

# **Mission Impossible !?**

## **On the Harmonization of National Allocation Plans under the EU Emissions Trading Directive**

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## **Downloadable Appendix: CGE Model Description**

([ftp://ftp.zew.de/pub/zew-docs/div/nap\\_cge.pdf](ftp://ftp.zew.de/pub/zew-docs/div/nap_cge.pdf))

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## A.1. Non-technical model description

For determining the country-specific assignment factors  $\lambda_r$ , as reported in Table 1 we make use of a 15-region, 9-sector computable general equilibrium (CGE) model for the EU economy calibrated to empirical data. At the sectoral level the model incorporates sufficient details on differences in factor intensities, degrees of factor substitutability and price elasticities of output demand in order to trace back the structural change induced by carbon abatement policies. The sectors in the model have been carefully selected to keep the most carbon-intensive sectors in the available data as separate as possible. Table A.1 provides an overview of the sectors represented in the model.

Table A.1: Overview of model regions and sectors (commodities)

EU member countries	Production sectors	
Austria	<i>Primary energy carriers</i>	
Belgium	COL	Coal
Germany	CRU	Crude oil
Denmark	GAS	Natural gas
Spain	<i>Energy-intensive sectors (EIS)</i>	
Finland	OIL	Refined oil products
France	ELE	Electricity
United Kingdom	ORE	Iron and steel
Greece	PPP	Paper, pulp, and printing
Ireland	NFM	Non-ferrous metals
Italy	<i>Remaining manufacturers and services</i>	
Luxembourg	ROI	Rest of Industry
Netherlands		
Portugal		
Sweden		

The energy goods identified in the model include primary carriers (coal, natural gas, crude oil) and secondary energy carriers (refined oil products and electricity). Furthermore, the model features three energy-intensive non-energy sectors (iron and steel; paper, pulp and printing; non-ferrous metals) whose installations – in addition to the secondary energy branches (refined oil products and electricity) – are subject to the EU emissions trading Directive. The remaining

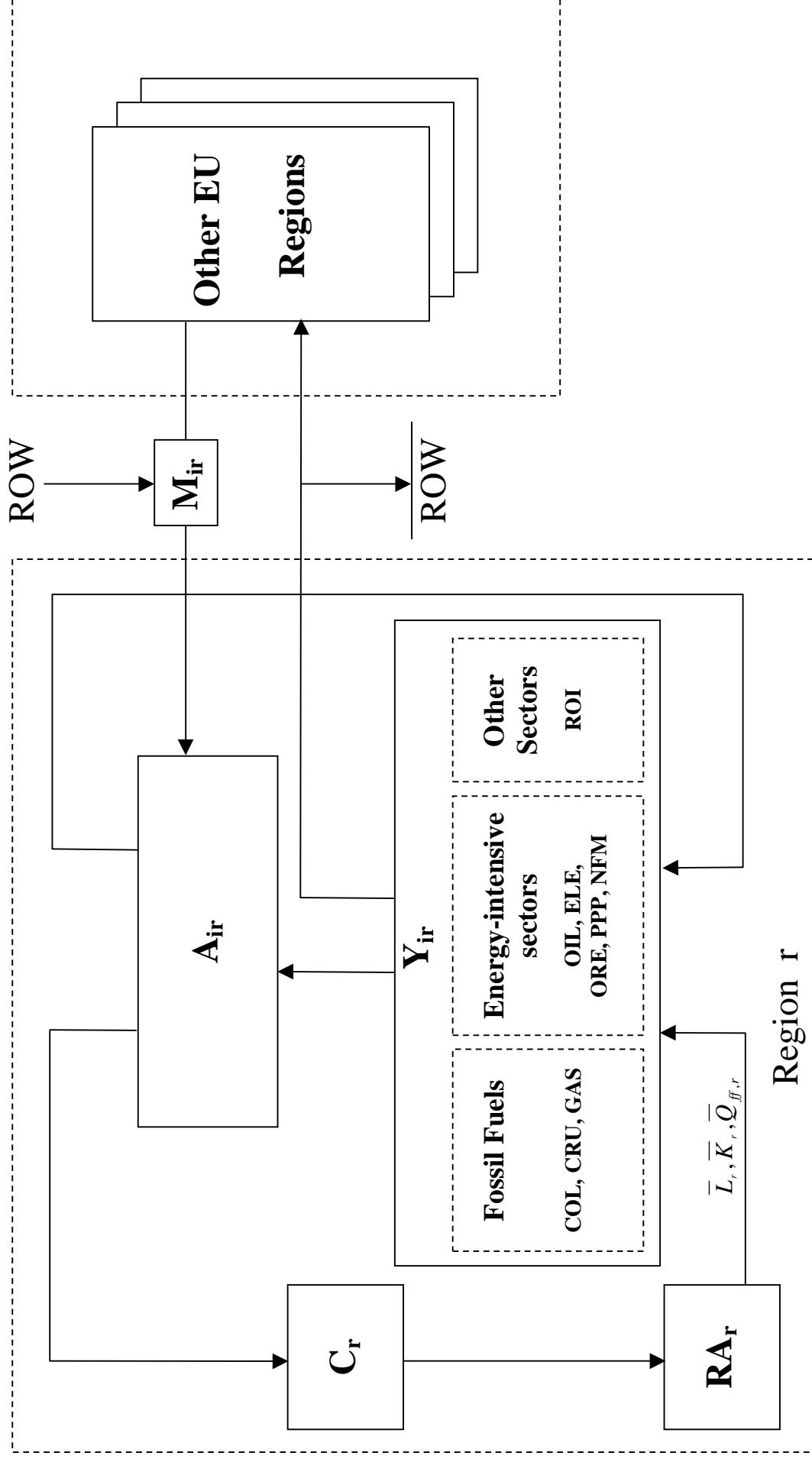
manufacturers and services are aggregated to a composite industry that produces a non-energy-intensive macro good.

Figure A.1 provides a diagrammatic overview of the model structure. Primary factors of each EU region  $r$  include labor  $\bar{L}_r$ , capital  $\bar{K}_r$ , and fossil-fuel resources  $\bar{Q}_{ff,r}$ . Labor and capital are assumed to be mobile across sectors within each region. In fossil fuel production, part of capital is treated as a sector-specific resource, resulting in upward sloping supply schedules consistent with exogenous own-price elasticities of supply.

Production  $Y_{ir}$  of commodities  $i$  in region  $r$ , other than primary fossil fuels, is captured by aggregate production functions which characterize technology through substitution possibilities between various inputs. Nested, separable constant elasticity of substitution (CES) cost functions with three levels are employed to specify the substitution possibilities in domestic production between capital, labor, energy and non-energy intermediate inputs, i.e. material. At the top level, material inputs are employed in fixed proportions with an aggregate of energy, capital and labor. At the second level, a CES function describes the substitution possibilities between the energy aggregate and the aggregate of labor and capital. The value-added composite is a CES function of labor and capital. The energy aggregate is produced with a CES function of a non-electric energy composite and electricity. The non-electric energy composite in turn is a CES function of coal, crude oil, refined oil, and natural gas. In the production of fossil fuels, all inputs, except for the sector-specific fossil fuel resource, are aggregated in fixed proportions at the lower nest. At the top level, this aggregate trades off with the sector-specific fossil fuel resource at a constant elasticity of substitution. The latter is calibrated in consistency with exogenous price elasticities of fossil fuel supply.

Final consumption demand  $C_r$  in each region is determined by a representative agent  $RA_r$ , who maximizes consumption subject to a budget constraint with fixed investment. Aggregate consumption of the representative agent is given as a CES composite which combines composite energy consumption with a non-energy consumption bundle.

Figure A.1: Diagrammatic model structure



Substitution patterns within the non-energy consumption bundle are reflected via Cobb-Douglas functions. The energy aggregate in final demand consists of the various energy goods trading off at a constant elasticity of substitution. Government demand within each region is fixed at exogenous real levels. Public goods and services are produced with a CES aggregation of commodity inputs. The expenditure for public good provision is handled through the budget constraint of the representative agent.

Trade between regions is specified using the Armington approach of product heterogeneity, so domestic and foreign goods of the same variety are distinguished by origin. The Armington composite  $A_{ir}$  for a traded good is a CES function of an imported composite  $M_{ir}$  and domestic production for that sector. The import composite is then a CES function of an EU import composite and imports from the rest of the world (ROW). The EU import composite of a specific EU region in turn is a CES function of production from all other EU countries. EU countries are assumed to be price-takers with respect to world market prices, i.e. ROW import-supply functions and ROW export-demand functions are perfectly elastic. There is an imposed balance of payment constraint to ensure trade balance between the EU and ROW through a flexible exchange rate. That is, the value of imports from the ROW to the EU must equal the value of exports from the EU to the ROW after accounting for the benchmark trade deficit or surplus of EU regions.

The effects of exogenous policy changes are measured with respect to a reference situation. In our comparative-static analysis, the reference situation is captured by economic transactions in a particular benchmark year (here: 1997). As is customary in applied general equilibrium analysis, benchmark quantities and prices – together with exogenous elasticities (see Table A.7 below) – determine the parameters of functional forms. For this model calibration, we employ the GTAP-5E database (McDougall 1999)<sup>14</sup> which provides most recent consistent accounts of regional production and consumption, bilateral trade and energy flows for up to 66 countries and 23 commodities.

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<sup>14</sup> McDougall, R.A. (1999), ed., *Global Trade, Assistance and Protection: The GTAP 5 Data Base*, Center for Global Trade Analysis, Purdue University, West Lafayette.

## A.2 Algebraic model description

The model is formulated as a system of nonlinear inequalities. These inequalities correspond to two classes of equilibrium conditions: zero profit and market clearance. The fundamental unknowns of the system are two vectors: activity levels and prices. In equilibrium, each of these variables is linked to one inequality condition: an activity level to a zero-profit condition and a commodity (factor) price to a market-clearance condition.

In the algebraic exposition below, the notation  $\Pi_{ir}^z$  is used to denote the (zero-)profit function of sector  $j$  in region  $r$  where  $z$  is the name assigned to the associated production activity. Differentiating the profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotellings's lemma), which appear subsequently in the market clearance conditions. We use  $i$  (aliased with  $j$ ) as an index for commodities (sectors) and  $r$  (aliased with  $s$ ) as an index for regions. The label  $EG$  represents the set of energy goods and the label  $FF$  denotes the subset of fossil fuels. Tables A.2 - A.7 explain the notations for variables and parameters employed within our algebraic exposition. Figures A.2 - A.6 provide a graphical exposition of the production and final consumption structure.

The implementation of cost-efficient National Allocation Plans across EU member states is equivalent to a comprehensive carbon trade equilibrium: Marginal abatement costs are equalized across all sectors and EU regions (to be achieved by domestic carbon taxes for non-trading sectors where the tax rate is set at the level of the international permit price for trading sectors). With lump-sum allocation of emission allowances to the trading sectors under the Directive and a representative agent per EU region, the revenues from carbon regulation – either in terms of tax revenues or the implicit value of free allowances – enter the budget constraint of the representative agent.

### Zero Profit Conditions

1. Production of goods except fossil fuels:

$$\prod_{ir}^Y = \left( \theta_{ir}^{XROW} p^{W^{1-\eta}} + (1-\theta_{ir}^X) p_{ir}^{1-\eta} \right)^{\frac{1}{1-\eta}} - \sum_{j \in EG} \theta_{jir} p_{jr}^A$$

$$- \theta_{ir}^{KLE} \left[ \theta_{ir}^E p_{ir}^E 1^{-\sigma_{KLE}} + (1-\theta_{ir}^E) \left( w_r^{\alpha_{jr}^L} v_r^{\alpha_{jr}^K} \right)^{1-\sigma_{KLE}} \right]^{\frac{1}{1-\sigma_{KLE}}} \leq 0 \quad i \notin FF$$

where  $Y_{ir}$  ( $i \notin ff$ ) is the associated activity variable.

2. Production of fossil fuels:

$$\prod_{ir}^Y = \left( \theta_{ir}^{XROW} p^{W^{1-\eta}} + (1-\theta_{ir}^X) p_{ir}^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

$$- \left[ \theta_{ir}^Q q_{ir}^{1-\sigma_{Qj}} + (1-\theta_{ir}^Q) \left( \theta_{Lir}^{FF} w_r + \theta_{Kir}^{FF} v_r + \sum_j \theta_{jir}^{FF} p_{jr}^A \right)^{1-\sigma_{Qj}} \right]^{\frac{1}{1-\sigma_{Qj}}} \leq 0 \quad i \in FF$$

where  $Y_{ir}$  ( $i \in ff$ ) is the associated activity variable.

3. Sector-specific energy aggregate:

$$\prod_{ir}^E = p_{ir}^E - \left\{ \theta_{ir}^{ELE} p_{\{ELE,r\}}^{A^{1-\sigma_{ELE}}} + (1-\theta_{ir}^{ELE}) \left[ \theta_{ir}^{COA} p_{\{COA,r\}}^{A^{1-\sigma_{COA}}} + (1-\theta_{ir}^{COA}) \left( \prod_{j \in LQ} p_{jr}^A \right)^{1-\sigma_{COA}} \right]^{\frac{1-\sigma_{ELE}}{1-\sigma_{COA}}} \right\}^{\frac{1}{1-\sigma_{ELE}}} \leq 0$$

where  $E_{ir}$  is the associated activity variable.

4. Armington aggregate:

$$\prod_{ir}^A = p_{ir}^A - \left[ \left( \theta_{ir}^A p_{ir}^{1-\sigma_A} + (1-\theta_{ir}^A) p_{ir}^{M^{1-\sigma_A}} \right)^{\frac{1}{1-\sigma_A}} + p_r^{CO2} a_i^{CO2} \right] \leq 0$$

where  $A_{ir}$  is the associated activity variable.

5. Aggregate imports across import regions:

$$\prod_{ir}^M = p_{ir}^M - \left( \sum_s \theta_{isr}^M p_{is}^X 1^{-\sigma_M} + \theta_{ir}^{MROW} p^W 1^{-\sigma_M} \right)^{\frac{1}{1-\sigma_M}} \leq 0$$

where  $M_{ir}$  is the associated activity variable.

6. Household consumption aggregate:

$$\Pi_r^C = p_r^C - \left( \theta_{Cr}^E p_{Cr}^E \cdot 1^{-\sigma_{EC}} + (1 - \theta_{Cr}^E) \left[ \prod_{i \notin FF} p_{ir}^{A_{ir}} \right] \right)^{1-\sigma_{EC}} \frac{1}{1-\sigma_{EC}} \leq 0$$

where  $C_r$  is the associated activity variable.

7. Household energy aggregate:

$$\Pi_{Cr}^E = p_{Cr}^E - \left[ \sum_{i \in FF} \theta_{iCr}^E p_{ir}^A \cdot 1^{-\sigma_{FF,C}} \right] \frac{1}{1-\sigma_{FF,C}} \leq 0$$

where  $E_{Cr}$  is the associated activity variable.

8. Investment:

$$\Pi_r^I = p_r^I - \sum_i \theta_{ir}^I p_{ir}^A \leq 0$$

where  $I_r$  is the associated activity variable.

### **Market Clearance Conditions**

9. Labor:

$$\bar{L}_r \geq \sum_i Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial w_r}$$

where  $w_r$  is the associated price variable.

10. Capital:

$$\bar{K}_r \geq \sum_i Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial v_r}$$

where  $v_r$  is the associated price variable.

11. Natural resources:

$$\bar{Q}_{ir} = Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial q_{ir}} \quad i \in FF$$

where  $q_{ir}$  is the associated price variable.



12. Output for internal markets:

$$Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p_{ir}} \geq A_{ir} \frac{\partial \Pi_{ir}^A}{\partial p_{ir}} + \sum_{s \neq r} M_{is} \frac{\partial \Pi_{is}^M}{\partial p_{ir}}$$

where  $p_{ir}$  is the associated price variable.

13. Sector-specific energy aggregate:

$$E_{ir} \geq Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p_{ir}^E}$$

where  $p_{ir}^E$  is the associated price variable.

14. Import aggregate:

$$M_{ir} \geq A_{ir} \frac{\partial \Pi_{ir}^A}{\partial p_{ir}^M}$$

where  $p_{ir}^M$  is the associated price variable.

15. Armington aggregate:

$$A_{ir} \geq \sum_j Y_{jr} \frac{\partial \Pi_{jr}^Y}{\partial p_{ir}^A} + C_r \frac{\partial \Pi_r^C}{\partial p_{ir}^A} + I_r \frac{\partial \Pi_r^I}{\partial p_{ir}^A}$$

where  $p_{ir}^A$  is the associated price variable.

16. Investment aggregate:

$$\bar{I}_r \geq I_r$$

where  $p_r^I$  is the associated price variable.

17. Household consumption:

$$C_r p_r^C = w_r \bar{L}_r + v_r \bar{K}_r + \sum_{j \in FF} q_{jr} \bar{Q}_{jr} + p_r^{CO_2} \bar{CO}_{2r} + p_r^I \bar{I}_r + \bar{B}_r$$

where  $p_r^C$  is the associated price variable.

18. Aggregate household energy consumption:

$$E_{Cr} = C_r \frac{\partial \Pi_r^C}{\partial p_{Cr}^E}$$

where  $p_{Cr}^E$  is the associated price variable.

19. Carbon emissions:

$$\overline{CO2}_r = \sum_i A_{ir} a_i^{CO2}$$

where  $p_r^{CO2}$  is the associated price variable.

20. Balance of payments:

$$\sum_{i,r} Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p^W} + \sum_r \bar{B}_r = \sum_{i,r} M_{ir} \frac{\partial \Pi_{ir}^M}{\partial p^W}$$

where  $p^W$  is the associated price variable.

Table A.2: Sets

I	Sectors and goods
J	Aliased with i
R	Regions
S	Aliased with r
EG	All energy goods: Coal, crude oil, refined oil, gas and electricity
FF	Primary fossil fuels: Coal, crude oil and gas
LQ	Liquid fuels: Crude oil and gas

Table A.3: Activity variables

$Y_{ir}$	Production in sector $i$ and region $r$
$E_{ir}$	Aggregate energy input in sector $i$ and region $r$
$M_{ir}$	Aggregate imports of good $i$ and region $r$
$A_{ir}$	Armington aggregate of good $i$ in region $r$
$C_r$	Aggregate household consumption in region $r$
$E_{Cr}$	Aggregate household energy consumption in region $r$
$I_r$	Aggregate investment in region $r$

Table A.4: Price variables

$p_{ir}$	Output price of good $i$ produced in region $r$ for domestic market
$p^W$	Real exchange rate with the rest of the world (ROW)
$p_{ir}^E$	Price of aggregate energy in sector $i$ and region $r$
$p_{ir}^M$	Import price aggregate for good $i$ imported to region $r$
$p_{ir}^A$	Price of Armington good $i$ in region $r$
$p_r^C$	Price of aggregate household consumption in region $r$
$p_{Cr}^E$	Price of aggregate household energy consumption in region $r$
$p_r^I$	Price of aggregate investment good in region $r$
$w_r$	Wage rate in region $r$
$v_r$	Price of capital services in region $r$
$q_{ir}$	Rent to natural resources in region $r$ ( $i \in \text{FF}$ )
$p_r^{\text{CO}_2}$	Shadow price of CO <sub>2</sub> unit in region $r$

Table A.5: Cost shares

$\theta_{ir}^{\text{XROW}}$	Share of ROW exports in sector $i$ and region $r$
$\theta_{jir}$	Share of intermediate good $j$ in sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^{\text{KLE}}$	Share of KLE aggregate in sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^E$	Share of energy in the KLE aggregate of sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\alpha_{ir}^T$	Share of labor ( $T=L$ ) or capital ( $T=K$ ) in sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^Q$	Share of natural resources in sector $i$ of region $r$ ( $i \in \text{FF}$ )
$\theta_{Tir}^{\text{FF}}$	Share of good $i$ ( $T=i$ ) or labor ( $T=L$ ) or capital ( $T=K$ ) in sector $i$ and region $r$ ( $i \in \text{FF}$ )
$\theta_{ir}^{\text{COA}}$	Share of coal in fossil fuel demand by sector $i$ in region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^{\text{ELE}}$	Share of electricity in energy demand by sector $i$ in region $r$
$\beta_{jir}$	Share of liquid fossil fuel $j$ in energy demand by sector $i$ in region $r$ ( $i \notin \text{FF}, j \in \text{LQ}$ )
$\theta_{isr}^M$	Share of imports of good $i$ from region $s$ to region $r$
$\theta_{ir}^{\text{MROW}}$	Share of ROW imports of good $i$ in region $r$
$\theta_{ir}^A$	Share of domestic variety in Armington good $i$ of region $r$

$\theta_{Cr}^E$	Share of fossil fuel composite in aggregate household consumption in region $r$
$\theta_{ir}^I$	Share of good $i$ in investment composite in region $r$
$\gamma_{ir}$	Share of non-energy good $i$ in non-energy household consumption demand in region $r$
$\theta_{iCr}^E$	Share of fossil fuel $i$ in household energy consumption in region $r$

Table A.6: Endowments and emissions coefficients

$\bar{L}_r$	Aggregate labor endowment for region $r$
$\bar{K}_r$	Aggregate capital endowment for region $r$
$\bar{Q}_{ir}$	Endowment of natural resource $i$ for region $r$ ( $i \in FF$ )
$\bar{B}_r$	Balance of payment deficit or surplus in region $r$ (note: $\sum_r \bar{B}_r = 0$ )
$\overline{CO_{2,r}}$	Endowment of carbon emission rights in region $r$
$a_i^{CO_2}$	Carbon emissions coefficient for fossil fuel $i$ ( $i \in FF$ )

Table A.7: Elasticities

$\eta$	Transformation between production for the domestic market and production for the export	4
$\sigma_{KLE}$	Substitution between energy and value-added in production (except fossil fuels)	0.5
$\sigma_{Q,i}$	Substitution between natural resources and other inputs in fossil fuel production calibrated consistently to exogenous supply elasticities $\mu_{FF}$ .	$\mu_{COA}=0.5$ $\mu_{CRU}=0.5$ $\mu_{GAS}=1.0$
$\sigma_{ELE}$	Substitution between electricity and the fossil fuel aggregate in production	0.3
$\sigma_{COA}$	Substitution between coal and the liquid fossil fuel composite in production	0.5
$\sigma_A$	Substitution between the import aggregate and the domestic input	2
$\sigma_M$	Substitution between imports from different regions	4
$\sigma_{EC}$	Substitution between the fossil fuel composite and the non-fossil fuel consumption aggregate in household consumption	0.8
$\sigma_{FF,C}$	Substitution between fossil fuels in household fossil energy consumption	0.3

For the sensitivity analysis reported in section 5 or the paper the lower and upper values of the uniform probability distributions for six key elasticities are as follows:

$$1 < \sigma_A < 4; 2 < \sigma_M < 8; 0.25 < \sigma_{KLE} < 0.75; 0.6 < \sigma_{EC} < 1; 0.25 < \mu_{CRU} < 1; 0.25 < \mu_{COL} < 1.$$

Figure A.2: Nesting in non-fossil fuel production

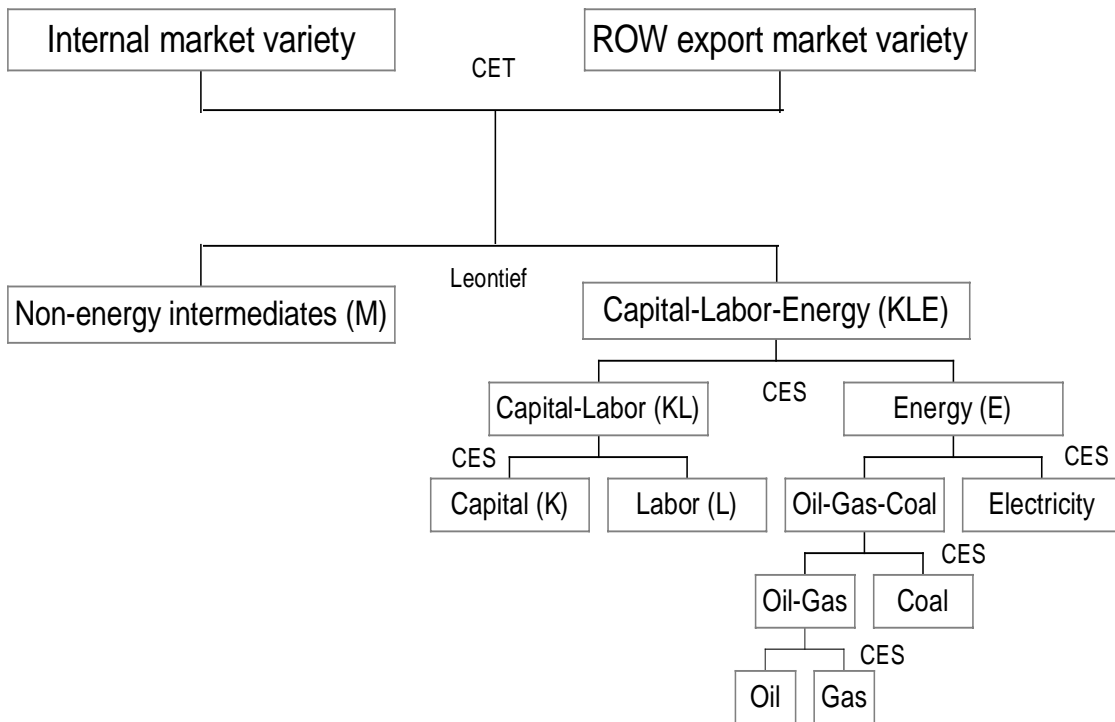


Figure A.3: Nesting in fossil fuel production

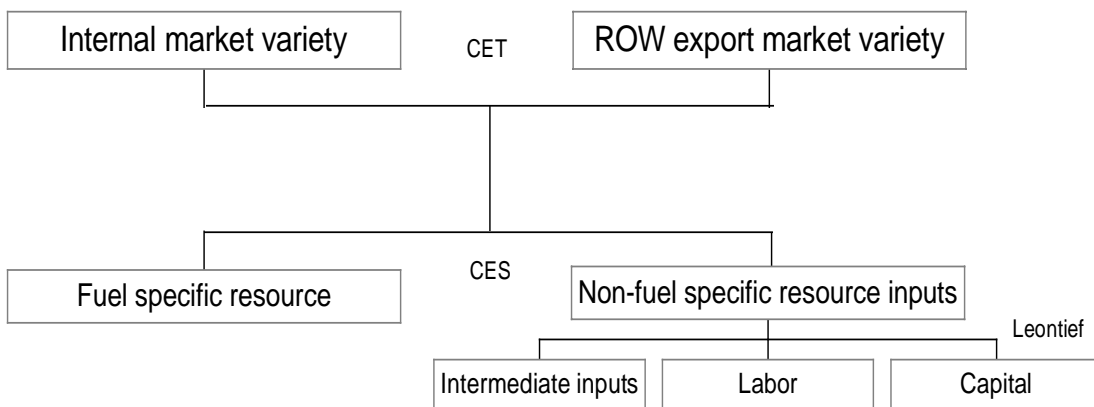


Figure A.4: Nesting in household consumption

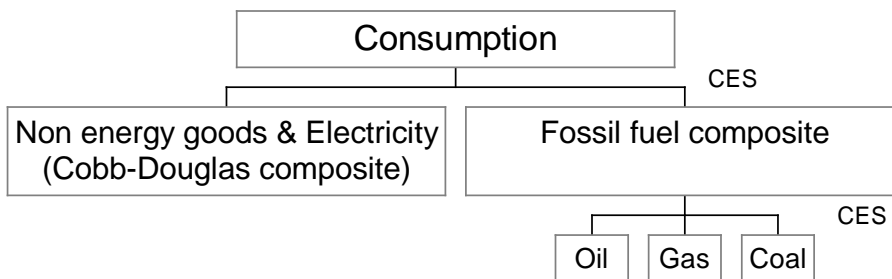


Figure A.5: Nesting in Armington production

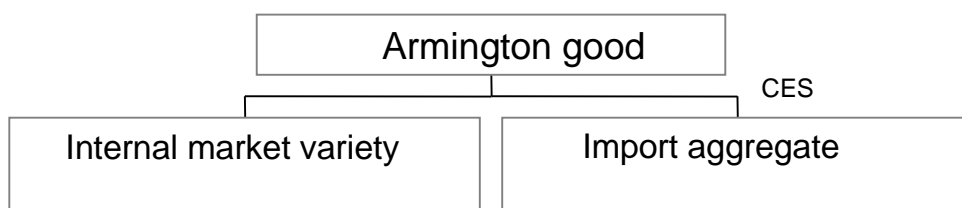


Figure A.6: Nesting in import aggregate

