Discussion Paper No. 13-025

Linking Price and Quantity Pollution Controls under Uncertainty

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and Andreas Löschel

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Non-Technical Summary

In some cases, linking of policies for the provision of international public goods can increase welfare on a global scale. One example is the international linking of national policies for the reduction of greenhouse gas emissions. By pooling of abatement options, economic efficiency of regulation can be increased since least-cost abatement options can be identified more easily. Linking of policies can lead to situations where abatement costs decrease in one country, but increase in another country. The magnitude of expected changes in abatement costs is an important determinant whether a country is interested in direct policy linking or not.

In this paper, we consider the case of two countries that have to produce an exogenously given amount of a public good (e.g. greenhouse gas abatement) under uncertain (abatement) costs or otherwise purchase the good on a market (e.g. international emissions trading). We compare the case when both countries choose regulation by quantities ("quantity-quantity linking") to a situation where one of the countries is choosing prices (e.g. a carbon tax) while the other country chooses quantities ("price-quantity linking"). To examine the effective costs of regulation for both countries under uncertainty and different policy scenarios, we numerically solve the model where uncertainty is modelled by assuming that abatement costs have a log-normal distribution.

It is shown that in most cases, total expected costs increase for the country that chooses prices under international emissions trading when compared to quantity regulation. The only case for which we found that the price-setting country would be better off choosing prices occurred when the quantity-setting country had a much more onerous target than the price-setting country. Countries will further be unable to identify a tax rate that minimizes costs because of uncertainty and unknown correlation of costs between countries. Large economies face higher expected cost increases relative to smaller economies when choosing regulation by prices under international emissions trading.

If there are specified quantity targets for the provision of an international public good, and international policy linkages exist, price regulation will usually lead to both higher costs and higher variance of costs when compared to quantity regulation. In this case, price regulation will not serve as a mechanism for cost containment.

Das Wichtigste in Kürze

In einigen Fällen kann die Verknüpfung von Politikmaßnahmen zur Bereitstellung internationaler öffentlicher Güter zu einer Steigerung der globalen Wohlfahrt führen. Ein Beispiel dafür ist die internationale Verknüpfung nationaler Politiken zur Vermeidung von Treibhausgasen. Dadurch werden zusätzliche kostengünstige Vermeidungsoptionen eröffnet deren Nutzung die globale Wohlfahrt erhöhen kann. Eine Verbindung von Politikmaßnahmen kann dabei zu Situationen führen in denen sich die Kosten für ein Land verringern, während die Kosten für ein anderes Land ansteigen. Das Ausmaß erwarteter Kostenänderungen bestimmt dabei unter anderem ob ein Land Interesse an einer Politikverknüpfung hat.

In diesem Papier wird der Fall zweier Länder betrachtet die eine exogen gegebene Menge eines öffentlichen Gutes (z.B. Treibhausgasminderung) unter unsicheren Kosten produzieren oder das Gut andernfalls auf dem Markt erwerben (internationaler Emissionshandel). Dabei werden die anfallenden Kosten verglichen falls beide Länder Mengenregulierung wählen (z.B. ein Emissionshandelssystem) und falls ein Land eine Preislösung implementiert (z.B. eine CO2-Steuer) während das andere Land eine Mengenlösung wählt. Um die Kostenentwicklung in beiden Fällen und unter Unsicherheit über die tatsächlich anfallenden Kosten zu ermitteln wird eine numerische Anwendung des Modells vorgenommen, wobei Unsicherheit durch eine log-Normalverteilung abgebildet wird.

Wie sich zeigt, steigen die erwarteten Kosten des Landes das eine Preislösung wählt (im Vergleich zur Mengenregulierung), wenn ein internationaler Emissionshandel besteht. Dem Land ist es zudem unmöglich einen optimalen Steuersatz zu ermitteln da Unsicherheit bezüglich der tatsächlichen Kosten sowie bezüglich der Korrelation der Kosten beider Länder besteht. Größere Länder müssen zudem höhere Kostensteigerungen bei Wahl einer Preislösung erwarten als kleinere Länder.

Falls vorgegebene Mengenziele bei der Bereitstellung eines öffentlichen Gutes bestehen und Verbindungen nationaler Politiken bestehen, dann wird die Preislösung im Vergleich zur Mengenlösung sowohl zu höheren erwarteten Kosten als auch zu einer höheren Varianz der Kosten führen. Die Preislösung ist daher in diesem Fall nicht zur Eindämmung oder Begrenzung der Kosten geeignet.

Linking Price and Quantity Pollution Controls under Uncertainty

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Abstract

This paper examines the linking of price-based and quantity-based provision of a public good by two parties in the example of pollution control under a global quantity constraint, using a stochastic partial-equilibrium model. One country chooses a price-based instrument (tax) and trades with another that lets its emissions price adjust. The expected cost for the price-setting country and the combined expected cost is higher than if both countries choose a quantity-based instrument, and the country with the quantity instrument stands to benefit in terms of expected net costs. The effect increases when the relative size of the country with the price-based constraint increases; and increases with respect to the degree of correlation in ex-ante uncertain abatement costs. While the quantity-setting country benefits from lower expected costs in most circumstances, the variance in cost can be much higher if its costs are correlated with the price-setting country. The optimal ex-ante tax rate differs from that under quantity-quantity linking. These results have important implications for instrument choice for the regulation of greenhouse gases and other pollutants and for the design of international agreements when there are domestic preferences for price regulation. The model is applicable to situations involving the provision of a fixed quantity of a public good beyond pollution control.

Keywords: Instrument Choice; Linking; Climate Policy; Prices vs. Quantities
JEL-Classification: Q52; H23; K32

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1 Introduction

The 'prices versus quantity' literature has found that reducing emissions through price control is preferable to quantity control if marginal costs are steeper compared to marginal benefits (Weitzman, 1974; Nordhaus, 1994; Newell & Pizer, 2003); that this holds under a wide range of circumstances (Kolstad, 1996; Hoel & Karp, 2002; Quirion, 2004; Pezzey & Jotzo, 2012); and that hybrid schemes of prices and quantities can reduce costs associated with uncertainty about benefits and costs (Roberts & Spence, 1976; Mandell, 2008), address problems of asymmetric information (Krysiak & Oberauner, 2010), and contribute to cost containment (Pizer, 2002). This literature pivots around the relationship between the marginal cost and marginal benefit of the provision of a public good under uncertainty.

In reality however, this consideration is unlikely to be of prominence in national policy-making for the global provision of environmental or other public goods. Where a regulating jurisdiction faces a defined standard, such as an agreed amount of emissions that must not be exceeded in a certain area and period of time, it may regard benefits as exogenous, and instead focus on how best to implement a commitment given national circumstances and preferences.

A commitment to produce a given amount of a public good resource can be fulfilled either through a quantity regulation or through a price instrument such as a tax. A tax may be preferred in some jurisdictions, while other jurisdictions may choose quantity instruments with flexible prices, such as cap-and-trade. Trading the public good provision between jurisdictions may be possible. Questions then arise as to the cost of one jurisdiction's choice of a price instrument and the distribution of costs between the two jurisdictions; trade-offs between expected costs and cost variability; the optimal tax rate; and the effect of parameters such as the relative size of jurisdictions and the correlation of uncertainties in costs.

We investigate these issues using a two-period, two-country model, which to our knowledge is the first such contribution in the literature. Countries face exogenous environmental standards (agreed in international negotiations), uncertainty about abatement costs, and choose policies (based on prices or quantities) before uncertainty is revealed. In the subsequent period, uncertainty is resolved and countries produce the public good according to the chosen policy. Results are derived numerically using a partial-equilibrium model that assumes log-normally distributed abatement costs. Environmental effectiveness is given by the assumption of full compliance with the exogenous environmental standard or abatement target. It is also assumed that countries have no incentive to abate more than the required target.

The archetypal application for this question is global greenhouse gas control. Some countries may choose a price instrument such as a carbon tax to reduce emissions, either because of a preference to make the marginal cost of production predictable where production costs are unknown ex ante, or for reasons of political economy. The resurgent discussion of a carbon tax for the United States (McKibbin *et al.* , 2012) is a point in case. Other countries rely on quantity instruments and leave their emissions price flexible, as in the emissions trading schemes in operation in Europe and Australia, and planned for China (Petherick, 2012; Lo, 2012) and South Korea (Tuerk *et al.* , 2013). Meanwhile, major countries have made quantitative commitments to limit their emissions to some given level at 2020 (Bodansky, 2011). The costs of reducing emissions are uncertain ex ante, and so will be the effect of a given tax. Consequently it would be logical for countries with price-based instruments to balance their actual emissions against their commitments by either buying emissions allowances from countries with quantity instruments or selling allowances to them. A fixed tax will differ from the ex-post optimum level, and from the emissions price in the 'quantity' country.

To summarise the model, the price-setting country assigns a fixed price but simultaneously aims to achieve a predetermined quantity under uncertain costs, so the likely outcome is a divergence of marginal costs and trading of allowances between the two countries. The price for allowances is equal to the marginal costs for the quantity-setting country; and indirectly determined by the tax rate in the price-setting country, which will affect the number of allowances traded.

While throughout this paper we consider two countries being confronted with an environmental standard with tradable property rights, our framework may also apply to other entities, such as states, cities, or firms, and to many situations of public good provision when there is no single social planner.

The paper is organised as follows. Section 2 outlines the framework for modelling price-quantity and quantity-quantity linking under fixed abatement targets. Section 3 contains the results from applying and numerically solving the model. These results include investigations of expected costs for different-sized countries; the ex-ante optimal choice of tax rate; and the variance in costs. Section 4 discusses the results, discusses how the results relate to some of the broader literature on prices vs. quantities, and concludes.

2 The Model

The model has two producing entities (countries), each of which has been allocated the production of a quantity of a public good (pollution reduction or abatement). Each country can produce the reductions itself, or can otherwise pay the other country to reduce its emissions by a greater amount (through international emissions trading). Countries face uncertainty about abatement costs. If a price-based instrument is chosen, the tax rate is set before uncertainty is resolved.

We assume that Country 1 sets a price (tax) and produces abatement up to a marginal cost equal to the tax rate, while Country 2 sets a quantity for its domestic abatement and lets its price be flexible. In order to meet its quantity target, Country 1 can purchase abatement from Country 2; or if it directly produced more abatement than its allocation, it can sell the excess to Country 2 ('price-quantity linking'). To assess the consequences from one country choosing prices, 'price-quantity linking' will be compared to the case when both countries choose quantities ('quantity-quantity linking').

2.1 Linking of Prices and Quantities

Country $i = 1, 2$ produces abatement q_i at costs c_i, which are ex-ante uncertain, so that

$$c_i = c_i(q_i, \theta_i)$$

where θ_i is a random variable that influences the cost for country i. Cost functions are continuously differentiable and each country has increasing marginal abatement costs so that, when $q_i > 0$,

$$\frac{\partial^2 c_i}{\partial q_i^2} > 0.$$

Each country faces an exogenous quantity target Q_i[1]. Country 1 sets a tax rate t, and produces a quantity $q_1(t, \theta_1)$ of the good that satisfies $c_1'(q_1, \theta_1) = t$, where $c_1' = \partial c_1/\partial q_1$ is Country 1's marginal cost function. The overall quantity goal is met by Country 2 producing whatever is required to meet the overall quantity goal $Q_1 + Q_2$.

If Country 1 produces less than Q_1 allowances, it purchases the difference in allowances $Q_1 - q_1$ from Country 2; if Country 1 produces more than Q_1 allowances, then is sells the difference to Country 2. However, Country 2 will be willing to buy no more than Q_2 allowances in total, so the amount of allowances sold from Country 1 to Country 2 will be given by

$$a = Q_2 - q_2 = \min\{q_1 - Q_1, Q_2\}. \tag{1}$$

The exogenous quantity target Q_2 is also a quantity restriction on the amount of allowances that Country 1 can sell to Country 2. The situation where $Q_2 < q_1 - Q_1$ will lead to a corner solution where there is more abatement than actually needed ($q_1 + q_2 > Q_1 + Q_2$) and the price of allowances becomes equal to zero. This could occur in practice, for example, if Country 2 is substantially smaller than Country 1, so that Q_2 is substantially smaller than Q_1. The situation where Country 2 is smaller or larger than Country 1 is examined in Section 3.2.

In order to meet its quantity requirement, Country 2 directly produces $q_2 = Q_2 + Q_1 - q_1$ of the good, except for when $Q_2 < q_1 - Q_1$, in which case Country 2 will produce none of the good. It is assumed that it is not possible to produce a negative quantity of the good and that both countries fully comply with their abatement targets. If Country 2 sells allowances to Country 1, then the price will be equal to the marginal cost of producing another unit of the good in country 2, i.e. $p = c_2'(Q_2 + Q_1 - q_1, \theta_2)$ when $Q_2 > q_1 - Q_1$, and zero otherwise. We assume that Country 1 is not able to exercise market power when trading allowances, so when $Q_2 < q_1 - Q_1$, allowances will be free.

[1]The model treats the quantity targets as exogenous, and does not model potential benefits from additional abatement. However, it is possible that countries who benefit from additional abatement could choose a high price (or tax) in order to achieve benefits from additional abatement. For the case of greenhouse gas abatement, benefits from additional abatement are limited by the public good character of abatement and strategic considerations (Beccherle & Tirole, 2011). While the costs for greenhouse gas abatement occur as private costs to a country, benefits occur on a global level where other countries can not be excluded from benefiting.

The total cost for Country 1 is the sum of the cost to directly reduce pollution, and the cost of paying Country 2 to produce the additional abatement required. It is given by

$$C_1 = c_1(q_1, \theta_1) - ac_2'(Q_2 - a, \theta_2). \tag{2}$$

Similarly, the total cost for Country 2 is given by

$$C_2 = c_2(Q_2 - a, \theta_2) + ac_2'(Q_2 - a, \theta_2). \tag{3}$$

When $Q_2 \geq q_1 - Q_1$ (the interior solution), costs are given by

$$\begin{aligned} C_1^{[Q_2 \geq q_1 - Q_1]} &= c_1(q_1(t,\theta_1),\theta_1) - \Big(q_1(t,\theta_1) - Q_1\Big)c_2'(Q_2 + Q_1 - q_1(t,\theta_1),\theta_2), \\ C_2^{[Q_2 \geq q_1 - Q_1]} &= c_2(Q_2 + Q_1 - q_1(t,\theta_1),\theta_2) \\ &+ \Big(q_1(t,\theta_1) - Q_1\Big)c_2'(Q_2 + Q_1 - q_1(t,\theta_1),\theta_2). \end{aligned} \tag{4}$$

When $Q_2 \leq q_1 - Q_1$ (the corner solution), costs are given by

$$C_1^{[Q_2 \leq q_1 - Q_1]} = c_1(q_1(t,\theta_1),\theta_1), C_2^{[Q_2 \leq q_1 - Q_1]} = 0. \tag{5}$$

When $Q_2 = q_1 - Q_1$, $C_1^{[Q_2 \geq q_1 - Q_1]} = C_1^{[Q_2 \leq q_1 - Q_1]}$, so either set of equations can be used.

2.2 Quantity-Quantity Linking

Assuming no market failure or other distortions, if both countries choose quantity regulation and trade allowances, marginal abatement costs will be equal. The situation with 'quantity-quantity linking' is used as the reference case that will be used to compare costs associated with price-quantity linking.

Suppose that Country 1 sells $\tilde{a}$ allowances to Country 2 (with $\tilde{a}$ being negative when Country 1 purchases allowances from Country 2) so that the countries have equal marginal costs. If $-Q_1 < \tilde{a}$ and $\tilde{a} < Q_2$, the price that the allowances trade at, $\tilde{p}$, will satisfy

$$\tilde{p} = c_1'(Q_1 + \tilde{a}, \theta_1) = c_2'(Q_2 - \tilde{a}, \theta_2). \tag{6}$$

The situation where $\tilde{a} = -Q_1$ corresponds to all abatement occurring in Country 2; and $\tilde{a} = Q_2$ corresponds to all abatement occurring in Country 1. Costs for each country under quantity-quantity linking are given by

$$\tilde{C}_1(\theta_1, \theta_2) = c_1(Q_1 + \tilde{a}, \theta_1) - \tilde{p}\tilde{a}, \tag{7}$$

$$\tilde{C}_2(\theta_1, \theta_2) = c_2(Q_2 - \tilde{a}, \theta_2) + \tilde{p}\tilde{a}. \tag{8}$$

2.3 Uncertain abatement costs

To examine interactions of price and quantity approaches more closely, further parameterisation is required. It is assumed that abatement costs are ex-ante uncertain and log-normally distributed. This is a suitable assumption when abatement costs are known to always be positive but subject to unknown changes in technology and markets[2].

Cost functions for country $i = 1, 2$ are given by

$$c_i(q_i, \theta_i) \quad = \quad \frac{1}{2}\theta_i \alpha_i q_i^2, \tag{9}$$

where α_i determines the slope of the marginal cost curves, and is constant. The random variable θ_i is log-normally distributed and has a mean value of 1. Marginal costs are derived by differentiating (9) with respect to q_i and are given by

$$c_i'(q_i, \theta_i) \quad = \quad \theta_i \alpha_i q_i. \tag{10}$$

When Country 1 chooses a tax rate t, it will produce a quantity of abatement q_1 that satisfies $c_1' = t$. From (10), q_1 can be expressed as a function of t and θ_1:

$$q_1(t, \theta_1) = \frac{t}{\theta_1 \alpha_1}. \tag{11}$$

Equation (11) implies that if Country 1 sets the tax rate to be sufficiently high relative to its *actual* abatement costs (after uncertainty is revealed), all abatement is born by Country 1. This is because it follows from (11) that

$$q_1 - Q_1 \leq Q_2 \text{ if and only if } t < \alpha_1(Q_1 + Q_2)\theta_1. \tag{12}$$

It is possible to obtain C_1 and C_2 as functions of t by substituting (11) into (2) and (3) [3].

Expected costs for Country 1 and Country 2 can be calculated by taking expectations of the above equations. In practice, abatement costs are likely to be partially correlated, for example due to global exogenous shocks or changes in technology that spill over. The random variables θ_1 and θ_2 are likely to be more highly correlated if changes in technology that affect abatement costs spill over between countries and occur on a global level, and if countries have similar economic features such as the composition of existing capital stock. For simplicity, rather than explicitly modelling partial correlation, this paper examines both the situation where the random variables θ_1 and θ_2 are independent; and the situation where θ_1 and θ_2 are perfectly correlated (and therefore equal).

Suppose that the random variables are independent. Suppose that θ is either θ_1 or θ_2. Because θ is log-normally distributed, its probability distribution depends on two parameters:

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[2]Prices of many commodities undergo 'geometric Brownian motion' where the price at a future point of time is log-normally distributed and depends on parameters that relate to the drift and volatility of the price (Hull, 2006).

[3]If $q_1 - Q_1 \leq Q_2$, then

$$C_1^{[Q_2 \geq q_1 - Q_1]} = \frac{1}{2} q_1(t, \theta_1)^2 \alpha_1 \theta_1 - \Big(q_1(t, \theta_1) - Q_1\Big)\Big(- q_1(t, \theta_1) + Q_1 + Q_2\Big)\alpha_2 \theta_2$$

and

$$C_2^{[Q_2 \geq q_1 - Q_1]} = -\frac{1}{2}\alpha_2 q_1(t, \theta_1)\Big(q_1(t, \theta_1) - 2Q_1\Big)\theta_2.$$

If $q_1 - Q_1 \geq Q_2$, then $C_1^{[Q_2 \leq q_1 - Q_1]} = c_1(q_1(t, \theta_1), \theta_1) = t^2/2\alpha_1\theta_1$ and $C_2^{[Q_2 \leq q_1 - Q_1]} = 0$.

a 'location parameter' μ; and a 'scale parameter' σ. The probability density function of θ is given by

$$f(\theta) = \frac{1}{\theta\sigma\sqrt{2\pi}} e^{-\frac{(\ln\theta-\mu)^2}{2\sigma^2}}.$$

Let v be the variance of θ. Because $E[\theta] = 1$, it follows from a routine calculation that $\mu = -1/2\ \ln(1+v)$, $\sigma = \sqrt{\ln(1+v)}$, and the probability density function of θ can be written as

$$f_i(\theta_i) = \frac{1}{\theta_i\sqrt{2\pi\ln(1+v_i)}} e^{-\frac{(\ln(\theta_i)-1/2\ln(1+v_i))^2}{2\ln(1+v_i)}}. \tag{13}$$

The relationship between the scale parameter σ and the variance v means that it is therefore a measure of the uncertainty in the marginal abatement cost. Because the expected values $E[\theta_1]$ and $E[\theta_2]$ are equal to 1, it also follows that the expected marginal abatement costs satisfy $E[c_1'] = \alpha_1 q_1$ and $E[c_2'] = \alpha_2 q_2$.

Recall that if X is a function of θ_1 and θ_2, then its expectation is given by the integral of X and the probability density function over θ_1 and θ_2. Thus, expected costs (including trading) are given by

$$E[C_i](t) \quad = \quad \int_{-\infty}^{\infty} C_i(t, \theta_1, \theta_2) f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2. \tag{14}$$

When the random variables θ_i are perfectly correlated, so that $\theta_1 = \theta_2 = \theta$, then with probability density function $f(\theta)$, expected costs are

$$E[C_i](t) \quad = \quad \int_{-\infty}^{\infty} C_i(t, \theta) f(\theta) d\theta. \tag{15}$$

A consideration regarding whether to use prices or quantities could be the variance in cost. It is given by

$$Var(c_i) \quad = \quad \int_{-\infty}^{\infty} (c_i(\theta) - E(c_i))^2 f(\theta) d\theta. \tag{16}$$

3 Application of the model

In this section, different parameter choices are investigated by numerically integrating equations (14) to (16) in order to calculate expected costs. These integrals are evaluated using the software package *Mathematica 8.0* (Wolfram Research, 2010). Mathematica is also used to find the choice of tax rate that minimises expected costs. This section also discusses some of the economic intuition behind the results.

Section 3.1 examines the case of ex-ante identical countries. Section 3.2 expands the analysis to the case of countries of different relative size. Section 3.3 focuses on optimal ex-post and ex-ante choices of the tax rate when Country 1 chooses regulation by prices, and examines the cost-variance for the price-setting country.

3.1 Ex-ante Identical Countries

Suppose that both countries are ex-ante identical, and so have the same expected marginal abatement costs, take on the same abatement commitment, and are of the same size. We compare relative costs under price and quantity linking, so that $\alpha_1 = \alpha_2 = 1$ and $Q_1 = Q_2 = 1$[4].

If there were no uncertainty about the costs of abatement, Country 1 could choose a tax per unit of emissions that results in the required amount of abatement Q_1. In this case, regulation by prices and regulation by quantities would be equivalent and either instrument could be used to generate abatement $Q_1 + Q_2$ at minimum cost.

Figure 1 shows the results from applying the model for ex-ante identical countries for: firstly, the case of Country 1 chooses regulation by prices when there is no uncertainty; secondly, Country 1 chooses regulation by prices under correlated and independent abatement costs, assuming a scale parameter $\sigma = 0.4$ ('price-quantity linking'); and thirdly, the case when both countries choose quantity regulation ('quantity linking'). Table 1 shows expected costs for Country 1, and expected total costs (for both countries), in terms of the scale parameter σ. It also shows the optimal tax rate for Country 1, the tax rate that will minimise the expected cost.

Uncertainty leads to higher expected costs for the price-setting country, and lower expected costs for the quantity-setting country. As uncertainty in abatement costs (and therefore σ) rises, expected costs for Country 1 increase under regulation by prices; expected costs for Country 2 decrease; and expected total costs for both countries increase, but not by as much. These results are intuitive because when the price-setting country trades allowances with the quantity-setting country, rents are transferred to the quantity-setting country who receives the permit price for each tonne of abatement, which will be greater than or equal to the incremental abatement costs when the quantity-setting country sells abatement, and less than or equal to abatement costs when the quantity-setting country buys abatement. Total costs are expected to be higher because of inefficiencies that arise when marginal abatement costs are not equalised between countries.

More uncertainty also leads to higher costs for Country 1 when θ_1 and θ_2 are correlated, compared to when they are independent. The ex-ante optimal tax rate decreases as uncertainty increases. The optimal ex-ante tax rate is slightly larger when abatement costs are correlated

[4] When countries are ex-ante identical, total (ex-post) costs for Country 1 are given by

$$C_1^{[Q_2 \geq q_1 - Q_1]} = \frac{t^2}{2\theta_1} + 2\theta_2 + \frac{t^2\theta_2}{\theta_1^2} - \frac{3t\theta_2}{\theta_1}$$

when $0 \leq t \leq 2\theta_1$; and

$$C_1^{[Q_2 \leq q_1 - Q_1]} = \frac{t^2}{2\theta_1}$$

when $2\theta_1 \leq t$. For Country 2,

$$C_1^{[Q_2 \geq q_1 - Q_1]} = -\frac{t^2\theta_2}{2\theta_1^2} + \frac{t\theta_2}{\theta_1}$$

when $0 \leq t \leq 2\theta_1$, and 0 when $2\theta_1 \leq t$.

compared to when they are independent, but this tax rate only differs by about 3 percent.

Under quantity-quantity linking, when abatement costs are correlated, expected costs are unchanged, but when abatement costs are independent, expected costs actually decrease. For both price-quantity linking and quantity-quantity linking, it is to be expected that there will be positive welfare impacts from abatement costs being independent, compared to being correlated, because 'gains from trade' arise from differences between the two countries.

The situation that has just been examined involved each country taking on an equivalent target. In this situation, price-quantity linking leads to an implicit welfare transfer from the price-setting country to the quantity-setting country because the quantity-setting countries's expected costs are lower compared to quantity-quantity linking. However, the following example illustrates a situation where a country can benefit from choosing prices.

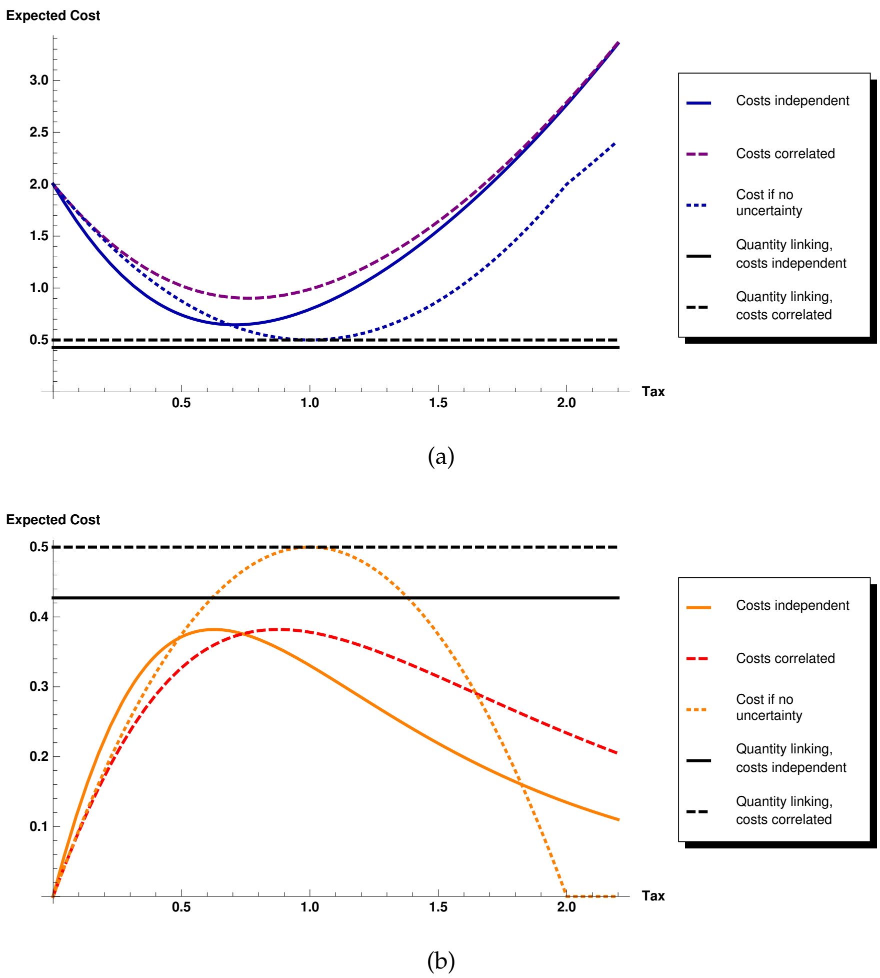


Figure 1: Expected costs for the 'price-setting country', Country 1, (a); and the 'quantity-setting country', Country 2, (b) under price-quantity linking and quantity-quantity linking with the degree of uncertainty corresponding to scale parameter $\sigma = 0.4$ (for both countries); and for comparison, when there is no uncertainty.

Suppose that Country 2 takes on a much more ambitious target than Country 1, but countries are similar otherwise (i.e. have the same ex-ante abatement costs), so that $\alpha_1 = \alpha_2 = 1$, $Q_1 = 1$, and $Q_2 = 10$. For the case of uncertain abatement costs and independent error terms

θ_i, Country 1 could benefit from choosing prices. Figure 2 shows expected costs for each country, and demonstrates that Country 1's expected costs from choosing prices are below costs under quantity-quantity linking. Thus, there are situations where Country 1 could have a strategic interest to choose price regulation to minimise its own expected costs while social costs would be minimised when both countries choose quantity regulations. While in most situations, the choice of price regulation by Country 1 leads to additional private costs for the country, the choice of price regulation by Country 1 leads to an increase in social costs in this special case. Thus, in some cases when abatement targets differ strongly, Country 1 could have the option to free-ride on Country 2 and domestic policy choice of Country 1 could have a negative impact on social welfare. This example will be discussed further in Section 3.3.

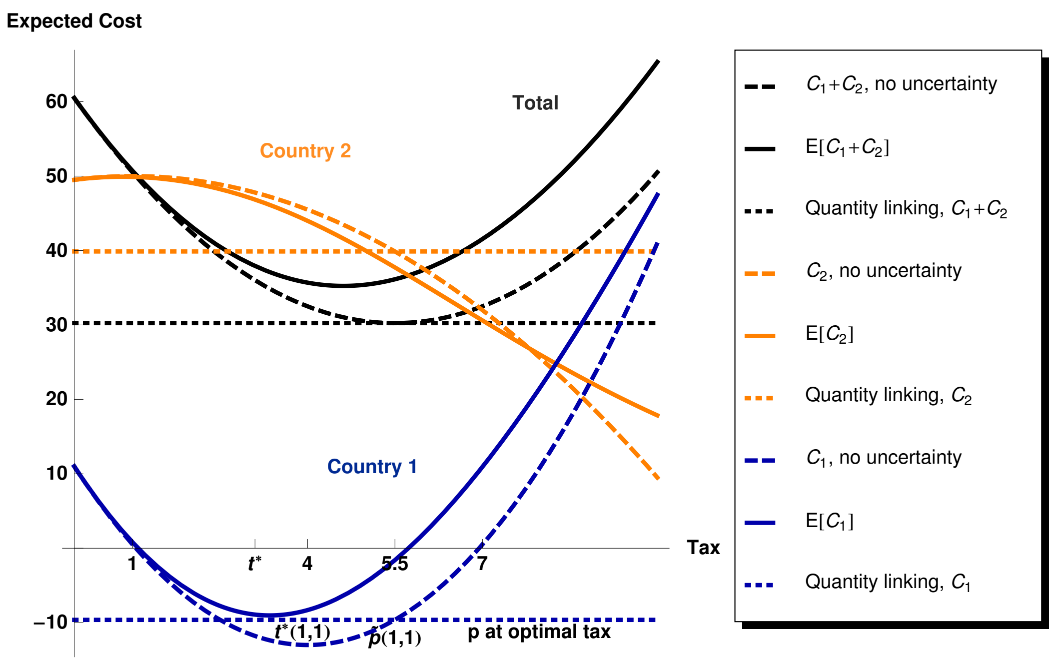


Figure 2: Expected costs for both countries (solid lines) when Country 2 takes on an ambitious abatement target ($Q_1 = 1, Q_2 = 10$). It is assumed that θ_1 and θ_2 are independent, and have log-normal distributions, with each country having a scale parameter $\sigma = 0.2$. The horizontal dotted line shows the expected total cost (for the same probability distribution) for quantity-quantity linking. The optimal tax rate in the case that there is no uncertainty ($t^*(1,1) = 4$), the price that allowances would trade at when there is no uncertainty ($\tilde{p}(1,1) = 5.5$), and the price that allowances would trade at at the optimal tax rate ($p = 7$) are also shown.

3.2 Countries of Different Size

This section investigates how the relative size of countries affects the welfare consequences of price-quantity linking. Suppose that the size of Country 2 is S times the size of Country 1, and both countries take on corresponding targets so that $Q_2 = SQ_1$, then countries have the same ex-ante marginal abatement costs (after adjusting for size) if $\alpha_2 = \alpha_1/S$. It is assumed that countries are ex-ante identical except for relative size S, and that parameters for Country 1 are normalised so that $\alpha_1 = 1$ and $Q_1 = 1$, then $\alpha_2 = 1/S$ and $Q_2 = S$. It is also assumed that the tax rate is chosen to be ex-ante optimal so that it minimises expected costs for Country 1.

Tables 2 and 3 show expected costs for Country 1 under both price-quantity linking and quantity-quantity linking, and the optimal tax rate for Country 1, with Table 2 corresponding to costs being independent; and Table 3 corresponding to costs being perfectly correlated. Figure 3 illustrates the cost impacts for different sized countries when abatement costs are independent. If the country that chooses prices is smaller than the country that chooses quantities (so that $S > 1$), expected welfare losses compared to quantity-quantity linking are lower than in the case of equal-sized countries ($S = 1$).

If Country 1 sets a tax and is large relative to Country 2, expected additional costs compared to quantity-quantity linking are considerable. This is to be expected, because if Country 2 is small, it will have less capacity to take up additional abatement. For the case when Country 1 is four times larger than Country 2 ($S = 0.25$), expected costs are about 1.7 times larger under price-quantity linking compared to quantity-quantity linking for $\sigma = 0.2$ and independent θ. As Country 1 becomes smaller, expected additional costs under prices decrease. For the case of Country 1 being four times smaller than Country 2 ($S = 4$), expected additional costs under price-quantity linking decrease to 1.17 times relative to quantity-quantity linking for $\sigma = 0.2$ and independent θ.

3.3 The Choice of Tax Rate

The optimal (ex-ante) tax rate will minimise the expected cost for the price-setting country, $E(C_1)$. Before examining the impact of uncertainty on the expected cost, it will be useful to calculate the optimal tax rate when there is no uncertainty. It will be shown that the optimal choice of tax rate is not necessarily the same as the price required for Country 1 to produce a given quantity of abatement; not necessarily the same as the price that allowances trade for at the optimal tax rate; and not necessarily the same as the price that allowances would trade at under quantity-quantity linking. This arises even when there is no uncertainty, and follows from these values each being the result of different optimisation problems.

Consider $C_1(\theta_1, \theta_2)$, the ex-post cost for Country 1. When there is no uncertainty, $E(C_1) = C_1(1, 1)$. Now if $Q_2 \geq q_1 + Q_1$, it follows from (4) that

$$C_1(\theta_1, \theta_2) = \frac{t^2}{2\alpha_1\theta_1} - \alpha_2 \left(Q_1 + Q_2 - \frac{t}{\alpha_1\theta_1} \right) \left(-Q_1 + \frac{t}{\alpha_1\theta_1} \right) \theta_2. \tag{17}$$

Thus, the optimal ex-post tax rate, $t^*(\theta_1, \theta_2)$ is derived by differentiating (17) and solving for t. It is given by

$$t^*(\theta_1, \theta_2) = \frac{(2Q_1 + Q_2)\,\alpha_1\alpha_2\theta_1\theta_2}{\alpha_1\theta_1 + 2\alpha_2\theta_2}. \tag{18}$$

In particular, when there is no uncertainty, the optimal tax rate is

$$t^*(1, 1) = \frac{(2Q_1 + Q_2)\,\alpha_1\alpha_2}{\alpha_1 + 2\alpha_2}. \tag{19}$$

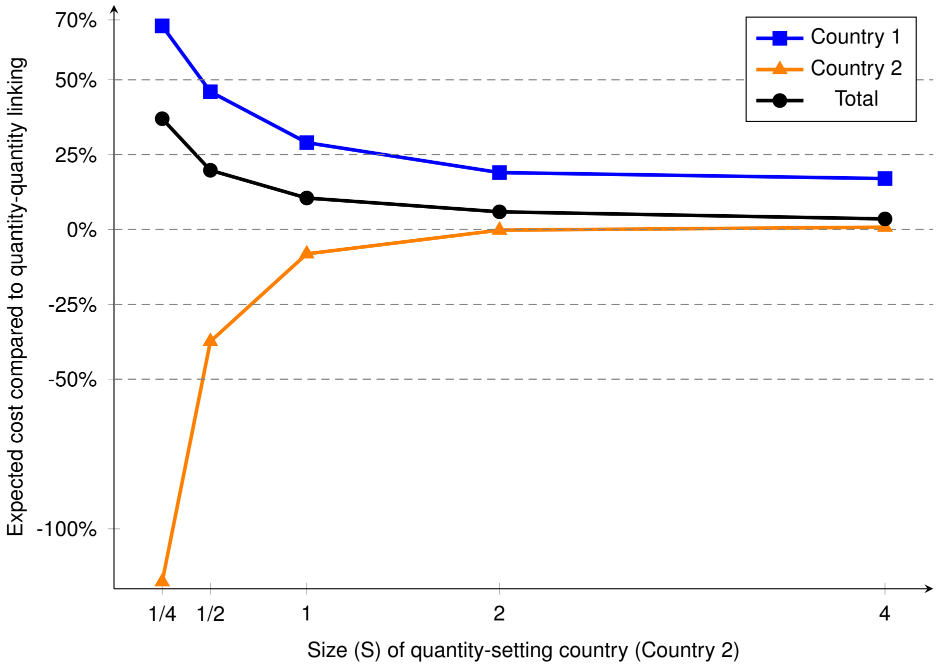


Figure 3: Expected costs for Country 1 under price-quantity linking compared to quantity-quantity linking for different-sized countries when costs for each country are independent and the degree of uncertainty corresponds to a scale parameter $\sigma = 0.2$.

The optimal choice of tax rate is therefore not necessarily the same as the price required for Country 1 to produce the quantity Q_1 of abatement, which is given by $c_1(q_1, \theta_1)$. Furthermore, the price that allowances trade for at the optimal tax rate could be different to the tax rate.

The ex-post price that allowances would trade at under quantity-quantity linking can be obtained by solving (6) and is given by

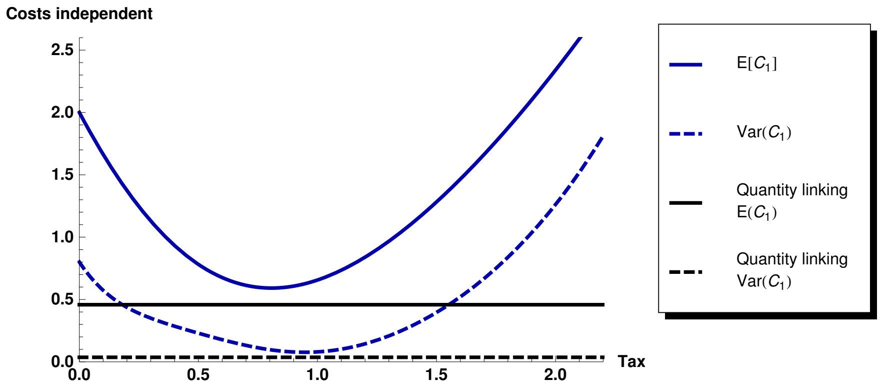
$$\tilde{p}(\theta_1, \theta_2) = \frac{(Q_1 + Q_2)\,\alpha_1\alpha_2\theta_1\theta_2}{\alpha_1\theta_1 + \alpha_2\theta_2}. \tag{20}$$

In particular, when there is no uncertainty, the price that allowances will be trade will be

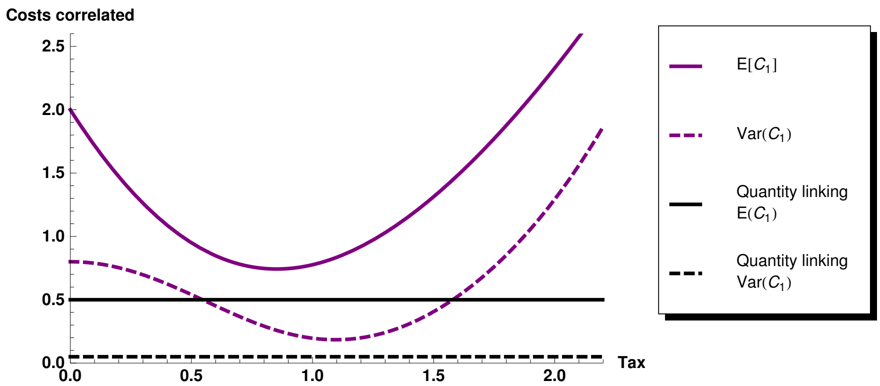
$$\tilde{p}(1,1) = \frac{(Q_1 + Q_2)\,\alpha_1\alpha_2}{\alpha_1 + \alpha_2}, \tag{21}$$

which is different to $t^*(1,1)$. The optimal choice of tax rate is therefore not necessarily the same as the price that allowances would trade at under quantity-quantity linking. Figure 2 illustrates a situation where the optimal choice of tax rate differs from both the price that allowances would trade at under quantity-quantity linking, and the price that allowances would trade at under price-quantity linking.

Tables 1, 2, and 3 show the impact of uncertainty on the optimal ex-ante tax rate. For all of the examples described in these tables (equally-sized countries, different-sized countries, abatement costs independent, and abatement costs correlated) greater levels of uncertainty in abatement costs (as quantified using the scale parameter σ) lead to lower optimal tax rates. When abatement costs are independent, equally-sized countries have the greatest reduction in tax rate; while when abatement costs are perfectly correlated, the greatest reduction in the optimal tax rate occurs when the quantity-setting country is large.



(a)



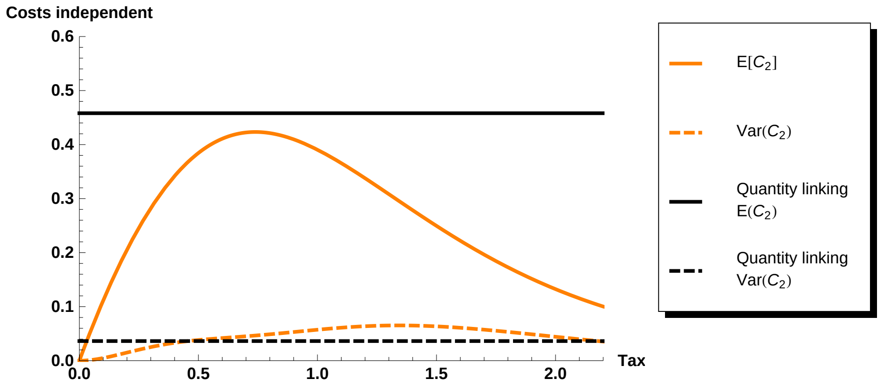
(b)

Figure 4: The expected cost and variance of cost for Country 1 under price-quantity linking and quantity-quantity linking, when countries are ex-ante identical ($\alpha_1 = \alpha_2$) and abatement costs are uncertain (with scale parameter given by $\sigma = 0.2$). (a) shows results when costs are independent; and (b) shows results when costs are perfectly correlated (so that $\theta_1 = \theta_2$).

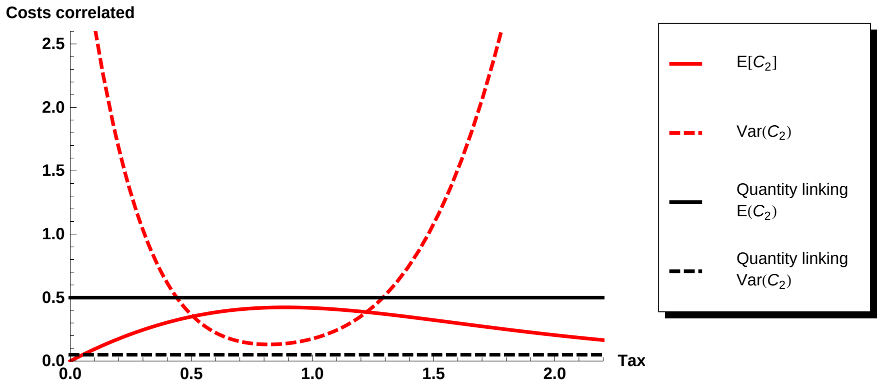
3.4 Variance of Costs under Price-Quantity Linking

A possible motivation for choosing prices rather than quantities, and in particular choosing a carbon tax, could be to contain costs by fixing the cost per unit of emissions for firms that are liable under the carbon tax. However, under price-quantity linking, the costs for the price-setting country are also affected by trade in allowances. This issue was investigated by numerically integrating equation (16).

Figure 4 shows the expected abatement cost, and the variance in abatement cost, for the price-setting country when there are equally-sized countries. The variance in cost, under



(a)



(b)

Figure 5: The expected cost and variance of cost for Country 2 under price-quantity linking and quantity-quantity linking, when countries are ex-ante identical ($\alpha_1 = \alpha_2$) and abatement costs are uncertain (with scale parameter given by $\sigma = 0.2$). (a) shows results when costs are independent; and (b) shows results when costs are perfectly correlated (so that $\theta_1 = \theta_2$).

price-quantity linking, is much higher than under quantity-quantity linking. So while setting a price will contain costs for liable firms, it will increase the uncertainty in the total costs for the price-setting country, including the cost of allowance purchases.

The tax rate that would minimise the variance of costs for Country 2 is in general above the tax rate that would minimise expected costs. As the scale parameter σ increases, the deviation of the tax rate that minimises expected costs and variance increases. For the case of perfectly correlated costs, the deviation is larger compared to the case of independent costs.

Figure 5 shows the expected abatement cost, and the variance in abatement cost, for the quantity-setting country when there are equally-sized countries. Under price-quantity linking the quantity-setting country stands to benefit in terms of expected net costs; but if costs are highly correlated between countries, the quantity-setting country will experience significantly greater variance in costs.

4 Discussion and Conclusion

This paper analysed the linking of price and quantity regulation under an exogenous quantity constraint by numerically solving a simple two country model with uncertain abatement costs. In most settings, uncertainty will lead to an expected welfare loss for the price-setting country and an expected welfare gain for the quantity-setting country. Overall, there will be an expected total welfare loss for both countries. Large countries in particular are more likely to have a greater welfare loss from setting a price instead of choosing to use a quantity-based mechanism. If a large country chooses prices, corner solutions, where the price that allowances trade at becomes equal to zero, are more likely.

However, as shown by the example in Section 3.3 there are situations where a country will be better off by choosing a price-based mechanism. In this example, the other country has a much more stringent target, and the probability distributions that determine actual costs for each country are independent.

The ex-ante optimal tax rate for the price-setting country will not necessarily be equal to the expected price that permits will trade at, or the expected marginal cost of abatement to meet the exogenous target in the quantity setting country. Greater levels of uncertainty will generally lead to a lower ex-ante optimal tax rate.

While the domestic price per unit of emissions is fixed if a country chooses a tax, total costs for meeting the exogenous target Q_i are still subject to uncertainty. Thus, the choice of price regulation could decrease cost uncertainty of domestic private entities that would have to pay a fixed tax rate t per unit of emissions, but will increase the uncertainty of total control costs of the country. Price regulation would not effectively contain a countries *total* control costs, but may be desirable to some emitters within a country, if they pay the cost of their emissions but do not pay the costs arising from international emissions trading.

In summary, this model allowed the issue of cost-correlation to be investigated in this context, and the choice of tax rate to be investigated. The results are relevant to domestic policy choices regarding the regulation of greenhouse gases; and to the design of international environmental agreements.

The results highlight the importance of international decisions for domestic policies. In the case discussed here, the design of the environmental standard also defines optimal (i.e. cost minimising) policy choice for a country. The design of an international agreement 'trickles down' and influences optimal policy choice for countries that participate in the agreement. It therefore may be better for international agreements to be flexible enough to take account of price-based approaches. However, the question of how to design an international agreement is a complex one that will need to take into account a much broader range of issues than the welfare losses that arise from countries choosing a different mechanism to that used by the agreement.

Policy decisions about whether to use prices or quantities depend on additional issues to those covered by the model presented here. Our results depend on the assumption that the overall quantity target is binding and that there is full compliance. This is equivalent to there being a very high penalty for either country not meeting their target. This assumption is not necessarily realistic in practice, since international agreements cannot impose strong penalties on sovereign countries, i.e. when sovereign countries are unwilling to sign up to such a treaty in the first place (Nordhaus, 2006). If there is instead a relatively low penalty for not complying with a quantity based target, price-based mechanisms and hybrid mechanisms could become more attractive. Montero (2002) has shown that quantity regulation can be preferable over prices for the case of incomplete enforcement when the costs and benefits of regulation are taken into account. However, if there is a single social planner, prices, or hybrid price-quantity approaches, are considered as a social optimal policy for greenhouse gas control (Nordhaus, 1994; Kolstad, 1996; Hoel & Karp, 2002; Newell & Pizer, 2003; Pezzey & Jotzo, 2012).

The model also assumes that any positive level of domestic abatement is possible. While this is not realistic in practice, it is necessary to be consistent with there being a very high penalty for non-compliance. But it is possible for countries to sequester greenhouse gas emissions, for example by changing land use, so the assumption is not completely unrealistic.

An additional issue not covered by the model is that uncertainty in permit prices can affect decision making about whether to make irreversible abatement investments (such as capital investments in renewable energy) or reversible abatement decisions (such as fuel switching). Zhao (2003) examined this issue using a general equilibrium model and found that incentives for investment in abatement decrease with increasing cost uncertainty, but that uncertainty would lead to greater reductions in investment under price-based mechanisms. Krysiak (2008) examined how instrument choice affects technology choices and found that price-based mechanisms lead to suboptimal technology choices compared to quantities. It should also be noted that both instruments are able to foster innovation in an efficient manner (Downing & White, 1986; Milliman & Prince, 1989; Jung *et al.* , 1996), with innovation incentives being greater when when allowances are auctioned rather than freely allocated (Milliman & Prince, 1989; Jung *et al.* , 1996; Requate, 2005).

The choice of mechanism also depends on factors such as institutional settings, preferences of governments and key stakeholders, and political feasibility (Barthold, 1994; Buchanan & Tullock, 1975; Hepburn, 2006; Howe, 1994). The welfare implications for a single country will depend on the design of individual policies and costs that are revealed ex-post. Goulder *et al.* (1999) has shown that pre-existing taxes can lead to significantly higher abatement costs and situations where prices and quantities are no more cost efficient, depending on the extent of pollution abatement.

We expect that many of the conclusions of this paper will generalise to the situation where there are more than two countries. This is because under the approach taken here, the situation where there are multiple countries that choose quantities and link with each other is equivalent to the situation where a single aggregate country chooses quantities.

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Appendix: Tables

Results for Country 1	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
Abatement costs are independent					
Optimal tax rate	1	0.88	0.81	0.75	0.71
Expected cost when using prices (at optimal tax rate)	0.5	0.55	0.59	0.62	0.65
Expected cost for quantity-quantity linking	0.5	0.48	0.46	0.44	0.42
Additional expected cost from using prices (as percentage of cost from using quantities)	0%	15%	29%	41%	51%
Abatement costs are correlated					
Optimal tax rate	1	0.91	0.83	0.77	0.72
Expected cost when using prices (at optimal tax rate)	0.5	0.64	0.74	0.83	0.90
Expected cost for quantity-quantity linking	0.5	0.5	0.5	0.5	0.5
Additional expected cost from using prices (as percentage of cost from using quantities)	0%	27%	49%	66%	81%

Total costs for both countries	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
Abatement costs are independent					
Expected cost when using prices (at optimal tax rate)	1	1.005	1.012	1.019	1.025
Expected cost for quantity-quantity linking	1	0.95	0.93	0.92	0.90
Additional expected cost from using prices (as percentage of cost from using quantities)	0%	5%	8%	11%	14%
Abatement costs are correlated					
Expected cost when using prices (at optimal tax rate)	1	1.09	1.16	1.23	1.28
Expected cost for quantity-quantity linking	1	1	1	1	1
Additional expected cost from using prices (as percentage of cost from using quantities)	0%	9%	17%	23%	28%

Table 1: **Expected abatement costs when countries are ex-ante identical**

a) Optimal tax rate

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	1	0.935	0.872	0.815	0.765
2	1	0.905	0.825	0.761	0.711
1	1	0.883	0.807	0.751	0.708
0.5	1	0.896	0.841	0.799	0.762
0.25	1	0.951	0.922	0.891	0.861

b) Country 1 expected cost

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0.5	0.479	0.465	0.457	0.452
2	0.5	0.504	0.515	0.526	0.537
1	0.5	0.551	0.592	0.622	0.646
0.5	0.5	0.622	0.695	0.745	0.784
0.25	0.5	0.713	0.824	0.903	0.966

c) Country 1 expected cost with quantity-quantity linking

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0.5	0.443	0.397	0.360	0.329
2	0.5	0.461	0.431	0.407	0.386
1	0.5	0.477	0.458	0.442	0.427
0.5	0.5	0.489	0.477	0.466	0.456
0.25	0.5	0.495	0.489	0.483	0.476

d) Additional expected cost from using prices for Country 1

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0	0.0361	0.0679	0.0968	0.123
2	0	0.0432	0.0838	0.120	0.151
1	0	0.0735	0.134	0.181	0.219
0.5	0	0.134	0.218	0.279	0.328
0.25	0	0.218	0.335	0.421	0.489

e) Additional expected cost from using prices for Country 1
(% of cost from using quantities)

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0%	8%	17%	27%	37%
2	0%	9%	19%	29%	39%
1	0%	15%	29%	41%	51%
0.5	0%	27%	46%	60%	72%
0.25	0%	44%	68%	87%	103%

Table 2: **Abatement costs are independent**

a) Optimal tax rate

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	1	0.909	0.833	0.769	0.715
2	1	0.909	0.835	0.774	0.724
1	1	0.913	0.852	0.804	0.762
0.5	1	0.940	0.909	0.879	0.849
0.25	1	1.007	1.009	0.999	0.983

b) Country 1 expected cost

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0.5	0.568	0.625	0.673	0.714
2	0.5	0.591	0.666	0.729	0.782
1	0.5	0.635	0.743	0.830	0.902
0.5	0.5	0.711	0.863	0.986	1.091
0.25	0.5	0.814	1.028	1.208	1.367

c) Country 1 expected cost with quantity-quantity linking

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0.5	0.5	0.5	0.5	0.5
2	0.5	0.5	0.5	0.5	0.5
1	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5
0.25	0.5	0.5	0.5	0.5	0.5

d) Additional expected cost from using prices for Country 1

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0	0.068	0.125	0.173	0.214
2	0	0.091	0.166	0.229	0.282
1	0	0.135	0.243	0.330	0.403
0.5	0	0.211	0.363	0.486	0.591
0.25	0	0.314	0.528	0.708	0.867

e) Additional expected cost from using prices for Country 1
(% of cost from using quantities)

Size Country 2	Scale factor (σ)				
	0	0.1	0.2	0.3	0.4
4	0%	14%	25%	35%	43%
2	0%	18%	33%	46%	57%
1	0%	27%	49%	66%	81%
0.5	0%	42%	73%	97%	118%
0.25	0%	63%	106%	142%	173%

Table 3: **Abatement costs are perfectly correlated**