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DISCUSSION PAPER

// ATABEK ATAYEV

Uncertain Product Availability in Search Markets

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Atabek Atayev †

ZEW—Leibniz Centre for European Economic Research in Mannheim

L 7, 1, 68161 Mannheim, Germany

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Abstract

In many markets buyers are poorly informed about which firms sell the product (product availability) and prices, and therefore have to spend time to obtain this information. In contrast, sellers typically have a better idea about which rivals offer the product. Information asymmetry between buyers and sellers on product availability, rather than just prices, has not been scrutinized in the literature on consumer search. We propose a theoretical model that incorporates this kind of information asymmetry into a simultaneous search model. Our key finding is that greater product availability may harm buyers by mitigating their willingness to search and, thus, softening competition.

JEL Classification: D43, D82, D83

Keywords: Consumer Search; Uncertain Product Availability; Information Asymmetry.

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†Corresponding author; e-mail: atabek.atayev@zew.de

1 Introduction

Initiated by Stigler (1961), theory of simultaneous search (also known as *nonsequential* search and *fixed-sample-size* search) has been widely employed to study observed inefficiencies in markets, where buyers wish to gather information quickly and the outcome of search is observed with a delay (Morgan and Manning (1985)).[1] The early studies in this field focus on how search costs mitigate buyers' incentive to discover prices (e.g., Salop and Stiglitz (1977), MacMinn (1980), Burdett and Judd (1983)). In many markets, however, buyers face uncertainty not only about prices but also product availability. A consumer looking for a loan does not know which banks will approve her application, a consumer planning a renovation does not know which companies provide the desired service, and a government agency soliciting bidders does not know which, and how many, bidders will participate in an auction. Later search models account for uncertain product availability (e.g., Janssen and Non (2009), Lester (2011)), yet they typically assume that an individual seller, just like buyers, does not know whether rivals supply the product. With the rise of data-gathering and -processing technologies, it has become evident that sellers have a good idea about their competitors, including whether they supply the product and are not capacity constrained.[2]

Despite the prevalence of information asymmetry on product availability in search markets, they remain understudied. With this paper we aim to narrow this gap. We derive novel results on the impact of product availability on market outcomes.

Our model is an extension of the canonical model of Burdett and Judd (1983).[3] The standard features of the model are that identical sellers produce homogeneous goods and

[1]Examples include consumer financial credit markets (e.g., Allen et al. (2013), Galenianos and Gavazza (2020)), labor markets (e.g., Gautier et al. (2016), Bagger and Lentz (2019)), and markets for consumer retail goods (e.g., Moraga-González and Wildenbeest (2008), Chandra and Tappata (2011), Honka (2014), Lin and Wildenbeest (2020), Murry and Zhou (2020)).

[2]Evidence suggests that sellers are likely to know which rivals supply the product. For instance, IBISWorld estimates that annual spending for acquisition of competitor information by companies in the United States was at \$ 2 billion in the first half of 2010s (see, Gilad (2015)). Most large companies have employees whose tasks are to gather and analyze information about their rivals (see, Billand et al. (2010)). These examples imply that, if firms invest in learning rival firms' business decisions, they are likely to have a good idea about product availability at their rivals.

[3]Burdett and Judd (1983)'s model of simultaneous search has been employed to study price competition in both homogeneous goods markets (e.g., MacMinn (1980), Janssen and Moraga-Gonzalez (2004)) and horizontally differentiated goods markets (e.g., Anderson et al. (1992), Moraga-Gonzalez et al. (2021)), competition by multiproduct firms (e.g., McAfee (1995)), price dynamics (e.g., Fershtman and Fishman (1992), Yang and Ye (2008)), and labor markets (e.g., Burdett and Mortensen (1998), Acemoglu and Shimer (2000)). It also serves as a convenient model for empirical studies employing structural estimation techniques (e.g., Hong and Shum (2006), Moraga-González and Wildenbeest (2008), Tappata (2009), De los Santos et al. (2012), Honka (2014), Galenianos and Gavazza (2020)).

compete on prices, while buyers need to engage in costly search to discover prices. We extend the model by assuming that n out of N potential sellers supply the product, where n is a random variable. We say that the product is more available if the probability of a high number of sellers offering the product rises and, accordingly, the probability of a low number of such sellers falls. Sellers know which of them offer the product and set prices conditional on the number of competitors. In contrast, buyers do not know which, and how many, sellers supply the product. Therefore, buyers cannot condition their search strategy on the number of actual sellers and will incur the search cost irrespective of whether the searched seller does or does not have the product.

Our main finding is that as the product becomes more available, buyers may be worse-off. We identify two effects. First, there is a direct competition effect. For a given buyer search strategy, the expected price declines with the realized number of sellers offering the product, i.e., with n. This is simply because sellers compete more intensely as they face more competitors. Since greater product availability means that they are more likely to face a larger number of competing sellers, buyers pay lower prices in expectation. There is, however, also an indirect search effect, which is anti-competitive. With greater product availability, buyers' search incentive declines. This happens for two reasons. First, it is simply easier to find the product and compare different deals. Second, the expected level of price dispersion falls. The (*ex-interim*) level of price dispersion has an inverse U- shape with respect to the realized number of sellers. Then, the expected (or *ex-ante*) level of price dispersion decreases if greater product availability means that a moderate number of sellers is less likely and a high number of sellers is more likely to be in the market. If, however, prices do not differ much across sellers in expectation, buyers have less incentive to compare deals. As buyers compare fewer deals by searching less intensely, sellers' market power for each realization of n increases. We show that this (indirect) anti-competitive effect of greater product availability dominates the (direct) competitive effect if the search cost is relatively small.

To check the robustness of our main result—greater product availability leading to less search and softer competition—we consider different extensions of our model. Specifically, we consider markets where buyers employ *noisy search* (Wilde (1977)). With noisy search, a searching buyer learns of offers of an unknown number of firms, but the more intensely she searches, the higher the probability is that she learns of more offers. Such modeling suits procurement markets where a government agency announces an auction via different platforms (e.g., newspapers, social networking websites such as LinkedIn), instead of soliciting bidders individually. The agency is not sure how many bidders it can attract through each platform, but it can reach more potential bidders if it makes announcements on more platforms. We demonstrate that our main result holds with noisy search. In another extension of our model, we allow for search cost heterogeneity by introducing a share of buyers who observe all offers for free. Such buyers can represent, for instance, consumers who use price-comparison websites. We report that greater product availability mitigates search incentives of buyers with positive search costs and decreases their well-being. Finally, we discuss information asymmetry on product availability in sequential search markets. We demonstrate that greater product availability may harm buyers. However, the mechanism behind this result is different from that in the model of simultaneous search and in line with that in traditional models of consumer search with search cost heterogeneity (e.g., Varian (1980), Stahl (1989)).

Our main result has implications in real-world markets. Products may be more available for a variety of reasons: an improvement in search/matching technology, a technological shock that makes production and logistics more efficient, or market entry. In online markets buyers rely on search engines to find the product. Online search platforms upgrade their algorithm regularly and—although we do not have direct evidence of this—we believe it is self-evident that with the time (namely, in the long run, such as a decade) search engines provide better matches to buyers' search requests. Literature on operations management suggests that many manufacturing companies opt for sharing relevant information with companies along the supply chain to ensure timely delivery of production resources (e.g., Fabbe-Costes and Jahre (2008), Kim (2009), Prajogo and Olhager (2012)). Clearly, timely delivery of production resources allows firms to relax capacity constraints. Regarding policy interventions, any policy that triggers entry makes the product more available to buyers.[4]

The rest of the paper is organized as follows. We discuss our paper's contribution to the literature in the following section. In Section 3 we present the model. We provide an equilibrium analysis in Section 4 and report our comparative statics results in Section 5. In Section 6 we analyze different extensions of our main model. The final section concludes.

2 Related Literature

Our paper contributes to several strands of the literature. One is the consumer search literature with uncertain product availability and, within this field, the studies by Janssen and Non (2009) and Lester (2011) are the closest to our paper. The main difference between these papers and ours is that they do not consider asymmetric information: an

[4] The impact of entry on market outcomes has been extensively studied in the literature on consumer search. Whereas the existing literature identifies sellers' pricing behavior as a leading mechanism behind the results, we show that buyers' search behavior is a key determinant of our results. Also the results of the existing studies crucially depend on search cost heterogeneity, whereas our results hold irrespective of this assumption. We provide a detailed discussion of these points in the next section.

individual seller, just like buyers, does not observe which other sellers offer the product. Therefore, sellers cannot condition their prices on the total number of sellers in the market. For instance, if there happens to be a single seller in the market, the monopolist simply does not know this fact and, in equilibrium, does not necessarily set the monopoly price. There are also other important differences. Specifically, Janssen and Non (2009) restrict their attention to two potential sellers, which makes the impact of greater product availability on competition straightforward. Precisely, the more available the product (or the more probable that there are two sellers rather than a monopolist), the stronger the competition.

Lester (2011) examines a model with exogenously given shares of consumers who observe a single price and those who compare multiple prices in markets where firms face limited production capacity. In the model, consumers do not really engage in search. Instead, they choose a seller from which to make a purchase, realizing that any seller may run out of stock if many buyers choose to buy from that seller. Each consumer who observes only one seller's price queues only at that seller. Given sellers' capacity constraints, price-comparing consumers "compete" to buy lower-priced products. Therefore, an (exogenous) increase in the share of price-comparing consumers spurs competition among these consumers for low-priced products, i.e., low-priced products become less available. Price-comparing buyers understand that their chance of making a purchase at a low price decreases and thus are ready to accept higher prices. This in turn increases firms' incentive to price the product high. Although a higher share of price-comparing consumers implies that firms compete more intensely to attract them, the competitive impact may be dominated by the above anti-competitive effect. Other papers which study uncertain product availability in consumer search markets, but without the information asymmetry, include Janssen and Rasmusen (2002), Rhodes (2011), Gomis-Porqueras et al. (2017) and Moraga-Gonzalez and Watanabe (2020).

There is a large body of literature that studies search frictions and uncertainty about product availability in labor markets. In these studies, uncertainty is related to availability of a job vacancy. The models employed in these studies are in the spirit of Lester (2011), where job-seekers choose to which companies to apply for a job after observing their offers. However, these studies do not consider information asymmetry on job availability. We refer to Wright et al. (2021) for an excellent review of the literature.

There is another strand of the consumer search literature that studies information asymmetries between buyers and sellers. Yet these studies focus on information asymmetry on either marginal cost of production (e.g., Benabou and Gertner (1993), Dana (1994), Tappata (2009), and Janssen et al. (2011)), or product quality (e.g., Hey and McKenna (1981), Pesendorfer and Wolinsky (2003), Wolinsky (2005), Fishman and Levy (2015)).

In a broader sense associating entry of firms with greater product availability, the paper also contributes to the literature that studies the effect of entry on competition (e.g., Janssen and Moraga-Gonzalez (2004), Chen and Riordan (2008), Gabaix et al. (2016), Moraga-Gonzalez et al. (2017), Chen and Zhang (2018)). The papers closest to ours are ones by Janssen and Moraga-Gonzalez (2004) and Moraga-Gonzalez et al. (2017). The crucial difference between these papers and ours is that, in these papers, both consumers and sellers know which sellers offer the product. In addition, Janssen and Moraga-Gonzalez (2004) let a share of consumers have zero search cost and the remaining consumers have a positive search cost which results in a bimodal search cost distribution, and Moraga-Gonzalez et al. (2017) allow for a continuous distribution of search costs. Janssen and Moraga-Gonzalez (2004) report that, if search costs are small (for buyers with a positive search cost) and the number of sellers is sufficiently high, the expected price rises with each additional entry. The reason for this is that competition for consumers with zero search costs increases with the number of firms. As the number of firms reaches a tipping point, firms find it optimal to ripoff buyers with a positive search cost, instead of competing intensely for consumers with zero search cost (as in traditional studies of Varian (1980) and Stahl (1989)). Similarly, Moraga-Gonzalez et al. (2017) demonstrate that, if search costs are relatively dispersed and the number of sellers is high enough, an additional firm entry results in higher prices. The reasoning behind this result is similar to that of Janssen and Moraga-Gonzalez (2004).

The only other paper that accounts for information asymmetry on product availability in search markets is one by Parakhonyak and Sobolev (2015). Whereas we analyze the question in a more traditional Bayesian game of incomplete information, they assume that buyers (who search sequentially) do not have a prior and want to minimize *regret* instead of maximizing utility. The authors report that the equilibrium expected price paid by buyers is invariant to changes in the number of sellers. Their reasoning is as follows. Given the presence of consumers with zero search cost and those with a positive search cost, an increase in the number of sellers has the same two (opposing) effects on prices as those in Janssen and Moraga-Gonzalez (2004). The interaction of these effects pushes up prices. However, there is also an additional effect. Consumers with positive search costs search more intensely as the number of sellers increases and, hence, are more likely to find a lower price. This effect offsets the interaction of the above two effects, so that the expected price paid by buyers does not change.

We finally note that the intuition behind our main result on the detrimental impact of more product availability on competition is similar to those in studies by Fershtman and Fishman (1994) and Armstrong et al. (2009). These papers report that price caps raise the expected price paid by buyers in non-sequential search markets. The reason for this is that an effective price cap limits the range of prices sellers can set. This reduces price dispersion, which in turn mitigates buyers' incentive to search. Just as price caps alleviate price dispersion, a higher probability of more sellers offering the product *may* make prices less dispersed in our model (recall the inverse-U-shaped relationship between the (*ex-interim*) level of price dispersion and the number of sellers).

3 Model

We now present our main model. In the first part of this section we simply lay out our model. In the second part we argue in favor of our model's assumptions.

3.1 Assumptions, Timing, and Equilibrium Concept

In the model there are $N \geq 3$ potential sellers, which we call *firms*. N is assumed to be finite. Nature chooses n number of entrants, or simply *sellers*, where $0 \leq n \leq N$. The probability with which nature chooses n entrants is given by θ_n, so that $\sum_{n=0}^{N} \theta_n = 1$. Let θ represent the probability mass function (PMF) and Θ stand for the corresponding cumulative distribution function (CDF) so that $\Theta(n)$ is the probability that there are at most n sellers in the market. Note that if $\theta_N = 1$, meaning that all firms are active sellers, the model collapses to the traditional model of simultaneous search as in Burdett and Judd (1983), yet with a finite number of sellers. Each firm observes who has entered the market. Sellers produce homogeneous goods at a marginal cost normalized to zero and compete on prices. Since mixed strategies are allowed, we let $x_{nj}(p)$ be the probability that seller j charges a price greater than p when there are n number of sellers in the market.

The demand side of the market is represented by a unit mass of buyers, or *consumers*. Each consumer has an inelastic demand for a unit of a product, which she values at $v > 0$.[5] *Ex-ante*, consumers do not know which (if any) firms are active sellers or what the sellers' prices are. In order to buy a product, a consumer has to engage in costly search and learn of at least one price. Search is simultaneous, meaning a consumer requests prices from k number of firms simultaneously, after which the search is terminated. We let $c > 0$ denote the search cost. Following the majority of literature on consumer search, we assume that searching one firm is free.[6] Finally, let q_k stand for the probability that buyers search k firms so that q represents the search probability distribution. An alternative explanation

[5]We can think of v as the effective reservation price of consumers. Specifically, if we let r be the actual reservation price and $w > 0$ be the outside option of each consumer, then $v = r - w$. Clearly, for $w \geq r$, buyers never participate in the market. The paper focuses on the interesting case of $r > w$.

[6]This assumption does not affect the main results qualitatively. If we allow each search to be costly, then in the trivial equilibrium stated in Proposition 1 buyers do not search and, thus, there is no trade.

is that q_k is the share of consumers who search k firms. As searching one firm is weakly dominant than not searching at all, we set $q_0 = 0$ for the rest of the paper.

The timing of the game is as follows. First, nature chooses a number of sellers that enter the market. Each firm observes whether itself or any other firms entered the market. Consumers do not have this information. Second, sellers simultaneously set prices. Third, without knowing prices, consumers choose the number of firms to visit. Consumers who observe at least one price may make a purchase.

The solution concept is Bayesian-Nash equilibrium (BNE). Let $x_{-jn}(p)$ represent the equilibrium pricing strategies of all sellers, but seller j, in a market with n sellers. Also let $\Pi_{nj}(p, x_{-jn}(p))$ denote the expected profit of seller j that charges p given pricing strategies of other sellers. Then, letting $\overline{\Pi}_{nj} \geq 0$ be some constant for each n and j, we define a BNE as a collection of price distributions $(x_{n1}, ..., x_{nn})_{n=1}^{N}$ and search probability distribution q such that for each n (a) $\Pi_{nj}(p, x_{-jn}(p)) \geq \overline{\Pi}_{nj}$ for all p in the support of $x_{nj}(p)$, $\forall j$, and (b) $\Pi_{nj}(p, x_{-jn}(p)) \leq \overline{\Pi}_{nj}$ for all p, $\forall j$; (c) each consumer searching k firms obtains no lower utility by searching any other number of firms for all $q_k > 0$ and (d) $\sum_{k=1}^{N} q_k = 1$.

Next, it is useful to note that the probability that a consumer observes m prices, when searching k firms in a market with n sellers, follows a hypergeometric PMF. This probability, denoted by $\alpha_{nk,m}$, is

$$\alpha_{nk,m} = \frac{\binom{N-n}{k-m}\binom{n}{m}}{\binom{N}{k}},$$

where for any two positive integers $I_1 < I_2$, we let $\binom{I_1}{I_2} = 0$. We also define

$$\alpha_{nk}(x) \equiv \sum_{m=0}^{n} \alpha_{nk,m} x(p)^m$$

to be the probability generating function, where $x(p)^m \equiv [x(p)]^m$. In the appendix we provide some properties of the hypergeometric PMF which we will employ in our analysis.

3.2 Discussion of Assumptions

We now discuss the following two assumptions of the model: goods are homogeneous and search is simultaneous. Regarding the first assumption, there are numerous real-world markets where products are fairly homogeneous across sellers. In financial credit markets, for instance, the main difference across lenders is the interest rate, and this is certainly a homogeneous good. In procurement markets, certain construction materials, such as ready-mixed concrete and plywood, or renovation services, such as painting walls and installing electric outlets, are fairly homogeneous across sellers.

Regarding our second assumption, it has been well established that simultaneous search is optimal in markets where a buyer wishes to gather information quickly and it takes time for firms to quote prices. Morgan and Manning (1985) demonstrate that this is so even if buyers learn through search not only of prices but also of market-fundamentals, such as product availability. To illustrate this argument, we consider a market where a government agency wishes to procure renovation services. Suppose that on average it takes a week for a renovation company to reply to the agency's request, e.g., to inspect the office rooms and quote a price. If the agency searches sequentially, it first contacts one of the companies, waits around a week to receive a reply from it, and only then decides whether to contact another company. Although sequential search allows the agency to better evaluate product availability with each search round, it is time-costly. In our example, it takes a month (on average) to contact four companies. However, the agency could contact all four companies on the same day and hence expect their replies within a week.

The time-efficiency of simultaneous search is especially relevant if the search has to be terminated at some point owing to some exogenous deadline.[7] Specifically we are referring to deadlines which may occur naturally or be a consequence of regulations. In a market for home renovation services, a consumer who starts searching at the end of autumn may set a tight deadline to find a service provider because she does not wish to undertake a renovation in cold weather. If, within the deadline, the consumer does not find a company that offers the desired service, she may prefer delaying the renovation until the following year. Deadlines in public procurement markets are set by law.[8]

Finally, empirical studies confirm that simultaneous search is prevalent in certain markets. For instance, De los Santos et al. (2012) and Honka and Chintagunta (2017) demonstrate that, in online markets for books and car insurance respectively, models of simultaneous search predict buyers' search behavior better than those of sequential search.

4 Equilibrium Analysis

We start our analysis by identifying consumers' search strategies which can be a part of a BNE. Subsection 4.1 serves this purpose. Here we first demonstrate the existence of a

[7]Implying a sharp rise in the search cost, deadlines are also consistent with the argument that the search cost is convex in the number of searches in sequential search markets (e.g., Ellison and Wolitzky (2012), Carlin and Ederer (2019)). It can be shown that under certain conditions, predictions of sequential search models with convex search coincide with those of simultaneous search models with linear search costs.

[8]Cursor (2019) reports that more than a quarter of the public procurement of medicines in the Russian Federation in 2019 was declared unsuccessful because no bid was submitted within the announced deadlines.

trivial equilibrium where consumers do not search more than one firm. As the existence of this equilibrium is fragile to model assumptions (as we show below) and the predictions of this equilibrium are not realistic, we focus on BNEs where some consumer search more than one firm, i.e., $q_1 < 1$. We call such search behavior an *active* search. We show that in any BNE with active search, buyers either search k firms where $2 \leq k \leq N-1$ or randomize over searching k and $k+1$ firms where $1 \leq k \leq N-1$.

We then proceed to construct those two types of BNEs with active search. To do that, we employ the following steps. In Subsection 4.2, we assume that consumers randomize between searching k and $k+1$ firms, and find the optimal pricing strategies of sellers. Given these pricing strategies, we check whether consumers indeed find it optimal to randomize over searching k and $k+1$ firms. In Subsection 4.3, we apply the same steps, but we consider a case in which all buyers search k firms.

We demonstrate that a BNE with active search definitely exists if the search cost is not too high. Generically, there is a multiplicity of equilibria. We focus on stable equilibria and moreover establish, in Subsection 4.4, that a locally stable BNE with active search is unique for sufficiently small search costs.

4.1 Preliminary Results

Our first result is that an equilibrium, where consumers search no more than one firm, always exists. If buyers do not search more than one firm, they do not compare prices. It is then optimal for sellers to charge the monopoly price v. Such pricing justifies the above search strategy of buyers, as buyers receive zero payoff whether or not they purchase a product, yet searching more than one firm is costly. This is a well-known result in models of both sequential (Diamond (1971)) and simultaneous search (Burdett and Judd (1983)), and is known as the *Diamond parardox*.

Proposition 1. *For any $c > 0$, there exists an equilibrium where sellers set the monopoly price v and consumers search no more than one firm: $q_1 = 1$.*

It is well-known that the existence of a Diamond-paradox type of equilibrium is fragile to small changes in the model assumptions. The equilibrium ceases to exist if, for instance, we introduce a very small share of consumers who learn of multiple offers during a single search (Wilde (1977)) or who have zero search cost (e.g., Salop and Stiglitz (1977), Stahl (1989)).[9] In contrast, equilibria where some consumers search more than one firm are robust to such assumptions, as we show in Section 6.

Therefore we turn our attention to equilibria where some consumers search at least two firms. Our next two results limit search strategies of buyers which can be a part

[9] Many studies show cases of how in equilibrium some of the ex-ante identical consumers observe multiple prices (Butters (1977), Robert and Stahl (1993), Atayev and Janssen (2019)).

of an equilibrium with active search. To state the results, we let $\underline{n} \geq 2$ represent the lowest number of oligopoly sellers that are drawn into the market with a strictly positive probability. This implies that $\theta_2 = ... = \theta_{\underline{n}-1} = 0$ while $\theta_{\underline{n}} > 0$.

Lemma 1. *For any $c > 0$ and $\underline{n} \geq 2$, in a BNE it cannot be that $\sum_{k=N-\underline{n}+2}^{N} q_k = 1$.*

The reasoning is by contradiction. Assume that, if there are at least $\underline{n}$ sellers in a market, consumers see two prices for certain, i.e., $\sum_{k=N-\underline{n}+2}^{N} q_k = 1$. Then, sellers optimally price as follows. The monopolist sets a price equal to v. Sellers price the product at the marginal cost of production if there are at least $\underline{n}$ number of sellers in the market. This argument stems from the observation that, in these markets, the price of an individual seller is always compared with at least one other price. Therefore, an individual seller does not want to be the highest-priced one and, in the case of a tie in prices, undercutting is profitable. Given the above pricing strategies of sellers, the consumer—who searches so as to make sure she observes at least two prices when the number of sellers in the market is at least $\underline{n}$—either faces a market with at most one seller and receives zero surplus (irrespective of whether or not she makes a purchase) or faces a market with at least $\underline{n}$ sellers and certainly makes a purchase, receiving a surplus equal to v from the purchase. However, she can search one firm less, receive the same surplus, and save on search cost. Thus we arrive at a contradiction.

An important implication of Lemma 1 is that, in any equilibrium with active search, sellers play mixed-strategy pricing in the market with $\underline{n}$ number of sellers. This is because some consumers search at least two firms and observe two prices, and consumers do not search so intensely that some buyers observe one price. On the one hand, sellers have an incentive to ripoff the latter type of buyers—also known as *captive* or *locked-in* consumers—by pricing the product high. On the other hand, sellers wish to price the product low to attract price-comparing consumers. The interaction of these two opposing incentives causes price dispersion. Using this fact, the following proposition narrows down even more the search strategies of buyers in an equilibrium with active search.

Proposition 2. *A BNE with $q_1 < 1$ exists if, and only if, $\theta_0 + \theta_1 < 1$. In any such BNE, there exists k such that for $2 \leq k \leq N - \underline{n} + 1$ it must be that $q_k = 1$, or for $1 \leq k \leq N - \underline{n} + 1$ it must be that $0 < q_k < 1$ and $q_k + q_{k+1} = 1$.*

It is straightforward that if there is at most one seller in the market, i.e., $\theta_0 + \theta_1 = 1$, consumers do not search actively. To understand the second part of the proposition, we first let X denote the ex-ante hypothetical price distribution, which we define as follows. This distribution assigns the probability weight of not finding the product to price v, as the buyer receives zero surplus both when she does not make a purchase and when she makes a purchase at price v. For instance, if at most one seller can be in the market (meaning that $\Theta(1) = 1$), X assigns all probability mass to v. X is non-degenerate as prices are dispersed in the market with $\underline{n}$ number of sellers, a fact implied by Lemma 1. Then, we can denote the added benefit of searching, say, the $k+1$th firm as the difference between the expected minimum of k hypothetical prices and that of $k+1$ hypothetical prices. As the distribution of the minimum of k hypothetical prices is $1-(1-X(p))^k$, the difference between the $k+1$th order statistic and the kth order statistic is $X(p)^k(1-X(p))$. This is clearly decreasing in k, meaning that the expected benefit of searching one more firm decreases with k. However, as the expected cost of searching one more firm is constant, it must be optimal for consumers to search the same number of firms or randomize over searching two adjacent numbers of firms.

We now present properties of price distribution in equilibrium with active search.

Proposition 3. *In any BNE with active search, the pricing strategy of sellers is symmetric and unique for each n. Furthermore, for each n the equilibrium price distribution is either degenerate with a unit probability mass at either v or 0, or is non-degenerate with the highest price being v and contains no mass points or flat regions in its support.*

The reasoning is as follows. Clearly, a monopolist seller always charges v, meaning there is a unit probability mass at the monopoly price. If all consumers search at least two firms (note that this does not mean that all consumers observe two prices for $n = \underline{n}$ and thus does not violate Lemma 1), consumers learn at least two prices when all firms happen to be active sellers, i.e., when $n = N$. In this case sellers charge a price equal to the production marginal cost in equilibrium, implying that there is a unit probability mass at $p = 0$. Notice that such pricing does not necessarily arise in equilibrium, e.g., there will be price dispersion for any $\underline{n} \leq n \leq N$ if buyers randomize between searching one firm and searching two firms. Also note that, for any realization of n where sellers play pure-strategy pricing, there is no asymmetry in pricing strategies.

Any asymmetry or multiplicity of optimal pricing strategy may then occur only in markets with price dispersion. Johnen and Ronayne (forthcoming) establish that mixed-strategy pricing of sellers in equilibrium is symmetric and unique if the share of buyers who exactly observe two prices is strictly positive. We show in the appendix that there is a strictly positive share of buyers who exactly observe prices for any n such that there is an equilibrium price dispersion. To ease the notation we drop seller-specific indices to simply write x_n and $\overline{\Pi}_n$ to denote the equilibrium price distribution and the equilibrium profit in a market with n sellers for the rest of the paper.

Moreover, any equilibrium non-degenerate price distribution x_n must be atomless because if it had an atom, undercutting would be beneficial owing to the strictly positive share of consumers who compare at least two prices. Also x_n cannot have a flat region in the support, otherwise an individual seller will not be indifferent between charging the lowest price and charging the highest price in that flat region. Furthermore, the highest price in the support of x_n must be equal to v. It cannot exceed v since a seller charging a price higher than v does not sell to anyone. The highest price in the support cannot be less than v because if it were, a seller could improve its profit by deviating to v, as its expected demand in both cases consists of only locked-in buyers.

The last two propositions provide us with a great deal of information about consumers' and sellers' strategies in equilibria with active search but relatively little information about conditions under which such equilibria may exist. The following two subsections address this issue.

4.2 Mixed Search Strategy

We start by considering the case where consumers play mixed strategies. Suppose that buyers randomize between searching k and $k+1$ firms, where $1 \leq k \leq N - \underline{n} + 1$. What is the optimal pricing strategy for sellers? Obviously, the monopolist seller always charges v. For $n \geq N - k + 2$, the sellers price at the production marginal cost as all consumers compare at least two prices. Finally, when there are n sellers in the market such that $2 \leq n \leq N - k + 1$, they set prices from price distribution x_n. The equilibrium strategy of sellers is such that an individual seller is indifferent between setting any price in the support of the equilibrium price distribution and must (weakly) prefer these prices to ones which are not in the support.

To derive price distribution x_n, we first note that a consumer searching k firms buys from seller j if she visits the seller and observes no lower price than the seller's price. Therefore, seller j pricing at p sells to this consumer with probability

$$\sum_{m=1}^{n} \frac{\binom{N-n}{k-m}\binom{n}{m} m}{\binom{N}{k} n} x_n(p)^{m-1} = \frac{1}{n} \sum_{m=0}^{n} \frac{\binom{N-n}{k-m}\binom{n}{m}}{\binom{N}{k}} m x_n(p)^{m-1} = \frac{\alpha'_{nk}(x_n(p))}{n}.$$

We next let $\beta_{nk}(x) \equiv q_k \alpha_{nk}(x) + (1-q_k)\alpha_{nk+1}(x)$ so that $\beta_{nk,m} \equiv q_k \alpha_{nk,m} + (1-q_k)\alpha_{nk+1,m}$ is the total share of consumers who observe m prices. Then, seller j that sets price p expects to earn

$$\Pi_{nj}(p, x_{-jn}(p)) = p \frac{(\beta_{nk,1} + 2\beta_{nk,2} x_n(p) + 3\beta_{nk,3} x_n(p)^2 + ...)}{n} = \frac{\beta'_{nk}(x_n(p))p}{n}.$$

As an individual seller is indifferent in terms of setting any price in the support of the equilibrium distribution function, it follows

$$p\beta'_{nk}(x_n(p)) = v\beta'_{nk}(x_n(v)). \tag{1}$$

This equation implicitly and uniquely defines equilibrium $x_n(p)$ (recall that uniqueness follows from Proposition 3). For convenience, we will use the inverse function $p_n(x_n)$, which in equilibrium satisfies $p_n(x_n) = n\overline{\Pi}_n/\beta'_{nk}(x_n)$. Then, the lower bound of the price distribution, denoted by $\underline{p}_n$, solves $\underline{p}_n = p_n(1)$.

We now have to check whether consumers indeed randomize between searching k firms and searching $k+1$ firms if sellers price the product as discussed above. To do so, we first note that as the density of the lowest of m prices is $-mx_n(p)^{m-1}x'_n(p)$, the expected price paid by a buyer searching k firms in a market with n sellers is

$$-\sum_{m=1}^{n}\int_{\underline{p}_n}^{v} p\alpha_{nk,m}mx_n(p)^{m-1}x'_n(p)dp = -\int_{\underline{p}_n}^{v} p\alpha'_{nk}(x_n(p))x'_n(p)dp = \int_0^1 p_n(x_n)\alpha'_{nk}(x_n)dx_n,$$

where we obtained the last equality by changing variables from p to $x_n(p)$. As not making a purchase is equivalent to paying price v, we define the expected *virtual* price paid by a consumer who searches k firms as

$$P_k \equiv (\theta_0+\theta_1)v + \sum_{n=\underline{n}}^{N-k+1}\theta_n\left(\alpha_{nk,0}v + \int_0^1 p_n(x_n)\alpha'_{nk}(x_n)dx_n\right).$$

Here, we used the fact that pricing policies of sellers in markets with n sellers such that $1 < n < \underline{n}$ are irrelevant for consumers as, by the definition of $\underline{n}$, we have $\theta_n = 0$ for all $1 < n < \underline{n}$. Next, we can express the expected virtual price paid by consumers who search $k+1$ firms the same way as above by changing the respective indices from k to $k+1$. Then, the incremental benefit of searching the $k+1$th firm is

$$\begin{aligned} P_k - P_{k+1} &= \sum_{n=\underline{n}}^{N-k+1}\theta_n\left((\alpha_{nk,0}-\alpha_{nk+1,0})v + \int_0^1 p_n(x_n)\left(\alpha'_{nk}(x_n)-\alpha'_{nk+1}(x_n)\right)dx_n\right) \\ &= -\sum_{n=\underline{n}}^{N-k+1}\theta_n\int_0^1 p'_n(x_n)\left(\alpha_{nk}(x_n)-\alpha_{nk+1}(x_n)\right)dx_n, \end{aligned}$$

where the second line follows from integration by parts and the facts that $\alpha_{nk}(0)=\alpha_{nk,0}$, $\alpha_{nk+1}(0)=\alpha_{nk+1,0}$, $p_n(0)=v$ and $\alpha_{nk}(1)=\alpha_{nk+1}(1)=1$. Clearly, in equilibrium, it must be the case that

$$P_k - P_{k+1} = c. \tag{2}$$

In the following lemma, we state properties of the expected benefit of searching the $k+1$th firm, and we provide the proof in the appendix.

Lemma 2. *$P_k - P_{k+1}$ is positive and strictly concave in $q_k \in (0,1)$ for each k where $1 \le k \le N-\underline{n}+1$.*

The lemma informs us that, if there exists a solution to (2), there must be either at least one or at most two solutions in $q_k \in (0,1)$. This is due to the concavity of the expected benefit of searching the $k+1$th firm with respect to the share of consumers who search k firms, q_k. The lemma also tells us that a solution(s) exists for a nonempty interval of search costs and the lower bound of this interval is positive. Let $\underline{c}_{k,k+1}$ and $\overline{c}_{k,k+1}$ represent the lower and upper bounds of that interval with $0 \leq \underline{c}_{k,k+1} < \overline{c}_{k,k+1}$. Then, according to the lemma, the bounds are determined as

$$\begin{aligned} \underline{c}_{k,k+1} &= \min\left\{(P_k - P_{k+1}|q_k = 0), (P_k - P_{k+1}|q_k = 1)\right\}, \\ \overline{c}_{k,k+1} &= \max_{q_k}\{P_k - P_{k+1}\}. \end{aligned} \tag{3}$$

This suggests that for values of the search cost in interval $(\underline{c}_{k,k+1}, \overline{c}_{k,k+1})$ there exist either one or two equilibria if buyers prefer searching either k or $k+1$ firms to not searching (i.e., searching one firm). The following proposition demonstrates that this is indeed the case.

Proposition 4. *Let $c \in (\underline{c}_{k,k+1}, \overline{c}_{k,k+1})$ for any $1 \leq k \leq N - \underline{n} + 1$, then there exist at least one and at most two BNEs where buyers randomize over searching k and $k+1$ firms. Such a BNE is given by $\left((x_n)_{n=1}^{N}, q\right)$, where the equilibrium price is v for $n = 1$, 0 for $n \geq N - k + 2$ and x_n is determined by (1) for $\underline{n} \leq n \leq N - k + 1$, and buyers' search strategy is determined by (2).*

Furthermore, $\underline{c}_{k,k+1} = 0$ for $k = 1$ and $k = N - \underline{n} + 1$.

The proof is in the appendix. The first part of the proposition shows the existence of search cost intervals under which equilibria in mixed strategies of buyers exist, but it does not tell us much about those intervals. The second part of the proposition partially addresses this issue (we address this issue fully in Proposition 5). Specifically, it implies that at least two equilibria exist for sufficiently small search costs: in one, buyers randomize over searching one and two firms, and in the other, they randomize over searching $N - \underline{n} + 1$ and $N - \underline{n} + 2$ firms.

4.3 Pure Search Strategy

Next, we consider the case where all consumers search k firms, where $2 \leq k \leq N - \underline{n} + 1$. It is easy to see that, if all consumers search k firms, the monopolist seller charges v and sellers price at the production marginal cost for $n \geq N - k + 2$ in equilibrium. For an intermediate number of sellers, $2 \leq n \leq N - k + 1$, price dispersion arises and equilibrium price distribution x_n is determined by

$$p\frac{\alpha'_{nk}(x_n(p))}{n} = v\frac{\alpha_{nk,1}}{n} = \overline{\Pi}_n > 0, \tag{4}$$

where the inequality is due to $\alpha_{nk,1} > 0$.

To check whether it is indeed optimal for buyers to visit k firms, given the above pricing strategies of sellers, it suffices to find conditions (if there are such) under which the following set of inequalities holds for $q_k = 1$:

$$\begin{aligned} P_k - P_{k+1} &\leq c, \\ P_{k-1} - P_k &\geq c. \end{aligned} \tag{5}$$

The following proposition shows that there exists a nonempty interval of search costs such that the set of inequalities is satisfied.

Proposition 5. *For any $v > 0$, $N \geq 3$, $\underline{n} \geq 2$, $0 \leq \theta_0 + \theta_1 < 1$ and k such that $2 \leq k \leq N - \underline{n} + 1$, there exists $[\underline{c}_k, \overline{c}_k] \subset [0, v]$ given by*

$$\begin{aligned} \underline{c}_k &= (P_k - P_{k+1} | q_k = 1), \\ \overline{c}_k &= (P_{k-1} - P_k | q_k = 1), \end{aligned} \tag{6}$$

such that for $c \in [\underline{c}_k, \overline{c}_k]$ there exists only one BNE where all buyers search k firms. The BNE is given by $\left((x_n)_{n=1}^{N}, q\right)$ where $p = v$ for $n = 1$, $p = 0$ for $n \geq N - k + 2$, x_n is determined by (4) for $2 \leq n \leq N - k + 1$ and $q_k = 1$.

The proof is in the appendix, and the intuition is similar to that behind Proposition 4. Therefore we omit the discussion of the intuition and, instead, we point out the relationship between cutoff values of search cost in the two propositions. Observe that in equilibrium in the buyers' mixed search strategy, where they randomize over searching k and $k+1$ firms, the share of consumers who search k firms can converge to one only if the value of search cost approaches $\underline{c}_k$. Similarly, in that equilibrium, the share of buyers who search $k+1$ firms can converge to one only if the value of search cost approaches $\overline{c}_{k+1}$. Formally, the former means that $\lim_{q_k \uparrow 1}(P_k - P_{k+1}) = \underline{c}_k$ and the latter implies that $\lim_{q_k \downarrow 0}(P_k - P_{k+1}) = \overline{c}_{k+1}$. These two observations, along with the second part of Proposition 4, mean that if the search cost is not very high, there always exists an equilibrium with active search.

Figure 1 graphically illustrates the idea behind the discussion (here $c_1 = 0.02, c_2 = 0.05$ and $c_3 = 0.11$). The horizontal axis represents q_n for $n \in \{1, 2, 3\}$. At each of the three points on the axis, i.e., at each q_n, we have $q_n = 1$. When we move to the left or right of that point along the axis, q_n decreases and q_{n-1} or q_{n+1}, respectively, increases. For example, start with point q_2 on the horizontal axis, which means that $q_2 = 1$ and $q_1 = q_3 = 0$. If we gradually move to the left along the axis, q_2 begins decreasing while q_1 begins increasing so that $q_1 + q_2 = 1$ and $q_3 = 0$. The vertical axis of the graph represents values of the search cost and the expected benefit of searching one more firm. The solid

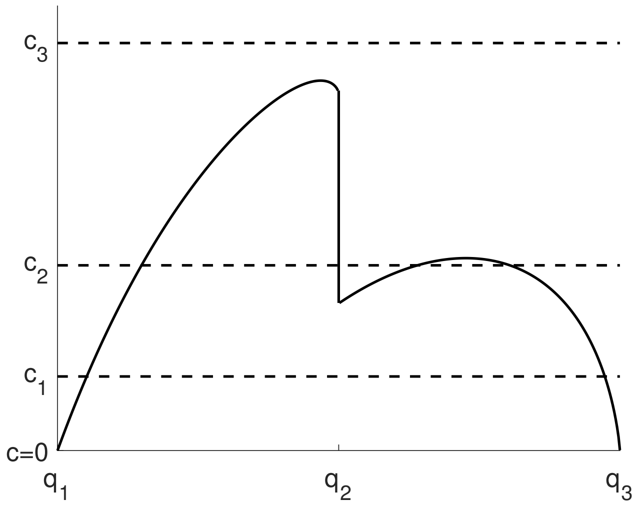

Figure 1: Illustration of BNEs for $N = 3$, $k = 2$, $v = 1$, $\theta_0 = 0$, $\theta_1 = \theta_3 = 0.05$, and $\theta_2 = 0.90$.

curve stands for a function of the incremental benefit of searching one additional firm. Observe that any point on the solid line that corresponds to a point between q_k and q_{k+1} on the horizontal axis represents the incremental benefit of searching the $k+1$th firm. The dashed lines stand for different levels of search cost. Each intersection of the solid curve and a dashed line represents an equilibrium for that particular value of search cost. Importantly, we note the following two points. The solid line is continuous over q_ns with $\lim_{q_2 \uparrow 1}(P_1 - P_2) = \overline{c}_2$ and $\lim_{q_2 \uparrow 1}(P_2 - P_3) = \underline{c}_2$. Moreover, the expected benefit of searching one more firm decreases as all buyers tend to search either one firm or three firms, i.e., $\lim_{q_1 \uparrow 1}(P_1 - P_2) = \underline{c}_{1,2} = 0$ and $\lim_{q_3 \uparrow 1}(P_2 - P_3) = \underline{c}_{2,3} = 0$. These points imply that an equilibrium with active search definitely exists if the search cost is not too high, e.g., if $c = c_1$ or $c = c_2$. For instance, there is no BNE with active search for a value of search cost equal to c_3.

4.4 Stability

Subsections 4.2 and 4.3 demonstrate a multiplicity of equilibria with active search. For instance, observe in Figure 1 that there are four BNEs with active search for a value of search cost equal to c_2. In this subsection we focus on locally stable equilibria, as we wish to undertake comparative static analysis later. We show that for sufficiently small search costs there exists a unique locally stable equilibrium with active search. This equilibrium is characterized by buyers' mixed search strategy.

We employ a notion of stability widely applied in consumer search literature (e.g., Burdett and Judd (1983), Fershtman and Fishman (1992), Janssen and Moraga-Gonzalez (2004), Atayev and Janssen (2019)). Specifically, we say that a BNE is locally stable if a small perturbation in search intensity of consumers around an equilibrium one leads to the convergence of the search intensity to the equilibrium one. We realize that this notion of stability focuses on out-of-equilibrium behavior of buyers only. In principle, one could take a different route: fix the search behavior of consumers and examine price adjustments to a small perturbation. Yet this notion of stability is difficult to conceive, as the equilibrium pricing is in mixed strategies over a compact support for certain realizations of n and convergence to such a mixed strategy is difficult to conceptualize.

Having formalized the notion of stability, we now state conditions under which there is a unique stable BNE with active search in the following corollary. The corollary is implied by propositions 4 and 5, and its proof is in the appendix.

Corollary 1. *For $c \in (0, \min\{\underline{c}_2, \underline{c}_{N-\underline{n}+1}\})$, there exists a unique locally stable BNE where consumers randomize over searching $N - \underline{n} + 1$ and $N - \underline{n} + 2$ firms.*

The intuition is as follows. In the appendix we first show that only two equilibria exist for sufficiently small search costs. Recall from the second part of Proposition 4 that, in one of the two equilibria, buyers randomize between searching one firm and searching two firms and, in the other equilibrium, they randomize over searching $N-\underline{n}+1$ and $N-\underline{n}+2$ firms. To show that these two are the only possible equilibria for sufficiently small search costs, we demonstrate that lower bounds of search cost intervals under which other than the above two equilibria exist are strictly positive. To see this point in Figure 1, note that the lower bound of the search cost interval where all buyers search two firms (which is represented by the lower point on a vertical segment of the solid curve) is strictly positive.

We next show in the appendix that of the above two equilibria, only an equilibrium where buyers randomize over searching $N - \underline{n} + 1$ and $N - \underline{n} + 2$ firms is stable. To do so, we first note that if the share of consumers who search one firm converges to one, the expected benefit of searching a second firm vanishes. This is intuitive because if all consumers search one firm, sellers optimally set the monopoly price and price dispersion vanishes. Similarly, if the share of consumers who search $N - \underline{n} + 2$ firms approaches one—meaning that they all observe at least two prices for certain when there are $\underline{n}$ or more sellers—price dispersion vanishes and an individual buyer is better-off searching one firm less (recall Lemma 1). In Figure 1, the former point is illustrated by the value of the solid curve at point q_1 and the latter by the value of the curve at point q_3.

Second, we know from Lemma 2 that the expected benefit of searching $k+1$th firm in an equilibrium where buyers randomize over searching k and $k+1$ firms is positive and concave in the share of buyers who search k firms. This, along with the above two limiting results, implies that the expected benefit of searching a second firm is increasing in the share of buyers who search two firms in an equilibrium where buyers randomize between searching one firm and searching two firms, whereas the expected benefit of searching

$N - \underline{n} + 2$ firms is decreasing in the share of buyers who search $N - \underline{n} + 2$ firms in an equilibrium where buyers randomize between searching $N - \underline{n} + 1$ firms and searching $N - \underline{n} + 2$ firms. Then, in the former equilibrium, if the actual share of consumers who search two firms is smaller (larger) than the equilibrium one, the expected benefit of searching a second firm is lower (higher) than the cost of doing so. Therefore, buyers have even less (more) incentive to search two firms, and buyers' search intensity diverges from the equilibrium one. In Figure 1, this equilibrium corresponds to the left-most intersection of the dashed line c_1 and the solid curve. We can apply a similar argument to see that an equilibrium where buyers randomize searching over $N - \underline{n} + 1$ and $N - \underline{n} + 2$ firms is stable. In Figure 1, this is illustrated by the right-most intersection of the dashed line c_1 and the solid curve.

5 Comparative Statics

In this section we examine the impact of changes in our two exogenous parameters on equilibrium market outcomes. The exogenous parameters of interest are θ and c, which represent product availability and search cost, respectively. The market outcomes of interest are expected price paid by consumers and their surplus. From now on, we will use the phrase *expected price paid* to refer to the expected price paid by consumers conditional on making a purchase. (In a BNE in mixed strategies of consumers, the expected price paid would mean the *average* expected price paid by consumers conditional on making a purchase when they randomize over searching two numbers of firms.) Notice that the expected price paid is not the same as the virtual price, as the virtual price assumes that a consumer who does not learn of any price and thus does not make a purchase pays price v. We focus on stable BNEs.

A change in θ can be a product of technological growth or government intervention, as we discussed in the introduction. Intuition tells us that greater product availability should benefit buyers, as they are more likely to find the desired product and, importantly, a greater number of sellers is usually associated with more intense competition. We demonstrate that this does not necessarily have to be the case. Specifically, greater product availability may not have any impact on market outcomes or may even harm buyers. The latter result is due to the detrimental effect of greater product availability on consumers' willingness to search.

Changes in c may represent technological advances, such as shops creating their websites so that consumers can find out whether a shop has a product by means of several clicks instead of visiting a brick-and-mortar store. Intuitively—and this turns out to be the case—a smaller search cost strengthens buyers' willingness to search, which in turn increases consumers' chances of comparing prices and thus triggers competition.

Following the order of our analysis in Section 4, we first undertake a comparative static analysis in BNEs where buyers use a mixed strategy. In Subsection 5.2 we examine markets where consumers play a pure strategy.

5.1 Consumers Playing a Mixed Strategy

We focus on a BNE that results when the search costs are sufficiently small. There are two reasons for this. One is that, following Corollary 1, we can see that there is a unique locally stable BNE with active search for small search costs. The other reason is that markets that have been mentioned in the introduction are generally characterized by low search costs relative to the value of the product.

We notice that, as $\sum_{n=0}^{N} \theta_n = 1$, an increase in the probability that there are i sellers (or θ_i) must be accompanied by a decrease in at least one other probability, e.g., θ_j where $j \neq i$. In other words, there are numerous ways of considering a change in the probability that there are i sellers. To understand the main mechanism through which a change in the probability of i sellers being in the market affects market outcomes, it is sufficient to focus on a change in θ_i that is associated with an opposite change in only a single θ_j. In this case, it only matters whether $2 \leq i, j \leq N-k+1$ or not, for $i \neq j$. Then, we need to consider only three cases: neither i nor j is in the set of integers in interval $[2, N-k+1]$, only i or j is in that set, and both i and j are in that set. We assume that $i > j$ such that an increase in θ_i with the associated equal decrease in θ_j implies that a product is more available.

In the following proposition, we state the main result of the section. Specifically, we identify sufficient conditions under which greater product availability has a detrimental effect on buyers' well-being.

Proposition 6. *In a stable BNE where consumers randomize between searching $N-\underline{n}+1$ and $N-\underline{n}+2$ firms for $2 \leq \underline{n} \leq N-1$, an increase in θ_i with a corresponding decrease in θ_j where $i > j$*

(i) causes weakly more search, decreases the expected price paid, and improves consumers' well-being for $j = 1$,

(ii) does not affect the search behavior of buyers, the expected price paid or their well-being for $j \geq \underline{n}+1$,

(iii) causes less search, raises the expected price paid and harms consumers' well-being for $j = \underline{n}$.

The reasoning behind (i) is straightforward. Note that, in equilibrium, prices are dispersed only in a market with $\underline{n}$ sellers and the price in a market with at least $\underline{n}+1$

sellers is equal to the production marginal cost. As the expected prices in markets for any realization of $n \geq 2$ are lower than the monopoly price, the direct effect of a decrease in the probability that there is a monopolist seller is a fall in the expected price paid. If this probability decreases at the expense of the probability of $\underline{n}$ sellers being in the market, there is an additional indirect effect on the expected price paid. Following such a change in product availability, consumers search more intensely as they are more likely to face a market with price dispersion. Therefore, sellers in a market with $\underline{n}$ sellers lose their market power because the share of price-comparing consumers rises. In addition, this change in product availability makes it easier to find the product. Despite the fact that more searching also leads to more resources spent on search, consumers are better-off.

The understanding behind (ii) is intuitive. Since the equilibrium price in a market with at least $\underline{n}+1$ sellers is equal to the production marginal cost, a decrease in the probability that there are at least $\underline{n}+1$ sellers, accompanied by an increase in the probability that there are even more sellers, does not change the expected price paid. Also, it does not affect consumers' search behavior, as this change in product availability does not affect the expected level of price dispersion. Finally, it does not make it more, or less, likely to find the product, as in markets with at least $\underline{n}+1$ sellers all consumers observe price offers of at least two sellers. Then, sellers' market power does not change for any realization of n, nor does consumers' well-being.

Finally, the intuition behind (iii) is as follows. First, there is a direct effect on the expected price paid. To show that, we keep consumers' and sellers' strategies fixed. Since prices in a market with $\underline{n}$ sellers are bounded above the production marginal cost and the equilibrium price in a market with more than $\underline{n}$ sellers is equal to zero, the expected price paid decreases. There is also an indirect effect. This effect takes into account how buyers and sellers respond to the change in product availability. Precisely, consumers search less following a decrease in the probability of $\underline{n}$ sellers being in the market because consumers are less likely to face a market with price dispersion. We know that less search raises the market power of sellers in a market with $\underline{n}$ sellers. Although consumers save on search costs, we demonstrate in the appendix that the negative effect of greater product availability on buyers' well-being dominates its positive effects.

To illustrate this idea, we present an example with $N = 3$, $c = 0.05v$, and $\theta_2 + \theta_3 = 1$, where θ_3 increases from 0.1 to 0.25. In a unique stable equilibrium with active search, buyers randomize between searching two and three firms. This means that the prices are dispersed in a duopoly market and triopoly sellers charge a price equal to the marginal cost of production. Following the increase in θ_3, the share of consumers who search all three firms drops from approximately 78% to 61%, which is around a 22% decrease. The expected price paid increases by around 54%—from $0.153v$ to $0.235v$. As a result, the consumer surplus (incorporating search costs) falls from approximately $0.758v$ to $0.685v$, which is a decrease of 9.6%.

We next discuss an extension of our analysis of the impact of greater product availability on market outcomes by putting more structure on the change of PMF θ. In assessing how any such changes in product availability affect market outcomes, we need to check which parts of Proposition 6 come into play. If different forces in different parts of the proposition affect the expected price paid and consumer well-being the same way (or do not affect market outcomes in different directions, such as parts (i) and (ii)), we can obtain clear-cut results. For instance, suppose that a market can be only an oligopoly, i.e., $\Theta(1) = 0$. This happens if, say, the entry costs of at least $\underline{n}$ firms are sufficiently small that they always enter the market and those of the remaining firms are high so that they randomize between entering the market and not entering it. Now assume that the product becomes more available such that it causes first-order stochastic dominance (FOSD) of CDF Θ. Then, it must be accompanied by a decrease in the probability that there are $\underline{n}$ sellers in the market and an increase in the sum of probabilities that there are more than $\underline{n}$ sellers in the market. Note that economic forces in parts (ii) and (iii) of Proposition 6 are relevant for this case, and thus consumers pay higher prices in expectation and are worse-off.

However, if a similar change in product availability takes place in a market where a seller can be a monopolist with a strictly positive probability (i.e., $\Theta(1) > 0$), it is ambiguous how market outcomes will react to this change in product availability. Buyers may face a monopolist seller if all firms have the same entry cost, so in equilibrium they all randomize between entering the market and not entering it. Then, the above change in product availability may reduce the probability that there is at most one monopolist seller and that there are $\underline{n}$ sellers in the market, but increase the sum of probabilities of more than $\underline{n}$ sellers being in the market. Note that in this case, economic forces in all three parts of Proposition 6 come into play. Which of these forces prevail depends on the way one models a change in product availability (which at the same time results in the FOSD of the CDF Θ). To offer some insights into such situations, we provide a numerical assessment of the impact of greater product availability for some commonly used probability distributions in the online appendix. There we use two versions of each probability distribution. One of these versions includes a strictly positive probability that there is at most one seller, and the other excludes this possibility. For each of the probability distributions, consumers are better-off in the former case and worse-off in the latter case.

Now we examine the impact of a change in c on market outcomes.

Proposition 7. *In any stable BNE characterized by consumers randomizing over searching k and $k+1$ firms, an increase in c causes less search and impairs consumers' well-being.*

It is fairly straightforward that an increase in search cost mitigates consumers' willingness to search. We know that less search is associated with greater market power of sellers. We also know consumers are less likely to find the product if they search less. Consumers still spend less resources on search costs. In the appendix we show that the former two negative effects of an increase in search cost on consumers' well-being dominate the latter positive effect.

5.2 Consumers Playing a Pure Strategy

We continue our comparative static analysis to examine BNEs where consumers play a pure strategy. Notice that there is a continuum of search costs under which such an equilibrium exists. This means that marginal changes in θ or c do not affect the search behavior of buyers or, therefore, pricing strategies of sellers. Nevertheless, these changes affect consumers' well-being as well as the total expected price paid by consumers, as we show in the following proposition.

Proposition 8. *In any stable BNE characterized by all consumers searching k firms,*

(i) an increase in θ_i with a corresponding decrease in θ_j does not affect consumers' search behavior and

(a) pushes down the average expected price paid and improves buyers' well-being for $j \leq N - k + 1$,

(b) has no impact on the expected price paid or on buyers' well-being for $j \geq N - k + 2$;

(ii) an increase in c does not affect consumers' search behavior or the expected price paid and impairs their well-being.

The intuition behind (i) is as follows. The expected price paid by a buyer, conditional on observing at least one price, is

$$\frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}} v.$$

The higher the fraction $\alpha_{nk,1}/(1 - \alpha_{nk,0})$, the greater the expected price paid. The proof shows that this fraction is decreasing in n, for $1 \leq n \leq N - k + 1$. This is intuitive as, for a given search strategy, consumers are more likely to compare prices when there are more sellers in the market. To illustrate the point, we present Figure 2, which depicts the above fraction (the vertical axis) for different values of n (the horizontal axis). However, note that sellers' pricing strategy for each realization of n and buyers' search strategy remain the same following changes in θ. Then, more product availability, as in (a), translates into a lower share of buyers who drop out of the market. This—along with the fact that buyers pay a lower expected price conditional on making a purchase—implies that buyers' well-being rises. However, this is not true if greater product availability is as in (b). In that case, all buyers in both markets—one with i number of sellers and the other with j number of sellers—make a purchase and pay a price equal to the marginal cost of production. Hence, the changes in θ_i and θ_j do not affect market outcomes. In Figure 2, this is illustrated by $j = 9$ and $i = 10$.

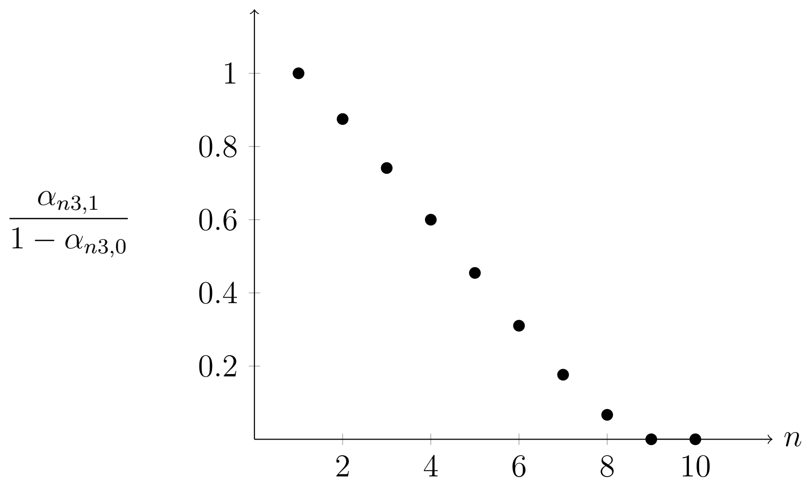

Figure 2: Fraction of consumers who observe exactly one price, conditional on observing at least one price, as a function of n for $N = 10$ and $k = 3$.

Part (ii) of the proposition is straightforward. An increase in search cost causes nothing but a rise in the total resources spent on search by consumers. As a result of this, buyers' welfare declines.

So far we have considered the impact of marginal changes in θ on market outcomes. Thus, to conclude the section, we provide some insights into the impact of substantial (as opposed to marginal) changes in product availability on market outcomes. From Section 4, it follows that there may be two stable BNEs for certain parameter regions. One of the BNEs occurs in pure strategies of consumers, whereas the other occurs in consumers' mixed strategies. Using numerical simulations, we report how market outcomes change in those two equilibria.

Figures 3 and 4 illustrate the impacts of greater product availability on the expected price paid by buyers and their well-being. We used the following parameter constellations: $N = 3$, $\theta_2 + \theta_3 = 1$, $v = 1$ and $c = 0.04$. The horizontal axes in the figures represent the value of θ_3, which we increase from 0.1 to around 0.5. The vertical axis in Figure 3 stands for the expected price paid by a random buyer, while in Figure 4 it stands for buyers' surplus. The solid lines represent the respective variables in a BNE where buyers play mixed strategies. The dashed lines correspond to the respective variables in a BNE characterized by pure strategies of buyers.

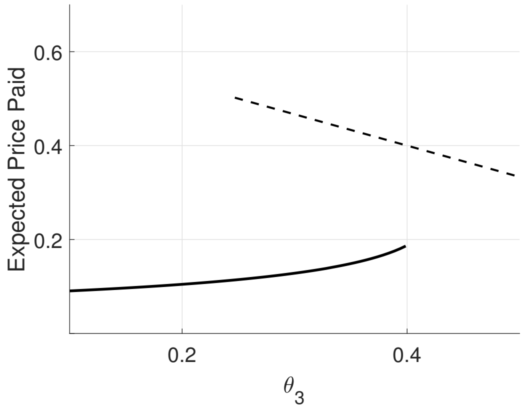

Figure 3: Impact of greater product availability on the expected price paid.

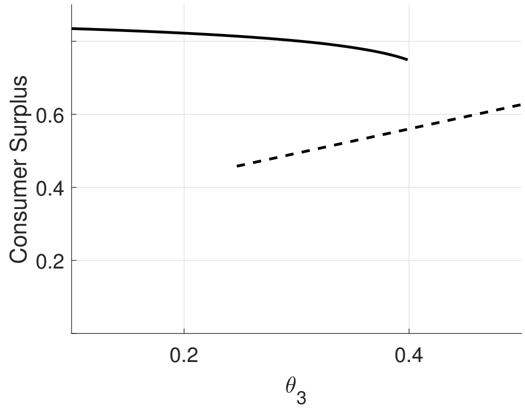

Figure 4: Impact of greater product availability on the consumer surplus.

From both figures we can see that for sufficiently small values of θ_3, there only exists a stable equilibrium in mixed strategies of consumers, with some buyers searching two firms and the remaining ones searching three firms. In contrast, for moderately high values of θ_3, only a stable equilibrium in pure strategies of buyers exists: all buyers search two firms. Finally, for moderate values of θ_3, both stable equilibria exist.

In Figure 3, the expected price paid rises with θ_3 in the equilibrium where consumers play a mixed strategy, whereas the expected price falls with θ_3 in the other equilibrium. Notice that for the value of θ_3 where both equilibria exist, the expected price is higher in the equilibrium characterized by buyers' pure strategies than in the other equilibrium. Figure 4 depicts a picture similar to Figure 3. Importantly, consumers' surplus is decreasing with greater product availability in the equilibrium where buyers play mixed strategies, while it is increasing in the equilibrium with buyers' pure strategies. For values of θ_3 where both types of equilibria exist, buyers are better off in the equilibrium with mixed strategies. This is not surprising, as in the equilibrium with consumers' mixed strategies they search more and impose more competitive pressure on sellers than in the equilibrium with pure strategies.

6 Extensions

In this section, we present extensions of our main model. Particularly, we introduce some "noise" to the search outcome so that, for instance, a consumer who searches once may obtain information about offers of more than one firm. Noisy search can represent procurement markets, where a government agency announces an auction on a platform and the number of potential bidders that will actually see the announcement and participate in the auction is uncertain. In subsection 6.2, we allow for search cost heterogeneity by introducing consumers with zero search cost. A common interpretation of such consumers is that they use price comparison websites. Finally, in subsection 6.3, we discuss the role of product availability in sequential search markets.

6.1 Noisy Search

For this subsection, we modify our main model by considering *noisy search* protocol as in Wilde (1977). With noisy search a buyer faces N different search technologies numbered $1, 2, ..., N$. Search technology $l \in \{1, ..., N\}$ entails cost $(l-1)c$ and provides information about product availability and prices (if the product is available) of k number of firms with probability δ_k^l so that $\sum_{k=l}^{N} \delta_k^l = 1$. Notice that if $\delta_l^l = 1$, noisy search collapses to simultaneous search as in our main model.

We make the following assumptions:

$$\begin{aligned}
&\delta_k^l = 0, \text{ for } k < l, \\
&\sum_{k=1}^{m} \delta_k^l \geq \sum_{k=1}^{m} \delta_k^{l+1}, \text{ for } m \geq l \text{ and } 1 \leq l \leq N-1, \\
&\sum_{k=l}^{N} \left(\delta_k^{l-1} + \delta_k^{l+1} - 2\delta_k^l\right) \alpha_{nk}(x) > 0, \text{ for } 2 \leq l \leq N-1, n \geq 2, x \in [0,1].
\end{aligned} \tag{7}$$

Assumptions in the first two lines reflect features of real-world markets fairly well. The first line means that a buyer searching according to search technology l receives information about at least l firms. The second line implies that a higher-numbered search technology is more likely to yield information about more firms than a lower-numbered search technology. The assumption in the third line is purely technical, and we need it to establish the existence of an equilibrium akin to one in Section 4.2.

We do not establish the uniqueness of an equilibrium, since our aim is to demonstrate that our results concerning the impact of greater product availability on market outcomes are robust to our modeling assumptions. We simply show the existence of an equilibrium similar to the unique (stable) one in the main body of the paper. Specifically, we demonstrate that for sufficiently small search costs there exists an equilibrium where consumers randomize over choosing two adjacent numbers of search technologies: $l(= N - \underline{n} + 1)$ and $l + 1(= N - \underline{n} + 2)$. In this equilibrium, sellers price the product in a way similar to Proposition 4. More specifically, a monopolist seller charges price v, sellers in a market with $\underline{n}$ number of sellers play mixed strategy pricing, and sellers in a market with more than $\underline{n}$ number of sellers charge a price equal to the production marginal cost. These observations are not surprising since simultaneous search is a special case of noisy search.

Since players' equilibrium behavior with noisy search is similar to that with simultaneous search, the impact of greater product availability on market outcomes is qualitatively the same as in the model with simultaneous search.

Proposition 9. *For any $N \geq 3$, $v > 0$, $\underline{n} \geq 2$, $\theta_0 + \theta_1 < 1$ and sufficiently small search cost $c > 0$, there exists a BNE where consumers randomize over choosing search technologies $N - \underline{n} + 1$ and $N - \underline{n} + 2$.*

Furthermore, greater product availability associated with an increase in θ_i and an equal decrease in θ_j where $i > j$

(i) causes weakly more search, reduces the expected price paid and improves consumers' well-being for $j = 1$,

(ii) does not affect consumers' search behavior, the expected price paid or consumers' well-being for $j \geq \underline{n} + 1$,

(iii) causes less search, raises the expected price paid and harms consumers' well-being for $j = \underline{n}$.

The proof is presented in the appendix. We omit the detailed discussion of the intuition, as it is similar to that of Proposition 6.

6.2 Search Cost Heterogeneity

In this subsection, we extend our model to address consumer heterogeneity. There are numerous ways to model consumer heterogeneity, but here we focus on heterogeneity of search costs. There is ample empirical evidence suggesting that buyers differ in terms of their search costs in real-world markets (e.g., Hong and Shum (2006), De los Santos et al. (2012), Honka and Chintagunta (2017)).

One common way of incorporating search cost heterogeneity into our model is the introduction of consumers who observe all prices in the market for free. With respect to search costs, we can think of these buyers as ones who use price comparison websites which list all available offers and prices (e.g., Varian (1980), Stahl (1989), Janssen and Moraga-Gonzalez (2004)). Thus, we assume that $\lambda \in (0, 1)$ is the (exogenously given) share of buyers who observe all prices. Call them *costless buyers*, and refer to the rest of the buyers as *costly buyers*. Since our aim is to show that the main mechanisms of our model exist in the presence of search cost heterogeneity, we will restrict our attention to cases where the probability that there are two sellers is strictly positive, or $\underline{n} = 2$. The rest of the model remains unchanged.

As in our main model, there is a unique stable BNE for sufficiently small search costs. In the equilibrium, costly buyers randomize over searching $N - 1$ and N firms. Then, it is easy to see that a monopolist seller sets price v, whereas sellers in a market with at least three sellers (namely, $n \geq 3$) price at the production marginal cost. Duopoly sellers play a mixed strategy. If we let $q \equiv q_{N-1}$ so that $q_N = 1 - q$, the expected profit of seller j that sets price p is

$$\Pi_{2j}(p, x_{-j}(p)) = \left[(1-\lambda)\left(\frac{q}{N} + \left(\frac{q(N-2)}{N} + (1-q)\right) x_2(p)\right) + \lambda x_2(p)\right] p.$$

It is easy to establish that the highest price in the support of $x_2(p)$ must be equal to v. Using this fact, we can derive the price distribution:

$$x_2(p) = \mu(q)\left(\frac{v}{p} - 1\right), \text{ with support } \left[\frac{\mu(q)}{1+\mu(q)} v, v\right], \tag{8}$$

where

$$\mu(q) = \frac{q(1-\lambda)}{N - 2q(1-\lambda)}$$

which is the ratio of locked-in consumers to that of price-comparing buyers (as in traditional models of Varian (1980) and Stahl (1989)).

Given the above pricing strategies of sellers, a costly buyer is indifferent between searching $N-1$ and N firms if

$$c = \frac{2\theta_2}{N}\left(E[p] - E[\min\{p_1, p_2\}]\right), \tag{9}$$

where $E[p]$ is the expected price and $E[\min\{p_1, p_2\}]$ is the expected minimum of the two prices in a duopoly market.

Our main concern is the impact of greater product availability on prices and costly consumers' well-being. The following proposition states the main result.

Proposition 10. *For any $v > 0$, $N \geq 3$, $\underline{n} = 2$ and $\lambda \in (0,1)$, there exists $\bar{c} > 0$ such that for $c \leq \bar{c}$ there exists a unique stable BNE given by $\left((x_n)_{n=1}^N, q\right)$, where $p = v$ for $n = 1$, $p = 0$ for $n \geq 3$, x_2 is given by (8), and q is implicitly determined by (9).*

Furthermore, an increase in θ_i with a corresponding decrease in θ_j

(i) causes weakly more search, reduces the expected price paid and improves costly consumers' well-being for $j = 1$,

(ii) does not affect consumers' search behavior, the expected price paid or costly consumers' well-being for $j \geq 3$,

(iii) causes more search, raises the expected price paid and harms costly consumers' well-being for $j = 2$.

The intuition is similar to that behind Proposition 6 and thus we omit it to avoid repetition. The only difference is that in the current proposition we report the impact of greater product availability on costly buyers' welfare only. Although analyzing the effect of a change in product availability on costless consumers would provide us with additional insight, such analysis turns out to be intractable.

6.3 Sequential Search

As a final extension of the model, we consider *sequential search* protocol. Sequential search is a type of search such that a consumer who has searched m firms so far, where $0 \leq m \leq N$, decides whether (i) to buy at the lowest of the observed prices if there are any, (ii) to search for one more firm if there are any non-searched firms, (iii) or to drop out of the market. We assume that buyers can engage in sequential search instead of simultaneous search, and the rest of the game and its timing are the same as in the main part of our paper.

Diamond (1971) shows that if all consumers face positive search costs (without uncertainty about product availability), the unique equilibrium is one where consumers do not search beyond the first firm and sellers price at v. This result holds in our model where consumers face uncertainty about product availability.

To obtain an equilibrium with active search and to provide some insights into the effect of greater product availability on that equilibrium, we consider a special case of sequential search in the working paper version of our study (Atayev (2019)). There we incorporate a share of consumers with zero search costs. We also assume that a buyer obtains information about a random firm and decides whether to search all of the remaining firms. By doing so, we reduce the number of search paths available to buyers. This search protocol is known as *newspaper search* (Varian (1980)) and is widely used in empirical studies examining gasoline markets (e.g., Tappata (2009), Chandra and Tappata (2011)). Furthermore, we restrict our focus to oligopoly markets with at least two and at most three sellers. This helps us to obtain a mixed pricing strategy that does not contain any atoms or flat regions. If there were a possibility of a monopolist seller being in the market, oligopoly sellers would want to charge the monopoly price with a strictly positive probability to signal an absence of competition. Finally, to ease analysis even further—and in line with the majority of literature on sequential consumer search—we impose *passive* out-of-equilibrium beliefs. This means that, if a consumer visits a seller and observes a price that is not a part of an equilibrium, she believes that other sellers price the product according to an equilibrium strategy.

Under these assumptions, we show that greater product availability may harm consumers with a positive search cost. Yet the mechanism behind this result is different from that in our model of simultaneous search, and is as follows. With each additional seller in the market, an individual seller's chance of selling to costless consumers falls more rapidly than their share of locked-in (costly) buyers. Thus, sellers' incentive to ripoff locked-in buyers by charging higher prices rather than competing fiercely for costless buyers rises with more product availability (as in Varian (1980) and Stahl (1989)). As a result, costly buyers pay higher prices in expectation.

7 Conclusion

We see the current paper as being the first to address information asymmetry between buyers and sellers on product availability in search markets. The results of the paper suggest that technological progress that makes product more available to buyers or policy intervention that stimulates entry do not necessarily benefit buyers. Thus, accounting for information asymmetry on product availability may help to better evaluate the impact of such technological progress and policy interventions on market outcomes.

We understand that the model is restricted in the sense that it considers a homogeneous goods market. In reality, goods are differentiated in many markets and buyers compare different deals not only on the basis of prices but also other products characteristics, such as match value. Therefore, a natural extension of the model is an incorporation of horizontal product differentiation as in Perloff and Salop (1985), Anderson et al. (1992) and Moraga-Gonzalez et al. (2021). In these models, uncertain product availability can be interpreted as inactive firms "supplying" a product with zero match value. Then, as products become more available, the dispersion of offers across sellers may either increase or decrease depending on the distribution of the actual products' match values. For instance, if the distribution of the actual match values is skewed towards left (right), more product availability reduces the probability that buyers will face inactive firms, or products with zero match values. Therefore, from buyers' perspective, the resulting distribution of match values is even more (less) skewed towards left and the dispersion of offers decreases (increases). As buyers' search intensity depends on the distribution of match values (among other things), the impact of greater product availability on buyers' incentive to search and, thus, on market outcomes is not straightforward.

A Hypergeometric Distribution

In this section, we discuss application of hypergeometric distribution and its properties that will be helpful for our proofs.

The distribution is widely applied in audits, e.g., quality control, election audit. Suppose that an auditor randomly selects k items from a population of N items and approves a product only if no more than m of the selected items are defect, or do not meet quality requirements. Assume that n number of items in the population are defect. As the probability that m of the k selected items follows a hypergeometric mass function, one can use the function to test what the probability with which the auditor approves the product.

We notice that our probability generating function $\alpha_{nk}(x)$ is closely related to the Gauss hypergeometric function. If

$$ {}_2F_1(a,b;c;x) \equiv \sum_{m=0}^{n} \frac{(a)_m (b)_m}{(c)_m m!} x^m(p) $$

represents the Gauss hypergeometric function, where $(a)_m = a(a+1)...(a+m-1)$, then it follows that

$$ \alpha_{nk}(x) = \frac{\binom{N-n}{k}}{\binom{N}{k}} {}_2F_1(-n,-k;N-n-k+1;x). \tag{A.1} $$

To obtain the equation, we first note that for any $n \geq 0$ and $0 \leq m \leq n$ we have

$$ \binom{n}{m} = \frac{(n-m+1)_m}{m!}, $$

so that

$$ \begin{aligned} \alpha_{nk}(x) &= \sum_{m=0}^{n} \frac{\binom{N-m}{n-m}\binom{n}{m}}{\binom{N}{k}} x(p)^m = \frac{\binom{N-n}{k}}{\binom{N}{k}} \sum_{m=0}^{n} \frac{\binom{N-m}{n-m}\binom{n}{m}}{\binom{N-n}{k}} x(p)^m \\ &= \frac{\binom{N-n}{k}}{\binom{N}{k}} \sum_{m=0}^{n} \frac{(N-n-(k-m)+1)_{k-m}(n-m+1)_m}{(N-n-k+1)_k} \frac{k!}{(k-m)!m!} x(p)^m \\ &= \frac{\binom{N-n}{k}}{\binom{N}{k}} \sum_{m=0}^{n} \frac{(N-n-(k-m)+1)_{k-m}(n-m+1)_m(k-m+1)_m}{(N-n-k+1)_k m!} x(p)^m. \end{aligned} $$

Next, as

$$ \frac{(N-n-(k-m)+1)_{k-m}}{(N-n-k+1)_k} = \frac{1}{(N-n-k+1)_m}, $$

it follows that

$$ \alpha_{nk}(x) = \frac{\binom{N-n}{k}}{\binom{N}{k}} \sum_{m=0}^{n} \frac{(n-m+1)_m(k-m+1)_m}{(N-n-k+1)_m m!} x(p)^m. $$

Finally, we rewrite

$$\begin{aligned}(n-m+1)_m(k-m+1)_m &= n(n-1)...(n-m+1)\times k(k-1)...(k-m+1)\\ &= (-n)(-n+1)...(-n+m-1)\times(-k)(-k+1)...(-k+m-1)\\ &= (-n)_m(-k)_m\end{aligned}$$

to obtain (A.1).

In the following lemma, we note a property of hypergeometric mass function, which we will use for our proofs.

Lemma 3. *For any $n \geq \underline{n}$ and $k \in \{1, ..., N-1\}$, it must be that $\sum_{m=0}^{l} \alpha_{nk,m} \geq \sum_{m=0}^{l} \alpha_{nk+1,m}$ for all $0 \geq l \geq k+1$.*

Proof. We prove the lemma with the help of the following four claims.

Claim 1. *For any $n \in \{0, ..., N\}$ and $k \in \{1, ..., N\}$, it must be that $\alpha_{nk,m}$ is single-peaked in m.*

Proof of Claim 1. It suffices to show that once $\alpha_{nk,m}$ starts decreasing in m, it never increases with m. Formally, it must be that $\alpha_{nk,m} \geq \alpha_{nk,m+1}$ if $\alpha_{nk,m-1} \geq \alpha_{nk,m}$ for $m \geq 1$. That $\alpha_{nk,m-1} \geq \alpha_{nk,m}$ for $m \geq 1$ means

$$\frac{\binom{n}{m-1}\binom{N-n}{k-m+1}}{\binom{N}{k}} \geq \frac{\binom{n}{m}\binom{N-n}{k-m}}{\binom{N}{k}} \text{ for } m \geq 1,$$

which simplifies to

$$\frac{1}{(n-m+1)(k-m+1)} \geq \frac{1}{m(N-n-k+m)} \text{ for } m \geq 1. \tag{A.2}$$

Then, it is indeed that $\alpha_{nk,m} \geq \alpha_{nk,m+1}$ if

$$\frac{\binom{n}{m}\binom{N-n}{k-m}}{\binom{N}{k}} \geq \frac{\binom{n}{m+1}\binom{N-n}{k-m-1}}{\binom{N}{k}},$$

which simplifies to

$$\frac{1}{(n-m)(k-m)} \geq \frac{1}{(m+1)(N-n-k+m+1)}.$$

The last inequality holds, as the denominator on its LHS is smaller than that on the LHS of (A.2) and the denominator on its RHS is greater than that on the RHS of (A.2). This proves that $\alpha_{nk,m} \geq \alpha_{nk,m+1}$ if $\alpha_{nk,m-1} \geq \alpha_{nk,m}$ for $m \geq 1$, which implies that $\alpha_{nk,m}$ is single- peaked in m. The proof of the claim is complete. □

Claim 2. *For any $k \in \{1, ..., N-1\}$, it must be $\alpha_{nk,0} > \alpha_{nk+1,0}$ for any $\underline{n} \leq n \leq N-k$ and $\alpha_{nk,1} > \alpha_{nk+1,1}$ for $n = N-k+1$.*

Proof of Claim 2. It is straightforward to calculate:

$$\alpha_{nk,0} - \alpha_{nk+1,0} = \frac{\binom{N-n}{k}}{\binom{N}{k}} - \frac{\binom{N-n}{k+1}}{\binom{N}{k+1}} = \frac{(N-n)!(N-k-1)!}{N!(N-n-k-1)!}\left(\frac{N-k}{N-n-k} - 1\right) > 0.$$

For $n = N - k + 1$, we have

$$\alpha_{nk,1} - \alpha_{nk+1,1} = \frac{\binom{k-1}{k-1}\binom{N-k+1}{1}}{\binom{N}{N-k+1}} - \frac{\binom{k-1}{k}\binom{N-k+1}{1}}{\binom{N}{k+1}} = \frac{\binom{k-1}{k-1}\binom{N-k+1}{1}}{\binom{N}{N-k+1}} > 0,$$

where the second equality is due to the fact that $\binom{k-1}{k} = 0$. □

Claim 3. *For any $1 \le k \le N - 1$, it must be* $\arg\max_m \alpha_{nk,m} \le \arg\max_m \alpha_{nk+1,m}$.

Proof of Claim 3. Note that $\alpha_{nk,m}$ is strictly positive for $0 \le m \le \min\{n, k\}$ and is equal to zero for the other values of m. Also $\alpha_{nk,m}$ is single-peaked, which follows from Claim 1. Then, $\alpha_{nk,m}$ achieves its maximum at the integer above value t_k which satisfies $\alpha_{nk,t_k} = \alpha_{nk,t_{k+1}}$. Clearly, the equality implies

$$\binom{N-n}{k-t_k}\binom{n}{t_k} = \binom{N-n}{k-t_k-1}\binom{n}{t_k+1}.$$

It is easy to check that this can be reduced to

$$(N - n - (k - t_k - 1))(t_k + 1) = (k - t_k)(n - t_k),$$

which yields

$$t_k = \frac{(k+1)(n+1)}{N+2} - 1. \tag{A.3}$$

Notice that it is possible that $t_k < 0$, and thus define $M(z)$ to be an operator which spits out z if z is a positive integer, the next high positive integer if z is a positive fraction, and 0 if $z < 0$. Then, the solution to $\arg\max_m \alpha_{nk,m}$ is t_k such that

$$t_k = M(t_k).$$

Similarly, the solution to $\arg\max_m \alpha_{nk+1,m}$ is

$$t_{k+1} = M(t_{k+1}),$$

where, it is easy to check that,

$$t_{k+1} = \frac{(k+1)(n+1)}{N+2}. \tag{A.4}$$

Then, $\arg\max_m \alpha_{nk,m} \le \arg\max_m \alpha_{nk+1,m}$ holds if

$$t_k = \frac{(k+1)(n+1)}{N+2} - 1 \le \frac{(k+1)(n+1)}{N+2} = t_{k+1},$$

which is certainly true. This completes the proof of the claim. □

Claim 4. *For any $k \in \{1, ..., N-1\}$, it must be* $\frac{\alpha_{nk,m+1}}{\alpha_{nk,m}} < \frac{\alpha_{nk+1,m+1}}{\alpha_{nk+1,m}}$ *for any* $0 \le m \le k$.

Proof of Claim 4. It is easy to calculate that

$$\begin{aligned}\frac{\alpha_{nk,m+1}}{\alpha_{nk,m}} &= \frac{\binom{N-n}{k-m-1}\binom{n}{m+1}}{\binom{N-n}{k-m}\binom{n}{m}} = \frac{(k-m)(n-m)}{(N-n-(k-m-1))(m+1)},\\ \frac{\alpha_{nk+1,m+1}}{\alpha_{nk+1,m}} &= \frac{\binom{N-n}{k+1-m}\binom{n}{m}}{\binom{N-n}{k-m}\binom{n}{m+1}} = \frac{(k+1-m)(n-m)}{(N-n-(k-m))(m+1)}.\end{aligned}$$

Then, $\frac{\alpha_{nk,m+1}}{\alpha_{nk,m}} < \frac{\alpha_{nk+1,m+1}}{\alpha_{nk+1,m}}$ is true if

$$\frac{k-m}{N-n-(k-m-1)} < \frac{k+1-m}{N-n-(k-m)},$$

which certainly holds because the numerator of the LHS is lower than that of the RHS and the denominator of the LHS is greater than that of the RHS. The proof of the claim is complete. □

Claim 1 shows that $\alpha_{nk,m}$ is single peaked m. Claim 2 implies that $\alpha_{nk,m}$ obtains its maximum in m for a weakly lower value of m than $\alpha_{nk+1,m}$. Claim 3 means that when $\alpha_{nk,m}$ is increasing in m, it increases more slowly than $\alpha_{nk+1,m}$; $\alpha_{nk,m}$ starts decreasing in m no later than $\alpha_{nk+1,m}$; and when both $\alpha_{nk,m}$ and $\alpha_{nk,m}$ $\alpha_{nk+1,m}$ decrease in m, the former decreases faster in m than the latter. These facts, along with Claim 4 and the fact that $\sum_{m=0}^{k} \alpha_{nk,m} = 1$, establish the the proof of the lemma. □

B Proofs

B.1 Proof of Proposition 3

For the proof of the proposition, it suffices to show that in any equilibrium with active search, the share of buyers who observe two prices is strictly positive for any n where sellers play mixed-strategy pricing. For the proof that sellers' equilibrium mixed strategy is symmetric and unique if the share of buyers comparing two prices is positive, we refer to Johnen and Ronayne (forthcoming).

We consider two cases: one where all buyers search the same k number of firms where $2 \leq k \leq N - \underline{n} + 1$, and the other where buyers randomize between searching k firms and searching $k+1$ firms where $1 \leq k \leq N - \underline{n} + 1$. In the former case, the share of consumers that compare two prices is $\binom{N-n}{k-2}\binom{n}{2}/\binom{N}{k}$, which is strictly positive if its numerator is greater than zero. As $\binom{N-n}{k-2} > 0$ for any $\underline{n} \leq n \leq N - k + 1$ and $2 \leq k \leq N - \underline{n} + 1$ and $\binom{n}{2} > 0$ for any $n \geq \underline{n}$, the numerator is indeed greater than zero, which means that the share of buyers who observe two prices is strictly positive. Next, suppose that buyers randomize over searching k and $k+1$ firms where $1 \leq k \leq N - \underline{n} + 1$. Then, the share of consumers who compare two prices is a weighted average of $\binom{N-n}{k-2}\binom{n}{2}/\binom{N}{k}$ and $\binom{N-n}{k-1}\binom{n}{2}/\binom{N}{k+1}$. It is easy to see that the numerator of at least one of these terms is greater than zero for any $\underline{n} \leq n \leq N - k + 1$, which shows that the share of buyers comparing two prices is strictly positive. Thus, in any equilibrium with active search, the share of buyers who observe two prices is strictly positive for any n where sellers play mixed-strategy pricing. This completes the proof of the proposition.

B.2 Proof of Lemma 2

That $P_k - P_{k+1}$ is positive follows directly from the stochastic dominance in Lemma 3. To show that $P_k - P_{k+1}$ is strictly concave, we note that

$$\frac{d(P_k - P_{k+1})}{dq_k} = v \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 \frac{\left(\alpha'_{nk}(x_n) - \alpha'_{nk+1}(x_n)\right)\left(\alpha_{nk,1}\alpha'_{nk+1}(x_n) - \alpha_{nk+1,1}\alpha'_{nk}(x_n)\right)}{\left[\beta'_{nk}(x_n)\right]^2} dx_n,$$

and

$$\frac{d^2(P_k - P_{k+1})}{dq_k^2} = -v \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 \frac{2\beta'_{nk}(x_n)(\alpha'_{nk}(x_n) - \alpha'_{nk+1}(x_n))^2 \left(\alpha_{nk,1}\alpha'_{nk+1}(x_n) - \alpha_{nk+1,1}\alpha'_{nk}(x_n)\right)}{\left[\beta'_{nk}(x_n)\right]^4} dx_n,$$

which is negative if $\Psi_n(x) \equiv \alpha_{nk,1}\alpha'_{nk+1}(x_n) - \alpha_{nk+1,1}\alpha'_{nk}(x_n) \geq 0$ for each $n \geq \underline{n}$ and all $x \in [0,1]$ and $\Psi_n(x) > 0$ for each $n \geq \underline{n}$ and some $x \in [0,1]$. To simplify the notation, we write x to imply x_n unless stated otherwise. Note that $\Psi_n(0) = \alpha_{nk,1}\alpha_{nk+1,1} - \alpha_{nk+1,1}\alpha_{nk,1} = 0$. Then, $\Psi_n(x) > 0$ for $x \in (0,1]$ if $d\Psi_n(x)/dx > 0$ for $x \in (0,1]$. To show that the derivative is positive, we first note that $\Psi_n(x)$ is $\bar{l} \equiv \min\{n-1, k\}$ times differentiable in x. Second, we point out that lth (where $l \leq \bar{l}$) derivative of the function is

$$\begin{aligned} \frac{d^l \Psi_n(x)}{dx^l} &= \alpha_{nk,1} \frac{d^l \alpha'_{nk+1}(x)}{dx^l} - \alpha_{nk+1,1} \frac{d^l \alpha'_{nk}(x)}{dx^l} \\ &= \alpha_{nk,1} \sum_{m=l}^{k+1} \frac{m!}{(m-l)!} \alpha_{nk+1,m} x^{m-l} - \alpha_{nk+1,1} \sum_{m=l}^{k} \frac{m!}{(m-l)!} \alpha_{nk,m} x^{m-l}. \end{aligned} \tag{B.1}$$

Third, we prove that $\frac{d^l \Psi_n(x)}{dx^l} > 0$ for each l such that $1 \leq l \leq \bar{l}$. We start by considering $\bar{l}$th derivative of $\Psi_n(x)$. For $n - 1 \geq k$, we have

$$\frac{d^k \Psi_n(x)}{dx^k} = \alpha_{nk,1}(k+1)!\alpha_{nk+1,k+1} > 0.$$

This means that $d^{k-1}\Psi_n(x)/dx^{k-1}$ in (B.1) is increasing in x. Then, however, it must be that $d^{k-1}\Psi_n(x)/dx^{k-1} > 0$ for $x \in (0,1]$ if $d^{k-1}\Psi_n(x)/dx^{k-1} \geq 0$ for $x = 0$. It is easy to see that

$$\left.\frac{d^{k-1}\Psi_n(x)}{dx^{k-1}}\right|_{x=0} = \alpha_{nk,1}k!\alpha_{nk+1,k} - \alpha_{nk+1,1}k!\alpha_{nk,k},$$

which is positive if $\alpha_{nk,1}\alpha_{nk+1,k} \geq \alpha_{nk+1,1}\alpha_{nk,k}$, which expands to

$$\frac{\binom{N-n}{k+1-1}\binom{n}{1}\binom{N-n}{k-k}\binom{n}{k}}{\binom{N}{k}\binom{N}{k+1}} \leq \frac{\binom{N-n}{k-1}\binom{n}{1}\binom{N-n}{k+1-k}\binom{n}{k}}{\binom{N}{k}\binom{N}{k+1}} \Rightarrow \binom{N-n}{k} \leq \binom{N-n}{k-1}(N-n),$$

or,

$$\binom{N-n}{k} \leq \binom{N-n}{k} \frac{(N-n)k}{N-n-(k-1)}.$$

This is true if $N - n - (k-1) \leq (N-n)k$. The last inequality can be simplified as $0 \leq (N-n+1)(k-1)$ which is clearly true. This shows that (B.1) for $l = k-1$ is positive for $x = 0$. Then, (B.1) for $l = k-1$ is strictly positive for any $x \in (0,1]$.

Now, we repeat similar steps to show that (B.1) holds for $l = k-2$. Namely, the fact that $d^{k-1}\Psi_n(x)/dx^{k-1} > 0$ for $x \in (0,1]$ means that $d^{k-2}\Psi_n(x)/dx^{k-2}$ is strictly increasing in $x \in (0,1]$. Then, $d^{k-2}\Psi_n(x)/dx^{k-2} > 0$ for all $x \in (0,1]$ if $d^{k-2}\Psi_n(x)/dx^{k-2}|_{x=0} \geq 0$. Instead of proving the last condition each time, we demonstrate that this holds for each l, which is true if $\alpha_{nk+1,1} l! \alpha_{nk,l} \leq \alpha_{nk,1} l! \alpha_{nk+1,l}$, or

$$\alpha_{nk+1,1}\alpha_{nk,l} \leq \alpha_{nk,1}\alpha_{nk+1,l}. \tag{B.2}$$

The inequality can be expanded as

$$\frac{\binom{N-n}{k}\binom{n}{1}\binom{N-n}{k-l}\binom{n}{l}}{\binom{N}{k}\binom{N}{k+1}} \leq \frac{\binom{N-n}{k-1}\binom{n}{1}\binom{N-n}{k+1-l}\binom{n}{l}}{\binom{N}{k}\binom{N}{k+1}},$$

which implies

$$\binom{N-n}{k}\binom{N-n}{k-l} \leq \binom{N-n}{k-1}\binom{N-n}{k+1-l},$$

or,

$$\binom{N-n}{k}\binom{N-n}{k+1-l}\left(\frac{k-l+1}{N-n-(k-l)}\right) \leq \binom{N-n}{k}\binom{N-n}{k+1-l}\left(\frac{k}{N-n-(k-1)}\right).$$

This reduces to $\frac{k-l+1}{N-n-(k-l)} \leq \frac{k}{N-n-(k-1)}$. The last inequality clearly holds as the numerator of the LHS is not greater than that of the RHS, and the denominator of the LHS is not smaller than that of the RHS. This proves that (B.2) holds for any l such that $1 \leq l \leq k$, which in its turn proves that (B.1) is positive for each l, including $l = 1$. Then, it means that $\Psi_n(x)$ is weakly increasing in $x \in [0,1]$ and strictly increasing in $x \in (0,1]$. Since $\Psi_n(0) = 0$, it follows that $\Psi_n(x) > 0$ for $x \in (0,1]$.

Now, it is left to consider the case where $n-1 < k$. Like in the previous case, it suffices to show that $d\Psi_n(x)/dx > 0$ for $x \in (0,1]$ since $\Psi_n(0) = 0$. For that, we apply the same method as for the case of $n-1 \geq k$. First, we note that if each of lth $(1 \leq l \leq n-1)$ derivative of $\Psi_n(x)$ with respect to x is positive (and it is strictly positive for $x \in (0,1]$), then $d\Psi_n(x)/dx > 0$. Second, we note that

$$\frac{d^{n-1}}{dx^{n-1}}\Psi_n(x) = \frac{\binom{N-n}{k-1}\binom{n}{1}\binom{N-n}{k+1-n}\binom{n}{n}}{\binom{N}{k}\binom{N}{k+1}}n! - \frac{\binom{N-n}{k}\binom{n}{1}\binom{N-n}{k-n}\binom{n}{n}}{\binom{N}{k}\binom{N}{k+1}}n!$$

which is strictly positive if

$$\binom{N-n}{k}\binom{N-n}{k+1-n}\left(\frac{k}{N-n-(k-1)}\right) > \binom{N-n}{k}\binom{N-n}{k+1-n}\left(\frac{k-n+1}{N-k}\right),$$

or $\frac{k}{N-n-(k-1)} > \frac{k-n+1}{N-k}$. Clearly, the inequality holds as the numerator of the LHS is greater than that of the RHS and the denominator of the LHS is smaller than that of the RHS. This demonstrates that $\frac{d^{n-1}}{dx^{n-1}}\Psi_n(x) > 0$ for all $x \in [0,1]$. The next step is to show that $\frac{d^l}{dx^l}\Psi_n(x)\mid_{x=0} \geq 0$ for each l such that $1 \leq l \leq n-2$, which proves that $\frac{d^l}{dx^l}\Psi_n(x) > 0$ for $x \in (0,1]$. The former is certainly true which follows from the proof of (B.2). This proves that $\Psi_n(x) > 0$ for $x \in (0,1]$ because $\Psi_n(0) = 0$. =

The proof of the lemma is now complete.

B.3 Proofs of Proposition 4

To prove the first part of the proposition, it suffices to show that buyers' participation constraint is satisfied, i.e., buyers prefer searching either k or $k+1$ firms to not searching at all. Fixing pricing strategies of firms in a BNE for $c \in \{\underline{c}_{k,k+1}, \overline{c}_{k,k+1}\}$, we note that an individual buyer's participation constraint is satisfied if $q_k(P_k+(k-1)c)+(1-q_k)(P_{k+1}+kc) = P_k+(k-1)c < v$, where we used (2) to obtain the equality. This is indeed the case as

$$\begin{aligned} P_k + (k-1)c &< P_{k-1} + (k-2)c < ... \\ &< P_1 = (\theta_0+\theta_1)v + \sum_{n=\underline{n}}^{N-k+1} \theta_n \left(\alpha_{n1,0}v - \alpha_{n1,0} \int_{\underline{p}_n}^{v} p x_n'(p)dp \right) \\ &< v. \end{aligned}$$

This proves that buyers' participation constraint is indeed satisfied, and completes the proof of the first part of the proposition.

We next prove the second part of the proposition. For that, it is enough to show that $P_1 - P_2$ and $P_{N-\underline{n}+1} - P_{N-\underline{n}+1}$ converge to zero as $q_1 \uparrow 1$ and $q_{N-\underline{n}+2} \uparrow 1$, respectively.

Since $\alpha_{n1}'(x_n) = \alpha_{n1,1}$, we have

$$\begin{aligned} \lim_{q_1\uparrow 1}(P_1 - P_2) &= \lim_{q_1\uparrow 1} \sum_{n=2}^{N} \theta_n v \left((\alpha_{n1,0} - \alpha_{n2,0}) + \int_0^1 \beta_{n1,1} \frac{(\alpha_{n1}'(x_n) - \alpha_{n2}'(x_n))}{\beta_{n1}'(x_n)} dx_n \right) \\ &= \sum_{n=2}^{N} \theta_n v \left((\alpha_{n1,0} - \alpha_{n2,0}) + \int_0^1 (\alpha_{n1}'(x_n) - \alpha_{n2}'(x_n))\, dx_n \right) = 0. \end{aligned}$$

Here, the first equality in the second line follows from the facts that

$$\begin{aligned} \lim_{q_1\uparrow 1} \beta_{n1}'(x_n) &= \lim_{q_1\uparrow 1} [q_1\alpha_{n1,1} + (1-q_1)(\alpha_{n2,1} + \alpha_{n2,2}x_n)] = \alpha_{n1,1}, \\ \lim_{q_1\uparrow 1} \beta_{n1,1} &= \alpha_{n1,1}, \end{aligned}$$

while the second equality in the same line follows from the facts that $\alpha_{n1}(1) = \alpha_{n2}(1) = 1$, $\alpha_{n1}(0) = \alpha_{n1,0}$, and $\alpha_{n2}(0) = \alpha_{n2,0}$. To evaluate $P_{N-\underline{n}+1} - P_{N-\underline{n}+1}$ as $q_{N-\underline{n}+2} \uparrow 1$, we note that, in a BNE where buyers randomize between searching $N-\underline{n}+1 (= k)$ and $N-\underline{n}+2 (= k+1)$ firms, only sellers in a market with $\underline{n}$ sellers play mixed strategy pricing. Also as $\alpha_{\underline{n}k,0} = \alpha_{\underline{n}k+1,0} = \alpha_{\underline{n}k+1,1} = 0$ meaning that $\lim_{q_{k+1}\uparrow 1} \beta_{\underline{n}k,1} = 0$ for $k = N-\underline{n}+1$, it follows that

$$\begin{aligned} \lim_{q_{k+1}\uparrow 1}(P_k - P_{k+1}) &= \lim_{q_{k+1}\uparrow 1} \theta_{\underline{n}} v \left(\alpha_{\underline{n}k,0} - \alpha_{\underline{n}k+1,0} + \int_0^1 \frac{\beta_{\underline{n}k,1}\left(\alpha_{\underline{n}k}'(x_{\underline{n}}) - \alpha_{\underline{n}k+1}'(x_{\underline{n}})\right)}{\beta_{\underline{n}k}'(x_2)} dx_{\underline{n}} \right) \\ &= 0. \end{aligned}$$

Then, it is indeed that $P_1 - P_2$ and $P_{N-\underline{n}+1} - P_{N-\underline{n}+1}$ converge to zero as $q_1 \uparrow 1$ and $q_{N-\underline{n}+2} \uparrow 1$, respectively. This completes the proof of the second part of the proposition.

B.4 Proof of Proposition 5

We first prove the existence of the cutoff values of the search cost. In the proof of Proposition 3, we showed that the benefit of searching the kth firm is greater than that of searching the $k+1$th firm (given a non-degenerate price distribution(s) for some $n \geq \underline{n}$). This means that $P_{k-1} + P_{k+1} - 2P_k > 0$. Due to strict inequality, it follows that there must a non-empty interval of search costs such that (5) holds, meaning that $0 \leq \underline{c}_k < \bar{c}_k \leq v$. Furthermore, by construction, it must be that

$$\begin{aligned} \lim_{q_k \uparrow 1} (P_k - P_{k+1}) &= \underline{c}_k, \\ \lim_{q_k \uparrow 1} (P_{k-1} - P_k) &= \bar{c}_k. \end{aligned} \tag{B.3}$$

We next show that buyers' participation constraint is satisfied. This is true if $P_k + (k-1)\bar{c}_k \leq v$. Given pricing policies of sellers in a BNE for $c = \bar{c}_k$, we have

$$\begin{aligned} P_k + (k-1)\bar{c}_k &< P_{k-1} - (k-2)\bar{c}_k < \dots \\ &< P_1 = (\theta_0 + \theta_1)v + \sum_{n=\underline{n}}^{N-k+1} \theta_n \left(\alpha_{n1,0} v + \alpha_{n1,1} \int_{\underline{p}_n}^{v} p x'_n(p) dp \right) \\ &< v. \end{aligned}$$

Thus, the consumers' participation constraint is indeed satisfied. This completes the proof of the proposition.

B.5 Proof of Corollary 1

It suffices to prove that (i) $\underline{c}_k > 0$ for any $2 \leq k \leq N - \underline{n} + 1$, (ii) $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ is increasing with $q_{N-\underline{n}+1}$ in the neighborhood of 0 and (iii) $P_1 - P_2$ is decreasing with q_1 in the neighborhood of 1.

For (i), we note that $\underline{c}_k$ is equal to $P_k - P_{k+1}$ evaluated at $q_k \uparrow 1$ in a BNE where consumers randomize over searching k and $k+1$ firms. Then, it suffices to show that $\lim_{q_k \uparrow 1} P_k - P_{k+1} > 0$ for any $2 \leq k \leq N - \underline{n} + 1$. It is easy to see that

$$\lim_{q_k \uparrow 1} (P_k - P_{k+1}) = -\lim_{q_k \uparrow 1} \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 p'_n(x_n) \left(\alpha_{nk}(x_n) - \alpha_{nk+1}(x_n) \right) dx_n.$$

As $p'_n(x_n) < 0$, the limiting expression is strictly positive if $\alpha_{nk}(x_n) - \alpha_{nk+1}(x_n) > 0$ for some n such that $\underline{n} \leq n \leq N - k + 1$. However, for $n = \underline{n}$, the inequality reduces to $\alpha_{\underline{n}k}(x) - \alpha_{\underline{n}k+1}(x) > 0$, which certainly holds due to the stochastic dominance in Lemma 3. Then, the limiting expression is indeed strictly positive.

For (ii), we recall from Lemma 2 that $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ is positive and strictly concave in $q_{N-\underline{n}+1}$. Also from the proof of Proposition 4, we know that

$$\lim_{q_{N-\underline{n}+1} \downarrow 0} (P_{N-\underline{n}+1} - P_{N-\underline{n}+2}) = 0.$$

These two observations mean that $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ must be increasing with $q_{N-\underline{n}+1}$ in the neighborhood of 0.

For (iii), we again recall from Lemma 2 that $P_1 - P_2$ is positive and strictly concave in q_1. In addition, we know from the proof of Proposition 4 that

$$\lim_{q_1 \uparrow 1}(P_1 - P_2) = 0.$$

These two facts imply that $P_1 - P_2$ must be decreasing in q_1 in the neighborhood of 1.

Points (i), (ii), and (iii) establish the proof of the corollary.

B.6 Proof of Proposition 6

First, we prove the impact of greater product availability on buyer' search intensity. For that, we note that $P_k = (\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk}(x)dx$, where $p_{\underline{n}}(x) = q_k \alpha_{\underline{n}k,1} v / \beta'_{\underline{n}k}(x_{\underline{n}})$ and $k = N - \underline{n} + 1$. Similarly, $P_{k+1} = (\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx$. Therefore, the indifference condition of an individual consumer is given by

$$\theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx = c. \tag{B.4}$$

We know that in equilibrium it must be that

$$\begin{aligned} \frac{d\theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx}{d\theta_{\underline{n}}} &= \int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx \\ &+ \theta_{\underline{n}} \frac{\partial \int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx}{\partial q_k} \times \frac{dq_k}{d\theta_{\underline{n}}} = 0. \end{aligned} \tag{B.5}$$

Note that, as $\int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx > 0$, it must be that

$$\frac{\partial \int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx}{\partial q_k} \times \frac{dq_k}{d\theta_{\underline{n}}} < 0.$$

As in the unique stable equilibrium the expected benefit of searching $k+1$th firm is increasing with q_k, the partial derivative is positive. Then, it must be that $dq_k/d\theta_{\underline{n}} < 0$. From this follows the proof of the effect of greater product availability on buyers' search intensity in all three parts of the proposition.

Next, we analyze the impact of more product availability on consumer welfare and the expected price paid. For that, we first observe that for any realization of $n > \underline{n}$ sellers price at the production marginal cost in equilibrium. To see that, replace k by $N - \underline{n} + 1$ such that $N - k + 2 = N - (N - \underline{n} + 1) + 2 = \underline{n} + 1$. From Proposition 4, it follows that sellers in a market with at least $N - k + 2 = \underline{n} + 1$ number of sellers price the product at the marginal cost of production. Parts (i) and (ii) of the proposition directly follows from these facts.

For the proof of part (iii) of the proposition, we note that buyers' total outlay is equal to

$$q_k P_k + (1 - q_k)P_{k+1} + (k + 1 - q_k)c = (\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx + (k+1)c, \tag{B.6}$$

where we used (B.4) to obtain the equality. The change in the outlay due to *lower* product availability, which is associated with an increase in $\theta_{\underline{n}}$, is given by

$$\frac{d\Big((\theta_0+\theta_1)v+\theta_{\underline{n}}\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx+(k+1)c\Big)}{d\theta_{\underline{n}}}=\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx$$
$$+\theta_{\underline{n}}\frac{\partial\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx}{\partial q_k}\times\frac{dq_k}{d\theta_{\underline{n}}}.$$

Since in equilibrium (B.5) must hold, the change in buyers' outlay can be rewritten as

$$\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx-\theta_{\underline{n}}\frac{\partial\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx}{\partial q_k}\left(\frac{\int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x)-\alpha'_{nk+1}(x)\right)dx}{\theta_{\underline{n}}\frac{\partial\int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x)-\alpha'_{nk+1}(x)\right)dx}{\partial q_k}}\right),$$

which is negative if the following holds

$$\frac{\partial\int_0^1 p_{\underline{n}}(x)\alpha'_{nk}(x)dx}{\partial q_k}\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx-\frac{\partial\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx}{\partial q_k}\int_0^1 p_{\underline{n}}(x)\alpha'_{nk}(x)dx<0.$$

The inequality is true if

$$\int_0^1\frac{\alpha_{nk,1}\alpha'_{nk}(x)\alpha'_{nk+1}(x)}{[\beta'_{nk}(x)]^2}dx\int_0^1\frac{q_k\alpha_{nk,1}\alpha'_{nk+1}(x)}{\beta'_{nk}(x)}dx-\int_0^1\frac{\alpha_{nk,1}\left[\alpha'_{nk+1}(x)\right]^2}{[\beta'_{nk}(x)]^2}dx\int_0^1\frac{q_k\alpha_{nk,1}\alpha'_{nk}(x)}{\beta'_{nk}(x)}dx<0,$$

or

$$\int_0^1\frac{\alpha'_{nk}(x)\alpha'_{nk+1}(x)}{[\beta'_{nk}(x)]^2}dx\int_0^1\frac{\alpha'_{nk+1}(x)}{\beta'_{nk}(x)}dx-\int_0^1\frac{\left[\alpha'_{nk+1}(x)\right]^2}{[\beta'_{nk}(x)]^2}dx\int_0^1\frac{\alpha'_{nk}(x)}{\beta'_{nk}(x)}dx<0.$$

Letting $h\equiv\alpha'_{nk}(x)/\beta'_{nk}(x)$, implying $(1-q_kh)/(1-q_k)=\alpha'_{nk+1}(x)/\beta'_{nk}(x)$, rewrite the inequality as

$$\int_0^1 h(1-q_kh)dx\int_0^1(1-q_kh)dx-\int_0^1(1-q_kh)^2dx\int_0^1 hdx<0.$$

This simplifies to

$$-\int_0^1 h^2dx+\left(\int_0^1 hdx\right)^2<0,$$

which is clearly true by Cauchy-Schwarz Inequality. This proves that buyers' outlay decreases with lower product availability; or equivalently, greater product availability harms buyers.

As a final step, we show that the expected price must increase with greater product availability. Notice that no buyer drops out of the market for any realization of $n\geq\underline{n}$. Also following an increase in product availability, buyers economize on their search costs as they search less. Hence, the only reason why buyers are worse-off due to greater product availability is that the expected price they pay must rise. This completes the proof.

B.7 Proof of Proposition 7

Clearly, a change in the search cost does not directly affect sellers. Hence, any changes in market outcomes caused by the change in the search cost must be through buyers' willingness to search. Recall that, in a stable equilibrium, $P_k - P_{k-1}$ is increasing in q_k. Then, an increase in the search cost, which must be accompanied with an increase in $P_k - P_{k-1}$, must raise q_k.

To see that buyers' welfare falls with an increase in c, first note that in equilibrium it must be

$$\frac{d(P_k - P_{k-1})}{dc} - 1 = 0. \tag{B.7}$$

Second, the change in the average expected virtual price, denoted by P, due to an increase in c can be written as

$$\begin{aligned} \frac{dP}{dc} &= \frac{d\left(P_{k+1} + q_k(P_k - P_{k+1})\right)}{dc} = \left(\frac{\partial P_{k+1}}{\partial q_k} + P_k - P_{k+1}\right)\frac{dq_k}{dc} + q_k \frac{d(P_k - P_{k-1})}{dc} \\ &= \left(\frac{\partial P_{k+1}}{\partial q_k} + c\right)\frac{dq_k}{dc} + q_k, \end{aligned}$$

where the second line is due to (2) and (B.7). Then, the corresponding change in consumer welfare is

$$\begin{aligned} \frac{d(v - P - [q_k(k-1) + (1-q_k)k]\,c)}{dc} &= -\left(\frac{\partial P_{k+1}}{\partial q_k} + c\right)\frac{dq_k}{dc} - q_k - (k - q_k) + \frac{dq_k}{dc}c \\ &= -\left(\frac{\partial P_{k+1}}{\partial q_k}\right)\frac{dq_k}{dc} - k. \end{aligned}$$

As $dq_k/dc > 0$, the derivative is negative if $\partial P_{k+1}/\partial q_k \geq 0$, or

$$v \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 \frac{\alpha'_{nk+1}(x)\left(\alpha_{nk,1}\alpha'_{nk+1}(x) - \alpha_{nk+1,1}\alpha'_{nk}(x)\right)}{\left[\beta'_{nk}(x)\right]^2} dx \geq 0.$$

However, we know from the proof of Proposition 4 that $\alpha_{nk,1}\alpha'_{nk+1}(x) - \alpha_{nk+1,1}\alpha'_{nk}(x)$ is positive for each n such that $\underline{n} \leq n \leq N - k + 1$, which means that the virtual price for n sellers market is indeed increasing in q_k. This shows that dP_{k+1} is increasing in q_k, meaning that the derivative of consumer welfare w.r.t. c is decreasing.

The proof is complete.

B.8 Proof of Proposition 8

(i) We prove that the average expected price paid conditional on buying decreases as the product becomes more available. This, along with the fact that with greater product availability the share of consumers who does not make purchase (because they do not find the product) decreases, will mean that buyers' welfare improves when product becomes more available.

Suppose buyers search k firms. Then, the expected price paid by buyers conditional on their observing at least one price in a market with n sellers is (recall (4))

$$\frac{\alpha_{nk,1}}{1-\alpha_{nk,0}}v.$$

To prove (a), we need to show that the fraction $\alpha_{nk,1}/(1-\alpha_{nk,0})$ is decreasing in n such that $1 \leq n \leq N-k+1$, or

$$\frac{\alpha_{nk,1}}{1-\alpha_{nk,0}} - \frac{\alpha_{n+1k,1}}{1-\alpha_{n+1k,0}} > 0.$$

Observe that the inequality certainly holds for $\alpha_{nk,1} \geq \alpha_{n+1k,1}$ because $\alpha_{nk,0} > \alpha_{n+1k,0}$ (which is easy to check).

Assume that $\alpha_{nk,1} < \alpha_{n+1k,1}$. Using the definition of $\alpha_{nk,m}$ and simplifying, it is easy to show that $\alpha_{nk,1} < \alpha_{n+1k,1}$ translates into $N-nk-k+1 > 0$. Next, simplify the inequality to be proven as

$$\frac{(1-\alpha_{n+1k,0})\alpha_{nk,1} - (1-\alpha_{nk,0})\alpha_{n+1k,0}}{(1-\alpha_{nk,0})(1-\alpha_{n+1k,0})} > 0.$$

The inequality holds if the numerator of its left-hand side is positive:

$$(1-\alpha_{n+1k,0})\alpha_{nk,1} - (1-\alpha_{nk,0})\alpha_{n+1k,0} > 0.$$

Employing the definition of $\alpha_{nk,m}$ expand the LHS of the inequality as follows:

$$\left[1 - \frac{\binom{N-n-1}{k}}{\binom{N}{k}}\right] \frac{\binom{N-n}{k-1}n}{\binom{N}{k}} - \left[1 - \frac{\binom{N-n}{k}}{\binom{N}{k}}\right] \frac{\binom{N-n-1}{k-1}(n+1)}{\binom{N}{k}} > 0,$$

or

$$\left[\binom{N}{k} - \binom{N-n-1}{k}\right]\binom{N-n}{k-1}n - \left[\binom{N}{k} - \binom{N-n}{k}\right]\binom{N-n-1}{k-1}(n+1) > 0,$$

Since

$$\binom{N-n-1}{k} = \binom{N-n}{k}\frac{N-n-k}{N-n}, \quad \binom{N-n}{k-1} = \binom{N-n-1}{k-1}\frac{N-n}{N-n-k-1},$$

simplify the inequality as

$$-\binom{N}{k}(N-nk-k+1) + \binom{N-n}{k}(N-k+1) > 0.$$

Divide both sides of the inequality by $N-nk-k+1(>0)$ and $N-k+1$ and rearrange to obtain

$$\frac{\binom{N-n}{k}}{N-nk-k+1} > \frac{\binom{N}{k}}{N-k+1},$$

or

$$\frac{(N-n)!}{(N-n-k)!(N-nk-k+1)} > \frac{N!}{(N-k)!(N-k+1)}. \tag{B.8}$$

Observe that for $n = 0$, the LHS and the RHS of the inequality are equal to each other. Then, the inequality holds for all $1 \leq n \leq N - k + 1$ if the LHS is increasing in n. To show that, we take the derivative of the LHS with respect to n and show that it is positive. As n is an integer, we apply Gamma function to take the derivative. First, we rewrite the LHS as

$$\frac{\Gamma(N-n+1)}{\Gamma(N-n-k+1)(N-nk-k+1)},$$

where Γ stands for Gamma function such that $\Gamma(x+1) = x!$. Noting that, for positive integer x,

$$\frac{d\Gamma(x+1)}{dx} = x!\left(-\gamma + \sum_{l=1}^{x} \frac{1}{l}\right),$$

where $\gamma = \lim_{x\to\infty}\left(-\ln(x) + \sum_{l=1}^{x} \frac{1}{l}\right)$ is the Euler-Mascheroni constant, the derivative of the LHS of the inequality is

$$\frac{(N-n-k)!(N-n)!\left\{-(N-nk-k+1)\left(-\gamma+\sum_{l=1}^{N-n}\frac{1}{l}\right)+(N-nk-k+1)\left(-\gamma+\sum_{l=1}^{N-n-k}\frac{1}{l}\right)+k\right\}}{[\Gamma(N-n-k+1)(N-nk-k+1)]^2}.$$

The derivative is positive if the term in the curly brackets in the numerator is positive, or $k - (N - nk - k + 1)\sum_{l=N-n-k+1}^{N-n}(1/l) > 0$. This is true as

$$\begin{aligned}\frac{k}{N-nk-k+1} &> \frac{k}{N-n-k+1} = \frac{1}{N-n-k+1} + \frac{1}{N-n-k+1} + \ldots + \frac{1}{N-n-k+1} \\ &> \frac{1}{N-n-k+1} + \frac{1}{N-n-k+2} + \ldots + \frac{1}{N-n} = \sum_{l=N-n-k+1}^{N-n}\frac{1}{l}.\end{aligned}$$

This demonstrates that the derivative of the LHS of (B.8) is positive. This, in its turn, implies that (B.8) is true for $n \geq 1$ and $N - nk - k + 1 > 0$, meaning that the expected price paid conditional on observing at least one price is decreasing with n for $\alpha_{nk,1} < \alpha_{n+1k,1}$. Then, the average expected price paid conditional on buying is decreasing with greater product availability.

To show that buyers' welfare increases with greater product availability, it suffices to demonstrate that the expected virtual price falls as the product becomes more available. The latter statement is true if the expected virtual price in a market with n sellers, P_k^n, decreases with n for $1 \leq n \leq N - k + 1$. Observe that

$$P_k^n = \alpha_{nk,0}v + (1 - \alpha_{nk,0})\frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}}v,$$

or the expected virtual price in a market with n sellers is a weighted average of the the monopoly price v and the expected price paid by buyers conditional on observing at least one price. First, it is easy to check that $\alpha_{nk,0}$ is decreasing in n. This means that monopoly price receives less weight while the expected price conditional on purchase receives more weight as n rises. Second, it has been proven above that $\alpha_{nk,1}/(1 - \alpha_{nk,0})$ decreases with n such that $1 \leq n \leq N - k + 1$. Then, these two effects must clearly cause a decrease in P_k^n as n rises. This proves that the virtual expected price falls, or that buyers' welfare rises, with greater product availability.

For (b), to prove that the expected price conditional on observing at least one price does not change with grater product availability for $j \geq N-k+2$, we need to demonstrate that

$$\frac{\alpha_{nk,1}}{1-\alpha_{nk,0}} - \frac{\alpha_{n+1k,1}}{1-\alpha_{n+1k,0}} = 0$$

for $n \geq N-k+2$. The equality holds as it is easy to check that $\alpha_{nk,1} = 0$ for $n \geq N-k+2$. Using similar steps as in the prove of (a), we can show that P_k^n does not change with $n \geq N-k+2$. This means that buyers' welfare does not change with greater product availability for $j \geq N-k+2$.

Proof of (ii) follows directly from discussion after the proposition in the main body of the paper.

The proof of the proposition is complete.

B.9 Proof of Proposition 9

We first show that for sufficiently small search costs, there exists a stable BNE where some consumers choose search technology $l \geq 2$ for $\underline{n} \geq 2$. Then, we prove the impact of greater product availability on market outcomes.

Existence of Equilibrium

For existence, we first assume that consumers randomize over choosing search technologies $l(= N-\underline{n}+1)$ and $l+1(= N-\underline{n}+1)$, where $1 \leq l \leq N-1$ and $\underline{n} \geq 2$, and derive pricing policies of sellers. Later, we show that given the above sellers' pricing decision, an individual consumer is indifferent between choosing search technologies l and $l+1$.

Thus, suppose that q share of consumers choose search technology l while the remaining consumers choose search technology $l+1$. Then, a monopolist seller sets price v, price in a market with at least $N-l+2$ sellers must be equal to zero, and prices in a market with $\underline{n}$ number of sellers must be dispersed. The equilibrium price distribution in the last market, which we denote by x, must satisfy:

$$\sum_{k=l}^{N} \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right) \alpha'_{\underline{n}k}(x(p))p = \sum_{k=l}^{N} \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right) \alpha'_{\underline{n}k}(x(v))v.$$

Here, the LHS is obtained from computing an individual seller's expected profit from setting price p in the support of the equilibrium price distribution x. The RHS of the equation represents the expected profit from selling to only locked-in consumers at a price equal to v. In addition, note that $\alpha'_{\underline{n}k} = 0$ for $k > \underline{n}$. Since the share of consumers who observe two prices is strictly positive, there is a unique solution in x that satisfies the above equation (see, Johnen and Ronayne (forthcoming)).

As $x(v) = 0$, $\sum_{k=l}^{N} \delta_k^l \alpha'_{\underline{n}k}(0) = \delta_l^l \alpha_{\underline{n}l,1}$ and $\sum_{k=l}^{N} \delta_k^{l+1} \alpha'_{\underline{n}k}(0) = 0$, the inverse function $p(x)$ (in equilibrium) can be written as

$$p(x) = \frac{q\delta_l^l \alpha_{\underline{n}l,1} v}{\sum_{k=l}^{N} \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right) \alpha'_{\underline{n}k}(x)}.$$

Next, we show that, given the above pricing policies by sellers and a sufficiently small search cost, consumers indeed find it optimal to randomize over choosing search technologies $l = N - \underline{n} + 1$ and $l + 1 = N - \underline{n} + 2$. This is certainly the case if an individual consumer is indifferent between those two options and prefer those options to any other search technology. To show that, we begin by proving that, for any non-degenerate price distribution for some $n \geq \underline{n}$, consumers cannot be indifferent over choosing $l-1$, l and $l+1$ search technologies where $2 \leq l \leq N-1$. As search technology $N - \underline{n} + 1$ yields an expected virtual price as follows

$$P_l = (\theta_0 + \theta_1)v + \sum_{n=\underline{n}}^{N} \theta_n \sum_{k=l}^{N} \delta_k^l \int_0^1 p(x_n)\alpha'_{\underline{n}k}(x_n)dx_n$$

where we used the fact that $\alpha_{\underline{n}\min\{k,\underline{n}\},0} = 0$ for $k \geq N - \underline{n} + 1$, a consumer is indifferent of choosing any of the three search technologies if

$$\sum_{n=\underline{n}}^{N} \theta_n \sum_{k=l-1}^{N} \left(\delta_k^{l+1} + \delta_k^{l-1} - 2\delta_k^l\right) \int_0^1 p(x)\alpha'_{\underline{n}k}(x_n)dx_n = 0.$$

However, we know that the LHS of the equation is strictly positive by assumption in (7). This shows that it cannot be that, given price dispersion for some $n \geq \underline{n}$, consumers cannot be indifferent of choosing any of three search technologies $l-1, l$ and $l+1$ for $2 \leq l \leq N-1$. However, this also means that, given non-degenerate price distribution for some $n \geq \underline{n}$, consumers can be indifferent over choosing at most two search technologies so that number of those two search technologies are adjacent.

We next establish that for sufficiently small search costs and the above pricing policies of sellers, consumers are indifferent of choosing between search technologies $l = N - \underline{n} + 1$ and $l + 1 = N - \underline{n} + 2$. Formally, this translates to

$$\theta_{\underline{n}} \sum_{k=l}^{N} \left(\delta_k^l - \delta_k^{l+1}\right) \int_0^1 p(x)\alpha'_{\underline{n}k}(x)dx = c, \tag{B.9}$$

where we used the fact that there is only price dispersion in a market with $\underline{n}$ number of sellers and consumers in that market observe at least one price, and we wrote simple x to mean $x_{\underline{n}}$.

We note the following two facts about the LHS of the equation. First, the LHS is positive for $0 < q < 1$. To see that, rewrite the LHS as

$$\begin{aligned}\sum_{k=l}^{N} \left(\delta_k^l - \delta_k^{l+1}\right) \int_0^1 p(x)\alpha'_{\underline{n}k}(x)dx &= \sum_{k=l}^{N} \left(\delta_k^l - \delta_k^{l+1}\right) \left(\underline{p}_{\underline{n}} - \int_0^1 p'(x)\alpha_{\underline{n}k}(x)dx\right) \\ &= \underline{p}_{\underline{n}} \sum_{k=l}^{N} \left(\delta_k^l - \delta_k^{l+1}\right) - \int_0^1 p'(x) \sum_{k=l}^{N} \left(\delta_k^l - \delta_k^{l+1}\right) \alpha_{\underline{n}k}(x)dx.\end{aligned}$$

Clearly, the first term is positive by the assumption in (7). The second term is positive if $\sum_{k=l}^{N} \left(\delta_k^l - \delta_k^{l+1}\right) \alpha_{\underline{n}k}(x) \geq 0$. To prove that this inequality holds, we use the following algorithm. We start at $k = N$. Three cases are possible: (i) both δ_N^l and δ_N^{l+1} are strictly positive, (ii) only δ_N^{l+1} is strictly positive, or (iii) both δ_N^l and δ_N^{l+1} are equal to zero. In case (i), it must be $\delta_N^l < \delta_N^{l+1}$ by assumption. In this case, we "take away" $\tau_{N-1} \in (0, 1]$ share of δ_{N-1}^l so that $\tau_{N-1}\delta_{N-1}^l + \delta_N^l - \delta_N^{l+1} = 0$, for which we have $\tau_{N-1}\delta_{N-1}^l\alpha_{\underline{n}k}(x) + (\delta_N^l -$

$\delta_N^{l+1})\alpha_{\underline{n}k}(x) > 0$ due to stochastic dominance in Lemma 3. If $\delta_{N-1}^l \delta_N^l - \delta_N^{l+1} < 0$, we in addition take $\tau_{N-2}\delta_{N-2}^l$ and add it to the LHS of the inequality so that it is zero; and if it is still negative, we repeat the procedure. As a second step, we undertake similar actions for $k = N-1$. We continue such steps until we get to step $k = l+1$. At this point, there must be left either $\delta_l^l \alpha_{\underline{n}l}(x) - \left((1-\tau_{l+1})\delta_{l+1}^l + \delta_{l+1}^{l+1}\right)\alpha_{\underline{n}l+1}(x)$ with $\delta_l^l + (1-\tau_{l+1})\delta_{l+1}^l - \delta_{l+1}^{l+1} = 0$ or $(1-\tau_l)\delta_l^l \alpha_{\underline{n}l}(x) - \delta_{l+1}^{l+1}\alpha_{\underline{n}l+1}(x)$ with $(1-\tau_l)\delta_l^l - \delta_{l+1}^{l+1} = 0$, both of which are strictly positive. In cases (ii) and (iii), we follow the same algorithm as we have laid out for case (i) with an exception that in case (iii) we simply skip step $k = N$. This proves that $\sum_{k=l}^N \left(\delta_k^l - \delta_k^{l+1}\right)\alpha_{\underline{n}k}(x) \geq 0$, or the LHS of (B.9) is positive.

The second fact is that, as $q \downarrow 0$, the LHS of (B.9) converges to zero:

$$\begin{aligned}
&\lim_{q\downarrow 0} \theta_{\underline{n}} \sum_{k=l}^N \left(\delta_k^l - \delta_k^{l+1}\right) \int_0^1 p(x)\alpha'_{\underline{n}k}(x)dx \\
= \;& \theta_{\underline{n}} \sum_{k=l}^N \left(\delta_k^l - \delta_k^{l+1}\right) \int_0^1 \lim_{q\downarrow 0} \left(\frac{q\delta_l^l \alpha_{\underline{n}l,1} v}{\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)} \right) \alpha_{\underline{n}k}(x)dx \\
= \;& 0,
\end{aligned}$$

where we used inverse functions $p(x)$ to obtain the first equality.

These two facts—that the LHS of (B.9) is positive and converges to zero as $q \downarrow 0$—mean that for sufficiently small search costs there exists $0 < q < 1$ that solves (B.9). Moreover, this equilibrium is locally stable as the LHS is strictly increasing in q for sufficiently small search costs. This completes the proof of the existence of a BNE.

Impact of Greater Product Availability

Now, we turn on proving the impact of greater product availability on buyers' search behavior, average expected price paid by them and their surplus.

Parts (i) and (ii) of Proposition 9 are straightforward, which is why we only prove part (iii) of the proposition. In equilibrium we have

$$\frac{dq}{d\theta_{\underline{n}}} = -\frac{\frac{\partial(P_{N-\underline{n}+1} - P_{N-\underline{n}+2})}{\partial\theta_{\underline{n}}}}{\frac{\partial(P_{N-\underline{n}+1} - P_{N-\underline{n}+2})}{\partial q}}, \tag{B.10}$$

which is negative as the numerator of the RHS is clearly positive and the denominator of the RHS is positive because of the above facts that the LHS of (B.9) (which is nothing but $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$) is positive for $0 < q < 1$ and converges to zero as $q \downarrow 0$. Next, consumers' expected outlay is equal to

$$qP_{N-\underline{n}+1} + (1-q)P_{N-\underline{n}+2} + (N - \underline{n} + 1 - q)c = P_{N-\underline{n}+2} + (N - \underline{n} + 1)c,$$

where we used the fact that in equilibrium we have $P_{N-\underline{n}+1} - P_{N-\underline{n}+2} = c$. We need to prove that this outlay is decreasing with $\theta_{\underline{n}}$ (and the associated equal decrease in θ_j where $j > \underline{n}$). This is true if the derivative of the outlay w.r.t. θ_n is negative. This derivative is equal to

$$\frac{\partial P_{N-\underline{n}+2}}{\partial\theta_{\underline{n}}} + \frac{\partial P_{N-\underline{n}+2}}{\partial q}\frac{dq}{d\theta_{\underline{n}}}.$$

Using (B.10), we can rewrite the derivative of the outlay w.r.t. θ_n as

$$\frac{\dfrac{\partial P_{N-\underline{n}+1}}{\partial q}\dfrac{\partial P_{N-\underline{n}+2}}{\partial \theta_{\underline{n}}}-\dfrac{\partial P_{N-\underline{n}+1}}{\partial \theta_{\underline{n}}}\dfrac{\partial P_{N-\underline{n}+2}}{\partial q}}{\dfrac{\partial (P_{N-\underline{n}+1}-P_{N-\underline{n}+2})}{\partial q}}. \tag{B.11}$$

As the denominator of the expression is positive, the expression in (B.11) is negative if its numerator is negative. The numerator is equal to

$$\theta(\delta_l^l \alpha_{\underline{n}l,1} v)^2 \int_0^1 \frac{\sum_{k=l}^N \delta_k^l \alpha'_{\underline{n}k}(x) \sum_{k=l}^N \delta_k^{l+1}\alpha'_{\underline{n}k}(x)}{\left(\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)\right)^2}dx \int_0^1 \frac{\sum_{k=l}^N \delta_k^{l+1}\alpha'_{\underline{n}\min\{k,\underline{n}\}}(x)}{\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)}dx$$

$$-\theta(\delta_l^l \alpha_{\underline{n}l,1} v)^2 \int_0^1 \frac{\sum_{k=l}^N \delta_k^l \alpha'_{\underline{n}k}(x)}{\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)}dx \int_0^1 \frac{\left(\sum_{k=l}^N \delta_k^{l+1}\alpha'_{\underline{n}k}(x)\right)^2}{\left(\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)\right)^2}dx.$$

To see that this is negative, let $h \equiv \frac{\sum_{k=l}^N \delta_k^l \alpha'_{\underline{n}k}(x)}{\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)}$ so that $(1-qh)/(1-q) = \frac{\sum_{k=l}^N \delta_k^{l+1}\alpha'_{\underline{n}k}(x)}{\sum_{k=l}^N \left(q\delta_k^l + (1-q)\delta_k^{l+1}\right)\alpha'_{\underline{n}k}(x)}$ as we did in the proof of Proposition 6, and simplify to obtain an expression which is the same as that on the LHS of the last inequality in the proof of Proposition 6, which is clearly negative. This means that the numerator of (B.11) is negative, which in turn proves that the derivative of consumers' outlay w.r.t. $\theta_{\underline{n}}$ is negative. As consumers search more and are better-off due to a decrease in product availability, associated with an increase in $\theta_{\underline{n}}$, the expected price paid must be lower. The proof of the proposition is now complete.

B.10 Proof of Proposition 10

We first prove the existence, and later prove the comparative static result.

Existence

We will use the following facts for the proof. First, $E[p] = v + \int_{\underline{p}}^v (1-x_2(p))dp$. Second, $E[\min\{p_1,p_2\}] = v - 2\int_{\underline{p}}^v (1-x_2(p))dp + \int_{\underline{p}}^v (1-x_2(p))^2 dp$, $k \neq l$ while

$$\begin{aligned} E[p] - E[\min\{p_1,p_2\}] &= \int_{\underline{p}}^v (1-x_2(p))dp - \int_{\underline{p}}^v (1-x_2(p))^2 dp \\ &= v\mu(q)\left((1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right)-2\right). \end{aligned}$$

Then, we can rewrite (9) as

$$\frac{c}{v} = \theta_2 \frac{2}{N}\mu(q)\left((1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right)-2\right). \tag{B.12}$$

The RHS of the equation is positive only if $(1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right) > 2$, or $\ln\left(1+\frac{1}{\mu(q)}\right) > \frac{2}{1+2\mu(q)}$. Note that when $\mu(q) \downarrow 0$ the LHS of the inequality goes to infinity while its RHS converges to 2, and when $\mu(q) \to \infty$ both the LHS and the RHS converge to 0. Then, it suffices to show that the derivative of LHS is more negative than that of the RHS for the inequality to hold. Indeed, the derivative of the LHS is $-\frac{1}{\mu(q)(1+\mu(q))} = -\frac{1+4\mu(q)+4\mu(q)^2}{\mu(q)(1+\mu(q))(1+2\mu(q))^2}$ while that of the RHS is $-\frac{4}{(1+2\mu(q))^2} = -\frac{4\mu(q)+4\mu(q)^2}{\mu(q)(1+\mu(q))(1+2\mu(q))^2}$, and the former is more negative than the latter. Summing up, this proves that the RHS of (B.12) is positive.

Now, we show that the RHS of (B.12) is inverse U-shaped in q, which is true only if it is inverse U-shaped which respect $\mu(q)$ as $\mu(q)$ is increasing in q with $\mu(0) = 0$ and $\mu(1) = (1-\lambda)/(N-2+2\lambda)$. The derivative of the RHS with respect to $\mu(q)$ is

$$\begin{aligned}
& \frac{2\theta_2}{N}\left((1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right) - 2 + \mu(q)\left[2\ln\left(1+\frac{1}{\mu(q)}\right) - \frac{1+2\mu(q)}{\mu(q)+\mu(q)^2}\right]\right) \\
= \; & \frac{2\theta_2}{N}\left(\frac{(1+5\mu(q)+4\mu(q)^2)\ln\left(1+\frac{1}{\mu(q)}\right) - 3 - 4\mu(q)}{1+\mu(q)}\right),
\end{aligned}$$

which is equal to zero only if its numerator is zero, or $M(\mu(q)) \equiv \ln\left(1+\frac{1}{\mu(q)}\right) - \frac{3+4\mu(q)}{(1+\mu(q))(1+4\mu(q))} = 0$. Thus, if $M(\mu(q)) = 0$ for only a single $\mu(q)$, then the RHS of (B.12) has one stationary point in $q \in (0,1)$. The following facts, along with the fact that $M(\mu(q))$ is continuous in $\mu(q) > 0$, prove that $M(\mu(q)) = 0$ has a single solution in $\mu(q) \in (0,1)$:

$$\begin{aligned}
& M(0) = +\infty, \text{ and } M(\infty) = 0, \\
& \frac{\partial M(\mu(q))}{\partial \mu(q)} = \frac{2\mu(q)-1}{\mu(q)(1+\mu(q))^2(1+4\mu(q))^2}\begin{cases} < 0 \text{ for } \mu(q) < \frac{1}{2}, \\ = 0 \text{ for } \mu(q) = \frac{1}{2}, \\ > 0 \text{ for } \mu(q) > \frac{1}{2}. \end{cases}
\end{aligned}$$

Thus, the RHS of (B.12) has a unique stationary point in $q \in (0,1)$. To see that the RHS is maximized at that stationary point, note that the derivative of the RHS is positive for some values of q close to 0 and negative for some values of q close to 1. This completes the proof that the RHS is inverse U-shaped and has a unique maximum in $q \in (0,1)$.

Next, we demonstrate that the RHS of (B.12) is less than 1. For that, we rewrite the RHS as

$$\begin{aligned}
\frac{c}{v} &= \frac{2}{N}\theta_2\left(\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right) + 2\mu(q)^2\ln\left(1+\frac{1}{\mu(q)}\right) - 2\mu(q)\right) \\
&= \frac{2}{N}\theta_2\left\{\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right) - 2\mu(q)\left[1-\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)\right]\right\}.
\end{aligned}$$

Next, we note that the first term in the large brackets, which is positive, is increasing in $\mu(q)$ and converges to 1 as $\mu(q) \to \infty$. Since the terms in the square brackets is positive and that the RHS is positive, the RHS must be less than 1.

Finally, we show that the RHS of (B.12) converges to zero as $q \to 0$, or $\mu(q) \to 0$.

Note that since

$$\lim_{\mu(q)\to 0} \mu(q) \ln\left(1+\frac{1}{\mu(q)}\right) = \lim_{z\to\infty} \frac{\ln(1+z)}{z} \overset{\text{l'Hopital}}{=} \lim_{z\to\infty} \frac{1}{1+z} = 0,$$

we have

$$\lim_{\mu(q)\to 0} \left(\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right) - 2\lim_{\mu(q)\to 0}\mu(q)\left[1-\lim_{\mu(q)\to 0}\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)\right]\right) = 0.$$

Therefore, the RHS of (B.12) converges to zero as $q \to 0$.

The facts that, for $0 < q < 1$ the RHS of (B.12) is positive for, inverse U-shaped in q, and converges to zero as $q \to 0$, there must be a stable BNE for sufficiently small search cost. The fact that the RHS (B.12) is less than one proves that $\bar{c} < v$.

Comparative Staics

Parts (i) and (ii) are straightforward which is why they are omitted.

We prove part (iii). For that, we note that in equilibrium it must be that

$$\frac{d}{d\theta_2}\left[\frac{2}{N}\theta_2\left(E[p]-E[\min\{p_1,p_2\}]\right)\right] = 0, \tag{B.13}$$

or

$$E[p]-E[\min\{p_1,p_2\}] + \theta_2\frac{\partial(E[p]-E[\min\{p_1,p_2\}])}{\partial\mu(q)} \times \frac{d\mu(q)}{d\theta_2} = 0,$$

so that

$$\frac{d\mu(q)}{d\theta_2} = -\frac{E[p]-E[\min\{p_1,p_2\}]}{\theta_2\frac{\partial(E[p]-E[\min\{p_1,p_2\}])}{\partial\mu(q)}}. \tag{B.14}$$

This derivative is negative as the denominator on the RHS of the equation is positive because we know that in a stable equilibrium $E[p]-E[\min\{p_1,p_2\}]$ is increasing in $\mu(q)$. As $\mu(q)$ is increasing in q, it follows that q is decreasing with θ_2.

Next, costly buyers are better-off with an increase in θ_2, associated with a decrease in θ_j for $j \geq 3$, if their outlay is decreasing with θ_2, or

$$\frac{d}{d\theta_2}\left[\theta_2\left(q\frac{2}{N}E[p]+q\frac{(N-2)}{N}E[\min\{p_1,p_2\}]) + (1-q)E[\min\{p_1,p_2\}]\right) + (N-1-q)c\right] < 0.$$

Using (9) to substitute the value of c, we obtain

$$\frac{d}{d\theta_2}\left[\theta_2\left(E[\min\{p_1,p_2\}] + \left(2-\frac{2}{N}\right)(E[p]-E[\min\{p_1,p_2\}])\right)\right] < 0. \tag{B.15}$$

Now, using (B.13), we rewrite (B.15) as

$$\frac{d\theta_2 E[\min\{p_1,p_2\}]}{d\theta_2} < 0,$$

so

$$\theta_2\left(E[\min\{p_1,p_2\}] + \frac{\partial E[\min\{p_1,p_2\}]}{\partial\mu(q)} \times \frac{d\mu(q)}{d\theta_2}\right) < 0.$$

We substitute the value of $d\mu(q)/d\theta_2$ from (B.14) and simplify to obtain

$$\frac{\partial E[p]}{\partial \mu(q)} E[\min\{p_1, p_2\}] - \frac{\partial E[\min\{p_1, p_2\}]}{\partial \mu(q)} E[p] < 0.$$

Employing the expression for $E[p]$ and $E[\min\{p_1, p_2\}]$, we obtain

$$v \frac{2\mu(q)\left(\mu(q)(1+\mu(q))\ln^2\left(1+\frac{1}{\mu(q)}\right)-1\right)}{1+\mu(q)} < 0, \tag{B.16}$$

which is true only if the terms in the large brackets in the numerator are negative, or

$$\ln^2\left(1+\frac{1}{\mu(q)}\right) < \frac{1}{\mu(q)(1+\mu(q))},$$

or

$$-\frac{1}{\sqrt{\mu(q)(1+\mu(q))}} < \ln\left(1+\frac{1}{\mu(q)}\right) < \frac{1}{\sqrt{\mu(q)(1+\mu(q))}}.$$

The left-hand inequality clearly holds for any $\mu(q) > 0$. Regarding the right-hand inequality, as both its sides go to infinity as $\mu(q) \to 0$ and converge to 0 as $\mu(q) \to \infty$, the inequality holds if the derivative of the LHS is less negative than that of the RHS. The derivative of the LHS is equal to $-\frac{1}{\mu(q)+\mu(q)^2}$ and that of the RHS is $-\frac{1}{\mu(q)+\mu(q)^2} \times \frac{1+2\mu(q)}{2\sqrt{\mu(q)+\mu(q)^2}}$. The former is less negative than the latter if $1+2\mu(q) < 2\sqrt{\mu(q)+\mu(q)^2}$, or

$$1 + 4\mu(q) + 4\mu(q)^2 > 4\mu(q) + 4\mu(q)^2$$

which is certainly true for $\mu(q) > 0$. This establishes that (B.16) holds. This means that costly buyers are better-off with an increase θ_2 at the expense of θ_j for $j \neq 3$.

Finally, as consumers search more and at the same time are better-off with an increase in θ_2 at the expense of θ_j for $j \neq 3$, they must pay lower expected price.

This completes the proof of the proposition.

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L 7,1 · 68161 Mannheim · Germany
Phone +49 621 1235-01
info@zew.de · zew.de