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DISCUSSION PAPER

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Mobilizing Credit for Clean Energy: De-risking and Public Loan Provision Under Learning Spillovers

Mobilizing Credit for Clean Energy: De-Risking and Public Loan Provision Under Learning Spillovers*

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Abstract

Policymakers regularly rely on public financial institutions and government bodies to provide loans to clean energy projects. However, the market failures that public loan provision addresses and the role it can play in a policy strategy that also features de-risking measures, such as interest rate subsidies, remain unclear. Here, we develop a model of banks providing loans to clean energy projects that use a novel technology. Early-stage loans build up financing experience that spills over to peers and hence is undersupplied by the market. In addition to this cooperation problem, bankability requirements can result in a coordination failure where the banking sector remains stuck in an equilibrium with no loans for the novel technology, although a preferable equilibrium with loans exists. Public provision of early-stage loans is inferior to de-risking instruments when solving the cooperation problem because it crowds out private banks' loan provision. However, public loan provision can more effectively resolve the coordination failure by pushing the banking sector to a better equilibrium, ideally in combination with additional de-risking measures to internalize learning spillovers.

Keywords: Energy transition, state investment bank, government loans, credit guarantees, multiple equilibria.

JEL Codes: G21, H81, Q48, Q55.

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1 Introduction

To mitigate dangerous climate change, investments in clean energy technologies have to grow considerably (IEA, 2022; Klaaßen & Steffen, 2023). The magnitude of necessary investments requires the mobilization of private sector financing (IPCC, 2022), including large amounts of debt for capital-intense technologies such as renewables. To achieve clean energy investment at a societal optimal level, economic theory suggests a combination of carbon pricing and research subsidies, internalizing the climate externality and the effect of knowledge spillovers (Acemoglu et al., 2012; Borenstein, 2012). Further, technology-specific subsidies can be an alternative if carbon pricing is not available (Abrell et al., 2019). In practice, however, policymakers regularly opt for financial measures to de-risk clean energy financing, such as interest rate subsidies or credit guarantees, and also provide debt finance directly to projects through public financial institutions or government bodies.

These financial measures are typically used in addition to other policy instruments that already address the climate externality. In the United States, for example, the Department of Energy's Loan Programs Office has been providing loans and credit guarantees to utility-scale clean energy projects, and the recently established USD 14 billion National Clean Investment Fund will capitalize national clean finance institutions under the Inflation Reduction Act (US EPA, 2023). In other OECD countries, state investment banks are remarkably active in renewable energy lending—particularly for higher-risk technologies such as offshore wind, where they feature in over 70% of all debt financing deals over the past two decades (Waidelich & Steffen, 2023). In addition, an increasing number of governments around the world have created public green banks that provide loans and de-risking measures (Whitney et al., 2020), including countries that typically hesitate to intervene much in financial markets, such as the United Kingdom.

However, despite the widespread use of public loan provision for the clean energy transition, the economic literature on the rationale for adding it to a policy strategy is sparse and has provided little guidance on when to favor the direct market activity of public banks over de-risking instruments. Previous sector-agnostic studies mainly discuss both policies in light of credit rationing arising from adverse selection and screening costs or unconsidered social externalities (Eslava & Freixas, 2021; Hainz & Hakenes, 2012; Williamson, 1994), from moral hazard for borrowers (Arping et al., 2010) and cyclical credit crunches (Eslava & Freixas, 2021; Mazzucato & Penna, 2016), or from adverse incentives due to legacy portfolios (Degryse et al., 2020; Minetti, 2011). Far less emphasis has been placed on the role of financiers *learning* about novel clean energy technologies through lending. By contrast, learning-by-doing processes at the technology level are increasingly prevalent in economic theory (Thompson, 2012) and numerical modeling (Gillingham et al., 2008). They are typically modeled via unit costs that decrease in cumulative production experience, which potentially spills over to competitors (Schauf &

Schwenen, 2021; Spence, 1981).

Empirical work has extended these concepts to clean energy financing, showing that increases in cumulative financing and the corresponding experience for financiers have coincided with substantial reductions in the cost of capital for solar photovoltaics and onshore wind (Egli, 2020; Egli et al., 2018). However, hitherto we lack a theoretical understanding of what this implies for optimal *policy* to mobilize financing for the clean energy transition. In particular, the existing literature lacks clarity on the need for financial policy measures if other policy interventions already sufficiently address technology-level and consumer-level market failures, such as climate externalities, knowledge spillovers, or lack of demand due to bounded rationality (Borenstein, 2012; Popp, 2019).

To address this gap, this paper investigates the potential and limitations of public loan provision and de-risking measures by developing a model of loans to clean energy projects using a novel technology that the banking sector is not (yet) familiar with. This accurately depicts the situation on the project loan market in key sectors for the energy transition, such as offshore wind and energy storage, in many regions. In the model, risky early-stage loans build up the banking sector's experience with the novel technology and thus improve future risk-adjusted returns by lowering uncertainties and transaction costs. Hence, early-stage credit to the novel technology causes a positive externality to other lenders, which results in two different market failures. First, uninternalized learning spillovers imply a *cooperation problem* between banks and lead to an undersupply of early-stage credit. Using public loan provision to address this problem, however, is inferior to de-risking instruments because it reduces the willingness of commercial banks to incur early-stage risk themselves and hence crowds out the private loan provision. Second, minimum risk-return requirements for a project to be "bankable" can result in a *coordination failure* where the banking sector remains stuck in a Nash equilibrium with no loans for the immature technology, although a better market equilibrium in which the novel technology receives loans is, in principle, possible. In this case, a sufficiently sized public loan provider, for example, in the form of a public green bank, can push the banking sector to a better equilibrium, particularly if combined with additional de-risking policies to internalize learning spillovers to other banks.

Therefore, this paper extends the argument by Rodrik (1996) that "when multiple equilibria exist, the role of government policy is to move the economy out of the bad equilibrium into the good one" to financial policies and public loan provision to novel technologies in particular. Importantly, our model does not require any market failures on the technology and consumer level to justify the policy intervention, thus clarifying the role of public loan provision and de-risking measures in a climate policy strategy that already features instruments such as carbon pricing and R&D subsidies.

The remainder of this paper is structured as follows. Section 2 summarizes the extant literature and clarifies the research gap we address through our model, whose general framework is introduced in Section 3. Section 4 compares the socially optimal loan financing amount to the market outcome without policy intervention before Section 5 introduces a de-risking instrument and public loan provision as two stylized policy options to address potential market failures. Finally, Section 6 discusses the policy implications of our findings and concludes.

2 Literature review

Clean energy technologies require substantial upfront financing due to their high capital intensity (Borenstein, 2012). Therefore, their cost-competitiveness for large-scale deployment strongly depends on their cost of capital (Hirth & Steckel, 2016; Stocks, 1984), which can be reduced substantially through debt financing (Schmidt et al., 2019)—particularly if higher leverage ratios can be obtained by using project finance (Steffen et al., 2018). However, this requires bank loans as long as technologies and firms have not matured sufficiently to tap bond markets (Berger & Udell, 1998). In consequence, accessible credit is key for the ramp-up of these technologies. However, it may be rationed due to financial market frictions (Stiglitz, 1993) and remaining externalities at the technology level (Popp, 2019). Indeed, there is empirical evidence that emerging clean energy technologies face financing constraints (cf. Haas and Kempa, 2023, for an overview).

Modern banking theory has extensively studied the potential of credit guarantees or interest rate subsidies to mitigate inefficient credit rationing (Arping et al., 2010; Hainz & Hakenes, 2012; Janda, 2011; Minelli & Modica, 2009; Philippon & Skreta, 2012). These insights on de-risking measures have been extended to the case of low-carbon technologies (Haas & Kempa, 2023), but there is less theoretical clarity about which role the public provision of loans to clean energy projects is supposed to play, if any. The extant literature is primarily centered around public (green) banks—which typically engage both in loan provision and de-risking (Eslava & Freixas, 2021; Whitney et al., 2020)—and suggests various reasons for how these institutions could limit the extent of credit rationing for low-carbon technologies.

One suggestion is that public loan programs and development banks provide counter-cyclical finance in times of credit crunches (Eslava & Freixas, 2021; Mazzucato & Penna, 2016). For the energy sector specifically, however, this notion has not been empirically confirmed (Waidelich & Steffen, 2023), and the question remains why economy-wide credit crunches should be addressed by sector-specific policy interventions instead of general counter-cyclical fiscal and monetary policy. Studies further cite high risk premia and discount rates of private banks as a rationale for public loan provision (Lehmann & Söderholm, 2018; Mazzucato & Penna, 2016). From an efficiency point of view, though, the preferences of market players per se cannot represent a market failure. Hence, this argument either requires remaining externalities at the technology level or the assumption that the optimal social discount rate is lower than the rate applied by private-sector financiers. Furthermore, the argument abstracts from well-established reasons why banks act more risk-averse than other types of investors, such as regulatory capital requirements or the risk of having to raise external finance due to unexpected deposit withdrawals (Diamond & Dybvig, 1983; Froot & Stein, 1998), while they, unlike equity investors, cannot participate in any project upsides and hence focus on mitigating default risks.

Another strand of literature argues that novel technologies threaten the value of banks' legacy positions and their information stock on incumbent technologies, calling for new institutions with a clean slate (Degryse et al., 2020; Minetti, 2011). This argument primarily motivates sufficient anti-trust policies for the banking sector and underlines the potential benefit of a new entrant bank. However, it provides little reason why the new bank should be public, particularly since dedicated green commercial banks are often important first movers for clean energy technologies (Zhang, 2020). More sector-agnostic studies have motivated the need for public loan provision based on two issues. First, the existence of projects with a negative net present value that are socially desirable—although, in the case of clean energy, this might be better addressed through first-best policies outside the financial sector; second, information asymmetries in the form of inefficiently low screening efforts if borrower types are unknown to banks and screening is costly, but project screening outcomes are observable to competitors (Eslava & Freixas, 2021; Hainz & Hakenes, 2012; Williamson, 1994).

Similar to this screening benefit argument, Geddes et al. (2018) highlight that, aside from providing loans and de-risking investments, public green banks often educate markets on novel technologies and provide strong signals on their economic viability. This behavior is motivated by the fact that novel technologies are not only subject to technological learning but also improve their risk-return profile as financing experience accumulates. Reasons for this include that an expanding credit track record reduces banks' uncertainty about the default probability of projects (Egli et al., 2018) and that more experienced debt providers can extract more value from pledged collateral, which reduces losses in case a borrower defaults (Minetti, 2011). More experience will also enable lenders to identify more relevant loan covenants and to reduce the transaction costs per loan since application reviews can be streamlined and contracts can be standardized (Umbeck & Chatfield, 1982). In the case of syndicated loans in project finance, the predominant financing structure for renewable energy technologies (IRENA, 2023), experience further allows for the standardization of deal structures, the conclusion of frame contracts, and the emergence of proven networks comprising financiers and financial/technical/legal advisors – reducing both transaction costs and necessary risk contingencies (Egli et al., 2018; Gatti, 2013). These findings from empirical interview studies highlight the need for a rigorous theoretical consideration of market failures and the need for policy if technology-level and consumer-level externalities have been sufficiently priced in.

Therefore, we formalize these considerations into a model for bank loans and account for learning spillovers by building on a recent strand of literature that incorporates learning effects into models of individual investors' technology investment decisions. In particular, Della Seta et al. (2012) model a novel technology whose marginal costs decrease in cumulative output and find that optimal investment involves significant initial losses that are compensated by later-stage gains, making the technology particularly prone to downside risk. Their model is extended by Sarkar and Zhang (2020), who introduce the option of debt-financing, which leads to more and earlier investment. They conclude that unless there are exogenous borrowing constraints, the optimal gearing ratio is higher if costs decrease faster in cumulative output. Moreover, Way et al. (2019) explore the optimal portfolio allocation between investments in two technologies under stochastic learning rates and risk aversion. Their model produces a trade-off between specializing in one technology to drive down costs and diversifying to hedge against downsides and requires numerical optimization to be solved as the learning feedback introduces multiple local optima. Finally, Lehmann and Söderholm (2018) review theoretical rationales for renewable energy support schemes in a partial equilibrium framework, including technological learning where second-period costs decrease convexly in first-period output. Among other things, they suggest that a subsidy scheme can also overcome financial market failures caused by inefficiently high risk aversion and discount rates by a private investor.

While these previous modeling studies of technology investment decisions take the perspective of a single equity investor, we study the interplay between multiple debt providers, including learning spillovers. By doing so, our paper suggests another important reason for credit rationing: a coordination failure between borrowers to gain sufficient experience with a novel clean energy technology, adding to the extensive literature on credit rationing. In this regard, our work is related to Haas and Kempa (2023), who explain credit rationing for clean energy technology firms with information asymmetries and unobservable project characteristics that can be addressed via de-risking. However, their model does not endogenize risks or financing experience. Therefore, neither their model nor, to the best of our knowledge, any other paper formally assesses public loan provision as a policy instrument and its role relative to de-risking measures in the context of learning effects.

3 General framework

We consider a two-period financial sector model that is populated by a discrete number N of banks. Banks are homogeneous, and in each period, face loan applications by projects using a novel clean energy technology. $l_{i,t}$ represents the overall amount of loan financing granted by bank i in period t, which is financed via deposits. We assume that the desired capacity expansion in the new (low-carbon) technology, and hence the total demand for loans, is determined exogenously by policy interventions in the energy sector (e.g., renewable portfolio standards, renewable energy auctions, or carbon prices) and denoted as D for the first period. In contrast, in the second period, it increases by an exogenous factor $\psi > 1$. To abstract from the issue of banking sector concentration, which has been studied extensively elsewhere (cf. Freixas and Rochet, 2023, for an overview), demand is allocated symmetrically across banks, such that

$$l_{i,1} \in [0, \frac{D}{N}], l_{i,2} \in [0, \frac{\psi D}{N}] \;\; \forall \;\; i = 1, ..., N \tag{1}$$

The two-period setup is motivated by two factors. First, the common bifurcation in financial markets, where technologies are either too novel (and hence risky) for debt finance or mature enough for debt finance, i.e., "bankable"; second, the fact that deployment in novel technologies, particularly under continued policy support, can ramp up considerably, which is represented by ψ. In our model, each period should be considered as representing multiple years such that loans are paid out at the beginning of each period and paid back with interest by the end of it. In the first, "early-stage" period, the novel clean energy technology is still financially immature and hence risky. However, its risk-return structure can improve in the second, "later-stage" period. Therefore, on every unit of early-stage loans $l_{i,1}$, bank i earns the following risk-adjusted net return

$$r - \bar{c} - r_D \tag{2}$$

where r denotes the risk-adjusted return that banks can earn on loans at full financial maturity.[1] The primary source of risk is each project's probability of default, which we do not model explicitly. Instead, we assume that the risk-adjusted return r increases monotonously in the expected return and decreases monotonously in the return variance and the banks' degree of risk aversion. $\bar{c}$ represents a strictly positive, constant penalty on the risk-adjusted return due to financial immaturity, comprising the risk premium related to novel technology and the higher screening costs due to a lack of experience assessing credit applications. r_D denotes the rate paid out to compensate deposit holders.

On every unit of later-stage loans $l_{i,2}$, bank i earns the following return

$$r - c\left(\tilde{L}_{i,1}\right) - r_D \tag{3}$$

where

[1]Here, we assume that there is no price feedback between the aggregate loan supply and the interest rate paid by projects. Relaxing this assumption for, say, a linear demand curve instead would effectively turn our model into a symmetric two-stage Cournot game, where, if N is finite, interest rate concerns further depress the number of loans that each bank is willing to supply.

$$\tilde{L}_{i,1} \coloneqq l_{i,1} + \gamma \sum_{j \neq i} l_{j,1} \tag{4}$$

denotes the financing experience from the first period gained by bank i through their own loan financing and the financing provided by their peers.[2] Therefore, early-stage loans to the novel technology at $t = 1$ cause a positive experience externality to other banks by improving their later-stage risk-adjusted return at $t = 2$. Without early-stage loan financing by any bank, no learning gains are realized, i.e., $c(0) = \bar{c}$. Due to diminishing returns to experience, we further assume that c decreases convexly in $\tilde{L}_{i,1}$ but remains non-negative.[3]. Learning spillovers between banks are imperfect, which is represented by $\gamma \in (0, 1)$ (cf. Fischer and Newell, 2008). A higher value of γ can denote that banks are more transparent about their financing experience, that their absorptive capacity is higher, or that they regard their peers as more competent and, hence, the financing decisions made by other banks as more instructive.

In this paper, we investigate market failures and policy interventions for novel clean energy technologies that have sufficient potential to become profitable from a lender's perspective at a later stage—but are not immediately attractive at an early stage due to lack of experience. Therefore, we assume a negative spread between the risk-adjusted return on loans at full financial immaturity and the deposit rate:

$$r < \bar{c} + r_D \tag{5}$$

By contrast, if all banks provide the full amount of early-stage financing, the spread would turn positive such that

$$r > c\left(D\frac{\tilde{N}}{N}\right) + r_D. \tag{6}$$

where

$$\tilde{N} \coloneqq 1 + \gamma(N - 1) < N. \tag{7}$$

The term $\frac{\tilde{N}}{N} < 1$ accounts for the loss of financing experience due to imperfect spillovers. As a result, the risk-adjusted return in the second, later-stage period $r - c(\tilde{L}_{i,1})$ is concavely increasing in $\tilde{L}_{i,1}$ and bounded between $r - \bar{c}$ and r, as displayed in Figure 1.

[2] Note that we use capitalized L for aggregates of loan amounts across banks and lowercase l for loan amounts of individual banks.

[3] Our restrictions that $c \geq 0, c' < 0, c'' > 0$ nest the most common functional forms for technological and financial learning curves in the literature (Della Seta et al., 2012; Egli et al., 2018; Samadi, 2018; Thompson, 2012).

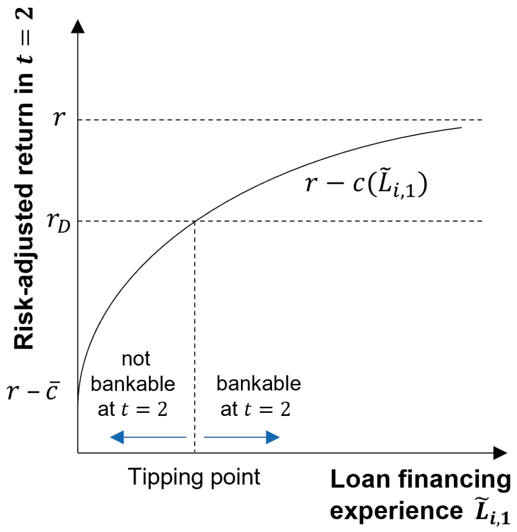

Figure 1: Risk-adjusted return on loans in $t = 2$

Combining the considerations above, the risk-adjusted profits of bank i discounted to $t = 1$ can be written as

$$\pi_i(l_{i,1}, l_{i,2}, \tilde{L}_{i,1}) = \underbrace{(r - \bar{c} - r_D)\, l_{i,1}}_{\text{Early-stage losses}} + \underbrace{\beta \left(r - c(\tilde{L}_{i,1}) - r_D \right) l_{i,2}}_{\text{Potential later-stage gains}} . \tag{8}$$

where $\beta \in (0, 1)$ is the discount factor common to all banks. To strike a profit, early-stage losses must be compensated by later-stage gains. Therefore, bank i will only provide loans to the novel technology at $t = 2$ if the financing experience from the first period $\tilde{L}_{i,1}$ is sufficiently high to push the immaturity-related cost premium $c(\tilde{L}_{i,1})$ below $r - r_D$. If this is the case, we will refer to the novel technology as being "bankable" at $t = 2$. In consequence, the loan decision of bank i at $t = 2$ only depends on whether the financing experience gained at $t = 1$ renders the novel technology bankable – and causes no externality to other banks. To avoid situations where banks are indifferent between outcomes, we assume throughout the paper that if two outcomes yield the same risk-adjusted return or profits, banks strictly prefer the one with less loan financing. This then gives us the following simple rule for the later-stage loan financing at $t = 2$:

Lemma 1. *Let $(l_{i,1}, l_{i,2})$ be the loan financing amounts for any bank i. Then*

$$l_{i,2} = \begin{cases} 0 & \text{if } r - c(\tilde{L}_{i,1}) \leq r_D \\ \psi \frac{D}{N} & \text{otherwise} \end{cases} \tag{9}$$

Proof. Combine Equation 20 in Appendix B.1 with the assumed strict preference for no loans if the return spread is exactly zero. □

Hence, in the later-stage period, banks either finance the technology's entire loan demand if the early-stage financing experience provides a positive return spread or refrain from any loan financing at $t = 2$.

Notably, our model is populated by banks only and hence does not feature any externalities at the technology or the consumer levels. This serves to clarify if and why market failures in the banking sector can arise even if other market failures are already addressed. However, our model can be easily extended to incorporate additional externalities that bank financing might have if project sponsors are unwilling to move forward without bank loans.

4 Social optimum and market outcome

In our model, the socially optimal solution maximizes the sum of present-value profits over all banks:

$$\max_{\{l_{i,1},l_{i,2}\}_{i=1}^{N}} \sum_{i\in\mathcal{N}} \pi_i(l_{i,1}, l_{i,2}, \tilde{L}_{i,1}) \qquad \text{s.t. } l_{i,1} \in [0, \frac{D}{N}], l_{i,2} \in [0, \psi\frac{D}{N}] \tag{10}$$

The full Karush-Kuhn-Tucker conditions are provided in Appendix B and reflect that in the unconstrained optimum, $l_{i,1}$ should be chosen to equate the return spread at $t = 1$ and the marginal learning gain such that

$$-\beta(\underbrace{c'(\tilde{L}_{i,1}^{SO})l_{i,2}^{SO}}_{\substack{\text{Learning gain}\\ \text{to bank } i}} + \underbrace{\gamma\sum_{j\neq i} c'(\tilde{L}_{j,1}^{SO})l_{j,2}^{SO}}_{\text{Learning gain to peers}}) = \underbrace{\bar{c} + r_D - r}_{\substack{\text{Initial return spread}\\ (=\text{ loss})}} \tag{11}$$

which might not hold in the constrained optimum if the demand or non-negativity constraint on $l_{i,1}$ binds. Note that the left-hand side of Equation 11 is positive since $c' < 0$.

An asymmetric solution to the optimization problem in Equation 10 cannot be ruled out entirely but significantly limits the analytic tractability of our model. Therefore, we impose symmetry on the social optimum, as is common in the literature (Eslava & Freixas, 2021). This means that

$$l_{i,t}^{SO} = l_t^{SO} \; \forall \; i = 1, ..., N \tag{12}$$

which by Lemma 1 also implies symmetry in the later-stage loan financing. This is a mild assumption because banks are homogeneous *and* because the profit of each bank $\pi_i(\cdot)$ is strictly concave in $\tilde{L}_{i,1}$. For this reason, allocating early-stage loan financing amounts asymmetrically between firms (which leads to a heterogeneous financing experience $\tilde{L}_{i,1}$ as $\gamma < 1$) is typically dominated by a symmetric allocation. Combining Lemma 1 with the first- and second-order conditions then results in the following proposition:

Proposition 1. *Let* $l_1^* := \tilde{N}^{-1}(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\beta\frac{\tilde{N}}{N}\psi D}\right)$ *denote the unconstrained symmetric solution to the first-order condition in Equation 11 under symmetry and* $(c')^{-1}(\cdot)$ *denote the inverse function of* c'.[4] *Then, the unique symmetric social optimum* (l_1^{SO}, l_2^{SO}) *is either*

- *an immediate financing scenario, in which every bank provides the full loan financing amount in the first and second period (*$\frac{D}{N}$ *and* $\psi\frac{D}{N}$*, respectively), or*
- *a gradual financing scenario, in which each bank provides* $l_1^* < \frac{D}{N}$ *at* $t = 1$ *before providing full loan financing at* $t = 2$*, or*
- *a no-financing scenario, in which banks do not provide any loan financing in either period.*

For the gradual or immediate financing scenario to exist, each bank must strike strictly positive present-value profits.

Proof. See Appendix B. □

Here, l_1^* denotes the loan financing amount for which marginal learning gains and the initial return spread balance prior to any demand or non-negativity constraints. At the asset level, comparative statics (see Appendix B) reveal that a more favorable risk-return profile, i.e., a higher risk-adjusted return at full maturity r and a lower initial immaturity penalty $\bar{c}$, make a no-financing optimum less likely and increase the socially optimal early-stage financing l_1^{SO}. At the financier level, the same holds if the initial loan demand by projects using the novel technology is higher ($D \uparrow$) and grows more strongly in the second period ($\psi \uparrow$), which increases the scope for learning effects, or if deposits are cheaper ($r_D \downarrow$). The socially optimal l_1^{SO} is also higher if banks are more patient ($\beta \uparrow$) and less secretive or more capable of learning from peers ($\gamma \uparrow$). However, a less concentrated banking sector ($N \uparrow$) will decrease the loan financing in the optimum because, ceteris paribus, this implies more spillover losses of financing experience as long as $\gamma < 1$.

Since the social optimum is symmetric and by Lemma 1, $l_{i,2}$ is a binary function of $l_{i,1}$, we can plot total profits $\sum_i \pi_i(\cdot)$ for all symmetric allocations as a function of $\tilde{L}_1 = \tilde{N}l_1$. In the left panel of Figure 2, we show this for the gradual-financing optimum (i.e., for a scenario under which total profits peak above zero for some $\tilde{L}_1 < \frac{\tilde{N}}{N}D$). For very low amounts of financing at $t = 1$, the risk-adjusted return on loans at $t = 2$ remains below r_D such that by Lemma 1, banks do not grant any loans and hence make zero profits from the new technology in the second period. At the same time, profits at $t = 1$ decrease linearly since for every unit of l_1, each bank loses the initial return spread $r_D + \bar{c} - r$. Therefore, small values of early-stage loan financing that are insufficient to render loans bankable at a later stage reduce overall profits below zero.

[4] Note that c' is monotonously increasing, and hence $(c')^{-1}$ exists and is monotonously increasing.

If l_1 increases further, however, the risk-adjusted return at $t = 2$ at some point equals the deposit rate r_D (blue line). Beyond this point, loans become profitable at $t = 2$, and hence banks will meet the entire loan demand such that $l_2 = \psi \frac{D}{N}$. Note that profits increase concavely because returns on financing experience are diminishing as c is convex. However, the positive profits at $t = 2$ do not immediately offset the incurred losses at $t = 1$. Hence, it takes some additional increase in l_1, i.e., further learning gains, until banks break even in present-value terms (grey line). As long as the marginal return on l_1 (i.e., the marginal learning gain to all banks plus $r - \bar{c}$) exceeds r_D, a higher l_1 increases profits further until marginal return and deposit rate equal in the social optimum (green line). Beyond this point, the marginal learning gain no longer compensates for the early-stage losses, and profits fall again.

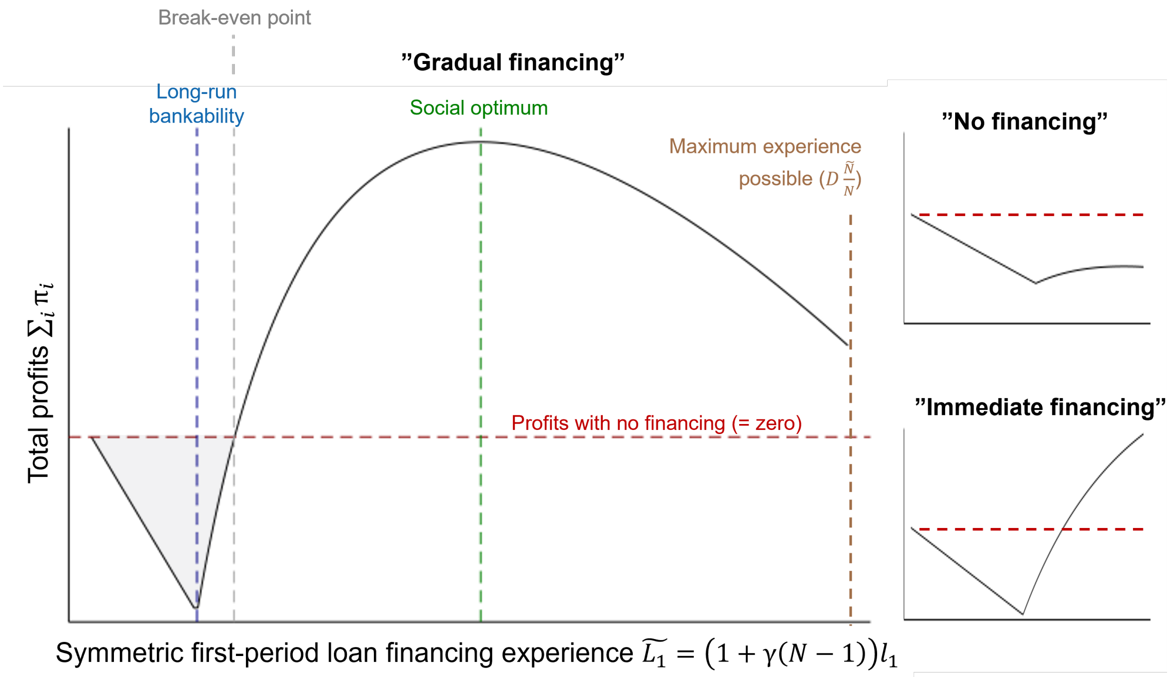

Figure 2: Aggregate bank profits over early-stage loan financing

Visually speaking, the gradual-financing optimum displayed in Figure 2 exists if the concave section of $\sum_i \pi_i(\cdot)$ peaks within the banking sector's available resources (for some $\tilde{L}_1 < \frac{\tilde{N}}{N}D$) and above zero. The other two potential optima in Proposition 1 have equally straightforward interpretations and are displayed in the right panel of Figure 2. The no-financing scenario is optimal if the concave section does not exceed zero for any $L_1 \in [0, \frac{\tilde{N}}{N}D]$. The immediate-financing optimum requires that the concave section only peaks after $\frac{\tilde{N}}{N}D$, but that total profits at this point already exceed zero.

By contrast, in a market outcome, each individually rational bank carries out the following profit maximization:

$$\max_{l_{i,1}, l_{i,2}} \pi_i(l_{i,1}, l_{i,2}, \tilde{L}_{i,1}) \qquad \text{s.t. } l_{i,1} \in [0, \frac{D}{N}], l_{i,2} \in [0, \psi \frac{D}{N}] \tag{13}$$

The first-order conditions, given in Appendix C.1, are similar to the one given in Equation 11—except that banks do not take into account how their own early-stage financing improves the later-stage risk-adjusted return for their peers.

However, the solution to the maximization problem of bank i still depends on their peers' behavior. For a fully fragmented banking sector, i.e., if $N \to \infty$, it is trivial to show that bank i's contribution to its own financing experience stock $\tilde{L}_{i,1}$ becomes negligible unless $\gamma \to 0$. Note that early-stage loans in the model come at a loss in $t = 1$ and can only be profitable through their impact on $\tilde{L}_{i,1}$ and hence on profits in $t = 2$. Therefore, for $N \to \infty$ where $l_{i,1}$ has no meaningful impact on $\tilde{L}_{i,1}$, the only possible market outcome is $(l_{i,1}, l_{i,2}) = (0, 0)\ \forall\ i = 1, ..., N$. However, banking sectors typically do not exhibit this perfect degree of competition (Freixas & Rochet, 2023; Stiglitz, 1993). Therefore, we consider a finite number of N, solve for possible Nash equilibria, and arrive at the following result:

Proposition 2. *The set of Nash equilibria under the market outcome can be characterized as follows:*

- *The possible Nash equilibria are all symmetric and feature a no-financing equilibrium, a gradual-financing equilibrium where each bank provides* $l_1^{*NE} := \tilde{N}^{-1}(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\beta\psi\frac{D}{N}}\right) < \frac{D}{N}$ *at* $t = 1$*, and an immediate-financing equilibrium.*
- *If the gradual-financing equilibrium exists, the immediate-financing equilibrium does not exist, and vice-versa. Both require strictly positive profits for each bank to exist.*
- *Both the gradual-financing and the immediate-financing equilibrium can co-exist with the no-financing equilibrium.*
- *The early-stage loan provision in any Nash equilibrium is strictly lower than the social optimum—except for the trivial case, in which both the social optimum and the Nash equilibrium are immediate financing or no financing.*

Proof. See Appendix C. □

The key intuition for the symmetry of the Nash equilibrium can be illustrated as follows: Consider the simplified case of only two banks i and j and an interior solution, and assume for contradiction that a Nash equilibrium with $l_{j,1} > l_{i,1}$ exists. This implies that bank j's learning experience $(l_{i,1}+\gamma l_{j,1})$ is greater than bank i's by exactly $(1-\gamma)(l_{j,1}-l_{i,1})$. However, note that i's and j's marginal learning gains must be equal in the optimum because both banks are homogeneous and face the same marginal first-period losses. Since marginal learning gains are strictly decreasing, that means that both banks' first-period learning experience must be identical, which requires that $l_{j,1} = l_{i,1}$ since $\gamma < 1$ (i.e., we have imperfect spillovers).

Notably, the closed-form expressions for the potential gradual-financing social optimum l_1^* and the gradual-financing Nash equilibrium l_1^{*NE} are almost identical. However, the latter features only the individual loan amount at $t = 2$ (i.e., $\psi\frac{D}{N}$) and not the overall loan amount net of spillover losses (i.e., $\frac{\tilde{N}}{N}D$). As a result, l_1^{NE} is weakly, but not strictly lower than l_1^{SO} because if no financing is socially optimal, this is the outcome the market will provide. In addition, it could theoretically be that the risk-return structure is so beneficial that immediate financing is not only the social optimum but a Nash equilibrium as well. However, the policy implications of such a setting extrapolate well from the more relevant setting in which only a gradual financing equilibrium exists, with the main exception that there is less of a rationale for de-risking measures. For this reason, we place less emphasis on the case where immediate financing is both the social optimum and a market equilibrium in the following discussion of market failures and policy instruments.

Since the market outcome must be symmetric, the conditions in Proposition 2 under which the different Nash equilibria exist have straightforward visual interpretations. We display the possible market outcomes as well as bank i's best response function under a gradual-financing social optimum in Figure 3 below. The no-financing equilibrium (left yellow ring) exists unless a single bank i can push beyond the "no-financing valley" and obtain positive profits by unilaterally providing loans to the novel technology at $t = 1$.[5] Therefore, the best response for bank i, if no other bank provides early-stage loans, is to forego loan financing as well, which is illustrated by the best response function in the lower panel. However, even if no financing is a possible Nash equilibrium, there might exist *another* equilibrium at $\tilde{L}_{i,1} = \tilde{N}l_1^{*NE}$ if and only if this point falls beyond the no-financing valley and provides above-zero profits. Once above-zero profits are in reach for bank i given the behavior of the other banks, the best response switches to providing early-stage loans until the (cumulative) learning experience reaches $\tilde{L}_{i,1} = \tilde{N}l_1^{*NE}$. Beyond that point, the deposit rate exceeds the marginal return on l at $t = 1$, excluding learning spillovers. As a result, bank i will no longer provide any early-stage loan financing, but it will still free-ride the other banks' financing experience by financing $l_{i,2} = \psi\frac{D}{N}$ in the second period. If the point $\tilde{L}_{i,1} = \tilde{N}l_1^{*NE}$ falls within the no-financing valley or violates the non-negativity constraint on l, then the gradual-financing Nash equilibrium does not exist because every bank would be better off by switching to the no-financing equilibrium instead.

[5]Note that Figure 3 rests on the assumption that all banks behave symmetrically, so the valley for such a unilateral financing provision is somewhat shorter because in this case, there would be no spillover losses of financing experience. In addition, the valley displayed here refers to bank financing for (large-scale) deployment and hence does not represent the conventional "valley of death" for the transition between laboratory and commercialization (Popp, 2019).

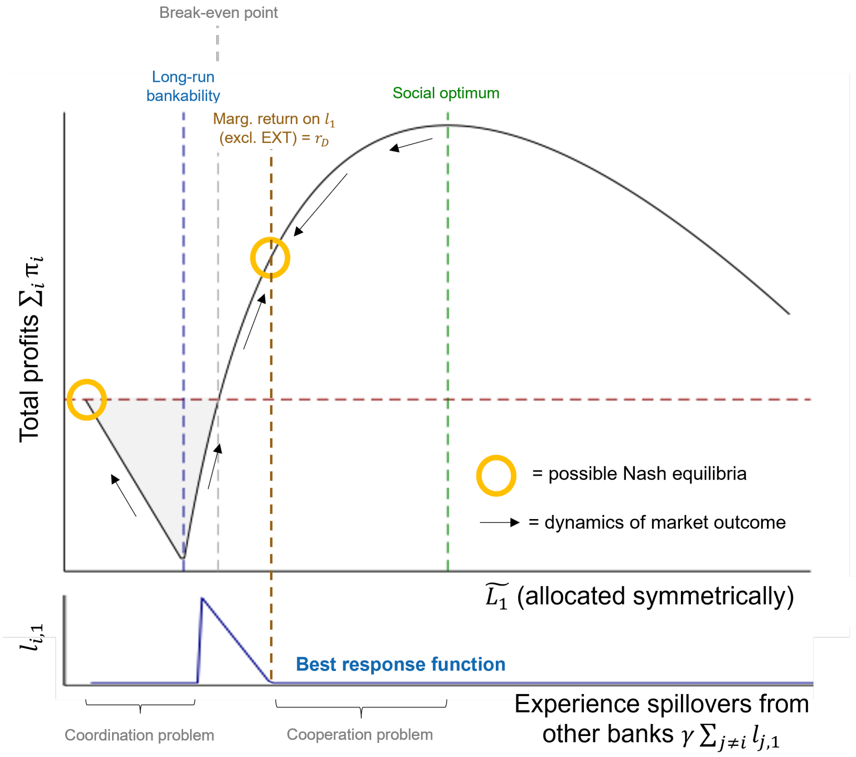

Figure 3: Possible Nash equilibria and best response function

Proposition 2 has several important implications regarding market failures in our model. First, if no financing is socially optimal, then no market failure exists. If gradual financing is optimal, then we can have two separate elements of market failures: First, a *cooperation problem* because banks ignore positive learning spillovers to their peers and hence choose a suboptimally low amount of early-stage loan financing (as visualized in Figure 3 above). Second, a potential additional *coordination failure* because even if a gradual-financing market outcome exists (which by definition must be profitable for every bank), the banking sector might remain stuck in the inferior no-financing equilibrium.

Since the closed-form expression for l_1^{*NE} is very similar to the one of the socially optimal l_1^*, the comparative statics for the gradual-financing social optimum similarly apply to the early-stage financing under the market outcome. More favorable conditions at the asset ($r \uparrow$, $\bar{c} \downarrow$) and financier level ($D \uparrow$, $\psi \uparrow$, $N \downarrow$, $r_D \downarrow$, $\beta \uparrow$, $\gamma \uparrow$) make it more likely that a gradual-financing Nash equilibrium exists and increase the financing amount in such a market outcome. Regarding the potential early-stage financing gap between the social optimum and the market outcome, we conclude as follows:

Lemma 2. *Let the social optimum be* $0 < l_1^{SO} < \frac{D}{N}$ *(gradual financing). Then, the minimum early financing gap between the market outcome and the social optimum is as follows:*

$$\frac{l_1^{SO}}{l_1^{*NE}} = \frac{(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\beta\psi\frac{D}{N}\tilde{N}}\right)}{(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\beta\psi\frac{D}{N}}\right)} \tag{14}$$

Ceteris paribus, the minimum early financing gap increases monotonously in γ.

Proof. Combine the expressions from Propositions 1–2 and take the partial derivative, keeping in mind that $(c')^{-1}$ is monotonously increasing and $\tilde{N} := 1 + \gamma(N-1)$. $\square$

Therefore, the gap between the market outcome and the gradual-financing optimum is higher if more of a bank's learning gains spill over to competitors or if competitors are more capable of absorbing these spillovers ($\gamma \uparrow$). In addition, if the marginal learning gain c' decreases more steeply in the cumulative financing experience, this reduces the financing gap because if returns to experience diminish more rapidly, then taking learning spillovers into account or not makes less of a difference.[6] However, these comparative statics only hold locally for limited changes in the given parameters since larger changes could also render the gradual-financing outcome sub-optimal from a societal point of view.

As discussed in Section 2, some papers have suggested that (inefficiently) high discount rates of private actors might prevent clean energy technologies from being financed (Lehmann & Söderholm, 2018; Mazzucato & Penna, 2016). While we focus on financing experience and the resulting coordination and cooperation problems here, we note that such discount rate considerations are easily integrated into our framework by assuming that banks use a discount factor $\phi\beta$ where $\phi \in (0,1)$, while β denotes the social discount factor. This would alter Equation 14 as follows:

$$\frac{l_1^{SO}}{l_1^{*NE}} = \frac{(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\beta\psi\frac{D}{N}\tilde{N}}\right)}{(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\phi\beta\psi\frac{D}{N}}\right)} \tag{15}$$

This expression shows that the minimum early financing gap between market outcome and social outcome increases in the time preference discrepancy between banks and society overall ($\phi \downarrow$).

5 Policy interventions

The previous section has established that if the social optimum is a gradual-financing (immediate-financing) outcome, the market outcome will (can) feature an inefficiently low provision of loan financing at $t = 1$ and might even fail to provide any loan financing. We consider two different policy interventions within the framework of our model to address this market failure. First, the government can improve the risk-adjusted return for banks, either by increasing the expected return of the loans or by reducing the volatility of returns (Polzin et al., 2019). Two of the most commonly discussed instruments to do so in the literature are interest rate subsidies and credit guarantees (Haas & Kempa, 2023). Due to

[6] The financing gap also decreases for a more concentrated market ($N \downarrow$) because if fewer peers benefit from spillovers, the positive externality is lower. However, this obviously increases the scope for competition-related market failures, from which our model set-up abstracts.

our framework of risk-adjusted returns, we can represent both of these options as stylized state-financed additive premia on the risk-adjusted return of all private banks in both periods denoted by $s_1, s_2 \geq 0$.[7]

Second, the government can provide loans directly to projects using the novel technology in both periods, with loan amounts denoted by $g_1, g_2 \geq 0$. Importantly, public loan provision also generates financial experience at $t = 1$ that partially spills over to the private banks at a rate $\gamma^g > 0$. Public green banks are known for actively sharing their expertise with the private sector (Geddes et al., 2018), which would imply $\gamma^g > \gamma$. However, γ^g might also be lower than γ if the public loan provider is perceived as less competent than a commercial bank, thus making banks hesitant to learn from the public sector's lending track record. In addition, public loan provision reduces the demand for loans that each bank faces since overall demand for loans by projects using the novel technology is policy-induced and hence fixed.[8]

Subject to the policy interventions, each bank i then carries out the following maximization problem:

$$\max_{l_{i,1},l_{i,2}} \pi_i = (r - \bar{c} - r_D + s_1)l_{i,1} + \beta \left(r - c(\gamma \sum_{j \neq i} l_{j,1} + \gamma^g g_1 + l_{i,1}) - r_D + s_2 \right) l_{i,2} \tag{16}$$

$$\text{s.t. } l_{i,1} \in [0, \frac{D - g_1}{N}], l_{i,2} \in [0, \psi \frac{D - g_2}{N}]$$

A direct takeaway from Equation 16 is that public loan provision at $t = 2$, i.e., once no further learning gains are possible, only factors into banks' decision by reducing the loan demand in $t = 2$ that they can serve if they do not opt for a no-financing strategy. Furthermore, we note that neither the de-risking instrument nor the public loan provision moderate our previous findings with respect to the deterministic rule of behavior for banks at $t = 2$, or the symmetric behavior of private banks in any possible Nash equilibrium. Therefore, Lemma 1 and the symmetry of the market outcome by Proposition 2 continue to hold (see Appendix D).

We first turn our discussion to the de-risking subsidy. Economic theory suggests that a subsidy should be calibrated to the magnitude of the unaccounted positive externality at the social optimum (Pigou, 1932)—which in our model only exists in $t = 1$. By incorporating such a well-calibrated de-risking subsidy into the first- and second-order conditions of individually rational banks, we arrive at the following proposition:

Proposition 3. *Let the social optimum be* $0 < l_1^{SO} < \frac{D}{N}$ *(gradual financing) and let* $s_1^* := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) > 0$ *be the optimally calibrated de-risking subsidy. Then,*

[7] In addition, the government could also adjust capital requirements for banks through a green-supporting factor (Campiglio et al., 2018), which in our model would have the same effect ($r \uparrow$).

[8] Such a "crowding-out" effect rests on the assumption that financing terms of public loan provision are usually concessional and hence out-compete the market rates charged by the banks in our model.

under $s_1 = s_1^*$ and for any $s_2 \geq 0$, the set of Nash equilibria can be characterized as follows:

- *A symmetric Nash equilibrium exists, in which banks behave like in the social optimum.*
- *Another Nash equilibrium with no financing by any bank exists if and only if no bank can unilaterally break even by providing loans. If such an equilibrium exists for $s_1 = s_2 = 0$ (i.e., without policy intervention), it also exists for $s_1 = s_1^*, s_2 = 0$.*
- *A sufficient condition for the no-financing equilibrium not to exist is $s_2 > \bar{c} + r_D - r$.*[9]

Proof. See Appendix D. □

Notably, under the optimal de-risking subsidy s_1^* (which is positive since $c' < 0$), there exists a gradual-financing or immediate-financing Nash equilibrium that coincides with the social optimum—even if prior to the policy intervention, the only possible Nash equilibrium featured no financing. However, such a well-calibrated subsidy does not necessarily rule out the coordination failure because even for $s_1 = s_1^*$, the return spread in the first period $r_D - (r - \bar{c} + s_1^*)$ remains strictly positive.[10] Hence, early-stage loans still come at a loss, albeit a smaller one, and if bank i cannot ensure bankability at $t = 2$ unilaterally, it cannot make a profit at $t = 2$ and no financing remains the best response. Visually speaking, a reduced initial return spread makes the no-financing valley in Figure 3 less steep without entirely removing it. Hence, if no bank is large enough to reach the tipping point unilaterally without any policy intervention, then introducing $s_1 = s_1^*$ will not remove the no-financing equilibrium. Furthermore, if the government were to set $s_1 > s_1^*$ to resolve the coordination problem, this subsidy would lead to an *oversupply* of early-stage loans unless the social optimum is an immediate-financing outcome.

However, Equation 36 demonstrates that the existence of the no-financing Nash equilibrium can always be ruled out via a sufficiently high de-risking subsidy at $t = 2$. The logic behind this is simple: If there is a profitable gradual-financing equilibrium (which is ensured by $s_1 = s_1^*$), the coordination failure only arises because, for low amounts of early-stage loan financing, banks are not fully committed to providing loans at a later stage and, therefore, withdraw to the non-financing Nash equilibrium to avoid losses. This no-financing equilibrium collapses once the de-risking measures at $t = 2$ improve the risk-return structure of loans such that unilateral financing of $l_{i,2} = \frac{\psi D}{N}$ suffices for bank i to make a profit in $t = 2$—even if all other banks do not grant any loans. A sufficient condition to ensure this is to set s_2 marginally above $r_D - (r - \bar{c})$, i.e., above the return spread at $t = 2$ if no bank provided any loan financing. Then, loans will always

[9] Note that this sufficient condition for s_2 is not a *necessary* one to rule out the no-financing equilibrium, but it is more tractable mathematically, thus facilitating policy comparisons.

[10] To show this, recall that in the social optimum, this expression equates the marginal learning gain of bank i, excluding spillovers, which is strictly positive.

be profitable at $t = 2$. Hence, banks commit to loan financing at $t = 2$ and always prefer the gradual-financing Nash equilibrium, which, due to the internalization of spillovers via $s_1 = s_1^*$, coincides with the social optimum.

Turning to a policy intervention in which the government provides loans directly instead of using the de-risking subsidy, the respective first- and second-order conditions lead us to the following proposition:

Proposition 4. *Let the social optimum be $0 < l_1^{SO} < \frac{D}{N}$ (gradual financing), let $l_1^{*NE}|g := (c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\phi\beta\psi\frac{D-g_2}{N}}\right) - \frac{\gamma^g}{N}g_1$ and let $g_1^* := \frac{1}{\gamma^g}c^{-1}(r - r_D)$. Under public loan provision in the absence of any de-risking subsidy (i.e., $s_1 = s_2 = 0$), the set of Nash equilibria can be characterized as follows:*

- *Both a zero-financing Nash equilibrium and a symmetric equilibrium where each bank provides $max\{0, l_1^{*NE}|g\}$ in $t = 1$ and $\psi\frac{D-g_2}{N}$ in $t = 2$ can exist.*
- *The higher g_2, the lower $l_1^{*NE}|g$, and the less likely it becomes that the Nash equilibrium with non-zero financing by each bank exists.*
- *The zero-financing Nash equilibrium cannot exist if g_1 exceeds g_1^* (marginally) as long as $g_2 \in [0, \psi D)$.*

Proof. See Appendix D.3. □

Hence, under public loan provision in $t = 1$, each bank provides only $\max\{0, l_1^{*NE} - \frac{\gamma^g}{N}g_1\}$ in $t = 1$ in the gradual-financing equilibrium, instead of l_1^{*NE} in the equilibrium without policy intervention. This is because the public loan provision does not alter the best response function of each bank. If the government provides early-stage loans on top of the gradual-financing Nash equilibrium, then for every unit of g_1, each bank reduces their own early-stage financing by $\frac{\gamma^g}{N}$ and instead benefits from the credit track record created by the public sector. The higher the spillover rate γ^g, the better the public loan provision substitutes banks' own financing experience, exacerbating this crowding-out dynamic. If $\frac{\gamma^g}{N}g_1 \geq l_1^{*NE}$, then the best response function of private banks flatlines at zero (see Figure 3) and private loan financing only occurs at $t = 2$. As a result, public loan provision is an inept policy instrument to close the gap between a market outcome with non-zero loan financing and the social optimum. Furthermore, public loan provision in $t = 2$ can only exacerbate the existing market failure. Reducing how much banks can loan at a later stage lowers the value of early-stage learning and hence the amount that banks are willing to loan ($l_1^{*NE}|g$)—and might even undermine the existence of a gradual-financing Nash equilibrium altogether.

The last part of Proposition 4, however, demonstrates that a certain minimum amount of public loan provision at $t = 1$ can overcome the coordination failure by ensuring that the no-financing equilibrium no longer exists. Note that for $g_1 = g_1^*$, the financing experience that spills over to banks is exactly the threshold for later-stage bankability since $r - c(\gamma^g g_1^*) = r_D$. Therefore, any $g_1 > g_1^*$ ensures that each bank provides $\psi\frac{D-g_2}{N}$ of loan financing at $t = 2$. Importantly, the required g_1^* decreases the more the public loan provider can diffuse its own financing experience to market players and the more willing private banks are to learn from the public sector ($\gamma^g \uparrow$). However, under $g_1 > g_1^*$ the cooperation problem not only continues to exist, such that the gradual-financing Nash equilibrium still falls short of the social optimum—the market outcome then features a strictly *lower* early-stage contribution by private banks due to free-riding.

By Proposition 3, however, a sufficient de-risking subsidy at $t = 2$ could reach a similar outcome—which poses the question of which of the two policy measures is more cost-effective in our model. A key difference between the two policies is that unlike for the de-risking subsidy, the money spent on public loan provision is (at least partially) recovered once loans are paid back, and the accumulated financing experience might be turned into further profits at $t = 2$, albeit at the cost of crowding out loans by commercial banks. To assess this, we define the costs of public loan provision as follows:

$$PC(g_1, g_2, l_1) := (r_D^g + \bar{c} - r^g)g_1 - \beta^g \left(r^g - c(g_1 + \gamma N l_1) - r_D^g\right) g_2 \tag{17}$$

where the parameters $r^g, r_D^g > 0, \beta^g \in (0, 1)$ have the same meaning as for private banks. Hence, policy costs can be understood as the negative of the public loan provider's profits. Note that here we allow for model parameters to vary between private banks and the public sector. For instance, the public loan provider might have a lower discount rate (such that $\beta^g > \beta$), a higher risk appetite (such that $r^g > r$), or access to capital at better rates than the private sector ($r_D^g < r_D$). By contrast, the opportunity cost of public money could also be higher since funds for public loan provision could otherwise be invested in core public responsibilities, such as military defense or education, with high, albeit non-financial returns (which could be reflected by $r_D^g > r_D$).

Furthermore, the cost of the de-risking subsidy paid at $t = 2$ in the absence of public loan provision is defined as follows:

$$PC(s_2) := s_2 \, \beta^g \psi D \, r_D^g \tag{18}$$

This reflects that if the coordination problem is resolved, this will lead to a policy-induced Nash equilibrium where $l_2^{NE} = \psi\frac{D}{N}$, and hence, the subsidy must be paid on all loans (ψD). Similar to the funds for public loans, the money for de-risking subsidies must be raised somewhere and hence comes at a cost r_D^g. Since policy costs only occur at $t = 2$, they are discounted at β^g.

We first compare the costs of the minimum public loan provision or second-period de-risking that necessarily rule out the coordination failure if the government parameters mirror the private sector's characteristics. Then we consider how deviations from this starting point change results, which results in the following finding:

Lemma 3. *Let $\epsilon > 0$ and $g_1 = g_1^* + \epsilon$ and $s_2 = r_D + \bar{c} - r + \epsilon$. Then, it holds that:*

- *If $g_1^* < \beta^g r_D^g \psi D$, the costs of the policy intervention g_1 are lower than the costs of the policy intervention s_2 if the costs of raising funds (r_D^g) and risk-adjusted loan return (r^g) for the public loan provision are identical to the rates faced by the banking sector.*

- *A higher r_D^g increases the policy costs of both measures, while the costs of providing g_1 also decrease r^g and γ^g.*

- *If the return spread for public loan provision in $t = 2$, i.e., $r^g - c(g_1 + \gamma N l_1) - r_D^g$, is positive (negative), the costs of this policy intervention decrease (increase) in g_2.*

Proof. See Appendix E.1. Comparative statics can be derived directly from the definitions above. □

Therefore, even if the public loan provider does not differ systematically from commercial banks, public loan provision will be the cheaper policy instrument unless the public loan financing required to resolve the coordination problem exceeds $\beta^g r_D^g \psi D$, i.e., the entirety of available loan demand at $t = 2$ plus financing costs discounted by one period. However, suppose the Nash equilibrium induced via public loan provision suffices to make loans at $t = 2$ profitable for the public sector. In that case, a continued public loan provision in the second period can reduce policy costs, particularly if the public loan provider has a lower risk aversion than private banks ($r^g \uparrow$). However, as discussed above, such later-stage loan provision would come at the cost of exacerbating the market failure. Furthermore, the more the public sector's learning gains spill over to private banks ($\gamma^g \uparrow$), the lower the required amount of public loan provision to resolve the coordination problem and, hence, the policy costs.

In conclusion, unlike de-risking subsidies, public loan provision cannot address the cooperation problem in our model, and later-stage loan provision even exacerbates market failures. However, early-stage loan provision can be used to overcome the coordination problem, i.e., to rule out the existence of an inferior no-financing Nash equilibrium, and is a more cost-effective measure to do so—unless the required loan financing amounts are excessively large, for example, if spillovers to the private banks are limited. Therefore, the case for this policy tool strongly depends on which Nash equilibrium policymakers consider as more likely to realize without any intervention, particularly since public loan provision will induce free-riding behavior by private banks to some degree—which would be exacerbated by including price feedbacks if the additional supply of loans reduces market rates.

These potential limitations of public loan provision as a stand-alone measure stem from banks' response function being unaffected, and learning spillovers to peers remain unaccounted for. This, however, can be addressed by combining public loan provision to rule out the zero-financing Nash equilibrium with the optimally calibrated de-risking subsidy at $t = 1$ in a policy mix:

Lemma 4. *Let $\epsilon > 0$ and the social optimum be $0 < l_1^{SO} < \frac{D}{N}$, and consider the following policy mix: $g_1 = g_1^* + \epsilon, s_1 = s_1^*, g_2 = s_2 = 0$. Then, it holds that:*

- *The unique Nash equilibrium is one where each bank provides $l_1^{SO} - \frac{\gamma^g}{\tilde{N}} g_1$ in $t = 1$ (i.e., less than in the social optimum) and $\psi \frac{D}{N}$ in $t = 2$*
- *The loan financing amount provided by each bank in the policy-mix equilibrium is higher than in the Nash equilibrium resulting from the same public loan provision $g_1 = g_1^* + \epsilon$ without the de-risking subsidy ($s_1 = 0$).*

Proof. See Appendix E.2. □

Such a policy mix, therefore, removes the risk of any no-financing Nash equilibrium and ensures a gradual-financing Nash equilibrium. From a private bank's perspective, this provides the same financing experience as in the social optimum, although some of the financing burden shifts from commercial banks to the public sector to avoid the coordination failure. How this affects overall profits and efficiency strongly depends on how the public loan provider and private banks differ in terms of their risk appetite, discount rate, and financing or opportunity costs. However, the loan financing amount provided by private banks under the policy mix of public loan provision and the optimally calibrated first-period de-risking subsidy is strictly higher compared to public loan provision as a stand-alone measure and is, therefore, more effective.

Based on our model framework, the relative merits of later-stage de-risking subsidies to address the coordination failure are primarily that more financing experience is generated directly within the private sector since no crowding-out occurs and that it induces no free-riding behavior. Therefore, the social optimum can be obtained, albeit at a relatively high policy cost, if the policy is successful. Beyond policy costs, however, direct loan provision has at least two distinctive advantages. First, an institutionalized public loan provider can easily be re-directed to other novel technologies as they emerge and pose new coordination problems for the financial sector. By doing so, institutions can leverage their previous financing experience even if high opportunity costs and crowding-out risks should force them to withdraw from matured technologies. If new technologies emerge and the institution's mandate is sufficiently flexible, then private debt markets for these technologies can be kickstarted without the need for introducing additional policies, which can accelerate the ramp-up of deployment. If no such technologies emerge, then selling the public loan provider to the private sector, as the UK did with its UK Green Investment Bank in 2017 (Whitney et al., 2020), can further provide an exit strategy to recover policy costs (partially) by monetizing the accumulated in-house experience.

Second, addressing the coordination failure through early-stage loan provision avoids the issue of time inconsistency on the government's side. By Proposition 3, a sufficiently high de-risking subsidy at $t = 2$ suffices to avoid an inferior no-financing equilibrium. However, once the second period begins, banks have already provided the required early-stage loan financing. Therefore, merely the *anticipation* of the support policy at $t = 2$ rules out the no-financing equilibrium. The actual payment of s_2 does not affect total profits and instead simply redistributes money from the public to the private sector. As a result, policymakers could be tempted to go back on their promises, which in turn will reduce policy effectiveness if banks assign a non-zero probability to such an outcome ex ante. Similar concerns, however, exist with respect to public loan provision at $t = 2$. A government that initially promised to phase out loan provision once the novel technology becomes bankable might be tempted to keep providing loans in $t = 2$ when they become profitable. These considerations seem particularly relevant for countries with lower institutional quality and low trust in the public sector or with highly bipartisan politics on climate change, such that elections pose severe risks of policy reversal. It also matters for countries with lower creditworthiness that might be forced to revoke expensive support policies by adverse macroeconomic shocks—as happened to renewable energy subsidies in Spain and Italy following the Euro crisis (Karneyeva & Wüstenhagen, 2017)—and for technologies with a lower later-stage demand potential ψ where public loan volumes can account for high market shares.

6 Conclusion

Over the last few years, direct loan provision to clean energy projects via government bodies and public investment banks has become increasingly popular, but the theoretical rationale behind this policy tool is not fully understood and lacks a coherent microeconomic framework. By analyzing bank loans to a novel clean energy technology in a model where cumulative financing experience improves risk-adjusted returns over time and spills over between banks, we show that the banking sector will not provide the socially optimal amount of risky early-stage financing due to two issues.

First, the positive learning externality leads to an undersupply of risky early-stage credit. This cooperation problem cannot be mitigated through public loan provision because public loans crowd out private investment and create no additional incentive for banks to provide risky early-stage loans. By contrast, introducing de-risking instruments, such as interest rate subsidies or credit guarantees, at an early stage can close the gap between the social optimum and a market equilibrium that involves some, albeit insufficient, early-stage loan financing. Second, the banking sector can remain stuck in an inferior Nash equilibrium featuring no loan financing due to a coordination failure. In this case, public loan provision serves to push the market to the preferable market equilibrium, which can be more cost-effective than resolving the coordination problem by using de-risking measures. However, public loan provision should always be paired with de-risking measures to minimize the gap between market outcome and social optimum and should be phased out at a later stage when the novel technology has become bankable to avoid exacerbating the market failures through crowding-out.

Since the findings presented here do not rely on market failures on the technology or consumer level, such as greenhouse gas emissions and R&D spillovers, they motivate financial policy intervention even if other first-best instruments, such as carbon pricing or renewable energy support schemes, are already in place. Therefore, they can guide policymakers in shaping the rules and mandates for public loan programs and investment banks that are targeting clean energy projects, such as the clean finance institutions to be established under the Inflation Reduction Act's National Clean Investment Fund.

While our model provides a clear framework to conceptualize the role of public loan provision for clean energy technologies, its simplicity also comes with limitations. First, by abstracting from externalities at the technology and consumer level, we risk painting a pessimistic picture of public loan provision as a policy instrument if first-best instruments cannot be easily implemented due to political constraints. Similarly, assuming a perfectly inelastic loan demand exacerbates crowding-out issues in our model, which would decrease in demand elasticity. Thus, we abstract from how supply-demand dynamics might impact risk-adjusted returns, whereas public loan provision could also address an undersupply of credit due to market power for a decreasing demand curve and a finite N. Lastly, while our model features a risk-adjusted return motivated by default risks and risk aversion, we do not account for within-portfolio correlations, uncertainties about key parameters, such as the learning rate or the growth potential of the novel technology (ψ), or for systemic risks and bank heterogeneity, which matter particularly for banking regulation (Freixas & Rochet, 2023).

Future research can address these limitations by extending our framework to multiple assets, explicitly incorporating uncertainties about c and ψ, and exploring how a falling loan demand curve and bank heterogeneity might moderate the findings presented here. Given the signaling role of public green banks suggested by qualitative studies (OECD, 2016), scholars could also model borrower projects explicitly to explore how co-investing with commercial banks can increase the policy impact of public loan providers or how incorporating herding dynamics can affect the conclusions presented here.

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Appendices

A Additional definitions

We introduce a variety of further definitions and conventions to make the subsequent proofs more concise:

- $\bar{L}_t \coloneqq \sum_{i=1}^{N} l_{i,t}$
- $(l_1, l_2)_{i=1}^{N}$ denotes any kind of symmetric outcome where each bank provides l_1 in $t = 1$ and l_2 in $t = 2$.
- L denotes the $N \times 2$ matrix with elements $l_{i,t}$ in its i-th row and t-th column.
- L_{-i} and $\bar{L}_{-i,t}$ denote the corresponding matrix and sum, respectively, if bank i is excluded from it.
- $\mathcal{L}^{NE}$ denotes the set of existing Nash equilibria under the market outcome, given model parameters.
- the notation $|g$, $|s$, and $|gs$ after a variable denotes the respective variable's value in the presence of a policy intervention (direct loan provision, de-risking subsidy, or both combined, respectively). For instance, $l_1^{NE}|g$ denotes the symmetric Nash equilibrium loan financing in $t = 1$ that results under direct loan provision.

As laid out in the main text, throughout the entire paper, we assume that the initial return spread over the deposit rate and transaction cost is negative but that for a full provision of early-stage financing, the return spread in $t = 2$ can become positive:

Assumption 1. *$r > r_D$, $r - \bar{c} < r_D$ and $r - c\left(\frac{\tilde{N}}{N}D\right) > r_D$, where $\tilde{N} \coloneqq 1 + \gamma(N-1) < N$.*

B Proofs for Proposition 1 (social optimum)

Using a more formal notation, Proposition 1 can be stated as

Proposition 1. *Let $l_1^* \coloneqq \tilde{N}^{-1}(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\beta\frac{\tilde{N}}{N}D}\right)$ where $(c')^{-1}(\cdot)$ denotes the inverse function of c'.*[11] *Then*

$$(l_1^{SO}, l_2^{SO}) = \begin{cases} (\frac{D}{N}, \frac{D}{N}) & \text{if } l_1^* \geq \frac{D}{N} \wedge \beta\left(r - c(\tilde{N}\frac{D}{N}) - r_D\right) > \bar{c} + r_D - r \\ (l_1^*, \frac{D}{N}) & \text{if } l_1^* < \frac{D}{N} \wedge \beta\left(r - c(\tilde{N}l_1^*) - r_D\right)\frac{D}{N} > (\bar{c} + r_D - r)\, l_1^* \\ (0,0) & \text{otherwise} \end{cases}$$

[11] Note that c' is monotonously increasing, and hence, $(c')^{-1}$ exists and is monotonously increasing.

Ceteris paribus, both l_1^* and the likelihood of the condition for $l_1^{SO}, l_2^{SO} > 0$ being satisfied increase monotonously in r, β, D, ψ, and γ, and decrease monotonously $\bar{c}$ and N.

Proof. See Appendices B.1 and B.2. □

B.1 First- and second-order conditions

The Lagrangian of the social maximization problem stated in Equation 10 is as follows:

$$\max_{l_{1,1},\dots l_{N,2}} \mathcal{L} = \sum_{i=1}^{N} (r - \bar{c} - r_D) l_{i,1} + \beta \left(r - c(\gamma \bar{L}_{-i,1} + l_{i,1}) - r_D \right) l_{i,2} + \mu_{i,1} \left(\frac{D}{N} - l_{i,1} \right) + \mu_{i,2} \left(\psi \frac{D}{N} - l_{i,2} \right) + \mu_{i,3} l_{i,1} + \mu_{i,4} l_{i,2}.$$

Note that here we redefine the Lagrangian multipliers applying to $t = 2$ as the original multiplier divided by β, which does not affect results since $\beta \in (0, 1)$, but simplifies the first-order conditions (FOCs).

The resulting Karush-Kuhn-Tucker conditions tell us that, for each bank i, the following conditions have to hold in the social optimum:

$$-r_D + r - \bar{c} - \beta \left(c'(\tilde{L}_{i,1}^{SO}) l_{i,2}^{SO} + \gamma \sum_{j \neq i} c'(\tilde{L}_{j,1}^{SO}) l_{j,2}^{SO} \right) - \mu_{i,1} + \mu_{i,3} = 0 \tag{19}$$

$$-r_D + r - c(\tilde{L}_{i,1}^{SO}) - \mu_{i,2} + \mu_{i,4} = 0 \tag{20}$$

$$\mu_{i,1} \left(\frac{D}{N} - l_{i,1}^{SO} \right) = \mu_{i,2} \left(\psi \frac{D}{N} - l_{i,2}^{SO} \right) = \mu_{i,3} l_{i,1}^{SO} = \mu_{i,4} l_{i,2}^{SO} = 0 \tag{21}$$

$$\mu_{i,1}, \mu_{i,2}, \mu_{i,3}, \mu_{i,4} \geq 0 \tag{22}$$

Obviously, the upper and lower bound restrictions on $l_{i,1}^{SO}$ and $l_{i,2}^{SO}$ are mutually exclusive. Hence, the complementary slackness conditions expressed by Equation 21 imply that:

$$\mu_{i,u} > 0 \rightarrow \mu_{i,v} = 0 \; \forall \; (u, v) \in \{(1, 3), (3, 1), (2, 4), (4, 2)\}$$

Keeping in mind that banks prefer no loan financing if the return spread is exactly zero, by Equation 20, we can rule out any scenario where $\mu_{i,2} = \mu_{i,4} = 0$, which gives us a simple rule for $l_{i,2}^{SO}$ given the optimal solution for all banks other than i, as expressed in Lemma 1 (see Section 3).

B.2 Social optimum under symmetry

As laid out in Section 4, we impose symmetry on the social optimum to maintain the analytical tractability of our model:

Assumption 2. *Let* $(l_{i,1}^{SO}, l_{i,2}^{SO})$ *be the solution to the maximization problem in Equation 10 for any bank* i*. Then*

$$l_{i,t}^{SO} = l_t^{SO} \ \forall\, i = 1, ..., N, \quad t = 1, 2$$

Based on Lemma 1, there are only six different ways that a given bank i can behave:

1. $l_{i,1}^{SO} = l_{i,2}^{SO} = 0$
2. $l_{i,1}^{SO} = \frac{D}{N}, l_{i,2}^{SO} = \psi\frac{D}{N}$
3. $l_{i,1}^{SO} \in \left(0, \frac{D}{N}\right), l_{i,2}^{SO} = \psi\frac{D}{N}$
4. $l_{i,1}^{SO} \in \left(0, \frac{D}{N}\right), l_{i,2}^{SO} = 0$
5. $l_{i,1}^{SO} = \frac{D}{N}, l_{i,2}^{SO} = 0$
6. $l_{i,1}^{SO} = 0, l_{i,2}^{SO} = \psi\frac{D}{N}$

Under symmetry, we can rule out the two cases involving $l_{i,1} > 0, l_{i,2} = 0$ because there is no point in providing early-stage financing if no one benefits from it in $t = 2$. Similarly, if all banks behave symmetrically, $l_{i,1} = 0, l_{i,2} = \psi\frac{D}{N}$ cannot be optimal because due to the absence of learning effects from $t = 1$, financing provision in $t = 2$ results in negative profits. Therefore, one of the three following cases must apply:

- "immediate financing": $(l_{i,1}^{SO}, l_{i,2}^{SO}) = (\frac{D}{N}, \psi\frac{D}{N}) \ \forall\, i = 1, ..., N$
- "gradual financing": $(l_{i,1}^{SO}, l_{i,2}^{SO}) = (l_1^{SO}, \psi\frac{D}{N}) \ \forall\, i = 1, ..., N$ with $l_1^{SO} \in (0, \frac{D}{N})$
- "no financing": $(l_{i,1}^{SO}, l_{i,2}^{SO}) = (0, 0) \ \forall\, i = 1, ..., N$

Furthermore, we can show that "immediate financing" and "gradual financing" are mutually exclusive as critical points of the Lagrangian:

Lemma 5. *Let* $\mathcal{L}^*$ *be the set of critical points satisfying the FOCs of the maximization problem in Equation 10. Then under Assumption 2, it holds that*

- $(l_1, \psi\frac{D}{N}) \in \mathcal{L}^*$ *for* $l_1 \in (0, \frac{D}{N}) \implies (\frac{D}{N}, \psi\frac{D}{N}) \notin \mathcal{L}^*$
- $(\frac{D}{N}, \psi\frac{D}{N}) \in \mathcal{L}^* \implies (l_1, \psi\frac{D}{N}) \notin \mathcal{L}^*$ *for* $l_1 \in (0, \frac{D}{N})$

Proof. Due to symmetry, the FOCs in Equations 19-20 simplify to:

$$-r_D + r - \bar{c} - \beta\tilde{N}l_2^{SO}\left(c'(\tilde{N}l_1^{SO})\right) - \mu_{i,1} + \mu_{i,3} = 0 \tag{23}$$

$$-r_D + r - c(\tilde{N}l_1^{SO}) - \mu_{i,2} + \mu_{i,4} = 0 \tag{24}$$

Note that $c' < 0$ such that $-\beta\tilde{N}l_2^{SO}\left(c'(\tilde{N}l_1^{SO})\right)$ is non-negative. Both under "immediate financing" and "gradual financing", $l_2^{SO} = \psi\frac{D}{N}$. Therefore, Equation 23 only differs in l_1^{SO} (and correspondingly $\mu_{i,1}$ and $\mu_{i,3}$) between the two outcomes. But if

$$\exists l_1 \in (0, \frac{D}{N}) : -\beta\tilde{N}\frac{D}{N}\left(c'(\tilde{N}l_1)\right) = r_D - (r - \bar{c})$$

then

$$-\beta\tilde{N}\frac{D}{N}\left(c'(\tilde{N}\frac{D}{N})\right) < r_D - (r - \bar{c})$$

as $c'' > 0$. By the same logic, Equation 23 cannot hold for $l_1^{SO} \in (0, \frac{D}{N})$ if it holds for $l_1^{SO} = \frac{D}{N}$. □

If Equation 23 holds for $\mu_{i,1} = \mu_{i,3} = 0$, this gives the following solution:

$$l_1^* := \tilde{N}^{-1}(c')^{-1}\left(-\frac{r_D - r + \bar{c}}{\beta\tilde{N}\psi\frac{D}{N}}\right)$$

By Lemma 5 and Assumption 2, there can at most be two critical points of the Lagrangian: one at $(0,0)_{i=1}^N$ and one either at $(l_1^*, \psi\frac{D}{N})_{i=1}^N$ or at $(\frac{D}{N}, \psi\frac{D}{N})_{i=1}^N$. Therefore, regarding the second-order condition (SOC) for an optimum at $(l_1, \psi\frac{D}{N})_{i=1}^N$ where $l_1 \in \{l_1^*, \frac{D}{N}\}$, it suffices to show that the objective function's value at this point exceeds the value at $(0,0)_{i=1}^N$. Since $\bar{\pi}_i(L = (0,0)_{i=1}^N) = 0$, then the SOC simply requires profits above zero:

$$N\left((r - \bar{c} - r_D)l_1 + \beta(r - c(\tilde{N}l_1) - r_D)\psi\frac{D}{N}\right) > 0 \tag{25}$$

Dividing by N and rearranging gives the condition in the Appendix version of Proposition 1.

Since $r - \bar{c} - r_D < 0$ by Assumption 1, this requires $r - c(\tilde{N}l_1) - r_D > 0$, which directly implies that $(l_1, \psi\frac{D}{N})_{i=1}^N$ also satisfies the FOC with respect to $l_{i,2}$:

Lemma 6. *Let $l_1 \in \{l_1^*, \frac{D}{N}\}$. If Equation 25 holds for l_1, then Equation 24 holds for $l_1^{SO} = l_1, \mu_{i,2} > 0$.*

Proof. For Equation 25 to hold, it must be that $r - c(\tilde{N}l_1) - r_D > 0$. Hence, Equation 24 can only hold if $\mu_{i,2} > 0$. □

Therefore, the conditions for a social optimum at $l_1 \in \{l_1^*, \frac{D}{N}\}$ given in the Appendix version of Proposition 1 only include the FOC with respect to $l_{i,1}$ and the SOC (which implies the FOC with respect to $l_{i,2}$). Taking partial derivatives of l_1^* and the respective SOC yields the comparative statics in the Appendix version of Proposition 1.

C Proofs for Proposition 2 (market outcome)

Using a more formal notation, Proposition 2 can be stated as

Proposition 2. *Let* $l_1^{*NE} := \tilde{N}^{-1}(c')^{-1}\left(-\frac{r_D+\bar{c}-r}{\beta\psi\frac{D}{N}}\right)$, *let* $\mathcal{L}^{NE}$ *be the set of possible Nash equilibria and let* $\bar{l}_1 := \min\{l_1^{*NE}, \frac{D}{N}\}$. *Then*

- $\mathcal{L}^{NE} = \{(0,0)_{i=1}^N\}$ *if* $\bar{l}_1 \leq 0$ *and otherwise* $\mathcal{L}^{NE} \in \mathcal{P}\left(\{(\bar{l}_1, \frac{D}{N})_{i=1}^N, (0,0)_{i=1}^N\}\right) \setminus \emptyset$
- $(0,0)_{i=1}^N \in \mathcal{L}^{NE}$ *if and only if*

$$\nexists\, l_{i,1} \in \{\tilde{N}l_1^{*NE}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \beta(r - c(l_{i,1}) - r_D)\psi\frac{D}{N} > (r_D + \bar{c} - r)l_{i,1} \tag{26}$$

- $(\bar{l}_1, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$ *if and only if*

$$\bar{l}_1 > 0 \wedge \beta(r - c(\tilde{N}\bar{l}_1) - r_D)\psi\frac{D}{N} > (r_D + \bar{c} - r)\bar{l}_1 \tag{27}$$

Furthermore, for any $(l_1, l_2)_{i=1}^N \in \mathcal{L}^{NE}$, *it must hold that* $l_1 \leq l_1^{SO}$, *with* $l_1 = l_1^{SO}$ *if and only if*

$$l_1^{SO} = 0 \vee \left(l_1^{SO} = \frac{D}{N} = l_1 = \bar{l}_1\right). \tag{28}$$

Proof. See Appendices C.2, C.3, C.4 and C.5. □

C.1 First-order conditions

The Lagrangian of the individual maximization problem of bank i stated in Equation 13 is as follows:

$$\begin{aligned}\max_{l_{i,1},l_{i,2}} \mathcal{L} =& (r - \bar{c} - r_D)l_{i,1} + \beta\left(r - c(\gamma\bar{L}_{-i,1} + l_{i,1}) - r_D\right)l_{i,2} \\ &+ \mu_{i,1}\left(\frac{D}{N} - l_{i,1}\right) + \beta\mu_{i,2}\left(\psi\frac{D}{N} - l_{i,2}\right) + \mu_{i,3}l_{i,1} + \beta\mu_{i,4}l_{i,2}.\end{aligned}$$

The resulting Karush-Kuhn-Tucker conditions that any Nash equilibrium must satisfy are as follows:

$$-r_D + r - \bar{c} - \beta c'(\tilde{L}_{i,1}^{NE}) l_{i,2}^{NE} - \mu_{i,1} + \mu_{i,3} = 0 \tag{29}$$

$$-r_D + r - c(\tilde{L}_{i,1}^{NE}) - \mu_{i,2} + \mu_{i,4} = 0 \tag{30}$$

$$\mu_{i,1}(\frac{D}{N} - l_{i,1}^{NE}) = \mu_{i,2}(\psi\frac{D}{N} - l_{i,2}^{NE}) = \mu_{i,3} l_{i,1}^{NE} = \mu_{i,4} l_{i,2}^{NE} = 0 \tag{31}$$

$$\mu_{i,1}, \mu_{i,2}, \mu_{i,3}, \mu_{i,4} \geq 0 \tag{32}$$

C.2 Best response function

Lemma 7. *Let* $l_{i,1} \in \left(0, \frac{D}{N}\right]$. *Then,*

$$(l_{i,1}, 0) \notin \arg\max_{l_i \in \mathcal{L}_i} \pi_i((L_{-i}, l_i)).$$

Proof. This is trivial because it cannot be individually optimal to play $l_{i,1} > 0$ without profiting from learning effects at $t = 2$ since by Assumption 1, $l_{i,1} > 0$ implies losses at $t = 1$. □

Lemma 8. *The individually profit-maximising amount* $l_{i,2}^* = \arg\max_{l_{i,2} \in \left[0, \psi\frac{D}{N}\right]} \pi_i(L_{-i}, (l_{i,1}, l_{i,2}))$ *follows a deterministic rule:*

$$l_{i,2}^* = \begin{cases} 0, & \text{if } r - c(\tilde{L}_{i,1}) \leq r_D \\ \psi\frac{D}{N}, & \text{otherwise.} \end{cases}$$

Proof. This follows directly from Equation 30 once we assume that banks prefer no loan financing if the return spread is exactly zero. □

Depending on the behavior of the other banks L_{-i}, we can then show that the best response of bank i can fall into four different categories:

1. "immediate financing" (IF): bank i finances the full amount of projects in both periods subject to its demand constraint
2. "gradual financing" (GF): bank i finances some but not all available projects at $t = 1$ and all projects at $t = 2$
3. "free-riding" (FR): bank i invests nothing at $t = 1$ but finances all available projects at $t = 2$
4. "no financing" (NF): bank i does not invest in neither period

Lemma 9. *Bank i's best response function* $BR_i : L_{-i} \mapsto \arg\max_{l_{i,1},l_{i,2}} \pi_i(L_{-i}, l_i)$ *is given by:*

$$BR_i(L_{-i}) = \begin{cases} \left(\frac{D}{N}, \psi\frac{D}{N}\right) & \text{if } r - \bar{c} - \beta c'(\tilde{L}_{-i,1} + \frac{D}{N})\psi\frac{D}{N} > r_D \land \\ & \quad r - c(\tilde{L}_{-i,1} + \frac{D}{N}) > r_D \land \\ & \quad \pi_i(L_{-i}, (\frac{D}{N}, \psi\frac{D}{N})) > \pi_i(L_{-i}, (0,0)) \\ \left(l^*_{i,1}, \psi\frac{D}{N}\right) & \text{if } \exists l_{i,1} \in [0, \frac{D}{N}) : \\ & r - \bar{c} - \beta c'(\tilde{L}_{-i,1} + l_{i,1})\psi\frac{D}{N} = r_D, \\ & r - c(\tilde{L}_{-i,1} + l_{i,1}) > r_D, \\ & \pi_i(L_{-i}, (l_{i,1}, \psi\frac{D}{N})) > \pi_i(L_{-i}, (0,0)) \\ \left(0, \psi\frac{D}{N}\right) & \text{if the prior conditions are not satisfied and in addition,} \\ & r - c(\tilde{L}_{-i,1}) > r_D \\ (0,0) & \text{otherwise} \end{cases}$$

with

$$l^*_{i,1} = (c')^{-1}\left(-\frac{r_D - r + \bar{c}}{\beta\psi\frac{D}{N}}\right) - \tilde{L}_{-i,1}.$$

Proof. This follows directly from the FOC in Equation 29 and Lemmas 7 and 8. □

C.3 Symmetry of Nash equilibrium

We can show that any Nash equilibrium must be symmetric:

Lemma 10. *In any Nash equilibrium, it holds that*

$$\neg\exists i \in \{1, ..., N\} : l_{i,1} = 0 \land l_{i,2} > 0.$$

Proof. Let $l_{i,1} = 0$ in a Nash equilibrium. Assume for contradiction that $l_{i,2} > 0$. By Lemma 8, this implies that $l_{i,2} = \psi\frac{D}{N}$.

Then, one of the following cases must hold:

- $\tilde{L}_{-i,1} = 0$. But by Assumption 1, this would imply that $r - c(\tilde{L}_{-i,1} + l_{i,1}) = r - \bar{c} < r_D$. Then, it is trivial to see that

  $$\pi_i((L_{-i}, (0,0))) > \pi_i((L_{-i}, (0, l_{i,2}))).$$

  Hence, $(0, l_{i,2})$ cannot be a best response to L_{-i} for bank i, and therefore this strategy cannot be part of a Nash equilibrium.

- $\tilde{L}_{-i,1} > 0$. This implies that there is another bank j with $l_{j,1} > 0$ and, concomitantly,

$l_{j,2} = \psi \frac{D}{N}$ and

$$\tilde{L}_{-j,1} = \gamma \bar{L}_{-i,j,1} < \gamma \bar{L}_{-i,j,1} + \gamma l_{j,1} = \tilde{L}_{-i,1}.$$

It follows immediately that $\tilde{L}_{j,1} - \tilde{L}_{i,1} = (1-\gamma) l_{j,1} > 0$. Note that $l_{j,1} > 0$ requires by j's best response function that

$$r - \bar{c} - \beta c'(\tilde{L}_{j,1}) \psi \frac{D}{N} \geq r_D,$$

while $l_{i,1} = 0$ requires by i's best response function that

$$r - \bar{c} - \beta c'(\tilde{L}_{i,1}) \psi \frac{D}{N} < r_D.$$

But since $-\beta c'(\cdot) \psi \frac{D}{N}$ is strictly decreasing, $\tilde{L}_{i,1} < \tilde{L}_{j,1}$ implies that

$$r - \bar{c} - \beta c'(\tilde{L}_{i,1}) \psi \frac{D}{N} > r - \bar{c} - \beta c'(\tilde{L}_{j,1}) \psi \frac{D}{N} \geq r_D,$$

which is a contradiction.

$\square$

Lemma 11. *In any Nash equilibrium with $\bar{L}_1 \in (0, D]$, it holds that*

$$l_{i,1} \neq 0 \; \forall \, i = 1, ..., N$$

Proof. $\bar{L}_1 \in (0, D]$ implies that there must be another bank j such that $l_{j,1} > 0$.

Assume for contradiction that there is a bank i such that $l_{i,1} = 0$. By Lemma 10, this implies that $l_{i,2} = 0$, which means that

$$\pi_i(L_{-i}, (0,0)) = 0.$$

Note that $l_{j,1} > 0$ implies that $l_{j,2} > 0$ by the contrapositive of Lemma 7, which implies by Lemma 8 that $l_{j,2} = \psi \frac{D}{N}$. Hence, bank j's profits must be

$$\pi_j(L_{-j}, (l_{j,1}, \psi \frac{D}{N})) = (r - \bar{c} - r_D) l_{j,1} + \beta \left(r - c(\tilde{L}_{-j,1} + l_{j,1}) - r_D \right) \psi \frac{D}{N}.$$

If the outcome is a Nash equilibrium, it has to hold by j's best response function that this yields positive profits—which is the profit made by bank i.

Also, note that

$$\tilde{L}_{-j,1} - \tilde{L}_{-i,1} = \gamma \bar{L}_{-i,j,1} - (\gamma \bar{L}_{-i,j,1} + \gamma l_{j,1}) = -\gamma l_{j,1} < 0,$$

which means that $\tilde{L}_{-j,1} < \tilde{L}_{-i,1}$. But since $\pi_i(\cdot)$ is weakly increasing in $\tilde{L}_{-i,1}$, this means that if bank i were to switch from $(0,0)$ to adopting bank j's strategy $(l_{j,1}, \psi \frac{D}{N})$, it must hold that

$$\pi_i(L_{-i}, (l_{j,1}, \psi\frac{D}{N})) \geq \pi_j(L_{-j}, (l_{j,1}, \psi\frac{D}{N})) > \pi_i(L_{-i}, (0, 0)).$$

By symmetry, $(l_{j,1}, \psi\frac{D}{N})$ must be feasible for bank i since it is feasible for bank j. Therefore, $(0, 0)$ cannot be a best response of bank i to L_{-i} and the outcome cannot be a Nash equilibrium. □

Lemma 12. *In any Nash equilibrium with $\bar{L}_1 \in (0, D]$, there is a $l_1^* \in \left(0, \frac{D}{N}\right]$ such that*

$$l_{k,1} = l_1^* \; \forall \; k = 1, ..., N.$$

Proof. In a Nash equilibrium with $\bar{L}_1 \in (0, D]$, by Lemma 11, it holds that $l_{i,1} > 0 \; \forall \; i$. Take any $i \neq j$ and assume for contradiction that $l_{j,1} > l_{i,1}$.

This implies that $\tilde{L}_{j,1} = \gamma\bar{L}_{-i,j,1} + \gamma l_{i,1} + l_{j,1}$ and $\tilde{L}_{j,1} = \gamma\bar{L}_{-i,j,1} + \gamma l_{j,1} + l_{i,1}$. Hence,

$$\tilde{L}_{j,1} - \tilde{L}_{i,1} = (1 - \gamma)(l_{j,1} - l_{i,1}) > 0.$$

Note that $l_{i,1}, l_{j,1} > 0$ implies by Lemma 7 that $l_{i,2} = l_{j,2} = \psi\frac{D}{N}$. Furthermore, $l_{j,1} > l_{i,1}$ implies that $l_{i,1} \in \left(0, \frac{D}{N}\right)$. Since the outcome is a Nash equilibrium, i's best response function then requires that

$$r_D = r - \bar{c} - \beta c'(\tilde{L}_{i,1})\psi\frac{D}{N}.$$

At the same time, $l_{j,1} > 0$ implies by j's best response function that

$$r_D \leq r - \bar{c} - \beta c'(\tilde{L}_{j,1})\psi\frac{D}{N}.$$

Combining both expressions yields the requirement that

$$-c'(\tilde{L}_{j,1}) \geq -c'(\tilde{L}_{i,1}).$$

However, since $-c(\cdot)$ is strictly decreasing and $\tilde{L}_{j,1} > \tilde{L}_{i,1}$, this cannot hold. □

Lemma 13. *In any Nash equilibrium, it holds that*

$$l_{i,t} = l_t^* \; \forall \; i, t.$$

Proof. Simply note that $\bar{L}_1 \in [0, D]$, which means that one of the following cases has to hold:

- $\bar{L}_1 = 0$. Then, trivially, $l_{i,1} = 0 \; \forall \; i$.
- $\bar{L}_1 \in (0, D]$. Then, by Lemma 12, $l_{i,1} = l_1^* \; \forall \; i$.

Since this implies that $\tilde{L}_{i,1} = (1+\gamma(N-1))l_1^* \; \forall \, i$, Lemma 8 tells us that $l_{i,2} = l_2^*$ for some $l_2^* \in \left\{0, \psi\frac{D}{N}\right\}$ for all i. □

C.4 Existence of Nash equilibria

Lemma 14. *Let* $l_1^{*NE} := \tilde{N}^{-1}(c')^{-1}(-\frac{r_D - r + \bar{c}}{\beta\psi\frac{D}{N}})$. *Then the symmetric no-financing Nash equilibrium* $l_{i,t} = 0 \; \forall \, i, t$ *exists if and only if*

$$\nexists \, l_{i,1} \in \{\tilde{N}l_1^{*NE}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \beta\,(r - c(l_{i,1}) - r_D)\,\psi\frac{D}{N} > (r_D - r + \bar{c})\, l_{i,1}.$$

Proof. **Only if**
By Assumption 1, the FOCs in Equation 29 and 30 hold for $l_{i,t} = 0 \; \forall \, i, t, \mu_{i,3}, \mu i, 4 > 0$, i.e., the symmetric no-financing outcome is always a critical point of the Lagrangian since both FOCs, in this case, reduce to $r - \bar{c} < r_D$.

By Lemma 9, $(l_{i,1}, l_{i,2}) = (0, \psi\frac{D}{N})$ cannot be a best response of bank i given $\bar{L}_{-i,1} = 0$. Therefore, by Lemma 9 the only possible best-response deviations for bank i from the symmetric no-financing outcome are

$$(l_{i,1}, l_{i,2}) = (\frac{D}{N}, \psi\frac{D}{N})$$

or

$$(l_{i,1}, l_{i,2}) = \left((c')^{-1}\left(-\frac{r_D - r + \bar{c}}{\beta\psi\frac{D}{N}}\right) - \tilde{L}_{-i,1}, \psi\frac{D}{N}\right)$$

which given $\bar{L}_{-i,1} = 0$ can be rewritten as

$$(l_{i,1}, l_{i,2}) = \left(\tilde{N}l_1^{*NE}, \psi\frac{D}{N}\right)$$

Assume that the symmetric no-financing outcome is not a Nash equilibrium. Then, one of these two possible deviations must be optimal for bank i given $\bar{L}_{-i} = 0$, which by the SOC requires that

$$\exists \, l_{i,1} \in \{\tilde{N}l_1^{*NE}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \pi_i\left(\bar{L}_{-i,1} = 0, (l_{i,1}, \psi\frac{D}{N})\right) > \pi_i\left(\bar{L}_{-i,1} = 0, (0, 0)\right) = 0$$

.

Inserting the expression for $\pi_i\left(\bar{L}_{-i,1} = 0, (l_{i,1}, \psi\frac{D}{N})\right)$ and rearranging gives us

$$\exists \, l_{i,1} \in \{\tilde{N}l_1^{*NE}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \beta\,(r - c(l_{i,1}) - r_D)\,\psi\frac{D}{N} > (r - \bar{c} - r_D)l_{i,1}$$

.

If

If
$$\nexists\, l_{i,1} \in \{\tilde{N} l_1^{*NE}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \beta\,(r - c(l_{i,1}) - r_D)\,\psi \frac{D}{N} \geq (r_D - r + \bar{c})\, l_{i,1}.$$

holds, then the SOC is violated for any $(l_{i,1}, l_{i,2}) \neq (0,0)$ that satisfies the best response function given by Lemma 9. Hence, the best response of bank i given $\bar{L}_{-i,1} = 0$ must be $(0, 0)$. □

Lemma 15. *Let* $l_1^{*NE} := \tilde{N}^{-1}(c')^{-1}\left(-\frac{r_D - r + \bar{c}}{\beta\psi\frac{D}{N}}\right)$. *Then the Nash equilibrium* $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N$ *exists if and only if*

$$l_1^{*NE} \in (0, \frac{D}{N}] \wedge \beta\left(r - c(\tilde{N} l_1^{*NE}) - r_D\right)\psi\frac{D}{N} > (r_D - r + \bar{c})\, l_1^{*NE} \tag{33}$$

Proof. **If**

First, we must show that $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N$ satisfies the FOC for each bank i. Plugging this into Equation 29 yields

$$-r_D + r - \bar{c} - \beta c'(\tilde{N} l_1^{*NE})\psi\frac{D}{N} - \mu_{i,1} + \mu_{i,3} = 0$$

.

Assuming $l_1^{*NE} \in (0, \frac{D}{N}) \implies \mu_{i,3} = \mu_{i,1} = 0$ and given the definition of l_1^{*NE}, this reduces to

$$0 = 0$$

Assuming $l_1^{*NE} = \frac{D}{N} \implies \mu_{i,3} = 0, \mu_{i,1} \geq 0$, the Equation still holds for $\mu_{i,1} = 0$, which does not violate the complementary slackness conditions. Hence, in both cases, the FOC is satisfied.

By Lemma 9, the only possible deviations for bank i from $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N$ are $(0, 0)$, $(0, \psi\frac{D}{N})$ and $(\frac{D}{N}, \psi\frac{D}{N})$. Then, the SOC requires that profits under these deviations are dominated by $(l_1^{*NE}, \psi\frac{D}{N})$.

First, note that

$$\beta\left(r - c(l_1^{*NE}) - r_D\right)\psi\frac{D}{N} > (r_D - r + \bar{c})\, l_1^{*NE}$$

implies that bank i makes an above-zero profit in the potential Nash equilibrium, and hence $(0, 0)$, which yields zero profits, cannot be an individually rational deviation from the potential Nash equilibrium.

For the other two deviations, it suffices to show that if $l_{i,2} = \psi\frac{D}{N}$, then for a given $L_{-i,1}$

$$\pi_i\left(\bar{L}_{-i,1}, (l_{i,1}, \frac{D}{N})\right) = (r - \bar{c} - r_D) l_{i,1} + \beta\left(r - c(l_{i,1} + \tilde{L}_{-i,1}) - r_D\right)\psi\frac{D}{N}$$

is strictly concave in $l_{i,1}$ since the first term is linear, hence weakly concave, in $l_{i,1}$ and $-c(\cdot)$ is strictly concave by the strict convexity of c. Therefore, any deviation from the critical point $(l_1^{*NE}, \psi\frac{D}{N})$ that still features $l_{i,2} = \psi\frac{D}{N}$ must result in strictly lower profits and cannot be a best response.

Only if

Assume that the Nash equilibrium $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^{N}$ exists. By Lemma 7, this then implies that

$$l_1^{*NE} \neq 0$$

and hence, given the demand constraint and the non-negativity condition

$$l_1^{*NE} \in (0, \frac{D}{N}]$$

.

However, if the Nash equilibrium $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^{N}$ exists, then by Lemma 9, this implies above-zero profits. □

Lemma 16. *The Nash equilibrium $(\frac{D}{N}, \psi\frac{D}{N})_{i=1}^{N}$ exists if and only if*

$$r - \bar{c} - \beta\psi\frac{D}{N}c'(\tilde{N}\frac{D}{N}) \geq r_D \wedge \beta\psi\left(r - c(\frac{\tilde{N}}{N}D) - r_D\right) \geq r_D - r + \bar{c}.$$

Proof. **If**

For this part of the proof, we follow the same steps as above for Lemma 15: For

$$r - \bar{c} - \beta\psi\frac{D}{N}c'\left(\tilde{N}\frac{D}{N}\right) \geq r_D$$

the FOC with respect to $l_{i,1}$ is satisfied and $l_1^{*NE} \geq \frac{D}{N}$, i.e., a deviation from the potential Nash equilibrium to $l_{i,1} = l_1^{*NE}$ would violate the demand constraint. By strict concavity of profits given $l_{i,2} = \psi\frac{D}{N}$, deviating away from the critical point l_1^{*NE} to $l_{i,1} = 0$ must yield strictly lower profits. By Lemma 9, the only remaining best response is $(0, 0)$, which cannot be optimal since

$$\beta\psi(r - c\left(\frac{\tilde{N}}{N}D\right) - r_D) \geq r_D - r + \bar{c}$$

implies profits above zero under the potential Nash equilibrium. Hence, the Nash equilibrium exists.

Only if

First, assume that profits of bank i under $(\frac{D}{N}, \psi\frac{D}{N})_{i=1}^{N}$ are non-positive. Since we assume that for zero profits, $(0,0)$ is preferred, then this directly implies that the best response must be $(0,0)$.

Alternatively, assume that

$$r - \bar{c} - \beta\psi\frac{D}{N}c'\left(\tilde{N}\frac{D}{N}\right) < r_D$$

. But then by Lemma 9, this implies that the best response cannot be $(l_{i,1}, l_{i,2}) = (\frac{D}{N}, \psi\frac{D}{N})$. □

Lemma 17. *Let $\mathcal{L}^{NE}$ be the set of Nash equilibria and let $l_1^{*NE} < \frac{D}{N}$. Then $(\frac{D}{N}, \psi\frac{D}{N})_{i=1}^N \notin \mathcal{L}^{NE}$ if $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$ and vice-versa.*

Proof. By Lemma 16, $(\frac{D}{N}, \psi\frac{D}{N})_{i=1}^N \notin \mathcal{L}^{NE}$ requires that

$$r - \bar{c} - \beta\psi\frac{D}{N}c'\left(\tilde{N}\frac{D}{N}\right) \geq r_D$$

whereas by Lemma 15, $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$ requires that

$$r - \bar{c} - \beta\psi\frac{D}{N}c'\left(\tilde{N}l_1^{*NE}\right) = r_D$$

As $l_1^{*NE} < \frac{D}{N}$ and $c'' > 0$, these conditions are mutually exclusive as

$$-\beta\psi\frac{D}{N}c'\left(\tilde{N}\frac{D}{N}\right) < -\beta\psi\frac{D}{N}c'(\tilde{N}l_1^{*NE})$$

. □

Lemma 18. *Let $\mathcal{L}^{NE}$ be the set of Nash equilibria. Then if $(0,0)_{i=1}^N \notin \mathcal{L}^{NE}$, it holds that*

$$(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE} \vee (\frac{D}{N}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$$

.

Proof. By Lemma 14, $(0,0)_{i=1}^N \notin \mathcal{L}^{NE}$ requires that

$$\exists\, l_{i,1} \in \{\tilde{N}l_1^{*NE}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \beta\left(r - c(l_{i,1}) - r_D\right)\psi\frac{D}{N} > \left(r_D - r + \bar{c}\right) l_{i,1}.$$

This condition holds under two cases:

Case 1:

$$\exists\, l_{i,1} = \tilde{N}l_1^{*NE} \in (0, \frac{D}{N}] : \beta\left(r - c(l_{i,1}) - r_D\right)\psi\frac{D}{N} > \left(r_D - r + \bar{c}\right) l_{i,1}.$$

Since this implies that $l_1^{*NE} \in (0, \frac{D}{N})$, both conditions in Lemma 15 are satisfied such that $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$.

Case 2:

$$\tilde{N}l_1^{*NE} \notin (0, \frac{D}{N}] \wedge \beta\psi \left(r - c(\frac{D}{N}) - r_D \right) \frac{D}{N} > (r_D - r + \bar{c}) \frac{D}{N}.$$

By Lemma 9, the latter expression (i.e., bank i can obtain positive profits unilaterally) requires that

$$r - \bar{c} - \beta\psi \frac{D}{N} c'(\frac{D}{N}) > r_D$$

Since $\tilde{N}l_1^{*NE}$ is implicitly defined by

$$r - \bar{c} - \beta\psi \frac{D}{N} c'(\tilde{N}l_1^{*NE}) = r_D$$

and $c'' > 0$, this implies that $\tilde{N}l_1^{*NE} > \frac{D}{N}$.

Now, we need to distinguish two further cases:

Case 2a: $l_1^{*NE} \leq \frac{D}{N}$.

We know that

$$\beta \left(r - c(\frac{D}{N}) - r_D \right) \psi \frac{D}{N} > (r_D - r + \bar{c}) \frac{D}{N}$$

and that $l_1^{*NE} \leq \frac{D}{N}$ while $\tilde{N}l_1^{*NE} > \frac{D}{N}$. Then it directly follows that

$$\beta \left(r - c(\tilde{N}l_1^{*NE}) - r_D \right) \psi \frac{D}{N} > (r_D - r + \bar{c})\, l_1^{*NE}$$

. since $c' < 0$ and by Assumption 1, $r_D - r + \bar{c} > 0$. Hence, both conditions in Lemma 15 are satisfied such that $(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$.

Case 2b: $l_1^{*NE} > \frac{D}{N}$.

Again, we can directly conclude that

$$\beta \left(r - c(\tilde{N}\frac{D}{N}) - r_D \right) \psi \frac{D}{N} > (r_D - r + \bar{c}) \frac{D}{N}$$

.

Hence, both conditions in Lemma 16 are satisfied such that $(\frac{D}{N}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$.

□

C.5 Early financing gap

Lemma 19. *Let $\mathcal{L}^{SO}$ be the set of socially optimal solutions and $\mathcal{L}^{NE}$ be the set of Nash equilibrium solutions. Then, if $(0,0)_{i=1}^N \in \mathcal{L}^{SO}$, then $\mathcal{L}^{NE} = \{(0,0)_{i=1}^N\}$.*

Proof. By the social planner's SOC, $(0,0)_{i=1}^N \in \mathcal{L}^{SO}$ implies that, for any $\bar{L}_1 \in [0, D]$, $\tilde{L}_1 \in$

$\left[0, \frac{\tilde{N}}{N}D\right]$,

$$0 \geq (r - \bar{c} - r_D)\bar{L}_1 + \beta(r - c(\tilde{L}_1) - r_D)\psi D.$$

Dividing both sides of this inequality by N, it is immediately clear that this is equivalent to

$$0 \geq (r - \bar{c} - r_D)l_1 + \beta(r - c(\tilde{L}_1) - r_D)\psi \frac{D}{N} \tag{34}$$

for any $l_1 \in \left[0, \frac{D}{N}\right]$, $\tilde{L}_1 \in \left[0, \frac{\tilde{N}}{N}D\right]$.

Now, assume for contradiction that there is a different Nash equilibrium. This implies that there is at least one bank i, for which $l_{i,1} \in \left(0, \frac{D}{N}\right]$ and $\tilde{L}_{-i,1} \in \left[0, \frac{\tilde{N}-1}{N}D\right]$. However, by bank i's best response function, this would require that

$$0 < (r - \bar{c} - r_D)l_{i,1} + \beta\left(r - c(\tilde{L}_{-i,1} + l_{i,1}) - r_D\right)\psi\frac{D}{N}.$$

But since $L_{-i,1} + l_{i,1} \in \left[0, \frac{\tilde{N}}{N}D\right]$, this would directly contradict the condition in Equation (34) above. □

Lemma 20. *Let the unique socially optimal solution be such that $\bar{L}_1^{SO} \in (0, D)$. Let $\mathcal{L}^{NE}$ be the set of Nash equilibrium solutions. Then, for any $L \in \mathcal{L}^{NE}$ and $N > 1$, it must hold that $\bar{L}_1 < \bar{L}_1^{SO}$.*

Proof. Assume for contradiction that $\bar{L}_1 \geq \bar{L}_1^{SO}$. By Lemma 13 (symmetry of the Nash equilibrium), this implies that $l_{i,1} = l_1 = \frac{\bar{L}_1}{N} \in \left(0, \frac{D}{N}\right]\ \forall\ i$. It also requires that there is at least one bank i with $l_{i,1}^{SO} \leq l_1$ and $\bar{L}_{-i,1} = \bar{L}_1 - l_1 \in \left[0, \frac{N-1}{N}D\right]$. Let i be this bank.

By the banks' best response function, this implies that

$$-c'(\gamma\bar{L}_{-i,1} + l_1) \geq \frac{r_D - r + \bar{c}}{\beta}\frac{N}{\psi D},$$

where the condition holds with strict inequality if and only if $l_1 = \frac{D}{N}$. On the other hand, the FOCs for bank i in the social planner's problem tell us that

$$-c'(\gamma\bar{L}_{-i,1}^{SO} + l_{i,1}^{SO}) \leq \frac{r_D - r + \bar{c}}{\beta}\frac{N}{\psi D}\frac{1}{\tilde{N}},$$

where the condition holds with strict inequality if and only if $l_{i,1}^{SO} = 0$.

As $N > 1$, we know that

$$\frac{r_D - r + \bar{c}}{\beta}\frac{N}{\psi D} > \frac{r_D - r + \bar{c}}{\beta}\frac{N}{\psi D}\frac{1}{\tilde{N}},$$

and hence that

$$-c'(\gamma \bar{L}_{-i,1} + l_1) > -c'(\gamma \bar{L}^{SO}_{-i,1} + l^{SO}_{i,1}).$$

Since $c(\cdot)$ is strictly decreasing and convex, $-c'(\cdot)$ is strictly decreasing, this implies that $\gamma \bar{L}_{-i,1} + l_1 < \gamma \bar{L}^{SO}_{-i,1} + l^{SO}_{i,1}$. Rearranging yields

$$\bar{L}_1 + \frac{1-\gamma}{\gamma}(l_1 - l^{SO}_{i,1}) < \bar{L}^{SO}_1.$$

Since $l_1 - l^{SO}_{i,1} \geq 0$, this implies that $\bar{L}_1 < \bar{L}^{SO}_1$, which contradicts $\bar{L}_1 \geq \bar{L}^{SO}_1$. □

Lemma 21. *Let $(\frac{D}{N}, \psi \frac{D}{N})_{i=1}^N$ be the unique socially optimal solution (immediate financing). Let $\mathcal{L}^{NE}$ be the set of Nash equilibrium solutions. Then, $(\frac{D}{N}, \psi \frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}$ if and only if*

$$-c'(\frac{\tilde{N}}{N}D) \geq \frac{r_D - r + \bar{c}}{\beta}\frac{N}{\psi D}. \tag{35}$$

Proof. If $(\frac{D}{N}, \psi \frac{D}{N})_{i=1}^N$ is the solution to the social maximization problem, the SOC implies that total profits exceed zero, i.e.,

$$\beta \left(r - c(\tilde{N}\frac{D}{N}) - r_D \right) \psi \frac{D}{N} > (r - \bar{c} - r_D)\frac{D}{N}$$

.

which satisfies the second condition in Lemma 16. The condition stated above is then simply the first condition in Lemma 16 restated. □

D Proofs for Propositions 3 and 4 (stand-alone policy interventions)

Using a more formal notation, Propositions 3 and 4 can be stated as follows:

Proposition 3. *Let $l^{SO}_1 \in (0, \frac{D}{N})$, let $s^*_1 := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l^{SO}_1) > 0$ and let $\mathcal{L}^{NE}|s$ be the set of possible Nash equilibria for a given $s_1, s_2 \geq 0$ and $g_1, g_2 = 0$. Then, it holds that*

- $(l^{SO}_1, l^{SO}_2)_{i=1}^N \in \mathcal{L}^{NE}|s \;\forall\; s_1 = s^*_1, s_2 \geq 0$
- $(0,0)_{i=1}^N \in \mathcal{L}^{NE}|s \;\forall\; s_1 = s^*_1, s_2 \geq 0$ *if and only if*

$$\begin{aligned} &\nexists\, l_{i,1} \in \{\tilde{N}l^{SO}_1, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \\ &\beta\,(r - c(l_{i,1}) - r_D + s_2)\,\psi\frac{D}{N} \geq (\bar{c} + r_D - r - s^*_1)\,l_{i,1} \end{aligned} \tag{36}$$

- $(0,0)_{i=1}^N \in \mathcal{L}^{NE}|s \; \forall \; s_1 = s_1^*, s_2 = 0$ *if*

$$(0,0) \in \mathcal{L}^{NE}|s \text{ for } s_1, s_2 = 0 \wedge r - r_D \leq c(\frac{D}{N}) \tag{37}$$

- $\mathcal{L}^{NE}|s = \{(l_1^{SO}, l_2^{SO})_{i=1}^N\} \; \forall \; s_1 = s_1^*, s_2 > \bar{c} + r_D - r$

Proof. See Appendix D.2. □

Proposition 4. *Let* $l_1^{SO} \in (0, \frac{D}{N})$ *and let* $\mathcal{L}^{NE}|g$ *be the set of possible Nash equilibria for a given* $g_1, g_2 \geq 0$ *and* $s_1, s_2 = 0$. *Let* $l_1^{*NE}|g := (c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\phi\beta\psi\frac{D-g_2}{N}}\right) - \frac{\gamma^g}{\tilde{N}}g_1$ *and let* $g_1^* := \frac{1}{\gamma^g}c^{-1}(r - r_D)$. *Then, it holds that*

- $\mathcal{L}^{NE}|g \in \mathcal{P}\left(\left\{\left(\max\{0, \min\{l_1^{*NE}|g, \frac{D-g_1}{N}\}\}, \psi\frac{D-g_2}{N}\right)_{i=1}^N, (0,0)_{i=1}^N\right\}\right) \setminus \emptyset$
- $(0,0)_{i=1}^N \notin \mathcal{L}^{NE}|g \; \forall \; g_1 \in (g_1^*, D), g_2 \in [0, \psi D)$
- $l_1^{*NE}|g$ *decreases in* g_2
- *The parameter space for which* $\left(\max\{0, \min\{l_1^{*NE}|g, \frac{D-g_1}{N}\}\}, \psi\frac{D-g_2}{N}\right)_{i=1}^N \in \mathcal{L}^{NE}|g$ *decreases in* g_2

Proof. See Appendix D.3. The comparative statics for $l_1^{*NE}|g$ follow directly from its definition, keeping in mind that $(c')^{-1}$ is monotonously increasing. □

D.1 First-order conditions

The first-order conditions for the maximization problem by bank i given in Equation 16 are as follows:

$$r - \bar{c} - r_D + s_1 - \beta c'\left(\gamma \sum_{j \neq i} l_{j,1}^{NE} + \gamma^g g_1 + l_{i,1}^{NE}\right) l_{i,2}^{NE} - \mu_{i,1} + \mu_{i,3} = 0 \tag{38}$$

$$r - c\left(\gamma \sum_{j \neq i} l_{j,1}^{NE} + \gamma^g g_1 + l_{i,1}^{NE}\right) - r_D + s_2 - \mu_{i,2} + \mu_{i,4} = 0 \tag{39}$$

$$\mu_{i,1}(\frac{D}{N} - l_{i,1}^{NE} - \frac{g_1}{N}) = \mu_{i,2}(\psi\frac{D}{N} - l_{i,2}^{NE} - \frac{g_2}{N}) = \mu_{i,3} l_{i,1}^{NE} = \mu_{i,4} l_{i,2}^{NE} = 0 \tag{40}$$

$$\mu_{i,1}, \mu_{i,2}, \mu_{i,3}, \mu_{i,4} \geq 0 \tag{41}$$

Considering Equations 38-39, $\forall s_1, s_1, g_1, g_2 \geq 0$, we could simply redefine $r^* := r + s_2$, $r_D^* := r_D - s_1 + s_2$ and $\bar{c}^* := c(\gamma^g g_1)$ and $c^*(x) = c(x + \gamma^g g_1)$ and then face the same maximization problem as before. Then one can follow the steps in Appendix C.3 again to derive that any resulting Nash equilibrium under the policy intervention must be symmetric and take the shape of a no-financing, gradual-financing or immediate-financing equilibrium—in addition, however, it is now also possible that $(0, \psi\frac{D-g_2}{N})_{i=1}^{N}$ is a Nash equilibrium if the policy intervention alone suffices to ensure bankability at $t = 2$. Nevertheless, the existence of this additional outcome does not affect the logical steps that underlie the symmetry of the Nash equilibrium:

Lemma 22. *Let $\mathcal{L}^{NE}|gs$ be the set of possible Nash equilibria for a given $s_1, s_2, g_1, g_2 \geq 0$. Then for any $(l_1^{NE}, l_2^{NE}) \in \mathcal{L}^{NE}|gs$ it holds that*

$$l_{i,t}^{NE} = l_t^{NE} \;\forall\; i = 1, ..., N, t \in \{1, 2\}$$

Proof. Omitted as this involves the same steps as for Lemma 13 with the redefined maximization problem. □

D.2 De-risking subsidy

Lemma 23. *Let $l_1^{SO} \in (0, \frac{D}{N})$. Then $s_1^* := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) < r_D - r + \bar{c}$.*

Proof. If $l_1^{SO} \in (0, \frac{D}{N})$, then by Equation 19 it holds that:

$$r - r_D - \bar{c} - \beta c'(\tilde{N}l_1^{SO})\psi\frac{\tilde{N}}{N}D = 0$$

Recalling that $\tilde{N} := 1 + \gamma(N-1)$, we can rewrite this as

$$-\beta c'(\tilde{N}l_1^{SO})\psi\frac{D}{N} - \beta c'(\tilde{N}l_1^{SO})\gamma(N-1)\psi\frac{D}{N} = r_D - r + \bar{c}$$

As $c' < 0$, both terms on the LHS equals are strictly positive. The second term on the LHS is equal to s_1^*. Therefore, $s_1^* < r_D - r + \bar{c}$. □

Lemma 24. *Let $l_1^{SO} \in (0, \frac{D}{N})$, let $s_1^* := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) > 0$ and let $\mathcal{L}^{NE}|s$ be the set of possible Nash equilibria for a given $s_1, s_2 \geq 0$ and $g_1, g_2 = 0$. Then $(l_1^{SO}, l_2^{SO}) \in \mathcal{L}^{NE}|s \;\forall\; s_1 = s_1^*, s_2 \geq 0$.*

Proof. Imposing symmetry by Lemma 22 and inserting $s_1 = s_1^*, g_1 = g_2 = 0$ into Equation 38 yields:

$$r - \bar{c} - r_D - \beta\gamma(N-1)l_2^{NE}c'(\tilde{N}l_1^{NE}) - \beta c'(\tilde{N}l_1^{NE})l_2^{NE} - \mu_{i,1} + \mu_{i,3} = 0$$

Note that this equals the FOC for the social optimum, such that we can replace (l_1^{NE}, l_2^{NE}) with (l_1^{SO}, l_2^{SO}) and, in addition, set $l_2^{SO} = \psi\frac{D}{N}$, which directly follows from $l_1^{SO} \in (0, \frac{D}{N})$:

$$r - \bar{c} - r_D - \beta\tilde{N}\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) - \mu_{i,1} + \mu_{i,3} = 0$$

Note that this is identical to Equation 19 at $l_2^{SO} = \psi \frac{D}{N}$. By the Appendix version of Proposition 1, $l_1^{SO} \in (0, \frac{D}{N})$ then implies that this satisfies the FOCs of banks. By the Appendix version of Proposition 1, the SOC of above-zero profits (Equation 25) must hold, which by symmetry implies above-zero profits for each bank i. Hence, both conditions in Lemma 15 are met and $(l_1^{SO}, l_2^{SO}) \in \mathcal{L}^{NE}|s$. □

Lemma 25. *Let $l_1^{SO} \in (0, \frac{D}{N})$, let $s_1^* := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) > 0$ and let $\mathcal{L}^{NE}|s$ be the set of possible Nash equilibria for a given $s_1, s_2 \geq 0$ and $g_1, g_2 = 0$. Then $(0,0) \in \mathcal{L}^{NE}|s \;\forall\; s_1 = s_1^*, s_2 \geq 0$ if and only if*

$$\nexists\, l_{i,1} \in \{\tilde{N}l_1^{SO}, \frac{D}{N}\} \cap (0, \frac{D}{N}] : \tag{42}$$

$$\beta\,(r - c(l_{i,1}) - r_D + s_2)\,\psi\frac{D}{N} > (r_D - r + \bar{c} - s_1^*)\,l_{i,1} \tag{43}$$

Proof. This simply reflects the condition in the Appendix version of Proposition 2 for the adjusted maximization problem allowing for $s_1, s_2 \geq 0$. Note that under $s_1 = s_1^*$ and for $l_{j,1} = 0 \;\forall\; j \neq i$, the loan financing amount, for which bank i's FOC w.r.t. $l_{i,1}$ holds with equality, is $\tilde{N}l_1^{SO}$—that is, the amount of loan financing (net of spillover losses) in the social optimum. Then, the remaining proof can be derived in the same steps as for Lemma 14. □

Lemma 26. *Let $l_1^{SO} \in (0, \frac{D}{N})$, let $s_1^* := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) > 0$ and let $\mathcal{L}^{NE}|s$ be the set of possible Nash equilibria for a given $s_1, s_2 \geq 0$ and $g_1, g_2 = 0$. Then, it holds that $(0,0) \in \mathcal{L}^{NE}|s \;\forall\; s_1 = s_1^*, s_2 = 0$ if*

$$r - c(\frac{D}{N}) \leq r_D$$

Proof. Inserting $s_2 = 0$ into the condition in Lemma 25 for the existence of a zero-financing Nash equilibrium yields:

$$\beta\,(r - c(l_{i,1}) - r_D)\,\psi\frac{D}{N} > (r_D - r + \bar{c} - s_1^*)\,l_{i,1}$$

By Lemma 23, the right-hand side is positive if $l_{i,1} > 0$. But by $r - c(\frac{D}{N}) \leq r_D$, the left-hand side is non-positive for all $l_{i,1} \in [0, \frac{D}{N}]$. Hence, the condition in Lemma 25 is satisfied as the inequality condition cannot hold for any $l_{i,1} \in [0, \frac{D}{N}]$. □

Lemma 27. *Let $l_1^{SO} \in (0, \frac{D}{N})$, let $s_1^* := -\beta\gamma(N-1)\psi\frac{D}{N}c'(\tilde{N}l_1^{SO}) > 0$ and let $\mathcal{L}^{NE}|s$ be the set of possible Nash equilibria for a given $s_1, s_2 \geq 0$ and $g_1, g_2 = 0$. Then, it holds that $\mathcal{L}^{NE}|s = \{(l_1^{SO}, l_2^{SO})\} \;\forall\; s_1 = s_1^*, s_2 > r_D + \bar{c} - r$*

Proof. By the Appendix version of Proposition 2, if $l_1^{SO} \in (0, \frac{D}{N})$ then

$$\mathcal{L}^{NE} \in \left\{ \{(0,0)_{i=1}^N\}, \{(0,0)_{i=1}^N, (l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N\}, \{(l_1^{*NE}, \psi\frac{D}{N})_{i=1}^N\} \right\}$$

The same steps can be followed for the re-defined maximization problem under policies in Equation 16 such that

$$\mathcal{L}^{NE}|s \in \left\{\{(0,0)_{i=1}^N\}, \{(0,0)_{i=1}^N, (l_1^{*NE}|s, \psi\frac{D}{N})_{i=1}^N\}, \{(l_1^{*NE}|s, \psi\frac{D}{N})_{i=1}^N\}\right\}$$

By Lemma 24, it holds that

$$(l_1^{SO}, \psi\frac{D}{N})_{i=1}^N \in \mathcal{L}^{NE}|s \;\forall\; s_1 = s_1^*, s_2 \geq 0$$

By Equation 39, the FOC with respect to $l_{i,2}$ requires that:

$$r - c(\gamma \sum_{j\neq i} l_{j,1}^{NE}|s + \gamma^g g_1 + l_{i,1}^{NE}|s) - r_D + s_2 - \mu_{i,2} + \mu_{i,4} = 0$$

which, since $c(\cdot) \leq \bar{c}$, can only hold for $g_1 = 0, s_2 > r_D - (r - \bar{c})$ if $\mu_{i,2} > 0$, i.e., for $l_{i,2}^{NE}|s = \psi\frac{D}{N}$. Hence, $(0,0)_{i=1}^N \notin \mathcal{L}^{NE}|s$. □

D.3 Public loan provision

Lemma 28. *Let $l_1^{SO} \in (0, \frac{D}{N})$ and let $\mathcal{L}^{NE}|g$ be the set of possible Nash equilibria for a given $g_1, g_2 \geq 0$ and $s_1, s_2 = 0$. Let $l_1^{*NE}|g := (c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\phi\beta\psi\frac{D-g_2}{N}}\right) - \frac{\gamma^g}{N}g_1$. Then, it holds that*

$$\mathcal{L}^{NE}|g \in \mathcal{P}\left(\left\{\left(\max\{0, \min\{l_1^{*NE}|g, \frac{D-g_1}{N}\}\}, \psi\frac{D-g_2}{N}\right)_{i=1}^N, (0,0)_{i=1}^N\right\}\right) \setminus \emptyset$$

Proof. By Lemma 22, every solution in $\mathcal{L}^{NE}|g$ must be symmetric. By Lemma 2, $l_1^{SO} < \frac{D}{N}$ implies that $l_1^{*NE} < \frac{D}{N}$, but for very low values of γ and very high values of g_1, it is still possible that the demand constraint $l_{i,1} \leq \frac{D-g_1}{N}$ binds.

Under public loan provision, the FOC with respect to $l_{i,2}$ now requires that

$$r - c(\tilde{L}_{i,1}^{NE}|g + \gamma^g g_1) - r_D - \mu_{i,2} + \mu_{i,4} = 0$$

The deterministic rule for $l_{i,2}^{NE}|g$ then becomes

$$l_{i,2}^{NE}|g = \begin{cases} 0, & \text{if } r - c(\tilde{L}_{i,1} + \gamma^g g_1) \leq r_D \\ \psi\frac{D-g_2}{N}, & \text{otherwise.} \end{cases} \tag{44}$$

Therefore, $\gamma^g g_1 \geq c^{-1}(r - r_D)$ implies that $l_2^{NE}|g = \psi\frac{D-g_2}{N}$.

Similarly, the best response function derived under the steps followed for Lemma 9 changes as follows:

$$
\mathrm{BR}_i(L_{-i})|g = \begin{cases} \left(\frac{D-g_1}{N}, \psi\frac{D-g_2}{N}\right) & \text{if } r-\bar{c}-\beta c'\left(\gamma L_{-i,1}+\gamma^g g_1+\frac{D-g_1}{N}\right)\psi\frac{D-g_2}{N} > r_D \wedge \\ & \quad r - c(\gamma L_{-i,1}+\gamma^g g_1 + \frac{D-g_1}{N}) > r_D \wedge \\ & \quad \pi_i(L_{-i}, (\frac{D-g_1}{N}, \psi\frac{D-g_2}{N})) > 0 \\ \left(l^*_{i,1}|g, \psi\frac{D-g_2}{N}\right) & \text{if } \exists l_{i,1} \in [0, \frac{D-g_1}{N}) : \\ & r-\bar{c}-\beta c'\left(\gamma L_{-i,1}+\gamma^g g_1+l_{i,1}\right)\psi\frac{D-g_2}{N} = r_D, \\ & r - c(\gamma L_{-i,1}+\gamma^g g_1 + l_{i,1}) > r_D, \\ & \pi_i(L_{-i}, (l_{i,1}, \psi\frac{D-g_2}{N})) > 0 \\ \left(0, \psi\frac{D-g_2}{N}\right) & \text{if the prior conditions are not satisfied and in addition,} \\ & r - c(\gamma L_{-i,1}+\gamma^g g_1) > r_D \\ (0,0) & \text{otherwise} \end{cases}
$$

with

$$
l^*_{i,1}|g = (c')^{-1}\left(\frac{r_D - r + \bar{c}}{\beta\psi\frac{D-g_2}{N}}\right) - \gamma L_{-i,1} - \gamma^g g_1.
$$

Steps to derive the possible Nash equilibria are the same as in Appendix C.2—with one important difference: By Equation 44, it is now possible that $(0, \psi\frac{D-g_2}{N})_{i=1}^N \in \mathcal{L}^{NE}|g$ if $\gamma^g g_1 > c^{-1}(r - r_D)$.

Deriving the symmetric amount of loan financing that solves the FOC with respect to $l_{i,1}$ yields:

$$
\tilde{N} l_1^{*NE}|g = (c')^{-1}\left(\frac{r_D - r + \bar{c}}{\beta\psi\frac{D-g_2}{N}}\right) - \gamma^g g_1.
$$

which, subject to the non-negativity and the demand constraint, leads to the possible definitions of $\mathcal{L}^{NE}|g$ in Lemma 28.

□

Lemma 29. *Let $l_1^{SO} \in (0, \frac{D}{N})$ let $\mathcal{L}^{NE}|g$ be the set of possible Nash equilibria for a given $g_1, g_2 \geq 0$ and $s_1, s_2 = 0$. Then the parameter space, for which $\left(\min\{l_1^{*NE}|g, \frac{D-g_1}{N}\}\}, \psi\frac{D-g_2}{N}\right)_{i=1}^N \in \mathcal{L}^{NE}|g$, decreases in g_2.*

Proof. Based on the best-response function $BR_i(L_i)|g$ above, the requirement for a Nash equilibrium with $l_1^{NE}|g > 0$ requires that

$$
r - \bar{c} - \beta c'\left(\tilde{N} l_1^{NE}|g + \gamma^g g_1\right)\psi\frac{D-g_2}{N} \geq r_D
$$

As $c' < 0$, this is, ceteris paribus, less likely to hold for $g_2 \uparrow$.

The FOC w.r.t. $l_{i,2}$ does not depend on g_2, while bank profits π_i ceteris paribus also decrease weakly monotonously in g_2, making the profitability condition in the best-response function less likely to hold as well. Therefore, all three conditions for a Nash equilibrium with $l_1^{NE}|g > 0$ are either less likely to hold or unaffected by $g_2 \uparrow$.

$\square$

Lemma 30. *Let $l_1^{SO} \in (0, \frac{D}{N})$ and let $\mathcal{L}^{NE}|g$ be the set of possible Nash equilibria for a given $g_1, g_2 \geq 0$ and $s_1, s_2 = 0$. Let $g_1^* := \frac{c^{-1}(r - r_D)}{\gamma^g}$. Then if $g_1 > g_1^*$, it holds that $(0,0) \notin \mathcal{L}^{NE}|g$.*

Proof. This follows directly from Equation 44 since $g_1 > g_1^*$ implies that $l_2^{NE}|g = \psi \frac{D - g_2}{N}$. Importantly, this is a sufficient but not a necessary condition for $(0,0) \notin \mathcal{L}^{NE}|g$ because a value for g_1 that is (slightly) below g_1^* might still enable an individual bank i to reach positive profits by deviating unilaterally from the $(0,0)$ Nash equilibrium. $\square$

E Proofs for Lemmas 3 and 4 (policy costs & policy mix)

E.1 Comparing the de-risking subsidy and public loan provision for addressing the coordination failure

Lemma 31. *Let $l_1^{SO} \in (0, \frac{D}{N})$, $g_1 = g_1^* + \epsilon, g_2 = s_1 = 0$ and $s_2 = r_D + \bar{c} - r + \epsilon$ where $\epsilon > 0$ is an infinitesimally small positive constant. Then, it holds that*

$$g_1^* < \beta^g r_D^g D \implies PC(g_1) < PC(s_2) \; \forall \; r_D^g = r_D, r^g = r, \beta^g \geq 0, g_2 \geq 0, l_1^{NE}|g \geq 0$$

Proof. If the characteristics of the public loan provider and private banks are identical and ϵ is negligible, then

$$PC(s_2) \approx (r_D + \bar{c} - r)\beta\psi D r_D$$

$$PC(g_1) = (r_D + \bar{c} - r)g_1 - \beta\left(r - c(g_1 + \gamma N l_1^{NE}|g) - r_D\right)g_2$$

Since g_2 can always be set to zero if second-period public loans are non-profitable, i.e., if $r - c(g_1 + \gamma N l_1^{NE}|g) \leq r_D$, this implies that

$$PC(g_1) \leq (r_D + \bar{c} - r)g_1$$

such that any $g_1 < \beta\psi D r_D$ satisfies $PC(g_1) < PC(s_2)$.

$\square$

E.2 Policy mix

Using a more formal notation, Lemma 4 can be stated as follows:

Lemma 32. *Let $l_1^{SO} \in (0, \frac{D}{N})$, let $\mathcal{L}^{NE}|gs$ be the set of possible Nash equilibria given $g_1 = g_1^* + \epsilon$ and $s_1 = s_1^*, g_2 = s_2 = 0$. Then, it holds that*

- $\mathcal{L}^{NE}|gs = \{(l_1^{SO} - \frac{\gamma^g}{\tilde{N}} g_1, \psi \frac{D}{N})_{i=1}^N\} \; \forall \; g_2 \geq 0$
- $l_1^{NE}|g < l_1^{NE}|gs < l_1^{SO}$

Proof. Note that the FOC with respect to $l_{i,2}$ in Equation 39 is unaffected by $s_1 = s_1^*$. Hence, Lemma equally applies such that $(0,0)_{i=1}^N \notin \mathcal{L}^{NE}|gs$ for $g_1 = g_1^* + \epsilon$.

By $l_1^{SO} \in (0, \frac{D}{N})$ and Equation 11, we know that

$$-\beta\psi \frac{D}{N} c'(\tilde{N} l_1^{SO}) \left(1 + \gamma(N-1)\right) = r_D + \bar{c} - r$$

Under the given policy mix, the FOC with respect to $l_{i,1}$ from the individual bank's profit maximization in Equation 38 then yields

$$-\beta\psi \frac{D}{N} c'(\tilde{N} l_1^{*NE}|gs + \gamma^g g_1) + s_1^* = r_D + \bar{c} - r$$

Inserting the definition of s_1^* then gives us

$$-\beta\psi \frac{D}{N} \left(c'(\tilde{N} l_1^{*NE}|gs + \gamma^g g_1) + \gamma(N-1) c'(\tilde{N} l_1^{SO}) \right) = r_D + \bar{c} - r$$

Since bank i takes g_1 as exogenous, combining this with the FOC from the social maximization problem requires

$$\tilde{N} l_1^{SO} = \tilde{N} l_1^{*NE}|gs + \gamma^g g_1$$

Note that by the Appendix version of Proposition 1, $l_1^{SO} \in (0, \frac{D}{N})$ implies strictly positive overall profits. Given the definition of g_1, this implies that

$$\tilde{N} l_1^{SO} > \gamma^g g_1$$

because for $\bar{L}_1 = 0, g_1 = g_1^* + \epsilon$, the return spread at $t = 2$ is zero, and banks make zero profits in both periods. Therefore,

$$l_1^{SO} - \frac{\gamma^g}{\tilde{N}} g_1 > 0$$

such that $l_1^{*NE}|gs \in (0, \frac{D}{N})$.

Regarding the second statement in Lemma 32, $l_1^{NE}|gs < l_1^{SO}$ follows directly from

$$l_1^{NE}|gs = l_1^{SO} - \frac{\gamma^g}{\tilde{N}} g_1$$

since $\gamma^g, g_1, \tilde{N} > 0$.

By the Appendix version of Proposition 4, the maximum value $l_1^{NE}|g$ that can take for any g_1 is $(c')^{-1}\left(-\frac{\bar{c}+r_D-r}{\phi\beta\psi\frac{D-g_2}{N}}\right)-\frac{\gamma^g}{\tilde{N}}g_1$. Given $g_2=0$, this is equivalent to $l_1^{*NE}-\frac{\gamma^g}{\tilde{N}}g_1$, where l_1^{*NE} is the unconstrained symmetric solution to the market outcome FOC w.r.t. $l_{i,1}$ in the absence of any policy intervention. From the Appendix version of Proposition 2, it follows that

$$l_1^{*NE} < l_1^{SO}$$

This implies that

$$l_1^{*NE} - \frac{\gamma^g}{\tilde{N}}g_1 < l_1^{SO} - \frac{\gamma^g}{\tilde{N}}g_1$$

which is equivalent to

$$l_1^{NE}|g < l_1^{NE}|gs$$

$\square$

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