Appendix: Detailed Algebraic Model Description

This section outlines the main characteristics of a generic static general equilibrium model of the world economy designed for the medium-run economic analysis of carbon abatement constraints. It is a well-known Arrow-Debreu model that concerns the interaction of consumers and producers in markets. Consumers in the model have a primary exogenous endowment of the commodities and a set of preferences giving demand functions for each commodity. The demands depend on all prices; they are continuous and non-negative, homogenous of degree zero in factor prices and satisfy Walras' Law, i.e. the total value of consumer expenditure equals consumer income at any set of prices. Market demands are the sum of final and intermediate demands. Producers maximize profits given a constant returns to scale production technology. Because of the homogeneity of degree zero of the demand functions and the linear homogeneity of the profit functions in prices, only relative prices matter in such a model. Two classes of conditions characterize the competitive equilibrium in the model: market clearance conditions and zero profit conditions. In equilibrium, price levels and production levels in each industry are such that market demand equals market supply for each commodity. Profit maximization under a constant returns to scale technology implies that no activity does any better than break even at equilibrium prices. The model is a system of simultaneous, non-linear equations with the number of equations equal to the number of variables.

A.1 Production

Within each region (indexed by the subscript *r*), each producing sector (indexed interchangeable by *i* and *j*) is represented by a single-output producing firm which chooses input and output quantities in order to maximize profits. Firm behavior can be construed as a two-stage procedure in which the firm selects the optimal quantities of primary factors *k* (indexed by *f*) and intermediate inputs *x* from other sectors in order to minimize production costs given input prices and some production level *Y* with $Y = j\ (k,x)$ the production functions. The second stage, given an exogenous output price, is the selection of the output level *Y* to maximize profits. The firm's problem is then:

$$\underset{y_{jir}, x_{jir}, k_{fir}}{Max}\ \Pi_{ir} = p_{ir} \cdot Y_{ir} - C_{ir}\left(p_{jr}, w_{fr}, Y_{ir}\right) \ \ s.t.\ \ Y_{ir} = j_{ir}\left(x_{jir}, k_{fir}\right) \qquad [1]$$

where Π denotes the profit functions, C the cost functions which relate the minimum possible total costs of producing Y to the positive input prices, technology parameters, and the output quantity Y, and p and w are the prices for goods and factors, respectively.

Production of each good takes place according to constant elasticity of substitution (CES) production functions, which exhibit constant returns to scale. Therefore, the output price equals the per-unit cost in each sector, and firms make zero profits in equilibrium (Euler's Theorem). Profit maximization under constant returns to scale implies the equilibrium condition:

$$p_{ir} = p_{ir} - c_{ir}(p_{jr}, w_{fr}) = 0 \quad \text{(zero profit condition)} \qquad [2]$$

where c and p are the unit cost and profit functions, respectively.

Demand functions for goods and factors can be derived by Shepard's Lemma. It suggests that the first-order differentiation of the cost function with respect to an input price yields the cost-minimizing demand function for the corresponding input. Hence, the intermediate demand for good j in sector i is:

$$x_{jir} = \frac{\partial C_{ir}}{\partial p_{jr}} = Y_{ir} \cdot \frac{\partial c_{ir}}{\partial p_{jr}} \qquad [3]$$

and the demand for factor f in sector i is:

$$k_{fir} = \frac{\partial C_{ir}}{\partial w_{fr}} = Y_{ir} \cdot \frac{\partial c_{ir}}{\partial w_{fr}} \qquad [4]$$

The profit functions posses a corresponding derivative property (Hotelling's Lemma):

$$x_{jir} = \frac{\partial \Pi_{ir}}{\partial p_{jr}} = Y_{ir} \cdot \frac{\partial p_{ir}}{\partial p_{jr}} \quad \text{and} \quad k_{fir} = \frac{\partial \Pi_{ir}}{\partial w_{fr}} = Y_{ir} \cdot \frac{\partial p_{ir}}{\partial w_{fr}} \qquad [5]$$

The variable, price dependent input coefficients, which appear subsequently in the market clearance conditions, are thus:

$$a_{jir}^{x} = \frac{\partial c_{ir}}{\partial p_{jr}} = \frac{\partial p_{ir}}{\partial p_{jr}} \quad \text{and} \quad a_{fir}^{k} = \frac{\partial c_{ir}}{\partial w_{fr}} = \frac{\partial p_{ir}}{\partial w_{fr}} \qquad [6]$$

The model captures the production of commodities by aggregate, hierarchical (or nested) constant elasticity of substitution (CES) production functions that characterize the technology through substitution possibilities between capital, labor, energy and material (non-energy) intermediate inputs (KLEM). Two types of production functions are employed: those for fossil fuels (in our case *v* = COL, CRU, GAS) and those for non-fossil fuels (in our case *n* = EIS, ELE, OIL, ROI).

Figure A.1 illustrates the nesting structure in non-fossil fuel production. In the production of non-fossil fuels *nr*, non-energy intermediate inputs *M* (used in fixed coefficients among themselves) are employed in (Leontief) fixed proportions with an aggregate of capital, labor and energy at the top level. At the second level, a CES function describes the substitution possibilities between the aggregate energy input *E* and the value-added aggregate *KL* (For the sake of simplicity, the symbols α, β, ϕ and θ are used throughout the model description to denote the technology coefficients.):

$$Y_{nr} = \min\left\{(1-q_{nr})M_{nr}, q_{nr} f_{nr}\left[a_{nr}E_{nr}^{r^{KLE}} + b_{nr}KL_{nr}^{r^{KLE}}\right]^{1/r^{KLE}}\right\} \qquad [7]$$

with $s^{KLE} = 1/(1-r^{KLE})$ the elasticity of substitution between energy and the primary factor aggregate and q the input (Leontief) coefficient. Finally, at the third level, capital and labor factor inputs trade-off with a constant elasticity of substitution s^{KL}:

$$KL_{nr} = f_{nr}\left[a_{nr}K_{nr}^{r^{KL}} + b_{nr}L_{nr}^{r^{KL}}\right]^{1/r^{KL}}. \qquad [8]$$

As to the formation of the energy aggregate *E*, we employ several levels of nesting to represent differences in substitution possibilities between primary fossil fuel types as well as substitution between the primary fossil fuel composite and secondary energy, i.e. electricity. The energy aggregate is a CES composite of electricity and primary energy inputs *FF* with elasticity $s^{E} = 1/(1-r^{E})$ at the top nest:

$$E_{nr} = f_{nr}\left[a_{nr}ELE_{nr}^{r^E} + b_{nr}FF_{nr}^{r^E}\right]^{1/r^E}. \qquad [9]$$

The primary energy composite is defined as a CES function of coal and the composite of refined oil and natural gas with elasticity $s^{COA} = 1/(1-r^{COA})$. The oil-gas composite is assumed to have a simple Cobb-Douglas functional form with value shares given by q:

$$FF_{nr} = f_{nr}\left[a_{nr}COA_{nr}^{r^{COA}} + b_{nr}\left(OIL^{q_{nr}} \cdot GAS^{1-q_{nr}}\right)^{r^{COA}}\right]^{1/r^{COA}}. \qquad [10]$$

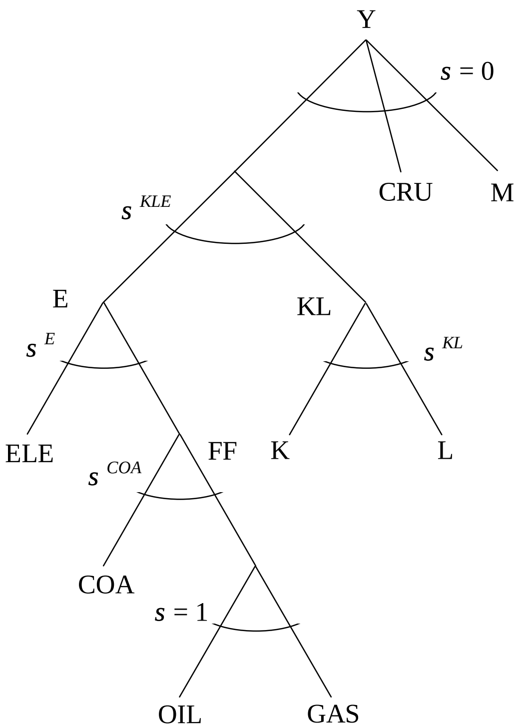

Figure A.1: Nesting structure of non-fossil fuel production

Fossil fuel resources v are modeled as graded resources. The structure of production of fossil fuels is given in Figure A.2. It is characterized by the presence of a fossil fuel resource in fixed supply. All inputs, except for the sector-specific resource R, are aggregated in fixed proportions at the lower nest. Mine managers minimize production costs subject to the technology constraint:

$$Y_{vr} = f_{vr}\left[a_{vr}R_{vr}^{r_v^f} + b_{vr}\left[\min\left(q_{vr}^K K_{vr}, q_{vr}^L L_{vr}, q_{vr}^E E_{vr}, q_{vr}^M M_{jvr}\right)\right]^{r_v^f}\right]^{1/r_v^f} \qquad [11]$$

The resource grade structure is reflected by the elasticity of substitution between the fossil fuel resource and the capital-labor-energy-material aggregate in production. The substitution elasticity between the specific factor and the Leontief composite at the top level is s_{vr}^{f} = 1/(1-r_{vr}^{f}). This substitution elasticity is calibrated in consistency with an exogenously given supply elasticity of fossil fuel e_{vr} according to

$$e_{vr} = \frac{1-g_{vr}}{g_{vr}} s_{vr}^{f} \qquad [12]$$

with γvr the resource value share.

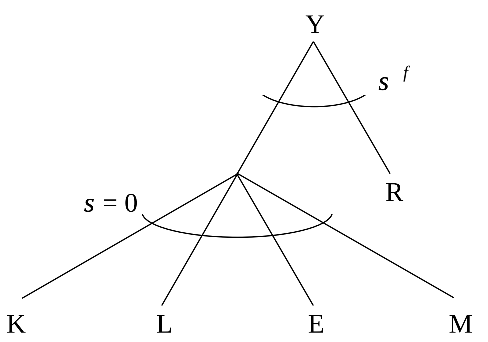

Figure A.2: Nesting structure for fossil fuel production

We now turn to the derivation of the factor demand functions for the nested CES production functions, taking into account the duality between the production function and the cost function The total cost function that reflects the same production technology as the CES production function for e.g. value added *KL* in non-fossil fuel production given by [8] is:

$$C_{nr}^{KL} = \frac{1}{f_{nr}} \left[a_{nr}^{s^{KL}} PK_{nr}^{1-s^{KL}} + b_{nr}^{s^{KL}} PL_{nr}^{1-s^{KL}} \right]^{1/\left(1-s^{KL}\right)} \cdot KL_{nr} \qquad [13]$$

where PK and PL are the per-unit factor costs for the industry including factor taxes if applicable. The price function for the value-added aggregate at the third level is:

$$PKL_{nr} = \frac{1}{f_{nr}}\left[a_{nr}^{s^{KL}} PK_{nr}^{1-s^{KL}} + b_{nr}^{s^{KL}} PL_{nr}^{1-s^{KL}}\right]^{1/\left(1-s^{KL}\right)} = c_{nr}^{KL} \tag{14}$$

Shepard's Lemma gives the price-dependent composition of the value-added aggregate as:

$$\frac{K_{nr}}{KL_{nr}} = f_{nr}^{s^{KL}-1}\left(a_{nr} \cdot \frac{PKL_{nr}}{PK_{nr}}\right)^{s^{KL}}, \quad \frac{L_{nr}}{KL_{nr}} = f_{nr}^{s^{KL}-1}\left(b_{nr} \cdot \frac{PKL_{nr}}{PL_{nr}}\right)^{s^{KL}} \tag{15}$$

In order to determine the variable input coefficient for capital and labor $a_{nr}{}^{K} = K_{nr} / Y_{nr}$ and $a_{nr}{}^{L} = L_{nr} / Y_{nr}$, one has to multiply [15] with the per unit demand for the value added aggregate KL_{nr} / Y_{nr}, which can be derived in an analogous manner. The cost function associated with the production function [7] is:

$$PY_{nr} = \left(1-q_{nr}\right) PM_{nr} + \frac{q_{nr}}{\tilde{f}_{nr}}\left[\tilde{d}_{nr}^{s^{KLE}} PE_{nr}^{1-s^{KLE}} + \tilde{b}_{nr}^{s^{KLE}} PKL_{nr}^{1-s^{KLE}}\right]^{\frac{1}{1-s^{KLE}}} \tag{16}$$

and

$$\frac{KL_{nr}}{Y_{nr}} = q_{nr}\, \tilde{f}_{nr}^{s^{KLE}-1}\left(\tilde{b}_{nr} \cdot \frac{PY_{nr}}{PKL_{nr}}\right)^{s^{KLE}} \tag{17}$$

with q_{nr} the *KLE* value share in total production. The variable input coefficient for e.g. labor is then:

$$a_{nr}^{L} = q_{nr}\, f_{nr}^{s^{KL}-1}\, \tilde{f}_{nr}^{s^{KLE}-1}\left(b_{nr} \cdot \frac{PKL_{nr}}{PL_{nr}}\right)^{s^{KL}}\left(\tilde{b}_{nr} \cdot \frac{PY_{nr}}{PKL_{nr}}\right)^{s^{KLE}} \tag{18}$$

A.2 Households

In each region, private demand for goods and services is derived from utility maximization of a representative household subject to a budget constraint given by the income level *INC*. The agent is endowed with the supplies of the primary factors of production (natural resources used for fossil fuel production, labor and capital) and tax revenues. In our comparative-static framework, overall investment demand is fixed at the reference level. The household's problem is then:

$$\underset{d_{ir}}{Max}\ W_r\left(d_{ir}\right)\ \ s.t.\ \ INC_r=\sum_f w_{fr}\,\overline{k}_{fr}+TR_r=\sum_i p_{ir}d_{ir} \tag{19}$$

where W is the welfare of the representative household in region r, d denotes the final demand for commodities, $\overline{k}$ is the aggregate factor endowment of the representative agent and TR are total tax revenues. Household preferences are characterized by a CES utility function. As in production, the maximization problem in [1] can thus be expressed in form of an unit expenditure function e or welfare price index pw, given by:

$$pw_r=e_r\left(p_{ir}\right) \tag{20}$$

Compensated final demand functions are derived from Roy's Identity as:

$$d_{ir}=\overline{INC}_r\,\frac{\partial e_r}{\partial p_{ir}} \tag{21}$$

with $\overline{INC}$ the initial level of expenditures.

In the model, welfare of the representative agent is represented as a CES composite of a fossil fuel aggregate and a non-fossil fuel consumption bundle. Substitution patterns within the latter are reflected via a Cobb-Douglas function. The fossil fuel aggregate in final demand consists of the various fossil fuels (fe = COL, OIL, GAS) trading off at a constant elasticity of substitution. The CES utility function is:

$$U_r=\left[a_r\left(\sum_{fe} b_{fe,r}C_{fe,r}^{r^F}\right)^{r^C/r^F}+f_r\left(\prod_{j\notin fe} C_{jr}^{q_j}\right)^{r^C}\right]^{1/r^C} \tag{22}$$

where the elasticity of substitution between energy and non-energy composites is given by s_C = 1/(1-r_C), the elasticity of substitution within the fossil fuel aggregate by s_{FE} = 1/(1-r_{FE}), and q_j are the value shares in non-fossil fuel consumption. The structure of final demand is presented in Figure A.3.

Total income of the representative agent consists of factor income, revenues from taxes levied on output, intermediate inputs, exports and imports, final demand as well as tax revenues from CO_2 taxes (TR) and a baseline exogenous capital flow representing the balance of payment deficits *B* less expenses for exogenous total investment demand *PI·I*. The government activity is financed through lump-sum levies. It does not enter the utility function and is hence exogenous in the model. The budget constraint is then given by:

$$PC_r \cdot C_r = PL_r \cdot \overline{L}_r + PK_r \cdot \overline{K}_r + \sum_v PR_{vr} \cdot \overline{R}_{vr} + TR_r + \overline{B}_r - PI_r \cdot I_r \qquad [23]$$

with *C* the aggregate household consumption in region *r* and *PC* its associated price.

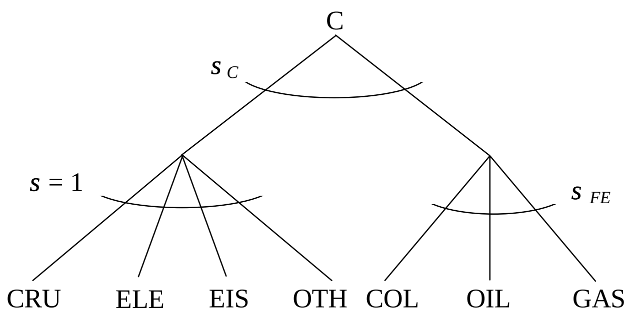

Figure A.3: Structure of household demand

A.3 Foreign Trade

All commodities are traded in world markets. Crude oil and coal are imported and exported as a homogeneous product, reflecting empirical evidence that these fossil fuel markets are rather integrated due to cheap shipping possibilities. All other goods are characterized by product differentiation. There is imperfect transformability (between exports and domestic sales of domestic output) and imperfect substitutability (between imports and domestically sold domestic output). Bilateral trade flows are subject to export taxes, tariffs and transportation costs and calibrated to the base year 1995. There is an imposed balance of payment constraint to ensure trade balance, which is warranted through flexible exchange rates, incorporating the benchmark trade deficit or surplus for each region.

On the output side, two types of differentiated goods are produced as joint products for sale in the domestic markets and the export markets, respectively. The allocation of output between domestic sales D and international sales X is characterized by a constant elasticity of transformation (CET) function. Hence, firms maximize profits subject to the constraint:

$$Y_{ir} = f_{ir}\left[a_{ir} D_{ir}^{h} + b_{ir} X_{ir}^{h}\right]^{1/h} \qquad [24]$$

with $s^{tr} = 1/(1 + h)$ the transformation elasticity.

Regarding imports, the standard Armington convention is adopted in the sense that imported and domestically produced goods of the same kind are treated as incomplete substitutes (i. e. wine from France is different from Italian wine). The aggregate amount of each (Armington) good A is divided among imports and domestic production:

$$A_{ir} = f_{ir}\left[a_{ir} D_{ir}^{r^D} + b_{ir} M_{ir}^{r^D}\right]^{1/r^D} \qquad [25]$$

In this expression $s^{D} = 1/(1\text{-}r^{D})$ is the Armington elasticity between domestic and imported varieties. Imports M are allocated among import regions s according to a CES function:

$$M_{ir} = f_{ir}\left[\sum_{s} a_{ir} X_{isr}^{r^M}\right]^{1/r^M} \qquad [26]$$

with X the amount of exports from region s to region r and $s^{M} = 1/(1\text{-}r^{M})$ the Armington elasticity among imported varieties. Intermediate as well as final demands are, hence, (nested CES) Armington composites of domestic and imported varieties.

The assumption of product differentiation permit the model to match bilateral trade with cross-hauling of trade and avoids unrealistically strong specialization effects in response to exogenous changes in trade (tax) policy. On the other hand, the results may then be sensitive to the particular commodity and regional aggregation chosen in the model.

A.4 Carbon emissions

Carbon emissions are associated with fossil fuel consumption in production, investment, government and private demand. Carbon is treated as a Leontief (fixed coefficient) input into production and consumption activities. Each unit of a fuel emits a known amount of carbon where different fuels have different carbon intensities. The applied carbon coefficients are 25 MT carbon per EJ for coal, 14 MT carbon per EJ for gas and 20 MT carbon per EJ for refined oil.

Carbon policies are introduced via an additional constraint that holds carbon emissions to a specified limit. The solution of the model gives a shadow value on carbon associated with this carbon constraint. This dual variable or shadow price can be interpreted as the price of carbon permits in a carbon permit system or as the CO2 tax that would induce the carbon constraint in the model. The shadow value of the carbon constraint equals the marginal cost of reduction. It indicates the incremental cost of reducing carbon at the carbon constraint. The total costs represent the resource cost or dead-weight loss to the economy of imposing carbon constraints. Carbon emission constraints induce substitution of fossil fuels with less expensive energy sources (fuel switching) or employment of less energy-intensive manufacturing and production techniques (energy savings). The only means of abatement are hence inter-fuel and fuel-/non-fuel substitution or the reduction of intermediate and final consumption.

Given an emission constraint producers as well as consumers must pay this price on the emissions resulting from the production and consumption processes. Revenues coming from the imposition of the carbon constraint are given to the representative agent. The total cost of Armington inputs in production and consumption that reflects the CES production technology in [25] but takes CO2 emission restrictions into account is:

$$C_{ir}^{A} = \left[\left(a_{ir}^{s^D} PD_{ir}^{1-s^D} + b_{ir}^{s^A} PM_{ir}^{1-s^D} \right)^{1/\left(1-s^D\right)} + t_r \cdot a_i \right] \cdot A_{ir} \qquad [27]$$

with a_i the carbon emissions coefficient for fossil fuel i and t the shadow price of CO_2 in region r associated with the carbon emission restriction:

$$\overline{CO2}_r = \sum_i A_{ir} \cdot a_i \qquad [28]$$

where $\overline{CO2}_r$ is the endowment of carbon emission rights in region r.

A.5 Zero Profit and Market Clearance Conditions

The equilibrium conditions in the model are zero profit and market clearance conditions. Zero profit conditions as derived in [2] require that no producer earns an “excess” profit in equilibrium. The value of inputs per unit activity must be equal to the value of outputs. The zero profit conditions for production, using the variable input coefficient derived above, is:

$$PK \cdot a_{ir}^{K} \cdot Y_{ir} + PL \cdot a_{ir}^{L} \cdot Y_{ir} + \sum_{j} PA_{j} \cdot a_{jir}^{M} \cdot Y_{ir} = PY_{ir} \cdot Y_{ir} \,. \qquad [29]$$

The market clearance conditions state that market demand equals market supply for all inputs and outputs. Market clearance conditions have to hold in equilibrium. Domestic markets clear, equating aggregate domestic output plus imports, i.e. total Armington good supply, to aggregate demand, which consists of intermediate demand, final demand, investment and government demand:

$$A_{ir} = \sum_{j} Y_{jr} \frac{\partial p_{jr}^{Y}}{\partial PA_{ir}} + C_{r} \frac{\partial e_{r}}{\partial PA_{ir}} \qquad [30]$$

with PA the price of the Armington composite. p_{ir}^{Z} is the per unit zero profit function with Z the name assigned to the associated production activity. The derivation of p_{ir}^{Z} , with respect to input and output prices, yields the compensated demand and supply coefficients, e.g. $\partial p_{jr}^{Y} / \partial PA_{ir} = a_{ijr}^{A}$ the intermediate demand for Armington good i in sector j of region r per unit of output Y. Output for the domestic market equals total domestic demand:

$$Y_{ir} \frac{\partial p_{ir}^{Y}}{\partial PD_{ir}} = \sum_{j} A_{jr} \frac{\partial p_{jr}^{A}}{\partial PD_{ir}} \qquad [31]$$

with PD the domestic commodity price. Export supply equals import demand across all trading partners:

$$Y_{ir} \frac{\partial p_{ir}^{Y}}{\partial PX_{ir}} = \sum_{s} M_{is} \frac{\partial p_{is}^{M}}{\partial PX_{ir}} \qquad [32]$$

with PX the export price. Aggregate import supply equals total import demand:

$$M_{ir} = A_{ir} \frac{\partial p_{ir}^{A}}{\partial PM_{ir}} \qquad [33]$$

where *PM* is the import price.

Primary factor endowment equals primary factor demand:

$$\overline{L}_r = \sum_i Y_{ir} \frac{\partial p^Y_{ir}}{\partial PL_r}, \qquad [34]$$

$$\overline{K}_r = \sum_i Y_{ir} \frac{\partial p^Y_{ir}}{\partial PK_r}, \qquad [35]$$

$$\overline{R}_{vr} = Y_{vr} \frac{\partial p^Y_{vr}}{\partial PR_{vr}}. \qquad [36]$$

An equilibrium is characterized by a set of prices in the different goods and factor markets such that the zero profit and market clearance conditions stated above hold.

Table A.1: Default values of key substitution and supply elasticities

	Description	Value
Substitution elasticities in non-fossil fuel production		
s^{KLE}	Energy vs. value added	0.8
s^{KL}	Capital vs. labor	1.0
s^{E}	Electricity vs. primary energy inputs	0.3
s^{COL}	Coal vs. gas-oil	0.5
Substituion elasticities in final demand		
s_{C}	Fossil fuels vs. non-fossil fuels	0.8
s_{FE}	Fossil fuels vs. fossil fuels	0.3
Elasticities in international trade (Armington)		
s^{D}	Substitution elasticity between imports vs. domestic inputs	4.0
s^{M}	Substitution elasticity between imports vs. imports	8.0
s^{tr}	Transformation elasticity domestic vs. export	2.0
Exogenous supply elasticities of fossil fuels *e*		
Crude oil		1.0
Coal		0.5
Natural gas		1.0