Managing Digital Piracy: Pricing and Protection Strategies

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Abstract: This paper studies the optimal choice of nonlinear pricing schedules and technology-based deterrence levels in a market with digital piracy, where sellers can influence the degree of piracy through their pricing policy, as well as by implementing digital rights management (DRM) systems. It is shown that a monopoly seller's optimal pricing schedule can be characterized as a simple combination of the zero-piracy pricing schedule, and a piracy-indifferent pricing schedule which makes all customers indifferent between legal usage and piracy. An increase in the threat of piracy, while lowering prices and seller profits, can result in substantial increases in total surplus. When sellers can optimally price-discriminate, these increases in surplus are from *legal* usage, and caused by an expansion in the fraction of legal users, as well as an increase in individual levels of legal usage.

When allowed to make endogenous choices of technology-based DRM protection, a seller who cannot price-discriminate chooses the technologically-maximal level, which maximizes the difference between the quality of the legal and pirated goods. However, when price-discrimination is feasible, a seller's optimal choice is always a strictly *lower* level of technology-based protection. From a welfare perspective, this shows that the feasibility of price discrimination induces a complementary change in the seller's choice DRM protection, and this change always increases total surplus. Moreover, if a DRM system weakens over time, due to its technology being progressively hacked, a seller's optimal strategic response may involve either increasing or decreasing their level of technology-based protection. This direction of change is related to whether the technology implementing each marginal reduction in piracy is increasingly less or more vulnerable to hacking.

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1. Introduction

Over the last decade, sellers of digital products have actively fought the availability of pirated copies of their products. Nevertheless, digital piracy rates are still high and increasing in many markets, despite a continuous increase in the availability and sophistication of copy protection and digital rights management technologies. Piracy concerns have expanded following the emergence of file-sharing networks like Gnutella and Kazaa, which have substantially increased the availability and exchange of high-quality illegal versions of software, music and digital video.

The sustained presence of piracy complicates the design of pricing schedules for sellers of digital goods. It also poses the new challenge of choosing an appropriate level of technology-based protection, and of responding strategically to the hacking of existing digital rights management systems. These are the issues addressed by the model in this paper.

The first part of the paper studies optimal pricing strategy. When faced with rising digital piracy, a seller's pricing power is increasingly limited by the quality and availability of pirated copies, which are imperfect substitutes for the legally available product. This effect of piracy is analyzed by developing a model of monopoly price discrimination in which heterogeneous customers can buy variable quantities of a digital good, and different customer types get differing value from their usage of the legal good, as well as from their usage of its pirated substitute. The seller's optimal pricing schedule is shown to be a simple combination of two contracts – the optimal pricing schedule in the absence of piracy (termed the *zero-piracy pricing schedule*), and a *piracy-indifferent pricing schedule,* which makes all customers indifferent between legal usage and piracy. An increase in the threat of piracy lowers prices and profits; however, there may be social benefits from piracy, realized through a expansion in both the fraction of customers who purchase the product *legally*, as well as the volume of legal usage.

The next part of this paper studies technology-based protection against digital piracy, which is typically achieved by implementing digital rights management (henceforth referred to as DRM) systems. Examples of DRM systems for digitally delivered products include SecureMedia Encryptonite (which is embedded in many digital televisions), Macrovision SafeCast, Apple's FairPlay, Microsoft Windows Media DRM Series, and Real Networks Helix. Other DRM systems aimed specifically at protecting the physical sources of the digital files shared illegally over the Internet include the Windows Media Data Session Toolkit, and Macrovision's Cactus Data Shield.

Unfortunately, making technological deterrence effective often necessitates a direct reduction in the value of the *legal product.* For instance, all DRM platforms for digital video and music involve encryption that increases file sizes, thereby lowering value by increasing download times for digitally delivered content. Textual content that is protected by Adobe's DRM partners can be electronically scanned by OCR software that takes PDF files as direct inputs and produces near-perfect scanned versions. Deterring this form of piracy will necessitate degrading fonts in rendered legal files, again lowering quality for legal users.

More importantly, implementing a DRM system often constrains the flexibility of usage for a legal user. In 2002, a number of music labels (most notably, Sony, with their Key2Audio scheme) introduced protection technology that would prevent their audio CD's from being played on personal computers. This technology affected value from legal usage by restricting inter-device portability; moreover, a substantial fraction of discs did not work on regular CD and DVD players[2]. Correspondingly, many online music services implement DRM by limiting the rendering of their MP3 files to a single device, and by placing related restrictions on the portability of these files. Highly restricted services like MusicNet and Rhapsody have not been well-received, and this is partly attributed to the fact that their protection schemes "...treat everyone like a potential criminal, and they take all the joy out of buying and playing music" (Mossberg, 2003). In contrast, the iTunes music service from Apple, whose choice of deterrence levels using the FairPlay platform has placed substantially fewer restrictions on a customers' rights to download, share and burn purchased MP3 files[3], has been a resounding success thus far.

Managing digital rights therefore often involves restricting the rights of usage that contribute to customer value, and reducing this value in the process. Consequently, when choosing the appropriate level of technology-based protection against piracy, the seller of a digital good needs to trade-off the effectiveness of *deterring piracy* with the *value reduction* of the legal product that is caused by the implementation of the DRM system. To study this trade-off, the model of pricing with digital piracy is extended to incorporate endogenous choices of technology-based protection. The *technologically-maximal* level of protection, which is the level of technology-based protection at which the quality difference between the legal good and the pirated good is maximized, is contrasted with the *profit-maximizing* level of protection. When the seller can price-discriminate, the profit-maximizing level of protection is shown to be *strictly lower* than the technologically-maximal level. The economic drivers of this result are explored in some detail, since it indicates that even after accounting for quality degradation on legal products, a DRM system that maximizes the quality gap

[2] For instance, an early release made under the Cactus format in Germany reportedly had a 4% return rate from people who found that these CDs didn't work on their normal CD players.

[3] iTunes allows users to burn their MP3 files to an unlimited number of CD's, copy them to an unlimited number of iPod MP3 players, play them on upto three computers, and stream them over a private LAN.

between the legal good and the pirated good is overprotecting the digital product. Additionally, as the effectiveness of a DRM system weakens over time (which typically occurs due to its technology being progressively hacked), a seller's optimal technological and pricing responses are examined. It is shown that the seller's optimal response may either be to decrease or to increase their level of technology-based protection. Conditions under which each of these responses is optimal are characterized, and the implications for preemptive under- or over-protection are discussed.

The model in this paper views pricing strategy and technological deterrence as alternative instruments for the monopolist to manage the problem of piracy, which is consistent with the model in Png and Chen (2003). When allowed to price-discriminate, this paper shows that a monopolist chooses a lower (and superior, both profit-wise and socially) level of DRM protection, suggesting that pricing policy and DRM technology can be complementary instruments for piracy deterrence, rather than necessarily being substitutes (Png and Chen, 2003). The monopolist's investment in technology-based deterrence is indeed excessive from a welfare-maximizing perspective (since any level of DRM protection is socially suboptimal), as established by Png and Chen (2003), though it is shown that admitting price-discrimination can mitigate the level of over-protection to some extent, as can the threat of DRM hacking (only in some cases – in other cases, it exacerbates it, as shown in Section 4.2). The threat of piracy also causes a price-discriminating monopolist's choice of pricing to subsidize legal purchases, and also results in differential subsidies to different customer types. In particular, price discrimination enables legal adoption by low-valuation customers who would otherwise resort to piracy. Apart from increasing total surplus, this has the potentially desirable welfare property of reducing the differences in surplus between different customer types.

Explicit taxes on copying devices or government subsidies on legal usage are not considered. Moreover, the technology-based controls modeled in this paper directly influence the quality of the pirated good, and the formation of sharing groups or software clubs (Gopal and Sanders, 1997, Bakos, Brynjolfsson and Lichtman, 1999) is not explicitly modeled. However, this focus allows a far richer demand and technology specification – usage in variable quantities by customers who value both legal usage and piracy differentially, simultaneously admitting a combination of second-degree price discrimination and variable technological protection against piracy, and explicitly considering the negative effect that digital rights management can have on the value from legal usage. A distinguishing aspect of this paper is therefore its deeper analysis of the economic effects of DRM technologies, and their role as technological deterrents to piracy that are controlled explicitly (and strategically) by a seller who can also use pricing policy to manage piracy.

An early model of strategic piracy deterrence is by Conner and Rummelt (1991) who established that increases in protection technology always increase firm profits, unless the product displays positive network effects. Related work on optimal strategic deterrence includes Yoon (2001). This paper extends their results significantly. For instance, it is shown that even in the absence of network effects, increases in protection technology can reduce firm profits, and the right response to an increase in the degree of piracy may be a weakening of protection levels, rather than a tightening of them. Other work on piracy-related peer-to-peer technologies include Duchene and Waelboeck (2001) who model the effects of these technologies on new-product introduction, and Gayer and Shy (2002) who explore their role as a marketing vehicle to spur in-store sales.

This paper also contributes to the literature on the economics of copying and piracy, which is surveyed and extensively supplemented in Watt (2000). This is part of a larger body of work on the economics of intellectual property and copyright; a good guide to the literature can be found in Besen and Raskind (1991). The papers by Liebowitz (1985) and Johnson (1985) are notable as early examples of the economics of piracy and copying technology, in contrast with much of the literature's focus on socially optimal copyright policy. More recently, Alvisi, Argentisi and Carbonara (2002) show that the presence of a pirated good may make it optimal for a monopolist to introduce a lower-quality substitute for their product. Belleflamme (2003) studies the interdependence between different producers' incentives to accommodate/deter the presence of a pirated good. Ben-Shahar and Jacob (2002) examine the entry-deterring properties of piracy, characterizing a monopolist's incentives to encourage piracy as a form of predatory pricing. Chellappa and Shivendu (2002) model pricing when buyers bear a 'moral cost' from using a pirated good derived from an evaluation version. The model in this paper builds on the approach of each these papers, by preserving their notion of the pirated good as an inferior (vertically differentiated) substitute for the legal good. Additionally, it generalizes their pricing analysis significantly, by modeling and deriving a menu of usage-based prices (rather than a single variable price, or a pair of prices for two quality-differentiated products) that explicitly take into account the differing value of pirated products to different customer types. This generalization is important, because it substantially alters results relating to the optimal level of technological protection, and to post-implementation protection and pricing trends – differences that would not be evident in a model with unit consumption and no price discrimination. This approach is also more likely to provide managerially relevant pricing guidelines since the results prescribe a straightforward way to actually design pricing schedules in the presence of digital piracy, as illustrated by a simple example in Section 5.

Unlike the piracy models in Conner and Rummelt (1991), Takeyama (1994) and Shy and Thisse (1999), positive network externalities are not considered, though their presence would directionally strengthen the results of this paper. These externalities are important in software markets; however, it is unlikely that there are substantial direct network effects in the industries more recently threatened by digital piracy – music, video and content. An extension incorporating indirect network effects is discussed briefly in Section 6.

The rest of this paper is organized as follows. Section 2 provides an overview of the model and describes the seller's optimal pricing schedule in the absence of piracy. Section 3 derives the optimal pricing strategy in the presence of different levels of digital piracy. Section 4 models the economic effects of using digital rights management systems, derives optimal technology-based protection levels, and characterizes strategic responses to changes in the effectiveness of an implemented DRM system. Section 5 presents a brief example. Section 6 discusses the managerial and welfare implications of the model's results, and concludes with an outline of open research questions.

2. Model

2.1. Seller and customers

The model involves an information good which may be used by consumers in continuously varying quantities[4]. The seller of this good (termed the *legal good*) is assumed to be a monopolist, by virtue of owning a copyright over the information good. Any fixed costs of production or IP protection are assumed to be sunk. Since the product is an information good, variable costs of production are zero. In addition to the legal good, there is also a *pirated good* available, which is a lower-quality substitute for the legal good, and is free.

Customers are heterogeneous, indexed[5] by their type $\theta \in [\alpha, \beta]$. The preferences of a customer of type θ for a good of *quality* z are represented by the multiplicatively separable utility function

$$u(q, \theta, z) = zU(q, \theta), \tag{2.1}$$

where q is the *quantity* of the good used by the customer. The function $U(q, \theta)$ is assumed to take

[4] This could either be a homogeneous good, or a bundle of related goods (such as a library of digital information, music or video), from which different customers use different goods. For instance, Apple's iTune music service offers access to a library of over half a million songs. Each user downloads a small fraction of this library, and different subscribers download different songs. In the context of this model, the unit of quantity would therefore be number of downloads.

[5] I have used $[\alpha, \beta]$ as the set of types rather than the more common $[\underline{\theta}, \overline{\theta}]$ because the lower bound of this interval often appears in expressions in the paper. Having $\underline{\theta}$ in these expressions makes them hard to read, and also makes it easy to confuse θ with $\underline{\theta}$.

the following form:

$$U(q,\theta) = \theta q - \frac{1}{2}q^2. \tag{2.2}$$

Numbered subscripts of functions represent partial derivatives with respect to the corresponding variable. For instance, the partial derivative of $U(q,\theta)$ with respect to q is denoted $U_1(q,\theta)$, and the cross-partial of $U(q,\theta)$ with respect to q and θ is denoted $U_{12}(q,\theta)$. This notation is preserved throughout the paper.

The following properties of $U(q,\theta)$ follow from equation (2.2):

1. For every θ, $U(q,\theta)$ has a finite maximum usage $\sigma(\theta) = \arg\max_q U(q,\theta) = \theta$.
2. $U_1(q,\theta) > 0$ for $q < \sigma(\theta)$, and $U_1(q,\theta) < 0$ for $q > \sigma(\theta)$.
3. $U_{11}(q,\theta) = -1$, and $U_{12}(q,\theta) = 1$. Therefore, $u(q,\theta,z)$ is strictly concave in q (diminishing marginal value from usage), and has the Spence-Mirrlees single crossing property.

The quality of the *legal good* is denoted by the variable v, and the quality of the *pirated good* is denoted by the variable s. The preferences of a customer of type θ for the legal good and the pirated good are therefore represented by the functions $vU(q,\theta)$ and $sU(q,\theta)$ respectively, where q is the quantity of the good (legal, pirated) used by the customer, and $U(q,\theta)$ is as defined in (2.2). Since the parameter s is related to how much customers value the pirated good, it is also interpreted as the *threat of piracy* faced by the seller.

These choices of quality v and s are initially exogenous (in Section 3), and then influenced by the seller's choice of DRM protection (in section 4). However, s is always assumed to be strictly less than v, implying that the pirated good is always strictly inferior to the legal good, and therefore, the seller can make a non-zero profit. The feasibility of digital replication of content might suggest that pirated goods and legal goods are perceived as being of equal quality by potential buyers; however, this is generally not the case. When attempting to access pirated content on peer-to-peer file-sharing networks like Gnutella and Kazaa, the exact title a user seeks is often not available. Even when available, locating this title can be slow and unreliable (due to the way pure peer-to-peer networks operate). Often, the contents of media files are not what they were supposed to be, partly due to the posting of decoy files by sellers[6]. Furthermore, the resolution of pirated digital content is uncertain (and often poor). Each of these factors can lead a user to view the legal good as being

[6] Ripley (2004) reports that this is an active deterrence strategy by movie companies, and that bogus postings by 'bored hackers' contributes further to this issue; for instance, downloads of pirated copies of what was purportedly *The Last Samurai* " turned out to be Scary Movie 3, Santa Clause 2 and a porn flick."

of higher quality. Additionally, the quality of pirated software is viewed as inferior due to more direct reasons (restrictions on functionality, lack of technical support).

Depending on the context, the parameter v may be normalized to 1, or may be replaced by the function $v(\rho)$. In addition, the parameter s may be replaced by the function $s(\rho)$, or the function $s(\rho, t)$. These functions will be explained in the appropriate section.

The maximum value that a customer of type θ can get from a pirated good of quality s is denoted $\hat{u}(\theta, s)$:

$$\hat{u}(\theta, s) = sU(\sigma(\theta), \theta) = \frac{s\theta^2}{2}. \tag{2.3}$$

Since the pirated goods are free, $\hat{u}(\theta, s)$ is the *reservation utility* of customer type θ.

The monopolist *does not observe* the type θ of any customer, but knows $F(\theta)$, the probability distribution of types in the customer population. For expositional simplicity, and since the hazard rate of the customer type distribution plays a significant role in subsequent analysis, we define the *inverse hazard rate* function $h(\theta)$:

$$h(\theta) = \frac{1 - F(\theta)}{f(\theta)}, \tag{2.4}$$

and the *cumulative inverse hazard rate* function $H(\theta)$:

$$H(\theta) = \int_{\alpha}^{\theta} \frac{1 - F(x)}{f(x)} dx. \tag{2.5}$$

The probability distribution of types is assumed to have the following properties:

1. $f(\theta) > 0$ for all θ, where $f(\theta)$ is the density corresponding to the distribution $F(\theta)$.

2. $\frac{\partial}{\partial\theta}[\frac{1-F(\theta)}{f(\theta)}] \leq 0$ for all θ: the inverse hazard rate is non-increasing in θ.

Each customer knows their own type θ. Without any loss in generality, the total number of customers in the market is normalized to 1.

2.2. Customer choice and pricing schedules

The seller offers a nonlinear pricing schedule (sometimes referred to as either a *contract* or a *pricing schedule*) which assigns a non-negative price to each feasible level of usage for the *legal good.* Rather than considering all possible pricing functions, the revelation principle ensures that we can restrict our attention to direct mechanisms – menus of quantity-price pairs $\{q(t), \tau(t)\}$, indexed by $t \in [\alpha, \beta]$ – which are incentive-compatible. The function $\tau(t)$ specifies the total price for a usage level $q(t)$;

the variable t has the same domain as the variable θ, and is used for notational clarity in equations (2.6) and (2.7) below. A pricing schedule $\{q(t), \tau(t)\}$, $t \in [\alpha, \beta]$ is said to be incentive-compatible if it satisfies:

$$\theta = \arg\max_t [vU(q(t), \theta) - \tau(t)], \text{ for all } \theta. \tag{2.6}$$

Given a pricing schedule $q(t), \tau(t)$ for the legal good, customers of type θ choose to purchase the legal good if their surplus from doing so is at least as much as the value they would derive from the (free) pirated good. Mathematically, if:

$$\max_t [vU(q(t), \theta) - \tau(t)] \geq \hat{u}(\theta, s), \tag{2.7}$$

then customers of type θ purchase the legal good. Therefore, an incentive-compatible pricing schedule $q(\theta), \tau(\theta)$ is said to *induce participation* from customer type θ if all customers of this type (weakly) prefer the legal good to the pirated good:

$$[vU(q(\theta), \theta) - \tau(\theta)] \geq \hat{u}(\theta, s). \tag{2.8}$$

The constraint (2.8) above is often referred to as the *piracy constraint* for type θ, since it becomes progressively harder to satisfy as s increases. Note that the piracy constraint is type-dependent[7]. In the special case of $s = 0$, since the pirated good has no value, $\hat{u}(\theta, s) = 0$, and constraint (2.8) above reduces to the standard individual rationality constraint of price discrimination problems.

Finally, the *optimal pricing schedule* $q^*(\theta, v, s), \tau^*(\theta, v, s)$ is the incentive compatible pricing schedule that maximizes the seller's profits, given v and s.

In general, the sequence of events is as follows: the seller announces their pricing schedule (and technological choices, if any), the customers make their purchase decisions (whether to use the legal good or the pirated good, and at what usage level) based on the pricing schedule, and each party gets their payoffs. An exact timeline is specified separately in each of the following sections.

2.3. Optimal pricing schedule in the absence of piracy

The optimal pricing schedule in the absence of piracy, termed the *zero-piracy pricing schedule*, is specified in this section. The zero-piracy pricing schedule benchmarks the analysis of pricing in the presence of piracy, and is also used in constructing the optimal pricing schedules.

[7] This is in contrast with most price discrimination or adverse selection models, where the RHS of the participation (individual rationality) constraint is normalized to zero across all types.

Lemma 1. *The zero-piracy pricing schedule* $q^{ZP}(\theta, v), \tau^{ZP}(\theta, v)$*, which is the optimal pricing schedule for the seller when* $s = 0$*, takes one of the following two forms.*

(a) If $h(\alpha) \leq \alpha$*, then the pricing schedule is designed to include all customer types The optimal contract is:*

$$q^{ZP}(\theta, v) = \theta - h(\theta); \tag{2.9}$$
$$\tau^{ZP}(\theta, v) = \frac{v[\alpha^2 - h(\theta)^2]}{2} + vH(\theta), \tag{2.10}$$

for all $\theta \in [\alpha, \beta]$.

(b) If $h(\alpha) > \alpha$*, then a set* $[0, \theta_{ZP}]$ *of customer types are priced out of the market, where* θ_{ZP} *is defined as:*

$$\theta_{ZP} = \theta : h(\theta) = \theta, \; \theta \in (\alpha, \beta). \tag{2.11}$$

The optimal contract is:

$$q^{ZP}(\theta, v) = \theta - h(\theta); \tag{2.12}$$
$$\tau^{ZP}(\theta, v) = \frac{v[h(\theta_{ZP})^2 - h(\theta)^2]}{2} + v[H(\theta) - H(\theta_{ZP})]. \tag{2.13}$$

for $\theta \in [\theta_{ZP}, \beta]$*, and*

$$q^{ZP}(\theta, v) = 0, \; \tau^{ZP}(\theta, v) = 0, \tag{2.14}$$

for $\theta \in [\alpha, \theta_{ZP}]$.

Unless otherwise specified, all proofs are available in Appendix A.

3. Pricing with digital piracy

This section analyzes pricing strategy when the seller faces digital piracy. The sequence of events modeled in this section is summarized in Figure 3.1.

3.1. Piracy-indifferent pricing schedule

This section specifies the incentive-compatible pricing schedule that implements *piracy-indifference*. Under this pricing schedule, all customer types are exactly indifferent between the legal good and the pirated good. This pricing schedule is important because it often forms a building block for the optimal pricing schedule.

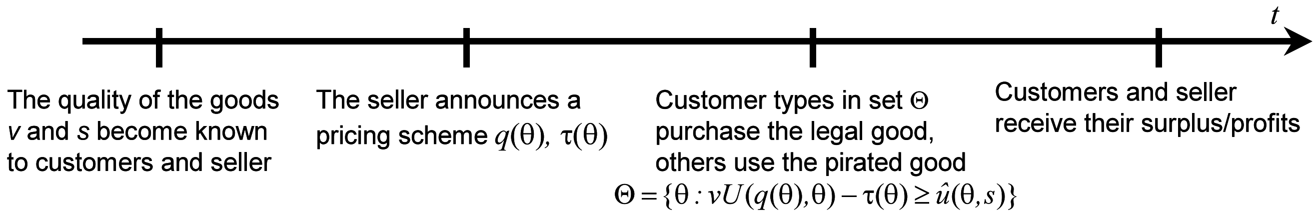


Figure 3.1: Timeline of events for Section 3

Lemma 2. *The unique incentive-compatible piracy-indifferent pricing schedule $q^{PI}(\theta, v, s), \tau^{PI}(\theta, v, s)$ for the legal good takes the following form:*

$$q^{PI}(\theta, v, s) = \frac{s\theta}{v}; \tag{3.1}$$

$$\tau^{PI}(\theta, v, s) = \frac{s[v-s]\theta^2}{2v}. \tag{3.2}$$

Under this pricing schedule, each customer type get the same surplus from their optimal usage of the legal good of quality v and their maximal usage of the pirated good of quality s.

Lemma 2 establishes that there is a unique piracy-indifferent pricing schedule, under which each customer type gets a net surplus exactly equal to their reservation utility – the value $\hat{u}(\theta, s)$ that they would get from their maximal usage of the pirated good. From equation (3.1), all customer types purchase positive quantities of the legal good under this pricing schedule. Moreover, their usage levels of the legal good are strictly increasing in the quality of the pirated good s. In addition, (3.2) indicates that so long as $s < v$, the total payment $\tau^{PI}(\theta, v, s)$ from each customer type θ is strictly positive. This establishes that the piracy-indifferent pricing schedule is strictly profitable for the seller.

3.2. Optimal price discrimination in the presence of piracy

This section describes the seller's optimal pricing schedule, and its main result is presented in the following result:

Theorem 1. *(a) When the threat of piracy is lower – that is, when $s \leq \dfrac{v[\alpha - h(\alpha)]}{\alpha}$ – the sellers optimal pricing schedule is a modified version of the zero-piracy pricing schedule, with total prices* *adjusted downwards by the same amount across all usage levels. The optimal contract is:*

$$q^*(\theta, v, s) = q^{ZP}(\theta, v); \tag{3.3}$$
$$\tau^*(\theta, v, s) = \tau^{ZP}(\theta, v) - \frac{s\alpha^2}{2}, \tag{3.4}$$

for all $\theta \in [\alpha, \beta]$, where $q^{ZP}(\theta, v)$ and $\tau^{ZP}(\theta, v)$ are as defined in equations (2.9) and (2.10).

(b) When the threat of piracy is higher – that is, when $s > \dfrac{v[\alpha - h(\alpha)]}{\alpha}$ – the seller's optimal pricing strategy is as follows:

(i) Customer types are partitioned into two sets $[\alpha, \hat{\theta}]$ and $[\hat{\theta}, \beta]$, where the transition type $\hat{\theta}$ is defined by:

$$\hat{\theta} = \theta : vh(\theta) = [v - s]\theta, \;\; \theta \in (\alpha, \beta). \tag{3.5}$$

(ii) The optimal pricing schedule for the lower set of customers is simply the piracy-indifferent pricing schedule:

$$q^*(\theta, v, s) = q^{PI}(\theta, v, s); \tag{3.6}$$
$$\tau^*(\theta, v, s) = \tau^{PI}(\theta, v, s), \tag{3.7}$$

for $\theta \in [\alpha, \hat{\theta}]$.

(iii) The optimal pricing schedule for the higher set of customers is an adjusted version of the zero-piracy pricing schedule, with total prices adjusted downwards by the same amount across all usage levels:

$$q^*(\theta, v, s) = q^{ZP}(\theta, v); \tag{3.8}$$
$$\tau^*(\theta, v, s) = \tau^{ZP}(\theta, v) - \left(vH(\hat{\theta}) - \frac{[v - s]\hat{\theta}^2}{2} + \frac{v\alpha^2}{2}\right), \tag{3.9}$$

for $\theta \in [\hat{\theta}, \beta]$, where $\tau^{ZP}(\theta, v)$ is as defined in part (a) of Lemma 1, in equation (2.10).

Theorem 1(a) establishes that when the quality of the pirated good is lower[8], the optimal pricing schedule is simply the zero-piracy pricing schedule, with a constant reduction in total price across all usage levels. The resulting usage levels of all consumers are unaffected by the presence of piracy,

[8] Clearly, the condition $s \leq \dfrac{v[\alpha - h(\alpha)]}{\alpha}$ of Theorem 1(a) does not just depend on the quality of the pirated good s, but also depends on v, α, and $h(\alpha)$. The statement 'when the quality of the pirated good is lower' is meant to indicate that for a fixed distribution, and fixed values of v and α, the proposition is more likely to apply at lower values of s.

and the reduction in total price across all customers is proportionate to s. An immediate corollary is that as the quality of the pirated good s increases, prices are strictly lower at all usage levels.

Theorem 1(b) establishes that with a higher threat of piracy, the portion of the optimal pricing schedule which is relevant to a lower set of customer types $[\alpha, \hat{\theta}]$ is simply the piracy-indifferent pricing schedule. Since $q^{PI}(\theta, v, s) > 0$, all these customer types purchase positive levels of the legal good. It can be shown that $\hat{\theta} > \theta_{ZP}$, and therefore any customer type who did not purchase in the absence of piracy is now a legal user, at their piracy-indifferent usage level $q^{PI}(\theta, v, s)$.

The presence of digital piracy can therefore have the socially beneficial effect of inducing *legal* usage from customers who may have otherwise been excluded by the seller's optimal price discrimination. While counter-intuitive, this result has a straightforward economic explanation. In the absence of piracy, when the seller is a monopolist, and there is no imperfect substitute for the seller's legal good, the seller finds it more favorable to price-discriminate in a manner that captures a higher level of surplus from the customer types $\theta \geq \theta_{ZP}$, at the cost of excluding customer types $[\alpha, \theta_{ZP})$ from the market. The only reason why customer types $\theta \in [\alpha, \theta_{ZP})$ are excluded is because the seller's optimal surplus extraction from higher customer types would not be feasible if there was any positive usage level affordable for the lower customer types. In the presence of piracy, each customer type who purchases the legal good must be provided with positive surplus of at least $\hat{u}(\theta, s)$ – their value from maximal usage of the pirated good. Since the seller is forced to provide this surplus level to the higher set, customer types in the lower set can now be offered positive and affordable usage levels, without affecting incentive-compatibility (and the seller's price-discrimination objectives).

It is straightforward to establish that $q^{ZP}(\theta, s) < q^{PI}(\theta, s)$ for $\theta \in [\alpha, \hat{\theta}]$. This implies that total usage either remains constant or goes up for all customer types, relative to the usage levels under the zero-piracy contract, and this increase is more pronounced at higher levels of s. As a consequence, the total value $vU(q^*(\theta, v, s), \theta)$ created by the usage of each customer type also increases, which in turn implies that total surplus is higher at higher levels of s. These observations are illustrated further in Figure 3.2. Additionally, under the optimal pricing schedule, all customer types get a surplus level which is *higher* than their reservation utility $\hat{u}(\theta, s)$ (which was also the surplus to each customer under the piracy indifferent pricing schedule). This surplus increase is due to the seller's desire to increase profits beyond the level obtained under the piracy-indifferent contract, by inducing higher usage across all customer types. Higher usage is necessarily accompanied by an increase in surplus for all types, in order to ensure incentive-compatibility.

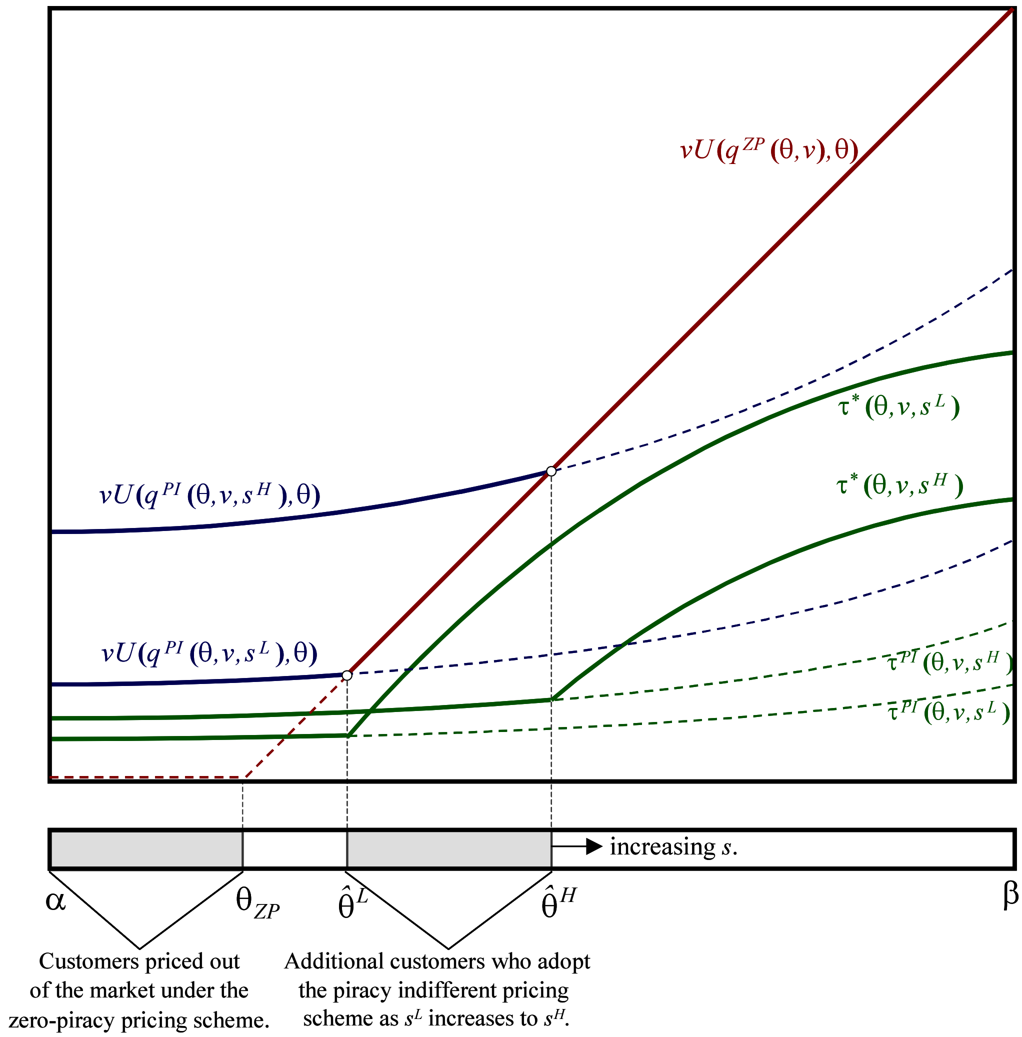


Figure 3.2: **(This figure is clearer when viewed electronically, or in color, since the curves are color-coded).** Illustrates the changes in pricing and surplus as the quality of the pirated good s changes For $s = s^L, s^H$, the piracy-indifferent pricing schedules are $q^{PI}(\theta, v, s), \tau^{PI}(\theta, v, s)$, and the total surplus generated by the usage of customer type θ is $vU(q^{PI}(\theta, v, s), \theta)$. The difference between $vU(q^{PI}(\theta, v, s), \theta)$ and $\tau^{PI}(\theta, v, s)$ is the minimum surplus that type θ must be provided in order to induce them to purchase the legal good, rather than using the pirated good. The thicker curves represent the optimal total prices and total surplus from optimal usage, while the dotted curves correspond to the portions of the 'building blocks' that are not part of the optimum. As shown, the optimal pricing schedule always involves the piracy-indifferent contract, for a subset of lower types $[0, \hat{\theta}]$. The set of types $[0, \theta_{ZP}]$ who would have been priced out of the market under the zero-piracy pricing schedule are now included. As s increases, the increase in $U(q^{PI}(\theta, v, s), \theta) - \tau^{PI}(\theta, v, s)$ forces the seller to expand the lower set of customer, and to lower prices for the higher types as well. Moreover, as s increases, the surplus $vU(q^*(\theta, v, s), \theta)$ generated by each customer type's consumption increases, which raises total surplus.

Theorem 1 is proved for a specific utility function, in which value (from both legal and pirated usage) is multiplicative separable into a quadratic function $U(q,\theta)$ and the quality parameter v (or s). However, the main result – that optimal pricing in the presence of piracy is a combination of the piracy-indifferent pricing schedule, and an adjusted version of the zero-piracy pricing schedule, with total prices adjusted downwards by the same amount across all usage levels – generalizes quite broadly. Loosely, the restrictions necessary for the result to hold are that utility $u(q,\theta,z)$ is strictly concave in usage, has a specific though not unusual kind of curvature (that $u_{12}(q,\theta,z)$ is strictly positive, increases weakly in q but decreases weakly in θ), and that the variation in specific marginal rates of changes in utility with quality z are positive at both $z = v$ and$z = s$. The mathematical details of the exact conditions under which the result generalizes are available to reviewers on request.

4. Digital rights management

This section studies digital rights management (DRM) systems that enable sellers to explicitly control their level of piracy protection. In addition to choosing a pricing schedule, the seller is now assumed to choose a level of technology-based protection $\rho \in [0,1]$, which affects the quality level of both the legal good and the pirated good, as described in Section 1. Specifically, at a level of technology-based protection ρ, the quality of the legal good is denoted $v(\rho)$ and the quality of the pirated good is denoted $s(\rho)$. The functions $v(\rho)$ and $s(\rho)$ are assumed to have the following properties:

1. $v(\rho) > s(\rho)$ for all ρ: The quality of the legal good is strictly higher than the quality of the pirated good, for all $\rho \in [0,1]$.

2. $v_1(\rho) < 0$, $s_1(\rho) < 0$: The quality of both the pirated good and the legal good are strictly decreasing in the level of technology-based protection ρ.

3. $s_1(0) < v_1(0)$: An increase in the level of technology-based protection initially reduces the quality of the pirated good more rapidly than the quality of the legal good. In other words, the DRM system is effective, at least initially.

4. $v_{11}(\rho) < s_{11}(\rho)$. The *rate* of quality degradation of the legal product increases relative to the *rate* of quality degradation of the pirated good, as ρ increases. This property implies diminishing returns from increasing technology-based protection ρ, and also ensures that $v_1(\rho) = s_1(\rho)$ at a unique point.

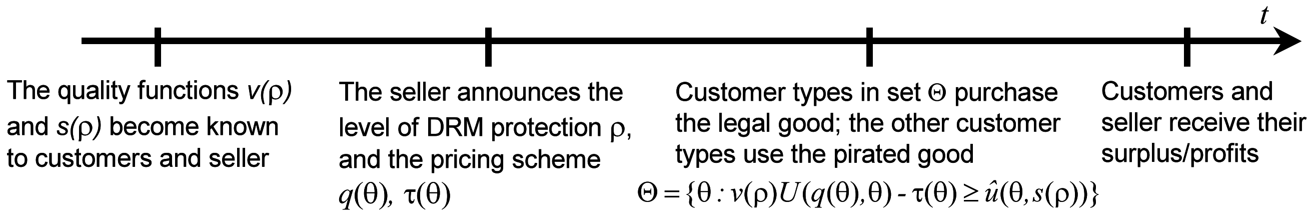


Figure 4.1: Timeline of events for Section 4

The costs to the seller of changing the level of protection ρ are assumed to be zero. This assumption is made in order to highlight the strategic and revenue effects of changes in technology-based protection levels. The sequence of events is summarized in Figure 4.1

4.1. Technologically-maximal protection level

Given a pair of quality functions $v(\rho)$ and $s(\rho)$, the *technologically-maximal level* of technology-based protection ρ^e is defined as the level of technology-based protection that maximizes the difference in quality between the legal good and the pirated good:

$$\rho^e = \arg\max_{\rho}[v(\rho) - s(\rho)]. \tag{4.1}$$

Under the properties of $v(\rho)$ and $s(\rho)$ assumed, the function $v(\rho) - s(\rho)$ is strictly concave in ρ, and therefore ρ^e is unique. The technologically-maximal level ρ^e is of interest because it seems like an intuitively natural choice for the seller, particularly in the absence of variable costs. It is also likely to be the level of protection marketed by a DRM vendor who is interested in highlighting the technologically effectiveness of their solution ('lots of reduction in piracy, only a minimal effect on product quality').

Additionally, if the seller does not price-discriminate, and chooses to charge each customer the same usage-independent fee T, then ρ^e is the optimal level of DRM protection. To see why this is the case: given values of T and ρ, the customer type θ who is indifferent between the legal good and the pirated good solves:

$$v(\rho)\frac{\theta^2}{2} - T = s(\rho)\frac{\theta^2}{2}, \tag{4.2}$$

which implies that the indifferent type θ at a price T and protection level ρ is:

$$\theta = \min[\alpha, \sqrt{\tfrac{2T}{v(\rho)-s(\rho)}}]. \tag{4.3}$$

In the former case (when all customers adopt),

$$T = [v(\rho) - s(\rho)]\frac{\alpha^2}{2}, \tag{4.4}$$

and the seller maximizes profits by maximizing $v(\rho) - s(\rho)$. In the latter case, the profit function that the seller maximizes when simultaneously choosing the optimal values of ρ and T is:

$$\Pi(\rho, T) = T[1 - F(\sqrt{\tfrac{2T}{v(\rho)-s(\rho)}})] \tag{4.5}$$

The first order condition $\Pi_1(\rho, T) = 0$ for the optimal ρ yields:

$$\left(T\sqrt{\tfrac{2T}{(v(\rho^*)-s(\rho^*))^3}}f(\sqrt{\tfrac{2T}{v(\rho^*)-s(\rho^*)}})\right)[v_1(\rho^*) - s_1(\rho^*)] = 0, \tag{4.6}$$

which implies that

$$v_1(\rho^*) - s_1(\rho^*) = 0,$$

since $T > 0$, $f(x) > 0$ for all x, and $v(\rho) > s(\rho)$.

This choice of technologically-maximal DRM protection when the seller charges all customers the same price has a simple intuitive explanation. Without the ability to price-discriminate, the seller's profits are driven entirely by the piracy constraint for the customer type θ who, at price T, is indifferent between the legal good and the pirated good. The difference in value between the legal and pirated good for this customer type θ is $[v(\rho) - s(\rho)]U(\sigma(\theta), \theta)$, which is also equal to the fixed fee that the seller charges all customers. Clearly, any increase in $v(\rho) - s(\rho)$ is therefore strictly profit-improving for the seller.

4.2. Optimal technology-based protection when the seller price-discriminates

This subsection characterizes the seller's optimal level of technology-based protection when the seller can price-discriminate, and shows that it is always strictly lower than the technologically-maximal level defined earlier.

Suppose that the relevant pricing schedule across the entire range $\rho \in [0, \rho^e]$ is as specified by Theorem 1(a). This occurs when $h(\alpha) \leq \frac{[v(\rho)-s(\rho)]\alpha}{v(\rho)}$ for all $\rho \in [0, \rho^e]$. Under this optimal pricing schedule, the seller's profits as a function of ρ are:

$$\Pi(\rho) = \Pi^L(\rho) = \int_{\alpha}^{\beta} \left(\frac{\alpha^2[v(\rho) - s(\rho)]}{2} + v(\rho)[H(\theta) - \frac{h(\theta)^2}{2}]\right) f(\theta)d\theta. \tag{4.7}$$

In contrast, under Theorem 1(b), the piracy constraint is binding for a positive fraction $[0, \hat{\theta}]$ of customers; moreover, the seller's increased pricing power from an increase in $v(\rho)$ only applies to the higher set $[\hat{\theta}, \beta]$ of customer types. For a specific ρ, define $\hat{\theta}(\rho)$ as the transition type between the two portions of the optimal pricing schedule derived in Theorem 1(b):

$$\hat{\theta}(\rho) = \theta : v(\rho)h(\theta) = \theta[v(\rho) - s(\rho)], \theta \in (\alpha, \beta) \tag{4.8}$$

This is identical to the definition of $\hat{\theta}$ in (3.5), simply indexed by ρ. Correspondingly, under the optimal pricing schedule specified by Theorem 1(b), the seller's profits as a function of ρ reduce to:

$$\begin{aligned} \Pi(\rho) &= \Pi^H(\rho) = \left(\frac{s(\rho)[v(\rho) - s(\rho)]}{v(\rho)}\right) \int\limits_{\alpha}^{\hat{\theta}(\rho)} \frac{\theta^2}{2} f(\theta)d\theta + v(\rho) \int\limits_{\hat{\theta}(\rho)}^{\beta} \left(H(\theta) - \frac{[h(\theta)]^2}{2}\right) f(\theta)d\theta \\ &\quad +[1 - F(\hat{\theta}(\rho))] \left(\frac{[v(\rho) - s(\rho)][\hat{\theta}(\rho)]^2}{2} - v(\rho)H(\hat{\theta}(\rho))\right). \end{aligned} \tag{4.9}$$

The next result shows that under either set of conditions, the optimal level of technology-based protection ρ^* is always strictly lower than the technologically-maximal level ρ^e:

Theorem 1. 2. *The profit-maximizing level of technology-based protection ρ^*:*

$$\rho^* = \arg\max_{\rho} \Pi(\rho), \tag{4.10}$$

is always strictly ***lower*** *than the technologically-maximal level of protection ρ^e. That is, $\rho^* < \rho^e$, where ρ^e is as defined in (4.1).*

Theorem 2 is a surprising result, since it indicates that at the optimal ρ^*, a small increase in technology-based protection would actually degrade the quality of the pirated good *more* than the quality of the legal good. However, it is not profitable for the seller to implement this increase in protection. The result is illustrated graphically in Figure 4.2

This result can be explained intuitively by examining its underlying economic effects. When the digital good is generally less replicable (that is, under the conditions of Theorem 1(a)), the piracy constraints defined in (2.8) are non-binding for all customer types $\theta > 0$. Therefore, the cost to the seller of a marginal increase in $s(\rho)$ is proportionate to the value the *lowest* type $\theta = 0$ gets from the pirated good. Simultaneously, the benefits of a marginal increase in $v(\rho)$ are two-fold. First, there is an increase in total price across all users due to the weakening of the piracy constraint for $\theta = 0$, which is identical in magnitude to the cost of the marginal increase in $s(\rho)$

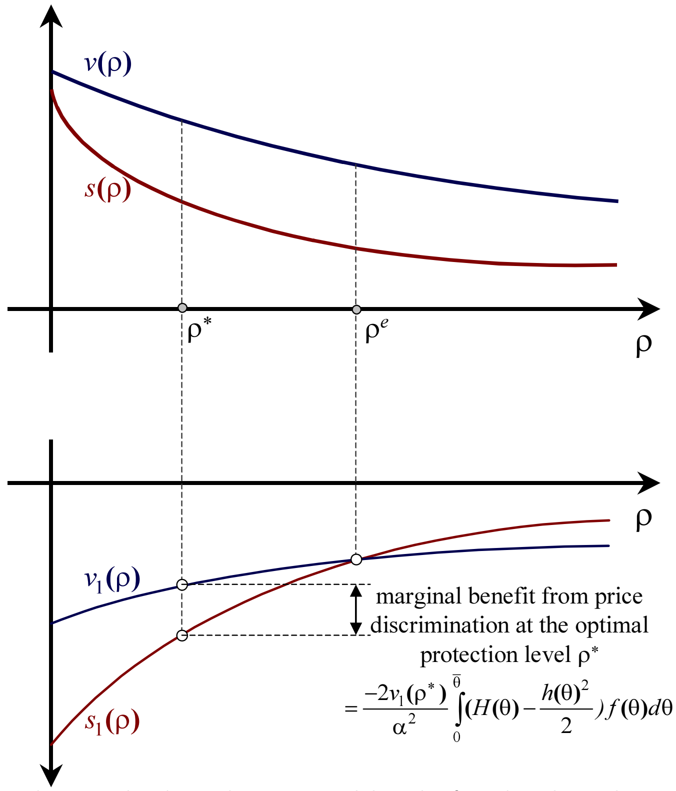


Figure 4.2: Illustrates the result that the optimal level of technology-based protection ρ^* is strictly lower than ρ^e, the technologically-maximal level . ρ^e occurs at the point where the difference between $v(\rho)$ and $s(\rho)$ is maximum, which is when $[v_1(\rho) - s_1(\rho)] = 0$. However, as illustrated, the first-order conditions for maximizing the sellers profits $\Pi^L(\rho)$ indicate that the optimal level ρ^* is at a point where $[v_1(\rho) - s_1(\rho)]$ is strictly positive, since there is an additional marginal benefit from increased pricing power when $v(\rho)$ is higher.

described above. In addition, there is a revenue change equal to the sum of the different changes in total price for different customer types which arise from optimally readjusting prices to satisfy incentive-compatibility. Under the optimal pricing schedule, the latter effect of a marginal increase in $v(\rho)$ is always positive.

Put simply, a small increase in $s(\rho)$ strengthens the piracy constraint, while a small increase in $v(\rho)$ weakens the piracy constraint (in an identical and opposite way), and also improves the seller's ability to price discriminate across all the customer types. Therefore, when the seller can price-discriminate, the benefit of a marginal increase in $v(\rho)$ is more than the cost of a corresponding marginal increase in $s(\rho)$. As a consequence, $\rho^* < \rho^e$.

Correspondingly, under the conditions of Theorem 1(b), a marginal increase in $s(\rho)$ strengthens the piracy constraint across all the customer types in the lower set $[0, \hat{\theta}(\rho)]$, whose usage levels are according to the piracy-indifferent contract. However, a marginal increase in $v(\rho)$ balances this effect exactly for each customer type. The result establishes that in addition, the marginal increase in $v(\rho)$ still has a net positive effect on the seller's pricing power for the higher set $[\hat{\theta}(\rho), \beta]$.

Clearly, Theorem 2 would continue to hold even if there was a direct fixed or variable cost to implementing DRM, so long as this cost was non-decreasing in the level of technology-based protection. Further implications of the result are discussed in Section 6.

4.3. Strategic responses to weakening DRM technology

Often, a DRM technology weakens over time, largely due to its being hacked by engineers who are trying to 'break' the protection scheme. This section investigates how a seller should alter their level of technology-based protection and their optimal pricing schedule in response to this progressive weakening of the DRM technology. The discussion in this subsection uses $\Pi(\rho) = \Pi^L(\rho)$. Comparable results hold for $\Pi(\rho) = \Pi^H(\rho)$, though the corresponding analysis is more involved.

The weakening of the DRM system is modeled as causing a gradual increase in the quality of the pirated good over time. Specifically, if the level of technology-based protection chosen is ρ, then the initial quality of the pirated good at time 0 – immediately upon implementing DRM – is denoted $s(\rho, 0)$, and the quality of the pirated good at time t is represented by the function $s(\rho, t)$, where $s_2(\rho, t) > 0$. In other words, as DRM protection weakens over time, the digital product becomes easier to replicate and the threat of piracy increases. Additionally, increasing the protection level ρ reduces the quality of the pirated good ($s_1(\rho, t) < 0$), as in Section 4.2.

It is assumed that the quality of the legal good $v(\rho)$ is not directly affected by the weakening of the DRM system. The seller's profit function therefore takes the following form:

$$\Pi^L(\rho, t) = \int_{\alpha}^{\beta} \left(\frac{\alpha^2[v(\rho) - s(\rho, t)]}{2} + v(\rho)[H(\theta) - \frac{h(\theta)^2}{2}] \right) f(\theta) d\theta, \tag{4.11}$$

and the optimal level of protection at time t, denoted $\rho^*(t)$, solves:

$$\rho^*(t) = \arg\max_{\rho} \Pi^L(\rho, t). \tag{4.12}$$

There are many ways in which $s(\rho, t)$ might evolve over time. An important clarification here is that there are no inter-temporal demand dependencies in the formulation. The discussion assumes a new set of customers at each point in time t, and examines the changes in protection that are optimal given the changes in the relationship between s and ρ, which are determined by $s(\rho, t)$.

Three specific scenarios are analyzed. The optimal technological and pricing responses prescribed for the seller are directionally different in each case, and these scenarios were chosen to highlight these differences. Under the first scenario, there is a constant upward drift in the quality of the pirated good, across all levels of protection:

$$s_2(\rho, t) > 0,\ s_{12}(\rho, t) = 0. \tag{4.13}$$

In this case, the marginal properties of $s(\rho, t)$ with respect to ρ do not change over time. That is, $s_1(\rho, t) = s_1(\rho, 0)$ for all t. Since $s_1(\rho, t)$ is constant over time, the optimal level of technology-based protection $\rho^*(t)$ is constant, and unaffected by the weakening of the technology. The seller should consequently maintain the same level of technology-based protection. While $v(\rho^*(t))$ remains unchanged, $s(\rho^*(t), t)$ increases over time, since $s_2(\rho, t) > 0$. Therefore, total prices should reduce over time for each usage level.

Under the second scenario, the weakening of the DRM system leads to *smaller* changes in the quality of the pirated good at higher levels of technology-based protection:

$$s_2(\rho, t) > 0,\ s_{12}(\rho, t) < 0. \tag{4.14}$$

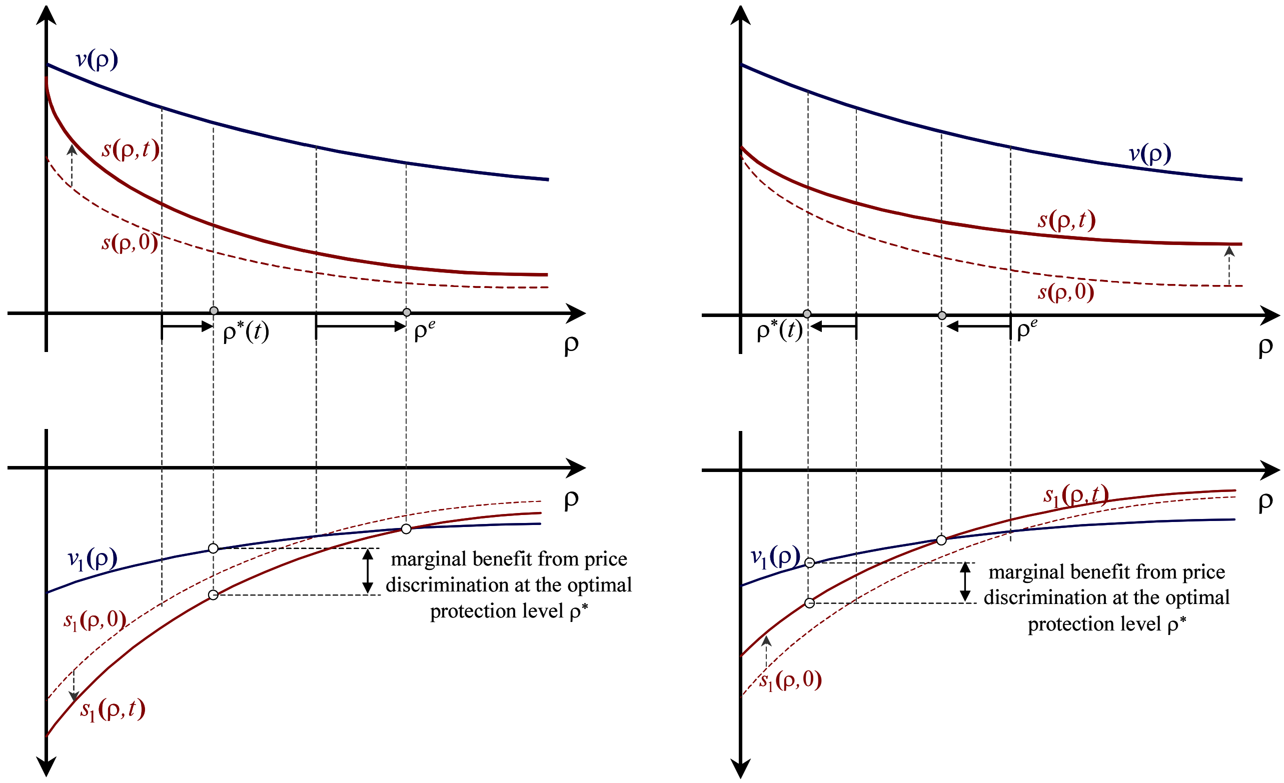
This type of change is illustrated in Figure 4.2 (a). It is characteristic of a DRM system under which higher levels of protection not only reduce the quality of the pirated good (by making it easier to replicate the digital good), but also make it increasingly difficult to hack the system.

Since $s_{12}(\rho, t) < 0$, the weakening of the DRM technology reduces the slope of $s(\rho, t)$ over time (that is, makes the slope more negative), thereby moving the function $s_1(\rho, t)$ downwards. As a consequence, the optimal level of protection shifts to the right, and the seller's optimal technological response is to *increase* their level of technology-based protection over time.

The optimal adjustment ρ from $\rho^*(0)$ to $\rho^*(t)$ causes a net decrease in the quality of the legal good to $v(\rho^*(t))$, accompanied by what is typically a net increase in the quality $s(\rho^*(t), t)$ of the pirated good[9] relative to the initial value $s(\rho^*(0), 0)$, The pricing schedule in Theorem 1 indicates that this should lead to a decrease in total prices. Therefore, in conjunction with their technological response, the seller's optimal pricing response to the weakening of the DRM system is to reduce prices across all customer types.

The third scenario is where the weakening of the DRM technology leads to *larger* changes in

[9] Even if there is a net decrease in the quality of the pirated good $s(\rho, t)$, it will be lower in magnitude than the corresponding decrease in the quality of the legal good $v(\rho)$. Therefore, prices will always reduce, as discussed in section 4.2.



(a) Changes in DRM protection when $s_{12}(\rho,t) < 0$ **(b) Changes in DRM protection when** $s_{12}(\rho,t) > 0$

Figure 4.3: Illustrates the changes in the optimal level of technology-based protection as the DRM system is progressively weakened by hackers over time.

the quality of the pirated good at *higher* levels of protection:

$$s_2(\rho, t) > 0,\ \ s_{12}(\rho, t) > 0. \tag{4.15}$$

This type of change is illustrated in Figure 4.2 (b). It is characteristic of a technology for which higher levels of protection reduce the quality level of the pirated good, but the technology that implements every marginal increase in the level of protection is increasingly vulnerable to hacking. Note that the assumption that $s_1(\rho, t) < 0$ is maintained, and so quality levels of the pirated good continue to be lower at higher levels of protection, even post-hacking.

In contrast with the earlier scenario, since $s_{12}(\rho, t) > 0$, the weakening of the DRM technology increases the slope of $s(\rho, t)$ over time, thereby moving the function $s_1(\rho, t)$ upwards. As a consequence, the optimal level of protection moves to the left, and the seller's optimal technological response is to *reduce* their level of technology-based protection over time.

As the optimal level of protection $\rho^*(t)$ decreases, there is a substantial increase in the quality of the pirated good $s(\rho^*(t), t)$. There is also an increase in the quality of the legal good (since $\rho^*(t)$

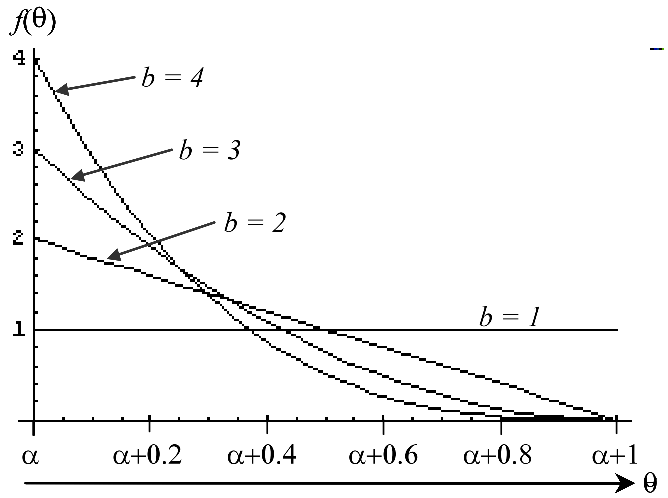


Figure 5.1: Illustrates the beta density function, used to characterize different customer type distributions in the example, with $a = 1$, and for different values of b.

decreases, $v(\rho^*(t))$ increases). It is clear that the increase in $s(\rho^*(t), t)$ is more than the increase in $v(\rho^*(t))$ – however, as discussed in Section 4.2, a marginal increases in the quality of the legal good increases prices and profits more than a corresponding marginal increase in the quality of the pirated good. Therefore the direction of the pricing response cannot be characterized in general. However, since the prices for lower customer types are progressively more affected by the quality of the pirated good (than prices for customer types higher than theirs), the pricing response will be progressively less favorable for higher customer types, independent of its direction.

Sometimes, implementing frequent changes to their level of technology-based protection ρ is costly for the seller. If the seller anticipates that there will be a weakening of the DRM system over time, it may be in their best interest to start out by overprotecting their legal good under scenario 2, and underprotecting it under scenario 3. This issue is discussed further in Section 6.

5. Example

In this section, the optimal pricing schedule is derived explicitly for a specific family of customer type distributions. This example, based on Theorem 1, further illustrates the effects of digital piracy on pricing, usage, and welfare, and highlights the effect of some properties of the customer type distribution.

The family of customer type distributions used in the example have a shifted beta density function[10] $B(\theta; a, b)$, with support $[\alpha, 1+\alpha]$, $a = 1$, and parametrized by $b \geq 1$. When $b = 1$, the distribution is the uniform distribution $U[\alpha, 1+\alpha]$. When $b > 1$, the distribution is positively skewed, and $f(\theta)$ is strictly decreasing in θ. The beta density function is illustrated in Figure 5.1, for a candidate set of b values.

When $a = 1$ and $b > 0$, the beta distribution has the distribution function $F(\theta) = 1-(1+\alpha-\theta)^b$ and density function $f(\theta) = b(1+\alpha-\theta)^{b-1}$. Accordingly, the inverse hazard rate function $h(\theta)$ and the cumulative inverse hazard rate function $H(\theta)$ take the following form:

$$h(\theta) = \frac{1-(\theta-\alpha)}{b}; \tag{5.1}$$

$$H(\theta) = \frac{2(\theta-\alpha)-(\theta-\alpha)^2}{2b}. \tag{5.2}$$

For the purpose of this example, the value of v is normalized to 1. The optimal pricing schedules can now be derived, based on the general expressions derived in Section 3.2. Since v has been normalized to 1, it is dropped as an argument of the pricing and usage functions.

Table 3.1 summarizes the relevant consumption and total pricing functions that define the optimal pricing schedule. When the quality of the pirated good is lower, an explicit pricing function can be easily derived:

$$p(q) = \left(\frac{2+b}{2b^2(b+1)} - \frac{\alpha(2+\alpha)}{2(1+b)} - \frac{\alpha^2 s}{2}\right) + \frac{2(1+\alpha)q - q^2}{2(b+1)}. \tag{5.3}$$

The optimal pricing schedule is therefore a *nonlinear two-part tariff*. Differentiating both sides of (5.3) with respect to q yields:

$$p_1(q) = \frac{(1+\alpha-q)}{(b+1)}; \tag{5.4}$$

$$p_{11}(q) = -\frac{1}{(b+1)}. \tag{5.5}$$

(5.5) implies that the variable portion of the optimal pricing schedule is strictly concave in q, for any $b > 0$. In addition, (5.4) indicates that the marginal price $p_1(q)$ is strictly increasing in α, and strictly decreasing in b. As one might expect, variable prices increase with an average increase in

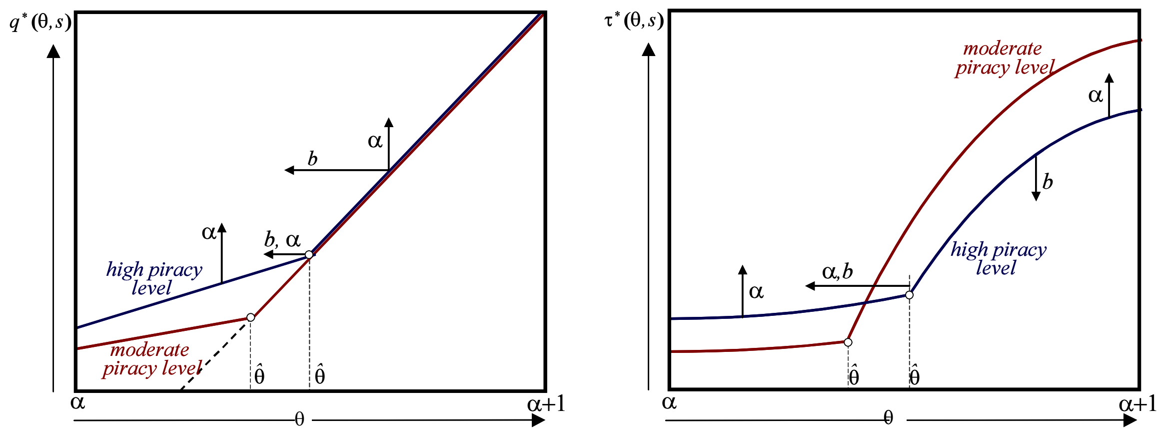
[10] The general form of the beta density function is

$$B(x; a, b) = \frac{x^{a-1}(1-x)^{b-1}}{\beta(a,b)},$$

where $\beta(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx$ is the beta function with parameters a and b.

Intermediate contracts	
Zero-piracy usage level:	$q^{ZP}(\theta) = \dfrac{\theta[1+b] - [\alpha+1]}{b}.$
Piracy-indifferent pricing schedule:	$q^{PI}(\theta, s) = s\theta;$ $\tau^{PI}(\theta, s) = \dfrac{s[1-s]\theta^2}{2}.$
Transition type $\hat{\theta}$:	$\hat{\theta} = \alpha + \dfrac{1-\alpha b[1-s]}{1+b[1-s]}.$
Optimal pricing schedule	
Lower quality of pirated good	
For $s \leq 1 - \dfrac{1}{\alpha b}$	$q^*(\theta, s) = \dfrac{\theta[1+b] - [\alpha+1]}{b};$ $\tau^*(\theta, s) = \dfrac{[1-s]b^2\alpha^2 - 1 + [1+b][2(\theta-\alpha) - (\theta-\alpha)^2]}{2b^2}.$
Higher quality of pirated good	
For $s \geq 1 - \dfrac{1}{bw}$, and $\theta \leq \hat{\theta}$	$q^*(\theta, s) = s\theta;$ $\tau^*(\theta, s) = \dfrac{s(1-s)\theta^2}{2}.$
For $s \geq 1 - \dfrac{1}{bw}$, and $\theta \geq \hat{\theta}$	$q^*(\theta, s) = \dfrac{\theta(1+b) - (\alpha+1)}{b};$ $\tau^*(\theta, s) = \frac{\hat{\theta}[1-s]}{2} + \frac{[1+b][2(\theta-\alpha)+(\theta-\alpha)^2]}{2b^2} - \frac{2(\hat{\theta}-\alpha)+(\hat{\theta}-\alpha)^2}{2b}$

Table 5.1: Optimal contracts and surplus expressions for the example



(a) Optimal usage $q^*(\theta,s)$ for each type θ

(b) Total price $\tau^*(\theta,s)$ for each type θ

Figure 5.2: **(This figure is clearer when viewed electronically, or in color, since the curves are color-coded).** Depicts the usage levels and total price for different customer types in the example, under the conditions of Proposition 3. The labeled arrows represent the direction in which the respective curves or points shift when the corresponding parameter increases (and while only one set of curves is labeled, the shift is directionally identical for both s values). For instance, an increase in α raises the usage levels and total price for all types, at all feasible levels of piracy.

value, and are lower when there is a higher proportion of the lower customer types.

When the quality of the pirated good is higher, Figure 5.2 illustrates the usage function $q^*(\theta, s)$ and total price function $\tau^*(\theta, s)$, for moderate and high piracy levels. The labeled arrows in the figures represent the direction in which the functions shift in response to a change in the corresponding parameter. As the quality of the pirated good s increases, the lower set of customer types expands, and there is a strict increase in each of their (piracy-indifferent) usage levels. The legal usage of the rest of the customers remains the same; however, the total price paid by each of these customer types is strictly lower.

An increase in α increases both the usage and total price across all customers, and also shifts a fraction of customers away from the piracy indifferent contract. An increase in the skewness of the distribution of customer types (an increase in b) increases the size of the higher customer type set. This increase in skewness does not affect consumption or pricing for those customers who are remain in the lower set – however, for all other customers, there is a strict increase in consumption, and a strict reduction in price. This is because as b increases, there is a larger density of customers at the lower end of the market, and it is in the seller's interest to increase usage for lower customer types, thereby increasing the surplus generated by their usage, and the seller's profit potential. This increase is at the expense of lower pricing power on the higher end of the market, which makes sense intuitively, since there are fewer higher customer types.

6. Discussion and conclusion

A number of new results relating to managing digital goods subject to piracy have been derived in Sections 3 and 4. This section discusses some of these results further, specifically highlighting pricing and technology-based protection guidelines, and some welfare issues. It concludes with an outline of open research questions raised by this paper.

6.1. Guidelines for pricing with digital piracy

With a positive but relatively low threat of piracy, Theorem 1(a) shows that piracy affects a seller's pricing power uniformly across its different customers segments. This is despite the fact that each of these segments may value the pirated good differently. Consequently, the seller's pricing strategy should be to design the optimal pricing schedule unconstrained by piracy, and then simply adjust total prices downwards across all usage levels, by an amount proportionate to the value their lowest customer type would get from the pirated good.

As piracy levels increase, sellers need to segment their customers more carefully, and pay closer attention to the differential value that customers may get from the pirated good. The optimal pricing adjustment in response to a higher threat of piracy often induces legal purchasing by a new set of customers who were previously priced out of the market. The corresponding part of the pricing schedule is based on the piracy-indifferent contract, which is a low-price, low-usage pricing schedule. Additionally, as piracy levels increase, pricing for the higher segment of the market should be lowered further, by the value the lowest customer type in this segment would derive from the pirated good.

As the market for a successful digital product matures, there is often an increase in desired usage levels across all customers in the market (which corresponds to a shift in the type distribution to the right, modeled as an increase in the value of α). In response, the seller should expand the fraction of customers included in their higher segment, and simultaneously increase prices. Alternately, the seller may observe a net increase in average desired usage, due to a progressive upward shift of lower-end customer types, which results in a flattening of the distribution of customer types. In this case, it is optimal for the seller to shrink the higher segment, move more customer types into the piracy-indifferent segment, and raise prices in the higher segment.

The careful reader may have noticed that the piracy-indifferent pricing may not be directly implementable, since total price is convex in quantity. In general, a menu of two-part tariffs is needed to approximate this convex schedule, as shown in Laffont and Martimort (2002). Additionally, there is often a fixed cost associated with administering the usage and billing of each individual customer (Sundararajan, 2003a). Sometimes, this cost may outweigh the revenue generated by the piracy-indifferent segment of customers, and the seller is better off excluding them. This exclusion will not reduce total surplus, since these customers will use the pirated good instead. These imperfections may explain the persistence of positive piracy rates over time, despite it being theoretically profitable for a monopolist to cover the entire market.

6.2. Guidelines for managing DRM-based piracy deterrence

Digital rights management is a valuable technological deterrent to piracy, and can improve a seller's profitability substantially. However, it is crucial that sellers consider the effects that their DRM implementation will have on the value of their legal product. The model in Section 4 highlights this issue, and provides guidelines for how to optimally balance value reduction with piracy deterrence. An immediate consequence of this trade-off is that excessive restrictions on legal usage that aim to deter piracy can result in a failure to create a viable market for the legal good. The success of Apple's iTunes music service which, at the risk of higher piracy levels, placed far fewer restrictions on legal usage than its online predecessors (like MusicNet and Rhapsody), is a recent illustration.

A more subtle result is that if the seller can price-discriminate, choosing the level of protection that balances marginal value reduction with marginal piracy deterrence – the technologically maximal level – is never optimal. Instead, the seller is always better off choosing a lower level of technology-based protection. When considering potential DRM solutions, it is natural for sellers to focus primarily on the ability of the technology to deter piracy. However, since the effect that DRM has on the value of the legal good is *more important* for profitability, sellers may need to realign their focus when evaluating these products. Correspondingly, when designing rights management technologies, vendors should focus more on how effective their solution will be in preserving the value provided to legal users, since this is the dominant profit driver for their corporate customers.

Even the best DRM technologies are unlikely to be hacker-proof. The results of this paper provide technological and pricing responses for sellers who must deal with this reality, and establish that is critical that sellers understand their DRM technology before they respond to the threat of hacking. As shown in Section 4.3, the nature of the interaction between the level of protection and the corresponding difficulty of breaking the protection scheme is what determines the optimal technological response. In implementing each marginal increase in protection, if the DRM system relies on technology that is increasingly fragile, then the seller is likely to need to reduce their protection levels over time. On the other hand, if the successive 'pieces' of the system are progressively more robust, the seller's best strategic response to hacking is to increase their level of technology-protection over time, and simultaneously reduce their prices.

While pricing responses are easy to implement, continuous variation of technology-based protection levels over time is often expensive, technologically difficult, and sometimes impossible to implement. As a consequence, it may be good strategy to preemptively implement a sub-optimal level of protection, based on the appropriate expectation of how future hacking will affect piracy levels. Again, a clear understanding of the details of the technology is crucial – whether an increase in protection levels makes the technology more robust or more vulnerable to hacking is an important determinant of whether to preemptively overprotect or underprotect.

6.3. Welfare and policy issues

The analysis in Section 3 suggests that the presence of digital piracy can have socially beneficial effects. Specifically, it may lead a seller to alter pricing in a manner that increases the legal usage of existing customers, and that includes lower-end customers who had been priced out of the market earlier. These changes raise total surplus, and the increase is a consequence of higher *legal* usage. These welfare benefits from piracy have not been highlighted in the literature thus far – the focus has been on surplus that might be generated from the use of the pirated product. Moreover, there may be a corresponding increase in distributional equity, since the total surplus generated is shared more evenly between customer types. Clearly, these increases in surplus come at the expense of a reduction in seller profits, which could affect incentives for the creation of content. This trade-off needs to be analyzed before concluding that piracy has unilateral welfare benefits. However, in many creative industries (such as music, art and literature), the creators' ability to capture rents from their creations may not be the primary driver of innovation, and the trade-off may not be especially important.

Digital piracy has also led to a much stronger emphasis on protecting intellectual property using technology, rather than the legal system. This has already led to substantial debate about the extent to which copyright owners should be able to control the usage of their products, especially in light of the somewhat overreaching legal protection that the 1998 Digital Millennium Copyright Act provides these owners. The social benefits of digital piracy that are outlined above strengthen recent arguments (Samuelson, 2003) that one needs to carefully assess the welfare implications of an enforcement system which increasingly relies on technology, thereby giving sellers a broader set of rights over their content in practice.Work on this front is discussed further in the concluding paragraph of this paper.

6.4. Concluding remarks

A natural set of questions raised by the results of this paper relate to their generalizability, if one were to change the functional form of the utility function, drop the assumption of multiplicative separability of quality and value, or use different functions to model the value from the legal and the pirated goods. As discussed in Section 3, under some assumptions about the curvature of the utility function, the main results of Theorem 1 continue to hold. Additionally, the higher impact of $v(\rho)$ on profits which leads to lower levels of technology-based protection, and the direction of technological responses to hacking are likely to hold under more general assumptions.

Another question that arises, especially in light of the past literature (Takeyama, 1994, Conner and Rummelt, 1991) relates to the relationship between network effects and piracy, while simultaneously admitting nonlinear pricing schedules and endogenous choices of protection technology. If anything, the presence of a positive network effect will strengthen the results of section 4.2, since the network benefits from increased gross usage would suggest the optimality of even lower levels of technology-based protection. The effects on pricing are less straightforward to predict. An extension based on Sundararajan (2003b) is being developed to investigate this. A different, and potentially more interesting externality is the *piracy-induced usage externality*. Since most pirated goods are 'produced' from legal copies of the product, their quality and availability is proportionate to the extent of legal usage of the product. Consider the example of pirated software – it is generally made available by legal users who crack the software's copy protection scheme, and therefore the quality (and availability) of the pirated software is likely to be higher when there are more potential pirates. Correspondingly, the variety of pirated music available on file-sharing networks depends on the variety of music legally purchased by the users who create the illegal copies. As the fraction of purchases via digital channels increase, this becomes an important usage externality associated with the consumption of the legal good that the seller needs to consider when pricing their product, or choosing a level of DRM technology. This is a negative externality from the point of view of the seller, but a positive externality for their customers. A parallel focus of ongoing work is endogenizing the presence of this externality into the model of this paper. Preliminary results from an multiperiod model indicate that as the seller attempts to internalizes the piracy-induced usage externality, total and average prices increase, though the changes in equilibrium levels of technology-based protection are governed entirely by demand interdependencies across the periods.

A companion paper under development studies the trade-offs between technology-based and law-based approaches to managing intellectual property, with a particular emphasis on the challenges raised by the digitization of a variety of products, and the accompanying issues of digital piracy.

To some extent, it is based on the approaches of Romer (1990) and Kwan and Lai (2003), though explicitly considering the degree to which technological capability affects the actual rights that users of digital goods get (either legally or otherwise), which is critical in this context. Preliminary results show that under a technology-based protection regime, inefficiency is caused by socially-suboptimal technology choices made by a profit-maximizing DRM intermediary; on the other hand, the inefficiency from a law-based regime stems from the divergence of equilibrium outcomes from the welfare-maximizing ones; both suboptimal production and suboptimal distribution contribute to this inefficiency, which also increases with the rate of technological progress under either regime. This analysis is still in progress, and I hope to complete it in the near future.

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A. Appendix: Proofs

Proof of Lemma 1

In the absence of piracy, all customer types have equal reservation utility of zero. The function $vU(q,\theta)$ and the inverse hazard rate $h(\theta)$ of the customer type distribution satisfy all the conditions necessary to apply the solution to the standard non-linear pricing model, where all types with non-zero allocations are separated at the optimum. See Salanié (1998), Chapter 2, or Maskin and Riley (1984), for an exposition of the theory, or Lemma 1 of Sundararajan (2002), which provides a complete proof of a more general version of this model (in the absence of piracy). Under this solution, the optimal allocation to each type $\theta \in [\alpha, \beta]$ satisfies:

$$q^{ZP}(\theta, v) = \max\{q(\theta, v), 0\}, \tag{A.1}$$

where, for each θ, $q(\theta, v)$ is the unique solution to:

$$vU_1(q(\theta, v), \theta) = vU_{12}(q(\theta, v), \theta)h(\theta), \tag{A.2}$$

and the optimal total price charged to type $\theta \in [\alpha, \beta]$ is

$$\tau^{ZP}(\theta, v) = vU((q^{ZP}(\theta, v), \theta) - \left(vU(q^{ZP}(\alpha, v), \alpha) + \int\limits_{x \in Q} vU_2(q^{ZP}(x, v), x)dx \right), \tag{A.3}$$

where

$$Q = \{\theta : q(\theta, v) \geq 0\}. \tag{A.4}$$

Recall that $U(q, \theta) = \theta q - \frac{1}{2}q^2$. Substituting the functional forms for $U_1(q, \theta)$ and $U_{12}(q, \theta)$ into (A.2) yields:

$$q(\theta, v) = \theta - h(\theta). \tag{A.5}$$

(a) Since $h_1(\theta) \leq 0$, if $h(\alpha) \leq \alpha$, it follows that $q(\theta, v) \geq 0$ for all θ. As a consequence, for each θ,

$$q^{ZP}(\theta, v) = \theta - h(\theta). \tag{A.6}$$

Substituting (A.6) into (A.3) and rearranging yields:

$$\tau^{ZP}(\theta, v) = v \left(\frac{\theta^2 - [h(\theta)]^2}{2} - \frac{\alpha^2 - [h(\alpha)]^2}{2} - \int\limits_{\alpha}^{\theta} [x - h(x)]dx \right),$$

which simplifies to the expression in (2.10).

(b) If $h(\alpha) > \alpha$, then $q(\alpha, v) < 0$. However, we know that $h(\beta) = 0$, and therefore, that

$q(\beta, v) = \beta > \alpha$. Since $h_1(\theta) \leq 0$, this implies that there is a unique $\theta_{ZP} \in (\alpha, \beta)$ such that:

$$\begin{aligned} \theta_{ZP} &= h(\theta_{ZP}); && \text{(A.7)} \\ q(\theta, v) &< 0 \text{ for } \theta < \theta_{ZP}; && \text{(A.8)} \\ q(\theta, v) &> 0 \text{ for } \theta > \theta_{ZP}. && \text{(A.9)} \end{aligned}$$

Consequently, from (A.1), it follows that:

$$\begin{aligned} q^{ZP}(\theta, v) &= 0 \text{ for } \theta \leq \theta_{ZP}; && \text{(A.10)} \\ q^{ZP}(\theta, v) &= \theta - h(\theta) \text{ for } \theta \geq \theta_{ZP}. && \text{(A.11)} \end{aligned}$$

Since $U(0, \theta) = 0$ for all θ, the expression for $\tau^{ZP}(\theta, v)$ in (A.3) reduces to:

$$\begin{aligned} \tau^{ZP}(\theta, v) &= 0 \text{ for } \theta \leq \theta_{ZP}; && \text{(A.12)} \\ \tau^{ZP}(\theta, v) &= v\left(\frac{\theta^2 - [h(\theta)]^2}{2} - \int_{\theta_{ZP}}^{\theta} [x - h(x)]dx\right) \text{ for } \theta \geq \theta_{ZP}. && \text{(A.13)} \end{aligned}$$

Simplifying (A.13) yields the expression in (2.13), which completes the proof.

Proof of Lemma 2

Recall that $\sigma(\theta) = \arg\max_q U(q, \theta) = \theta$. The conditions for a contract $q(\theta), \tau(\theta)$ to be both incentive-compatible and piracy-indifferent are:

$$\theta = \arg\max_x v(U(q(x), \theta) - \tau(x); \tag{A.14}$$

$$vU(q(\theta), \theta) - \tau(\theta) = sU(\sigma(\theta), \theta) \text{ for all } \theta. \tag{A.15}$$

(A.14) ensures incentive-compatibility, and (A.15) ensures piracy indifference. First-order conditions for (A.14) for each θ yield:

$$vU_1(q(\theta), \theta)q_1(\theta) = \tau_1(\theta) \text{ for all } \theta. \tag{A.16}$$

Now, differentiating (A.15) with respect to θ, and using the fact that $U_1(\sigma(\theta), \theta) = 0$, one gets:

$$vU_1(q(\theta), \theta)q_1(\theta) + vU_2(q(\theta), \theta) - \tau_1(\theta) = sU_2(\sigma(\theta), \theta). \tag{A.17}$$

Substituting (A.16) into (A.17) yields:

$$U_2(q(\theta), \theta) = \frac{sU_2(\sigma(\theta), \theta)}{v}, \tag{A.18}$$

which upon substitution of the functional form of $U(q,\theta)$ yields

$$q(\theta) = \frac{s\theta}{v}. \tag{A.19}$$

Substituting (A.19) into (A.15) and rearranging, one gets:

$$\tau(\theta) = \frac{s[v-s]\theta^2}{2v} \tag{A.20}$$

Therefore, the simultaneous requirements of incentive-compatibility and piracy indifference yield a unique contract. Consequently, the unique piracy-indifferent contract is:

$$q^{PI}(\theta, v, s) = \frac{s\theta}{v}, \tag{A.21}$$

$$\tau^{PI}(\theta, v, s) = \frac{s[v-s]\theta^2}{2v}, \tag{A.22}$$

which completes the proof.

Proof of Theorem 1

The proof introduces some new notation, which follows Jullien (2000) for the most part. Define

$$\hat{h}(\gamma, \theta) = \frac{\gamma - F(\theta)}{f(\theta)}. \tag{A.23}$$

Clearly, $\hat{h}(1,\theta) = h(\theta)$. Next, define:

$$l(\gamma, \theta) = \arg\max_q vU(q,\theta) - \hat{h}(\gamma,\theta)U_2(q,\theta) \tag{A.24}$$

First-order conditions for (A.24) yield:

$$l(\gamma,\theta) = \theta - \hat{h}(\gamma,\theta). \tag{A.25}$$

Finally, define the set Θ:

$$\Theta = \{\theta : l(1,\theta) \leq q^{PI}(\theta, v, s) \leq l(0,\theta)\}, \tag{A.26}$$

where $q^{PI}(\theta, v, s)$ is as defined in (3.7) of Lemma 2. It is easily shown that $l(0,\theta) > q^{PI}(\theta, v, s)$ for all θ, and therefore the latter inequality in (A.26) is redundant.

The following two intermediate results – Lemma 3 and Lemma 4 – are used in the proof of this theorem:

Lemma 3. *If $h(\theta)$ is non-increasing for all θ, then $\hat{h}_2(\gamma,\theta) \leq 0$ for all θ, γ such that $\hat{h}(\gamma,\theta) \geq 0$.*

Proof. Assume the converse – that for some γ, $\hat{h}(\gamma,\theta)$ is increasing in some interval $[\theta_1, \theta_2]$.

This implies that

$$\frac{\gamma - F(\theta_1)}{f(\theta_1)} < \frac{\gamma - F(\theta_2)}{f(\theta_2)}. \tag{A.27}$$

Since $F(\theta_1) < F(\theta_2)$, this implies that $f(\theta_1) > f(\theta_2)$, which in turn implies that

$$\frac{1-\gamma}{f(\theta_1)} < \frac{1-\gamma}{f(\theta_2)} \tag{A.28}$$

Adding (A.27) and (A.28) yields $\frac{1-F(\theta_1)}{f(\theta_1)} < \frac{1-F(\theta_2)}{f(\theta_2)}$, which contradicts the fact that $h(\theta)$ is non-increasing, and the result follows. ■

Lemma 4. *For all γ, θ such that $\hat{h}(\gamma,\theta) \geq 0$, $l_2(\gamma,\theta) > q_1^{PI}(\theta, v, s)$.*

Proof. Differentiating (A.25) with respect to θ yields:

$$l_2(\gamma,\theta) = 1 - \hat{h}_2(\gamma,\theta), \tag{A.29}$$

which implies that $l_2(\gamma,\theta) \geq 1$, since $\hat{h}_2(\gamma,\theta) \leq 0$. Similarly, differentiating (3.7) with respect to θ yields:

$$q_1^{PI}(\theta, v, s) = \frac{s}{v} < 1, \tag{A.30}$$

and the result follows. ■

Under Lemmas 3 and 4, all the conditions for Proposition 3 of Jullien (2000) to apply are met. Given that $l(0,\theta) > q^{PI}(\theta, v, s)$, the optimal contract is therefore specified by:

$$q^*(\theta, v, s) = q^{PI}(\theta, v, s) \text{ for } \theta \in \Theta \tag{A.31}$$

$$q^*(\theta, v, s) = l(1,\theta) \text{ for } \theta \notin \Theta \tag{A.32}$$

From (A.25) and (3.7),

$$l(1,\theta) = \theta - h(\theta); \tag{A.33}$$

$$q^{PI}(\theta, v, s) = \frac{s\theta}{v}. \tag{A.34}$$

Part (a): Under the conditions of part (a) of the theorem, $h(\alpha) \leq \dfrac{[v-s]\alpha}{v}$. Since $h_1(\theta) \leq 0$, this implies that

$$h(\theta) \leq \frac{\theta[v-s]}{v} \tag{A.35}$$

for all θ, which when combined with (A.33) and (A.34), implies that:

$$l(1,\theta) > q^{PI}(\theta, v, s), \tag{A.36}$$

and the set Θ is therefore empty. Comparing (A.33) and (2.9), and using (A.32) yields:

$$q^*(\theta, v, s) = q^{ZP}(\theta, v) \text{ for all } \theta. \tag{A.37}$$

The expression for $\tau^*(\theta, v, s)$ follows from imposing profit maximization, incentive compatibility, and the participation constraint for $\theta = \alpha$.

Part (b): Under the conditions of part (b) of the theorem, $h(\alpha) > \frac{[v-s]\alpha}{v}$. This implies that $l(1,0) < q^{PI}(0, v, s)$. Recall that the support of $f(\theta)$ is the interval $[\alpha, \beta]$. Now, since $h(\beta) = 0$, it is clear that $l(1, \beta) > q^{PI}(\beta, v, s)$. Using the fact that $h_1(\theta) \leq 0$, it is easily shown that Θ is an interval $[\alpha, \hat{\theta}]$, where

$$\hat{\theta} = \theta : l(1, \theta) = q^{PI}(\theta, v, s). \tag{A.38}$$

Substituting the expressions for $l(1, \theta)$ and $q^{PI}(\theta, v, s)$ into (A.38) and rearranging yields:

$$\hat{\theta} = \theta : \theta[v - s] = vh(\theta). \tag{A.39}$$

Consequently, from (A.31), (A.32) and (A.39), it follows that

$$q^*(\theta, v, s) = q^{PI}(\theta, v, s) \text{ for } \theta \leq \hat{\theta} \tag{A.40}$$

$$q^*(\theta, v, s) = q^{ZP}(\theta, v) \text{ for } \theta \geq \hat{\theta}, \tag{A.41}$$

where θ is as defined in (A.39). The expressions for $\tau^*(\theta, v, s)$ for $\theta \leq \hat{\theta}$ follow immediately from Lemma 2. For $\theta \geq \hat{\theta}$, the $\tau^*(\theta, v, s)$ expressions follow from simultaneously imposing profit maximization and incentive compatibility, and accounting for the participation constraint of customer type $\hat{\theta}$. This completes the proof.

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The proofs of Theorem 2 uses Lemma 5, which follows directly from Theorem 1.

Lemma 5. $\int_{\hat{\theta}}^{\beta} [H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}] f(\theta) d\theta > 0$, *where* $\hat{\theta}$ *is as defined in (3.5) in Theorem 1(b)*

Proof. Assume the converse:

$$\int_{\hat{\theta}}^{\beta} [H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}] f(\theta) d\theta \leq 0 \tag{A.42}$$

Now, for $\theta \geq \hat{\theta}$, the optimal pricing schedule under Theorem 1(b) can be rearranged as

$$\tau^*(\theta) = \frac{[v - s]\hat{\theta}^2}{2} + v[H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}], \tag{A.43}$$

and the seller's profits from the customer types in $[\hat{\theta}, \beta]$are:

$$\frac{[v-s]\hat{\theta}^2}{2}[1-F(\hat{\theta})] + v\int_0^\beta [H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}]f(\theta) \tag{A.44}$$

(A.42) and (A.44) imply that the seller's profits from $[\hat{\theta}, \beta]$ are lower than $\frac{[v-s]\hat{\theta}^2}{2}[1-F(\hat{\theta})]$. However, if the seller were to offer customers in $[\hat{\theta}, \beta]$ the fixed-fee contract

$$T = \frac{[v-s]\hat{\theta}^2}{2}, \tag{A.45}$$

then all of these customer types would participate, yielding profits of $\frac{[v-s]\hat{\theta}^2}{2}[1-F(\hat{\theta})]$ from the segment $[\hat{\theta}, \beta]$. Moreover, the fixed-fee contract would not affect incentive compatibility for $\theta < \hat{\theta}$. This means that the seller can (weakly) improve the contract derived in Theorem 1(b), which contradicts the fact that this is the unique optimal contract. The result follows.

Note that as $\hat{\theta}$ tends to $\underline{\theta}$, Theorem 1(a) becomes applicable, and in the limit, the lemma also implies that

$$\int_0^\beta [H(\theta) - \frac{h(\theta)^2}{2}]f(\theta) > 0. \tag{A.46}$$

(A.46) can also be derived using an argument similar to the proof of this Lemma, but in the context of Theorem 1(a). ■

Proof of Theorem 2

Recall again that the support of $f(\theta)$ is the interval $[\alpha, \beta]$.

First, consider the case when $h(\alpha) \leq \frac{[v(\rho) - s(\rho)]\alpha}{v(\rho)}$. From (4.7),

$$\Pi(\rho) = \Pi^L(\rho) = \int_\alpha^\beta \left(\frac{\alpha^2[v(\rho) - s(\rho)]}{2} + v(\rho)[H(\theta) - \frac{h(\theta)^2}{2}] \right) f(\theta)d\theta. \tag{A.47}$$

Differentiating (A.47) with respect to ρ yields:

$$\Pi_1^L(\rho) = \frac{\alpha^2[v_1(\rho) - s_1(\rho)]}{2} + v_1(\rho)\int_\alpha^\beta [H(\theta) - \frac{h(\theta)^2}{2}]f(\theta)d\theta. \tag{A.48}$$

The optimal value of ρ^* must satisfy the first-order condition $\Pi_1^L(\rho^*) = 0$. Rearranging (A.48):

$$v_1(\rho^*) - s_1(\rho^*) = \frac{-2v_1(\rho^*)}{\alpha^2} \int_{\alpha}^{\beta} [H(\theta) - \frac{h(\theta)^2}{2}] f(\theta) d\theta. \tag{A.49}$$

Using the fact that $v_1(\rho) < 0$ for all ρ, and equation (A.46) from Lemma 4, (A.49) implies that:

$$v_1(\rho^*) - s_1(\rho^*) > 0. \tag{A.50}$$

Since $v_{11}(\rho) - s_{11}(\rho) < 0$, (A.50) implies that $\rho^* < \rho^e$, which completes the proof for lower levels of piracy.

Next, consider the case when $h(\alpha) \geq \dfrac{[v(\rho) - s(\rho)]\alpha}{v(\rho)}$. From (4.9),

$$\begin{aligned} \Pi^H(\rho) &= \left(\frac{s(\rho)[v(\rho) - s(\rho)]}{v(\rho)}\right) \int_{\alpha}^{\hat{\theta}(\rho)} \frac{\theta^2}{2} f(\theta) d\theta + v(\rho) \int_{\hat{\theta}(\rho)}^{\beta} \left(H(\theta) - \frac{[h(\theta)]^2}{2}\right) f(\theta) d\theta \\ &\quad + [1 - F(\hat{\theta}(\rho))] \left(\frac{[v(\rho) - s(\rho)][\hat{\theta}(\rho)]^2}{2} - v(\rho) H(\hat{\theta}(\rho))\right). \end{aligned} \tag{A.51}$$

Differentiating both sides of (A.51) with respect to ρ, cancelling out common terms, and rearranging substantially yields the following expression:

$$\Pi_1^H(\rho) = f^A(\rho) + f^B(\rho) + f^C(\rho) + f^D(\rho) + f^E(\rho), \tag{A.52}$$

where:

$$f^A(\rho) = \frac{s_1(\rho)[v(\rho) - s(\rho)]^2}{v(\rho)^2} \int_{\alpha}^{\hat{\theta}(\rho)} \frac{\theta^2}{2} f(\theta) d\theta; \tag{A.53}$$

$$f^B(\rho) = [v_1(\rho) - s_1(\rho)] \left[[1 - F(\hat{\theta}(\rho))] \frac{[\hat{\theta}(\rho)]^2}{2} + \frac{s(\rho)^2}{v(\rho)^2} \int_{\alpha}^{\hat{\theta}(\rho)} \frac{\theta^2}{2} f(\theta) d\theta \right]; \tag{A.54}$$

$$f^C(\rho) = v_1(\rho) \int_{\hat{\theta}(\rho)}^{\beta} [H(\theta) - H(\hat{\theta}) - \frac{h(\theta)^2}{2}] f(\theta) d\theta; \tag{A.55}$$

$$f^D(\rho) = \hat{\theta}_1(\rho)[1 - F(\hat{\theta}(\rho)][\hat{\theta}(\rho)[v(\rho) - s(\rho)] - v(\rho) h(\hat{\theta}(\rho)]; \tag{A.56}$$

$$f^E(\rho) = \hat{\theta}_1(\rho) f(\hat{\theta}(\rho)) \left[\frac{v(\rho)^2 h(\hat{\theta}(\rho))^2 - [v(\rho) - s(\rho)]^2 [\hat{\theta}(\rho)]^2}{2v(\rho)} \right]. \tag{A.57}$$

From the definition of $\hat{\theta}(\rho)$, we know that

$$[v(\rho) - s(\rho)]\hat{\theta}(\rho) = v(\rho)h(\hat{\theta}(\rho)). \tag{A.58}$$

Substituting (A.58) into (A.56) and (A.57) yields $f^D(\rho) = f^E(\rho) = 0$ for all ρ. Also, since $s_1(\rho) < 0$, it follows that $f^A(\rho) < 0$ for all ρ. Moreover, Lemma 5 and the fact that $v_1(\rho) < 0$ imply that $f^C(\rho) < 0$ for all ρ. Now, the optimal value of ρ^* must satisfy the first-order condition $\Pi_1^H(\rho^*) = 0$. Rearranging (A.52), this condition reduces to:

$$f^B(\rho^*) = -[f^A(\rho^*) + f^C(\rho^*)] \tag{A.59}$$

Since we have established that $f^A(\rho) < 0, f^C(\rho) < 0$ for all ρ, (A.59) implies that $f^B(\rho^*) > 0$. For any $\hat{\theta}(\rho) < \beta$, the term in square parentheses on the RHS of (A.54) is strictly positive. In conjunction with (A.59), this in turn implies that

$$v_1(\rho^*) - s_1(\rho^*) > 0. \tag{A.60}$$

Since $v_{11}(\rho) - s_{11}(\rho) < 0$, (A.60) implies that $\rho^* < \rho^e$, which completes the proof.