait time bid is lowered, since it enables relatively more patient infra-marginal individuals to compete with marginal individuals (see Theorem 4).

The rest of the paper is organized as follows: Section 2 reviews the relevant literature. Section 3 introduces the model, while Section 3.1 outlines the primary axioms. Section 4 compares the status quo mechanism with the wait time auction. Section 5.1 shows that wait times in the proposed auction are zero in equilibrium when individuals incur small application costs. Section 5.2 examines the impact of limiting bids on the overall effectiveness of screening. Finally, Section 6 concludes.

2 Literature

This paper analyses the subsidies from a market design perspective (see e.g. Sönmez (2023)). An axiomatic approach is employed and subsidy programs are modeled using contracts (Hatfield and Milgrom, 2005; Hatfield and Kojima, 2010).

Furthermore, our paper adds to the literature on screening. This literature discusses two possible routes for screening, either through tagging or costly signaling. The first approach, tagging, involves screening individuals based on observable characteristics (Akerlof, 2005). There is some evidence that tagging might not be very effective in practice. However, evidence suggests tagging may be ineffective in practice. For instance, Fowlie et al. (2018) find that the US federal Weatherization Assistance Program's costs outweigh its benefits.

Our proposal uses the second approach of costly signaling. Costly signaling often involves imposing ordeals on relevant individuals, as discussed by Zeckhauser (2021) in the context of health care, where waiting times play a prominent role. Dworczak et al. (2023) studies a stylized problem where a designer allocates a fixed budget of money to individuals with differing privately observed marginal values for money, achieving screening through an unspecified ordeal. Condorelli (2012) studies when its optimal to use queuing or lotteries, when individuals have a maximum willingness to wait in line to obtain an certain object like affordable housing. Here, all individuals are assumed to be marginal, and thus the goal is to give them to individuals with a high valuation instead of targeting individuals with a sufficiently low valuation.

In our paper, we propose a novel auction mechanism where individuals bid for subsidies with wait times. This approach contrasts with the mechanism analyzed by Globus-Harris (2020), which uses fixed wait times to screen out infra-marginal individuals. The key distinction lies in the endogeneity of wait times in our auction design: a small application cost ensures wait times drop to zero in equilibrium, a feature that cannot be replicated with fixed ex ante wait times. Related, Alatas et al. (2016) show that in practice application costs can effectively deter infra-marginal individuals from applying in the context of Indonesia's Conditional Cash Transfer program.

While many papers in the literature focus on maximizing welfare directly, our focus is on maximizing the total number of products purchased. This approach is also adopted by DeShazo et al. (2017), who assesses alternative rebate designs for plug-in electric vehicles. Moreover, our modeling assumption of individuals sharing a common discount factor aligns with Burkett and Woodward (2021), who show that a seller cannot benefit from screening on discount rate. In fact, in our model a common discount factor implicitly provides a theory on impatience, where impatient individuals are those who value a given product more highly.

3 Model

There is a finite set of **individuals** $I = \{1, \ldots, n\}$. A single **product**, with at least n identical copies, is sold at a **price** $p > 0$. Each individual $i \in I$ draws a **valuation** for the product from a continuous cumulative distribution function F, where the lowest and the highest value in the support of F are denoted by $\underline{v}$ and $\overline{v}$, respectively, with $0 \leq \underline{v} < p < \overline{v} < \infty$. Each individual's valuation v_i is private information. A **valuation**

profile is denoted by $v = (v_i)_{i \in I}$ and the set of all profiles is denoted by V. A valuation profile for everyone except individual $i \in I$ is denoted by $v_{-i} = (v_{i'})_{i' \in I \setminus \{i\}}$ and the set of all such profiles is denoted by V_{-i}.

Absent any subsidies, an individual can either choose to buy the product, represented by the **buy action** a_1, or not buy the product, represented by the **not buy action** a_0. We assume that individuals have quasi-linear utility. Thus, the former action results in a utility of $u(a_1, v_i) = v_i - p$, while the latter action results in a utility of $u(a_0, v_i) = 0$. Note that, each individual $i \in I$ has an optimal action $a_i^* \equiv \arg\max_{a \in \{a_1, a_0\}} u(a, v_i)$ which we refer to as the individual's **outside option**. This partitions the set of individuals into two different types: **Infra-marginal individuals** $I^{Inf} \equiv \{i \in I : a_i^* = a_1\}$ find it optimal to buy a product absent any subsidies, while **marginal individuals** $I^{Mar} \equiv \{i \in I : a_i^* = a_0\}$ will not buy any product without additional monetary incentive.

A subsidy program has a fixed amount of **budget** $B \in \mathbb{R}_0^+$ available to distribute among individuals applying for subsidies. This budget can be used to hand out subsidy contracts. A **subsidy contract** $x = (s, t)$ specifies an amount of subsidy $s \in \mathbb{R}^+$ paid, conditional on buying product at time $t \in \mathbb{R}_0^+$. For a given subsidy contract x we refer to its elements by s_x and t_x. The set of all possible subsidy contracts is denoted by $X \equiv \mathbb{R}^+ \times \mathbb{R}_0^+$.

We assume that individuals have a common discount factor δ, with $0 < \delta < 1$. Thus, if an individual $i \in I$ executes a contract $x \in X$ it receives utility $u(x, v_i) = \delta^{t_x}(v_i - p + s_x)$. Furthermore, it will be helpful to specify what happens to individuals that end up without a contract, denoted by $\varnothing$. That is, such individuals will simply execute their outside option, resulting in utility $u(\varnothing, v_i) \equiv u(a^*, v_i)$.

In summary, a **subsidy problem** is a tuple $\langle I, p, v, B \rangle$. Fixing the variables I, p, B throughout the paper, a subsidy problem is simply denoted by v .

3.1 Mechanisms and the desired properties

An **outcome** is a profile $o = (o_i)_{i \in I}$ where for all $i \in I$ we have $o_i \in X \cup \{\varnothing\}$, that is, each individual $i \in I$ is assigned a subsidy contract or nothing. An outcome is **feasible** if the paid subsidies remain within the budget, i.e., $\sum_{x \in X_o} s_x \leq B$. The set of all feasible outcomes is denoted by O. When defining properties, it will be useful to introduce the following notation: For a given outcome $o \in O$, the set of individuals assigned a contract is denoted by $I_o = \{i \in I : o_i \neq \varnothing\}$, the set of infra-marginal individuals assigned a contract is denoted by $I_o^{Inf} \equiv I_o \cap I^{Inf}$, and the set of marginal individuals assigned a contract is denoted by $I_o^{Mar} \equiv I_o \cap I^{Mar}$.

Given the revelation principle (Myerson, 1982), we restrict our attention to direct mechanisms. Formally, a **direct mechanism** is a function $\psi : V \mapsto O$. Next, let us discuss the desired properties for any mechanism used for the distribution of subsidies.

First, a mechanism should assign individually rational subsidies, ensuring that every individual will execute their assigned contract. That is, any individual that is assigned a contract under the mechanism, must weakly prefer to execute its assigned contract over executing its outside option.

Definition 1 (Individually rational subsidies)**.** A mechanism ψ assigns individually rational subsidies if for all $v \in V$, and for all $i \in I_{\psi(v)}$ we have

$$u(\psi(v)_i, v_i) \geq u(a_i^*, v_i).$$

Second, a mechanism should be incentive compatible, ensuring that no individual can benefit from misreporting its valuation. That is, any individual must weakly prefer reporting its true valuation over reporting any other valuation.

Definition 2 (Incentive compatible)**.** A mechanism ψ is incentive compatible if for all $i \in I$, $v_{-i} \in V_{-i}$, and $v_i, \hat{v}_i \in V_i$ we have

$$u(\psi(v_i, v_{-i})_i, v_i) \geq u(\psi(\hat{v}_i, v_{-i})_i, v_i).$$

Third, a mechanism should be locally marginal-maximal, locally maximizing the number of marginal individuals being assigned a contract. That is, there should be no marginal individual without a contract, that would execute the contract assigned to any infra-marginal individual.

Definition 3 (Locally marginal-maximal)**.** A mechanism ψ is locally marginal-maximal, if for all $v \in V$, there does not exist $i \in I^{Mar} \setminus I^{Mar}_{\psi(v)}$ and $i' \in I^{Inf}_{\psi(v)}$ such that

$$u(\psi(v)_{i'}, v_i) > u_i(a_i^*, v_i) = 0$$

4 Exogenous participation: Status quo mechanism and wait time auction

Apart from the just defined requirements on mechanisms, the wait time imposed on individuals will also determine the desirability of any proposed mechanism.

We will show that the overall wait time does not only depend on the mechanism in place but more broadly on the decision of individuals to apply to a subsidy program in the first place. To fix ideas, we first discuss the status quo mechanism, and the new class of wait time auctions in a vacuum, i.e., assuming that every individual will participate in the mechanism.

4.1 Status Quo mechanism

Most subsidy programs operate on a first come, first serve basis, with subsidies simply reimbursing some fraction $\alpha \in (0, 1)$ of the product's price p without imposing any wait times. That is, there is a single subsidy contract $x = (s, 0)$ with $s = \alpha \times p$. Furthermore, the **first come first serve order** of the assignment process will be represented by a (strict simple) order π over I, where $i\pi i'$ represents i applying before i'.[1] With all this in mind we are ready to define the status quo mechanism:

[1] A strict simple order is transitive, asymmetric, and complete.

Status quo mechanism. *Go through the following steps:*

i) First, deny all applications of individuals for whom the subsidy amount is too low, i.e., $v_i + s < p$.

ii) Second, among the remaining individuals, accept the application of individuals one by one, following the first come first serve order π*, until accepting one more would go over the budget. Then deny the applications of the remaining individuals.*

iii) Third, every individual who got the application accepted is assigned contract $x = (s, 0)$*, all who have their application denied are assigned no contract* $\varnothing$.

The status quo mechanism is incentive compatible while only handing out subsidies to those who will use them, though clearly is not designed to screen out any infra-marginal individuals.

Theorem 1. *The status quo mechanism* ψ^{SQ} *assigns individually rational subsides and is incentive compatible, but fails to be locally marginal-maximal.*

Let us now compare the status quo mechanism to the wait time auction. In practice, a wait time would inquire individuals to bid the maximum amount of time they are willing to wait for receiving a specified amount of subsidy. As we specify everything in terms of direct mechanisms, one first needs to translate the reported valuations into wait time bids. That is, given a valuation and specified subsidy amount, the corresponding **wait time bid** makes the individual indifferent between its outside option and getting the specified subsidy amount. Formally, bids are defined as follows:

$$b(v_i, s) = \begin{cases} \log_\delta\left(\frac{v_i - p}{v_i + s - p}\right) & \text{if } v_i - p \geq 0 \\ \infty & \text{if } v_i - p < 0 \text{ and } v_i - p + s \geq 0 \end{cases}$$

Note that bids are undefined for individuals that will, given the subsidy amount, never buy the product. Given the bids, a **modified order** $\pi(v, s)$ is constructed, where individuals with higher bids are given higher priority: For all distinct $i, i' \in I$ we have $i\pi i'$ if and only if $b(v_i, s) > b(v_{i'}, s)$, or $b(v_i, s) = b(v_{i'}, s)$ and $i\pi i'$.

Wait auction. *Go through the following steps:*

i) *First, deny all applications of individuals for whom the subsidy amount is too low, i.e.,* $v_i + s < p$*.*

ii) *Second, among the remaining individuals, accept the application of individuals one by one, following the* ***modified order*** $\pi(v, s)$*, until accepting one more would go over the budget. Then deny the applications of the remaining individuals.*

iii) *Every individual who got the application accepted is assigned contract* $x = (s, t)$*, where the* ***wait time is set to the highest unsuccessful bid, or no wait time if there is no such bid****. All individuals who have their application denied are assigned no contract* $\varnothing$*.*

The wait time auction is both incentive compatible while only handing out subsidies to those who will use them, though improves upon the status quo mechanism by screening out infra-marginal individuals.

Theorem 2. ψ^W *is individually rational, incentive compatible, locally marginal-maximal, and does not distort the market.*

Of course, taking individuals participation in the mechanism as exogenously given, screening out infra-marginal individuals through a wait time auction comes at the cost of imposing undesirable wait times on successful applicants. Any analysis of subsidy program mechanisms must ultimately take into account the decision of individuals to apply in the first place.

5 Endogenous participation

In practice, applying for a subsidy program is costly, among other things, requiring individuals to fill out the relevant administrative forms. Clearly, any marginal individual that finds the subsidy amount too low to buy the subsidized product, would therefore never apply in the first place. Moreover, strategically sophisticated individuals will abstain from applying if their chances of success are sufficiently low.

5.1 Strategic participation

In this section we define and analyze the participation game induced by a given mechanism. We focus on participation games under direct and incentive compatible mechanisms, implicitly assuming that once individuals decide to partake in a mechanism they simply submit their true valuation.

The idea is straightforward: individuals choose whether to apply for subsidies or refrain from doing so. Individuals deciding not to apply get a payoff as determined by their outside option. Individuals that do apply incur a small application cost, and otherwise their payoff is determined by the subsidy mechanism, run only with the subset of participating individuals.

Definition 4 (Participation game)**.** Formally, a participation game is defined for a given subsidy problem $\langle I, p, v, B \rangle$ and induced by a direct and incentive compatible mechanism ψ:

1. Each individual $i \in I$ is a **player**.
2. For each $i \in I$, let $s_i \in S_i \equiv \{s_1, s_0\}$ denote i's **strategy** of applying or not applying for subsidies. Let $s = (s_i)_{i \in I} \in S$ denote a strategy profile. Slightly abusing notation, for a given strategy profile, let $I^1 = \{i \in I : s_i = s_1\}$ be the set of applicants and $v^1 = (v_i)_{i \in I^1}$ their reported types. The individuals not applying under a given strategy.profile are denoted by $I^0 \equiv I \setminus I^1$.
3. Incurring a small application cost $\epsilon > 0$, an individual's **payoff** is as follows:

$$u_i(s) = \begin{cases} u(\psi(v^1)_i, v_i) - \epsilon & \text{if } i \in I^1_{\psi(v(s))} \\ u(a_i^*, v_i) - \epsilon & \text{if } i \in I^1 \setminus I^1_{\psi(v^1)} \\ u(a_i^*, v_i) & \text{otherwise.} \end{cases}$$

Next, we analyze the equilibrium of the participation game.

Definition 5 (Nash Equilibrium)**.** Formally, a strategy profile s is a **Nash equilibrium**

if $u_i(s = s^1) \geq u_i(s = s^0)$ for all $i \in I^1$ and $u_i(s = s^0) \geq u_i(s = s^1)$ for all $i \in I^0$.

We establish the following result:

Theorem 3. *Consider any participation game induced by a wait time auction ψ^W.*

i) Wait times in any Nash equilibrium are zero.

Under full participation let i be the lowest successful bidder and i' be the highest unsuccessful bidder.

ii) If $i \in I^{Mar}$, there exists a unique Nash equilibrium with $I_{\psi^W(v)} = I_{\psi^w(v^1)}$ if and only if $b(v_{i'}, s) \leq log_\delta(\frac{v_i - p + \epsilon}{v_i - p + s})$.

iii) If $i \in I^{Inf}$, there exists a unique Nash equilibrium with $I_{\psi^W(v)} = I_{\psi^w(v^1)}$ if and only if $b(v_{i'}, s) \leq log_\delta(\frac{\epsilon}{v_i - p + s})$.

Corollary 1. *A necessary condition for the existence of a unique Nash equilibrium requires the budget to cover all targetable marginal individuals $B \geq |\{i \in I^{Mar} : v_i + s \geq p\}| \times s$.*

That is, in equilibrium individuals will only apply for subsidies if their application will be successful. Therefore, there will be no highest unsuccessful bid, leading to zero wait times.

If individuals do not manage to coordinate on the equilibrium, wait times are still zero as long as individuals are on the cautious side and only relatively few individuals apply. On the flip-side, if coordination fails and the subsidy program remains over-demanded, wait times can still be substantial. A possible simple solution to mitigate this problem is discussed in the next section.

5.2 Fail-safe

In this section, we point out the option to implement a wait time auction with a maximum bid. While doing so comes at the cost of lowering the effectiveness of screening it also limits the wait times imposed on individuals during the auction.

Let b^{max} denote the maximum allowed bid during a wait time auction. Then the adjusted bid becomes $b'(v_i, s) = \min(b(v_i, s), b^{max})$. Given the bids, an **adjusted modified order** $\pi'(v, s)$ is constructed, where individuals with higher bids are given higher priority: For all distinct $i, i' \in I$ we have $i\pi i'$ if and only if $b'(v_i, s) > b'(v_{i'}, s)$, or $b'(v_i, s) = b'(v_{i'}, s)$ and $i\pi i'$.

Then the **generalized wait time auction** is defined equivalently to the wait time auction with adjusted modified order $\pi'(v, s)$.

We note that, in this case, the status quo mechanism is just a wait time auction with a maximum bid of zero.

Naturally all the results from before hold except the wait time auction fails to be locally marginal-maximal. Though, screening still works, with higher maximum bids being more effective. Next, let us define the idea of screening effectiveness formally, in terms of the relative increase in marginal individuals ending up with a contract.

Definition 6 (Marginal-domination)**.** A mechanism ψ marginal-dominates a mechanism ψ', if for all $v \in V$,

$$I^{Mar}_{\psi'(v)} \subseteq I^{Mar}_{\psi(v)}$$

We establish the following result.

Theorem 4. *Consider any two generalized wait time auctions with $b^{max} > b^{max\prime}$ then ψ^{GW} marginal-dominates $\psi^{GW\prime}$.*

Note that, if the maximum wait time bids are restricted to be zero, the wait time auction is identical to the status quo mechanism. We conclude that introducing at least some wait time within the horizon of the subsidy program is sensible, ensuring that, at the very least, the most impatient infra-marginal individuals are screened out.

6 Conclusion

This paper introduces a novel approach to subsidy allocation through the use of wait time auctions. By leveraging wait times, the mechanism effectively screens out infra-marginal individuals who would purchase the eco-friendly product without additional incentives. A key design feature is that wait times are determined based on submitted bids, ensuring that no additional wait time is imposed in equilibrium. Furthermore, the paper explores implementing a fail-safe to safeguard the system against deviations from equilibrium behavior, thereby enhancing its practical robustness. Overall, the findings shed light on the inefficiencies of current subsidy programs and present a promising alternative for more targeted and effective subsidy allocation, with minimal deviation from existing practices.

A Mathematical appendix

A.1 Theorem 1

Proof. **Individually rational subsidies**: By definition of the mechanism's step i), for any individual being assigned a subsidy contract $i \in I_{\psi^{SQ}(v)}$, we have that $v_i + s \geq p$.

Case 1: Among individuals being assigned a subsidy contract, take any infra-marginal individual $i \in I^{Inf}_{\psi^{SQ}(v)}$. We have that $u(\psi^{SQ}(v), v_i) = v_i + s - p > v_i - p = u(a_i^*, v_i)$ as $s > 0$.

Case 2: Among individuals being assigned a subsidy contract, take any marginal individual $i \in I^{Mar}_{\psi^{SQ}(v)}$. We have that $u(\psi^{SQ}(v), v_i) = v_i + s - p \geq 0 = u(a_i^*, v_i)$ as $v_i + s \geq p$.

Thus, executing the assigned contract is preferred by every individual over its outside option, i.e., $u(\psi^{SQ}(v), v_i) \geq u(a_i^*, v_i)$ for all $i \in I_{\psi^{SQ}(v)}$ concluding the proof. □

Proof. **Incentive compatible**: Note that there are only two outcomes for each individual. Either it is assigned a subsidy contract or it executes its outside option.

Case 1: Suppose $i \in I_{\psi(v)}$, that is, i is assigned a subsidy contract. By individually rational subsidies, it directly follows that i cannot be made better off, i.e., $u(\psi^{SQ}(v)_i, v_i) \geq u(\psi^{SQ}(\hat{v}_i, v_{-i})_i, v_i)$ for all $\hat{v}_i \in V_i$.

Case 2: Suppose $i \in I \setminus I_{\psi(v)}$ and $i \pi i'$ for some $i' \in I_{\psi(v)}$, that is, i is not assigned a subsidy contract but has high enough priority to qualify. In this case, by definition of the mechanism step i), we have $v_i + s < p$. That is, executing the outside option is preferred to executing the subsidy contract, i.e., $u(\psi^{SQ}(v)_i, v_i) = u(a_i^*, v_i) \geq u(\psi^{SQ}(\hat{v}_i, v_{-i})_i, v_i)$ for all $\hat{v}_i \in V_i$.

Case 3: Suppose $i \in I \setminus I_{\psi(v)}$ and $i' \pi i$ for all $i' \in I_{\psi(v)}$, that is, i is not assigned a subsidy contract due to her priority being too low. In this case, the construction of the algorithm implies that i will remain unassigned regardless of which valuation she submits, i.e. $u(\psi^{SQ}(v)_i, v_i) = u(\psi^{SQ}(\hat{v}_i, v_{-i})_i, v_i) = u(a_i^*, v_i)$ for all $\hat{v}_i \in V_i$..

Cases 1-3 imply that for all $i \in I$ and for all $\hat{v}_i \in V_i$ we have $u(\psi^{SQ}(v)_i, v_i) \geq$

$u(\psi^{SQ}(\hat{v}_i, v_{-i})_i, v_i)$, concluding the proof.

□

Proof. **Not locally marginal-maximal**: We show that ψ^{SQ} is not marginal-maximal by counterexample. That is, consider any $v \in V$ s.t. there is at least one individual with a subsidy contract $i' \in I_{\psi(v)}$, and at least one individual without a contract due to having too low priority, i.e., $i \in I \setminus I_{\psi(v)}$ and $i' \pi i$ for all $i' \in I_{\psi(v)}$. Now, consider a modified valuation draw v^{mod} where

1. $v_{i'}^{mod} = p + \epsilon$ with $\epsilon \in (0, s)$
2. $v_{i}^{mod} = p - \epsilon$ with $\epsilon \in (0, s)$,
3. $v_{i''}^{mod} = v_{i''}$ for all $i'' \in I \setminus \{i, i'\}$.

By construction, the outcome of the status quo mechanism remains unchanged, i.e., $\psi^{SQ}(v) = \psi^{SQ}(v^{mod})$. Moreover, we have reached a violation of locally marginal-maximality as i' is an infra-marginal individual that is assigned a subsidy contract acceptable to the marginal individual i, i.e., there exists $i' \in I^{Inf}_{\psi^{SQ}(v^{mod})}$, $i \in I^{Mar} \setminus I^{Mar}_{\psi^{SQ}(v^{mod})}$ such that $u(\psi^{SQ}(v^{mod})_{i'}, v_i) > u(a_i^*, v_i) = 0$.

□

A.2 Theorem 2

Proof. **Individually rational subsidies**: By definition of the mechanism's step i), for any individual being assigned a subsidy contract $i \in I_{\psi^W(v)}$, we have that $v_i + s \geq p$.

Case 1: Among individuals being assigned a subsidy contract, take any infra-marginal individual $i \in I^{Inf}_{\psi^W(v)}$. By definition of the mechanism's step iii) any infra-marginal individual assigned a contract is assigned a wait time $t \leq \log_\delta(\frac{v_i - p}{v_i + s - p})$. Thus, it follows that $u(\psi^W(v), v_i) = \delta^t(v_i + s - p) \geq \frac{v_i - p}{v_i + s - p}(v_i + s - p) = v_i - p = u(a_i^*, v_i)$.

Case 2: Among individuals being assigned a subsidy contract, take any marginal individual $i \in I^{Mar}_{\psi^W(v)}$. By definition of the mechanism's step iii) any marginal individual assigned a contract is assigned a wait time $t \leq \infty$. We have that $u(\psi^W(v), v_i) = \delta^t(v_i + s - p) \geq$

$0 = u(a_i^*, v_i)$ as $v_i + s \geq p$.

Thus, executing the assigned contract is preferred by every individual over its outside option, i.e., $u(\psi^W(v), v_i) \geq u(a_i^*, v_i)$ for all $i \in I_{\psi^W(v)}$ concluding the proof. □

Proof. **Incentive compatible**: Note that there are only two outcomes for each individual. Either it is assigned a subsidy contract with wait times equal to the lowest unsuccessful bid or it executes its outside option.

Case 1: Suppose $i \in I_{\psi(v)}$, that is, i is assigned a subsidy contract. By individually rational subsidies, it directly follows that i cannot be made better off, i.e., $u(\psi^W(v)_i, v_i) \geq u(\psi^W(\hat{v}_i, v_{-i})_i, v_i)$ for all $\hat{v}_i \in V_i$.

Case 2: Suppose $i \in I \setminus I_{\psi(v)}$ and $i \pi i'$ for some $i' \in I_{\psi(v)}$, that is, i is not assigned a subsidy contract but has high enough priority to qualify. In this case, by definition of the mechanism step *i*), we have $v_i + s < p$. That is, executing the outside option is preferred to executing the subsidy contract regardless of wait time, i.e., $u(\psi^{SQ}(v)_i, v_i) = u(a_i^*, v_i) \geq u(\psi^W(\hat{v}_i, v_{-i})_i, v_i)$ for all $\hat{v}_i \in V_i$.

Case 3: Suppose $i \in I \setminus I_{\psi(v)}$ and $i' \pi i$ for all $i' \in I_{\psi(v)}$, that is, i is not assigned a subsidy contract due to her priority being too low.

Case 3.1: If i is infra-marginal, then the construction of the algorithm implies that the distributed subsidy contracts have an associated wait time $t \geq \log_\delta(\frac{v_i - p}{v_i + s - p})$. Suppose that i can submit a valuation $\hat{v}_i$ that results in a subsidy contract. As any such successful valuation must result in weakly higher wait times than t this is never strictly preferred, i.e., $u(\psi^W(\hat{v}_i, v_{-i}), v_i) = \delta^t(v_i + s - p) \leq \frac{v_i - p}{v_i + s - p}(v_i + s - p) = v_i - p = u(a_i^*, v_i)$.

Case 3.2: If i is marginal, then i has already submitted the highest possible bid. Thus no other valuation can change that i is not assigned a contract, i.e., $u(\psi^W(v)_i, v_i) = u(\psi^{SQ}(\hat{v}_i, v_{-i})_i, v_i) = u(a_i^*, v_i)$ for all $\hat{v}_i \in V_i$.

Cases 1-3 imply that for all $i \in I$ and for all $\hat{v}_i \in V_i$ we have $u(\psi^W(v)_i, v_i) \geq u(\psi^W(\hat{v}_i, v_{-i})_i, v_i)$, concluding the proof.

□

Proof. **Locally marginal-maximal**:

Suppose that the mechanism ψ^W is not locally marginal-maximal, that is, there exists $i \in I^{Mar} \setminus I_{\psi(v)^W}$ and $i' \in I^{Inf}_{\psi^W(v)}$ such that $u(\psi(v)_{i'}, v_i) > u_i(a_i^*, v_i) = 0$. By construction of the mechanism, if any infra-marginal individual is assigned a contract, all marginal individuals that submitted a bid are also assigned a contract, i.e. all $i \in I$ s.t. $v_i - p < 0$ and $v_i - p + s \geq 0$ submit a bid $b(v_i, s) = \infty$. Thus, the only marginal individuals without a contract have $v_i - p + s < 0$. That is, we have reached a contradiction as $u(\psi(v)_{i'}, v_i) = \delta^t(v_i - p + s) < 0 = u_i(a_i^*, v_i)$.

□

A.3 Theorem 3

Proof. **i) Zero wait times**: Note that, for all $i \in I$ choosing to participate without being assigned a subsidy contract is not an equilibrium strategy as then $u_i(s^1) = u(a^*, v_i) - \epsilon < u_i(s^0) = u(a^*, v_i)$. It directly follows that there is no highest unsuccessful bin in any Nash equilibrium. By the definition of the wait time auction, in this case wait times are zero. □

Proof. **ii) Uniqueness if**: Consider the lowest successful bidder $i \in I$, which by assumption is infra-marginal, and note that i has a dominant strategy to always participate. That is, the highest possible wait time under any strategy profile s is $t \leq log_\delta(\frac{v_i - p + \epsilon}{v_i - p + s})$, and thus participation results in a payoff $u_i(s^1) \geq [\frac{v_i - p + \epsilon}{v_i - p + s}(v_i - p + s)] - \epsilon = v_i - p = u_i(s^0)$. Following this, it's easy to check that participation is a dominant strategy for all bidders that are successful under full participation, i.e., for all $i \in I_{\psi^W(v)}$. Given this, participation is never optimal for the remaining $i' \in I \setminus I_{\psi^W(v)}$. Thus, there exists a unique Nash equilibrium with $I_{\psi^W(v)} = I_{\psi^w(v^1)}$. □

Proof. **ii) Uniqueness only if**: We prove the contrapositive: That is, if the bid of the highest unsuccessful bidder is $b(v_{i'}, s) > log_\delta(\frac{v_i - p + \epsilon}{v_i - p + s})$, then there exists no unique Nash equilibrium. Specifically, its easy to check that there are at least two Nash equilibria:

1. Consider s s.t. $I^1 = I_{\psi^W(v)}$. It is easy to verify that this is a Nash equilibrium,
2. Consider s s.t. $I^1 = I_{\psi^W(v)} \cup \{i'\} \setminus \{i\}$, where for i changing from $s_i = s_0$ to $s_i = s_1$ results in a lower payoff as even though i is assigned a contract, the resulting wait time is too high given the participation cost, i.e., $u_i(s_1) < [\frac{v_i - p + \epsilon}{v_i - p + s}(v_i - p + s)] - \epsilon = v_i - p = u_i(s_0)$.

□

Proof. **iii) Uniqueness if**: Consider the lowest successful bidder $i \in I$, which by assumption is marginal, and note that i has a dominant strategy to always participate. That is, the highest possible wait time under any strategy profile s is $t \leq log_\delta(\frac{\epsilon}{v_i - p + s})$, and thus participation results in a payoff $u_i(s^1) \geq [log_\delta(\frac{\epsilon}{v_i - p + s})(v_i - p + s)] - \epsilon = 0 = u_i(s^0)$. Following this, it's easy to check that participation is a dominant strategy for all the remaining bidders that are successful under full participation and are all marginal individuals, i.e., for all $i \in I_{\psi^W(v)}$. Given this, participation is never optimal for the remaining $i' \in I \setminus I_{\psi^W(v)}$. Thus, there exists a unique Nash equilibrium with $I_{\psi^W(v)} = I_{\psi^w(v^1)}$. □

Proof. **iii) Uniqueness only if**: We prove the contrapositive: That is, if the bid of the highest unsuccessful bidder is $b(v_{i'}, s) > log_\delta(\frac{\epsilon}{v_i - p + s})$, then there exists no unique Nash equilibrium. Specifically, it's easy to check that there are at least two Nash equilibria:

1. Consider s s.t. $I^1 = I_{\psi^W(v)}$. It is easy to verify that this is a Nash equilibrium,
2. Consider s s.t. $I^1 = I_{\psi^W(v)} \cup \{i'\} \setminus \{i\}$, where for i changing from $s_i = s_0$ to $s_i = s_1$ results in a lower payoff as even though i is assigned a contract, the resulting wait time is too high given the participation cost, i.e., $u_i(s_1) < [\frac{\epsilon}{v_i - p + s}(v_i - p + s)] - \epsilon = 0 = u_i(s_0)$

□

A.4 Theorem 4

Proof. Fix any $v \in V$ and consider any two generalized wait time auctions ψ^{GW} and $\psi^{GW\prime}$, with $b^{max} > b^{max\prime}$, and let their corresponding adjusted modified orders be $\pi'(v, s)$

and $\pi''(v,s)$. Note that both mechanisms give out subsidy contracts to the same number of individuals equal to $\lfloor \frac{B}{s} \rfloor$. That is, $I_{\psi^{GW}(v)} = \{i \in I : |i' \in I : i'\pi'(v,s)i| < \lfloor \frac{B}{s} \rfloor\}$ and $I_{\psi^{GW\prime}(v)} = \{i \in I : |i' \in I : i'\pi''(v,s)i| < \lfloor \frac{B}{s} \rfloor\}$. Towards a contradiction, suppose that there exists $i \in I^{Mar}_{\psi^{GW\prime}(v)}$ but $i \notin I^{Mar}_{\psi^{GW}(v)}$.

Case 1: We have $i \notin I^{Mar}_{\psi^{GW}(v)}$ as $v_i + s < p$; but then this directly implies that $i \in I^{Mar}_{\psi^{GW\prime}(v)}$, a contradiction.

Case 2: We have $i \notin I^{Mar}_{\psi^{GW}(v)}$ as $|i' \in I : i'\pi'(v,s)i| \geq \lfloor \frac{B}{s} \rfloor$. Note that, as i is marginal, and thus must have submitted the maximum bid. Furthermore, we have that all individuals with a contract have also submitted the maximum bid and must have a higher priority than i, i.e., for all $i' \in i \in I^{Mar}_{\psi^{GW}(v)}$ we have $b'(v_{i'}, s) = b^{max}$ and $i'\pi i$. But, the same must hold true for a lower maximum bid $b^{max\prime} < b^{max}$ under $\psi^{GW\prime}$ and therefore $i \notin I^{Mar}_{\psi^{GW\prime}(v)}$.

Case 1 and 2 conclude the proof. □

References

G Akerlof. The economics of 'tagging'as applied to the optimal income tax, welfare programs and manpower planning'. *Explorations in Pragmatic Economics*, page 100, 2005.

Vivi Alatas, Ririn Purnamasari, Matthew Wai-Poi, Abhijit Banerjee, Benjamin A Olken, and Rema Hanna. Self-targeting: Evidence from a field experiment in indonesia. *Journal of Political Economy*, 124(2):371–427, 2016.

Justin Burkett and Kyle Woodward. Intertemporal allocation with unknown discounting. 2021.

Daniele Condorelli. What money cant buy: Efficient mechanism design with costly signals. *Games and Economic Behavior*, 75(2):613–624, 2012.

JR DeShazo, Tamara L Sheldon, and Richard T Carson. Designing policy incentives for cleaner technologies: Lessons from california's plug-in electric vehicle rebate program. *Journal of Environmental Economics and Management*, 84:18–43, 2017.

Piotr Dworczak et al. Equity-efficiency trade-off in quasi-linear environments. Technical report, Working paper, 2023.

Meredith Fowlie, Michael Greenstone, and Catherine Wolfram. Do energy efficiency investments deliver? evidence from the weatherization assistance program. *The Quarterly Journal of Economics*, 133(3):1597–1644, 2018.

Kenneth Gillingham, Amelia Keyes, and Karen Palmer. Advances in evaluating energy efficiency policies and programs. *Annual Review of Resource Economics*, 10(1):511–532, 2018.

Isla Globus-Harris. Waiting periods as a screening mechanism for environmental subsidies. *Journal of the Association of Environmental and Resource Economists*, 7(6):1151–1180, 2020.

John William Hatfield and Fuhito Kojima. Substitutes and stability for matching with contracts. *Journal of Economic Theory*, 145(5):1704–1723, 2010.

John William Hatfield and Paul R Milgrom. Matching with contracts. *The American Economic Review*, 95(4):913–935, 2005.

Roger B Myerson. Optimal coordination mechanisms in generalized principal–agent problems. *Journal of mathematical economics*, 10(1):67–81, 1982.

Tayfun Sönmez. Minimalist market design: A framework for economists with policy aspirations. *arXiv preprint arXiv:2401.00307*, 2023.

Richard Zeckhauser. Strategic sorting: the role of ordeals in health care. *Economics & Philosophy*, 37(1):64–81, 2021.

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