The *Service Sentiment Indicator* — A Business Climate Indicator for the German Business–Related Services Sector

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Summary: No other area of the German economy has developed so emphatically in the past ten years as have business–related services. Regardless their growing importance, business–related services do still play a minor role in official statistics. Above all, official statistics do not provide up–to–date information on the state of the business cycle of this sector. In a situation where such quantitative information is lacking, data obtained from business surveys give important guidelines on the state of this part of the economy. In this paper we show how a reliable compounded business climate indicator for business–related services can be constructed from both business survey and national accounts data.

1 Introduction

In the past year, hardly any another sector of the German economy industry has developed as dynamically as the service sector. The structural change from manufacturing to services has deepened in the last decade. Moreover, structures are not only altering amongst the economic sectors, but also within the service sector itself. Business–related services are gaining in importance, whereas traditional services, such as retail and wholesale trade, are losing their importance.

The strong growth of the service sector and especially of business–related services has led to a heightened attentiveness as far as the public, the media and politics are concerned. In official statistics, however, business–related services plays a rather subordinate role. As far as these official statistics are concerned, around 80 separate statistics are available in total, covering varying characteristics such as the number of businesses, firms and employees, yet a systematic recording of services is missing.[1] As a result, it is not astonishing that no exact reports can be released which constitute the overall economic importance of the business–related services sector. Even the Federal Statistical Office has at its disposal almost up–to–data and precise information on this sector.[2] This lack of data has recently been criticized by Hax (1998) and the Council of Economic Advisors (Sachverständigenrat, 1998).

To compensate for the lack of up-to- date data for business–related services, the Center for European Economic Research (ZEW) is carrying out a quarterly business survey in this sector in cooperation with Germany's largest credit rating agency CREDITREFORM since June 1994.

Since there is little information on business–related services available in official statistics, the ZEW/CREDITREFORM business survey has in some ways gained similar importance

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[1] Link (1996) refers to the usability of official statistics for the service sector in great detail.

[2] The Federal Statistical Office calculates the Gross National Product for the sector "services not noted elsewhere — among them are business–related services — on the basis of the value–added statistics which are available yearly with a delay of 18 months and the employment statistics, which are processed quarterly with delays of from six to seven months.

as the well known business surveys of the ifo institute at Munich which were developed after World War II to modernize and supplement the official statistics and which have been the only available information on the state of the economy for a long time (Strigel, 1997).

A further advantage of the ZEW/CREDITREFORM survey is that the results can already be publicized between one and a half to two weeks after the survey's data collection period. According to Oppenländer (1997), this quick data procession is an important reason as to why survey data should at least bear equal importance as quantitative data.

In this paper, we show the construction of the *Service Sentiment Indicator*, a business climate index which was first released in the second quarter of 1998 and which is gaining increasing recognition in the media, among financial institution and in economic policy.

2 The ZEW/CREDITREFORM business survey

The definition of business–related services is controversially discussed in the literature. To our knowledge, no clear–cut and generally accepted definition exists. We follow the convention proposed by Hass (1995), Klodt et al. (1997) and Strambach (1995), who define business–related services by enumeration of certain sectors. In this paper, the following sectors are defined as business–related services (WZ93 code in the second column of the table):[3]

Sector	WZ '93
Computer services	72100, 72201–02, 72301–04, 72601–02, 72400
Legal & book–keeping activities	74123, 74127, 74121–22
Business management	74131–32, 74141–42
Architectural activities	74201–04
Technical testing & planning	74205–09, 74301–04
Advertising	74844, 74401–02
Vehicle renting	71100, 71210
Machine renting	45500, 71320, 71330
Cargo handling & storing	63121, 63403, 63401
Waste & refuse disposal	90001–90007

According to Hass (1995), business–related services represent an important link between the secondary and tertiary sectors. They support the quality and distribution of products. Furthermore, in literature it is presumed that business–related services ease the way to innovation, as is indicated in Licht et al. (1997). Additionally, as Steil (1997) points out, business–related services exhibit a well above–average firm foundation rate. In the last ten years, business–related services profited from distinct outsourcing–activities of the manufacturing sector. This can be partly lead back to its close relationship to the production sector. Some authors such as Audretsch and Yamawaki (1991), Hass (1995)

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[3]The WZ93 code is a classification system which was developed by the German Federal Statistical Office in accordance with the European standard NACE, Rev. 1. The aim of this code is to be able to classify every firm with regard to its main economic activity.

and Lichtblau et al.(1996) use Input-Output tables to show that the relationship between services — here, particularly the business–related services — and the industry is in no way substitutive, as Fourastié (1950) amongst others had stated. The relationship between both areas is instead complementary.

The ZEW sends out a single–sided questionnaire every three months to about 4000 firms belonging to the ten economic sectors. The survey is constructed as a panel. It is stratified random sample, stratified with respect to the ten sectors, regional affiliation (East/West Germany) and five size classes (two for East, three for West Germany). Details on the survey design and on the way expansion factors are constructed are given in Kaiser et al. (1999).

The data collection period starts three weeks prior to the end of a quarter. The questionnaires are mostly sent back to the ZEW by fax. After two weeks, those firms who still have not replied are sent a reminder. Altogether, the response rate amounts to about 30 per cent. As a thankyou for filling out the questionnaire, the participating firms receive a four pages report. In addition, they can call up further information over the internet.[4]
The questionnaire is divided up into two parts. In the first part, firms are asked to indicate on a three–point Likert scale whether their sales, prices, demand, profits and number of employees have, in the respective previous quarter, either increased, stayed the same, or decreased. Moreover, they are supposed to give an assessment for the coming quarter. The second part of the survey is dedicated to present–day economic and political issues.[5] The following lines are exclusively concerned with the first part of the survey.

3   Dimensions of the business–related services sector

Out of the 80 single statistics which the Central Statistics Office makes available, four are potentially able to report on growth and overall economic importance of business–related services. These are: (1) the Mikrozensus,[6] (2) the value–added statistic,[7] (3) the four–annual cost–structure statistic,[8] and (4) the employment statistic[9]. Although all four statistics are not primarily constructed for the recording of the service sector, they can be used as secondary statistics. In general there is the problem that all four statistics are only available with considerable delays and are therefore only partly suitable for economic observation. Another problem is the fact that the four statistics are not really comparable, since they are based on different levels of investigation. The Mikrozensus is a household survey, the value–added statistic collects data on the firm level and both the four–annual cost–structure and the value–added statistic are based on plant–level data. Services can therefore be described either in terms of a functional or a sectoral perspective. In the functional approach, activities performed by individuals stand in the foreground.

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[4]The internet address is: http://www.zew.de/aktuell/branchenreport/wb-BreportStart.html.

[5]On request we are happy to send a copy of the questionnaire.

[6]The Mikrozensus is an annual household survey conducted by the Federal Statistical office.

[7]The value–added statistic is collected and processed by the Federal Statistical Office.

[8]The four–annual cost–structure statistic should not be mixed up with the quarterly cost–structure statistics in the construction and manufacturing sector.

[9]The employment statistic is collected by the Federal Labor Office and are processed by the Institute for Labor Research (IAB).

This facilitates the contemplation of service activities in all sectors, including the producing sector. If the analysis is based on sector level, every company is categorized into a certain sector according to its main economic activity. This classification scheme forms the basis of the economy branch system WZ93. In contrast to the sectoral approach, the data material available for the functional analysis is sketchy and based on household surveys. Also, databases which integrate the sectoral and the functional perspective are rare and are associated with high costs.

Due to these reasons, the sectoral perspective forms the basis of this section. The WZ93-Code is used on a three–digit–level since all the included statistics — except for the value–added statistic — are coded on the WZ93 three–digit level. As a result, a detailed ordering of the branches such as in the ZEW/CREDITREFORM survey is not possible. Aside from the problems which we have dealt with up until now, the recorded secondary statistics differ considerably over the course of time, with regard to their periodicity and up–to–dateness.

Keeping these this difficulties in mind, let us now turn to some figure on the dimensions and growth of the service sector. According the employment statistic, it turns out that business–related services had a share of six percent in total employment (employment above the social security level) in Germany in 1997.[10] The growth rate is 1.5 percentage points between 1996 and 1997 and would be considerably higher if there was not the recession in the construction sector which caused the growth rate of architects and technical planning to become negative. More importantly, computer services are missing since they are not explicitly defined by the classification scheme used in the employment statistic.

In their ZEW–report for the Federal Ministry of Science and Research on the innovative activities of the service sector, Ebling et al. (1999) have searched various databases in order to describe growth and dimensions of the service sector. The number of firms in the entire service sector increased by 3.1 percentage points between 1994 and 1996. Computer services experienced the largest growth rates in this time period. They have grown by 17.7 percentage points. Growth rates of technical services (architecture, technical advice and planning) were 11.9 percentage points, these of the transportation sector 17 percentage points and these of heterogeneous segment other business–related services 3.9 percentage points. In comparison, the number of firms in the manufacturing sector decreased by 8.7 percentage points. The entire service sector makes 85.9 percent of the total number of firms in Germany. The share of software, transport and technical services is 15,3 percent. Table 1 gives an overview of the number of firms in the service sector.

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[10]Note that the employment statistic uses a different classification scheme than the WZ93 code. Therefore, the employment statistic code was manually transferred into the WZ93 code.

Table 1: Number of firms and growth in business–related services

	1994		1996		change
	total	in %	total	in %	in%
services	372,762	100	384,411	100	3.1
of it:					
— Wholesale trade	42,789	11.5	40,062	10.4	-6.4
— Retail trade	128,070	34.4	127,568	33.2	-0.4
— Cargo handling & storing	47,463	12.7	55,529	14.4	17.0
— Banks, insurance	7,313	2.0	7,033	1.8	-3.8
— Software	9,059	2.4	10,659	2.8	17.7
— Technical services	1,822	0.5	2,039	0.5	11.9
— other business–related services	136,246	36.6	141,521	36.8	3.9
manufacturing	68,331		62,843		-8.7

It has to be stressed at this point that all figures presented in this section are based on a three–digit classification scheme whereas the ZEW/CREDITREFORM business survey is based on a five–digit code. Therefore, the tables overestimate growth and economic importance of the sectors under consideration in the ZEW/CREDITREFORM business survey.

4 Construction of the Service Sentiment Indicator

4.1 A three stages approach

In this section we describe how we constructed the SSI in a three stages approach. The SSI is a compounded indicator which consists of both survey and national accounts data. As Kaiser (1998) has shown, the ZEW/CREDITREFORM business survey data are quite responsive to political events implying that such events may influence the response patterns of the survey participants. In order to dampen these influences, we decided to additionally use national accounts data in the construction of the indicator. Our approach is in this respect comparable to Kranendonk and Jansen (1997) who construct a leading indicator for The Netherlands.
The construction of the SSI therefore proceeds in three steps. In a first step, we construct a partial indicator based on the ZEW/CREDITREFORM business survey data. This partial indicator is mixed in with national accounts data, namely with the gross value added (GVA) in "services not noted elsewhere". The ten sectors investigated in the ZEW/CREDITREFORM business survey are among this sector. There are no data which are more better up–to–date GVA in "services not noted elsewhere available on a quarterly basis for this sector. The national accounts data are collected by the Federal Statistical office and are published quarterly with a lag of one quarter. Hence, the current GNP in services not noted elsewhere is forecasted in a second step of the SSI–construction. In the third step, we attach weights to each of the two partial indicators stemming from the business survey and from the national accounts data.

4.2 The partial indicator based on the survey data

In a pre–step to the construction of the indicator, the individual responses of the partici-
pating firms have to be aggregated. As usual for this kind of data, we use sales (demand,
employment, profits) balances. Balances denote the share of firms reporting increased
sales minus the share of firms reporting decreased sales.[11]

In the ZEW/CREDITREFORM business survey, it is asked for both realized changes in
sales and for expectations for the next quarter. In order to merge both information, we
proceed analogously to the ifo–business climate indicator. Our aggregates index for sales
(likewise for the other variables) is

$$ UI \; = \; \left((\text{Sales judgement}+200) \; \cdot \; (\text{Sales sales expectation}+200)\right)^{1/2} \; - \; 200, \qquad (1) $$

where UI denotes the sales index. Equation (1) is the geometric mean of sales expec-
tations and sales judgements. In order to avoid negative values in the square root, 200
is added. The time series used for the calculation of UI are seasonally adjusted with
an approach described in appendix A. The aggregated indices for the other variables are
constructed using the same procedure.

In the following, we use first differences of the aggregated indices since even survey data
may have non–stationary components (Entorf and Kavalkakis, 1992).

How can the information contained in the aggregated indices of the four variables be
condensed to a single indicator? In principle, one could attach arbitrary weights to the
individual variables. However, especially in the present case, the variables in question a
highly correlated with one another as becomes apparent from Table 3.

*Table 3: Correlations between the first differences of the aggregated indices of the survey
variables*

	Sales	Profits	Demand	Employment
Sales	1.0000			
Profits	0.7532	1.0000		
Demand	0.8012	0.6581	1.0000	
Employment	0.4554	0.5396	0.6016	1.0000

The construction of the partial indicator from the ZEW/CREDITREFORM business
survey therefore follows Rao's (1971) proposal which was successfully implemented in the
construction of the G–Mind (Szczesny et al., 1997). Rao suggests a regression–like ap-
proach to determine the weights used in a business climate indicator. Rao's suggestion
is to transform the variables in question — in our case the aggregated sales, demand,
profit, prices and employment indices — into variables with mean 0 and variance 1. The
variance–covariance matrix $(X'X)^{-1}$ is then identical to the correlation matrix. If an
appropriate proportionality vector r is defined, it can be shown that the weights of the
individual components can be uniquely defined. Rao's approach leads to correlation–
adjusted weights for the survey variables.

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[11]The price variables are left out here since, as Entorf (1993) has shown for the ifo data, price
variables are often not very reliable in business surveys.

As a consequence of the high correlation of the survey variables, Rao's correlation–adjusted weights may become negative which is economically implausible. Therefore, we decided to just consider the sales and employment variables since these exhibit the weakest correlation with one another.

We define the proportionality vector as $r = (1, .75)'$ since sales is the more important variable for an employee. After adjustment for correlation and normalization, the final weights for the two components of the partial indicator of the survey data are 0.6909 for the sales variable and 0.3091 for the employment variable. These weights have an immediate interpretation: The first difference of the aggregated sales variables have a weight of 69.09 percent whereas the employment variables' weight is 30.91 percent.

Since the proportionality vector r is arbitrarily defined, we conducted a sensitivity analysis where we tried alternative combinations of the elements of r. Figure 1 plots the different partial indicator yield from alternative weighting of the sales and employment variables against a time axis. It turns out that the partial indicator calculated from the survey data is quite robust against different weighting schemes. Major differences only arise if the employment variable receive twice the value in r as the sales variables. These differences, however, diminish after the fourth quarter 1995.

$$\boxed{\textbf{Insert Figure 1 at about here}}$$

4.3 The partial indicator from national accounts data

Hier kommt Herbie!!!.

4.4 Merging both partial indicators

In the third step of the SSI–construction, both components are merged into one single indicator. The weighting scheme between the partial indicator from the survey and from the national accounts data is .8 for the survey part and .2 for the national accounts part, $w = (.8, .2)'$. These weights were chosen since they guarantee a smooth course of the indicator of the time period under investigation. Figure 2 shows a sensitivity analysis of the weights attached to the partial indicators. Both partial indicators are measured in first differences so that we decided to cumulate the compounded indicator's values. Figure 2 thus presents *cumulated* values of the compounded indicator.

It turns out from Figure 2 that the *shapes* of the curves are very similar while the levels differ considerably. the level, however, is meaningless in a business–cycle analysis context. Thus, our indicator is quite robust against alternative weighting schemes.

$$\boxed{\textbf{Insert Figure 2 at about here}}$$

One major difficulty of the indicator we have developed so far is its interpretation. It mixes first differences of aggregated survey data with first differences of national accounts data so that the indicator's dimension is undetermined. Therefore, we decided to put the indicator's value on an ordinal scale which starts from zero. The maximum remains undetermined since the time series are too short to cover an entire business cycle. The

ordinal indicator takes on the value 1 (2, 3, ...) if the compounded cumulated indicator's value is less than -3 (between -3 and -2.9, between -2.9 and -2.8, and so forth in .1–steps). An upward direction of the final indicator, the *SSI*, reflects an expansion period whereas an downward directed indicator reflects a recessive period. If there are no changes, the situation in the business–related services sector remains unchanged — note that all variables used in the indicator's construction were seasonally adjusted and hence is the indicator seasonally adjusted. As long as the indicator has not passed through a complete business cycle, it can neither reflect overheating nor recessive tendencies. As soon as sufficient data is available, we will be able to attach bands to the indicator.

4.5   In search of a reference indicator

A final step in the construction of business climate indicators usually is to compare it to some reference time series. However, as we have discussed in detail, there is no such time series available for business–related services despite the national accounts data which were included in the indicator's construction and thus cannot be used.

In order to give at least a hint with regard to the indicator's performance, we have plotted the SSI against a number of time series related to manufacturing industries. Business–related service are closely connected to manufacturing industries so that the development of the manufacturing sector should influence business–related services with or without a lag. Figure 3 displays the relation between the SSI and the ifo capacity utilization index of commodities. Both time series roughly show the same pattern, and it seems as if there is no lead/lag relationship between them. If the ifo capacity utilization index for manufacturing is used, is turns out that the ifo index leads by two quarters until 1996. The lead–relationship then diminishes so that at the margin there also is neither a lead nor a lag relationship. Similar patterns occur if the SSI is compared to the sales index of manufacturing industries, the net production index of manufacturing industries, investment indices, commodity indices and consumption indices.

$$\boxed{\textbf{Insert Figure 3 at about here}}$$

It is clearly still desirable to compare the SSI to a time series which actually represents the state of the economy in the business–related services sector. As long as no such data are provided by official statistics, there is, however, no way to test the indicator against actual outcomes.

5   Summary and conclusion

This paper showed how a reliable business climate indicator can be constructed even when the data material at hand is weak. To our best knowledge, this is the first approach to construct a business climate indicator for business–related services.

The indicator is constructed in three steps. In the first step, data from a business survey in the business–related services sector are condensed into a single (partial) indicator. Since business survey data are often subject to political news and short–time influences,

we constructed a compounded indicator. National accounts data on the sector "services not noted elsewhere were used as a second component of the indicator in a second step. Since these national accounts data are only available with a lag of one quarter, we have to forecast the current value of the national accounts data by a linear regression. In a third step, both partial indicators are merged to the *Service Sentiment Indicator*.

A drawback of the insufficient availability of data on the services sector is that a reference time series of our indicator is missing. Therefore, we have conducted several sensitivity analysis at various steps in the construction of the indicator. The indicator proved to be very robust against alternative weighting schemes.

Figure 1: Attaching alternative weighting schemes to the partial indicator from the survey data: A sensitivity analysis
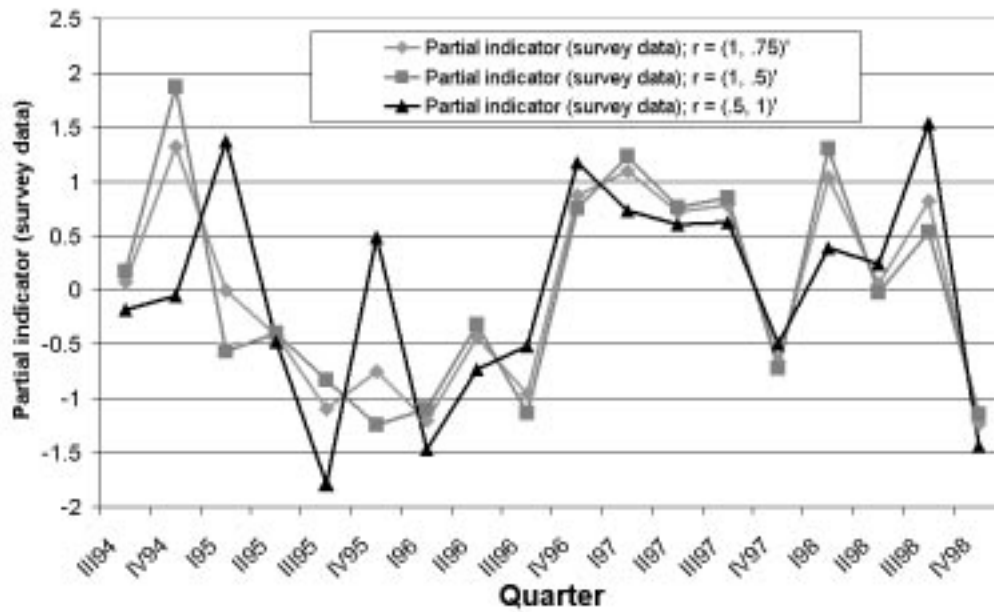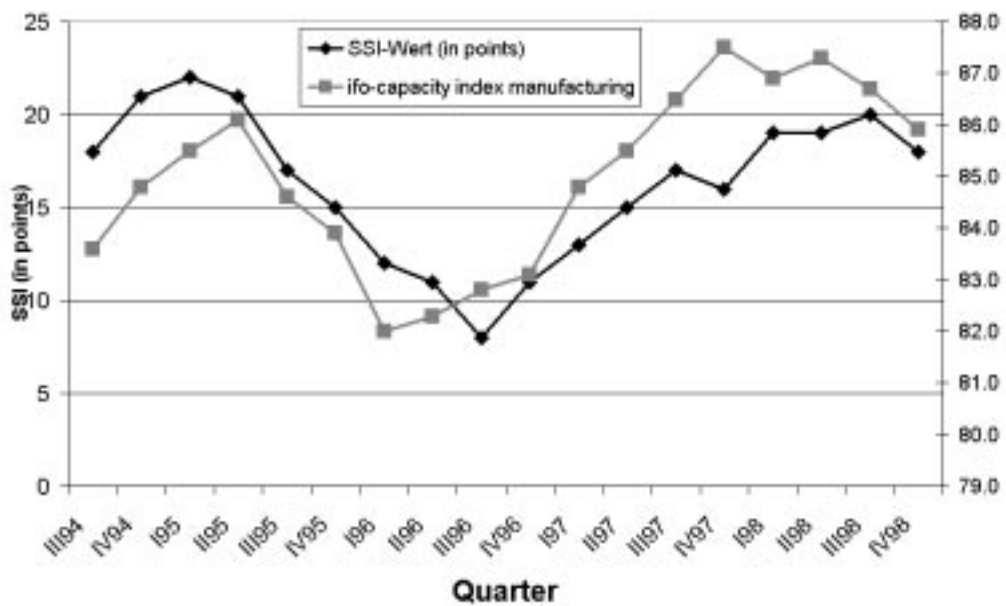

The figure compares different weighting schemes with one another. $r = (1, .75)'$ means that the weight of the sales variable is 100 percent into the partial indicator from the survey data and that the weight of the employment variable is 75 percent, and so forth.

Figure 2: Attaching alternative weighting schemes to compounded indicator: A sensitivity analysis
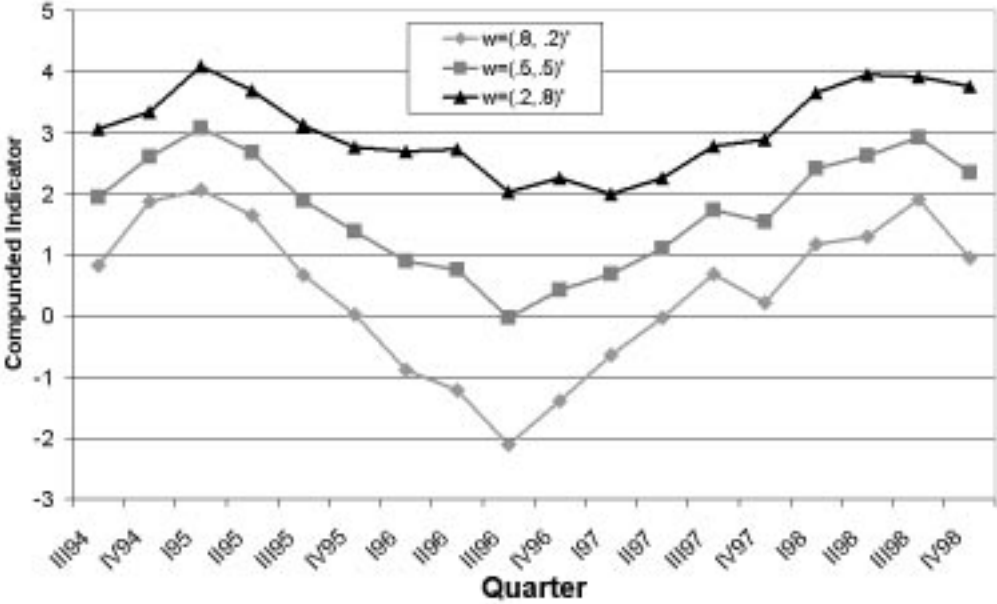
The figure compares different weighting schemes with one another. $w = (.8, .2)'$ means that the weight of the partial indicator of the business survey if 80 percent and that the weight of the national accounts partial indicator is 20 percent.

Figure 3: The SSI and the ifo capacity utilization index commodities

The figure compares different weighting schemes with one another. $w = (.8, .2)'$ means that the weight of the partial indicator of the business survey if 80 percent and that the weight of the national accounts partial indicator is 20 percent.

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A   Seasonal adjustment of the survey data

One difficulty with the ZEW/CREDITREFORM business survey data is that the time series — especially the sales series — show strong seasonal patterns. Therefore, seasonal adjustment is needed. The classical approaches to seasonal adjustment do not apply here since the time series in question are too short. We thus present a seasonal adjustment method which can be used even for short time series.

If the seasonal patterns are independent of a time trend, the individual components of a time series X_t — time trend T_t, the cyclical component C_t, the seasonal component S_t and an irregular component u_t (where the subscript t denotes the corresponding point in time) — is given by

$$X_t \;=\; T_t \;+\; C_T \;+\; S_t \;+\; u_t \tag{2}$$

Alternative approaches using the ZEW/CREDITREFORM business survey data lead to the conclusion that their seasonal patterns are best captured by seasonal dummies. Since it is a quarterly survey, we include three quarterly dummies, S_i $(i \;=\; 1,\; 2,\; 3)$ and a constant term, a. Our regression equation is

$$X_t \;=\; a \;+\; b_1\, S_1 \;+\; b_2\, S_2 \;+\; b_3\, S_3 \;+\; \epsilon_t, \tag{3}$$

where the b_i's denote the coefficients to be estimated and ϵ_t is an i.i.d. normal distributed error term which includes besides the irregular components u_t also the cyclical component C_t. From applying an ordinary least squares regression on equation (3), the residuals $\hat{\epsilon}_t$ are obtained by

$$\hat{\epsilon}_t \;=\; X_t \;-\; \hat{X}_t, \tag{4}$$

where $\hat{X}_t$ denotes the fitted value of X_t. Since $\hat{\epsilon}_T$ contains beside the error term (which has a mean value of 0 and a constant variance by construction) also the cyclical component one additional step needs to be undertaken to yield the seasonally adjusted time series. Since the original time series did not exhibit a time trend, its mean value is added to $\hat{\epsilon}_t$ for all elements of the original time series:

$$\tilde{X}_t \;=\; \frac{1}{t}\sum_{t=1}^{T} X_t \;+\; \hat{\epsilon}_t, \tag{5}$$

where T is the total number of time periods. $\tilde{X}_t$ denotes the seasonally adjusted time series.