

# **Economic Implications of Alternative Allocation Schemes for Emission Allowances**

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## **Downloadable Appendix**

<ftp://ftp.zew.de/pub/zew-docs/div/allocation.pdf>

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## Appendix A: Algebraic Model Summary

This appendix provides an algebraic summary of the equilibrium conditions for our comparative-static model designed to investigate the economic implications of alternative allowance allocation rules. For the generic model the following assumptions apply:

- In the core model version with competitive factor and commodity markets all production activities exhibit non-increasing returns to scale. Goods are produced with capital, labor, energy and material (KLEM). Nested separable constant elasticity of substitution (CES) functions characterize the use of inputs in production. In the production of commodities, other than primary fossil fuels, non-energy inputs are employed in fixed proportions at the top level with an aggregate of energy, capital and labor. At the second level, a CES function describes the substitution possibilities between the energy aggregate and the aggregate of labor and capital. Finally, at the third level, capital and labor trade off with a constant elasticity of substitution. As to the formation of the energy aggregate, we allow sufficient levels of nesting to permit substitution between primary energy types, as well as substitution between the primary energy composite and secondary energy, i.e. electricity. In the production of fossil fuels all inputs, except for the sector-specific fossil fuel resource, are aggregated in fixed proportions at the lower nest. At the top level, this aggregate trades off with the sector-specific fossil fuel resource at a constant elasticity of substitution. The latter is calibrated in consistency with an exogenously given price elasticity of fossil fuel supply.
- A representative agent (RA) in each region is endowed with three primary factors: natural resources (used for fossil fuel production), labor and capital. The RA maximizes utility from consumption of a CES composite subject to a budget constraint with fixed investment demand (i.e. fixed demand for the savings good). The aggregate consumption bundle combines demands for fossil fuels, electricity and non-energy commodities. Total income of the RA consists of factor income and taxes (including revenues from carbon taxes or auctioned carbon permits).
- Supplies of labor, capital and fossil-fuel resources are exogenous. Labor and capital are mobile within domestic borders but cannot move between regions; natural resources are sector specific.
- All goods are differentiated by region of origin. Constant elasticity of transformation functions (CET) characterize the differentiation of production between production for the domestic markets and the export markets. Regarding imports, nested CES functions characterize the choice between imported and domestic varieties of the same good (Armington).
- Goods from regions which are not explicitly represented (rest of the world – ROW) are differentiated, and a set of horizontal export demand and import supply functions determine the trade between ROW and the regions whose production and consumption patterns are described in detail. In other words, the representation of ROW is reduced to import and export

flows with the explicit regions of the model where the latter are assumed to be price-takers with respect to ROW import and export prices.

As is customary in applied general equilibrium analysis, benchmark quantities and prices – together with exogenous elasticities (see Table A.6) – determine the parameters of the functional forms. The model calibration is based on GTAP5 data which provides consistent accounts of national production and consumption, trade, and energy flows for 1997 across 66 regions and 57 sectors.

For our quantitative analysis of carbon abatement under alternative permit allocation schemes, we employ 10 composite sectors. Energy goods in the model include primary energy carriers (coal, gas, crude oil) and secondary energy carriers (refined oil products and electricity). This disaggregation is essential in order to distinguish energy goods by carbon intensity and by degree of substitutability. Moreover, we represent other energy-intensive (non-energy) sectors (iron and steel, non-ferrous, metals, paper, pulp, and printing) that – in addition to the secondary energy carriers – are the candidates for a restricted allowance trading system as envisaged within the EU. The remaining production and services are summarized as a composite industry that produces a non-energy-intensive macro good (ROI).

With respect to the regional disaggregation, we restrict our core simulations to a single-region small-open-economy setting: Detailed domestic production and consumption patterns are described for one region (selected from the set of 57 regions in the GTAP5 database) while all other regions are condensed to *Rest of the World* with infinitely elastic export supply and import demand.

The model is formulated as a system of nonlinear inequalities. These inequalities correspond to two classes of equilibrium conditions: zero profit and market clearance. The fundamental unknowns of the system are two vectors: activity levels and prices. In equilibrium, each of these variables is linked to one inequality condition: an activity level to a zero-profit condition and a commodity (factor) price to a market-clearance condition.

In the algebraic exposition below, the notation  $\Pi_{ir}^z$  is used to denote the (zero-)profit function of sector  $j$  in region  $r$  where  $z$  is the name assigned to the associated production activity. Differentiating the profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotellings's lemma), which appear subsequently in the market clearance conditions. We use  $i$  (aliased with  $j$ ) as an index for commodities (sectors) and  $r$  (aliased with  $s$ ) as an index for regions. The label *EG* represents the set of energy goods and the label *FF* denotes the subset of fossil fuels. Tables A.1-A.6 explain the notations for variables and parameters employed within our algebraic exposition. Figures A.1-A.5 provide a graphical exposition of the production and final consumption structure.

### Zero Profit Conditions

1. Production of goods except fossil fuels:

$$\prod_{ir}^Y = \left( \theta_{ir}^{XROW} p^{W^{1-\eta}} + (1-\theta_{ir}^X) p_{ir}^{1-\eta} \right)^{\frac{1}{1-\eta}} - \sum_{j \in EG} \theta_{jir} p_{jr}^A$$

$$- \theta_{ir}^{KLE} \left[ \theta_{ir}^E p_{ir}^E 1^{-\sigma_{KLE}} + (1-\theta_{ir}^E) \left( w_r^{\alpha_{jr}^L} v_r^{\alpha_{jr}^K} \right)^{1-\sigma_{KLE}} \right]^{\frac{1}{1-\sigma_{KLE}}} \leq 0 \quad i \notin FF$$

where  $Y_{ir}$  ( $i \notin ff$ ) is the associated activity variable.

2. Production of fossil fuels:

$$\prod_{ir}^Y = \left( \theta_{ir}^{XROW} p^{W^{1-\eta}} + (1-\theta_{ir}^X) p_{ir}^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

$$- \left[ \theta_{ir}^Q q_{ir}^{1-\sigma_{Qj}} + (1-\theta_{ir}^Q) \left( \theta_{Lir}^{FF} w_r + \theta_{Kir}^{FF} v_r + \sum_j \theta_{jir}^{FF} p_{jr}^A \right)^{1-\sigma_{Qj}} \right]^{\frac{1}{1-\sigma_{Qj}}} \leq 0 \quad i \in FF$$

where  $Y_{ir}$  ( $i \in ff$ ) is the associated activity variable.

3. Sector-specific energy aggregate:

$$\prod_{ir}^E = p_{ir}^E - \left\{ \theta_{ir}^{ELE} p_{\{ELE,r\}}^{A^{1-\sigma_{ELE}}} + (1-\theta_{ir}^{ELE}) \left[ \theta_{ir}^{COA} p_{\{COA,r\}}^{A^{1-\sigma_{COA}}} + (1-\theta_{ir}^{COA}) \left( \prod_{j \in LQ} p_{jr}^{A^{\beta_{jir}}} \right)^{1-\sigma_{COA}} \right]^{\frac{1-\sigma_{ELE}}{1-\sigma_{COA}}} \right\}^{\frac{1}{1-\sigma_{ELE}}} \leq 0$$

where  $E_{ir}$  is the associated activity variable.

4. Armington aggregate:

$$\prod_{ir}^A = p_{ir}^A - \left[ \left( \theta_{ir}^A p_{ir}^{1-\sigma_A} + (1-\theta_{ir}^A) p_{ir}^{M^{1-\sigma_A}} \right)^{\frac{1}{1-\sigma_A}} + p_r^{CO2} a_i^{CO2} \right] \leq 0$$

where  $A_{ir}$  is the associated activity variable.

5. Aggregate imports across import regions:

$$\prod_{ir}^M = p_{ir}^M - \left( \sum_s \theta_{isr}^M p_{is}^X 1^{-\sigma_M} + \theta_{ir}^{MROW} p^W 1^{-\sigma_M} \right)^{\frac{1}{1-\sigma_M}} \leq 0$$

where  $M_{ir}$  is the associated activity variable.

6. Household consumption aggregate:

$$\Pi_r^C = p_r^C - \left( \theta_{Cr}^E p_{Cr}^E l^{-\sigma_{EC}} + (1 - \theta_{Cr}^E) \left[ \prod_{i \in FF} p_{ir}^{A^{Y_{ir}}} \right]^{l^{-\sigma_{EC}}} \right)^{\frac{1}{1-\sigma_{EC}}} \leq 0$$

where  $C_r$  is the associated activity variable.

7. Household energy aggregate:

$$\Pi_{Cr}^E = p_{Cr}^E - \left[ \sum_{i \in FF} \theta_{iCr}^E p_{ir}^A l^{-\sigma_{FF,C}} \right]^{\frac{1}{1-\sigma_{FF,C}}} \leq 0$$

where  $E_{Cr}$  is the associated activity variable.

8. Investment:

$$\Pi_r^I = p_r^I - \sum_i \theta_{ir}^I p_{ir}^A \leq 0$$

where  $I_r$  is the associated activity variable.

### ***Market Clearance Conditions***

9. Labor:

$$\bar{L}_r \geq \sum_i Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial w_r}$$

where  $w_r$  is the associated price variable.

10. Capital:

$$\bar{K}_r \geq \sum_i Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial v_r}$$

where  $v_r$  is the associated price variable.

11. Natural resources:

$$\bar{Q}_{ir} = Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial q_{ir}} \quad i \in FF$$

where  $q_{ir}$  is the associated price variable.

12. Output for internal markets:

$$Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p_{ir}} \geq A_{ir} \frac{\partial \Pi_{ir}^A}{\partial p_{ir}} + \sum_{s \neq r} M_{is} \frac{\partial \Pi_{is}^M}{\partial p_{ir}}$$

where  $p_{ir}$  is the associated price variable.

13. Sector-specific energy aggregate:

$$E_{ir} \geq Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p_{ir}^E}$$

where  $p_{ir}^E$  is the associated price variable.

14. Import aggregate:

$$M_{ir} \geq A_{ir} \frac{\partial \Pi_{ir}^A}{\partial p_{ir}^M}$$

where  $p_{ir}^M$  is the associated price variable.

15. Armington aggregate:

$$A_{ir} \geq \sum_j Y_{jr} \frac{\partial \Pi_{jr}^Y}{\partial p_{ir}^A} + C_r \frac{\partial \Pi_r^C}{\partial p_{ir}^A} + I_r \frac{\partial \Pi_r^I}{\partial p_{ir}^A}$$

where  $p_{ir}^A$  is the associated price variable.

16. Investment aggregate:

$$\bar{I}_r \geq I_r$$

where  $p_r^I$  is the associated price variable.

17. Household consumption:

$$C_r p_r^C = w_r \bar{L}_r + v_r \bar{K}_r + \sum_{j \in FF} q_{jr} \bar{Q}_{jr} + p_r^{CO2} \bar{CO2}_r + p_r^I \bar{I}_r + \bar{B}_r$$

where  $p_r^C$  is the associated price variable.

18. Aggregate household energy consumption:

$$E_{Cr} = C_r \frac{\partial \Pi_r^C}{\partial p_{Cr}^E}$$

where  $p_{Cr}^E$  is the associated price variable.

19. Carbon emissions:

$$\overline{CO2}_r = \sum_i A_{ir} a_i^{CO2}$$

where  $p_r^{CO2}$  is the associated price variable.

20. Balance of payments:

$$\sum_{i,r} Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial p^W} + \sum_r \bar{B}_r = \sum_{i,r} M_{ir} \frac{\partial \Pi_{ir}^M}{\partial p^W}$$

where  $p^W$  is the associated price variable.

Table A.1: Sets

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I	Sectors and goods
J	Aliased with $i$
R	Regions
S	Aliased with $r$
EG	All energy goods: Coal, crude oil, refined oil, gas and electricity
FF	Primary fossil fuels: Coal, crude oil and gas
LQ	Liquid fuels: Crude oil and gas

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Table A.2: Activity variables

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$Y_{ir}$	Production in sector $i$ and region $r$
$E_{ir}$	Aggregate energy input in sector $i$ and region $r$
$M_{ir}$	Aggregate imports of good $i$ and region $r$
$A_{ir}$	Armington aggregate of good $i$ in region $r$
$C_r$	Aggregate household consumption in region $r$
$E_{Cr}$	Aggregate household energy consumption in region $r$
$I_r$	Aggregate investment in region $r$

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Table A.3: Price variables

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$p_{ir}$	Output price of good $i$ produced in region $r$ for domestic market
$p^W$	Real exchange rate with the rest of the world (ROW)
$p_{ir}^E$	Price of aggregate energy in sector $i$ and region $r$
$p_{ir}^M$	Import price aggregate for good $i$ imported to region $r$
$p_{ir}^A$	Price of Armington good $i$ in region $r$
$p_r^C$	Price of aggregate household consumption in region $r$
$p_{Cr}^E$	Price of aggregate household energy consumption in region $r$
$p_r^I$	Price of aggregate investment good in region $r$
$w_r$	Wage rate in region $r$
$v_r$	Price of capital services in region $r$
$q_{ir}$	Rent to natural resources in region $r$ ( $i \in \text{FF}$ )
$p_r^{\text{CO}_2}$	Shadow price of $\text{CO}_2$ unit in region $r$

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Table A.4: Cost shares

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$\theta_{ir}^{\text{XROW}}$	Share of ROW exports in sector $i$ and region $r$
$\theta_{jir}$	Share of intermediate good $j$ in sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^{\text{KLE}}$	Share of KLE aggregate in sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^E$	Share of energy in the KLE aggregate of sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\alpha_{ir}^T$	Share of labor ( $T=L$ ) or capital ( $T=K$ ) in sector $i$ and region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^Q$	Share of natural resources in sector $i$ of region $r$ ( $i \in \text{FF}$ )
$\theta_{Tir}^{\text{FF}}$	Share of good $i$ ( $T=i$ ) or labor ( $T=L$ ) or capital ( $T=K$ ) in sector $i$ and region $r$ ( $i \in \text{FF}$ )
$\theta_{ir}^{\text{COA}}$	Share of coal in fossil fuel demand by sector $i$ in region $r$ ( $i \notin \text{FF}$ )
$\theta_{ir}^{\text{ELE}}$	Share of electricity in energy demand by sector $i$ in region $r$
$\beta_{jir}$	Share of liquid fossil fuel $j$ in energy demand by sector $i$ in region $r$ ( $i \notin \text{FF}$ , $j \in \text{LQ}$ )
$\theta_{isr}^M$	Share of imports of good $i$ from region $s$ to region $r$
$\theta_{ir}^{\text{MROW}}$	Share of ROW imports of good $i$ in region $r$
$\theta_{ir}^A$	Share of domestic variety in Armington good $i$ of region $r$



$\theta_{Cr}^E$	Share of fossil fuel composite in aggregate household consumption in region $r$
$\theta_{ir}^I$	Share of good $i$ in investment composite in region $r$
$\gamma_{ir}$	Share of non-energy good $i$ in non-energy household consumption demand in region $r$
$\theta_{iCr}^E$	Share of fossil fuel $i$ in household energy consumption in region $r$

Table A.5: Endowments and emissions coefficients

$\bar{L}_r$	Aggregate labor endowment for region $r$
$\bar{K}_r$	Aggregate capital endowment for region $r$
$\bar{Q}_{ir}$	Endowment of natural resource $i$ for region $r$ ( $i \in FF$ )
$\bar{B}_r$	Balance of payment deficit or surplus in region $r$ (note: $\sum_r \bar{B}_r = 0$ )
$\overline{CO2}_r$	Endowment of carbon emission rights in region $r$
$a_i^{CO_2}$	Carbon emissions coefficient for fossil fuel $i$ ( $i \in FF$ )

Table A.6: Elasticities

$\eta$	Transformation between production for the domestic market and production for the export	2
$\sigma_{KLE}$	Substitution between energy and value-added in production (except fossil fuels)	0.8
$\sigma_{Q,i}$	Substitution between natural resources and other inputs in fossil fuel production calibrated consistently to exogenous supply elasticities $\mu_{FF}$ .	$\mu_{COA}=0.5$ $\mu_{CRU}=1.0$ $\mu_{GAS}=1.0$
$\sigma_{ELE}$	Substitution between electricity and the fossil fuel aggregate in production	0.3
$\sigma_{COA}$	Substitution between coal and the liquid fossil fuel composite in production	0.5
$\sigma_A$	Substitution between the import aggregate and the domestic input	4
$\sigma_M$	Substitution between imports from different regions	8
$\sigma_{EC}$	Substitution between the fossil fuel composite and the non-fossil fuel consumption aggregate in household consumption	0.8
$\sigma_{FF,C}$	Substitution between fossil fuels in household fossil energy consumption	0.3

Figure A.1: Nesting in non-fossil fuel production

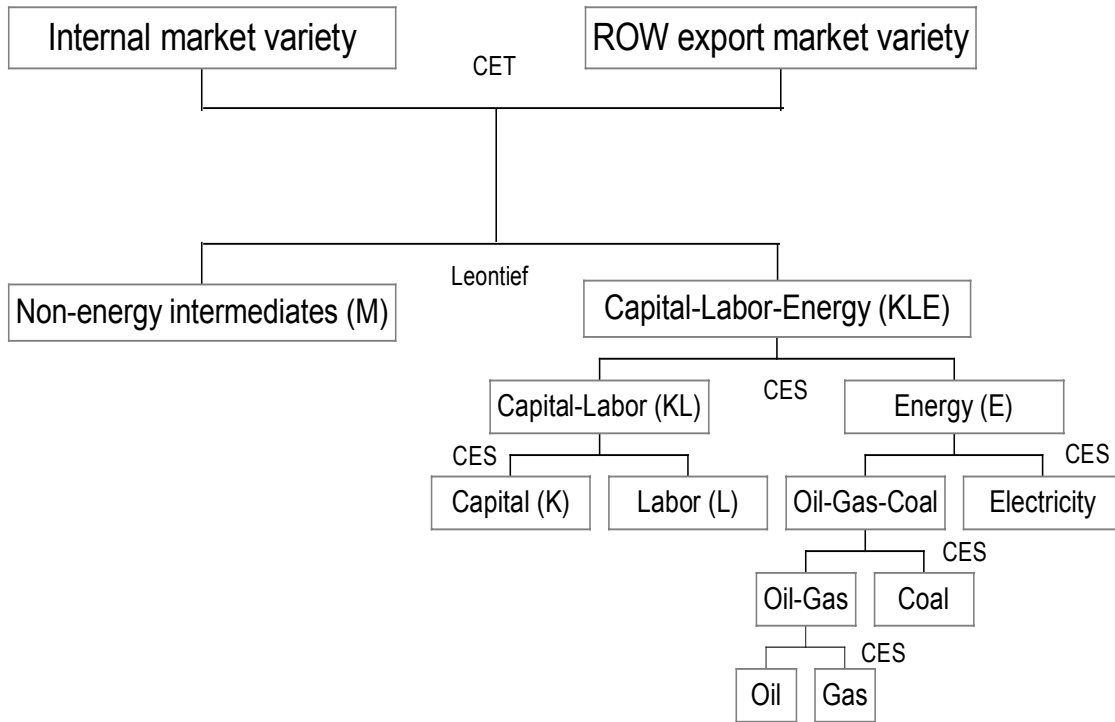


Figure A.2: Nesting in fossil fuel production

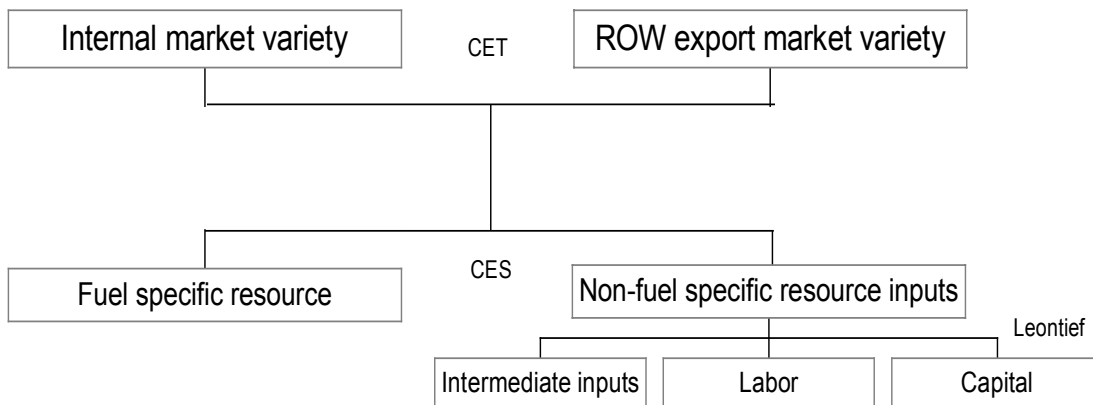


Figure A.3: Nesting in household consumption

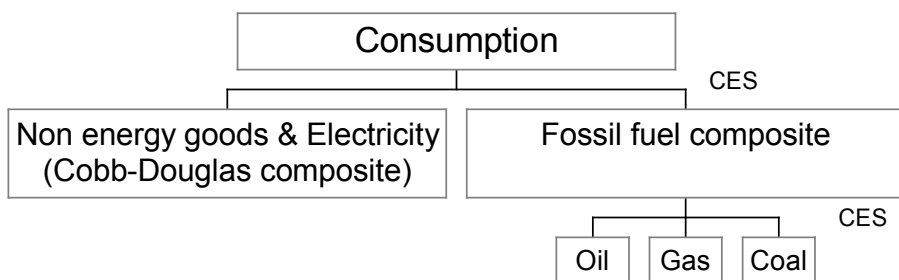


Figure A.4: Nesting in Armington production

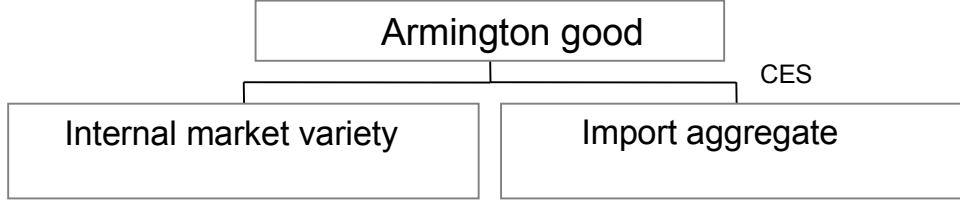
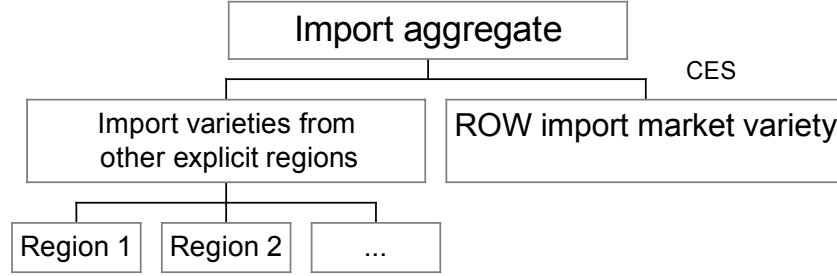


Figure A.5: Nesting in import aggregate



### ***Implementation of Allowance Allocation Rules***

In our simulations on alternative allowance allocation rules, the price of a unit of CO<sub>2</sub> for an industry  $i$  or the household  $C$  depends on whether the respective segment of the economy is eligible for carbon trade (denoted  $T$ ). To distinguish carbon prices by sector, we must explicitly account for carbon demands within the zero-profit conditions characterizing the sector-specific energy aggregate and the household energy aggregate (instead of the Armington aggregate). Carbon demands by segments  $i$  or  $C$  then reads as:

$$CO2_{ir} = E_{ir} \sum_{j \in J} \frac{\partial \Pi_{ir}^E}{\partial (p_{jr} + a_j^{CO2} p_{jr}^{CO2})} \quad \text{and} \quad CO2_{Cr} = E_{Cr} \sum_{j \in J} \frac{\partial \Pi_r^C}{\partial (p_{jr} + a_j^{CO2} p_{jr}^{CO2})} .$$

Allowances can be traded internationally at an exogenous world market price. In the algebraic formulation, two additional zero profit conditions must be added to specify carbon import activities  $IM_{zr}^{CO2}$  and carbon export activities  $EX_{zr}^{CO2}$  for segments  $z$  of the economy that are open to international trade ( $z \in T$ ):

$$\bar{e}p^W \geq p_{zr}^{CO2} \quad (\text{imports}) \quad \text{and} \quad p_{zr}^{CO2} \leq \bar{e}p^W \quad (\text{exports})$$

where  $\bar{e}p^W$  denotes the international price for a unit of CO<sub>2</sub> in domestic currency. Revenues from exports of emission allowances or, likewise, expenditures for imports of carbon emission rights enter the balance of payment constraints. The market clearance condition for those segments that form part of allowance trading reads as:

$$\overline{CO2}_r^T \geq \sum_{z \in T} (CO2_{zr} + IM_{zr} - EX_{zr}) .$$

We distinguish three rules how emission allowances are assigned to different segments of the economy: (i) auctioning, (ii) emission-based assignment, and (iii) output-based assignment. Our exposition of generic equilibrium conditions in sections A.2 and A.3 cover the case of (non-distortionary) auctioning. Under emission-based or output-based assignment, the value of freely allocated emission rights constitutes a subsidy that enters the zero-profit condition of sectoral production.

For the output-based rule, where allowances per-unit of output are allocated to eligible sectors in proportion to the benchmark emission intensity  $\frac{\overline{E}_{ir}}{\overline{Y}_{ir}}$  the implicit ad-valorem output subsidy  $s_{ir}$  can be written as:

$$s_{ir} \geq \lambda_r^y \left( \frac{\overline{E}_{ir}}{\overline{Y}_{ir}} p_{ir}^{CO2} \right) / p_{ir} ,$$

where  $\lambda_r^y$  denotes the endogenous average emission assignment factor per unit of output. This factor is determined by the associated “emission budget” constraint:

$$\overline{CO2}_r^T \geq \lambda_r^y \sum_{i \in T} \frac{\overline{E}_{ir}}{\overline{Y}_{ir}} Y_{ir} .$$

For the emission-based rule, where allowances per-unit of output are allocated to eligible sectors in proportion to their emissions  $CO2_{ir}$  the equivalent input subsidy  $\tau_{ir,r}$  reads as:

$$\tau_{ir} = \lambda_r^E ,$$

where  $\lambda_r^E$  denotes the endogenous average emission assignment factor per unit of emission. This factor is determined by the associated “emission budget” constraint:

$$\overline{CO2}_r^T \geq \lambda_r^E \sum_{i \in T} CO2_{ir} .$$

## Appendix B: Sensitivity Analysis

### B.1 Benchmark Data

Figure B.1.1a: Consumption (in % change from *BaU*) – France

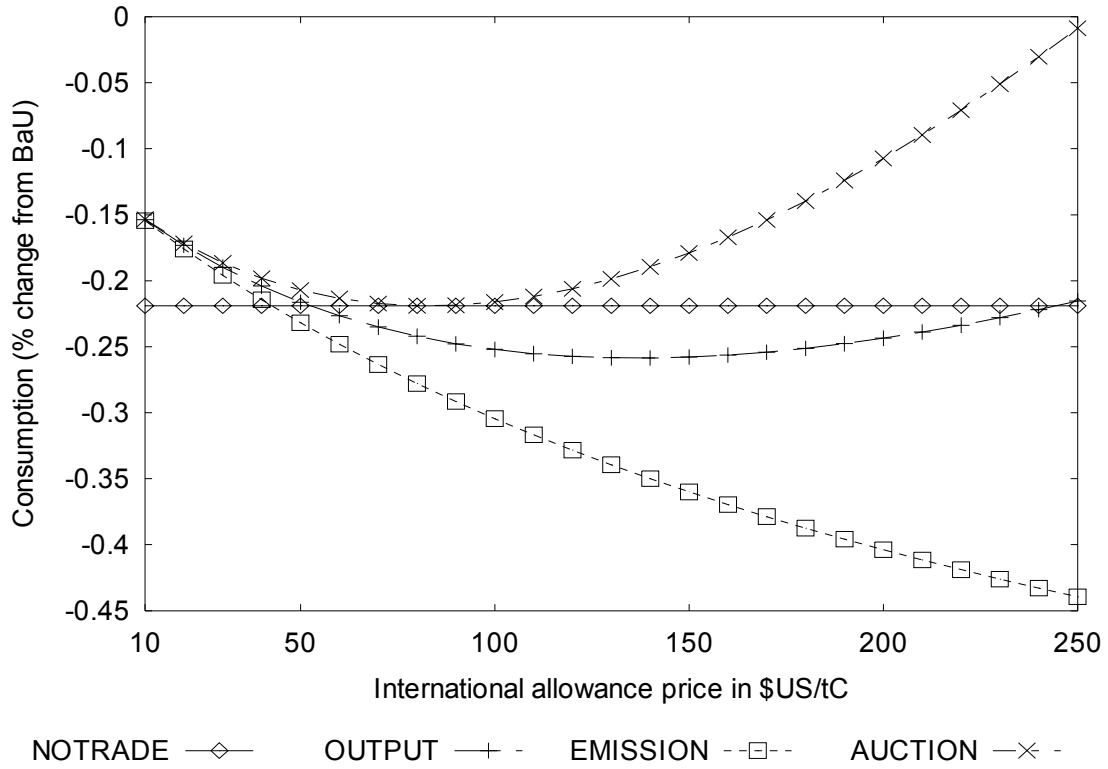


Figure B.1.1b: Employment in energy-intensive industries (in % change from *BaU*) – France

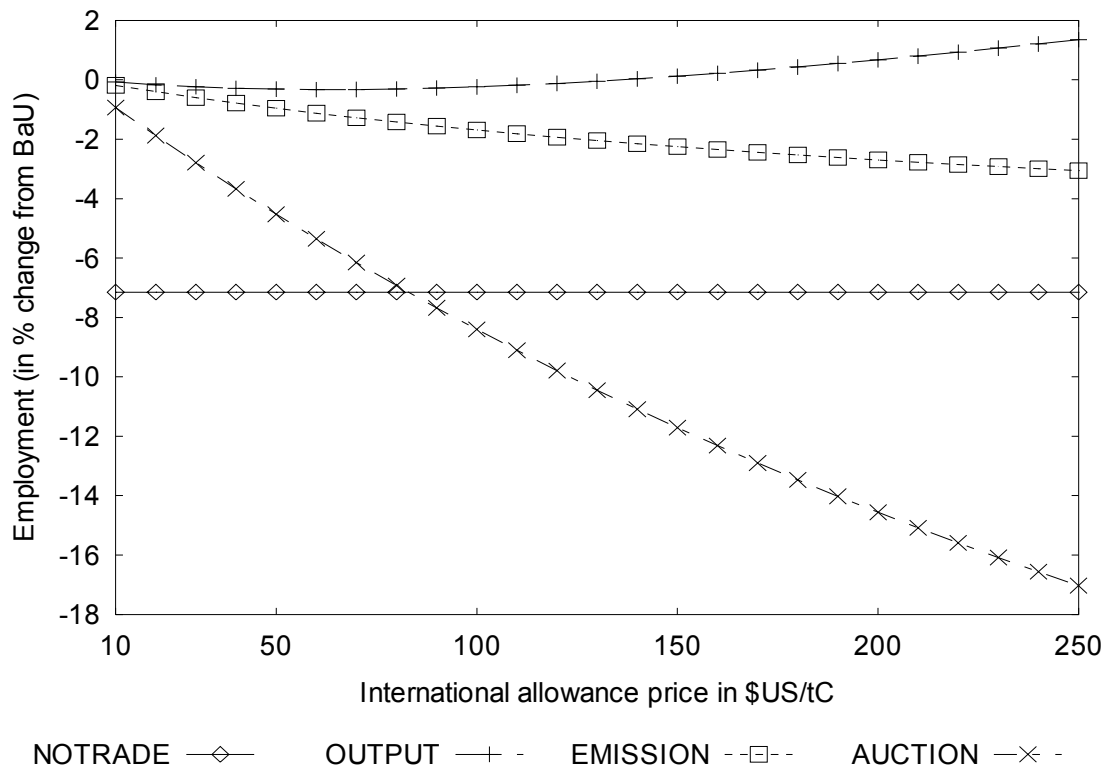


Figure B.1.2a: Consumption (in % change from *BaU*) – Great Britain

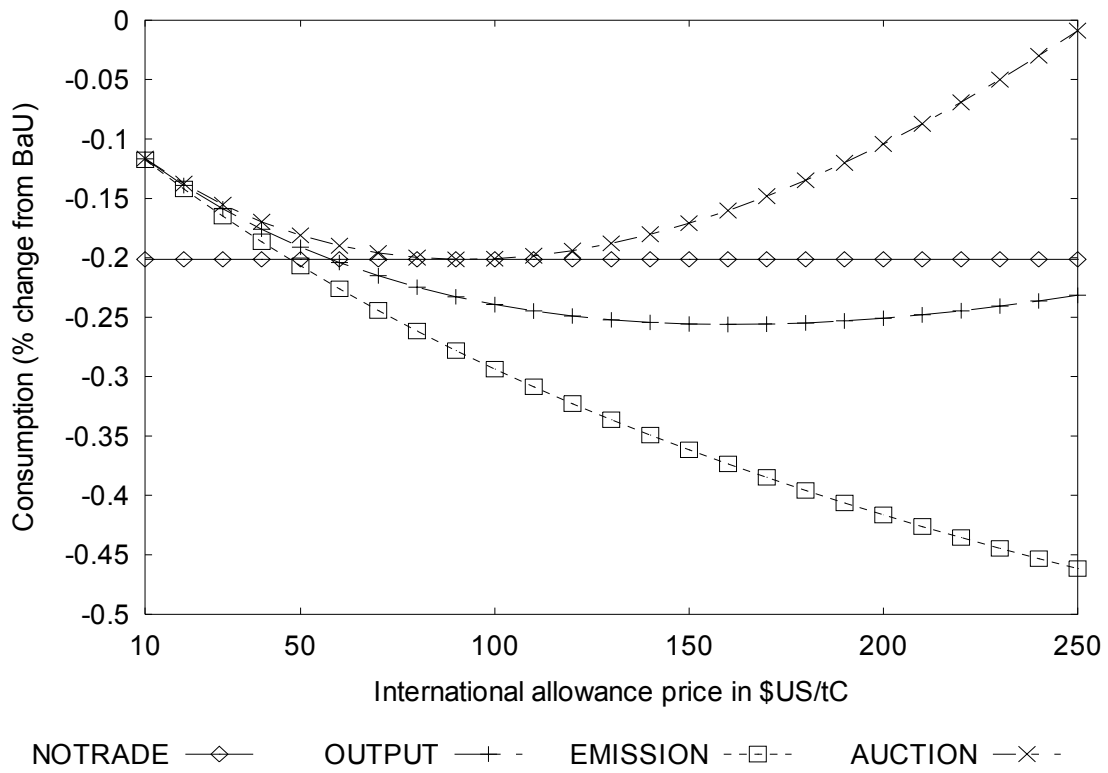


Figure B.1.2b: Employment in energy-intensive industries (% change from *BaU*) – Great Britain

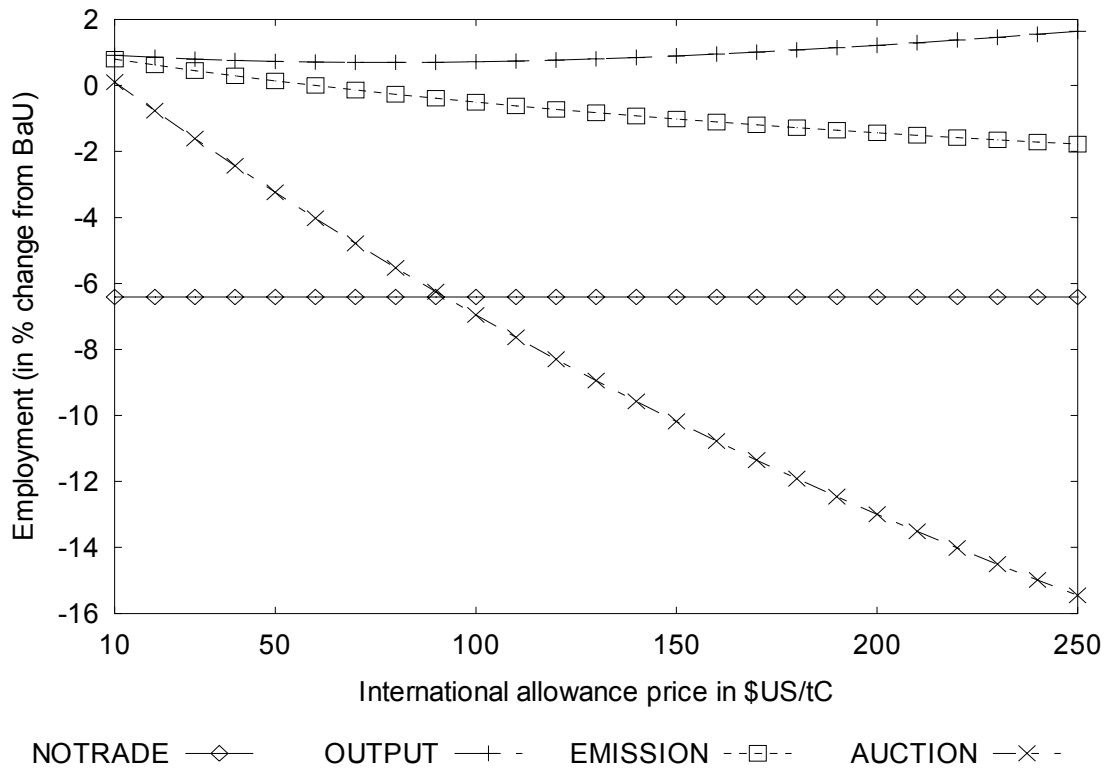


Figure B.1.3a: Consumption (in % change from *BaU*) – Italy

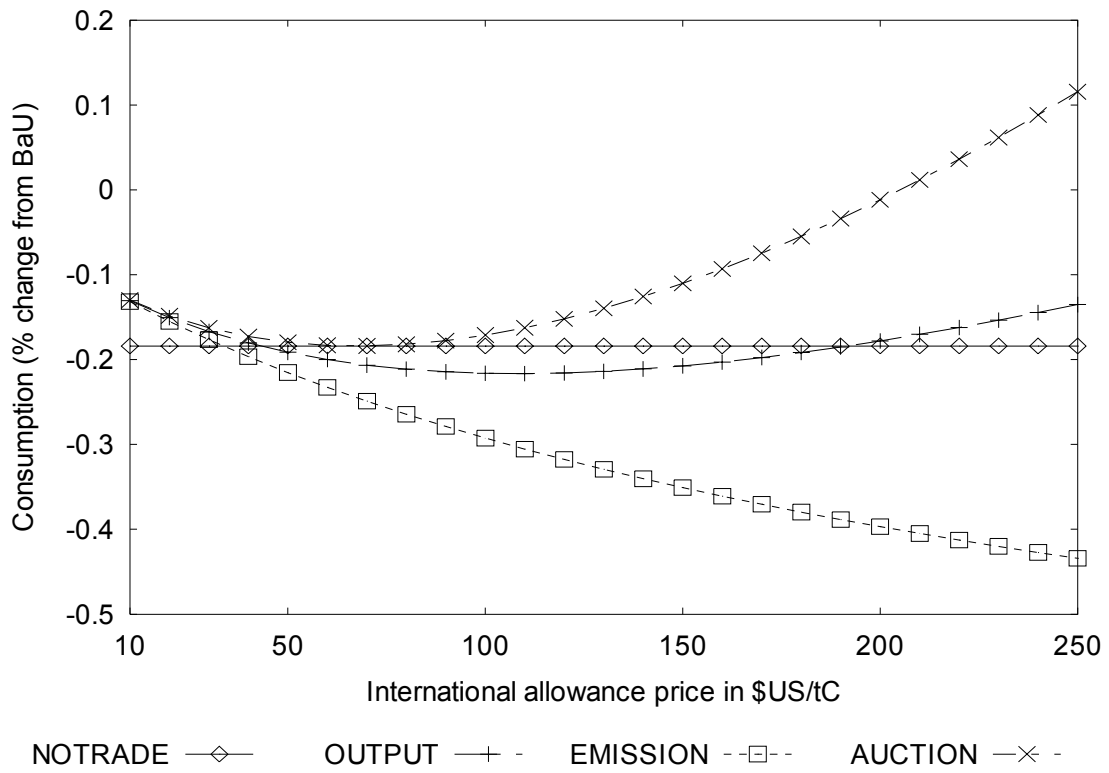


Figure B.1.3b: Employment in energy-intensive industries (% change from *BaU*) – Italy

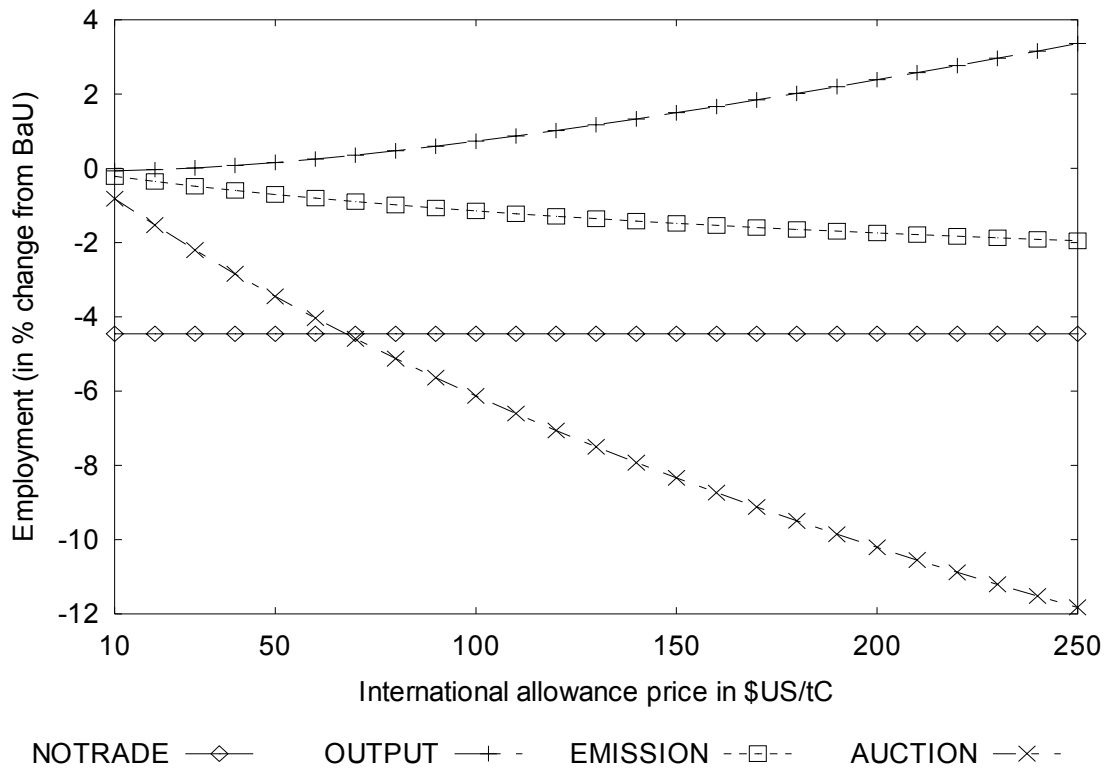


Figure B.1.4a: Consumption (in % change from *BaU*) – Sweden

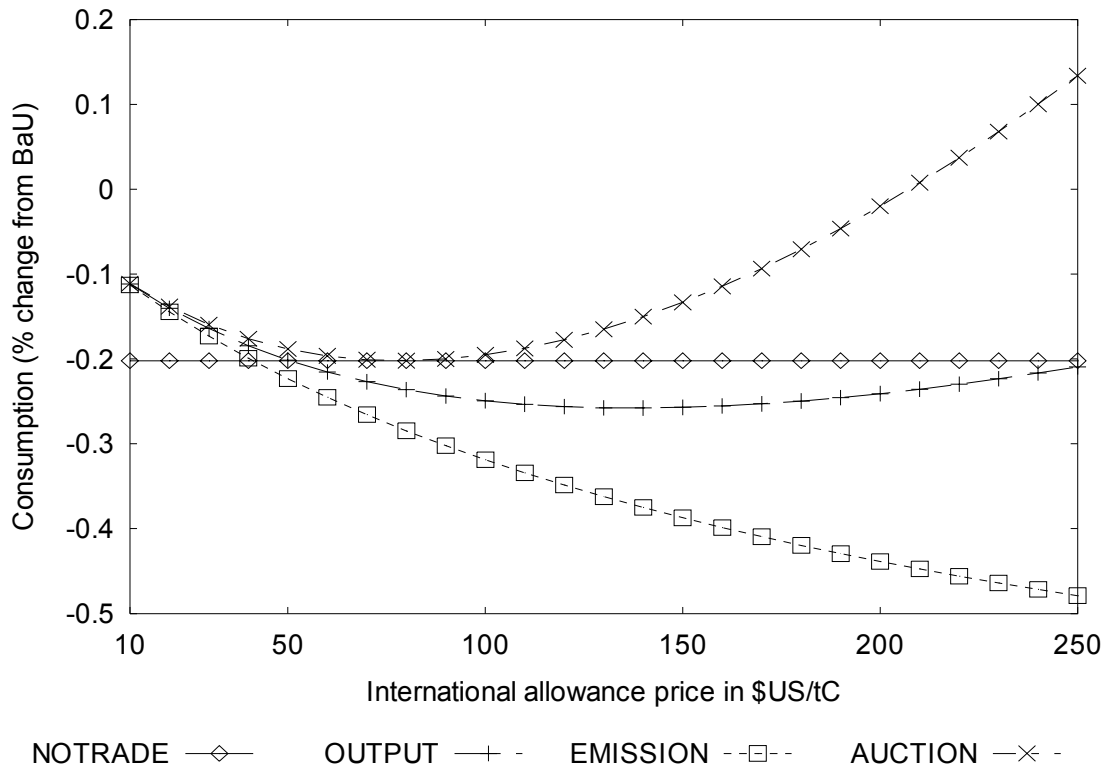


Figure B.1.4b: Employment in energy-intensive industries (in % change from *BaU*) – Sweden

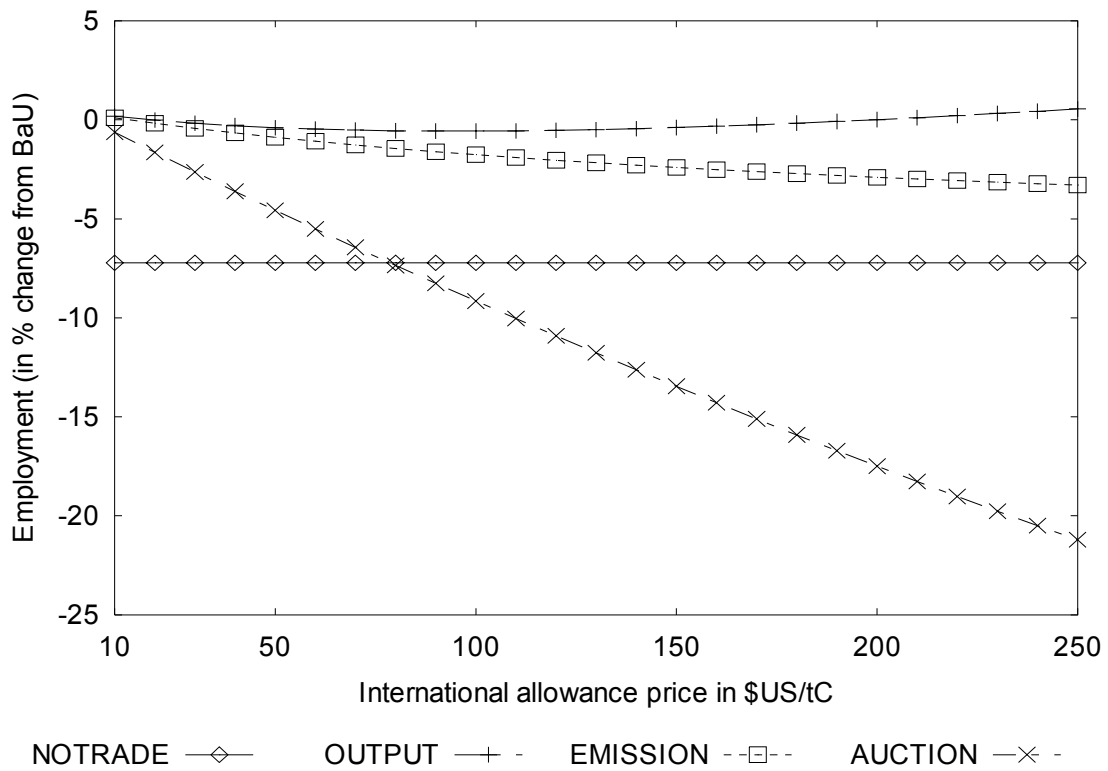




Figure B.1.5a: Consumption (in % change from *BaU*) – Japan

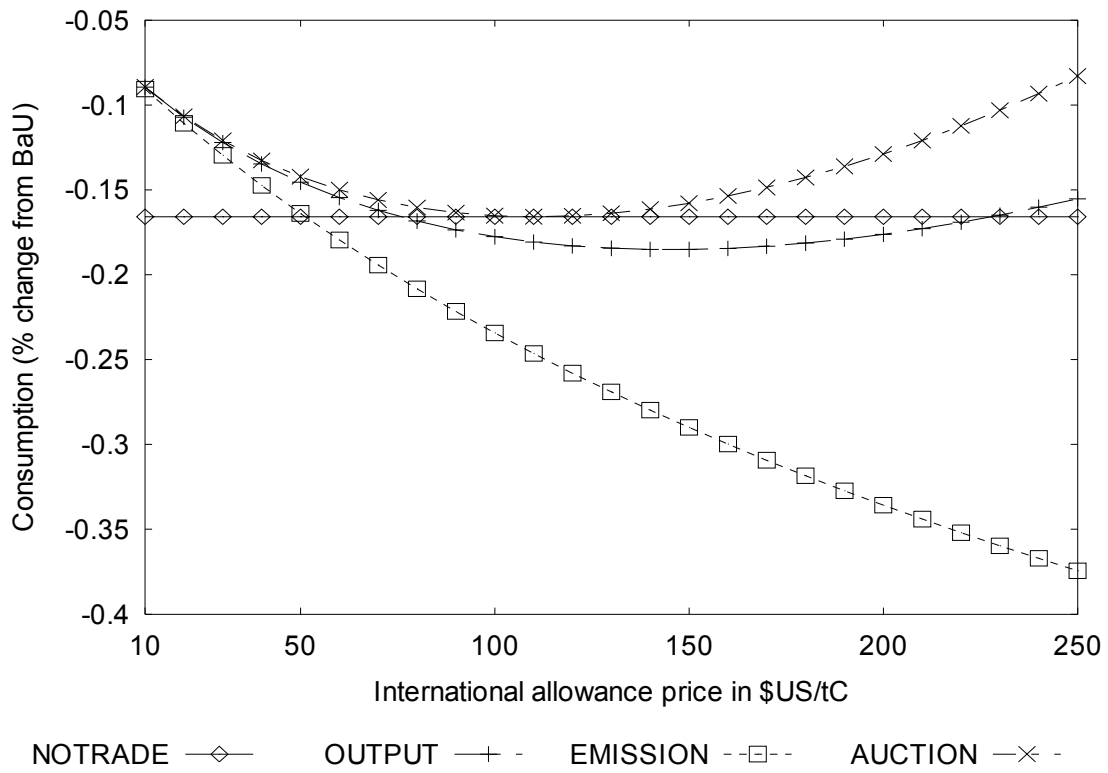


Figure B.1.5b: Employment in energy-intensive industries (% change from *BaU*) – Japan

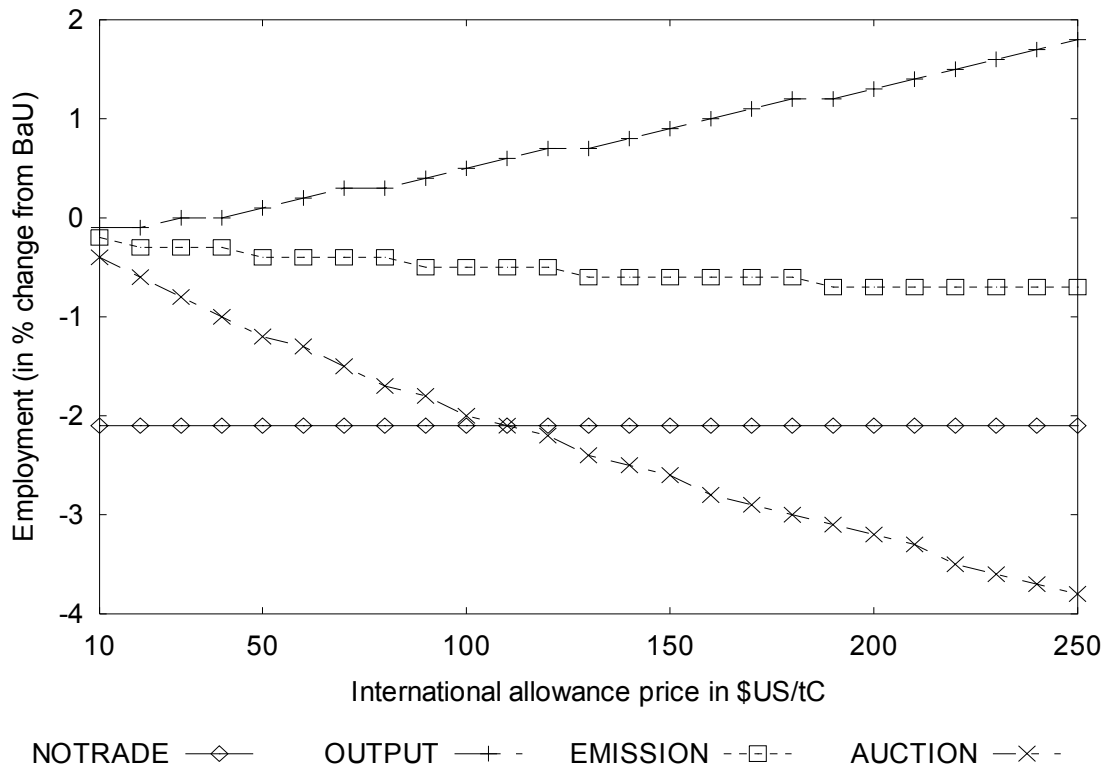


Figure B.1.6a: Consumption (in % change from *BaU*) – USA

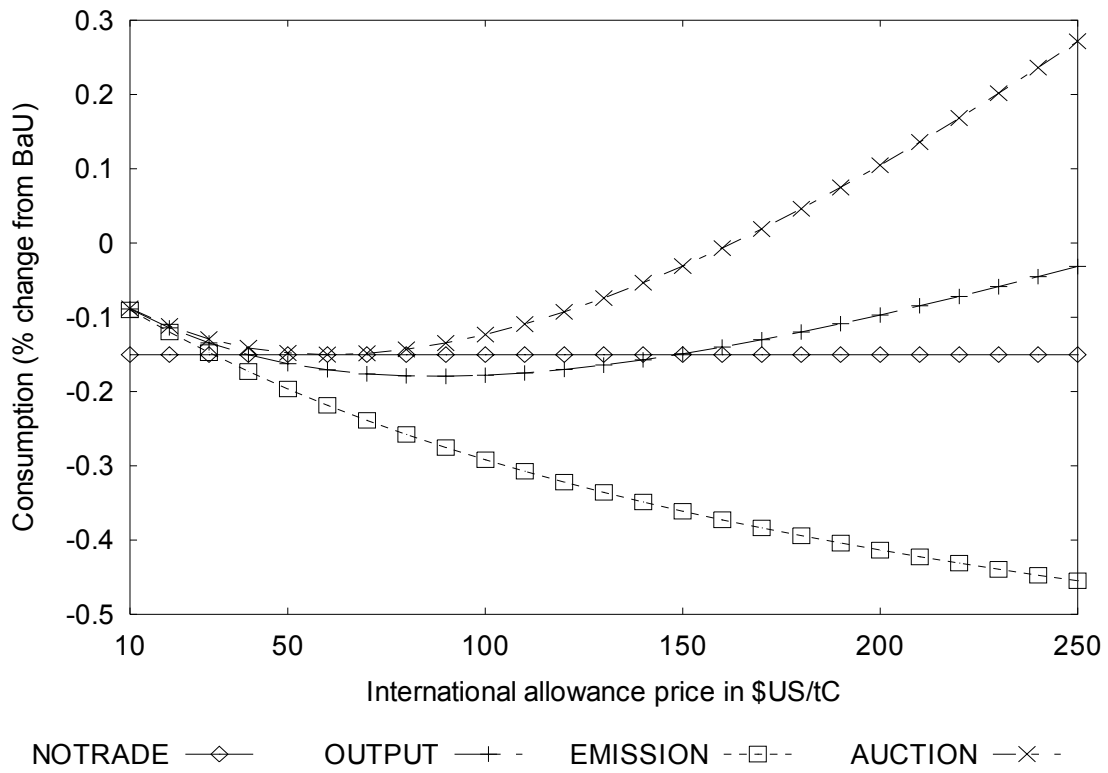
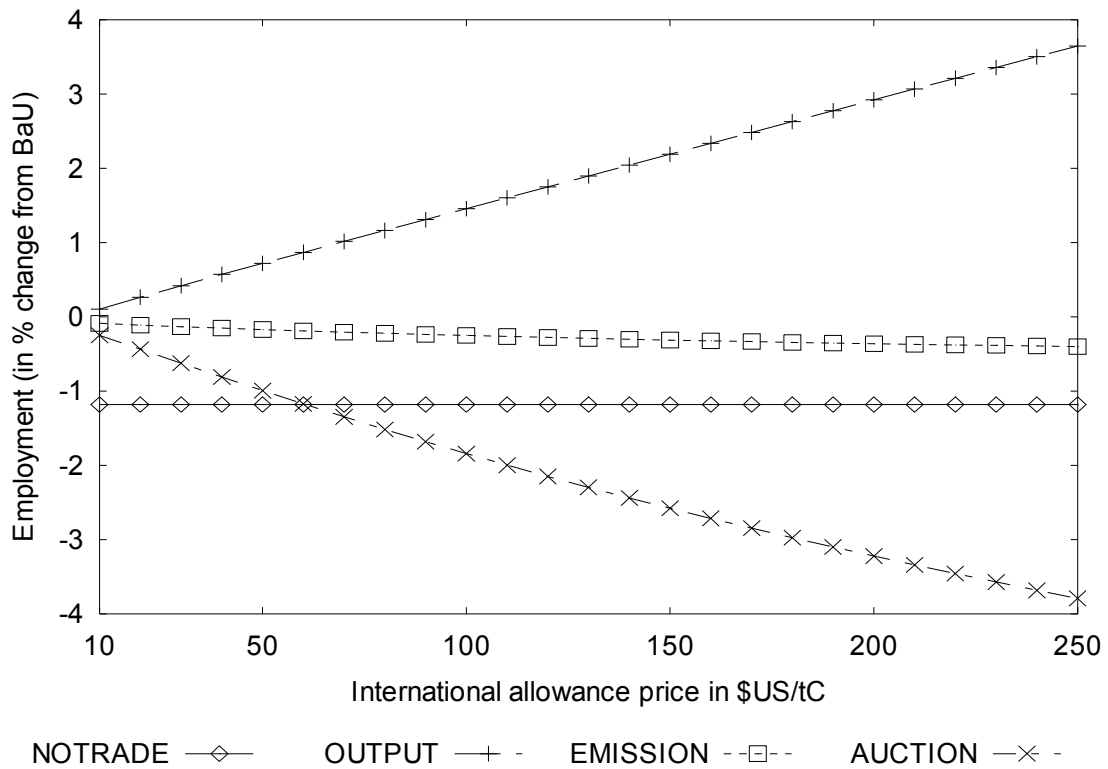


Figure B.1.6b: Employment in energy-intensive industries (% change from *BaU*) – USA



## B.2 Reduction Targets

Figure B.2a: Consumption gains (in %) of output-based allocation compared to emission-based allocation for alternative emission reduction targets (5 – 30% emission reduction vis-à-vis *BaU* emission level) – Germany

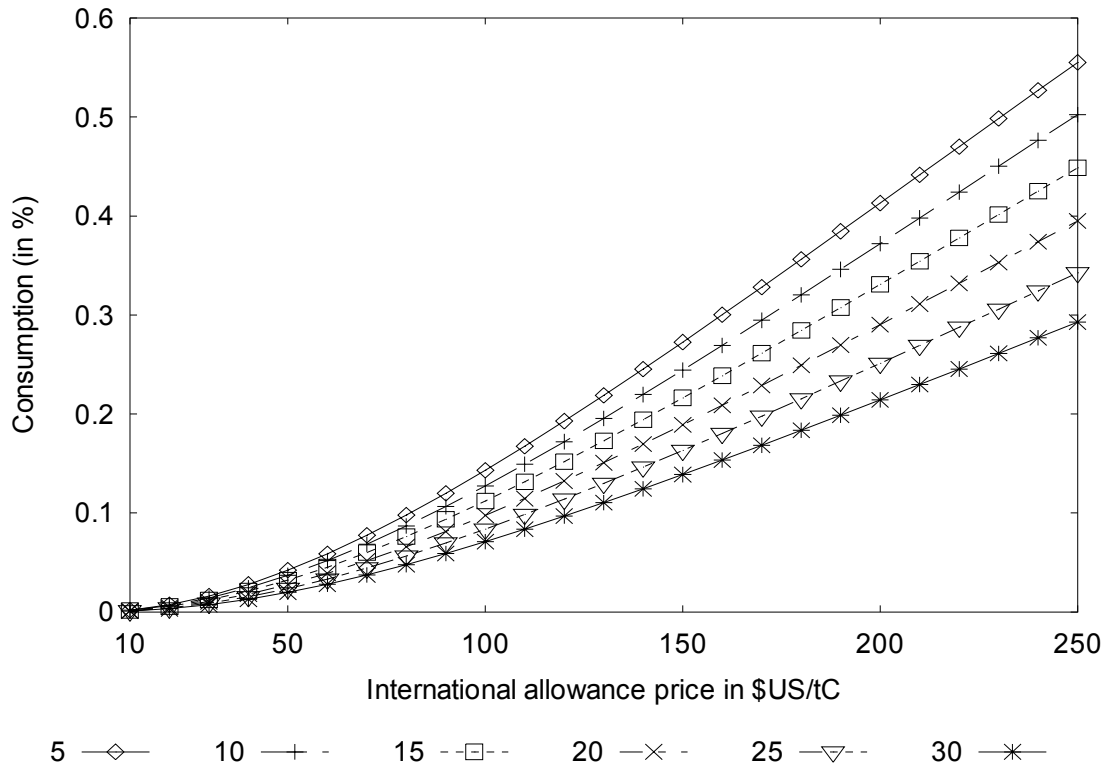
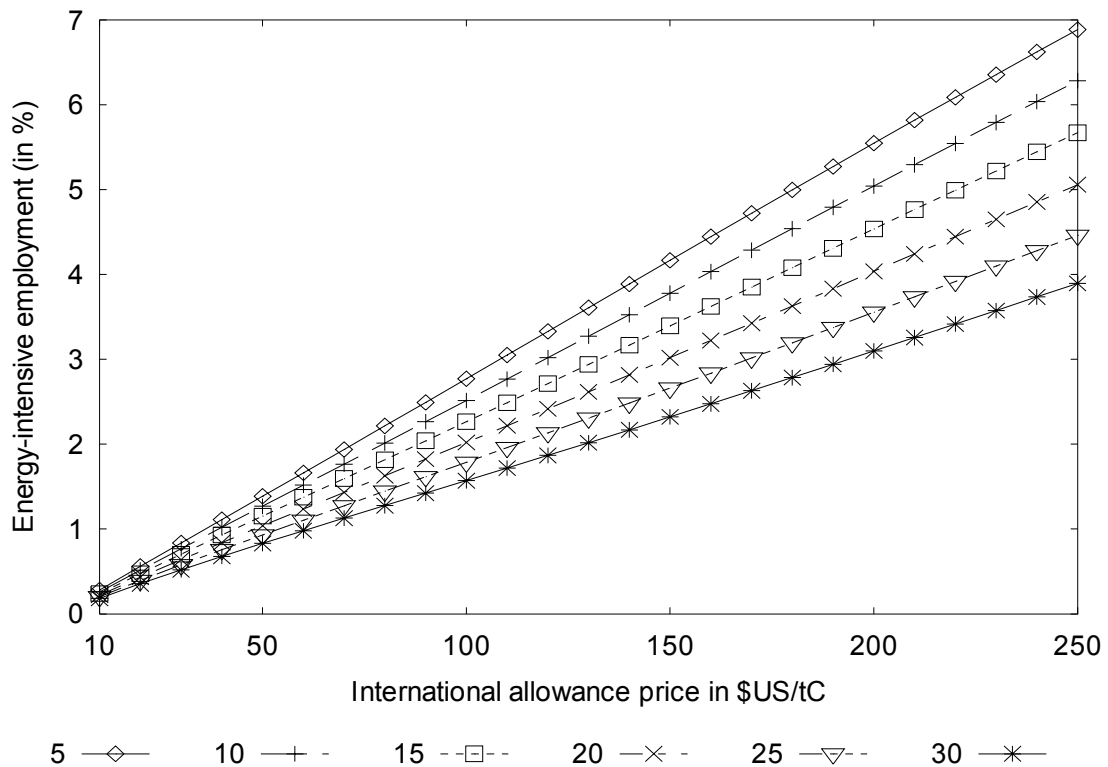


Figure B.2b: Employment gains (in %) of output-based allocation compared to emission-based allocation for alternative emission reduction targets (5 – 30 % emission reduction vis-à-vis *BaU* emission level) – Germany



### B.3 Multilateral Abatement

Figure B.3a: Consumption (in % change from *BaU*) – Germany

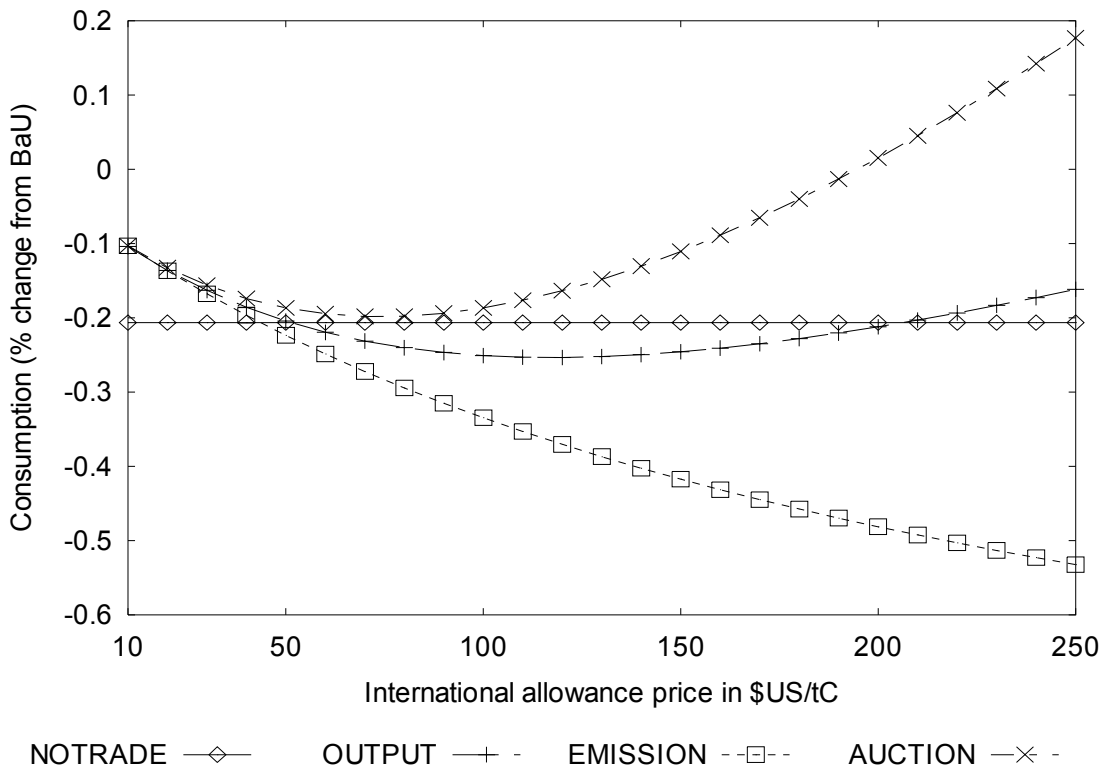
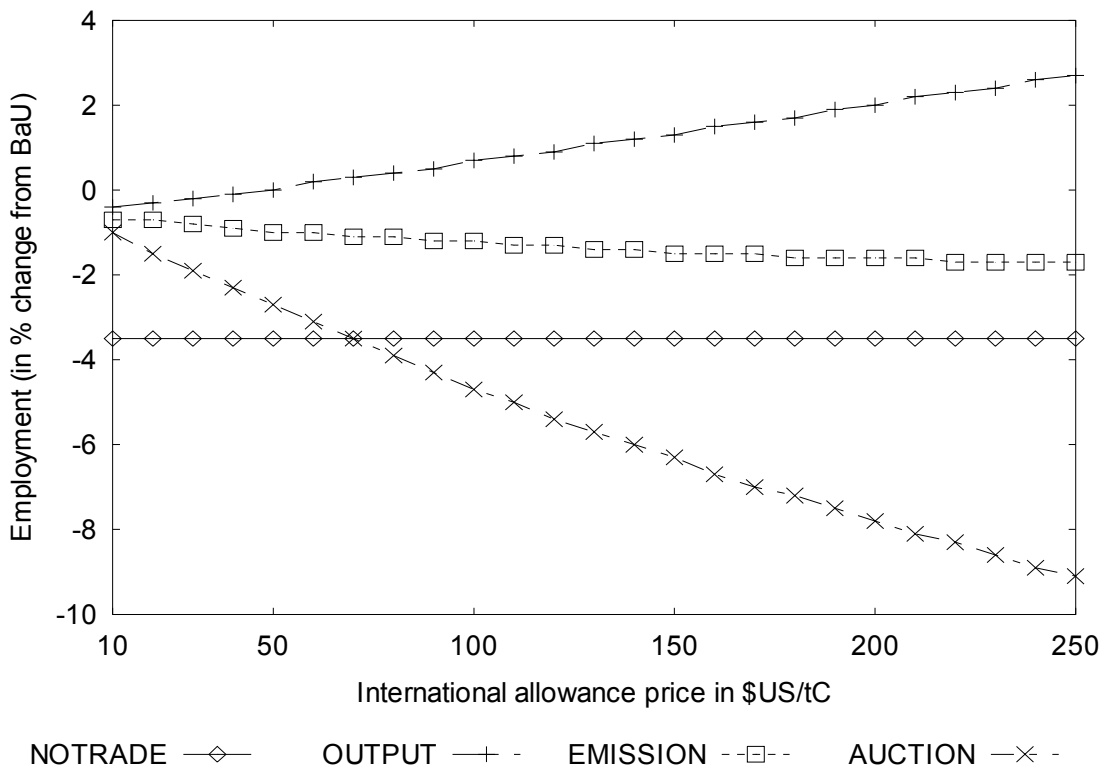


Figure B.3b: Employment in energy-intensive industries (% change from *BaU*) – Germany



### B.4 Market Structure (Cournot Competition in Energy-intensive Industries)

Figure B.4a: Consumption (in % change from *BaU*) – Germany

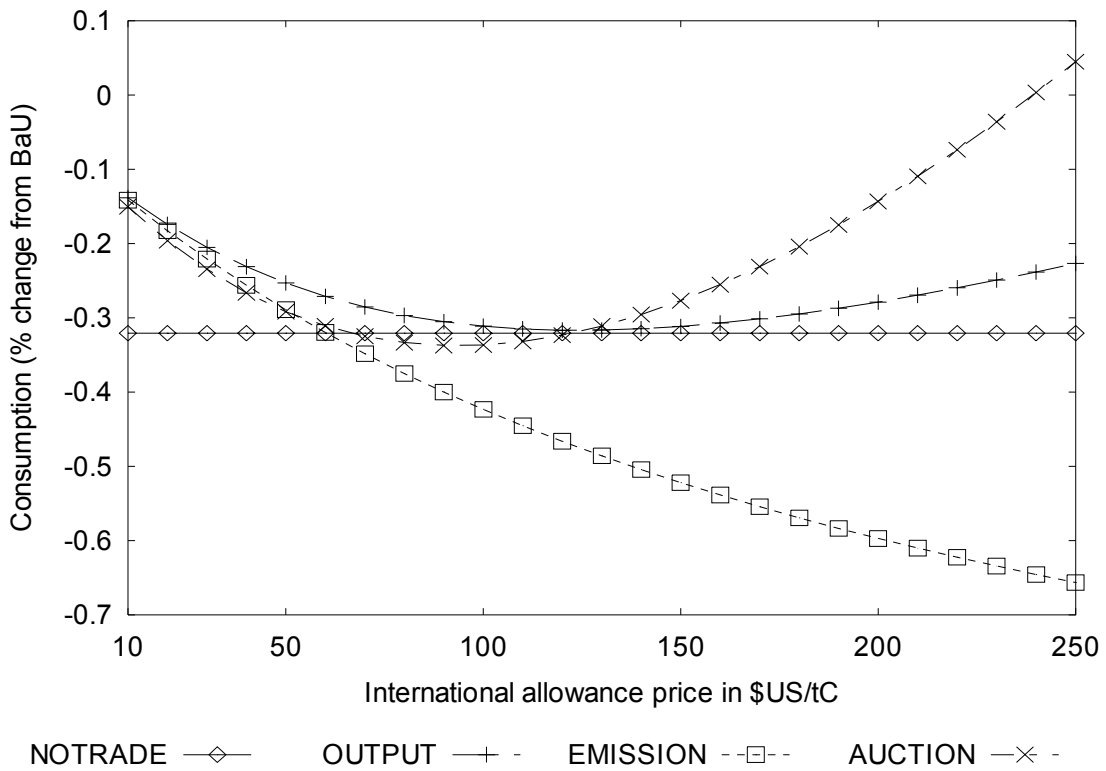


Figure B.4b: Employment in energy-intensive industries (% change from *BaU*) – Germany

