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Mean-Risk Hedging Strategies in Electricity Markets with Limited Liquidity

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Abstract

This article investigates mean risk hedging with respect to limited liquidity and studies the impact of different risk measures on the hedging strategies. For motivation and application purposes hedging in electricity markets is chosen, because the relevant hedging markets are characterized by limited liquidity. We enhance the approach in Woll and Weber (2015) to a mean-risk optimization under limited liquidity, including the risk measures absolute and relative Value and Conditional Value at Risk (VaR and CVaR). It can be shown that for position independent measures (Variance, relative VaR, relative CVaR) liquidity has no influence on the minimum risk hedging strategies, whereas for position dependent measures (absolute VaR, absolute CVaR) liquidity has an impact on the minimum risk hedging strategies. The article gives the mathematical formulations of the problems and discusses the economic relevance of the different models. In addition, we apply the analyzed concepts to the German Electricity markets.

Keywords : optimization; electricity, liquidity; electricity trading; mean-risk-model
JEL-Classification : C61, G11, Q40

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1 Introduction

The maximization of profits is the overall objective of any company. Attaining this objective nearly always implies decision making under uncertainty. This uncertainty generates risk for the company. In the case of a power producer, he has the goal to sell the energy produced generating the best possible profit. For energy trading, several markets and several products exist. The relevant markets are the electricity markets and the related commodity markets, such as coal, gas or CO_2 markets. These markets are subject to different kinds of uncertainties. In this article the focus is on the electricity markets, because they are most important for a power producer. Prices on electricity markets depend, inter alia, on current demand, outages of the plants, fuel prices, temperature or current wind and solar power production, which all are stochastic. The calculation of the optimal trading strategy thus is a stochastic problem. The share of profits exposed to uncertainty corresponds to the economic risk incurred by the power plant operator. Maximizing the profit and minimizing the corresponding risk are hence complementary objectives for a power producer. A closer look at the electricity markets and the available products for trading electricity leads to the distinction of spot and futures markets. Spot market products are more flexible than futures market products, but risk on spot markets is in general much higher than on futures markets. So, operators usually engage in hedging on futures markets as an instrument for minimizing overall risk (cf. e.g. RWE AG (2014) p. 8). But the futures markets for electricity show a limited liquidity. This has an important implication: Power producers are not necessarily price takers on futures markets, but they can rather impact prices on futures markets by their own trading activity. Due to this strategic aspect, it is difficult to determine optimal hedging strategies in electricity markets with limited liquidity. An approach to calculate optimal hedging strategies under limited liquidity is proposed by Woll and Weber (2015). They calculate the optimum based on a modified version of the classical mean-variance approach going back to Markowitz. Measuring risk by the variance, however, is not always appropriate. In practice, many decision makers for example prefer downside risk measures for their risk management. Thus, the question arises whether and how optimal hedging strategies depend on the applied measure of risk in markets with limited liquidity. We give an answer to this question by first reviewing the relevant literature on mean-risk approaches in section two and the relevant risk measures in section three. Section four develops the modelling framework and section five contains the application to electricity markets. The article ends with a conclusion of the main results and their economic impacts.

2 Mean-Risk Hedging Strategies in Literature

Hedging comprises trading strategies with the objective to minimize the risk of a company. Usually these trading strategies create a portfolio consisting of different hedging products or trading activities of one hedging product over time, or both. Thus, finding the optimal hedging strategy leads to a portfolio optimization problem. There also exists a broad literature on mean-risk hedging in the context of asset portfolios and option pricing, such as Föllmer and Sondermann (1986), Schweitzer (1992) or Gourieroux et al. (1998). This kind of hedging is related to asset potfolios and focusses on the terminal value of a portfolio. Furthermore liquidity is not regarded in these articles. In this article we will focus on optimal hedging decisions.

The literature on portfolio optimization is going back to Markowitz (1952), who provides a general mean-variance portfolio selection problem. Here the objective is to maximize the risk-adjusted return of a portfolio and the returns are assumed to follow a multivariate normal distribution. Many works on the role of risk in portfolio selection have followed. The work of Sharpe (1964) links the portfolio selection to the CAPM and Tobin (1958) investigates liquidity preferences in the sense of accounting liquidity. Baumol (1963) is the first who criticises the variance as a measure of risk and proposes the expectation minus the K-weighted standard deviation, with K being any real number as an alternative risk measure.[1] More recently, Alexander and Baptista (2002) compare the mean-variance approach with a mean-VaR approach. They show that it could be efficient to select portfolios with larger standard deviations when switching from variance to VaR as a measure of risk and thus emphasise that "VaR is not an unqualified improvement over variance as a measure of risk". Rockafellar and Uryasev (2000) present the Conditional Value-at-Risk as "a new approach to optimizing or hedging a portfolio of financial instruments to reduce risk". They focus on computational issues and give some applications.

With regard to electricity markets, there are several works on mean-risk optimization in the context of operational portfolio management, such as Eichhorn et al. (2005), Xu et al. (2006) and Woll and Weber (2015). Whereas the first two articles focus on optimal power plant scheduling, the latter deals with trading forward contracts for hedging purposes. In addition, Woll and Weber (2015) address the case of limited market liquidity and its impact on optimal hedging strategies. Lo et al. (2003) have done some work on the impact of liquidity to the efficient frontier of a portfolio selection model, but without a direct link to hedging strategies over time.

To complement the literature on mean-risk hedging strategies, we want to analyze the impact of mean-VaR and mean-CVaR optimization on hedging in the case of illiquid markets.

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[1] For K= q_α^N this leads to the mean-absolute VaR optimization and for K= $\frac{-\varphi(q_\alpha^N)}{\alpha}$ to the mean-absolute CVaR optimization.

Before giving the mathematical formulations for the optimization, we compare the risk measures used in this article.

3 Comparison of different risk measures

A classical measure for quantifying risk of an uncertain objective *Z,* such as cash flows, profits, or returns is the variance given as

$$\mathrm{Var}(Z) = \int_{-\infty}^{+\infty} (\mathrm{z} - \mathrm{E(z)})^2 \cdot \mathrm{f(z)dz} \quad (1)$$

with *f(z)* the corresponding probability density. $\mathrm{E}(Z) = \int_{-\infty}^{+\infty} \mathrm{z} \cdot \mathrm{f(z)dz}$ is the expectation or rather the mean of *Z*. The variance and its square root, the standard deviation (Std(Z)), are measures for the statistical spread of a random variable *Z* and are often used in economic models, e.g. the mean-variance models going back to Markowitz (1952). The variance is a symmetric risk measure.

Due to the fact that in most situations only the uncertainty of one side of a distribution is relevant, downside risk measures have been developed. One of the most important downside risk measures is the Value at Risk (VaR). According to Jorion (2001) the VaR is the "[...] expected maximum loss (or worst loss) over a target horizon with a given level of confidence [...]".

The VaR is hence the quantile $\mathrm{q}_{\alpha}(Z) = \inf_{z \in \Re} \{\mathrm{F}_Z(z) \geq \alpha\}$ of the distribution *F(Z)* corresponding to a given confidence level α. With the use of the quantile function as the inverse of the distribution function the VaR can be written as

$$\mathrm{VaR}_{1-\alpha}(Z) = -F_Z^{-1}(\alpha) \quad (2)$$

(see Jorion (2001)). This formulation is also called the absolute Value at risk (VaR^{abs}) because the Value at Risk is measured as the distance to zero. Due to the fact that loss is sometimes defined as the deviation from the expectation of *Z,* a relative Value at Risk (VaR^{rel}) may also be defined as

$$\mathrm{VaR}_{1-\alpha}^{rel}(Z) = E(Z) + \mathrm{VaR}_{1-\alpha}^{abs}(Z) \tag{3}$$

In many applications, the random variable *Z* can be assumed to be normally distributed $Z \sim N(\mu, \sigma^2)$. Then an analytical expression for the VaR exists (see Dowd 1998).

$$\mathrm{VaR}_{1-\alpha}^{abs}(Z) = -(q_\alpha^N \cdot \sigma + \mu) \qquad \mathrm{VaR}_{1-\alpha}^{rel}(Z) = -q_\alpha^N \cdot \sigma \tag{4}$$

with q_α^N the corresponding quantile of the standard normal distribution *N*(0,1).

Another relevant downside risk-meausure is the Conditional Valuet at Risk (CVaR). In contrast to the VaR, the CVaR considers also information about the losses exceeding the VaR. Following the distinction of the VaR in absolute and relative VaR, the CVaR can be defined analogously (according to Strohbücker 2011).

$$\mathrm{CVaR}_{1-\alpha}^{abs}(Z) = -E(Z \,|\, Z \le -\mathrm{VaR}_{1-\alpha}^{abs}(Z)) \tag{5}$$

$$\mathrm{CVaR}_{1-\alpha}^{rel}(Z) = -E(Z - E(Z) \,|\, Z \le -\mathrm{VaR}_{1-\alpha}^{abs}(Z)) \tag{6}$$

Assuming again a normal distribution for Z, the analytic expressions for the CVaR are

$$\mathrm{CVaR}_{1-\alpha}^{abs}(Z) = -\left(\frac{-\varphi(q_\alpha^N)\cdot\sigma}{\alpha} + \mu\right) \qquad \mathrm{CVaR}_{1-\alpha}^{rel}(Z) = -\frac{-\varphi(q_\alpha^N)\cdot\sigma}{\alpha} \tag{7}$$

It is obvious that the following relations will hold for every random variable *Z*,

$$\mathrm{VaR}_{1-\alpha}^{abs}(Z) \le \mathrm{CVaR}_{1-\alpha}^{abs}(Z) \text{ and } \mathrm{VaR}_{1-\alpha}^{rel}(Z) \le \mathrm{CVaR}_{1-\alpha}^{rel}(Z) \tag{8}$$

and for positive *E(Z)* the relative VaR and CVaR are stronger risk measures than the absolute ones.

An important property of risk measures is whether the measure is dependent or independent from the position of the distribution of *Z*. This means that for a position dependent risk measure, the amount of the loss is crucial. An example for such a risk measure is the absolute VaR. By contrast, the relative VaR is a position independent measure. Figure 1 illustrates this difference with respect to position dependence.

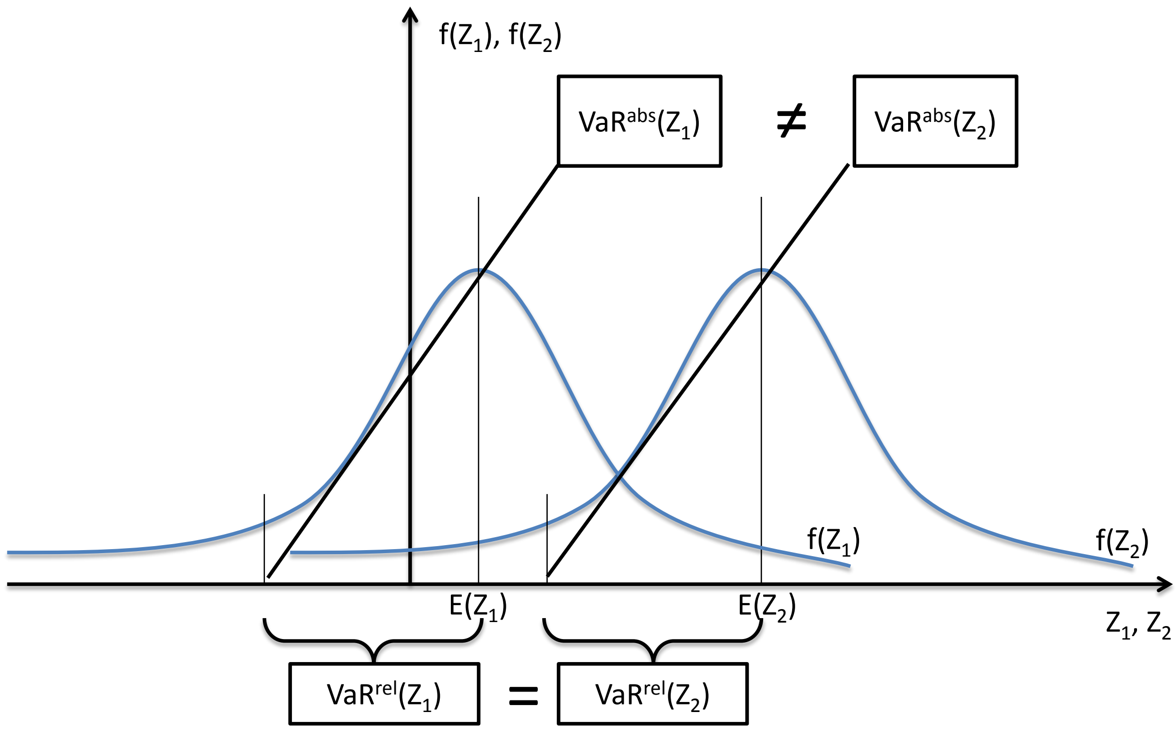

Figure 1: Comparison of VaR^{abs} and VaR^{rel} for two different distributions of *Z* with the same variance but different means.

Z_1 and Z_2 have a different position, nevertheless they have the same variance, but different means $E(Z_1)$ and $E(Z_2)$. The relative VaR as the difference from the α-quantile to the mean is the same for both distributions. But the absolute value of the α-quantile is different.

Using a position independent risk measure, the risk obtained e.g. for two different strategies may be the same, even though the absolute risk of one alternative be much larger implying a larger loss, than for the other alternative. This illustrates the need for an appropriate choice of the risk measure depending on the application. Consequently the implications of the different measures need to be investigated.

4 Mean-Risk Hedging Strategies with Limited Liquidity

As mentioned in the introduction, hedging in energy markets and especially in electricity markets has to cope with limited market liquidity on forward markets. Therefore, we need a modelling framework taking into account that trading activities derived from mean-risk hedging strategies will have an influence on the market price of hedging products. Woll and Weber (2015) develop such a framework for mean-variance hedging strategies under limited liquidity. We take this modelling framework as a starting point and extend it with regard to different risk measures, absolute and relative VaR and CVaR. Thereby, we show that depending on the measure of risk, hedging strategies may exist which outperform the naïve risk-minimizing strategy of full hedging at the first time-step both in terms of profit and risk.

4.1 General Mean-Risk Hedging with Limited Liquidity

Following Woll and Weber (2015) limited liquidity is considered through a linear price-impact function $p_t = p_{u,t} - \beta x_t$ with x_t the trading volume in a certain time step t, and β the illiquidity parameter of the market. $p_{u,t}$ is the price at sales quantity zero and β the slope of the inverse residual demand (or price-sales) function.[2] Thus, the higher the liquidity in the market, the lower the value of β. The price $p_{u,t}$ is assumed to be a normally distributed random variable, whereas β shall be non-stochastic in order to keep the problem quadratic.

[2] β may be estimated by dividing half the bid-ask spread of historical forward prices for electricity by the corresponding average trading volumes.

We label V_0 the size of the portfolio. In the general mean-risk case, optimal hedging strategies under limited liquidity can be derived by maximizing the risk adjusted return:

$$\max_{x} \left\{ \mu - \gamma\rho \middle| V_0 = \iota' x \right\} \tag{9}$$

This is equivalent to the minimization problem:

$$\min_{x} \left\{ \rho - \frac{1}{\gamma}\mu \middle| V_0 = \iota' x \right\} \tag{10}$$

The set of efficient hedging strategies – i.e. the efficient frontier in a μ- ρ diagram - may then be determined by solving the following optimization problem for different values of μ *(*cf. Merton (1972):

$$\min_{x} \left\{ \rho \middle| \mu = (p_0\iota - \beta x)' x; V_0 = \iota' x \right\} \tag{11}$$

Here, μ considers the linear price-impact function. This leads to the following Lagrangian function and the corresponding first order conditions.

$$L(x, \lambda_1, \lambda_2) = \rho + \lambda_1(\mu - (p_0\iota - \beta x)' x) + \lambda_2(V_0 - \iota' x) \tag{12}$$

$$\frac{\partial L}{\partial x} = \frac{\partial \rho}{\partial x} + \lambda_1(-p_0\iota + 2\beta x) - \lambda_2\iota = 0 \tag{13}$$

$$\frac{\partial L}{\partial \lambda_1} = \mu - (p_0\iota - \beta x)' x = 0 \Leftrightarrow \mu = (p_0\iota - \beta x)' x \tag{14}$$

$$\frac{\partial L}{\partial \lambda_2} = V_0 - \iota' x = 0 \Leftrightarrow V_0 = \iota' x \tag{15}$$

4.2 Mean-Variance Hedging Strategies with Limited Liqudity

Woll and Weber (2015) consider the optimal hedging strategy for an electricity sales volume over *T* discrete time steps with respect to the variance (σ^2) as the measure of risk. This leads to the following optimization problem:

$$\min_x \left\{\sigma^2 \middle| \mu = (p_0\iota - \beta x)'x; V_0 = \iota' x\right\} \tag{16}$$

Iota ι is thereby the unit vector. The price vector $p_u = (p_{u,1}, \ldots, p_{u,T})'$ is assumed to be multivariately normally distributed, with $p_u \sim N(p_0\iota, C)$. The expected price at sales quantity zero for all future periods is set equal to p_0, to avoid systematic incentives for arbitrage trading. This also means that the price process for $p_{o,t}$ is assumed to fulfil the martingale property, i.e. $E[p_{u,t}] = p_0$.

The solution of this optimization problem is derived using the Lagrange method. The optimal hedging strategy in Woll and Weber (2015) is given by $x = V_0 \frac{M^{-1}(\lambda_1)\iota}{\iota' M^{-1}(\lambda_1)\iota} \iota$ and the corresponding mean and variance are

$$\mu = p_0 V_0 - \beta \frac{V_0^2 \iota' M^{-2}(\lambda_1)\iota}{\left(\iota' M^{-1}(\lambda_1)\iota\right)^2} \text{ and } \sigma^2 = V_0^2 \frac{\iota' M^{-1}(\lambda_1) C \left(M^{-1}(\lambda_1)\iota\right)}{\left(\iota' M^{-1}(\lambda_1)\iota\right)^2} \tag{17}$$

Here, C is the covariance matrix, $M(\lambda_1) := [2C + 2\lambda_1 \beta I]$ and λ_1 is the Lagrangian multiplier corresponding to the equation for the given value of the expected return. Figure 2 summarizes the results of Woll and Weber (2015). The main result is that limited liquidity reduces profits. Furthermore, the quantity to be hedged has an impact on the optimal solution: The larger the sales quantity, the later one should hedge.

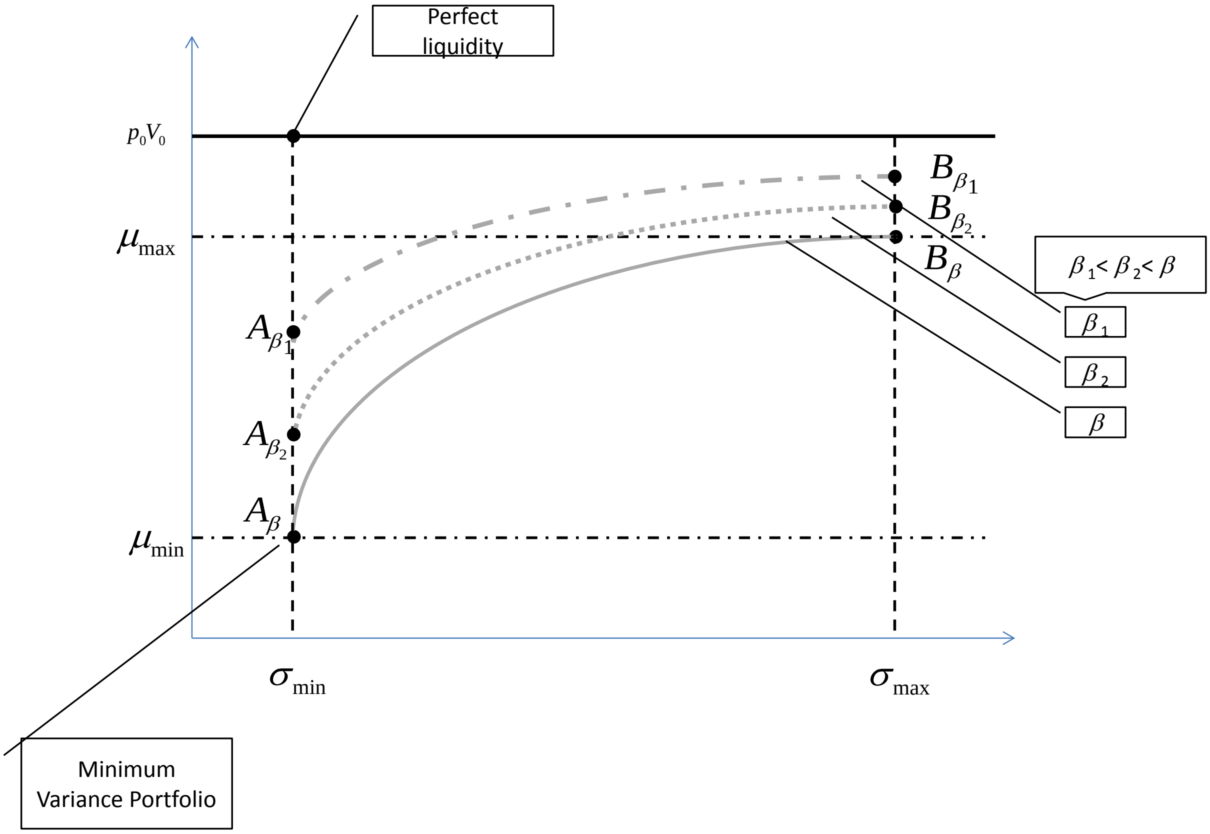

Figure 2: Results of Woll and Weber (2015)

Here A denotes the minimum risk strategy of selling the entire volume at the first time-step and B denotes the point with maximum profit – at constant liquidity over time this corresponds to a uniform partition of the whole sales volume to the regarded trading months. The figure shows that the efficient frontier is monotonously increasing and that an increasing liquidity, i. e. smaller β, leads to a flatter efficient frontier. Thus, for a given target return on risk (cost of risk capital), the optimal μ- σ tradeoff will move towards point A and therefore lead to earlier hedging. In the case of perfect liquidity ($\beta = 0$), only the point of selling everything immediately is optimal.

4.3 Mean-Risk Hedging Strategies with Limited Liqudity

For the investigation of mean-risk hedging with limited liquidity our focus concentrates on the analysis of the risk measures absolute and relative VaR and and CVaR. From equations (4) and (7) is it obvious, that these risk measures can be written as

$$\rho = k_1\sigma - k_2\mu \qquad (18)$$

with k_2=0 for the relative measures and k_2=1 for the absolute measures. Using this functional relation in (18) the optimization problem can be reformulated to:

$$\min_x \left\{ k_1\sigma - k_2\mu - \frac{1}{\gamma}\mu \middle| V_0 = \iota' x \right\} \qquad (19)$$

Using again the Merton approach it is obvious, that equation (14) and (15) from section 4.1 are independent of the risk measure. Thus, the differences in the optimal solutions by choosing a different risk measure only depend on the derivative of the risk measure with respect to *x*. Under the assumption of a normal distribution for $p_o \sim N(p_0\iota, C)$, these derivatives may be derived explicitly. For the absolute Value at Risk this leads to

$$\frac{\partial\rho}{\partial x} = \frac{\partial \mathrm{VaR}_{1-\alpha}^{abs}((p_0\iota - \beta x)' x)}{\partial x} = -q_\alpha^N \cdot \frac{Cx}{\sqrt{x' Cx}} - (p_0\iota - 2\beta x)\,. \qquad (20)$$

And the derivative for the corresponding relative VaR is given by

$$\frac{\partial\rho}{\partial x} = \frac{\partial \mathrm{VaR}_{1-\alpha}^{rel}((p_0\iota - \beta x)' x)}{\partial x} = -q_\alpha^N \cdot \frac{Cx}{\sqrt{x' Cx}}\,. \qquad (21)$$

Similar calculations can be performed for the CVaR measures, with the absolute CVaR's derivative

$$\frac{\partial\rho}{\partial x} = \frac{\partial C\mathrm{VaR}_{1-\alpha}^{abs}((p_0\iota - \beta x)' x)}{\partial x} = \frac{\varphi(q_\alpha^N)}{\alpha} \frac{Cx}{\sqrt{x' Cx}} - (p_0\iota - 2\beta I x) \qquad (22)$$

And the relative CVaR's derivative

$$\frac{\partial\rho}{\partial x} = \frac{\partial \mathrm{CVaR}^{rel}_{1-\alpha}((p_0\iota-\alpha x)'x)}{\partial x} = \frac{\varphi(q^N_\alpha)}{\alpha}\frac{Cx}{\sqrt{x'Cx}}\,. \qquad (23)$$

A comparison of these derivatives reveals that all of them are closely related to the derivative $\frac{Cx}{\sqrt{x'Cx}}$ of the standard deviation of *X*. In fact they differ at most by a scaling parameter plus a linear function of the original vector *x.* This follows directly from the functional realtion (18) and the use of the Merton approach and implies that for a given value of μ the same optimal solution for *z* will occur.

For the position independent risk measures relative VaR and relative CVar, the risk is only a scaling of the standard deviation. Hence minimizing ρ in equation (11) leads to the same optimal hedging strategy *z* as minimizing σ. The efficient frontiers are hence only shifted and stretched in the risk dimension. Figure 3 sketches this rescaling for the relative VaR and the relative CVaR, which both stretch the standard deviation (for $\alpha < \Phi^{-1}(-1)$) and shift it to the right.

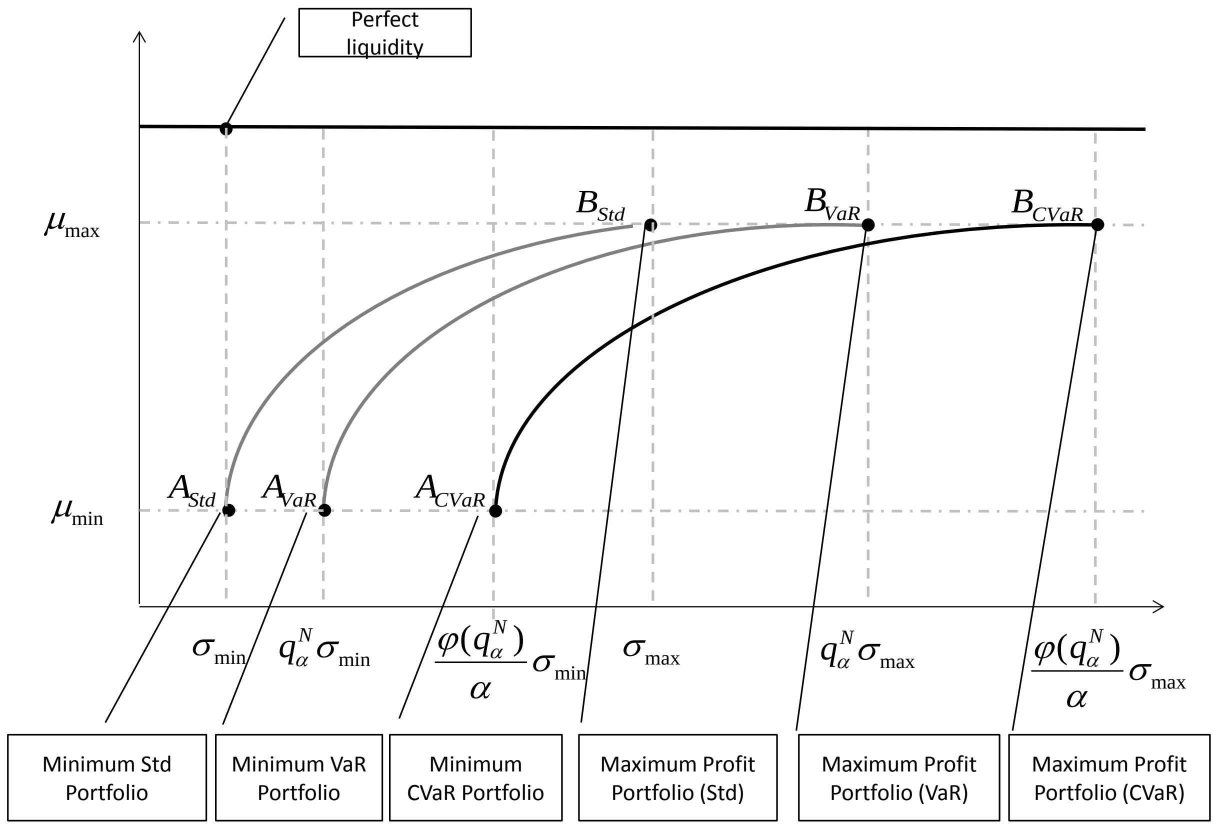

Figure 3: Efficient frontiers of position independent risk measures

Here *A* and *B* again denote the points of selling everything immediately (*A*) and the uniform partition of the portfolio (*B*).

In contrast, the risk function includes a term with the mean value for position dependent risk measures such as the absolute variants of VaR and CVaR. Therefore, the shape of the efficient frontier changes. To get a better understanding, the analytical expressions for the mean and the risk in the optimum are computed. Since the optimal solution for *x* is identical for each risk measure, we take the expression from Woll and Weber (2015) $x = M^{-1}(\tilde{\lambda}_1)\left(\tilde{\lambda}_1 p_0 + \tilde{\lambda}_2\right)\iota$. With this expression, we are able to derive the following analytical expression for mean and risk, here exemplarily for the absolute CVaR:

$$\mu = p_0 V_0 - \beta \frac{V_0^2 \iota' M^{-2}(\tilde{\lambda}_1)\iota}{\left(\iota' M^{-1}(\tilde{\lambda}_1)\iota\right)^2} \tag{24}$$

$$\rho = -\left(\frac{\varphi(q_\alpha^N)}{\alpha} V_0 \sqrt{\frac{\iota' M^{-1}(\tilde{\lambda}_1) C \left(M^{-1}(\tilde{\lambda}_1)\iota\right)}{\left(\iota' M^{-1}(\tilde{\lambda}_1)\iota\right)^2}} + p_0 V_0 - \beta \frac{V_0^2 \iota' M^{-2}(\tilde{\lambda}_1)\iota}{\left(\iota' M^{-1}(\tilde{\lambda}_1)\iota\right)^2} \right). \tag{25}$$

In this case it is obvious that the risk ρ depends on the illiquidity parameter β. It is already included in the expression for the mean and the mean is part of the risk term. This is a remarkable difference to the case of position independent measures, where ρ is only a multiple of the standard deviation σ (see equation (9) and (18)). However, the most important difference is that the minimum risk hedging strategy may change. Figure 4 illustrates the shapes of the efficient frontiers for the case of position dependent risk measures.

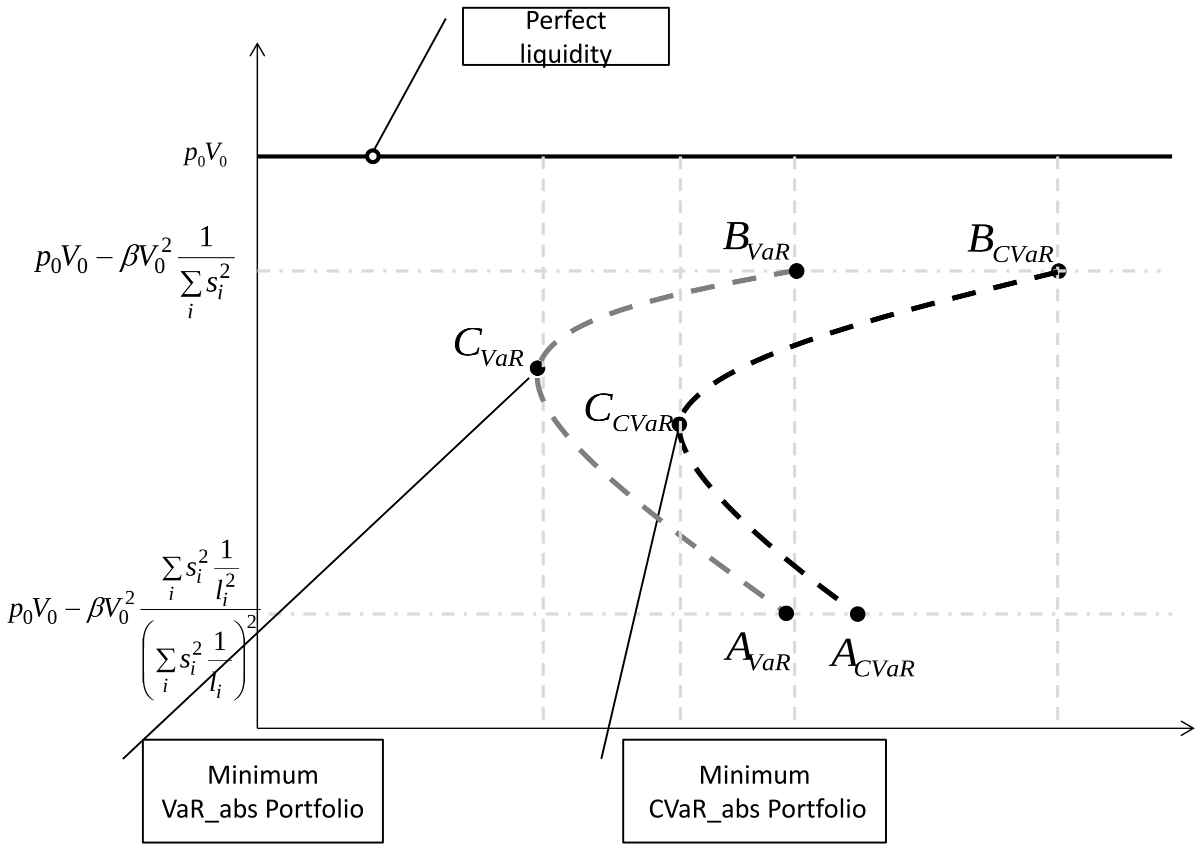

Figure 4: Efficient frontiers of position depended risk measures

It is obvious that in this case the efficient frontier may no longer be expressed as a function *$\mu(\rho)$*, because two different profit levels exist for some risk levels and the point A, corresponding to hedging immediately the entire volume at the first time-step, is no longer the point with minimum risk. The point with minimum risk is labelled here C and B is again the point with maximum profit and the corresponding uniform distribution of sales over time.

Given the form of the efficient frontier the optimal hedging strategy should be selected with special circumspection. In classical mean-variance optimization, the Sharpe-ratio (c.f. Sharpe (1964)) (and possibly the CAPM) are often used to derive the optimal portfolio. When changing the measure of risk, the link to the Sharpe-ratio is however no longer obvious. Therefore we focus in the following on the minimum risk strategy as a key element for decision support (cf. e.g. Perold and Sharpe (1988)).

The minimum risk strategy for position independent risk measures is the same for all risk measures. It is the strategy of hedging the entire volume at the first time-step, because risk increases over time. In the case of position dependent risk measures, strategies exist with lower risk and higher mean, so that the immediate hedging strategy is obviously inefficient with respect to the chosen risk measure.

A necessary condition for an interior risk minimum is that the derivative of the risk with respect to the mean is equal to zero. Using the realtion (18) this derivative can be computed as

$$\frac{\partial \rho}{\partial \mu} = \frac{\frac{\partial \rho}{\partial \lambda_1}}{\frac{\partial \mu}{\partial \lambda_1}} = \frac{\frac{\partial (k_1 \sigma + k_2 \mu)}{\partial \lambda_1}}{\frac{\partial \mu}{\partial \lambda_1}} = \frac{k_1 \frac{\partial \sigma}{\partial \lambda_1} - k_2 \frac{\partial \mu}{\partial \lambda_1}}{\frac{\partial \mu}{\partial \lambda_1}} = k_1 \frac{\frac{\partial \sigma}{\partial \lambda_1}}{\frac{\partial \mu}{\partial \lambda_1}} - k_2 = k_1 \frac{\partial \sigma}{\partial \mu} - k_2 \tag{26}$$

With this expression of the derivative of risk with respect to the mean, the minimum risk portfolio is given when

$$\frac{\partial\rho}{\partial\mu}=0 \Leftrightarrow k_1\frac{\partial\sigma}{\partial\mu}-k_2=0 \Leftrightarrow \frac{\partial\sigma}{\partial\mu}=\frac{k_2}{k_1}. \qquad (27)$$

Using this formulation, the minimum risk portfolio for the position dependent risk measures can be linked back to the efficient frontier of the mean-variance case. Thus, for the position independent case (k_2=0), the minimum risk portfolio is obtained when the inverse slope of the efficient frontier of the mean-variance case is equal to zero. This confirms the results for relative VaR and CVaR that the optimal solutions are identical. On the other side, comparing this condition for the position dependent case (k_2=1),

$$\frac{\partial \mathrm{VaR}_{1-\alpha}^{abs}}{\partial\mu}=0 \Leftrightarrow \frac{\partial\sigma}{\partial\mu}=\frac{k_2}{k_1}=\frac{1}{-q_\alpha^N} \qquad \frac{\partial \mathrm{CVaR}_{1-\alpha}^{abs}}{\partial\mu}=0 \Leftrightarrow \frac{\partial\sigma}{\partial\mu}=\frac{k_2}{k_1}=\frac{\varphi(q_\alpha^N)}{\alpha} \qquad (28)$$

it can be confirmed that the optimal hedging solutions for the risk measures absolute VaR and CVaR are different than for the position independent measures and also compared to each other. Since the confidence level α for the downside risk measure is between zero and 0.5, the relation $0\le\frac{\varphi(q_\alpha^N)}{\alpha}\le\frac{1}{-q_\alpha^N}$ holds. Since $\frac{\partial\sigma}{\partial\mu}$ increases with μ, the absolute CVaR leads to a minimum risk portfolio with lower mean and, thus, using the results from Woll and Weber (2015), to earlier hedging activity.

A further difference is that the minimum risk depends on liquidity in the case of position dependent risk measures, but not in the case of position independent measures. For the impact of liquidity on the minimum risk portfolio the conditions in equation (23) and the results from Woll and Weber (2015) may be used again. Equation (23) implies that the minimum risk portfolio in the position dependent case is found at the point of the mean-sigma efficient frontier with a slope equal to $-q_{\alpha}^{N}$ for the absolute VaR and $\frac{\alpha}{\varphi(q_{\alpha}^{N})}$ for the absolute CVaR respectively. According to Woll and Weber (2015) the slope of the mean-sigma efficient frontier will increase with decreasing liquidity of the market. Lower liquidity implies hence that hedging is deferred to later time steps under the minimum risk strategy for the position dependent risk measures. Also the result on the impact of the hedging volume on the hedging strategy can be generalized to minimum risk portfolios. The larger the total volume, the later the hedges will be done in the minimum risk portfolio under all risk measures.

For the position independent risk measures, the minimum risk strategy will be the same for all risk measures: it is the strategy with selling the entire volume in the first time-step. This means that liquidity has no influence on the minimum risk strategy for position independent risk measures. The results for the impact of limited liquidity as in Woll and Weber (2015) (see section 4.1) will hold for strategies with a higher risk than the minimum risk strategy, because the shape of the efficient frontiers will only be shifted and stretched by liquidity limitations.

5 Application

We follow Woll and Weber (2015) for the setting for the application. We consider the case of hedging a given volume of electricity V_0 by selling the forward product for a continuous band delivery for one year, called yearly base product. We start hedging 12 months before delivery and consider 13 time steps for hedging, including immediate hedging and hedging once per month in the 12 remaining months. The values for the parameters are given in the following table.

Price at sales quantity zero [€/MWh] p_0	Liqudity [€/MWh^2] β	Portfolio size [MW] V_0	Std for covariance matrix [€/MWh] σ_1	Confidence level α
52	0.0035	1000	2.85	0.1

Table 1: Parameter values for the application

The covariance matrix *C* is constructed according to Woll and Weber (2015) as

$$C_1 = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \sigma_1{}^2 & \sigma_1{}^2 & \cdots & \sigma_1{}^2 \\ 1 & \sigma_1{}^2 & 2\sigma_1{}^2 & \cdots & 2\sigma_1{}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \sigma_1{}^2 & 2\sigma_1{}^2 & \cdots & t\sigma_1{}^2 \end{pmatrix}, \qquad (29)$$

with the rows and columns representing the different time steps for trading activities. The parameters are derived from historical data. For the estimation procedures see again Woll and Weber (2015).

With these parameters, we compute the corresponding optimal hedging strategies and minimum risk portfolios for the different risk measures according to equation (24), (25) and (28). Figure 5 illustrates the different efficient frontiers for the different measures.

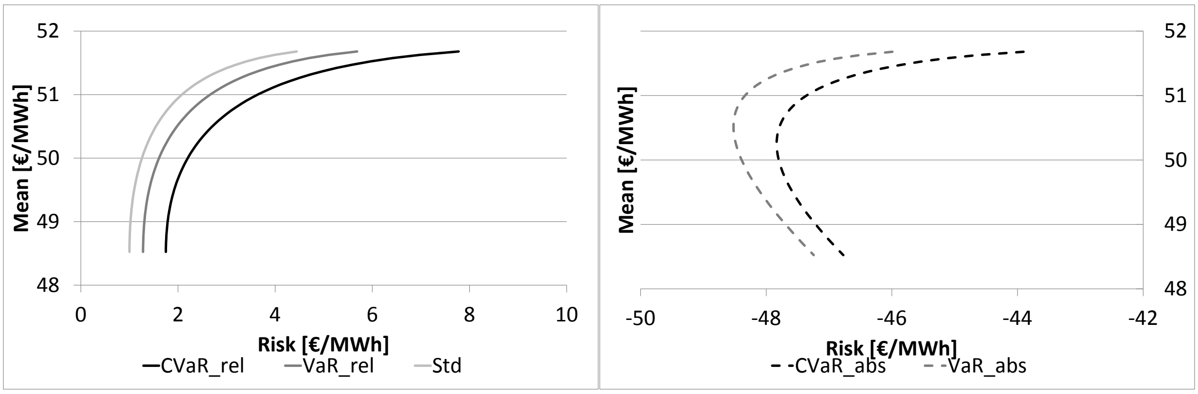

Figure 5: Efficient frontier position independent risk measures (left); Efficient frontier position dependent risk measures (right)

The figure shows the results derived in the previous section. The efficient frontiers for the position independent risk measures are only shifted and stretched, whereas the frontier in the position dependent case includes inefficient strategies with higher risk but less profit. In this case, risk is plotted in terms of losses. A negative value for the risk thus indicates a positive profit and therefore highly negative values correspond to low risk. For a comparison of the efficient frontiers of both position dependent and independent measures, we readjust the risk scale, using for each risk measure ρ^0, the minimum risk for the perfect liquidity case, as a reference value. This risk value is obtained when the whole hedging volume is sold in the first time step with a liquidity parameter β=0. Subsequently, the deviation of the risk on the different efficient frontiers from this ρ^0 is calculated. The rescaled risk value is then given by $\rho^{trans}(\mu) = \rho(\mu) - \rho^0$.

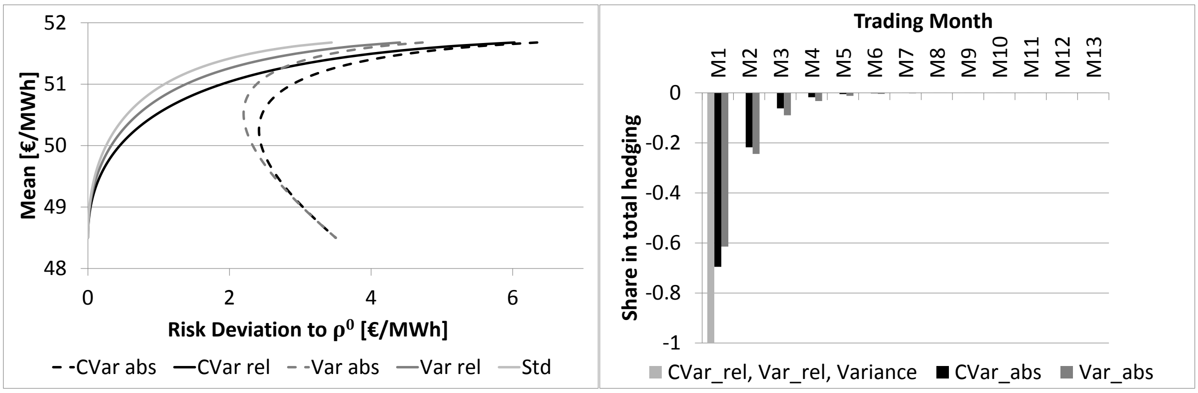

Figure 6: Efficient frontier comparison (left); Optimal hedging strategy in minimum risk case (right)

For the position independent risk measures, the range of the risk between the portfolios with highest and lowest expected profit is according to Figure 6 much higher than for the corresponding dependent ones. The optimal hedging strategies in the minimum risk portfolio in the right part illustrate the result of section 4.2 that the minimum risk strategy leads to immediate full hedging for the position independent measures. For the position dependent measures, the stronger risk measure, i. e. the absolute CVaR, leads to earlier hedging. Or vice versa, the lower the (measured) risk, the more hedging is shifted to the future.

Since the minimum risk strategy is invariant under changes in liquidity and portfolio size for position independent risk measures, the following sensitivity analysis for the liquidity parameter β, the portfolio size V_0, and the price at sales quantity zero p_0 are only performed for the position dependent case and illustrated for the absolute CVaR.

The sensitivity analysis for the liquidity parameter β shows that minimum risk decreases with an increasing liquidity of the market. The corresponding mean in the minimum risk strategy increases simultaneously. Figure 7 shows these results.

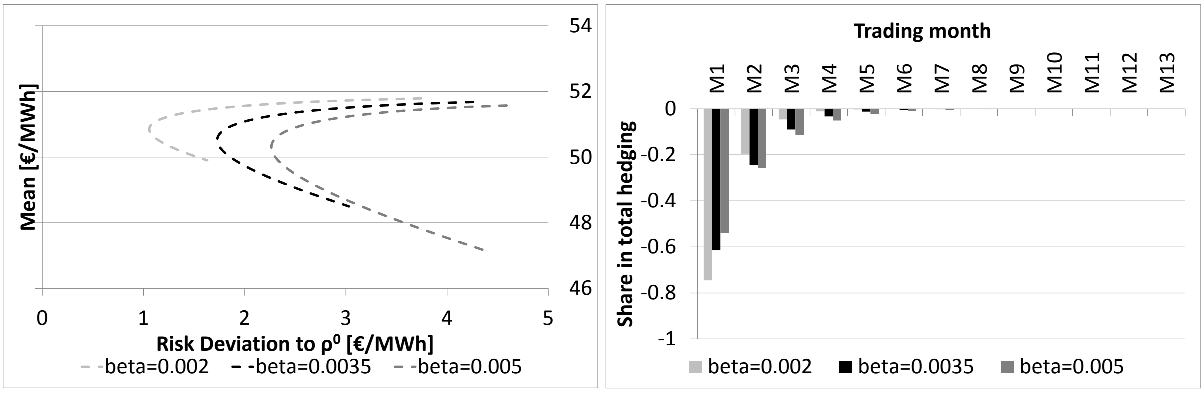

Figure 7: Sensitivity analysis for the liquidity parameter. Efficient frontier comparison (left); Optimal hedging strategy in the minimum risk case (right)

The optimal hedging strategies indicate that with an increase in β, corresponding to a more illiquid market, the hedging strategy in the minimum risk case is to postpone hedging.

A sensitivity analysis for the total hedging volume V_0 shows similar results (cf. Figure 8). The minimum risk increases with an increasing quantity and the corresponding mean decreases.

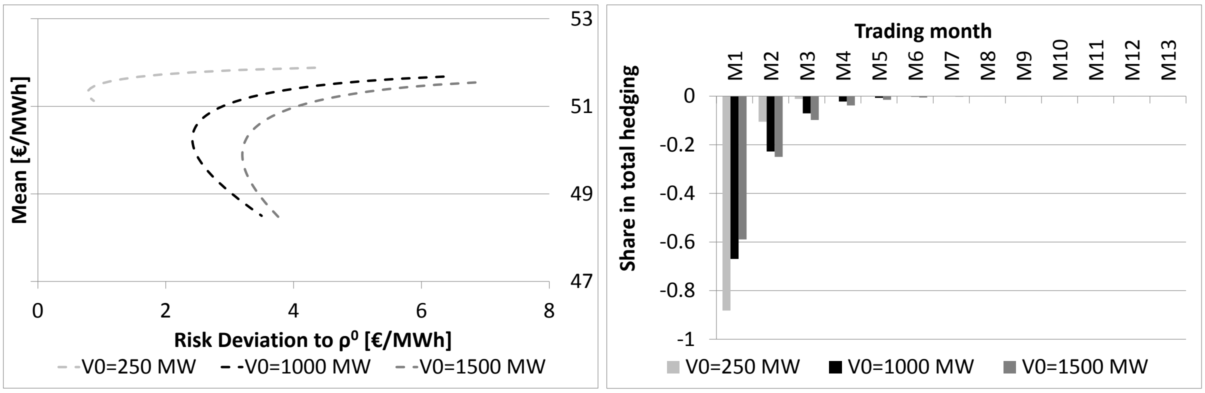

Figure 8: Sensitivity analysis for the hedging volume. Efficient frontier comparison (left); Optimal hedging strategy in the minimum risk case (right)

The hedging strategies corresponding to the different minimum risk cases explain these results. The hedging share in the first month decreases with an increasing total volume. Price reactions are then larger and it is hence optimal to sell later for the producer.

Figure 9 shows a sensitivity analysis with respect to the price level p_0. Obviously the price level has no influence on the minimum risk strategy, when we assume that the price level has no impact on the liquidity parameter β. Here the mean of the different cases is normalized in relation to the maximum mean for reasons of comparability.

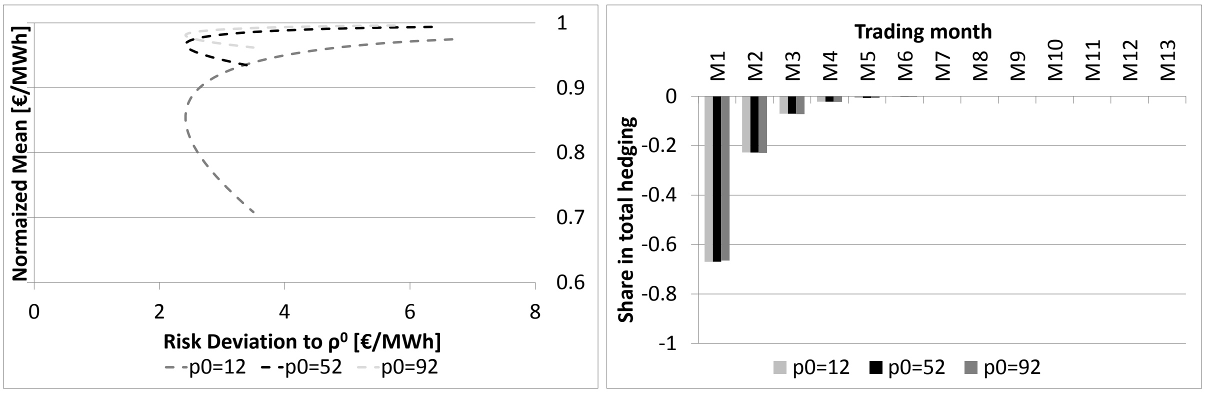

Figure 9: Sensitivity analysis with respect to the price level. Efficient frontier comparison (left); Optimal hedging strategy in minimum risk case (right)

A sensitivity analysis with respect to the standard deviation σ_1 used for the calculation of the covariance matrix is shown in Figure 10. As expected, the riskier the market the earlier the hedging will be done.

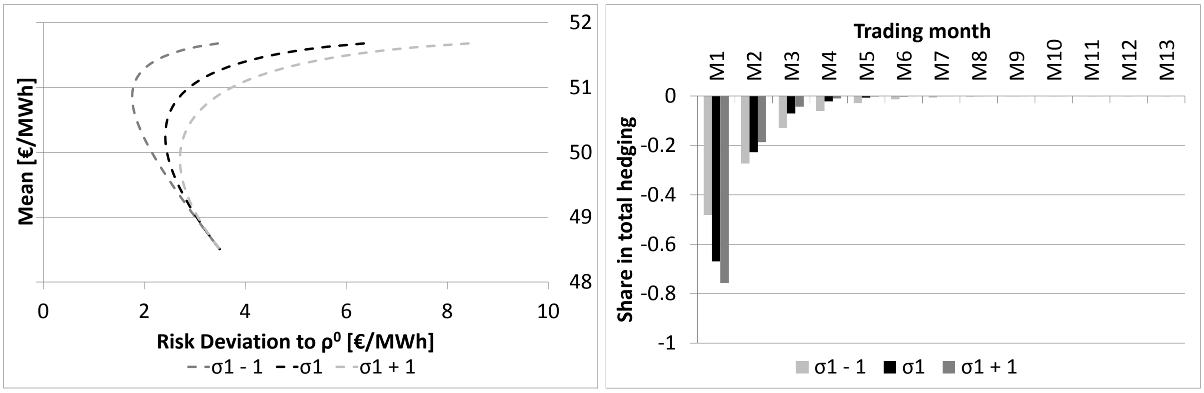

Figure 10: Sensitivity analysis with respect to the standard deviation σ1. Efficient frontier comparison (left); Optimal hedging strategy in minimum risk case (right)

6 Implications for Practical Financial Risk Management

A major goal of practical risk management is sustainable growth of the company value. This value is often measured as the shareholder value based on discounted cash flows and can be increased by reducing the risk and/or improving the returns for a company. Often, the ratio of risk and return is chosen for measuring the performance of risk management, because the two objectives are typically assumed to be countervailing. The corresponding measures are called risk-adjusted performance measures.

One of these measures in the context of portfolio theory is the Sharpe ratio (in the mean-variance case), which has been addressed in section 4. Analogous ratios exist for the mean-risk cases. The major implication of the results in this article for practice is that reducing risk and improving returns or profits is not always a trade off, but also depends on the risk measure used. This means that the strategy focusing on the lowest standard deviation is not always the strategy with the lowest risk. When choosing a position dependent risk measure, such as absolute VaR and CVaR, strategies with higher expected profits and lower risk may exist. In this case a strategy with higher variance may decrease the overall risk. The results of section 4 additionally highlight that with decreasing market liquidity, the share of more risky operations (i. e. deferred hedging) should be higher.

Approaches widely used in practice for risk-adjusted performance measurement and integral risk-return management are the economic value added (EVA) and the risk adjusted return on capital (RORAC). They are often used in practice as instruments for value based risk management. EVA is an absolute performance measure and is calculated as

$$EVA = Expected\ Profits - r_H \cdot RiskCapital,$$

(see Diers 2011) with r_H an internal hurdle rate and *RiskCapital* measured as risk of the profits with a certain risk measure, such as VaR or CVaR. The RORAC, in contrast, is a relative performance measure and is calculated as

$$RORAC = \frac{Expected\ Profits}{RiskCapital},$$

(see Diers 2011) with the *RiskCapital* again measured as risk of the profits.

Within these frameworks the *RiskCapital* is usually interpreted as absolute loss and thus position dependent risk measures, such as the absolute VaR and CVaR, are used for calculation (see Scherpereel 2006). A further aspect why absolute measures are used for calculating the *RiskCapital* is the possibility of capital allocation, which is important for the planning of the optimal capital structure of a company and the determination of specific risk limits for different business units. Therefore the characteristic of absolute VaR and CVaR being translation invariant and, in the normally distributed case, subadditive is important because of the diversification effects of risk (see Scherpereel 2006). In contrast, the relative VaR and CVaR do not have the property of being translation invariant. Adding a risk free element to the portfolio will then not reduce the risk by the value of this risk free element.

For practical risk management using *EVA* and *RORAC* the results from the previous section are very useful. The *RORA*C corresponds to the slope of the μ- ρ diagram. Considering a given hurdle rate for the *RORAC* the strategy would be to increase risk until the expected return is covered. Usually this meets a higher risk than the minimum risk. In order to derive an optimal hedging strategy for a perspective hurdle rate, this strategy can be determined analogous to the determination of the optimal portfolio in Woll and Weber (2015).Thus the slope of the efficient frontier has to be numerically calculated and the optimal strategy corresponds to the first point with a slope smaller than the perspective hurdle rate. This implies that capital is not scarce.

7 Conclusion

This article investigates mean-risk hedging strategies under limited liquidity and studies the impact of using different risk measures for the resulting hedging strategy. The risk measures are distinguished in position independent measures (Variance, relative VaR, relative CVaR) and position dependent measures (absolute VaR, absolute CVaR). A key result is that the minimum risk strategy for the position independent measures is not affected by market liquidity. In this case, the minimum risk strategy always corresponds to the immediate hedging of the entire open position. In contrast, liquidity has an impact on the minimum risk strategy when position dependent measures are employed. Due to the dependence of the absolute risk measures on the mean, there exist strategies with lower risk and higher mean than the immediate full hedging strategy. As a third result, our modelling framework enables us to link back the the minimum risk strategy for all investigated risk measures to the efficient frontier in the mean-variance case. This allows computation of the corresponding minimum-risk strategies. In addition, the results on the impact of limited liquidity in the mean-variance case from Woll and Weber (2015) are found to hold analogously for these more general mean-risk cases. Notably higher liquidity leads to earlier hedging in the minimum risk strategy and the total hedging volume has an influence on the minimum risk strategy. In addition, for practical risk management the results of the article emphasises how the choice of the risk measure can influence instruments for value based risk management.

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