Growth, Skill-Mismatch and Search Unemployment*

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Abstract

The paper examines the impact of growth generated skill-mismatch on equilibrium unemployment. We discuss the empirically observed shift of the Beveridge curve in the theoretical framework of the Mortensen & Pissarides (1998) vintage matching model. With non-renovable job matches, the introduction of skill-mismatch reverses the results of previous models of this type for a range of relevant parameter values. Growth reduces the matching rate and reduces the growth rate of the outside option of a worker. This reduces wage growth and increases the profitability and the duration of a match. Consequently growth leads to longer job duration and lower equilibrium unemployment.

JEL code: J24, J31 and J42

Keywords: Technical progress, skills and matching

1 Introduction

Empirical literature has pointed out a considerable increase in the level of unemployment for given vacancies since the early 1970's in most European economies. An apparent outward shift of the relationship between unemployment and vacancies, represented by the Beveridge curve (U/V curve), has been observed (e.g. Layard et al.1991, Blanchard and Diamond 1989). The empirical studies also point to the observation that whereas the shift of the Beveridge curve has been a continuing phenomenon in Europe during the last decades, there has lately been a shifting back inwards in the United States.

Proposed explanations to this empirical evidence relate to search and matching in the labor market and frictions involved in the matching process[1]. Attempts to explain the shift of the Beveridge curve have discussed a fall in the effectiveness or intensity of search of the unemployed or the firms (e.g. Pissarides 2000). The intensity of search is endogenously determined by the agents optimizing decisions, and include everything the agents do during the search process. Another approach is to consider the possibility of increased mismatch between the unemployed workers and vacancies posted. Increased mismatch is unrelated to individual search decisions. The shift of the Beveridge curve is considered to be caused by factors unrelated to the search process such as technological progress or structural shocks (e.g. Lilien 1982). Mismatch in these studies is related to a skill-bias between the skills required by employers and the skills

*The paper is work in progress. All comments and suggestions are welcome.

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[1]See e.g. Bean (1994) for survey of theoretical discussion.

possessed by workers. Mismatch as an explanation for the shift of the Beveridge curve has been criticized and said to explain only little of the shift. Overall, the literature finds no single variable or combination of variables accounting for the fall in the matching rate during the past decades. The deterioration of the matching rate is attributed to unmeasured elements of the unemployment insurance system and mismatch (Petrongolo and Pissarides 2000).

This paper discusses growth and technological progress[2] as a factor determining the shifting of the Beveridge curve. We attempt to provide a theoretical framework fitting the empirical observations on the Beveridge curve. We introduce growth generated skill-mismatch into the Mortensen and Pissarides (1998) and Pissarides (2000) vintage matching framework and study it's impact on the labor market. When job matches can not be updated, introducing growth generated mismatch reverses the outcome of these models for a range of relevant parameter values.

In Mortensen and Pissarides (1998), Aghion and Howitt (1994, 1998) and Pissarides (2000) the technological frontier and productivity of new jobs increase in time, but the technical skills and productivity of a worker in a match remain constant. The competing new jobs pay higher wages due to their higher productivity. A feature of these models is the assumption of perfect mobility. This means that workers are assumed to able to change jobs to more recent technology vintages, irrespective of the fact that their skills represent an older, already obsolete technology vintage.When this is the case, the new more productive jobs raise the outside option of a worker and raise the wage in existing matches as well, eventually over their productivity. This leads to zero profits and to the destruction of the match. The models show growth and the wage growth implied by it to lead to shorter job duration and to produce a higher rate of equilibrium unemployment when existing matches cannot be updated.

In this study we introduce imperfect mobility between different vintages by skill mismatch. When growth generated skill-mismatch is introduced into the model, faster technical progress can reduce the growth rate of the outside option of a worker instead of increasing it. This occurs because a faster rate of technical progress increases the growth in the difference between the technological frontier and the constant technical skills of a worker of a previous vintage. This affects the mobility of a worker between jobs, as new jobs require skills that the workers in old matches do not have. The consequently reduced growth rate of the outside option leads the optimal value of a job and labor market tightness to increase with growth, implying a lower rate of job creation and destruction, longer job duration and less reshuffling of jobs (fig. 1). When the increase in job duration is high enough, growth leads to lower equilibrium unemployment. This is the opposite to what happens in previous studies.

The different components of the outside option of the worker play a conclusive role in determining whether growth reduces or increases equilibrium unemployment.[3] The condition derived shows that the training investments of the firm must be sufficiently high (low) relative to the unemployment benefit and search costs for a higher rate of technological progress to reduce (increase) equilibrium unemployment. In this paper we do not study the case presented

[2]Technical progress and growth will be used as close synonymes in this paper, as growth and productivity increases are assumed to depend on technical progress.

[3]In the case of match updating, Mortensen and Pissarides (1998) show that matches with a high idiosyncratic component are updated. They take the idiosyncratic or match specific component of a job-worker pair as exogenously given. In this study, growth produces specificity to the matches and we get the positive impact endogenously. Mortensen and Pissarides (1998) show that the set of matches for which updating is optimal decreases with growth. In our framework growth would probably increase this set. At this stage we study only the case of no renovation. The renovation case will be considered in a following paper.

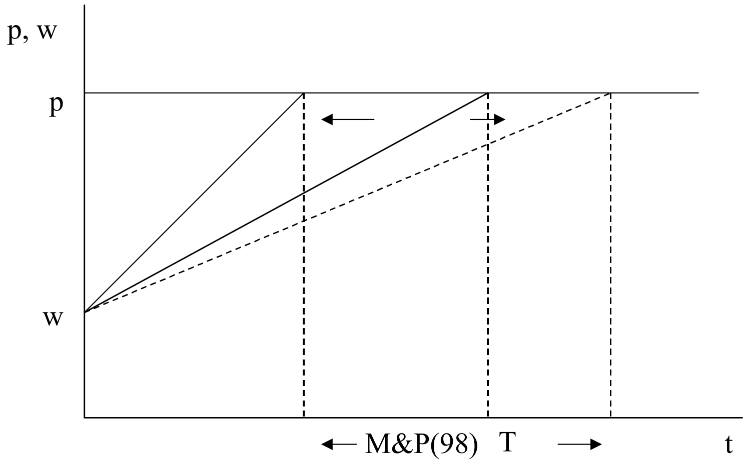

Figure 1: Match duration and growth

in Mortensen and Pissarides (1998) where it is possible to update existing matches. This case is important and it will be studied in future work.

This paper contributes to the matching literature by considering growth as a source of mismatch in a vintage model. Introducing the mismatch parameter makes it possible to take account of the skill-bias and its' implications in labor markets. The model combines the vintage models studying growth and unemployment to the mismatch literature and the discussion on the Beveridge curve. This paper attempts to contribute to the discussion on the theoretical explanation of the empirically observed shift of the Beveridge curve.

The structure of the paper is the following. Section 2 presents the model. First we provide a brief discussion on the role of skill mismatch and the introduction of a skill parameter into the matching function. Then we study the firm's and worker's optimizing behavior and thus job creation and wage determination. Section 3 studies the equilibrium solution to the model. Of particular interest is the impact on equilibrium unemployment of the different components determining the outside option of the worker. Section 4 concludes the paper and presents directions for future research. An appendix on training and search investments is presented at the end of the paper.

2 The Model

The structure of the model follows the Mortensen and Pissarides (1998) vintage matching model.[4] New technology is introduced into the economy through the creation of new job matches. The productivity of a match is constant and it is determined by the cutting-edge technology adopted at the date of creation of the match. Once the technology has been chosen, it can not be altered. The rate of technical progress or growth is $p(t) = e^{gt}$, where t is time. Technology is assumed to be embodied in new capital equipment, which means that new technology benefits only the productivity of jobs that invest in new technology. Because technology is embodied and there are no spillovers, new more productive technology doesn't raise

[4] A similar framework has been used to study the impact of technological progress and growth on equilibrium unemployment by Aghion and Howitt (1994, 1998), see also Pissarides (2000).

the productivity of existing matches. Newly created jobs reduce the relative productivity of job matches of previous vintages. The economy thus consists of job matches of different vintages, created at different time periods, each vintage representing the level of technology of their date of creation.

The wage increases throughout the duration of the match. Technical progress introduces new more productive vacancies into the labor market, and these jobs pay higher wages. These competing jobs increase the outside option of workers, leading to wage growth in existing matches of previous vintages.

The surplus produced by the match is shared in fixed proportions between the firm and the worker. While the productivity of a match is constant and wages are rising, eventually technological progress and the wage bidding process drive the product of the match to zero. At some point of time it becomes optimal to destroy the existing match and create a new match using new technology. Firms invest in more productive technology and employ a worker to form a new match. The Schumpeterian notion of "creative destruction" is used to describe this idea (Aghion and Howitt 1994, 1998)[5]. In addition job destruction arises due to exogenous shocks.

We will first discuss the matching process of firms and workers is affected by mismatch, then consider how wages are determined by the optimizing behavior and Nash bargaining of firms and workers. We then study the impact of growth and technical progress on job value and labor market tightness and how this affects the optimal duration of a job match. Finally we consider the determination of equilibrium unemployment.

2.1 The matching function

Each match is formed by a firm and a worker who meet through a random matching process. In most of the theoretical literature an aggregate matching function is assumed to exist and the general properties of it are assumed to be known, but the matching function itself has been left as a black box[6]. The search process is captured by an aggregate matching function

$$m = am\left(u, v\right),$$

where the matching rate m is assumed to be increasing in both of its arguments, $\partial m/\partial u > 0$ and $\partial m/\partial v > 0$, concave and usually homogeneous of degree one. a is a shift parameter characterizing the efficiency of the matching process. This general functional form gives the benefit of aggregating the matching frictions in a simple manner. The average probabilities that a vacancy gets filled and a worker finds a job during a period of unit length are respectively, $m\left(u, v\right)/v$ and $m\left(u, v\right)/u$.

Early aggregate studies converged to what has remained so far the most common specification in the applied literature, namely the Cobb-Douglas matching function with constant returns to scale $m\left(u, v\right) = au^{\alpha}v^{1-\alpha}$.[7] Attempts to provide microeconomic foundations for the stylized fact of the Cobb-Douglas matching function have been numerous in the literature. However the theoretical results have not been convincing enough to explain why this specification emerges

[5]Mortensen and Pissarides (1998) study also the case where updating of the technology of a job is possible. In this study we only consider the case of technology adoption through creative destruction, i.e. by destruction of obsolete matches and creation of new matches.

[6]For examples of matching function specifications see Pissarides & Petrongolo (2000), Manacorda & Petrongolo (1999), Acemoglu (1997).

[7]Blanchard and Diamond (1989) find empirical support for this type of a function with the coefficients in the range of 0.5-0.7, although they do not provide a specific matching process leading to the function.

and no model has reached universal support and thus no dominant microfoundation has risen from the literature[8]. We will adopt the Cobb-Douglas specification because of the empirical support and it's convenient properties and focus on the mismatch parameter.

The empirically observed outwards shift of the Beveridge curve (the u/v curve)[9] reflects a deterioration in the matching process of unemployed workers and posted vacancies (fig. 2). That is, for a given number of vacancies, a higher number of unemployed are required to keep the matching rate at a given level.

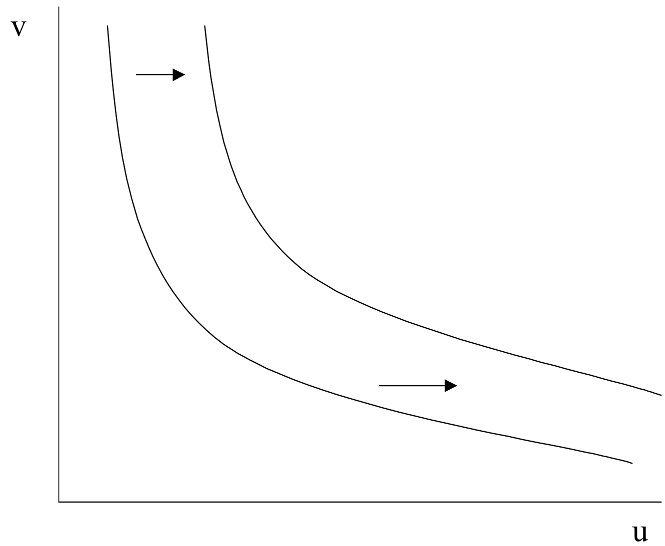

Figure 2: Beveridge curve

We consider the deteriorated matching to be determined by technical progress. Contrary to previous studies we assume imperfect labor mobility between jobs of different vintages. We will consider a shift parameter a in the matching function which accounts for the skills demanded by firms posting vacancies relative to the skills of unemployed workers. a is a measure of skill match. We want to characterize the impact of growth on the skill differential between offered and required skills, and its effect on the equilibrium outcome of the model. Differing from the models of endogenous intensity of search, the effectiveness of search in this paper is determined by growth.[10] The individual actions of agents do not affect the effectiveness of search.

Vacancies are always created at most advanced known technological level. Workers employed to these jobs will work with this technology from the date of creation τ until the date of destruction T of the match. Meanwhile, new more productive technologies enter the labor market at the rate $p(t) = e^{gt}$ and the relative technology level and productivity of the current match of the worker fall. When the job is eventually destroyed, the skill level of the unemployed

[8] See Petrongolo and Pissarides (2000) for a survey on the matching function and its microfoundations.

[9] The Beveridge curve reflects equilibrium unemployment, that is equality of job creation and destruction rates, for different degrees of labor market tightness. Labor market tightness is described by the number of vacancies v relative to that of the unemployed u, $\theta = \frac{v}{u}$.

[10] The growth rate is considered as exogenous in this study.

worker corresponds to an obsolete technology that is $\tau+T$ periods old, while vacancies are posted at the technological cutting edge. Thus technological progress produces a skill gap between the skills possessed by unemployed workers and those required by firms posting vacancies. We model the skill parameter as

$$a = e^{-gT}.$$

The measure of relevant skills a is negatively related to the rate of technical progress g and the duration T of a job match in the economy. For simplicity we assume all matches to have equally long duration. This specification states that the skill gap grows at the same rate as technical progress, which implies that the skills of workers are assumed not to develop while being employed at some specific match. With the Cobb-Douglas specification for the matching function we obtain the augmented matching function

$$m = am\left(u, v\right) = e^{-gT}u^{\alpha}v^{1-\alpha}. \tag{1}$$

For intuition, an analogy can be drawn from growth theory: technological progress is input-augmenting in the production function of a growth model, it is input-downgrading in the matching function. For given inputs (v, u) the matching function produces less output, that is matches. In unemployment-vacancy space this means a shift outwards of the Beveridge curve.

To simplify notation, throughout most of the analysis we will denote e^{-gT} by a and denote the matching function in general form in terms of labor market tightness. Following standard notation in the literature, the rate of vacancy filling for a firm is denoted $aq\left(\theta\right) = am\left(u, v\right)/v$ and the rate of moving from unemployment to employment for a worker is $a\theta q\left(\theta\right) = am\left(u, v\right)/u$.

2.2 Job Creation

The firms asset value equations for a job at time t of vintage τ and a vacancy at date t, with a worker an appropriate level of skills are given by

$$rJ\left(\tau, t\right) = p\left(\tau\right)x - w\left(\tau, t\right) - \lambda J\left(\tau, t\right) + \dot{J}\left(\tau, t\right) \tag{2}$$

$$rV\left(t\right) = -p\left(t\right)c + aq\left(\theta\right)\left[J\left(t, t\right) - V\left(t\right) - p\left(t\right)K\right] + \dot{V}\left(t\right). \tag{3}$$

Equation (2) states that the flow capital cost rJ of a filled job equals the real output of the match $p\left(\tau\right)x$ minus the wage $w\left(\tau, t\right)$ payed to the worker minus λJ, which is the risk of job destruction by an exogenous shock times the value of match surplus. The productivity of the match is constant and determined at the date of creation of the match. The cost of posting an open vacancy is $p\left(t\right)c$ and $aq(\theta)\left[J\left(t, t\right) - V\left(t\right)\right]$ is the probability of the vacancy changing into a filled job times the value of the status change. c is the cost involved in searching for a worker. $aq(\theta)p\left(t\right)K$ is the expected training cost for a realized match. The model assumes free entry which implies zero profits for posting vacancies $V = 0$. For simplicity only steady state solutions are considered so $\dot{J} = 0$ and $\dot{V} = 0$. Equation (3) states that the capital cost of a vacancy equals its return.

Substituting the assumptions concerning $\dot{J}$, $\dot{V}$ and V into (3) gives

$$\frac{c}{aq(\theta)} + K = \frac{J(t,t)}{p(t)}. \tag{4}$$

Equation (4) states that the sum of the expected cost of filling a vacant job and the training cost of the hired worker equals the value of a job match expressed in terms of productivity of the frontier technology. For an individual firm $1/aq(\theta)$ is the expected duration of a vacancy i.e. the duration of search.

2.3 Workers

The asset value equations for the worker are

$$rW(\tau,t) = w(\tau,t) - \lambda[W(\tau,t) - U(t)] + \dot{W}(\tau,t), \tag{5}$$

$$rU(t) = p(t)b + a\theta q(\theta)[W(\tau,t) - U(t)] + \dot{U}(t). \tag{6}$$

These equations can be interpreted in an analogous manner to equations (2) and (3). $U(t)$ and $W(\tau,t)$ denote the present-discounted value of the expected income stream of an employed and unemployed worker. The permanent income of employed workers rW equals the wage plus the expected capital gain $\lambda[W(\tau,t) - U(t)]$. So the permanent income includes the risk of unemployment (job destruction) λ times the value of unemployment. $p(t)b$ is a real income that an unemployed worker enjoys while being unemployed, e.g. unemployment benefit. This income is lost when the worker gets employed. $a\theta q(\theta)$ is the probability of an unemployed worker finding a job and $W(\tau,t) - U(t)$ is the value of the change in status. Thus $a\theta q(\theta)[W(\tau,t) - U(t,\delta)]$ is the expected capital gain from the change of state. The skill differential between a firm and a worker reduces the probability of an indivudual worker getting a job. This is because from the point of view of the firm, the worker has to be trained in any case, so it can choose anyone from the pool of unemployed. This has a negative effect on the position of the worker relative to the firm.

2.4 Wage determination

The net returns from a match to a firm and a worker are respectively, $J - V$ and $W - U$. The firm must give up V for J and the worker must give up U for W. As $V = 0$ the net return from a job match for the firm is equal to the value of creating a job, J. The wage w is determined from the generalized Nash bargaining solution. w maximizes the weighted product of the firm's and worker's net return from the match at every t

$$w(\tau,t) = \arg\max\,[J(\tau,t) - V(t)]^{1-\beta}[W(\tau,t) - U(t)]^{\beta}, \tag{7}$$

where β represents the workers share of the match surplus after the match is formed ($0 \leq \beta \leq 1$). In symmetric situations $\beta = 0.5$. Because of free entry the value of holding a job vacant for a firm in equilibrium is zero, $V(t) = 0$, we get the first-order maximization condition

$$W(\tau,t) - U(t) = \beta\left[J(\tau,t) + W(\tau,t) - U(t)\right]. \tag{8}$$

This implies that the value of a job match for a worker equals a constant share β of the total match surplus. By substituting from equations (2)-(8), the wage at time t on a job of vintage τ is given by

$$w(\tau,t) = \beta p(\tau)x + (1-\beta)p(t)\omega(\theta(t)) \tag{9}$$

where

$$\omega(\theta) = b + \frac{\beta}{1-\beta}\left[c\theta + a\theta q(\theta)K\right]. \tag{10}$$

Wages in (9) consist of two terms, one characterizing the match product and the other characterizing the workers outside option. $\beta p(\tau)x$ is the workers share of the match product. The productivity of a match is determined at the date of creation τ of the match and is fixed. This follows from the assumption that technology is embodied, so that higher productivity is embodied to the creation of new matches using new technology. There are thus no spillovers from new matches to old matches.

The second term $(1-\beta)p(t)\omega(\theta(t))$ characterizes the impact of the reservation wage or outside option $\omega(\theta(t))$ of the worker on the wage. As opposed to the productivity term, the outside option term increases with the technological frontier rate of productivity $p(t) = e^{gt}$[11]. Rewriting the second term by substituting $\omega(\theta)$ from (10) into the second term of (9) we obtain

$$w(\tau,t) = \beta p(\tau)x + (1-\beta)p(t)b + \beta p(t)\left[c\theta + a\theta q(\theta)K\right]. \tag{11}$$

This way of presenting the wage equation shows more clearly the way the outside option or reservation wage $\omega(\theta)$ affects the wage. The second term on the left hand side of equation (11) is the impact of the unemployment benefit. It is assumed to be rising with the frontier rate of growth of productivity $p(t)$. The same goes for the third term of the left hand side, which represents the share β appropriable by the worker of the firms job creation investments. This term can be interpreted as a reward to the worker for saving of job creation costs that the firm enjoys when a job is formed (Pissarides 2000). The job creation investments consist of search costs c and training costs K. Labor market tightness θ enters the wage equation through the bargaining power of the firm and worker. By the properties of the matching technology the elasticity of $\theta q(\theta)$ is positive. A higher θ means that jobs arrive to workers at a higher rate than workers arrive to vacant jobs, relative to a lower equilibrium. Both terms in the square brackets increase with θ. This implies a higher bargaining power for the worker which leads to a higher wage rate.

The skill parameter a, enters the wage equation as a multiplier of the firm's training investments K. The higher the rate of technical progress, the less relevant or appropriate are the skills possessed by workers relative to those required by firms posting vacancies, and the lower is a $(\partial a/\partial g < 0)$. A higher growth rate reduces wage growth because the bargaining strength of the worker is lower when she is less suitable for a job from the viewpoint of the firm.

[11] Note that in the productivity term we have $p(\tau)$ which is the productivity of a match determined at the date of creation and in the outside option term $p(t)$ which is the productivity at the current technological frontier.

The productivity of a match is fixed and determined by the date of match creation. However, due to the effects of technological progress through the outside option the wage rises throughout the lifetime of the match. Consequently, at some point the job will be destroyed because the profits of the match turn negative.

When skill-mismatch is included in the model, the impact of technical progress on wage growth becomes ambiguous. On the one hand the outside option and thus the wage increases with $p(t)$, but on the other hand a higher growth rate implies a larger skill gap, a lower a, which has a negative impact on the outside option and the wage.

3 Steady-state unemployment

Solving the equilibrium unemployment rate involves solving the optimal lifetime $T^0$of a match and labor market tightness θ^0. The firm chooses the lifetime of a job T^0 so that the value the job is maximized throughout the lifetime of the match. T^0 is such that the reservation wage to a worker equals the product of the match at the date of destruction of the match. The equilibrium solution to the model is given by a pair of job value and labor market tightness $\left(J^0, \theta^0\right)$ that solves conditions (given below) of job creation and job destruction for the firm. These conditions maximize the value of a job throughout the lifetime of a match. Having determined the optimal lifetime T^0 of a match and optimal labor market tightness θ^0, the equilibrium unemployment rate can be determined using job flows, that is the rates out of unemployment into employment and the rate of destruction of jobs.

We will analyze how growth affects the equilibrium values of J, θ and T and the consequences on equilibrium unemployment. We derive a condition under which job value and labor market tightness are positively affected by growth, $\partial J^0/\partial g > 0$ and $\partial \theta^0/\partial g > 0$. As a consequence the optimal lifetime of a job is also positively affected, $\partial T^0/\partial g > 0$. Due to these effects we have a condition under which a higher growth rate produces lower equilibrium unemployment.

To determine the optimal life T of a job, the firm maximizes the value of a job at all dates after creation $t \geq \tau$. Solving for $J(\tau, t)$ in equation (2) and integrating from the current time t to the time of destruction $\tau + T$ of a job we obtain

$$J(\tau, t) = \max_T \left\{ \int_t^{T+\tau} \left[p(\tau)\, x - w(\tau, s)\right] e^{-(r+\lambda)(s-t)} ds \right\}. \tag{12}$$

By substituting the wage equation (9) into this equation we get

$$J(\tau, t) = \max_T \left\{ (1-\beta) \int_t^{T+\tau} \left[p(\tau)\, x - p(s)\, \omega(\theta(s))\right] e^{-(r+\lambda)(s-t)} ds \right\}.$$

The value of a new job is proportional to productivity on the technology frontier, $J(t,t) = p(t)\, J = e^{gt} J$. When we consider a job that is created at current time s.t. $\tau = t$ and we further normalize $t = 0$ we have

$$J = \max_T \left\{ (1-\beta) \int_0^T \left[x - e^{gt}\omega(\theta)\right] e^{-(r+\lambda)t} dt \right\}. \tag{13}$$

From equation (4) above we get

$$\frac{c}{aq(\theta)} + K = J. \tag{14}$$

The equilibrium solution to the model is given by a pair of job value and market tightness (J^0, θ^0) that solves equations (13) and (14). These conditions will be referred to as the job destruction and job creation condition below. The optimal lifetime of a job is chosen by maximizing equation (13). The lifetime of a job match is chosen such that the shadow wage available to the worker is equal to the match product at the date of destruction. The job is destroyed at the point where its value drops to zero. Equation (13) is downward sloping in the job value - labor market tightness space. Equation (14) states that in equilibrium the expected profit from a new job is equal to the expected cost of hiring a worker. It is upward sloping in the job value - labor market tightness space (Fig. 3.).

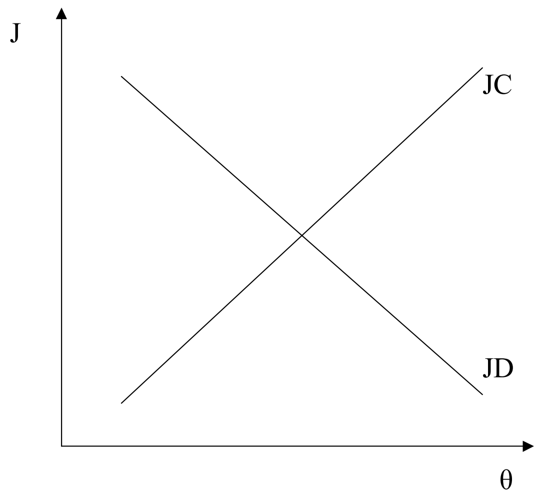

Figure 3: Job creation and destruction

3.1 Equilibrium job value and labor market tightness

When skill mismatch is introduced into the model, growth enters both the job creation and job destruction conditions. This is not the case in Mortensen and Pissarides (1998), where growth affects the equilibrium only through the job destruction condition by raising the outside option of the worker. When growth affects both the job creation and job destruction conditions, and when it affects the latter in a two-fold way, the impact of growth on the labor market becomes more involved.

3.1.1 Job creation and destruction

The job creation condition is influenced by the growth rate through a, the skill parameter. As was discussed above, the relevant skills of a worker decrease with the growth rate, so a higher growth rate increases the gap between the skills possessed by an unemployed worker and the skills demanded by the job. This decreases a, which reduces the matching rate and prolongs the expected duration of search. As the expected profit from a new job is defined to equal the creation costs, J^0 increases with growth. Intuitively this is to be considered as an increase of the required value of a match for it to be created. The value of a job required to cover the costs rises.

Equilibrium labor market tightness θ^0 is reduced by growth and the increased mismatch of skills (lower a). As the costs involved in creating a job rise, the incentives to create jobs decline. When the required value of a job to cover costs rises, fewer jobs fill this requirement and less vacancies are posted which reduces vacancies relative to the unemployed. However, if a job is created, it has a higher value due to the specific investments made in the match.

The impact of growth on job creation thus differs from Mortensen and Pissarides (1998) where, for given labor market tightness, the matching rate is independent of the growth rate and thus job creation costs are independent of growth. Instead of leaving the job creation curve immune to growth, the introduction of growth induced mismatch makes (14) shift up and to the left.

The impact of growth the job destruction condition (13) is more involved. There are several channels through which growth has an impact on job value and labor market tightness. Growth enters the job destruction condition through the outside option term of the worker which we rewrite here[12]

$$p(t)\,\omega(\theta) = e^{gt}\left(b + \frac{\beta}{1-\beta}\left[c\theta + a\theta q(\theta) K\right]\right).$$

The outside option depends on the growth rate through the growth factor e^{gt}. A higher growth rate $p(t) = e^{gt}$ accelerates the rise of wages in competing jobs. The general rise in wages raises the workers reservation wage and thus the wage in the current match. Faster increasing wages reduce the surplus of the match and thus reduce job value J^0. The profitability of job match decreases with wage rises and consequently labor market tightness θ^0 is reduced as well.

The growth rate affects the outside option and thus the profitability of the match also through the skill parameter $a = e^{-gT}$. The skill parameter features as a multiplier of the training investments of firms. Together with the matching rate $\theta q(\theta)$ and the workers bargaining power β the skill parameter a determines the fraction of the firms training investments that are appropriable by the worker. As growth reduces the relevant skills of a worker for a new job, $\partial a/\partial g < 0$, an increase in the growth rate reduces the chances of a worker finding a job once unemployed. This shown by a reduction in a. The reduced probability of finding a new job reduces the outside option of the worker and thus reduces the share of training costs that the worker can appropriate from the firm.

The impact of growth on the equilibrium values of θ and J is derived from job destruction (13) and job creation (14) equations:

$$\frac{\partial\theta^0}{\partial g} = -\frac{\partial JD/\partial g - \partial JC/\partial g}{\partial JD/\partial\theta - \partial JC/\partial\theta} \gtrless 0$$

$$\frac{\partial J^0}{\partial g} = \frac{\partial JC}{\partial\theta}\frac{\partial\theta^0}{\partial g} + \frac{\partial JC}{\partial g} \gtrless 0$$

[12] To ease notation some parameters have not been carried along throughout the analysis.

and we obtain

$$\frac{\partial \theta^0}{\partial g} = \frac{-\left(1-\beta\right)\omega\left(\theta\right)\mu - a'\beta\theta q\left(\theta\right)K\gamma - \left(-\frac{a'}{a}\right)\frac{c}{aq(\theta)}}{\beta\gamma\left[c + aK\left(q\left(\theta\right) + \theta q'\left(\theta\right)\right)\right] - \frac{c}{aq(\theta)}\frac{q'(\theta)}{q(\theta)}} \gtrless 0 \tag{15}$$

$$\frac{\partial J^0}{\partial g} = -\frac{c}{aq\left(\theta\right)}\frac{q'\left(\theta\right)}{q\left(\theta\right)}\frac{\partial \theta^0}{\partial g} + \left(-\frac{a'}{a}\right)\frac{c}{aq\left(\theta\right)} \gtrless 0 \tag{16}$$

where $\gamma = \int_0^T e^{t(g-r-\lambda)}dt = \left(\frac{1}{g-r-\lambda}\right)\left[e^{T(g-r-\lambda)} - 1\right]$ is the lifetime impact of an argument on job value. γ is also positive. $\mu = \frac{\partial}{\partial g}\int_0^T e^{t(g-r-\lambda)}dt = -\left(\frac{1}{(g-r-\lambda)^2}\right)\left[e^{T(g-r-\lambda)} - 1\right] + \left(\frac{1}{g-r-\lambda}\right)Te^{T(g-r-\lambda)}$ is the percentage change in the reservation wage throughout the lifetime of a match that arises through the growth factor e^{gt}. μ is also positive.

The sign of the effect of growth on the equilibrium θ and J is ambiguous because growth induces effects in opposite directions in the job destruction condition (13)[13]. Whether the impact of growth on job value in this case is positive or negative depends on the impact of growth on the outside option arising through the general growth factor $p\left(t\right) = e^{gt}$ and that arising through higher mismatch (lower a). This gives importance to the relative magnitudes of the different components of the outside option, K, c and b.

Determining the sign of these expressions reduces to determining the numerator in (15). When $\partial J^0/\partial g < 0$ and $\partial\theta^0/\partial g < 0$ the model produces qualitatively a similar outcome to Mortensen and Pissarides (1998) and is thus not worth further discussion in this paper. We will focus on the case where both $\partial J^0/\partial g > 0$ and $\partial\theta^0/\partial g > 0$. Using (10), $a = e^{-gT}$, $q\left(\theta\right) = \theta^{-\alpha}$and rearranging the numerator can be presented as

$$\left(T\gamma - \mu\right)e^{-gT}\beta\theta^{1-\alpha}K > \left(1-\beta\right)b\mu + \beta c\theta\mu + T\frac{c}{e^{-gT}\theta^{-\alpha}}. \tag{17}$$

To abstract from notation and to give a more intuitive picture of this expession we denote it as

$$\frac{\partial\Omega_K}{\partial g} > \left(\frac{\partial\Omega_b}{\partial g} + \frac{\partial\Omega_c}{\partial g}\right) + \frac{\partial JC}{\partial g} \tag{18}$$

where $\frac{\partial\Omega_K}{\partial g}$, $\frac{\partial\Omega_c}{\partial g}$, and $\frac{\partial\Omega_b}{\partial g}$ represent the change in equilibrium job value J^0 and labor market tightness θ^0 produced by training investments K, search costs c and the unemployment benefit b respectively over the whole duration of the match. These effects come through the profitability of a job in the job destruction condition (13). $\frac{\partial JC}{\partial g}$ is the impact of growth on job creation costs in the job creation equation (14).

Training investments K have an overall positive effect on J^0 and θ^0. The the reduced power to appropriate the training investments of the firm that reduces wage growth dominates the effect arising through competing jobs. Search costs c and the unemployment benefit b reduce J^0 and θ^0. Mismatch does not affect these variables, and thus they are only affected by the

[13]Without training costs K the model would unambiguously produce $\partial J^0/\partial g < 0$ and $\partial\theta^0/\partial g < 0$.

wage increasing effect through competing jobs. $\frac{\partial JC}{\partial g}$ affects J^0 and θ^0 negatively as search costs increase the required value for a job to be created.

Condition (18) postulates that for growth to increase equilibrium job value J^0 and labor market tightness θ^0, the impact of training costs must outstrip the effects that arise through search costs and the unemployment benefit. The reduction in the outside option due to reduced appropriability must be larger than the effects on K, c and b that arise due to general wage growth. In addition to these the effect of higher job creation costs must be outstripped.

3.1.2 Example[14]

The rather involved condition (17) states that the effect arising from training costs K must be large enough relative to that arising through unemployment benefits b and search costs c for $\partial J^0/\partial g > 0$ and $\partial \theta^0/\partial g > 0$. There is a large number of factors influencing this condition and an analytical solution cannot be presented. Therefore, we discuss the conditions in terms of a numerical solution and its' sensibility to parameter values. The base-line parameter values are based roughly on values used in the literature

Table 1: Baseline parameter values and specifications

Training cost	$K = 0.4$
Search cost	$\frac{c}{aq(\theta)} = 0.1$
Unemployment benefit	$b = 0.35$
Worker's share in bargaining	$\beta = 0.5$
Workers contribution to matching	$\alpha = 0.5$
Rate of technical progress/growth rate	$g = 0.03$
Interest rate	$r = 0.02$
Exogenous job destruction rate	$\lambda = 0.2$
$\gamma = \frac{1}{g-r-\lambda}\left(e^{T(g-r-\lambda)} - 1\right)$	
$\mu = -\frac{1}{(g-r-\lambda)^2}\left(e^{T(g-r-\lambda)} - 1\right) + \left(\frac{1}{g-r-\lambda}\right) Te^{T(g-r-\lambda)}$	
Skill parameter	$a = e^{-gT}$
Matching function	$q(\theta) = \theta^{-\alpha}$

The above baseline values stated above provide a benchmark case for which the condition (17) holds and for which $\partial J^0/\partial g > 0$ and $\partial \theta^0/\partial g > 0$. In the special cases when $b = 0$ or $c = 0$, the condition is insensitive to moderate and even large changes in parameter values. However as these grow the condition becomes increasingly sensitive to the other parameters of the model. High unemployment benefits b and search costs c both raise the right hand side of (18) and thus raise the magnitude of K required for the condition to hold. This reduces the generality of the condition. Thus, for growth to have a positive impact on the equilibrium J^0 and θ^0 training investments must be sufficiently high relative to the unemployment benefits and search costs in determining the outside option of workers. Investing in training costs rather than search raises J^0 and θ^0. If one considers the firm as having the possibility of either investing in search to find an appropriate worker, or to make small investments in search, employ a less-qualified worker and train her, the latter option is to be chosen for high equilibrium J^0 and θ^0. As will

[14] The computations are preliminary at this stage.

be discussed in forthcoming work, this will indeed happen[15]. In the presence of frictions in the labor market the firm has incentives to invest in training of their workers rather than search, regardless of whether the trained skills are specific or general.

The unemployment benefit parameter b plays an important role as a policy parameter. The choices made between investment in search or training are made by optimizing firms. The choice between these investments may however be limited due to technological constraints. b, however, can be influenced by policy. If the right hand side of (18) is larger than the left hand side and the choices of K and c are constrained, the choice of the level of unemployment benefits may be used to influence equilibrium job value and labor market tightness, that is the supply of vacancies relative to the unemployed.

3.2 Optimal job duration

The firm seeks to maximize the value of a job. The optimal destruction date T^0 is such that it maximizes the value of the match at all dates after creation. T^0 is obtained with the first-order condition of the right hand side of (13). It states that the shadow wage available to searching worker equals match product at date of destruction

$$\exp\left(gT^0\right)\omega(\theta^0) = x \tag{19}$$

By substituting (19) into (14) we get a relation between the optimal value of a job J^0 and the optimal duration of a job T^0

$$J^0 = (1-\beta)\, x \int_0^T \left[1 - \exp\left(g\left(t - T^0\right)\right)\right] e^{-(r+\lambda)t} dt \tag{20}$$

The right hand side is increasing in both g and T^0. The impact of growth on the optimal duration of a job is determined by the effect of growth on equilibrium job value J^0. As the right hand side is increasing in both g and T^0, an increase in job value due to an increase in the growth rate, $\partial J^0/\partial g > 0$, implies an increase in optimal job duration T^0.

In the Mortensen and Pissarides (1998) model, where there is no parameter for the growth induced mismatch, growth unambiguously reduces the value of a job, $\partial J^0/\partial g < 0$, which implies $\partial T^0/\partial g < 0$. This is because growth accelerates the growth of wages to the level where it equals productivity. The faster this process is, the shorter the optimal lifetime of the match[16].

When skill-mismatch is introduced to the model we get the opposite result. Mismatch implies $\partial J^0/\partial g > 0$ and $\partial \theta^0/\partial g > 0$ and the optimal job-lifetime T^0 increases. The growth of the wage in the match is slowed down by growth. The higher matching friction reduces the outside option of the worker more than the general rise in wages increases it. Thus when technical progress is fast, the latter effect dominates and the time it takes for a job to become non-profitable is prolonged. Job duration increases.

[15] See appendix for preliminary discussion of the determination of training and search investments.

[16] The same qualitative impact arises when the mismatch parameter is introduced and there are no training costs.

3.3 Equilibrium unemployment

To determine the equilibrium unemployment level, we will examine the job destruction and job creation rates. There are two reasons for job destruction. They can be destroyed due to exogenous shocks or endogenously due to obsolescence. λ is the fraction destroyed by exogenous shocks. The fraction of job creation flow that survive until obsolescence is $\exp\left(-\lambda T^0\right)$. The total job-destruction flow, JD, is then

$$JD = \lambda n + JC \exp\left(-\lambda T^0\right), \tag{21}$$

where n is level of employment and JC is job creation flow. The job creation flow is determined by the rate of unemployed moving to employment $\theta q\left(\theta\right)$ and the rate of unemployment u, so the job creation flow is

$$JC = a\theta^0 q\left(\theta^0\right) u. \tag{22}$$

In steady state job creation is equal to job destruction, so we have

$$a\theta^0 q\left(\theta^0\right) u = \lambda n + JC \exp\left(-\lambda T^0\right). \tag{23}$$

The rate at which unemployed workers move to employment times the unemployment rate equals job destruction by exogenous shocks plus endogenous job destruction due to obsolescence. From the previous equation and with $u = 1 - n$, we obtain the Beveridge equation which gives the steady state unemployment rate

$$u = \frac{\lambda}{\lambda + \left[1 - \exp\left(-\lambda T^0\right)\right] a\theta^0 q\left(\theta^0\right)} \tag{24}$$

Equilibrium unemployment is determined by the exogenous destruction rate λ, the rate of unemployed moving to employment $a\theta^0 q\left(\theta^0\right)$ and the optimal lifetime of a job T^0. In Mortensen and Pissarides (1998) $\partial J^0/\partial g < 0$ and $\partial \theta^0/\partial g < 0$, i.e. growth shifts both the job creation curve and the job destruction curve to the left. $\partial J^0/\partial g < 0$ implies a lower age of destruction T^0 for jobs. A lower T^0 means a higher rate of endogenously determined job destruction. As T^0, labor market tightness θ^0 decrease with growth and a is not present, the denominator in (24) decreases, and growth unambiguously leads to higher equilibrium unemployment.

When skill-mismatch is introduced to the model and $\partial J^0/\partial g > 0$ and $\partial \theta^0/\partial g$, both T^0 and θ^0 increase as was discussed above. The optimal lifetime of jobs increases as well as vacancies relative to the unemployed increase. These increase the denominator in the Beveridge equation which reduces the equilibrium unemployment rate. For this outcome to arise condition (17) must hold. As was discussed in the previous section, this depends on the relative values of the components determining the reservation wage, that is the relative values of the unemployment benefit b, the search cost c and the training cost K as well as the relative bargaining power of the firm $(1 - \beta)$ and the worker β. The unemployment benefits and search costs must be sufficiently low relative to training costs in determining the outside option of workers for the condition to hold and for a higher growth rate to produce lower equilibrium unemployment.

However, a higher growth rate reduces the skill parameter $a = e^{-gT}$, which reduces the denominator and increases the equilibrium unemployment rate. The relative impact θ, T and a determine whether growth induces higher or lower equilibrium unemployment.

With the Cobb-Douglas specification for the matching function and the skill parameter defined as $a = e^{-gT}$, the denominator increases and equilibrium unemployment decreases with growth when

$$\frac{\partial T}{\partial g} > \left(T - \frac{1-\alpha}{\theta}\frac{\partial \theta}{\partial g} \right) \frac{e^{\lambda T} - 1}{\lambda}$$

For the baseline parameter values this condition holds for sufficiently small values of T. In other words growth reduces unemployment in sectors of the economy where job duration is short. This result can be explained intuitively by the fact that when job duration is short, the technological frontier does not move very far away during the employment period. Thus the skill-mismatch doesn't get very large and it doesn't affect the matching process in a too serious manner. On the contrary, when job duration is long, the technological frontier has escaped from the worker so far away, that the finding of a new job becomes practically impossible. Thus, there is a critical level under which the prolongation of job duration reduces the reshuffling of jobs without downgrading the matching process so much that it leads to higher unemployment. When this critical value of T is exceeded, technical progress still prolongs jobs, but it leads to so high skill-mismatch and downgrades the matching process to a so large measure, that unemployment increases.

Alternatively, if growth and mismatch produce a large increase in job duration ($\partial T/\partial g$ is large), the original value of T has less importance. However, extremely large increases in T due to increased friction may not be very realistic.

4 Concluding remarks

The impact of technological progress on equilibrium unemployment is twofold when it involves skill mismatch between firms posting vacancies and unemployed workers. The introduction of new technology into the economy involves creation of new more productive and better payed jobs and destruction of obsolete ones. The higher wages in new jobs increase the outside options of workers working at old productivity levels, thus raising wages even in existing old jobs. This reduces the profitability of existing jobs and leads to a higher rate of job destruction and a lower rate of job creation (posting vacancies), which increase the equilibrium unemployment level.

This study suggests, however, that technological progress leads to a mismatch between the skills of unemployed workers and those required by firms posting vacancies. This mismatch reduces the suitability of the worker from the viewpoint of the firm, and reduces the bargaining position of the worker. Thus, while technological progress does raise productivity and wages in alternative new jobs, the mismatch related in changing jobs limits the workers options of changing jobs and thus also reduces her outside option. This increases the profitability of the match, leading to higher rate of posted vacancies relative to the unemployed and lower job destruction. However as matching decreases with the skill-mismatch, the overall effect on unemployment remains ambiguous and depends on the parameters of the model.

The overall effect of technological progress on equilibrium unemployment depends on the relative strength of these two effects that increase and decrease the outside option of the worker.

The model produces a condition under which the latter effect dominates and consequently technological progress produces a lower level of equilibrium unemployment. This condition highlights the importance of the relative values of the components determining the outside option. For technological progress to produce a lower level of equilibrium unemployment the workers bargaining position and the values of the unemployment benefit and search costs must be relatively low with respect to training investments of the firm. When there are frictions in the labor market the firm has incentives to invest in training of workers rather than search costs. This is because the appropriable share of the training investments by the worker is smaller than that of the search costs.

Of interest is also the role of unemployment benefits. If the choice between job creation investments is limited due to e.g. technological constraints, the magnitude of unemployment benefits plays a crucial role. For growth to have a negative impact on equilibrium unemployment, the unemployment benefit must be sufficiently low relative to job creation costs. This is because the appropriability of the creation costs by the worker are reduced by growth, whereas the unemployment benefit affects the outside option only in by increasing it.

5 Appendix: Investment in training and search[17]

A firm can obtain a sufficiently skilled worker in two ways. It has the choice of either searching for a worker with appropriate skills for the job available, or it can take any worker that comes along and train that worker the relevant skills.

5.1 Training investments and friction

We follow Acemoglu (1997) to discuss the implications of friction in the labor market when friction is increasing in skills. When $w'(\delta) < f'(\delta)$, friction increases with skills and reduces the wages of skilled workers relative to unskilled workers. This produces incentives for the firms to provide general training.

New jobs require skills that match the new technology and unemployed workers have skills of an earlier vintage. Thus newly created jobs always involve a skill gap. At a low rate of growth g or low job duration T the skill gap will be minimal. As these increase, so does the matching friction.This nature of the labor market implies that there are always frictions at the job creation level, the higher the growth rate, the higher the friction. When growth produces frictions to the labor market through skill mismatch, it produces incentives for firms to provide for general training for their workers. The firm avoids the hold-up problem related to providing general training.

The firm provides the worker with training at the creation time of the match before production. The productivity of a worker with training δ is $p(t)\,x(\delta,\sigma)$ in every period and her wage is $w(t,\tau,\delta,\sigma)$. σ denotes the skill level the unemployed worker already has. $p(t)\,x(\delta,\sigma)$ is increasing in δ and concave. Without any training the workers productivity is normalized to $p(t)\,x(0) = 0$. $K(\delta)$ denotes the training cost, which will be incurred by the firm. $K(\delta)$ is assumed to be strictly increasing, differentiable and convex with $K'(0) = K(0) = 0$ and $\lim_{\delta\to\infty} K'(\delta) = \infty$. The optimal level of training is given by the condition $K'\left(\delta^0\right) = p(t)\,x'\left(\delta^0,\sigma\right)$, and from the assumptions of the cost of training it follows that $\delta^0 > 0$.

[17] This section is preliminary at this stage of the work.

For the optimal training level (number of trained skill units δ^0) for given search investments the firm maximizes the value of creating a job. For a given period t, the level of training that maximizes the value of an occupied job minus costs involved in hiring, is given by

$$\max_{\delta} \left\{ J - \frac{c(\sigma)}{aq(\theta)} - K(\delta) \right\} \tag{25}$$

The asset value equation for an occupied job was given above, into which we substitute the wage equation (9) to get

$$\max_{\delta} \left\{ \left(\frac{1}{r+\lambda} \right) \left[\begin{array}{c} (1-\beta)\, p(t)\, x(\delta,\sigma) - \\ (1-\beta)\, p(t) \left(b + \frac{\beta}{1-\beta} \left(c(\sigma)\,\theta + a\theta q(\theta)\, K(\delta) \right) \right) \end{array} \right] - \frac{c(\sigma)}{aq(\theta)} - K(\delta) \right\}. \tag{26}$$

The first-order condition takes the form

$$(1-\beta)\, p(\tau)\, x'(\delta,\sigma) = \left(p(t)\, \beta a\theta q(\theta) + p(\tau)\,(r+\lambda) \right) K'(\delta)\,. \tag{27}$$

For all $\beta < 1$ and $\beta a\theta q(\theta) < \infty$, the firm will choose a positive level of training, $\delta^0 > 0$, as $K'(0) = 0$. It must be that $\beta < 1$, for otherwise the worker would appropriate all the surplus, and there would be no incentives for the firm to train. The features that $r < \infty$ and $\lambda < 1$ mean that the future is taken account of and that the job will not be destroyed for sure. That $a\theta q(\theta) < 1$ means that there are frictions in the matching process. When $a\theta q(\theta) \longrightarrow 1$, the labor markets are perfectly competitive and the worker finds a job immediately[18]. This last condition is the requirement for labor market imperfections for firms to invest in general training. So the lower the job matching rate is or the higher is skill mismatch (lower a), the higher are the incentives for the firm to train.

When growth produces friction to the matching process, firms find it optimal to invest in the general training of their workers, general or specific. K gets thus a positive value. The greater the friction, the higher is the optimal level of K, $\partial K^0/\partial g > 0$.

5.2 Search costs and friction

In a similar manner we can determine the optimal level of search for given training. The search cost $c(\sigma)$ depends on the level of existing skills of the worker σ that is required as the outcome of the search. From (26) we have the first-order condition

$$(1-\beta)\, p(\tau)\, x'(\delta,\sigma) = \left(p(t)\, \beta\theta + p(\tau)\,(r+\lambda) \frac{1}{aq(\theta)} \right) c'(\sigma)\,. \tag{28}$$

Here a higher rate of technical progress and thus higher skill mismatch (lower a) induce a lower optimal level of investment in search. When the skill gap between skills required by firms and those possessed by unemployed workers is large, search is more costly as it implies a longer time of search for an appropriate worker. Alternatively the firm can search more intensively. These are both costly. Thus, higher skill mismatch in the labor market reduces the optimal level of search, $\partial c^0/\partial g < 0$.

[18] This would produce the Beckerian (1964) outcome as a special case where firms would not invest in training.

5.3 Mismatch and choice of job creation costs

For the firm to start production with a worker, the worker has to have some skill level defined by the current technology. We assume that there are no technological constraints so that the firm can choose any combination of search and training costs to get a worker with appropriate or sufficiently high skills. The firm chooses a combination of training and search such that the marginal cost of training equals the marginal cost of search, that is

$$[p(t)\beta a\theta q(\theta) + p(\tau)(r+\lambda)] K'(\delta) = \left(p(t)\beta\theta + p(\tau)(r+\lambda)\frac{1}{aq(\theta)} \right) c'(\sigma). \tag{29}$$

As growth reduces the marginal cost of training and increases the marginal cost of search, growth induces the firm to increase the share of training and reduce the share of search in job creation costs to maximize profits. We have $\partial K^0/\partial g > 0$ and $\partial c^0/\partial g < 0$.

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