

DISCUSSION

// NO.22-056 | 11/2022

DISCUSSION PAPER

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The Value of Decentralization Using the Blockchain

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October 25, 2022

Abstract

Although blockchain technology and cryptocurrencies have grown in popularity over the past years, there does not seem to be a consensus if they bring any value to economic interactions. In this paper, I argue that a fundamental value the blockchain provides is commitment. I develop a model of an entrepreneur, who can create a network for her users. She can decide to retain control of the network with centralized implementation through a regular company, or surrender control over the network with a decentralized implementation through the blockchain. Users that join the network are subject to a locked-in effect. I show that a decentralized implementation of the network is (i) preferred by the entrepreneur and (ii) a Pareto improvement, if and only if the size of the locked-in effect is sufficiently large.

Keywords: Blockchain, Smart Contracts, Decentralization, Cryptocurrency, Commitment, Networks

JEL Classification: C70, D00, D2, D4, L2

*I thank Thomas Tröger for his continued support. I also thank Piotr Dworzak, Vitali Gretschko, Carl-Christian Groh, Federico Innocenti, Scott Duke Kominers, Volker Nocke, Marion Ott, Jonas von Wangenheim and audiences at the 2022 DICE Winter School for Applied Micro Theory, the 2022 CRC TR 224 Young Researchers Workshop and the 10th CRC TR 224 Retreat for insightful comments. This work was supported by the University of Mannheim's Graduate School of Economic and Social Sciences. Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project B01) is gratefully acknowledged.

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1 Introduction

AWS (Amazon Web Services), Google, Facebook, Spotify, and Twitter are some of the largest tech companies that have billions of users worldwide. Suppose an entrepreneur wanted to create a competitor to these companies. Should she do so by founding an ordinary company, or should she follow in the footsteps of Filecoin¹, Internet Computer², Presearch³, Decentralized Social⁴, Minds⁵ and Audius⁶, and use a decentralized implementation that leverages blockchain technology?⁷ What is the value of decentralization, and what are an entrepreneur's incentives to decentralize?

Vitalik Buterin, co-founder of the Ethereum blockchain, argues that decentralization is, among other things, useful for “Collusion resistance — it is much harder for participants in decentralized systems to collude to act in ways that benefit them at the expense of other participants, whereas the leaderships of corporations and governments collude in ways that benefit themselves but harm less well-coordinated citizens, customers, employees, and the general public all the time.”⁸ This sentiment is shared throughout the white papers of several of the networks listed above. For example, the Presearch white paper states⁹: “With traditional search engines, not only do we often lose control of information about ourselves, we allow others to monetize that information for their own benefit.” Internet Computer asserts that “the IC platform provides an alternative to the consolidation driven by large technology companies that are monopolizing the Internet.”¹⁰ Decentralized Social shares concerns that “A handful of private companies effectively control public discourse, and earn monopoly profits off of content that they don't even create.”¹¹

The contribution of this paper is to develop a theoretical model that determines when an entrepreneur should implement a network in a centralized manner and when it is optimal to decentralize through the use of a blockchain. With that, I provide an answer to a question that is frequently raised when it comes to the topic of blockchain and cryptocur-

¹<https://www.filecoin.io/> is “a decentralized storage network designed to store humanity's most important information”.

²<https://www.internetcomputer.org/> is a “public blockchain that hosts smart contracts [...]”.

³<https://www.presearch.io/> is a “decentralized search engine”.

⁴<https://www.deso.org/> is “the decentralized social blockchain”.

⁵<https://www.minds.com/> is an “open source social network dedicated to Internet freedom”.

⁶<https://audius.org/> is a “decentralized protocol for audio content”.

⁷Other examples of centralized companies with decentralized counterparts include: 1) payment processors such as Visa and various cryptocurrencies 2) centralized finance providers such as Banks and decentralized finance (DeFi) applications and centralized exchanges such as Binance and Coinbase and decentralized exchanges (Dex) such as Uniswap, Pancakeswap and others.

⁸<https://medium.com/@VitalikButerin/the-meaning-of-decentralization-a0c92b76a274>

⁹<https://presearch.io/vision.pdf>, page 7

¹⁰<https://dfinity.org/whitepaper.pdf>, page 3

¹¹<https://docs.deso.org/about-deso-chain/readme>

rencies: *Why should anybody use it?* Specifically, I show that an entrepreneur may prefer a decentralized implementation of a network through the use of the blockchain and smart contracts¹² to generate commitment. As the core friction at play, I assume that users of the network are subject to a locked-in effect due to switching costs. Shapiro and Varian (1998) remark that “switching costs are the norm, not the exception, in the information economy”, strengthening the case for a wide range of applications of my model. If the frictions that arise due to the potential abuse of this locked-in effect by the entrepreneur are sufficiently large, I show that an entrepreneur prefers decentralizing her network. As a result, she effectively gives up control over the network and thus generates commitment to not abuse the locked-in effect of the users.

Achieving such commitment can lead to a Pareto improvement compared to a centralized implementation of a network through an ordinary company. That is, both the entrepreneur who creates the network, and the users may be better off if the network is decentralized. However, decentralization also comes at a cost for the entrepreneur: she surrenders the control over the network to the users and, to align incentives, engages in revenue sharing. Therefore, there is a trade-off between the costs of centralization and decentralization. I show that if the locked-in effect is small, an entrepreneur should implement her network in a centralized manner. On the other hand, if the locked-in effect is sufficiently large, an entrepreneur should implement her network in a decentralized manner. Given this result, the list of companies with decentralized counterparts is not surprising. Arguably, users of AWS, Facebook, and other tech companies are subject to particularly large locked-in effects.

In the model, an entrepreneur (she) creates a network for her (potential) users (he). The users need the network to interact or achieve a goal. However, they lack the ability to develop a technological solution that suits their needs. The entrepreneur, on the other hand, possesses the necessary skills to build a network that fits the users’ needs. At the start of the game, the entrepreneur decides between a centralized implementation of the network through a regular company and a decentralized implementation using the blockchain. In both implementations, the network can be monetized (for example through advertisement, sale of user data, or other means), and any revenues that are raised can be shared between the entrepreneur and the users. The entrepreneur and the users interact with each other through the network over an infinite time horizon. If the entrepreneur chooses centralized governance, she can change monetization and revenue sharing in every period. Each period, the existing users of the network have the choice to stay in the network or leave the network. Further, new users arrive every period and can choose to join the network. If the entrepreneur chooses decentralized governance,

¹²A brief explanation of blockchain, smart contracts, and some examples can be found in section 4

revenue sharing is decided by the entrepreneur through the tokenomics¹³ at the start of the game. Then, the users decide on monetization in every period through decentralized governance.¹⁴ As in centralized governance, each period, the existing users of the network have the choice to stay in the network or leave the network, and new users arrive who have the choice to join the network.

There is complete information, and the full history of the game is observed by both the entrepreneur and the users. The entrepreneur is purely interested in generating revenue through monetization, while the users' utility is threefold: First, they derive utility from using the network. Second, they dislike monetization such as advertisements, and third, they benefit from any revenue that is shared with them. I use sub-game perfect equilibria to analyze the game. Therefore, an entrepreneur using a centralized implementation of the network is unable to credibly commit to future levels of monetization and revenue sharing. Instead, her choice of monetization and revenue sharing has to be sequentially optimal for every history of the game given the strategy of the users.

The analysis of the model is broken down into three subsections. First, the sub-game of centralized governance. Second, the sub-game of decentralized governance and third, determining the optimal governance structure for the network.

In the analysis of centralized governance, I show that the equilibrium of the game features two distinct phases. A growth phase in which new users join the network, and an exploitation phase in which no new users join the network and the entrepreneur exploits the locked-in effect of the existing user base through increased monetization and decreased revenue sharing. The transition between the two phases crucially depends on the network's future growth and is characterized by the point at which the entrepreneur is indifferent between attracting new users and foregoing growth to exploit the locked-in effect of the existing user base. In equilibrium, the users anticipate being locked-in to the network and have to be compensated up front to be incentivized to join the network in the first place.¹⁵ The compensation equals the discounted value of the switching costs that lead to the locked-in effect. Thus, as the severity of the locked-in effect increases, it becomes increasingly harder for the entrepreneur to attract users in the first place. I show that for a sufficiently large locked-in effect, no users join the network in equilibrium, resulting in zero revenues for the entrepreneur. This highlights the commitment prob-

¹³Tokenomics is a mix of the two words token and economics. "Token" refers to a digital asset. The tokenomics then describe the underlying economics of that particular token, such as supply, distribution, vesting and other parameters.

¹⁴In practice, there are many mechanisms for on-chain governance. In the model, I use majority voting, where 1 unit of the token equals 1 vote, and an even split of tokens among the users.

¹⁵This property of the equilibrium is nicely summarized in Shapiro and Varian (1998)'s advice to buyers that anticipate becoming locked-in: "Bargain hard at the outset of the lock-in cycle for a sweetener or some form of long-term protection before you become locked in"

lem, that an entrepreneur may try to solve with decentralization through the blockchain.¹⁶

If the entrepreneur chooses decentralized governance, the degree of monetization is decided by the users. Unlike the entrepreneur, the users internalize the negative effects of monetization through their utility function. As a result, the locked-in effect of the network will not be abused when it is controlled by the users. To align incentives, the entrepreneur engages in revenue sharing with the users. Further, the network grows every period, unlike in centralized governance. However, decentralized governance has two drawbacks. First, the entrepreneur surrenders control over the network, such that she cannot choose the degree of monetization she prefers. Second, because users choose the degree of monetization, the entrepreneur has to engage in revenue sharing to align incentives. In the optimum, the entrepreneur shares half the revenues with the users, which imposes a significant cost onto decentralization.

Finally, I determine the optimal governance structure of the network by comparing centralized governance to decentralized governance. I show that for minimal locked-in effects, an entrepreneur is always better off choosing centralized governance. In contrast, for a sufficiently large locked-in effect, decentralized governance is preferred, as the entrepreneur is unable to attract any users when choosing centralized governance. To determine the optimal mode of governance for an arbitrary size of the locked-in effect, I show that the revenue that the entrepreneur can achieve with centralized governance is a decreasing function of the size of the locked-in effect. In contrast, the revenue that the entrepreneur can raise with decentralized governance is independent of the size of the locked-in effect. Thus, there exists a threshold size, such that the entrepreneur should decentralize her network if and only if the locked-in effect is sufficiently severe.

This paper contributes to the literature on the economics of blockchains. It most closely relates to papers that have discussed blockchain technology regarding commitment and competition. Similar to Sockin and Xiong (forthcoming), I consider an entrepreneur who can exploit the platform's users and show that creating commitment through the blockchain may be beneficial for the entrepreneur. My paper contributes relative to theirs as follows: First, they consider a one shot interaction between the entrepreneur and the users on the platform. As such, in centralized governance, exploitation occurs for sure since there is no ongoing relationship between the entrepreneur and the users. I contribute by considering a repeated interaction between the entrepreneur and the users, and show that the problem of exploitation persists even in repeated interactions. Cru-

¹⁶An alternative solution to creating commitment for a centralized network could be contracting over monetization and revenue sharing. However, it is likely that these contracts would be incomplete. Thus, contracting may face issues such as renegotiation, as discussed in the literature on incomplete contracts (e.g., Hart and Moore (1988), Hart and Moore (1999)), and fail to be a suitable solution.

cially, I consider the potential for user growth in the network, and show that user growth can be a substitute for commitment when future growth is strong, but fails to generate commitment when future growth is sufficiently low. Further, the longer time horizon allows me to consider locked-in effects and show that the entrepreneur decentralizes her network if and only if the locked-in effect is sufficiently large.

Goldstein et al. (2019) investigate the use of initial coin offerings (ICOs) and tokenized platforms for commitment. They argue that using an ICO and committing to the free resale of tokens can enable a monopolistic entrepreneur to commit to competitive pricing. My paper complements their contribution by focussing on the importance of locked-in effects in platforms. Both papers demonstrate that commitment through the blockchain may improve welfare. However, Goldstein et al. (2019) show that committing to the free resale of tokens through an ICO yields lower profits for an entrepreneur compared to operating the network in a traditional, centralized manner. In contrast, I show that an entrepreneur can increase her revenue by implementing her network through the blockchain, if the costs of centralization are too large. Further, I contribute by adding network growth and showing that growth can be a substitute for commitment at first, but fails to be a substitute for commitment, when growth slows down over time.

Another paper that addresses monopoly pricing is Huberman et al. (2021). They focus on bitcoin as a payment system (BPS), which can be considered as a network in the terms of my model, and show that user surplus in the BPS is larger compared to a monopolist payment provider. However, the incentives for a monopolist to set up a decentralized network such as bitcoin remain unclear. What further differentiates their work from my paper is that I focus on a broader range of applications that is not restricted to payment systems.

Catalini and Gans (2018) focus on entrepreneurs that are capital constrained and need to raise capital through an ICO to fund their network. They show that competition among consumers gives rise to the value of the tokens issued in the ICO. As another possible explanation for the popularity of ICOs, Bakos and Halaburda (2018), Li and Mann (2018) and Cong et al. (2021), show how ICOs can mitigate coordination failures in the users' decision to join or not join a particular network. These types of coordination failures are not a salient feature of my model. Rather, it offers a complementary rationale for the use of the blockchain without such coordination failures.

Finally, Howell et al. (2020), Adhami et al. (2018) conduct empirical assessments of ICOs. Howell et al. (2020) examine the characteristics that determine the success of an ICO. They find that "success is associated with disclosure, credible commitment to the network, and quality signals", underlining the value of commitment as argued in my paper.

Adhami et al. (2018) show that ICOs are more successful if “the code source is available, when a token presale is organized, and when tokens allow contributors to access a specific service (or to share profits)”. Connecting this empirical finding to the theory, profit sharing also is a key feature for the success of the decentralized implementation of the network in my paper.

Arruñada and Garicano (2018) and Chen et al. (2021) investigate the details of decentralized governance more closely. Further, this paper also relates to the literature of blockchain consensus, as it shares some intersections with blockchain governance. Contributions include Abadi and Brunnermeier (2018), Biais et al. (2019), Catalini et al. (2020) and Saleh (2021). Decentralization through the blockchain gives users decision power in the network. Thus, my paper also shares some commonalities with the literature on common ownership in traditional corporations. Magill et al. (2015) argue how common ownership, implemented by employee and consumer rights, may improve welfare compared to shareholder value maximization. Cres et al. (2020) solve for the optimal distribution of voting rights such that the Cournot-Walras equilibrium allocations are Pareto optimal. Azar and Vives (2021) establish conditions under which common ownership leads to market concentration, and discuss how such markets should be regulated.

Another strand of the literature that connects to my model is the literature on (two-sided) platforms, as all the applications I have mentioned are platforms. Important contributions include for example Katz and Shapiro (1985), Rochet and Tirole (2003) and Armstrong (2006). They focus on equilibrium pricing and competition between platforms. As such, my paper is complementary to this literature, as my model features neither competition between platforms nor focuses on prices for either side of the market. I focus on the value of commitment for the entrepreneur as a function of the size of the locked-in effect of the platform. For an overview of the literature, see for example Farrell and Klemperer (2007).

Conceptually, my model connects to the literatures on dynamic and repeated games. It resembles an infinitely repeated game, as the users who are already present in the platform repeatedly interact with each other and with the entrepreneur. However, it differs from the ordinary setup consisting of a super game that is a series of infinitely repeated stage games. In my model, new users keep arriving to join the network as the network grows. This adds new players to the game at any point in time. Further, as users join or leave the network, the network size and with that, the payoffs of the users and the entrepreneur vary. In section 3, I address equilibrium multiplicity that is similar to what is observed through various folk theorems (e.g., Abreu (1983), Abreu et al. (1986), Fu-

denberg and Maskin (1990)). However, I argue that unlike the equilibrium presented in the main section of the paper, these alternative equilibria are susceptible to coordination failures: They are not robust to small uncertainties in equilibrium selection on the users' side in response to small deviations by the entrepreneur.

The rest of the paper is structured as follows: Section 2 consists of the model and the results that outline when decentralization through the blockchain is preferable to centralization. Section 3 offers a discussion of alternative equilibria of the model. Section 4 gives a brief overview over the blockchain technology, some use cases, and how it enables an entrepreneur to generate commitment. Readers that are interested in learning more about blockchains and smart contracts may want to read section 4 before proceeding with the model. Section 5 concludes.

2 Model

The model is a sequential game with infinitely many periods $t = 0, 1, 2, \dots$ between an entrepreneur (she) and a continuum of users (he), indexed by i . The entrepreneur creates a network for the users in $t = 0$ and the mass of users in the network at time t is denoted by μ_t . In $t = 1, 2, 3, \dots$ the network can be monetized (for example through advertisement, sale of user data, or other means), and any revenues that are raised can be shared between the entrepreneur and the users. The revenue from monetization can be decomposed into two parts. First, there is a certain level of monetization of the network $\pi_t \in \mathbb{R}_+$. This variable represents the intensity with which the network is monetized, such as how often or how many advertisements are displayed, or how much of the user data is sold. Second, given a measure of users μ_t and a level of monetization π_t , the revenue generated by the network equals $\pi_t \phi(\mu_t)$ where ϕ is an increasing, continuously differentiable function with $\phi(0) = 0$. $\phi(\mu_t)$ represents the rate an advertiser is willing to pay for advertisements or for user data. The fraction of revenue that the entrepreneur keeps is denoted by α_t , while the leftover fraction of revenue $(1 - \alpha_t)$ is shared with the users.

How monetization and revenue sharing are chosen depends on the mode of governance of the network. At the beginning of the game, in $t = 0$, the entrepreneur chooses the mode of governance (centralized or decentralized). If the entrepreneur chooses centralized governance, she can change monetization π_t and revenue sharing α_t in every period $t = 1, 2, \dots$. Each period, users have a binary choice. The existing users of the network have the choice to stay in the network or leave the network. Further, new users arrive every period and can choose to join or not join the network. If the entrepreneur chooses decentralized governance, she commits to a fixed percent-

age¹⁷ α of revenue sharing in $t = 0$ through the tokenomics of the network. She achieves this through the appropriate distribution of the network's token between herself and the users.¹⁸ In every period $t = 1, 2, \dots$ the users of the network determine the amount of monetization π_t through *on-chain governance*. As in centralized governance, each period, users have a binary choice. The existing users of the network have the choice to stay in the network or leave the network. Regardless of the mode of governance, users that decide to leave the network or newly arriving users who decide not to join the network drop out of the game and realize the value of their outside option.

The arrival of new users is governed by a growth function $g(\mu_{t-1})$: Let μ_{t-1} be the mass of users present in the network at period $t - 1$. Then, in period t there will be $g(\mu_{t-1}) - \mu_{t-1} \geq 0$ potential new users that arrive to the network. If those new users join, the new measure of users in the network is equal to $g(\mu_{t-1})$. If they do not join, the network remains at μ_{t-1} users. The growth function g is continuously differentiable and the mass of users in period 0 is equal to $\mu_0 = 0$. If the network loses all its users within a period, no new users will arrive at any point in the future. This assumption rules out cyclical equilibria in which the entrepreneur continuously "starts over". There is complete information and both the entrepreneur and the users observe the full history of the game.

The entrepreneur is strictly interested in revenue: her utility in a particular period t is equal to her revenue share α_t multiplied by the revenue raised by monetization $\pi_t \phi(\mu_t)$: $u_t^E = \alpha_t \pi_t \phi(\mu_t)$. The utility a user receives from participating in the network has three components: First, a user derives utility $V(\mu_t)$ from using the network. I assume that V is increasing, continuously differentiable and that $V(0) = 0$. Second, as a result of the monetization of the network, π_t , the user's utility decreases by $k\pi_t^2$, where $k > 0$ describes the user's aversion to monetization. This represents the decrease in utility a user suffers when being forced to watch advertisements, through the sale of his data, or other detrimental effects of monetization. As a third component, a user may potentially receive a share of the revenues that the network generates. I assume that this share is equally split between all users, such that each user receives a fraction $\frac{1-\alpha_t}{\mu_t}$ of the revenue. The utility function of a user thus equals $u_t = V(\mu_t) - k\pi_t^2 + \frac{(1-\alpha_t)}{\mu_t} \pi_t \phi(\mu_t)$.

A user who newly arrives in the network can decide to join the network and realize the utility as described above. If the user decides not to join the network, he realizes

¹⁷In an extension in appendix A.9, I allow the entrepreneur to pre-commit to a path for revenue sharing and show that she chooses a constant percentage of revenue sharing. Thus, considering a fixed percentage throughout the main body of the paper is without loss of generality.

¹⁸For the example of Uniswap, 60% of the token supply has been allocated to users, while the other 40% is split between the Uniswap team, investors, and advisors. For details, see <https://uniswap.org/blog/uni>.

an outside option that is normalized to 0. A user who has already taken part in the network for at least one period can decide to stay in the network, realizing the utility of participating, or leave the network. However, the outside option for these users is equal to $-u < 0$. Users that already take part in the network suffer from a *locked-in effect*. They have gotten used to the network and derive less utility from their outside option than before. This assumption represents the idea that users in a social network may have formed connections with others that they do not want to lose, or users have spent time interacting with the network, such that its algorithm has adapted to their needs.¹⁹ An equivalent interpretation is that the value of the outside option has remained constant, but users encounter a switching cost equal to u when leaving the network in favor of the outside option.

Both the entrepreneur and the users maximize the sum of their discounted utilities. Future utilities are discounted by a common discount factor $\delta \in (0, 1)$. I divide the analysis into subsections dedicated to the sub-games of centralized and decentralized governance. Within those sections, I give a detailed description of the structure of the sub-games of centralized and decentralized governance. Then I derive the sub-game perfect Nash equilibria and discuss their properties. Finally, I determine the optimal decision of the entrepreneur at the start of the game: to choose centralized or decentralized governance for her network.

2.1 Centralized Governance

If the entrepreneur chooses centralized governance, every period $t = 1, 2, \dots$ has the following timing:

1. The entrepreneur chooses a level of monetization π_t and a fraction of revenue sharing α_t
2. Users make a simultaneous choice:
 - (a) Users that arrived in period t choose to join or not to join
 - (b) Users who are already present in the network choose to stay or leave
3. Utilities realize

A centralized entrepreneur retains full control over the monetization and revenue sharing of the network. However, she lacks the ability to commit to the levels of monetization

¹⁹For example, Google's search algorithm learns from a user's past searches and improves its search results. Spotify's algorithm learns a user's taste in music, improving the likelihood of playing music that the user likes.

and revenue sharing for future periods because her strategy has to be sequentially optimal. Now I can define strategies for the entrepreneur and the users in more detail. For that, define by h_t a history of the game up to period t . Then a strategy is defined as a mapping from the set of possible histories into the possible actions. Specifically, for the entrepreneur, a strategy maps any possible history into some degree of monetization π_t and revenue sharing α_t . For the users, a strategy maps into the binary decisions of join or don't join at their time of arrival in the network, or, if already present in the network, into a binary decision of staying or leaving. I impose the following tie-breaking rules: Newly arriving users that are indifferent between two strategies, such that one prescribes joining the network and one prescribes not joining the network will join the network. Users that are indifferent between two strategies in some period t such that one strategy prescribes leaving the network and another strategy prescribes not leaving in the network will choose to remain in the network.

As a preliminary step in the analysis, it is useful to think about efficient choices of monetization and revenue sharing within a given period. That is, what choice of monetization and revenue sharing maximizes the entrepreneur's revenue, given that the users should receive some arbitrary level of utility \hat{u} ?

Lemma 1 *Consider the entrepreneur's problem to maximize revenue myopically in a single period t while ensuring utility \hat{u} for users when the network size is μ_t .*

1. If $\frac{\phi(\mu_t)^2}{4k\mu_t^2} + V(\mu_t) < \hat{u}$ the entrepreneur is unable to ensure utility \hat{u} for the users.
2. If $\frac{\phi(\mu_t)^2}{4k\mu_t^2} + V(\mu_t) \geq \hat{u}$ and

(a) $\left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 \geq \frac{V(\mu_t) - \hat{u}}{k}$, the optimal π_t, α_t are given by

$$\pi_t = \frac{\phi(\mu_t)}{2k\mu_t} \tag{2.1}$$

$$\alpha_t = \frac{1}{2} + \frac{2k\mu_t^2(V(\mu_t) - \hat{u})}{\phi(\mu_t)^2} \tag{2.2}$$

The entrepreneur's revenue is equal to

$$\mu_t V(\mu_t) + \frac{\phi(\mu_t)^2}{4k\mu_t} - \mu_t \hat{u} \tag{2.3}$$

(b) $\left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 < \frac{V(\mu_t) - \hat{u}}{k}$, the optimal π_t, α_t are given by

$$\pi_t = \sqrt{\frac{V(\mu_t) - \hat{u}}{k}} \quad (2.4)$$

$$\alpha_t = 1 \quad (2.5)$$

The entrepreneur's revenue is equal to

$$\sqrt{\frac{V(\mu_t) - \hat{u}}{k}} \phi(\mu_t) \quad (2.6)$$

Proof. The proof follows from a standard constrained optimization problem. For details, see appendix A.1 ■

From now on, I will denote the entrepreneur's revenue for a network of size μ_t with user utility level \hat{u} , as defined in the lemma above by $\psi(\mu_t, \hat{u})$. This function $\psi(\mu_t, \hat{u})$ will be crucial for the analysis of centralized governance. There are two main possibilities that can occur. First, if \hat{u} is relatively large, the entrepreneur is unable to ensure that level of utility for the users. That is, even if she decided to distribute all the revenue she has raised in the particular period to her users, the revenue would be insufficient to guarantee utility \hat{u} for the users. Second, if \hat{u} is relatively small, the entrepreneur can ensure that level of utility for the network's users. Then there are two subcases: Either it is optimal to share some revenue with users or it is optimal for the entrepreneur to keep all revenues for herself. For example, if the parameter that measures the users' aversion to monetization, k , is relatively small, it is beneficial for the entrepreneur to increase monetization and share some revenues with her users. If the users are highly averse to monetization, it is optimal for the entrepreneur to choose a relatively small amount of monetization and keep all the revenues for herself. Note that the entrepreneur's revenue is strictly decreasing in the level of utility \hat{u} that the users receive. As such, there is a conflict of interest between the entrepreneur and the users. Next, I impose a regularity assumption to ensure that larger networks generate higher revenues:

Assumption 1 $\frac{\phi(\mu_t)}{\mu_t}$ is non-decreasing in μ_t .

Assumption 1 ensures that the entrepreneur's revenue $\psi(\mu_t, \hat{u})$ is increasing in the size of the network μ_t . As such, the entrepreneur prefers a larger size network and there are no structural incentives for the entrepreneur to limit the size of her network. Clearly, the assumption holds if $\phi(\mu_t)$ increases at least linearly in μ_t . For example, this holds true in cost-per-view and cost-per-click advertisement that is commonly used in online advertisement. If c is the cost per click/view and a fraction $\gamma \in [0, 1]$ of the users interacts with advertisement, it holds that $\frac{\phi(\mu_t)}{\mu_t} = c\gamma$, which is constant in μ_t .

To derive the equilibrium of the centralized governance sub-game, it is instructive to consider the entrepreneur's incentives to grow her network. Every period, new, potential users arrive to take part in the network. For the network to grow, joining the network has to be weakly beneficial for a newly arriving user. That is, joining the network has to yield at least utility equal to 0. Instead of growing the network, the entrepreneur can exploit her existing user base. Given that existing users are locked into the network and have an outside option that is valued at $-u < 0$, the entrepreneur can potentially achieve a higher level of revenue when focusing on extracting additional revenue from existing users. To quantify the revenue that an entrepreneur generates when she decides to exploit the users in her network, consider some period t . The existing user base at the start of the period is equal to μ_{t-1} . If she exploits the existing user base forever, the present value of the stream of her discounted future revenue equals

$$\frac{1}{1-\delta}\psi(\mu_{t-1}, -(1-\delta)u) \quad (2.7)$$

Note that the entrepreneur provides a per-period utility of $-(1-\delta)u$ to the users, such that the discounted utility is equal to $-u$, keeping the users indifferent between staying and leaving. To grow the network, the entrepreneur has to provide enough utility to the users, such that they are better off joining the network in the first place. If the entrepreneur grows the network one last time in some period t before focussing on exploiting existing users, she has to provide utility δu to the last users who are to join the network. The entrepreneur's revenue from growing the network one more time and then exploiting the network's users from that point onward equals

$$\psi(g(\mu_{t-1}), \delta u) + \frac{\delta}{1-\delta}\psi(g(\mu_{t-1}), -(1-\delta)u) \quad (2.8)$$

The point at which the entrepreneur will be indifferent between growing the network one last time and exploiting the existing users in her network will be crucial for the analysis of the equilibrium:

Definition 1 *Denote the solution to the following equation by $\bar{\mu}$:*

$$\frac{1}{1-\delta}\psi(\bar{\mu}, -(1-\delta)u) = \psi(g(\bar{\mu}), \delta u) + \frac{\delta}{1-\delta}\psi(g(\bar{\mu}), -(1-\delta)u) \quad (2.9)$$

It is exactly at the network size $\bar{\mu}$ where the entrepreneur is indifferent between growing the network one last time and then exploiting it in the future, and exploiting the network right away. It highlights the trade-off between exploiting the locked-in effect of a smaller mass μ_{t-1} of users starting today, or, growing the network at the cost of providing utility δu to the users and to then exploiting a larger size network with $g(\mu_{t-1})$ users starting

tomorrow.

For the definition of $\bar{\mu}$ to be meaningful, the existence of such a value $\bar{\mu}$ has to be assured. Further, to keep the model concise, $\bar{\mu}$ should be unique. To this end, I impose a sufficient assumption on the model as the amount of users in the network grows:

Assumption 2 *For existence:*

1. As $\mu_t \rightarrow \infty$ it holds that $g(\mu_t) - \mu_t \rightarrow 0$
2. $\psi(g(\mu_t), \hat{u}) - \psi(\mu_t, \hat{u})$ is decreasing in μ_t for all \hat{u}

For uniqueness:

1. $\sqrt{2kV'(\mu_t)\mu_t} < \frac{\phi(\mu_t)}{\mu_t}$ for all $\mu_t > 0$

This assumption represents the idea that user growth will slow down over time and there are decreasing returns to the entrepreneur's revenue when growing the network. As the size of the network increases, fewer new users will arrive. This assumption should be satisfied in many applications, as the potential user base of a network is limited. Further, the assumption imposes a regularity on the difference between the revenue that the entrepreneur generates. As the network grows, the gap between the revenue created from a network that has grown one more time and a network that has not, shrinks.

Mathematically, the assumption requires that the revenue function ψ , which depends on the functions V and ϕ , is not too convex in the network size μ_t , in relation to the rate at which the network growth slows down over time. To illustrate the point, consider an example with $V(\mu_t)$ constant and $\phi(\mu_t) = \mu_t$. Note that in this specification ψ is a linear function in μ_t , and as such at the extreme end of "not too convex" functions that can be considered. Then, for any growth function of the form $g(\mu_t) = \mu_t + \gamma(\mu_t)$ where $\gamma(\mu_t)$ is a strictly decreasing, strictly positive function that approaches 0 as $\mu \rightarrow \infty$ it is easy to confirm that the assumption above holds. A more detailed discussion of the assumption can be found in appendix A.2.

Proposition 1 *Assumption 2 is sufficient to guarantee the existence and uniqueness of $\bar{\mu}$.*

Proof. See appendix A.2. ■

To see why $\bar{\mu}$ exists, note that $\psi(\mu_t, \cdot)$ is continuous in μ_t and that at $\mu_t = 0$ it holds that $\psi(0, \cdot) = 0$, such that the revenue generated from exploiting the users in a network of size 0 is equal to 0. In contrast, since $g(0) > 0$, the revenue generated from growing the network at least once is strictly positive. Therefore, without any users, the entrepreneur

prefers growing the network over starting to exploit the users today. Further, as $\mu_t \rightarrow \infty$ assumption 2 ensures that the additional revenue from growing the network becomes negligible. Thus, the entrepreneur will prefer to exploit the users, when μ_t grows sufficiently large. Therefore, a value $\bar{\mu}$ at which the entrepreneur is indifferent exists.

For the following analysis, suppose that

$$\frac{\phi(\bar{\mu})^2}{4k\bar{\mu}^2} + V(\bar{\mu}) \geq \delta u \quad (2.10)$$

This condition relates back to lemma 1. It ensures that it would be feasible to the entrepreneur to ensure the utility level δu to a network of size $\bar{\mu}$. Later, I will discuss what happens when this condition is not satisfied.

For a better understanding of the equilibrium that will follow shortly, I want to emphasize that the level of user utility \hat{u}_t that is implied by a degree of monetization π_t and revenue sharing α_t is a function of the amount of users μ_t that are present in the network at the end of period t . For example, a particular tuple (π_t, α_t) implies different user utility levels \hat{u}_t when $\mu_t = 0$ compared to when $\mu_t > 0$. Now, the intuition of the trade-off between growing the network and exploiting the existing user base can be condensed into an equilibrium:

Proposition 2 *Suppose condition 2.10 is satisfied. Then the following strategies constitute a sub-game perfect Nash equilibrium:*

Entrepreneur's strategy:

- If $\mu_{t-1} < g^{-1}(\bar{\mu})$, set π_t and α_t to maximize revenue as given in Lemma 1 with user utility level $\hat{u}_t = 0$ for a network size $\mu_t = g(\mu_{t-1})$
- If $g^{-1}(\bar{\mu}) \leq \mu_{t-1} < \bar{\mu}$, set π_t and α_t to maximize revenue as given in Lemma 1 with user utility level $\hat{u}_t = \delta u$ for a network size $\mu_t = g(\mu_{t-1})$
- If $\bar{\mu} \leq \mu_{t-1}$ set π_t and α_t to maximize revenue as given in Lemma 1 with user utility level $\hat{u}_t = -(1 - \delta)u$ for a network size $\mu_t = \mu_{t-1}$

Users' strategy:

- In the period of arrival, join the network iff
 1. $\mu_{t-1} < g^{-1}(\bar{\mu})$ and π_t, α_t are such that user utility level $\hat{u}_t \geq 0$ for a network size $\mu_t = g(\mu_{t-1})$
 2. $g^{-1}(\bar{\mu}) \leq \mu_{t-1}$ and π_t, α_t are such that user utility level $\hat{u}_t \geq \delta u$ for a network size $\mu_t = g(\mu_{t-1})$

- *If already locked in to the network, stay in the network iff π_t, α_t are such that user utility level $\hat{u}_t \geq -(1 - \delta)u$ for a network size $\mu_t \geq \mu_{t-1}$*

Proof. See appendix A.3 ■

The equilibrium features the cutoff $\bar{\mu}$, at which the entrepreneur switches from growing the network to exploiting the existing users in the network. The entrepreneur's strategy has three distinct parts. If $\mu_{t-1} < g^{-1}(\bar{\mu})$, the entrepreneur will grow the network in the next period, as $g(\mu_{t-1}) < \bar{\mu}$. Thus, the entrepreneur sets user utility equal to $\hat{u}_t = 0$ and the users are willing to join the network. Note that in these periods, the entrepreneur has basically regained commitment to not abuse the locked-in effect of the users. The entrepreneur refrains from exploiting the locked-in effect of existing users in the network with the aim to grow the network larger. At $g^{-1}(\bar{\mu}) \leq \mu_{t-1} < \bar{\mu}$, the entrepreneur reaches the limits of how far she is willing to grow the network. If the entrepreneur grows the network, it holds that $\mu_t = g(\mu_{t-1}) > \bar{\mu}$, such that in the future, the entrepreneur will be better off with exploiting the locked-in effect of the users compared to growing the network any further. However, to attract users to the network, the entrepreneur has to offer a utility level equal to $\hat{u}_t = \delta u$. In the last part, when $\bar{\mu} \leq \mu_{t-1}$, the entrepreneur is better off exploiting the locked-in effect of the network's existing users compared to growing the network any further.

The users' strategies are rather simple. When they newly arrive at the network, they do not suffer from a locked-in effect. They observe the network size and if $\mu_{t-1} < g^{-1}(\bar{\mu})$, anticipate that the entrepreneur will grow the network further in the future, such that it is optimal for them to join the network if $\hat{u}_t \geq 0$. If $g^{-1}(\bar{\mu}) \leq \mu_{t-1} < \bar{\mu}$, they know that they arrived in the last period of growth of the network. As such, they require a level of utility at least equal to δu to join the network. If they are already locked into the network, they will remain in the network iff $\hat{u}_t \geq -(1 - \delta)u$, as this implies that the discounted value of their future utility is at least equal to the value of their outside option $-u$. Note that no profitable deviations exists for neither the entrepreneur nor the users. In equilibrium, newly arriving users are indifferent between joining and not joining the network, while users that are already locked into the network strictly prefer staying in the network before the entrepreneur starts exploiting the users and are indifferent between staying and leaving when the entrepreneur starts exploiting the network. For the entrepreneur, deviations that increase the users' utility level are not profitable, since it does not change the users actions on the equilibrium path and her revenues are decreasing in the users' utility levels. Decreasing the utility offered to the users at any point in time will cause the users to leave the network, resulting in 0 revenues, thus not being a profitable deviation.

Now reconsider what happens if

$$\frac{\phi(\bar{\mu})^2}{4k\bar{\mu}^2} + V(\bar{\mu}) < \delta u \quad (2.11)$$

Then, the entrepreneur cannot pay the compensation utility δu in the last period where she will grow the network. If the entrepreneur sets a utility level of less than δu , no new users will join, as the value of joining is below the outside option of 0. However, if the entrepreneur is unable to attract any new users, she should maximize revenues from the existing users of the network. That is, setting user utility equal to $-(1 - \delta)u$ instead. Denote this last period of potential growth in which this issue occurs as t^* . Then, users should anticipate that the entrepreneur will exploit the locked-in effects not starting from period $t^* + 1$ onward, but from period t^* . Then, the users who arrive at period $t^* - 1$ need to be provided utility level δu , for them to be incentivized to join the network. However, note that at period $t^* - 1$ the size of the network is necessarily smaller than at t^* . Thus, since the network's revenues are increasing in the mass of user μ_t , it is also not feasible for the entrepreneur to provide utility level δu to the users in period $t^* - 1$. This logic carries forward until the first period, such that no users should join the network at all. To further examine when this issue occurs, I define:

Definition 2 Define by $\underline{\mu}$ the solution to the equation

$$\frac{\phi(\underline{\mu})^2}{4k\underline{\mu}^2} + V(\underline{\mu}) = \delta u \quad (2.12)$$

Intuitively speaking, $\underline{\mu}$ is the minimum required size of the network, such that the entrepreneur can afford to give utility δu to the users. Now, if $\bar{\mu} \geq \underline{\mu}$, the entrepreneur can attract users to her network. However, if $\bar{\mu} < \underline{\mu}$, the entrepreneur is unable to attract any users to her network. The entrepreneur's main issue in the network with centralized governance is her lack of commitment to not abusing the locked-in effect of the users. Thus, I focus on the effects of the severity of the locked-in effect u on $\underline{\mu}$ and $\bar{\mu}$.

Lemma 2 $\underline{\mu}$ strictly increases in u . As $u \rightarrow \infty$ it holds that $\underline{\mu} \rightarrow \infty$.

To see why the lemma holds true, consider equation 2.12. When u increases the RHS of the equation increases. Then the lemma clearly holds true, as the LHS of the equation is increasing in $\underline{\mu}$ since $\frac{\phi(\underline{\mu})^2}{4k\underline{\mu}^2}$ is increasing in $\underline{\mu}$ by assumption A.2 (i.e. $\frac{\phi(\underline{\mu})}{\underline{\mu}}$ is increasing in $\underline{\mu}$) and $V(\underline{\mu})$ is also increasing in $\underline{\mu}$ by assumption.

Next, consider $\bar{\mu}$. Note that $\bar{\mu}$ is only implicitly defined in definition 1. It is the size of the network that makes the entrepreneur indifferent between growing the network once more today and exploiting the users in the future vs. exploiting the users starting today. As such, I employ the implicit function theorem to show the following lemma:

Lemma 3 $\bar{\mu}$ strictly decreases in u . As $u \rightarrow 0$ it holds that $\bar{\mu} \rightarrow \infty$.

Proof. See appendix A.4. ■

As the size of the locked-in effect grows, the entrepreneur stops growing the network and start exploiting the existing users earlier. With a larger locked-in effect, there is more to gain by exploiting the existing user base. To sum things up, I have shown that $\underline{\mu}$ is strictly increasing in u and that $\bar{\mu}$ is strictly decreasing in u . Therefore, as u increases, the following two effects take place. First, the entrepreneur needs a larger size network to be able to raise enough revenue to guarantee users a utility level δu in the last period of growth. Second, as u increases, the entrepreneur is more tempted to exploit the existing user base of the network. Therefore, the following corollary formalizes that when u grows too large, the entrepreneur is unable to attract any users to her network:

Corollary 1 *There exists some value u^* such that the entrepreneur is unable to attract any users to the network if $u > u^*$. Consequently, the equilibrium revenue of the network with centralized governance is 0.*

The corollary follows by defining u^* as the value of u for which $\underline{\mu} = \bar{\mu}$. Then for all $u > u^*$ it holds that $\bar{\mu} < \underline{\mu}$. As the size of the locked-in effect grows too large, the entrepreneur will more readily exploit users who are already in the network, rather than growing the network by attracting new users. However, in equilibrium, this is anticipated by any users that arrive at the network, such that no users join the network at all. This highlights the commitment problem of the entrepreneur. If she was able to commit to not abusing the locked-in effect of the users, she would be able to attract users to her network and generate revenues. Note that this corollary establishes a sufficiency result. When the size of the locked-in effect is sufficiently large, it is better to decentralize the network, if the entrepreneur can attract at least some users in decentralized governance. In section 3 I show that this result carries over more generally, by determining the cutoff size for the locked-in effect such that the entrepreneur prefers to decentralize the network if and only if the locked-in effect is sufficiently severe. Before that, the next section discusses the sub-game of decentralized governance.

2.2 Decentralized Governance

If the entrepreneur chooses decentralized governance, every period $t = 1, 2, \dots$ has the following timing:

1. Users make a simultaneous choice:
 - (a) Users who are not present in the network choose to join or not to join
 - (b) Users who are already present in the network choose to stay or leave

2. Users collectively choose π_t
3. Utilities realize

This section focuses on the sub-game of decentralized governance. First, the entrepreneur chooses a permanent revenue split α . Then, users that have newly arrived have the choice to join or not join the network. Existing users have the choice to stay or leave the network. Afterward, users vote on the degree of monetization π_t for the period and utilities realize. When analyzing the voting equilibria, I will restrict the equilibrium analysis to weakly undominated strategies. In voting games, the strategy of voters has to be optimal, conditional on being pivotal. As no single voter is ever pivotal when there is a continuum of users, basically any strategy can be played in an equilibrium. Therefore, I restrict the users' strategies to be weakly undominated, that is, they choose to vote for a degree of monetization π_t *as if* they were pivotal. This leads to the following equilibrium:

Proposition 3 *There is a sub-game perfect equilibrium such that every period the users of the network will vote for a degree of monetization*

$$\pi_t^* = \frac{1 - \alpha \phi(\mu_t)}{2k \mu_t} \quad (2.13)$$

The network will grow every period. The entrepreneur shares half of the revenue with the users.

Proof. See appendix A.5. ■

The equilibrium highlights that decentralized governance is an effective commitment tool for the entrepreneur. In contrast to centralized governance, the users can be certain that their locked-in effect will not be exploited by the entrepreneur. Thus, users will continue to join the network every period. However, for the entrepreneur this commitment comes at a substantial cost: she shares half the revenues of the network with her users. Nonetheless, it is necessary for her to share revenue with her users. If she would not share any revenue, the users would subsequently vote to stop the monetization of the network. As a result, the entrepreneur would not receive any revenue. Therefore, the sharing of revenue in a decentralized implementation of the network is necessary, as it aligns the incentives of the entrepreneur and the incentives of the network's users.

One potential point of contention in decentralized governance could be conflicts of interest between existing and newly arriving users. The users' utility function equals $V(\mu_t) - k\pi_t^2 + \frac{1-\alpha}{\mu_t}\phi(\mu_t)$. The share of revenue that each user gets in the network is $\frac{1-\alpha}{\mu_t}$. As such, newly arriving users will dilute the revenue shares of existing users in the network. However, note that the users' per period utility in the equilibrium equals $V(\mu_t) + \frac{\phi(\mu_t)^2}{8k\mu_t^2}$. Assumption 1 ensures that $\frac{\phi(\mu_t)}{\mu_t}$ is non-decreasing and thus implies that

the equilibrium utility is increasing in μ_t . Intuitively speaking, the network effects that accompany the entry of new users sufficiently compensate the dilution of the revenue share of existing users. Thus, there is no incentive for existing users to try to prevent entry from newly arriving users to avoid dilution of their revenue shares.

2.3 Optimal Governance

The two preceding sections have solved the sub-games of centralized and decentralized governance. Now the question remains, which form of governance the entrepreneur should choose when she creates her network? As has been shown in proposition 2, centralized governance will result in the entrepreneur eventually stopping growing the network and starting exploiting the locked-in effect of the users. This change from network growth to exploiting the users is inherent in centralized governance, as the entrepreneur is unable to commit to future monetization and revenue sharing. Subsequently, corollary 1 showed that, when the locked-in effect is sufficiently large, the entrepreneur is unable to attract any users to the network, yielding her 0 revenue in equilibrium. This threshold of the locked-in effect serves as a sufficient condition for when it is optimal to decentralize. However, the question that remains is a more complete comparison between the entrepreneur's revenue in centralized and decentralized governance. That is, what is the optimal mode of governance for any arbitrary size of the locked-in effect? To answer this question, I start by considering the opposite extreme of what was discussed in the corollary, namely when the locked-in effect is very small. Then, I move to locked-in effects of arbitrary size.

For small locked-in effects, the commitment problem of the entrepreneur becomes less and less severe, and in the limit of $u = 0$, disappears entirely. Comparing centralized and decentralized governance for $u = 0$ is rather straightforward. When $u = 0$, there is no locked-in effect that can be abused by the entrepreneur in the future. Thus, users will join the network every period, resulting in growth in any period in the centralized network. In comparison, note that the decentralized network also featured growth in every period. As such, the potential revenues that can be generated in both modes of governance are the same. However, in centralized governance, the entrepreneur stays in control and can generate maximum amounts of revenue for herself, while she surrenders control over the network in decentralized governance and has to engage in revenue sharing to align the users' preferences with hers. Thus, centralized governance is superior when the locked-in effect is small. This intuition is condensed in the following lemma:

Lemma 4 *As $u \rightarrow 0$ centralized governance is always preferred over decentralized governance.*

Proof. See appendix A.6 ■

So far, I have established comparisons of centralized and decentralized governance at

both extremes of the size of the locked-in effect. Namely, for minimal locked-in effects ($u \rightarrow 0$), centralized governance is optimal for the entrepreneur, while for sufficiently large locked-in effects ($u > u^*$), decentralized governance is optimal for the entrepreneur. For intermediate values, the optimal mode of governance is hard to compute explicitly, as the revenue of the entrepreneur in the centralized network is only given implicitly, through the implicit definition of the maximum network size $\bar{\mu}$. However, what can be shown is a monotonicity result. That is, as the size of the locked-in effect increases, the entrepreneur's revenue in centralized governance decreases. As a result, there is a clear cutoff in the size of the locked-in effect, such that decentralized governance is preferred if and only if the size of the locked-in effect is larger than this cutoff. This idea is condensed into the following proposition:

Proposition 4 *There exists a well-defined size of the locked-in effect, u^{**} , such that decentralized governance is preferred by the entrepreneur if and only if $u > u^{**}$.*

Proof. See appendix A.7. ■

The idea of the proof is as follows. First, recall that I have shown that at the two extremes of minimal and very large locked-in effects, the entrepreneur prefers centralized and decentralized governance respectively. Next, note that the entrepreneur's revenue with decentralized governance is independent of the size of the locked-in effect u . This holds as the users decide the level of monetization in the network with decentralized governance, and their optimal decision does not depend on u . The final step of the proof shows, that the entrepreneur's revenue with centralized governance is decreasing in the size of the locked-in effect u . Together, these observations imply the result, as they imply that the functions of the revenue under centralized and decentralized governance can cross at most once.

To realize why the entrepreneur's revenue with centralized governance is decreasing in u , consider the effect of a change in the size of the locked-in effect. In the centralized network, revenue is generated in three different phases. First, is the growth phase in which the entrepreneur provides 0 period utility to the users. Second, the last period of growth in which the entrepreneur provides utility equal to δu to the users, and lastly, the periods of exploiting where the entrepreneur provides utility equal to $-(1 - \delta)u$ to the users. Consider the immediate effect of an increase in u . The revenues of the first phase of the network are independent of u and remain unchanged. Second, the required period utility of the users in the last phase of growth, δu increases, resulting in decreased revenue for the entrepreneur. Finally, the utility level in the exploitation phase, $-(1 - \delta)u$ decreases and leads to increased revenues for the entrepreneur. However, the entrepreneur's revenue is a function that is concave in the utility level (c.f. $\psi(\mu_t, \hat{u}) = \mu_t V(\mu_t) + \frac{\phi(\mu_t)^2}{4k\mu_t} - \mu_t \hat{u}$ or $\psi(\mu_t, \hat{u}) = \sqrt{\frac{V(\mu_t) - \hat{u}}{k}} \phi(\mu_t)$) that is extracted from the users. As a result, the additional

cost of providing additional utility in the last period of growth does not outweigh the additional benefit from the extra revenue the entrepreneur generates in the exploitation phase. Thus, the immediate effect on the entrepreneur's revenue of an increase in the size of the locked-in effect is negative.

As a secondary effect, an increase in the size of the locked-in effect u , decreases the maximum size of the network $\bar{\mu}$, as was shown in Lemma 3. Note that the change in the maximum size of the network size is only relevant for the last period of growth and the following period of exploitation, but not for the first periods of network growth. As such, the smaller amount of users that the entrepreneur has to provide utility level δu to in the last period of growth is offset by an equally smaller amount of users that the entrepreneur can exploit by providing utility level $-(1 - \delta)u$ in the following periods. Further, the entrepreneur's revenue is increasing in the size of the network, such that a decrease in the network size decreases the entrepreneur's revenue. As both the immediate and secondary effects on the entrepreneur's revenue from an increase in the size of the locked-in effect are negative, the total effect is negative. Thus, the entrepreneur's revenue with centralized governance is decreasing in u .

2.4 Welfare

Finally, I want to address the welfare implications of the governance decisions. In particular: When does decentralization improve welfare? It turns out, that this question can be answered with the analysis that has been conducted so far. First, note that users in the centralized implementation of the network are always indifferent between joining the network and their outside option ex-ante. In contrast, users receive strictly positive utility in the decentralized implementation of the network. Thus, users always prefer decentralized governance. For the entrepreneur, proposition 4 has established that she prefers decentralization if and only if the size of the locked in effect u is larger than the threshold u^{**} . Therefore, the following corollary can be established:

Corollary 2 *Decentralized governance of the network is a Pareto improvement over centralized governance if and only if the size of the locked-in effect u is larger than u^{**}*

As an alternative measure of welfare, one might consider utilitarian welfare. Then trivially, utilitarian welfare is also increased through decentralization if decentralization constitutes a Pareto improvement, i.e. if the size of the locked-in effect u is larger than u^{**} . Note that the statement for utilitarian welfare is not an if and only if statement. It is not generally obvious whether it would improve welfare to force an entrepreneur, who does not find it beneficial for herself to decentralize her network, to decentralize her network

regardless. Doing so creates two welfare effects with opposing signs: the decrease in welfare through the decrease in revenue for the entrepreneur, and the increase in welfare through the increase in utility for the users. The sign of the aggregate of these two effects will generally depend on the parametrization of the model.

3 Discussion: Equilibrium Multiplicity

Section 2 has discussed the implications of centralized governance for an equilibrium in which the entrepreneur grows the network up to a particular size and then stops growing the network to exploit the locked-in effect of its users. However, there exist other, albeit less convincing, equilibria, in the sub-game of centralized governance. In particular, when δ is sufficiently large, there exists the following folk-theorem type of equilibrium:

Users' strategy: Existing users leave the network and newly arriving users do not join the network if the level of utility implied by any revenue sharing α_t and monetization π_t in the history of the game at any time t is strictly lower than the level $\hat{u}_t = V(g(\mu_{t-1})) - (1 - \delta)u$, conditional on all existing users staying and new users joining the network.

Entrepreneur's strategy: In every period t , set revenue sharing α_t and monetization π_t such that the level of utility for the users is equal to \hat{u}_t conditional on all existing users staying and new users joining the network. If the entrepreneur is being "punished" by the users, set utility equal to $-(1 - \delta)u$ conditional on 0 (measure) users being in the network.

First, consider why these strategies constitute a sub-game perfect equilibrium by checking for one shot deviations.

Deviations by the entrepreneur: Given the users strategies, and the fact that the entrepreneur's revenue is decreasing in \hat{u}_t , clearly there are no profitable deviations for the entrepreneur. Increasing \hat{u}_t lowers her revenue without changing the users' behavior on the equilibrium path. Decreasing \hat{u}_t causes all users to leave the network, resulting in 0 revenues for the entrepreneur. When the entrepreneur is being punished and there are no users in the network, the entrepreneur is indifferent between all of his choices, such that there is no incentive to deviate.

Deviations by the users: Fix the strategies of the entrepreneur and the users. Now consider some arbitrary user i . For sub-game perfection, the user cannot have any incentive to (one-shot) deviate from the equilibrium strategy at any history of the game.

First, consider histories of the game such that the entrepreneur has offered at least utility level \hat{u}_t in every period. Suppose user i is already locked into the network. If user i leaves, his utility will be equal to $-u$. If he stays, his utility will be equal to $\sum_{t=0}^{\infty} (\delta^t V(g^{(t)}(\mu_t))) - u$ which is larger than $-u$, such that leaving is not a profitable

deviation. Now consider the case where user i is newly arriving to the network. Again, his utility is $\sum_{t=0}^{\infty} (\delta^t V(g^{(t)}(\mu_t)) - u)$. This will be larger than 0 for δ large enough, such that there is no incentive to deviate.

Next, consider histories of the game such that the entrepreneur is offering a utility level $\tilde{u}_t < \hat{u}_t$ in some period t . If user i leaves, his utility will be equal to $-u$. If user i stays on the other hand, his utility will be equal to

$$\tilde{u}_t - V(g(\mu_{t-1})) - \delta u \tag{3.1}$$

Staying is optimal iff

$$\tilde{u}_t - V(g(\mu_{t-1})) - \delta u > -u \tag{3.2}$$

$$\iff \tilde{u}_t > V(g(\mu_{t-1})) - (1 - \delta)u \tag{3.3}$$

Which cannot hold since $V(g(\mu_{t-1})) - (1 - \delta)u = \hat{u}_t > \tilde{u}_t$. Therefore, staying in the network is not a profitable deviation for user i .

Now, while this type of equilibrium exists, it is particular demanding in terms of coordination between the users. To illustrate this point, consider the following notion for stability of an equilibrium akin to trembles considered by Selten (1975). Suppose that the entrepreneur deviates and instead offers utility level $\hat{u}_t - \epsilon$ for some arbitrarily small ϵ . Since the utility level of the deviation is arbitrarily close to \hat{u}_t , suppose that user i is not entirely certain whether all other users will follow the equilibrium strategy and punish the entrepreneur by leaving the network/not joining the network. User i assigns probability p to the event that all other users unexpectedly stay in the network, for example because the trigger strategy they follow is slightly more lenient towards the entrepreneur than expected. With probability $1 - p$ all other users leave the network as prescribed by the equilibrium. An equilibrium can be considered unstable, if, for a degree of uncertainty of punishment p , there is a small deviation ϵ in the utility offered by the entrepreneur such that any user i is better off staying in the network and not punishing the entrepreneur.

Proposition 5 *The alternative equilibrium discussed in this section is unstable for any degree of uncertainty $p > 0$. In contrast, the equilibrium of the main body of the paper, i.e., in proposition 2, is stable for all degrees of uncertainty.*

Proof. See appendix A.8 ■

Intuitively speaking, the folk-theorem style equilibrium has the feature that a particular user i will want to follow through with punishing the entrepreneur for deviating

only if all other users also follow through. He wants to avoid punishing the entrepreneur, if the other users do not follow suit. Thus, this kind of equilibrium requires an incredibly large degree of coordination. In contrast, the equilibrium presented in the main paper has the feature that a particular user i will want to leave the network (punish the entrepreneur) *regardless* of whether the other users also leave. Thus, no degree of coordination is necessary.

4 Explanation of Blockchain, Smart Contracts and the Creation of Commitment

This section provides a brief overview over blockchains, smart contracts and some examples of projects that leverage this technology. It is intended to provide sufficient background information for this paper, but a thorough treatment of the topic itself is outside the scope of this paper. For a basic introduction to the topic of see for example Lewis (2021). For some further information and more current research, see for example the contributions on <https://www.cber-forum.org/>.

4.1 Blockchain

The blockchain is a ledger that allows for the storage of information. In this paper, the focus lies on decentralized blockchains, i.e., those that are permissionless, and public. They are updated and maintained decentrally by their users through a consensus mechanism. The two most common consensus mechanisms are Proof of Work and Proof of Stake.²⁰ For a more detailed introduction to Blockchain, and its consensus mechanisms, see for example Saleh (2021). For this paper, the focus lies on the implications of the blockchain for economic interactions. As it is permissionless, there is no central authority that can censor access to the blockchain. As such, an entrepreneur that leverages a decentralized blockchain finds herself unable to interfere with the users' ability to use the blockchain. Further, it is tamper-proof, i.e. the entrepreneur and any single user are unable to change records on the blockchain. As the blockchain is public, anyone can publicly observe – and trust in – the current consensus of information on the blockchain.²¹ Bitcoin is probably the most well-known blockchain to date. It was created in 2009 by Satoshi Nakamoto.²² The Bitcoin blockchain securely stores account balances and facilitates transactions be-

²⁰In practice, blockchains are updated by a subset of their users. In Proof of Work blockchains, this subset is commonly referred to as miners. In Proof of Stake blockchains, they are commonly referred to as validators.

²¹There are a variety of explorers that allow for easier reading of blockchains. For example, <https://etherscan.io/> covers the Ethereum blockchain.

²²Satoshi Nakamoto is a pseudonym. The real name of the bitcoin founder is unknown. Furthermore, it is unknown if Satoshi Nakamoto is a single person or a group of people.

tween its users.

4.2 Smart Contracts

From a technological standpoint, Bitcoin is not as advanced as many newer blockchains. Most notably, it is not smart contract compatible.²³ Essentially, a smart contract is a piece of code that can be executed on the blockchain. Smart contracts have first been formalized by Szabo (1997). The first smart contract compatible blockchain, Ethereum was conceived in a white paper by Vitalik Buterin in 2014.²⁴ Smart contract compatible blockchains offer vast possibilities for interactions between economic agents in a trustless environment. For example, they can be programmed to facilitate the exchange of cryptocurrencies between two economic agents, without the need for trust in each other or a central party as an intermediary. To date, the top 10 cryptocurrencies by market capitalization consist of Bitcoin, three stablecoins, and six smart contract compatible blockchains.²⁵ This highlights the growing importance of smart contract compatible blockchains.

4.3 Creating commitment through smart contracts: the example of Uniswap

One of the simplest examples of networks that rely on smart contracts to govern the economic interactions between its users is decentralized exchanges. The largest decentralized exchange to date is Uniswap²⁶. It was founded in November 2018 by Hayden Adams and deployed on the Ethereum blockchain. Uniswap allows its users to exchange different cryptocurrencies in a trustless environment using smart contracts as intermediaries. As of September 2022, it has facilitated the exchange of roughly \$1.1 trillion worth of cryptocurrencies in 110 million trades. As the exchange is facilitated by smart contracts, which are immutable once deployed to the blockchain, the terms of the exchange remain unchanged at a 0.3% fee, regardless of how popular it has become.²⁷ It is entirely impossible for Adams to change the terms of the smart contracts governing Uniswap to extract additional rents from its sizeable user base. Changes to the Uniswap protocol are facilitated through a decentralized governance mechanism that uses UNI “governance

²³As pointed out in the Ethereum white paper, technically Bitcoin can perform some computations, but it is severely limited. For example, it is not *Turing complete*.

²⁴<https://ethereum.org/en/whitepaper/ethereum>

²⁵At the date of writing the top 10 cryptocurrencies are: Bitcoin, 3 stablecoins (USDT, USDC, BUSD) and 6 smart contract compatible blockchains (Ethereum, Binance Smart Chain, Ripple, Cardano, Solana and Dogecoin). A stablecoin is a cryptocurrency pegged to a fiat currency, most commonly the US Dollar.

²⁶<https://uniswap.org/>

²⁷There are other Uniswap smart contracts available with fees of 0.01%, 0.05% and 1% respectively.

tokens”.²⁸ Such an arrangement is also referred to as a *Decentralized Autonomous Organization (DAO)*. Changes to the protocol are then voted on in a majority vote where 1 token equals 1 vote.

4.4 Creating commitment when smart contracts are not sufficient: the example of Presearch

For some networks, it is not feasible to contain the entire interaction between agents within a smart contract. Consider the example of Presearch, a decentralized search engine. When a user searches on a search engine, a simplified workflow is as follows: 1) The user issues a search request and sends it to the search engine, 2) the search engine computes the search results and sends them back to the user. If one were to try to contain this interaction in a smart contract, there would be at least two serious challenges. First, the block creation times on current blockchains range from minutes (Bitcoin) to seconds (Ethereum) to several hundred milliseconds (Solana). As such, the execution of a search through a smart contract would simply be too slow to be practical. Second, interaction with a smart contract requires the user to pay for “gas fees”²⁹. With Ethereum, these gas fees are typically in the range of several dollars.³⁰ As such, they are too high to facilitate millions to billions of searches a day.³¹ Therefore, for many networks, at least some interactions have to happen “off-chain”.

To see how this works in practice, consider an entrepreneur who wishes to create a search engine. In a centralized implementation, she develops the code and sets up a data center with the computing infrastructure to handle the users’ search requests. To monetize her search engine, she allows advertisers to place advertisements within the search results. The entrepreneur starts off with minimal advertisement to attract new users. As users are locked-in to her search engine, she increases the number of advertisements she displays with the search results. Suppose the users anticipate this behavior by the entrepreneur and that it is necessary for the entrepreneur to be able to commit. How can she create commitment through decentralization and the blockchain?

Instead of operating the search engine through her own infrastructure, she decides to

²⁸The UNI governance token is a digital asset. A digital asset is referred to as a cryptocurrency if it has its own underlying blockchain. If it utilizes another blockchain, it is referred to as a token. UNI exists under the ERC-20 token standard on the Ethereum blockchain

²⁹Gas fees are transaction fees that have to be paid to interact with a smart contract on a blockchain. They are necessary to ensure that computations finish within a finite amount of time and keep malicious actors from impeding the operation of the blockchain through endless smart contract calculations.

³⁰Current Ethereum gas fees can be found using <https://etherscan.io/gastracker>

³¹For example, Google handles around 5-6 billion search requests a day.

distribute the code of the search engine freely and asks her users to set up the infrastructure (a so-called node) for the search engine. Now, suppose the entrepreneur tries to update the software to increase the advertisement on the search engine as the users have become locked in. Since the users are effectively operating the search engine, they can simply refuse to install the software update that the entrepreneur has put forward. Thus, the entrepreneur is unable to abuse the locked-in effect of the users. So far, this does not necessarily require the use of the blockchain. However, to compensate the users for the costs of operating the infrastructure, the entrepreneur promises to share part of the advertisement revenue with them. In this interaction, an opportunity for blockchain technology to mitigate economic frictions arises.

Suppose the entrepreneur has promised the users 50% of advertisement revenues. Further, suppose there are two potential advertisers that are willing to pay \$100 to advertise on the search engine, but it is only possible to display advertisements from one of the advertisers. The willingness to pay is known to the entrepreneur, but not the users. The payment of the advertisers to the entrepreneur is not publicly observable. If everybody behaves honestly, competition will drive the advertisers to pay \$100 for the advertisement, and the entrepreneur and the users will receive \$50 each. Now suppose that the entrepreneur and one of the advertisers decide to collude: The entrepreneur proposes that she will tell the users that the advertiser was only willing to pay \$50 for the advertisement. The other \$50 will be split 30-20 between the entrepreneur and the advertiser. Such collusion between the entrepreneur and the advertiser is profitable for both, since now, the entrepreneur pockets \$55 and the advertiser gets to advertise on the search engine for \$80 instead of \$100. If the users anticipate such collusion, it may be optimal for them to refrain from operating a node in the first place.

This situation can be remedied through the use of the blockchain: when setting up her search engine, the entrepreneur employs a smart contract on the blockchain. It is structured such that advertisers pay the smart contract for the advertisement. The software of the search engine is programmed, such that it displays the advertisement for the highest paying advertiser in the smart contract. Revenues are distributed 50/50 between the entrepreneur and the users using the smart contract. Now collusion between the entrepreneur and one of the advertisers is no longer possible: Suppose the entrepreneur and one of the advertisers agree to pay \$50 for advertising into the smart contract and again split the other \$50 between each other. Now the second advertiser can simply deposit \$51 into the smart contract to have their advertisement displayed, breaking the possibility of collusion between the other advertiser and the entrepreneur.

In this example, the decentralized network run by the users serves as a commitment device for the entrepreneur to not abuse their locked-in effect through increased adver-

tisement. The blockchain serves as a commitment device for the entrepreneur to honor her revenue-sharing agreement with the users.

5 Discussion and Conclusion

Before concluding, I want to briefly discuss some further points of interest. First, the reader may wonder if this model implies that an established network such as Google or Facebook should decentralize their business through the blockchain. Such a conclusion cannot be drawn from this model, as these networks have already established a large user base (e.g. Facebook already has around 3 billion users³²). As such, the value of extracting additional revenues from existing users that are already locked-in may outweigh the value of commitment that is offered by a decentralized implementation. However, the model provides insights on the optimal governance of newly founded competitors.

Second, it may be plausible that locked-in effects become larger when there are more users. When the network size is small, growth has already been shown to be a substitute for commitment in section 2.1. Smaller locked-in effects would leave this result unchanged. Further, when the network size, and thus the locked-in effect, would be large, the entrepreneur will find it even more beneficial to stop growing the network and exploit the existing users. Therefore, such an extension will leave the model qualitatively unchanged.

Last, consider the possibility that the entrepreneur may treat newly arriving and already existing users differently. For example, she could try to treat newly arriving users or early adopters favorably. However, if this also implies that she can treat existing users less favorably, this change would exacerbate the commitment problem of the entrepreneur when choosing centralized governance even further. That is, it would be sequentially optimal to exploit the locked-in effect of all users as soon as possible. Therefore, commitment should become even more valuable for the entrepreneur.

To summarize, this paper provides an answer to a question that is frequently raised when it comes to the topic of blockchain and cryptocurrencies: *Why should anybody use it?* As the main result, I showed that (i) an entrepreneur prefers to decentralize her network and (ii) decentralization is a Pareto improvement, if and only if the locked-in effect is sufficiently large. To broaden our understanding of further implications of decentralization, I believe that further research is needed, especially regarding the economics of decentralized governance.

³²Meta Earnings Presentation Q2, 2022, p.14

A Appendix

A.1 Proof of Lemma 1

The proposition follows from the following maximization problem:

$$\max_{\alpha_t \pi_t} \alpha_t \pi_t \phi(\mu_t) \quad (\text{A.1})$$

$$\text{s.t. } V(\mu_t) - k\pi_t^2 + \frac{1 - \alpha_t}{\mu_t} \pi_t \phi(\mu_t) = \hat{u} \quad (\text{A.2})$$

$$1 \geq \alpha_t \geq 0 \quad (\text{A.3})$$

The problem can be solved through a standard KKT approach. The FOCs associated with the resulting Lagrangian with the complementary slackness conditions then reads

$$\frac{\partial}{\partial \alpha_t} = \pi_t \phi(\mu_t) + \lambda_1 \left(\frac{-\pi_t}{\mu_t} \phi(\mu_t) \right) - \lambda_2 + \lambda_3 = 0 \quad (\text{A.4})$$

$$\frac{\partial}{\partial \pi_t} = \alpha_t \phi(\mu_t) + \lambda_1 \left(-2k\pi_t + \frac{1 - \alpha_t}{\mu_t} \phi(\mu_t) \right) = 0 \quad (\text{A.5})$$

$$\frac{\partial}{\partial \lambda_1} = V(\mu_t) - k\pi_t^2 + \frac{1 - \alpha_t}{\mu_t} \pi_t \phi(\mu_t) - \hat{u} = 0 \quad (\text{A.6})$$

$$\frac{\partial}{\partial \lambda_2} \lambda_2 = (1 - \alpha_t) \lambda_2 = 0 \quad (\text{A.7})$$

$$\frac{\partial}{\partial \lambda_3} \lambda_3 = \alpha_t \lambda_3 = 0 \quad (\text{A.8})$$

First, focus on the case where $\alpha_t \in (0, 1)$, such that $\lambda_2, \lambda_3 = 0$. Then straightforward calculations yield that

$$\pi_t = \frac{\phi(\mu_t)}{2k\mu_t} \quad (\text{A.9})$$

$$\alpha_t = \frac{1}{2} + \frac{2k\mu_t^2(V(\mu_t) - \hat{u})}{\phi(\mu_t)^2} \quad (\text{A.10})$$

And the entrepreneur's revenue equals

$$\left(V(\mu_t) + \frac{\phi(\mu_t)}{4k\mu_t^2} - \hat{u} \right) \mu_t \quad (\text{A.11})$$

Note that $\alpha_t \in (0, 1)$ requires that

$$\alpha_t > 0 \quad (\text{A.12})$$

$$\iff \frac{\phi(\mu_t)^2}{4k\mu_t^2} + V(\mu_t) \geq \hat{u} \quad (\text{A.13})$$

and

$$1 > \alpha_t \tag{A.14}$$

$$\iff \left(\frac{\phi(\mu)}{2k\mu_t} \right)^2 > \frac{V(\mu_t) - \hat{u}}{k} \tag{A.15}$$

Next, consider the possible solution with $\alpha_t = 1$. Then it follows that

$$\pi_t = \sqrt{\frac{V(\mu_t) - \hat{u}}{k}} \tag{A.16}$$

The entrepreneur's revenue then equals

$$\sqrt{\frac{V(\mu_t) - \hat{u}}{k}} \phi(\mu_t) \tag{A.17}$$

Last, consider the possible solution where $\alpha_t = 0$. Notice that in this case, the entrepreneur's revenue is equal to 0, regardless of the choice of π_t . The choice of π_t that maximizes the users' utility is $\pi_t = \frac{\phi(\mu_t)}{2k\mu_t}$. Then it is not possible to ensure utility \hat{u} for the user if

$$V(\mu_t) - k \left(\frac{\phi(\mu_t)}{2k\mu_t} \right)^2 + \frac{\phi(\mu_t)}{2k\mu_t} \frac{\phi(\mu_t)}{\mu_t} < \hat{u} \tag{A.18}$$

$$\iff V(\mu_t) + \frac{\phi(\mu_t)^2}{4k\mu_t^2} < \hat{u} \tag{A.19}$$

A.2 Proof of Proposition 1 and Discussion of Assumption 2

For reference, first I restate the assumption:

Assumption 2:

For existence:

1. As $\mu_t \rightarrow \infty$ it holds that $g(\mu_t) - \mu_t \rightarrow 0$
2. $\psi(g(\mu_t), \hat{u}) - \psi(\mu_t, \hat{u})$ is decreasing in μ_t for all \hat{u}

For uniqueness:

1. $\sqrt{2kV'(\mu_t)\mu_t} < \frac{\phi(\mu_t)}{\mu_t}$ for all $\mu_t > 0$

A.2.1 Proof of proposition 1

Recall the definition of $\bar{\mu}$ as the value that solves the equation

$$\frac{1}{1-\delta} \psi(\bar{\mu}, -(1-\delta)u) = \psi(g(\bar{\mu}), \delta u) + \frac{\delta}{1-\delta} \psi(g(\bar{\mu}), -(1-\delta)u) \tag{A.20}$$

As argued in the main body of the paper, at $\mu = 0$ it holds that LHS of equation $<$ RHS of the equation. Evaluating at $\mu \rightarrow \infty$ implies LHS of equation $>$ RHS of the equation. Given the continuity of all functions involved, an application of the intermediate value theorem implies existence. To show the unique cutoff, consider the first derivative of the difference of the RHS and the LHS with respect to μ :

$$g'(\mu)\psi_\mu(g(\mu), \delta u) + g'(\mu)\frac{\delta}{1-\delta}\psi_\mu(g(\mu), -(1-\delta)u) - \frac{1}{1-\delta}\psi_\mu(\mu, -(1-\delta)u) \quad (\text{A.21})$$

$$= g'(\mu)\psi_\mu(g(\mu), \delta u) - \psi_\mu(\mu, -(1-\delta)u) + \frac{\delta}{1-\delta}(g'(\mu)\psi_\mu(g(\mu), -(1-\delta)u) - \psi_\mu(\mu, -(1-\delta)u)) \quad (\text{A.22})$$

What is to be shown is that this first derivative is negative. To this end, I show the intermediate result that under the assumption that $\sqrt{2kV'(\mu)\mu} < \frac{\phi(\mu)}{\mu}$ for all $\mu > 0$ it holds that $\frac{\partial\psi^2}{\partial\mu\partial\hat{u}} < 0$ for all $\mu > 0$.

Lemma 5 $\sqrt{2kV'(\mu)\mu} < \frac{\phi(\mu)}{\mu}$ for all $\mu > 0$ implies $\frac{\partial\psi^2}{\partial\mu\partial\hat{u}} < 0$ for all $\mu > 0$.

Proof. Note that

$$\frac{\partial\psi^2}{\partial\mu\partial\hat{u}} = \begin{cases} -1 & \text{if } \left(\frac{\phi(\mu)}{2k\mu}\right)^2 \geq \frac{V(\mu)-\hat{u}}{k} \\ -\frac{\phi'(\mu)}{2\sqrt{k}}(V(\mu)-\hat{u})^{-0.5} + \frac{V'(\mu)}{4\sqrt{k}}(V(\mu)-\hat{u})^{-1.5}\phi(\mu) & \text{if } \left(\frac{\phi(\mu)}{2k\mu}\right)^2 < \frac{V(\mu)-\hat{u}}{k} \end{cases} \quad (\text{A.23})$$

Therefore I focus on showing that the second case is negative:

$$-\frac{\phi'(\mu)}{2\sqrt{k}}(V(\mu)-\hat{u})^{-0.5} + \frac{V'(\mu)}{4\sqrt{k}}(V(\mu)-\hat{u})^{-1.5}\phi(\mu) < 0 \quad (\text{A.24})$$

$$\iff -2\phi'(\mu)(V(\mu)-\hat{u}) + V'(\mu)\phi(\mu) < 0 \quad (\text{A.25})$$

Note that to be in this second case, \hat{u} is bounded above such that $\hat{u} < -\left(\frac{\phi(\mu)}{2k\mu}\right)^2 k + V(\mu)$. Therefore, it holds that

$$-2\phi'(\mu)(V(\mu)-\hat{u}) + V'(\mu)\phi(\mu) < -2\phi'(\mu)\left(\frac{\phi(\mu)}{2k\mu}\right)^2 k + V'(\mu)\phi(\mu) \quad (\text{A.26})$$

This is smaller than 0 if

$$-2\phi'(\mu)\left(\frac{\phi(\mu)}{2k\mu}\right)^2 k + V'(\mu)\phi(\mu) < 0 \quad (\text{A.27})$$

$$\iff 2k\mu^3V'(\mu)\frac{\phi(\mu)}{\phi'(\mu)\mu} < \phi(\mu)^2 \quad (\text{A.28})$$

Note that assumption 1 guarantees that $\frac{\phi(\mu)}{\phi'(\mu)\mu} \leq 1$. This implies that the inequality below is a sufficient condition for A.28

$$\sqrt{2kV'(\mu)\mu} < \frac{\phi(\mu)}{\mu} \quad (\text{A.29})$$

Which is the uniqueness part of assumption 2 ■

Now, I revisit the initial derivative

$$g'(\mu)\psi_\mu(g(\mu), \delta u) - \psi_\mu(\mu, -(1-\delta)u) + \frac{\delta}{1-\delta} (g'(\mu)\psi_\mu(g(\mu), -(1-\delta)u) - \psi_\mu(\mu, -(1-\delta)u)) \quad (\text{A.30})$$

Using the lemma derived above, note that $\psi_\mu(\mu, \delta u) < \psi_\mu(\mu, -(1-\delta)u)$. Thus, it holds hat

$$g'(\mu)\psi_\mu(g(\mu), \delta u) - \psi_\mu(\mu, -(1-\delta)u) + \frac{\delta}{1-\delta} (g'(\mu)\psi_\mu(g(\mu), -(1-\delta)u) - \psi_\mu(\mu, -(1-\delta)u)) \quad (\text{A.31})$$

$$< g'(\mu)\psi_\mu(g(\mu), \delta u) - \psi_\mu(\mu, \delta u) + \frac{\delta}{1-\delta} (g'(\mu)\psi_\mu(g(\mu), -(1-\delta)u) - \psi_\mu(\mu, -(1-\delta)u)) \quad (\text{A.32})$$

Further, the assumption that $\psi(g(\mu_t), \hat{u}) - \psi(\mu_t, \hat{u})$ is decreasing in μ_t for all \hat{u} implies that

$$g'(\mu)\psi_\mu(g(\mu), \hat{u}) - \psi_\mu(\mu, \hat{u}) \leq 0 \quad (\text{A.33})$$

Using this implies that expression A.32 is smaller than 0 which finishes the proof.

A.2.2 An example with a general growth function and linear revenues:

First off, I show that the specification of $V(\mu)$ constant and $\phi(\mu) = \mu$ with $g(\mu) = \mu + \gamma(\mu)$ and γ being strictly decreasing, strictly positive and approaching 0 as $\mu \rightarrow \infty$ satisfies assumption 2. Clearly, as $\mu \rightarrow \infty$ it holds that $g(\mu_t) - \mu_t \rightarrow 0$ as $\gamma(\mu) \rightarrow 0$ as $\mu \rightarrow \infty$. Next, consider the difference $\psi(g(\mu_t), \hat{u}) - \psi(\mu_t, \hat{u})$. Plugging in V and ϕ yields that $\psi(\mu_t, \hat{u})$ is a linear function of μ_t . Now for the assumption to hold, consider the first derivative of the difference $\psi(g(\mu_t), \hat{u}) - \psi(\mu_t, \hat{u})$:

$$\frac{\partial}{\partial \mu_t} (\psi(g(\mu_t), \hat{u}) - \psi(\mu_t, \hat{u})) = g'(\mu_t)\psi_{\mu_t}(g(\mu_t), \hat{u}) - \psi_{\mu_t}(\mu_t, \hat{u}) \quad (\text{A.34})$$

$$= g'(\mu_t)\psi_{\mu_t}(\mu_t, \hat{u}) - \psi_{\mu_t}(\mu_t, \hat{u}) \quad (\text{A.35})$$

$$= \gamma'(\mu_t)\psi_{\mu_t}(\mu_t, \hat{u}) < 0 \quad (\text{A.36})$$

The condition for uniqueness can be easily confirmed.

A.2.3 An example with a general revenue function and growth that slows abruptly:

For another example, consider the opposite end of the spectrum. That is, consider a growth function $g(\mu)$ such that

$$g(\mu) = \begin{cases} g(0) > 0 & \text{if } \mu = 0 \\ \mu & \text{if } \mu > 0 \end{cases} \quad (\text{A.37})$$

and arbitrary functions $V(\mu)$ and $\phi(\mu)$. Then clearly we have

$$\psi(g(\mu), \hat{u}) - \psi(\mu, \hat{u}) > 0 \quad (\text{A.38})$$

if $\mu = 0$ and the difference equals 0 otherwise. Intuitively speaking, this growth function allows the network to grow for exactly 1 period at the start, and then in future periods no new users arrive. Restricting the growth function in this way allows for maximum freedom regarding the functions V and ϕ .³³

To recap, assumption 2 relies on a balance between the convexity of the revenue function ψ in relation to the growth function g . For the minimum degree of convexity of ψ , i.e., when ψ is linear when V is constant and $\phi(\mu) = \mu$ it is possible to allow very general growth functions g . On the other end, it is possible to allow very general functions V and ϕ , implying very general shapes on the revenue function ψ , if growth slows down extremely fast, that is, decreases to 0 within 1 period. In general, appropriate functions for V , ϕ and g can be found by keeping in mind the trade-off between relatively more convex revenue functions ψ (as calculated by V and ϕ) for growth functions g that slow down relatively faster and vice-versa.

A.2.4 Weakening the assumption:

What is important for the proofs in the paper is that $\bar{\mu}$ exists and is unique. For this, assumption 2, laid out sufficient conditions. However, they are not necessary. Alternatively, it is possible to assume that

$$\psi(g(\mu), \delta u) + \frac{\delta}{1-\delta} \psi(g(\mu), -(1-\delta)u) - \frac{1}{1-\delta} \psi(\mu, -(1-\delta)u) \quad (\text{A.39})$$

³³Note that this definition of g includes a discontinuity. To use such a g in the model, one would have to extend g to a continuous function or use a slightly more general definition of $\bar{\mu}$, both of which can be accommodated fairly easily.

is

1. Increasing up to some value $\tilde{\mu}$
2. Strictly decreasing for any $\mu > \tilde{\mu}$

Note that this assumption also guarantees the existence of a unique $\bar{\mu}$ and that it is more general in the sense that it contains assumption 2 for the case where $\tilde{\mu} = 0$. However, it is considerably more challenging to calculate examples that satisfy this assumption.

A.3 Proof of Proposition 2

To check for profitable deviations by the entrepreneur or the users, I employ the one-shot deviation principle (see for example Theorem 4.2 in Fudenberg and Tirole (1991)). Note that the one-shot deviation principle applies, as the game is obviously continuous at infinity.³⁴ Therefore, it is sufficient to check that there is no single period profitable deviation.

A.3.1 Deviations by the entrepreneur:

Consider a history of the game up to some period t that results in a mass of users μ_{t-1} at the start of the period. Then there are two cases:

Case 1 ($\mu_{t-1} \leq \bar{\mu}$): First, note that deviations that increase the utility of the users are not profitable, since the equilibrium path remains unchanged and the entrepreneur's revenue is decreasing in the utility level she provides to the users. Now, consider a deviation that decreases the utility level the entrepreneur provides for the users. Given the users' strategies, a large decrease in the utility level below $-(1 - \delta)u$ will cause all users to leave the network and not be profitable. A small decrease will cause existing users to remain in the network and newly arriving users to not join the network. Therefore, the most profitable deviation would be to a utility level of $-(1 - \delta)u$. The entrepreneur's revenue for this deviation is $\psi(\mu_{t-1}, -(1 - \delta)u)$ plus the discounted revenue of the continuation of the initial strategy starting in the next period. If the entrepreneur had not deviated, she would receive the value of the continuation of the initial strategy starting this period. Note that this value depends on how many more periods the entrepreneur will grow the network according to the initial strategy. I show that the deviation is not profitable by induction on the number of periods of future growth. First, consider the case with 1

³⁴c.f. Definition 4.1 and explanation in Fudenberg and Tirole (1991): A game is continuous at infinity if for each player i the utility function u_i satisfies $\sup_{h, \tilde{h} \text{ s.t. } h^t = \tilde{h}^t} |u_i(h) - u_i(\tilde{h})| \rightarrow 0$ as $t \rightarrow \infty$. It is satisfied if the overall payoffs are a discounted sum of per-period payoffs and the per period payoffs are uniformly bounded.

period of future growth. Then the deviation is not profitable if

$$\psi(\mu_{t-1}, -(1-\delta)u) + \delta \left(\psi(g(\mu_{t-1}), \delta)u + \frac{\delta}{1-\delta} \psi(g(\mu_{t-1}), -(1-\delta)u) \right) \quad (\text{A.40})$$

$$\leq \psi(g(\mu_{t-1}), \delta)u + \frac{\delta}{1-\delta} \psi(g(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.41})$$

$$\iff \frac{1}{1-\delta} \psi(\mu_{t-1}, -(1-\delta)u) \leq \psi(g(\mu_{t-1}), \delta)u + \frac{\delta}{1-\delta} \psi(g(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.42})$$

Which holds true since $\mu_{t-1} \leq \bar{\mu}$. Now suppose that it is not profitable to deviate when there are T periods of future growth. Next, I show that it is not profitable to deviate with $T + 1$ periods of future growth. A deviation with $T + 1$ periods of future growth is not profitable if

$$\psi(\mu_{t-1}, -(1-\delta)u) + \delta \left(\sum_{s=0}^{T-1} \delta^s \psi(g^{(s)}(\mu_{t-1}), 0) + \delta^T \psi(g^{(T)}(\mu_{t-1}), \delta)u + \frac{\delta^{T+1}}{1-\delta} \psi(g^{(T)}(\mu_{t-1}), -(1-\delta)u) \right) \quad (\text{A.43})$$

$$\leq \sum_{s=0}^{T-1} \delta^s \psi(g^{(s)}(\mu_{t-1}), 0) + \delta^T \psi(g^{(T)}(\mu_{t-1}), \delta)u + \frac{\delta^{T+1}}{1-\delta} \psi(g^{(T)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.44})$$

$$\iff \frac{1}{1-\delta} \psi(\mu_{t-1}, -(1-\delta)u) \quad (\text{A.45})$$

$$\leq \sum_{s=0}^{T-1} \delta^s \psi(g^{(s)}(\mu_{t-1}), 0) + \delta^T \psi(g^{(T)}(\mu_{t-1}), \delta)u + \frac{\delta^{T+1}}{1-\delta} \psi(g^{(T)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.46})$$

Since by induction the assertion holds true for T periods of future growth, it suffices to show that the RHS of the inequality above for T periods of future growth is smaller than the RHS of the inequality above for $T + 1$ periods of future growth, since the LHS is identical in both cases. Thus, I have to show that

$$\sum_{s=0}^{T-2} \delta^s \psi(g^{(s)}(\mu_{t-1}), 0) + \delta^{T-1} \psi(g^{(T-1)}(\mu_{t-1}), \delta)u + \frac{\delta^T}{1-\delta} \psi(g^{(T-1)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.47})$$

$$\leq \sum_{s=0}^{T-1} \delta^s \psi(g^{(s)}(\mu_{t-1}), 0) + \delta^T \psi(g^{(T)}(\mu_{t-1}), \delta)u + \frac{\delta^{T+1}}{1-\delta} \psi(g^{(T)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.48})$$

$$\iff \delta^{T-1} \psi(g^{(T-1)}(\mu_{t-1}), \delta)u + \frac{\delta^T}{1-\delta} \psi(g^{(T-1)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.49})$$

$$\leq \delta^{T-1} \psi(g^{(T-1)}(\mu_{t-1}), 0) + \delta^T \psi(g^{(T)}(\mu_{t-1}), \delta)u + \frac{\delta^{T+1}}{1-\delta} \psi(g^{(T)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.50})$$

Now note that $\psi(g^{(T-1)}(\mu_{t-1}), \delta u) < \psi(g^{(T-1)}(\mu_{t-1}), 0)$. Then this implication and some rearranging yields

$$\frac{1}{1-\delta}\psi(g^{(T-1)}(\mu_{t-1}), -(1-\delta)u) \leq \psi(g^{(T)}(\mu_{t-1}), \delta u) + \frac{\delta}{1-\delta}\psi(g^{(T)}(\mu_{t-1}), -(1-\delta)u) \quad (\text{A.51})$$

Which holds true since this is precisely the condition that it is optimal to grow $T + 1$ times. Therefore, one-shot deviations by the entrepreneur to abuse the locked-in effect of the users are not profitable.

Case 2($\mu_{t-1} > \bar{\mu}$): For this case, deviations that decrease the user utility are not profitable, since they will result in all users leaving the network and zero revenues. Now consider deviations that increase the users' utility. First, marginal increases will not change the user behavior on the equilibrium path and are not profitable. Second, the smallest deviation that changes the users' behavior on the equilibrium path is to increase the utility sufficiently to grow the network one more time. However, by definition of $\bar{\mu}$ such deviations are not profitable when $\mu_{t-1} > \bar{\mu}$.

A.3.2 Deviations by a user:

Newly arriving users: First, consider any histories on the equilibrium path. Then, there is no profitable deviation, since users are exactly indifferent between joining and not joining the network. Now, consider deviations off the equilibrium path. For any histories that offer more utility than the equilibrium path, clearly it is still optimal to join the network, such that not joining is not a profitable deviation. In contrast, any histories that have reduced utility imply that it is optimal to not join the network, such that joining is not a profitable deviation.

Users that are locked-in: First, consider any histories on the equilibrium path. There are two cases. Before the exploitation phase begins, there are no profitable deviations since remaining in the network provides 0 utility, while leaving gives utility $-u < 0$. During the exploitation phase, the users are indifferent between staying and leaving, such that leaving is not a profitable deviation.

Second, consider histories off the equilibrium path. Histories that result in increased user utility obviously do not offer profitable deviations. Now, consider histories such that the user's utility is reduced. Leaving the network provides $-u$ utility, while remaining in the network provides the user a utility level smaller than $-(1-\delta)u$ for the period in which he is alone in the network and utility $-\delta u$ from leaving the network the next period. Total utility is thus smaller than $-(1-\delta)u - \delta u = -u$, such that the deviation is not profitable.

A.4 Proof of Lemma 3

First, I show that the implicit function theorem is applicable in this situation. In particular, it has to be shown that the revenue function is differentiable. Clearly, it is piece-wise differentiable. However, it has to be shown that it is also differentiable at the point where the entrepreneur stops revenue sharing, i.e., when

$$\left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 = \frac{V(\mu_t) - \hat{u}}{k} \quad (\text{A.52})$$

The two pieces of the function are

$$\mu_t V(\mu_t) + \frac{\phi(\mu_t)^2}{4k\mu_t} - \mu_t \hat{u} \quad (\text{A.53})$$

and

$$\sqrt{\frac{V(\mu_t) - \hat{u}}{k}} \phi(\mu_t) \quad (\text{A.54})$$

Consider differentiability regarding \hat{u} . The derivatives regarding \hat{u} are

$$-\mu_t \quad (\text{A.55})$$

and

$$-\frac{1}{2\sqrt{k}} \frac{1}{\sqrt{V(\mu_t) - \hat{u}}} \phi(\mu_t) \quad (\text{A.56})$$

It is straightforward to verify algebraically that the two derivatives are equal to each other when $\left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 = \frac{V(\mu_t) - \hat{u}}{k}$

Next, I consider the derivatives regarding μ_t . They are

$$V(\mu_t) + \mu_t V'(\mu_t) + \frac{2\phi'(\mu_t)\phi(\mu_t)4k\mu_t - 4k\phi(\mu_t)^2}{(4k\mu_t)^2} - \hat{u} \quad (\text{A.57})$$

and

$$\frac{1}{\sqrt{k}} \left(\frac{V'(\mu_t)}{2} \frac{1}{\sqrt{V(\mu_t) - \hat{u}}} \phi(\mu_t) + \sqrt{V(\mu_t) - \hat{u}} \phi'(\mu_t) \right) \quad (\text{A.58})$$

Using the identity $\left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 = \frac{V(\mu_t) - \hat{u}}{k}$ at the point of interest we can simplify the two

derivatives to

$$\left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 k + \mu_t V'(\mu_t) + \frac{\phi'(\mu_t)\phi(\mu_t)}{2k\mu_t} - \left(\frac{\phi(\mu_t)}{2k\mu_t}\right)^2 k = \mu_t V'(\mu_t) + \frac{\phi'(\mu_t)\phi(\mu_t)}{2k\mu_t} \quad (\text{A.59})$$

and

$$\frac{1}{\sqrt{k}} \left(\frac{V'(\mu_t)}{2\sqrt{k}} \frac{2k\mu_t}{\phi(\mu_t)} \phi(\mu_t) + \frac{\phi(\mu_t)\sqrt{k}}{2k\mu_t} \phi'(\mu_t) \right) = \mu_t V'(\mu_t) + \frac{\phi'(\mu_t)\phi(\mu_t)}{2k\mu_t} \quad (\text{A.60})$$

respectively, which are equal to each other. Therefore, the implicit function theorem applies. To shorten notation define

$$F := \psi(g(\mu_t), \delta u) + \frac{\delta}{1-\delta} \psi(g(\mu_t), -(1-\delta)u) - \frac{1}{1-\delta} \psi(\mu_t, -(1-\delta)u) \quad (\text{A.61})$$

and by the implicit function theorem it holds that

$$\frac{\partial \bar{\mu}}{\partial u} = - \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial \mu_t}} \Bigg|_{\bar{\mu}, u} \quad (\text{A.62})$$

For the denominator, notice that the derivative is negative by the definition of $\bar{\mu}$.

For the numerator, notice that at $u = 0$ it holds that $F > 0$. Moreover, note that the three parts of F are decreasing and concave, increasing and concave, and decreasing and convex with respect to u respectively. In order for F to be equal to 0 at $(\bar{\mu}, u)$, the derivative of F regarding u has to be negative for at least some values of U . However, note that when the derivative of F turns negative, it will remain negative. This holds, as the middle part of F is increasing and concave, such that its growth slows down. When the derivative turns negative, the third part of F , $-\frac{1}{1-\delta} \psi(\mu_t, -(1-\delta)u)$ alone will keep the derivative negative, as $\mu_t < g(\mu_t)$. Thus, the negative slope is steeper than the positive slope of $\frac{\delta}{1-\delta} \psi(g(\mu_t), -(1-\delta)u)$. In particular, this implies that the slope of F regarding u at $(\bar{\mu}, u)$ is negative. Therefore, the numerator is negative and the fraction as a whole is negative.

A.5 Proof of Proposition 3

The degree of monetization follows from a simple optimization problem. Namely,

$$\max_{\pi_t} V(\mu_t) - k\pi_t^2 + \frac{1-\alpha}{\mu_t} \pi \phi(\mu_t) \quad (\text{A.63})$$

The equilibrium is confirmed by an application of the one-shot deviation principle. First, no user has an incentive to deviate in the degree of monetization in weakly dominant strategies. Second, as all users receive strictly positive utility from participation in the

network, there is no incentive to deviate into not joining.

Last, the entrepreneur's optimization problem in $t = 0$ equals

$$\max_{\alpha} \sum_{t=1}^{\infty} \left(\alpha \frac{1 - \alpha \phi(g^{(t)}(\mu_0))}{2k} \frac{\phi(g^{(t)}(\mu_0))}{g^{(t)}(\mu_0)} \phi(g^{(t)}(\mu_0)) \right) \quad (\text{A.64})$$

Where $g^{(t)}$ denotes the t -time chaining of the growth function. From this, it is straightforward to derive $\alpha^* = 0.5$

A.6 Proof of Lemma 4

Note that at $u = 0$ the strategy of the entrepreneur is to ensure 0 utility for the users in every period. Further, it holds that there is no value of $\bar{\mu}$ that makes the entrepreneur indifferent between growing the network once more and exploiting the users in the future and exploiting the users right away. Namely, it will always be better to grow the network as $g(\mu) - \mu \geq 0$. Therefore, at $u = 0$ the network will grow every period, as it does with decentralized governance. However, since the choice set regarding monetization and revenue sharing is larger in centralized governance than it is in decentralized governance, her revenues are necessarily higher with centralized governance.

A.7 Proof of Proposition 4

Corollary 1 established that decentralized governance is preferred over centralized governance if u is sufficiently large, i.e. $u > u^*$. Further, lemma 4 established that centralized governance is preferred if u is sufficiently small. To derive the result of the proposition, note that the entrepreneur's revenue with decentralized governance is independent of u . Thus, it is sufficient to show that centralized revenue is decreasing in u to prove the proposition. Now, consider the change in the entrepreneur's revenue with centralized governance as u increases. Note that the entrepreneur does not exploit the locked-in effect in the first periods of growth, that is, she sets $\hat{u}_t = 0$ for all periods of growth except the last period. Now, consider the last period of growth and the following periods of exploiting the locked-in effect. Note that the size of the network in all of those periods is the same. Then the first order effect from increasing the size of the locked-in effect is equal to

$$\delta \psi_u(\mu, \delta u) - (1 - \delta) \frac{\delta}{(1 - \delta)} \psi_u(\mu, -(1 - \delta)u) \quad (\text{A.65})$$

This is negative if

$$\psi_u(\mu, \delta u) \leq \psi_u(\mu, -(1 - \delta)u) \quad (\text{A.66})$$

Now there are three options to compare. They are 1) both sides of the equation are in the linear part of ψ . 2) The LHS is in the linear part and the RHS is in the concave part of ψ . 3) Both sides are in the concave part of ψ . The first case holds trivially. The second case holds as

$$-\mu \leq -\frac{1}{2\sqrt{k}} \frac{1}{\sqrt{V(\mu) + (1 - \delta)u}} \phi(\mu) \quad (\text{A.67})$$

$$\iff \left(\frac{\phi(\mu)}{2k\mu} \right)^2 < \frac{V(\mu) + (1 - \delta)u}{k} \quad (\text{A.68})$$

Which is a true statement, as it is precisely the condition from lemma 1 that ensured that the RHS is in the concave part of the function.

Last, I show that the inequality holds if both the RHS and the LHS of the equation are in the concave part of ψ .

$$-\frac{1}{2\sqrt{k}} \frac{1}{\sqrt{V(\mu) - \delta u}} \phi(\mu) \leq -\frac{1}{2\sqrt{k}} \frac{1}{\sqrt{V(\mu) + (1 - \delta)u}} \phi(\mu) \quad (\text{A.69})$$

$$\iff u \geq 0 \quad (\text{A.70})$$

For the second order effect, note that the maximum network size $\bar{\mu}$ is dependent on u . In particular, lemma 3 showed that $\bar{\mu}$ is decreasing in u . Further, the entrepreneur's revenue ψ is increasing in μ , such that the decrease in the maximum size of the network decreases the entrepreneur's revenues. Thus, the total effect of an increase in u on the entrepreneur's revenues is negative.

A.8 Proof of proposition 5

Consider the equilibrium of section 3 and the incentive of a user i to deviate from punishing the entrepreneur. For that, compare his utility $-u$ from leaving with the utility of

staying

$$p \sum_{t=0}^{\infty} \delta^t \hat{u}_t + (1-p)(\hat{u}_t - V(g(\mu_{t-1})) - \delta u) - \epsilon < -u \quad (\text{A.71})$$

$$\Rightarrow p \sum_{t=0}^{\infty} \delta^t (V(g^{(t)}(\mu_0) - (1-\delta)u) + (1-p)(-(1-\delta)u - \delta u) - \epsilon < -u \quad (\text{A.72})$$

$$\Rightarrow p \sum_{t=0}^{\infty} \delta^t (V(g^{(t)}(\mu_0) - (1-\delta)u) + (1-p)(-(1-\delta)u - \delta u) - \epsilon < -u \quad (\text{A.73})$$

$$\Rightarrow p \sum_{t=0}^{\infty} \delta^t (V(g^{(t)}(\mu_0)) < \epsilon \quad (\text{A.74})$$

Which is a contradiction for ϵ small enough, i.e. the user is better off when deviating to staying in the network.

Now consider the equilibrium of the main body of the paper. Consider user i 's utility when staying. If all other users unexpectedly stay in the network, the discounted utility of user i is strictly less than the value of his outside option, since he receives utility $-(1-\delta)u - \epsilon$ today and discounted future utility equal to $-\delta u$. Thus the utility of staying is equal to $-u - \epsilon < -u$. If, on the other hand, all other users leave the network and user i remains in the network alone, his utility is less than $-(1-\delta)u - \delta u - \epsilon < -u$ for any $\epsilon > u$. Therefore, user i prefers to stick to the initial equilibrium, regardless of the level of uncertainty p .

A.9 Extension: Pre-commitment to revenue sharing path in decentralized governance

Suppose that the entrepreneur can pre-commit to the full path of revenue sharing for all periods $t = 1, 2, \dots$ at the start of the game in $t = 0$. Now, note that for any pre-committed level of α_t , the user's optimal choice of monetization π_t is derived analogously to the optimal monetization π_t^* for a fixed percentage of revenue sharing, and thus equals

$$\frac{1 - \alpha_t}{2k} \frac{\phi(\mu_t)}{\mu_t} \quad (\text{A.75})$$

and that the user's utility level for the period thus is

$$V(\mu_t) + \frac{1}{4k} \left((1 - \alpha_t) \frac{\phi(\mu_t)}{\mu_t} \right)^2 \geq 0 \quad (\text{A.76})$$

such that the user’s choice of monetization implies that it is always optimal for new users to join. Then the entrepreneur’s maximization problem in $t = 0$ is equal to

$$\max_{\{\alpha_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \left(\delta^t \frac{\alpha_t(1 - \alpha_t)}{2k} \frac{\phi(\mu_t)}{\mu_t} \right) \quad (\text{A.77})$$

Now, straight forward maximization over the α_t implies that in the optimum $\alpha_t = \alpha = 0.5$ for all $t = 1, 2, \dots$

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