Network competition with price discrimination: 'Bill-and-Keep' is not so bad after all

Carlo Cambini (Politecnico di Torino)*

and

Tommaso M. Valletti (Imperial College London and CEPR)**

January 2003

Abstract

This paper studies the impact that access charges have on the incentives to invest in competing telecommunications networks that can deliver faster speed, lower congestion, or wider coverage. It demonstrates that high (above marginal cost) access charges soften competition in the investment stage and can be used to sustain higher profits under competition with two-part tariffs and termination-based price discrimination. On the contrary, below-cost charges (such as 'bill and keep' arrangements) have a positive impact on operators' willingness to invest.

Keywords: Telecommunication, Interconnection, Competition policy, Investments.

JEL Classification: L41, L96.

* DSPEA – Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Turin, Italy. Tel +39 0115647292, Fax +39 011 5647299, e-mail: carlo.cambini@polito.it.

** Corresponding author. Imperial College London, 53 Prince's Gate, Exhibition Road, London SW7 2PG, UK, Tel. +44-20-7594 9215, Fax +44-20-7823 7685, e-mail t.valletti@ic.ac.uk, http://www.ms.ic.ac.uk/tommaso.

1. Introduction

Many liberalized telecommunications markets can be described as oligopolies with only a few players competing against each other. When operators use their own network facilities, there is an interesting problem of "two way" access since all operators need to seek access to rivals' networks to terminate calls that are originated by their own customers but destined to rival customers. In this environment, regulation is usually kept at a minimum level. Regulators do not impose any requirement over retail prices, but may monitor the way access charges are set. The concern that arises is that competing firms, having a mutual need to terminate calls on each other's network, may use access charges as an instrument of collusion, i.e., agree on particular charges to induce high retail prices.

This question has been addressed in the seminal papers of Armstrong (1998) and Laffont *et al*. (1998) (ALRT hereafter). Their answer is that indeed two-way access charges can be used to generate a 'raise-each-other's-cost' effect. This result, however, depends on firms competing in linear prices and does not extend to more realistic pricing policies. If firms compete in two-part uniform prices, access charges have a neutral effect on profits since any extra revenue from termination would feed into an equal change in the fixed fee offered to customers.

The result of profit neutrality does not hold if firms offer two-part tariffs with network-based price discrimination (i.e., call charges depend on the destination network). In this context, firms' profits are increased by agreeing on below-cost reciprocal access charges. This surprising result has been shown by Gans and King (2001) and can be understood by noting that below-cost access charges softens price competition: firms make losses every time they terminate a call, hence they are more reluctant to fight for market share. This result in some respect goes against conventional wisdom of regulators, since they are typically concerned that operators negotiate access charges that are too high and not too low.

Armstrong, in the conclusions of his extensive survey on access pricing, writes that "an important next step for the theoretical research in this area is the analysis of the dynamics of access pricing" (Armstrong, 2002, p. 381). In a companion paper (Valletti and Cambini, 2002), we introduce a quality parameter in the ALRT framework, where quality is related to a costly fixed investment of operators and it positively affects customers' usage (one can think of speed, capacity, coverage), and we ask how access pricing affects the firms' incentives to invest. We show that the regulators' conventional wisdom is restored when firms compete in uniform two-part prices: firms would prefer to negotiate above-cost access charges because, by doing so, they avoid competing excessively against each other over their investments.

The purpose of this paper is to see if such a collusive underinvestment result is robust when firms compete in two-part discriminatory prices, in the light of the result of Gans and King (2001). We show how, once quality is taken exogenously, the result of Gans and King is quite robust and can be generalized to an asymmetric context. However, once investments are endogenized, the underinvestment result of our companion paper still holds true: above-cost access charges would induce firms to invest less, and this effect may well be the prevailing one such that firms would agree on above-cost reciprocal charges to increase their profits.

2. The model

There are two network operators differentiated à la Hotelling. As in ALRT, a unit mass of consumers is uniformly located on the segment [0,1], while the network operators are located at the two extremities. We denote by 1 (respectively 2) the firm located at the origin (respectively at the end) of the line. Network operators compete in two part tariffs discriminating between on-net and off-net calls, i.e., operator i charges $T_i(Q_i,\tilde{Q}_i) = F_i + p_iQ_i + \tilde{p}_i\tilde{Q}_i$, where the fixed fee F_i can be interpreted as a subscriber line charge, p_i and $\tilde{p}_i$ as the marginal price for on-net (Q_i) and off-net calls ($\tilde{Q}_i$), respectively. When a consumer located at x buys from firm i located at x_i, he enjoys a utility given by:

$$y + v_0 - |x - x_i|/(2\sigma) + V_i(p_i) + \tilde{V}_i(\tilde{p}_i) - F_i$$

where y is the income, v_0 is a fixed surplus component from subscribing and $V_i(p_i)$ and $\tilde{V}_i(\tilde{p}_i)$ are the indirect utilities derived from making on-net and off-net calls, once connected to network i. The parameter σ represents an index of substitutability between the networks (it is the inverse of transport cost), and its function is to regulate the intensity of price competition.

We introduce in the above framework a new element related to the quality of a network. Let $k_i \geq \underline{k} > 0$ denote operator i's quality, where $\underline{k}$ is some minimum quality level that operators have to guarantee. We assume that both quantity and indirect utility are increasing in k_i; in particular, we assume that that they can be expressed in a multiplicative form, i.e., $Q_i(p_i) = k_iq_i(p_i)$, $\tilde{Q}_i(\tilde{p}_i) = k_i\tilde{q}_i(\tilde{p}_i)$ and $V_i(p_i) = k_iv(p_i)$, $\tilde{V}_i(\tilde{p}_i) = k_i\tilde{v}(\tilde{p}_i)$.

Quality is increasing in an operator's investment. This notion captures the idea that investment is something that consumers enjoy, ceteris paribus, since they will have easier access, faster delivery, less congestion problems, and so on. In this paper we assume that the quality supplied for on-net and off-net services is the same, meaning that the rival's quality does not affect a consumer behavior, once the subscription decision has already occurred.[1]

The consumer indifferent between the networks determines the market share α_i of firm i:

$$\alpha_i = \alpha(p_i, \tilde{p}_i, F_i) = \frac{1}{2} + \sigma(w_i - w_j)$$

and $w_i = \alpha_i v_i(p_i) + \alpha_j \tilde{v}_i(\tilde{p}_i) - F_i \ = \alpha_i k_i v(p_i) + (1-\alpha_i) k_i \tilde{v}(\tilde{p}_i) - F_i$, $i, j = 1,\ 2,$ is the net surplus for customers subscribing to network i. The market share of network 1 thus becomes:

$$\alpha_1 = \alpha = \frac{1/2 + \sigma\left(k_1 \tilde{v}(\tilde{p}_1) - k_2 v(p_2)\right) - \sigma\left(F_1 - F_2\right)}{1 + \sigma\left[k_1\left(\tilde{v}(\tilde{p}_1) - v(p_1)\right) + k_2\left(\tilde{v}(\tilde{p}_2) - v(p_2)\right)\right]}. \tag{1}$$

Both networks have full coverage. Serving a customer involves a fixed cost f of connection and billing. Each call has to be originated and terminated. The marginal cost is c per call at the originating end and t at the terminating end. The total marginal cost for a call is thus $c + t$. Networks pay each other a reciprocal two-way access charge, denoted by a, for terminating each other's off-net calls. Finally, network i incurs a convex cost $I(k_i)$ to provide a service of quality k_i, with $I'(\cdot) > 0$ and $I''(\cdot) > 0$.

We are mostly interested in cost-based regulation of access, which is the current regulatory benchmark in many countries. Our status quo is cost-based regulation ($a = t$) and we analyze how a small departure from such benchmark affects firms' profits and consumer surplus. The understanding is that, should our results show that both firms' profits are positively (respectively negatively) affected by setting reciprocal charges above cost, firms would then negotiate them above (respectively below) cost if they are left unregulated.

The timing of the game is as follows. First the operators invest in quality and interconnection terms are set. Then operators compete in two-part discriminatory prices.

3. Price competition

Network 1 has to solve (expressions for network 2 are symmetric):

[1] In Valletti and Cambini (2002) we consider the alternative case where the quality for off-net services depends on the minimum quality provided by the two networks, i.e., quality has bottleneck features and influences consumer behavior after their subscription decision. This alternative scenario is more appropriate to describe situations where quality is related to the simultaneity of a conversation. On the other hand, the case analyzed here is suitable when quality describes features such as network coverage or bandwidth.

$$\max_{F_1, p_1, \tilde{p}_1} \Pi_1 = \max_{F_1, p_1, \tilde{p}_1} \pi_1 - I(k_1)$$

$$\pi_1 = \alpha\left\{\alpha\left(p_1 - c - t\right)k_1 q(p_1) + (1-\alpha)(\tilde{p}_1 - c - a)k_1 \tilde{q}(\tilde{p}_1) + F_1 - f\right\} + \alpha(1-\alpha)(a-t)k_2 \tilde{q}(\tilde{p}_2)$$

Since competition is in two-part prices, firms set the usage charges equal to the perceived marginal cost of a call. The intuition for this result is simple: these are in fact the call prices that maximize the joint surplus for a firm and its customers, fixed fees are then used to distribute such surplus according to the intensity of competition. Equilibrium usage charges are thus $p_1 = p_2 = p^* = c + t$, $\tilde{p}_1 = \tilde{p}_2 = \tilde{p}^* = c + a$. Using this result, one can also derive the equilibrium fixed component of the two-part tariff:

$$F_1^* = \frac{\alpha\Theta}{\sigma} + f - (1-2\alpha)(a-t)k_2 \tilde{q}\left(\tilde{p}^*\right)$$

where $\Theta = 1 + \sigma\left[k_1\left(\tilde{v}(\tilde{p}_1) - v(p_1)\right) + k_2\left(\tilde{v}(\tilde{p}_2) - v(p_2)\right)\right] = 1 + \sigma\left(k_1 + k_2\right)\left(\tilde{v}(\tilde{p}^*) - v(p^*)\right)$.

After substitution into eq. (1) and rearranging, we have the following expression for the market share of firm 1 at equilibrium:[2]

$$\alpha_1^* = \frac{1}{2} + \frac{\sigma}{2} \frac{(k_1 - k_2)(\tilde{v} + v)}{\left\{3\left[1 + \sigma(\tilde{v} - v)(k_1 + k_2)\right] + 2\sigma\tilde{q}(a-t)(k_1 + k_2)\right\}} \tag{2}$$

Finally, in equilibrium, the profit (gross of investment) of network operator 1 is given by:

$$\pi_1^* = \frac{(\alpha_1^*)^2}{\sigma}\Theta + (\alpha_1^*)^2(a-t)k_2\tilde{q}(\tilde{p}^*)\,. \tag{3}$$

3.1 Comparative statics in stage II

As we have argued before, a typical regulatory benchmark is cost-based regulation. Then, it is interesting to analyze the impact of access charges and quality (stage-I parameters) on the equilibrium previously defined when access charges are slightly increased above (or decreased below) termination costs.

[2] We assume the existence of a shared equilibrium. This is ensured as long as σ is low enough and a is close to the marginal cost t. The proof of existence would follow the same lines of Laffont *et al.* (1998), as extended to asymmetric settings by de Bijl and Peitz (2002).

From now onwards, as in ALRT, we will assume that the functional form for unit indirect utility is the same for on-net and off-net calls, i.e., $v(p)=\tilde{v}(p)$. We have the following:

Proposition 1. If $k_1 \neq k_2$, a small variation of the access charge above the marginal cost of termination has a local effect on market shares:

$$\left.\frac{\partial \alpha_i^*}{\partial a}\right|_{a=t} = -\frac{\sigma q(k_i-k_j)}{6}\left[1-\frac{2}{3}\sigma v(k_i+k_j)\right] \neq 0,\ i=1,2; \tag{4}$$

In addition, when access is regulated at cost, market shares are affected by quality:

$$\left.\frac{\partial \alpha_i^*}{\partial k_i}\right|_{a=t} = \sigma v/3 = -\left.\frac{\partial \alpha_i^*}{\partial k_j}\right|_{a=t},\ i,j=1,2. \tag{5}$$

Proof. Denote with $\Omega = 3\left[1+\sigma(\tilde{v}-v)(k_1+k_2)\right]+2\sigma\tilde{q}(a-t)(k_1+k_2)$. Then, from (2) we have:

$$\frac{\partial \alpha_i^*}{\partial a} = \frac{-\sigma\tilde{q}(k_1-k_2)(2\Omega)-2\sigma(k_1-k_2)(\tilde{v}+v)\left(\partial\Omega/\partial a\right)}{4\Omega^2}$$

with $\partial\Omega/\partial a = -\sigma\tilde{q}(k_1+k_2)+2\sigma(a-t)(k_1+k_2)\tilde{q}'$.
Given that $\tilde{v}\big|_{a=t} = v\big|_{a=t}$, we can simplify $\Omega\big|_{a=t}=3$, $\partial\Omega/\partial a\big|_{a=t} = -\sigma\tilde{q}(k_1+k_2)$ and rearranging, we obtain (4). Similarly:

$$\frac{\partial \alpha_i^*}{\partial k_1} = \frac{2\Omega\sigma(\tilde{v}+v)-2\sigma(k_1-k_2)(\tilde{v}+v)\left(\partial\Omega/\partial k_1\right)}{4\Omega^2}. \tag{6}$$

Since $\partial\Omega/\partial k_1 = 3\sigma(\tilde{v}-v)+2\tilde{q}\sigma(a-t)$ and so $\partial\Omega/\partial k_1\big|_{a=t}=0$, we obtain (5). *Q.E.D.*

Notice how the sign of eq. (4) is not obvious a priori, while the impact of quality is unambiguous: a higher quality allows a firm to obtain a bigger share of the market (eq. (5)) when access is regulated at cost. We are now in a position to state the following result:

Proposition 2. In a neighborhood of $a = t$, a small increase in the access charge always decreases the gross profit of the high-quality network. Sufficient conditions for reducing the gross profit of the low-quality network are: a) the quality differential is small or b) the minimum quality level is sufficiently high.

Proof. From eq. (3) we have:

$$\frac{\partial \pi_1^*}{\partial a} = 2\frac{\alpha_1^*\Theta}{\sigma}\frac{\partial\alpha_1^*}{\partial a}+\frac{(\alpha_1^*)^2}{\sigma}\frac{\partial\Theta}{\partial a}+(\alpha_1^*)^2\tilde{q}k_2+(a-t)k_2\frac{\partial\left[(\alpha_1^*)^2\tilde{q}\right]}{\partial a}$$

Since $\Theta|_{a=t} = 1$, $\partial\Theta/\partial a|_{a=t} = -\sigma q(k_1 + k_2)$ and rearranging, it results:

$$\left.\frac{\partial \pi_1^*}{\partial a}\right|_{a=t} = -\alpha_1^*\big|_{a=t}\, q\left[\frac{(k_1 - k_2)}{3} + \frac{\sigma v}{9}(k_1 - k_2)^2 + \frac{k_1}{2} - \frac{\sigma v}{9}(k_1 - k_2)k_2\right] \tag{7}$$

Firstly, note that in a symmetric equilibrium, i.e., $k_1 = k_2$, the expression is always negative: $\left.\frac{\partial \pi_1}{\partial a}\right|_{\substack{a=t\\ k_1=k_2=k}} = -\frac{qk}{4} < 0$. By continuity, the expression is negative for small quality differences.

Secondly, since $\alpha_1^*\big|_{a=t} = \frac{1}{2} + \frac{\sigma v}{3}(k_1 - k_2)$, for a shared equilibrium to exist it has to be $0 < \Delta < 3/(2\sigma v)$, where $\Delta = |k_1 - k_2|$.

Thirdly, imagine firm 1 has a better quality, i.e., $k_1 > k_2$. The bracket in eq. (7) can be rewritten as $\sigma v \Delta^2 / 9 + \Gamma$, where $\Gamma = \Delta/3 + k_1/2 - \sigma v k_2 \Delta / 9$. Since Γ is linear in Δ, it suffices to show that Γ is always positive when Δ is at the extremities in the plausible range. This is trivial when $\Delta = 0$. When $\Delta = 3/(2\sigma v)$ then $\Gamma = 1/(2\sigma v) + k_1/2 - k_2/6 > 0$.

Lastly, imagine firm 2 has a better quality, i.e., $k_1 < k_2$. The bracket in eq. (7) can be rewritten as $\Lambda = 2\sigma v \Delta^2/9 + 2\Delta(\sigma v k_1 - 3)/18 + k_1/2$. Since $\partial\Lambda/\partial\Delta = 4\sigma v\Delta/9 + (\sigma v k_1 - 3)/9$, there are two cases. If $k_1 > 3/(\sigma v)$ then Λ is positive. If $k_1 < 3/(\sigma v)$ then Λ has a minimum in Δ = $3/(\sigma v)$ - $k_1/4$, and, at this minimum, $\Lambda = \left(-k_1^2 + 42k_1 - 1/(\sigma v)\right)/72$. The last expression is positive when $0.215/(\sigma v) \approx 3(7 - 4\sqrt{3})/(\sigma v) < k_1 < 3/(\sigma v)$. Hence if $\underline{k} > 0.215/(\sigma v)$ then Λ is positive everywhere when an interior solution exists. *Q.E.D.*

Proposition 2 shows that, in a symmetric equilibrium, the result of Gans and King (2001) is confirmed. Network operators prefer to negotiate an access charge below marginal cost. In fact, when $a < t$, the price for off-net calls is lower than the price for on-net calls, hence subscribers prefer to belong to the smaller network. Networks do not compete hard for market shares and this increases their profits. Indeed, we have shown how this result is quite robust also once asymmetries are introduced, i.e., for a <u>given</u> level of investments.

However, the type of asymmetries considered here arise from the endogenous decision of firms to invest in quality. Since access charges do not change very frequently (they have to be filed with regulators and approved), it is more realistic to think that access charges have an impact on endogenous investment levels. This is what we turn to study in the next section.

4. Investment decision

In stage I, given eq. (3), the FOC w.r.t. quality of firm 1 takes this expression:

$$\frac{\partial \Pi_1}{\partial k_1} = 2\frac{\alpha_1^*}{\sigma}\frac{\partial \alpha_1^*}{\partial k_1}\Theta + \frac{(\alpha_1^*)^2}{\sigma}\frac{\partial \Theta}{\partial k_1} + 2(a-t)k_2\tilde{q}\alpha_1^*\frac{\partial \alpha_1^*}{\partial k_1} - I'(k_1) = 0\,. \tag{8}$$

Since $\partial\Theta/\partial k_1 = \sigma(\tilde{v} - v)$ and so $\partial\Theta/\partial k_1\big|_{a=t} = 0$, and using eq. (5), in a neighborhood of a = t, eq. (8) implies that the optimal investment level is given by the following condition:

$$\frac{2v}{3}\alpha_i^*\Big|_{a=t} = I'(k_i),\ i, j = 1,2\,. \tag{9}$$

Our interest here is to analyze the impact of interconnection charges (around the cost-based benchmark) on networks' investing decisions. In particular, we want to see if a firm would increase or decrease quality for a given rival's quality when the access charge is slightly increased above cost. We are able to obtain this result:

Proposition 3. Imagine the access charge is slightly increased above cost. Then the optimal investment level of firm *i* changes in the following way:

$$\frac{\mathrm{d}k_i}{\mathrm{d}a}\bigg|_{a=t} = -\frac{-\frac{\sigma v q}{9}(6k_i - 5k_j) - \frac{5}{12}q - \frac{\sigma^2 v^2 q}{9}(k_i - k_j)(k_i - \frac{5}{3}k_j)}{\frac{2}{9}\sigma v^2 - I''(k_i)},\ i, j = 1,2\,. \tag{10}$$

In particular, in a symmetric equilibrium we have:

$$\frac{\mathrm{d}k_i}{\mathrm{d}a}\bigg|_{\substack{a=t\\ k_1=k_2=k}} = q\frac{\frac{1}{9}\sigma v k + \frac{5}{12}}{\frac{2}{9}\sigma v^2 - I''(k_i)} < 0\,. \tag{11}$$

Proof. See the Appendix.

The result implies that firms tend to underinvest in quality if they can negotiate reciprocal termination charges above cost. Intuitively, when termination charges are above cost, an increase of own quality relative to the rival creates an access deficit that makes an operator reluctant to invest, confirming the result of Valletti and Cambini (2002) in a price discrimination setting. To understand the firms' incentives to negotiate access charges above or below cost, we have to analyze the overall effect of access charges on networks' profits, in a neighborhood of $a = t$. In equilibrium, the profit of a network, say network 1, is given by:

$$\Pi_1 = \Pi_1\left(k_1^*(a), k_2^*(a), a\right)$$

where $k_i^*(a)$ is the optimal level of investment, given an access charge a, defined by eq. (9).

The total effect of a is thus:

$$\frac{\mathrm{d}\Pi_1}{\mathrm{d}a}=\frac{\partial\Pi_1}{\partial k_1}\frac{\mathrm{d}k_1}{\mathrm{d}a}+\frac{\partial\Pi_1}{\partial k_2}\frac{\mathrm{d}k_2}{\mathrm{d}a}+\frac{\partial\Pi_1}{\partial a} \tag{12}$$

The first term in (12) is zero for the envelope theorem. The sign of the above expression is not obvious a priori in a symmetric equilibrium. On the one hand, the direct effect of access charge is negative ($\left.\frac{\partial\Pi_1}{\partial a}\right|_{\substack{a=t\\k_1=k_2}}=-\frac{qk}{4}<0$) due to the increase in access deficit (Proposition 3). On the other hand, the strategic effect deriving from investment is positive as investment decisions are strategic substitutes:

$$\frac{\partial\Pi_1}{\partial k_2}=2\frac{\alpha_1^*}{\sigma}\frac{\partial\alpha_1^*}{\partial k_2}\Theta+\frac{(\alpha_1^*)^2}{\sigma}\frac{\partial\Theta}{\partial k_2}+(a-t)\tilde{q}\frac{\partial\left[k_2\alpha_1^*\right]}{\partial k_2}\Rightarrow\left.\frac{\partial\Pi_1}{\partial k_2}\right|_{\substack{a=t\\k_1=k_2}}=-\frac{v}{3}$$

and $\left.\frac{\mathrm{d}k_2}{\mathrm{d}a}\right|_{a=t}<0$. The overall sign of (12) depends on the functional form of the investment function $I(\cdot)$. Assume that the investment function takes a power form: $I(k)=k^\beta/\beta$, $\beta>1$. From eq. (9) it results in a symmetric equilibrium k_1 = k_2 = k^* where $(k^*)^{\beta-1}=v/3$. Moreover, from Proposition 3, we obtain $\left.\frac{\mathrm{d}k_i}{\mathrm{d}a}\right|_{\substack{a=t\\k_i=k_j}}=q\frac{\frac{1}{9}\sigma vk^*+\frac{5}{12}}{\frac{2}{9}\sigma v^2-(\beta-1)\frac{v}{3k^*}}<0$.

Then, in this example, it is easy to verify that eq. (12) simplifies to $-qv\frac{(24+10k^*\sigma v-9\beta)}{108\left[\frac{2}{9}\sigma v^2-(\beta-1)\frac{v}{3k^*}\right]}$. The denominator is negative because of the SOC w.r.t. k. Hence a sufficient condition for a small variation of a above cost to have an overall positive impact is $\beta<8/3$. In this case above cost termination leads the rival network to reduce its investment; in turn, this enhances the profit of network 1 more than the reduction in profits due to the increase in the access deficit. The condition on the convexity of the investment cost function is intuitive. For strategic substitutability to prevail over the stage-II effect, it must be the case that investments are responsive to a local change in a, which happens if the investment function is not "too" convex (see eq. (11), the denominator increases with I'').

Our findings show that it is possible to obtain a result of "tacit collusion" where firms negotiate above cost access charges, shown by ALRT in linear pricing context, even in a symmetric model with two-part discriminatory pricing. This is detrimental for social welfare.[3]

5. Conclusions

Our results suggest that private negotiations over reciprocal access charges would not be efficient, and that firms would often prefer to set access charges above termination costs once their investment decisions are endogenous. On the contrary, in order to induce firms to invest in an efficient manner, access charges should be set below costs. Then, the current policies carried on by many regulators – primarily based on the Long Run Incremental Cost - may not be right in a dynamic perspective. However, it is not obvious how to set the "right" level of access charges below cost. All in all, it then looks interesting a regime based on "bill and keep" arrangements, i.e., a zero interconnection rate, not because the optimal interconnection need be zero, but because it would be simple to put in practice and it would give higher incentives to invest. A regime based on "bill and keep" has recently been advocated as a way of sharing efficiently the value created by a call when both callers and receivers benefit from it (DeGraba, 2003). Our results, based on incentives to invest, reinforce their good properties.

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[3] Inefficiencies arise for two reasons: access charges different than costs induce a misallocation of off-net calls since the usage charge reflects a perceived cost different than the true cost. In addition, the socially efficient investment level is $k_i^{**} = I'^{-1}(v/2)$. When firms negotiate $a > t$ they invest less than the level shown in eq. (9), hence $k_i^*\big|_{\substack{a>t \\ k_i=k_j}} < k_i^*\big|_{\substack{a=t \\ k_i=k_j}} = I'^{-1}(v/3) < k_i^{**}$.

Appendix

Proof of Proposition 3. We start by implicitly differentiating the FOC given by eq. (8):

$$\begin{aligned}
&\mathrm{d}k_1\left[\frac{2\Theta}{\sigma}\left(\frac{\partial\alpha_1^*}{\partial k_1}\right)^2+\frac{2\Theta\alpha_1^*}{\sigma}\frac{\partial^2\alpha_1^*}{\partial k_1^2}+\frac{4\alpha_1^*}{\sigma}\frac{\partial\alpha_1^*}{\partial k_1}\frac{\partial\Theta}{\partial k_1}+\frac{\left(\alpha_1^*\right)^2}{\sigma}\frac{\partial^2\Theta}{\partial k_1^2}+\right.\\
&\qquad\left.+2(a-t)k_2\tilde{q}\left(\partial\left\{\left(\partial\alpha_1^*/\partial k_1\right)\alpha_1^*\right\}/\partial k_1\right)-I''(k_1)\right]+\\
&+\mathrm{d}a\left[\frac{2\Theta}{\sigma}\frac{\partial\alpha_1^*}{\partial k_1}\frac{\partial\alpha_1^*}{\partial a}+\frac{2\Theta\alpha_1^*}{\sigma}\frac{\partial^2\alpha_1^*}{\partial k_1\partial a}+\frac{2\alpha_1^*}{\sigma}\frac{\partial\alpha_1^*}{\partial k_1}\frac{\partial\Theta}{\partial a}+\frac{2\alpha_1^*}{\sigma}\frac{\partial\alpha_1^*}{\partial a}\frac{\partial\Theta}{\partial k_1}+\frac{\left(\alpha_1^*\right)^2}{\sigma}\frac{\partial^2\Theta}{\partial k_1\partial a}+\right.\\
&\qquad\left.+2k_2\tilde{q}\frac{\partial\alpha_1^*}{\partial k_1}\alpha_1^*+2k_2\tilde{q}(a-t)\left(\partial\left\{\left(\partial\alpha_1^*/\partial k_1\right)\alpha_1^*\right\}/\partial a\right)\right]=0
\end{aligned}\tag{a1}$$

We want to calculate all the previous expressions at $a = t$, and we compute some terms to simplify calculations. In particular, from eq. (6) it results:

$$\begin{aligned}
\frac{\partial^2\alpha_1^*}{\partial k_1^2}=&\frac{4\Omega^2\left[2\sigma(\tilde{v}+v)\dfrac{\partial\Omega}{\partial k_1}-2\sigma(\tilde{v}+v)\dfrac{\partial\Omega}{\partial k_1}-2\sigma(\tilde{v}+v)(k_1-k_2)\dfrac{\partial^2\Omega}{\partial k_1^2}\right]}{16\Omega^4}+\\
&-\frac{8\Omega\dfrac{\partial\Omega}{\partial k_1}\left[2\Omega\sigma(\tilde{v}+v)-2\sigma(k_1-k_2)(\tilde{v}+v)\left(\partial\Omega/\partial k_1\right)\right]}{16\Omega^4}
\end{aligned}\tag{a2}$$

From Proposition 1, we have that $\left.\partial\Omega/\partial k_1\right|_{a=t}=0$; in addition it is simple to verify that $\left.\partial^2\Omega/\partial k_1^2\right|_{a=t}=0$. Then, in $a = t$ eq. (a2) becomes $\left.\dfrac{\partial^2\alpha_1^*}{\partial k_1^2}\right|_{a=t}=0$.

In addition $\left.\partial\Theta/\partial k_1\right|_{a=t}=0$ and $\left.\partial^2\Theta/\partial k_1^2\right|_{a=t}=0$. We still have to compute two terms in (a1) (the second cross partial derivatives). First it results $\left.\dfrac{\partial^2\Theta}{\partial k_1\partial a}\right|_{a=t}=-\sigma q$. From eq. (6), we have:

$$\begin{aligned}
\frac{\partial^2\alpha_1^*}{\partial k_1\partial a}=&\frac{4\Omega^2\left[-2\sigma\tilde{q}\Omega+2\sigma(\tilde{v}+v)\dfrac{\partial\Omega}{\partial a}+2\sigma\tilde{q}(k_1-k_2)\dfrac{\partial\Omega}{\partial k_1}-2\sigma(\tilde{v}+v)(k_1-k_2)\dfrac{\partial^2\Omega}{\partial k_1\partial a}\right]}{16\Omega^4}+\\
&-\frac{8\Omega\dfrac{\partial\Omega}{\partial a}\left[2\Omega\sigma(\tilde{v}+v)-2\sigma(k_1-k_2)(\tilde{v}+v)\left(\partial\Omega/\partial k_1\right)\right]}{16\Omega^4}
\end{aligned}\tag{a3}$$

Given that $\left.\dfrac{\partial^2\Omega}{\partial k_1\partial a}\right|_{a=t}=-\sigma q,$ evaluating (a3) in $a = t$, after some simplifications it results:

$$\left.\frac{\partial^2 \alpha_1^*}{\partial k_1 \partial a}\right|_{a=t} = \frac{\sigma q}{18}\left(4\sigma v k_1 - 3\right)$$

We assume that, in $a = t$, the SOC holds (Π_1 is concave in k_1):

$$\left.\frac{\partial^2 \Pi_1}{\partial k_1^2}\right|_{a=t} = \frac{2}{9}\sigma v^2 - I''(k_1) < 0 .$$

Rearranging (a1) and evaluating in $a = t$, we obtain:

$$\mathrm{d}k_1\left[\frac{2}{9}\sigma v^2 - I''(k_1)\right] +$$

$$+\mathrm{d}a\left[\frac{2v}{3}\left.\frac{\partial \alpha_1^*}{\partial a}\right|_{a=t} + \frac{q\left.\alpha_1^*\right|_{a=t}}{9}\left(4\sigma v k_1 - 3\right) - \frac{2\sigma v q (k_1 + k_2)\left.\alpha_1^*\right|_{a=t}}{3} - q\left(\left.\alpha_1^*\right|_{a=t}\right)^2 + \frac{2}{3}k_2 \sigma v q \left.\alpha_1^*\right|_{a=t}\right] = 0$$

Finally, using eq. (4) and rearranging, we obtain eqs. (10) and (11) in Proposition 3.