

Would You Like to Be a Prosumer?

Information Revelation, Personalization and Price Discrimination in Electronic Markets

Martin Bandulet¹, Universität Augsburg
Karl Morasch², Universität der Bundeswehr München

This version: March 2003

Abstract

Electronic commerce and flexible manufacturing allow personalization of initially standardized products at low cost. Will customers provide the information necessary for personalization? Assuming that a consumer can control the amount of information revealed, we analyse how his decision interacts with the pricing strategy of a monopolist who may abuse the information to obtain a larger share of total surplus. We consider two scenarios, one where consumers have different tastes but identical willingness to pay and another with high and low valuation customers. In both cases full revelation may only result if the monopolist can commit to a maximum price before consumers decide about disclosure.

JEL-code: D82, D42, L14

Keywords: E-Commerce, Personalization, Asymmetric information, Price discrimination

¹Address: Dipl.-Volksw. Martin Bandulet, Wirtschaftswissenschaftliche Fakultät, Universität Augsburg, D-86135 Augsburg, Germany, Tel. ++49-821-5984197, Fax ++49-821-5984230, e-mail: martin.bandulet@wiwi.uni-augsburg.de

²Corresponding Author — Address: PD Dr. Karl Morasch, Fakultät für Wirtschafts- und Organisationswissenschaften, Universität der Bundeswehr München, D-85577 Neubiberg, Germany, Tel. ++49-89-60044201, Fax ++49-89-6014693, e-mail: karl.morasch@unibw-muenchen.de

1 Introduction

The concept of a “prosumer” has been introduced in 1980 by the futurist Alvin Toffler in his book “The Third Wave” as a blend of producer and consumer. He imagined a future type of consumer becoming involved in the design and manufacture of products in a way that they could be made to individual specification. Basing product design decisions appropriately on implicit or explicit information about customer preferences has always been a key factor for economic success in any business. Also personalization has been the traditional way of production in many areas of craftsmanship as for for example taylors, shoemakers and the like. However, a manufacturer or a national service provider has until recently been restricted to sell a standardized product or at most a limited number of differentiated goods or services. Electronic commerce, partially in conjunction with flexible manufacturing, now provides the opportunity to obtain the information necessary for personalization from customers all over the world at low cost and, specifically in the case of digital products, to tailor general-purpose goods or services to the specific needs of each customer (“mass customization”).

While personalization enhances the value of a product for the consumer it is not necessarily in his interest to reveal his personal information. Besides privacy issues (see *Varian*, 1997) it are the conflicting interests of buyers and sellers that may make the consumer reluctant to become a prosumer: He must fear that the seller may be able to abuse the information to obtain a bigger share of the total gains from trade. In our paper we will discuss this problem in the context of an asymmetric information game between a potential buyer with private information about his taste and his willingness to pay and a monopolist that uses information obtained from the customer to personalize the product and to optimize his pricing policy.

Our paper is closely related to some other work that deals with issues in electronic commerce: As in *Bakos* (1997) who considers the impact of a reduction in search costs caused by electronic coordination we assume that product characteristics are located on a Salop circle. The versioning paper by *Varian* (2000) is similar to our analyses insofar as consumers may differ with respect to their willingness to pay. However, while *Varian* analyses the incentives to sell goods of different quality, in our analysis the buyer may signal his valuation by revealing some specific amount

of private information. Another related paper is *Acquisti/Varian* (2001) where the behavior of market participants is analysed in a setting with suppliers that are able to observe (for example by way of cookies) whether a potential buyer is a new customer or not. Our central theme, the interaction of personalization, mass customization and price discrimination is analysed in two papers by Ulph and Vulkan (*Ulph/Vulkan*, 2000 and 2001): In both papers consumers differ with respect to their most desired product and knowledge about the characteristics of their customers allows a firm to charge different prices. There are two main differences to our analysis: It is assumed that firms already know the specific taste of each consumer and there are two firms in the market that compete in price strategies. In *Ulph/Vulkan* (2000) each firm is located on another end-point of an Hotelling line and is only able to price discriminate (but not to personalize the product). Mass customization (personalization) is then introduced in *Ulph/Vulkan* (2001) and it is shown that in equilibrium firms often choose both mass customization and price discrimination although both sellers would be better off by not adopting the two technologies.

Our work is complementary to the papers by Ulph und Vulkan insofar as we consider the incentives of consumers (instead of producers) to accept a personalization strategy and to disclose the relevant information. To keep the analysis tractable we do assume that the product is provided by a monopolist (instead of considering duopoly competition) which in our opinion also helps to highlight the specific aspects of the decision by the consumer. We deal with two distinct effects of information revelation: (i) Information about the taste of the consumer enables the monopolist both to customize the product appropriately and to base his pricing decision on the information obtained. (ii) Consumers may not only differ with respect to the location in product space but also with respect to their general willingness to pay for this kind of good. In this setting, revealing more or less information about the location may also deliver a signal about the valuation of the customer.

In section 2 we analyse the first effect in isolation and derive the optimal amount of information disclosure as a function of the given uniform willingness to pay for a perfectly personalized product. We extend the analysis to incorporate the second effect in section 3 where we consider a situation with high and low valuation consumers to study how signalling aspects affect revelation incentives. In section 4 we show how the results change if the monopolist is able to commit to a maximum price

before information is disclosed by the buyer. Section 5 summarizes our findings and presents some suggestions for further research.

2 Consumers with identical willingness to pay

We consider a game between a risk neutral monopolist and a continuum of risk neutral consumers located uniformly on a Salop circle (see *Salop, 1979*). The actual position l_c of a consumer is private information. The monopolist may customize his product by choosing any location l_f . To simplify matters and to enable concentration on information revelation and signalling issues, production and personalization is assumed to be costless. The perimeter of the Salop circle is normalized to a value of two which yields a maximum distance $|l_c - l_f|$ equal to one.

The consumer may (partially) reveal his position to the monopolist. This is done by choosing some value $i \in [0, 1]$ that changes the Salop circle by multiplying the perimeter by $1 - i$. Therefore, $i = 0$ indicates that no information is revealed while $i = 1$ stands for telling the monopolist the exact location l_c . For values of i between zero and one, the maximum distance between l_c and l_f is reduced so that the product while not perfectly personalized is at least more likely to be closer to the customer's location.

The time structure is as follows: In a first step the consumer determines the information revelation parameter i . The monopolist then arbitrarily chooses some location l_f on the modified Salop circle and sets the monopoly price p^M . Finally the consumer decides about buying the given product at the specified price by comparing this price with the utility derived by a product at location l_f .

Gross utility of a consumer is given by $u = v_{max} - |l_c - l_f|$ where v_{max} indicates the valuation for a perfectly personalized product, i. e. a product where locations l_c and l_f coincide. The consumer will accept the offer of the monopolist whenever $u \geq p$. Without information revelation $|l_c - l_f|$ is distributed uniformly between zero and one. The probability of trade is then given by $Pr(u \geq p) = v_{max} - p$ for $p \in [\max\{0, v_{max} - 1\}, v_{max}]$. The monopolist will set p^M in order to maximize expected return $p Pr(u \geq p) = p(v_{max} - p)$. For $i > 0$ we obtain a modified demand schedule $Pr(u \geq p) = 1/(1 - i)(v_{max} - p)$ — for each price it is now more likely

that the consumer accepts the monopolist's offer. At $i = 1$ the product is perfectly personalized and for $p^M \leq v_{max}$ trade occurs with certainty.

In figure 1 we attempt to visualize the effects of information revelation: We consider a consumer with valuation $v_{max} = 1/2$ and display inverse demand curves for different values of i . All inverse demand curves start on the price axis at v_{max} . Note that in our analysis "demand" stands for the probability of trade — a given consumer either wants to buy one unit of the good or nothing. For $i = 0$ demand is given by a falling straight line with a slope of -1 , the line gets flatter with rising i and finally becomes horizontal at $i = 1$.

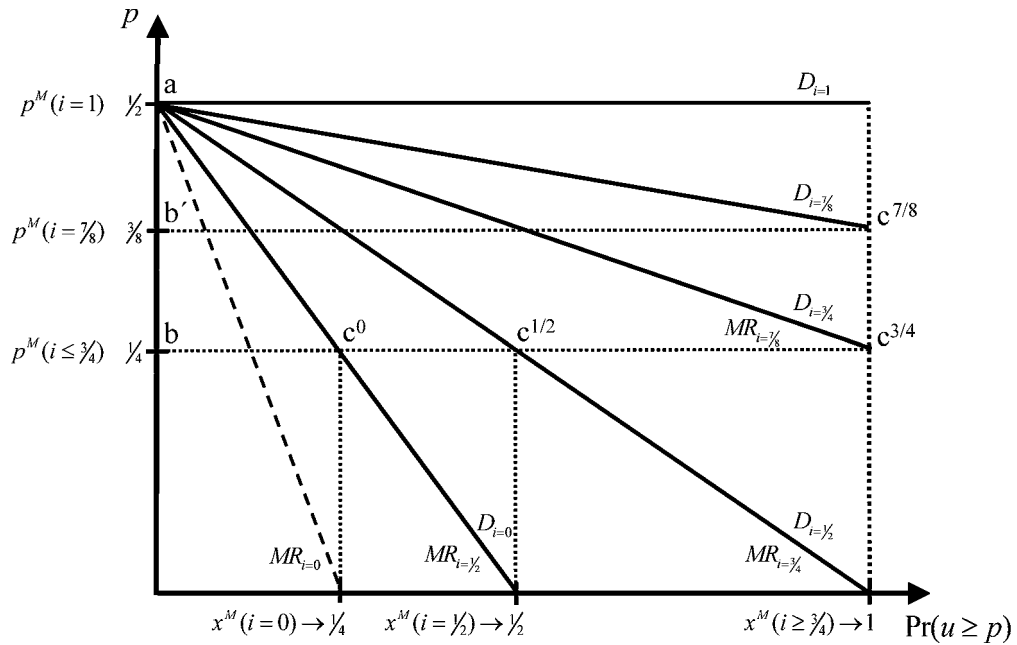


Figure 1: Consumer with $v_{max} = 1/2$: Information revelation and pricing

Based on the linear inverse demand curves it is straightforward to determine the monopoly solution: For $Pr(u \geq p) < 1$ the marginal revenue MR of the risk neutral monopolist is given by a straight line with twice the slope of the respective demand curve while at $Pr(u \geq p) = 1$ the marginal revenue becomes zero. We can now discuss the implications of the optimal strategies in the second stage of the game:

- For $i \in [0, 1/2)$ gains from trade are not assured even if the monopolist chooses

the competitive price $p = 0$.

- For $i \in [0, 3/4)$ the monopolist maximizes profits by setting a price $p^M = 1/4$ and at this price the probability of trade is always smaller than one (it starts at $1/4$ for $i = 0$ and approaches 1 at $i = 3/4$). Revelation of information up to $i = 3/4$ thus not only increases the possible gains from trade but also reduces the monopoly distortion. As can be seen by a look at the consumer surplus triangles abc^i and the profit rectangles below, consumer and monopolist equally share the gains from information revelation.
- Beyond $i = 3/4$ the monopolist chooses the highest possible price that guarantees trade for any l_c — for example $p^M = 3/8$ at $i = 7/8$. While this pricing policy ensures an efficient solution, it also helps the monopolist to increase its share of total surplus which in turn yields an absolute reduction of expected consumer surplus relative to the situation at $i = 3/4$: Triangle $abc^{3/4}$ is bigger than triangle $ab'c^{7/8}$. Finally, at $i = 1$ social surplus will be maximized (each product sold is perfectly personalized), however, all gains from trade are appropriated by the monopolist.

Given our game structure, the subgame perfect equilibrium is given by partial disclosure of information $i^* = 3/4$ and a resulting monopoly price $p^M(i^*) = 1/4$. While at this equilibrium monopoly distortions in the pricing decision are eliminated, some asymmetry of information remains and thus total surplus is not maximized.

Having started with a situation where in the initial situation without information revelation gains from trade are not assured, we will now discuss how higher valuations v_{max} will affect revelation incentives. In figure 2 we compare the results obtained for $v_{max} = 1/2$ with two additional situations: For $v_{max} = 1$ trade would take place even without any information disclosed at the competitive price $p^C = 0$, however, monopoly pricing yields a probability of trade below one. This monopoly distortion will be absent at $v_{max} = 2$ (and also for higher values of v_{max}).

As can easily be seen, the amount of information revelation in equilibrium decreases with rising consumer valuation because the elimination of the monopoly distortion is assured for lower values of i : For $v_{max} = 1$ we obtain equilibrium strategies $(i^*, p^*) = (1/2, 1/2)$ and for $v_{max} \geq 2$ information revelation is no longer in the

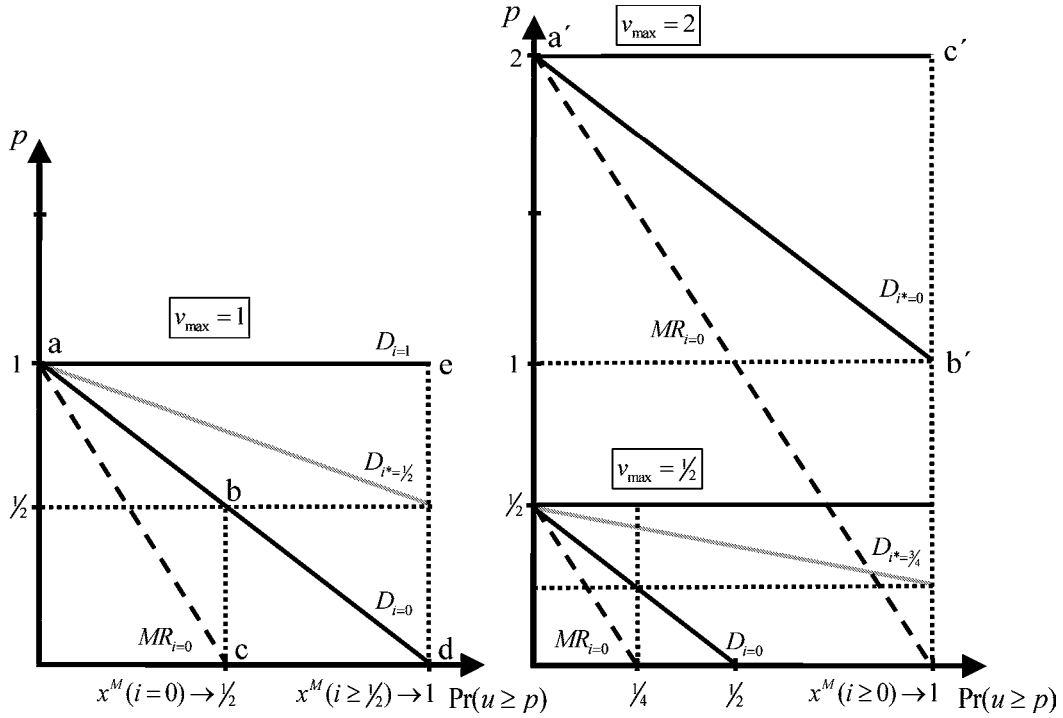


Figure 2: Comparing information revelation and pricing for $v_{max} = 1/2, 1$ and 2

interest of the consumer and $(i^*, p^*) = (0, v_{max} - 1)$. The amount of information disclosed in equilibrium can be written as a function of v_{max} :

$$i^*(v_{max}) = 1 - \frac{v_{max}}{2} \quad \text{for } v_{max} \in [0, 2] \quad (1)$$

A second interesting point that can be visualized in figure 2 is the maximum improvement in total surplus that may be realized by information revelation. While the relative change is larger the smaller v_{max} , the absolute effect by revealing all information is maximized at $v_{max} = 1$: The potential gain is given by the area $abcde$ which is bigger than the triangle $a'b'c'$ that represents the same effect for $v_{max} = 2$ and, exceeding $1/2$, is also bigger than the effect for $v_{max} = 1/2$.

To sum up our results in this section: (i) While disclosing all information would allow to perfectly personalize the product and thus maximize total surplus, choosing this strategy is not in the interest of the consumer because the monopolist would fully appropriate this surplus. (ii) Partial revelation of information is the optimal strategy for the consumer as long as the probability for trade in the game without information revelation is below one. (iii) The lower the valuation of a consumer,

the more information she will disclose in equilibrium. (iv) The relative change in total surplus that may be realized by revealing all information is larger the lower the valuation of the consumers, however, the absolute change approaches a maximum at $v_{max} = 1$

3 Two types of consumers and signalling

After having discussed the situation with uniform valuation for the perfectly personalized product, we do now consider the case where consumers differ with respect to their willingness to pay. To keep the analysis tractable we assume that there are only two types of consumers, $t = \{h, l\}$, with both types equally likely and h indicating high valuation consumers with $v_{max} = 1$ and l referring to low valuation customers with $v_{max} = 1/2$. We also restrict the strategy space of buyers: The amount of information revelation may either be zero (labeled i_n for “no information revealed”), $i = 1/2$ (i_s for “some information revealed”) or $i = 1$ (i_a for “all information revealed”). We have chosen these values for the following reasons: (i) $i = 1/2$ represents the optimal value for the high valuation consumer and existence of a pure strategy separating equilibrium seems to be most likely at this optimal value. (ii) To reveal all information has been included as an option because there might exist a pooling equilibrium where this strategy is chosen. Also, as will be shown in section 4, disclosing all information is an equilibrium action for the high valuation consumer if the monopolist can commit in advance to some maximum price.

3.1 Strategies, Beliefs and Payoffs

Following the standard backward induction logic we start at the end of the game, where the consumer has to decide whether he will buy the product at the specified price. Note that the consumer’s buying decision not only depends on the price but also on his type and on the distance between the locations l_c and l_f . Because in general neither the exact location l_c nor the type of consumer is known to the monopolist, he faces a demand curve that provides him with the (subjective) probability of trade for a given price. While the monopolist cannot directly charge different prices for different types, the price may depend on the signal observed. When receiving

signal i , the monopolist will assign some probability that the sender of such a signal is of type h . Assume now that these beliefs are correct (as they will be on the equilibrium path) and denote by θ_i the actual proportion of type h consumers as a fraction of all customers that choose some signal i . Under this assumption we obtain (conditional) demand functions $x_i(p_i, \theta_i)$ that denote the probability for a sender of information i to actually buy the product at price p_i :

$$x_n(p_n, \theta_n) = \begin{cases} \frac{1}{2} + \frac{1}{2}\theta_n - p_n & \text{if } 0 \leq p_n \leq \frac{1}{2} \\ \theta_n(1 - p_n) & \text{if } \frac{1}{2} < p_n \leq 1 \end{cases} \quad (2)$$

$$x_s(p_s, \theta_s) = \begin{cases} 1 - 2p_s(1 - \theta_s) & \text{if } 0 \leq p_s \leq \frac{1}{2} \\ (2 - 2p_s)\theta_s & \text{if } \frac{1}{2} < p_s \leq 1 \end{cases} \quad (3)$$

$$x_a(p_a, \theta_a) = \begin{cases} 1 & \text{if } 0 \leq p_a \leq \frac{1}{2} \\ \theta_a & \text{if } \frac{1}{2} < p_a \leq 1 \end{cases} \quad (4)$$

The demand curves are kinked at $p_i = 1/2$ because at prices above $1/2$ only customers of type h may buy the product. We get the total expected demand Q by summing up the products $w_i x_i$ where w_i denotes the share of consumers that have chosen to send signal i : $Q = \sum_i w_i x_i$. Note that w_i has no effect on the pricing decision of the monopolist since we assume that he will set his price after receiving signal i : Conditional on the signal $i \in \{i_n, i_s, i_a\}$ he has been observing, the monopolist will choose a price p_i^M to solve the profit maximization problem:

$$p_i^M = \arg \max_{p_i} \pi_i^M = w_i \cdot p_i \cdot x_i(p_i, \theta_i) \quad (5)$$

While w_i has no effect on p_i^M , the profit maximizing price at some signal i depends on the respective proportion of high valuation customers, θ_i . As we have stated above, this actual proportion has to match the monopolist's belief for each information set (that is, signal i) which will be reached in the game with positive probability. Solving the the maximization problem for each signal we obtain

$$p_n^M(\theta_n) = \frac{1 + \theta_n}{4} \quad (6)$$

$$p_s^M(\theta_s) = \begin{cases} \frac{1}{2} & \text{if } \theta_s > \frac{1}{2} \\ \frac{1}{4 - 4\theta_s} & \text{if } \theta_s \leq \frac{1}{2} \end{cases} \quad (7)$$

$$p_a^M(\theta_a) = \begin{cases} 1 & \text{if } \theta_a > \frac{1}{2} \\ \frac{1}{2} & \text{if } \theta_a \leq \frac{1}{2} \end{cases} \quad (8)$$

Based on these considerations we are now able to determine the expected utility $E[u^t|i = i] \equiv u_i^t$ of a type t consumer who has chosen strategy i in the first stage of the game. This expected utility depends on the probability of gains from trade $Pr(u^t \geq p_i)$ and the expected consumer surplus if trade occurs which is given by $(v_{max}^t - p_i)/2$:

$$u_i^t = \frac{Pr(u^t \geq p_i)(v_{max}^t - p_i)}{2} \quad (9)$$

The consumers' expected utility is affected by p_i in two ways: First the price has an influence on the purchasing decision as higher prices will reduce the probability of gains from trade. Moreover, in case of buying the product, consumer surplus will be lower the higher the price. Given the information about the pricing decision of the monopolist from equations (6) to (8) we can derive the expected utility for both types of consumers at any strategy $i \in \{i_n, i_s, i_a\}$ as a function of the belief θ_i of the monopolist. For the low valuation customer we receive

$$u_n^l(\theta_n) = \frac{1}{32} (1 - 2\theta_n + \theta_n^2) \quad (10)$$

$$u_s^l(\theta_s) = \begin{cases} \left(\frac{1-2\theta_s}{4-4\theta_s}\right)^2 & \text{if } \theta_s \leq \frac{1}{2} \\ 0 & \text{if } \theta_s > \frac{1}{2} \end{cases} \quad (11)$$

$$u_a^l(\theta_a) = 0 \quad (12)$$

while the payoffs for a buyer with high willingness to pay are given by

$$u_n^h(\theta_n) = \frac{1}{32} (9 - 6\theta_n + \theta_n^2) \quad (13)$$

$$u_s^h(\theta_s) = \begin{cases} \frac{2-3\theta_s}{4-4\theta_s} & \text{if } \theta_s \leq \frac{1}{2} \\ \frac{1}{4} & \text{if } \theta_s > \frac{1}{2} \end{cases} \quad (14)$$

$$u_a^h(\theta_s) = \begin{cases} \frac{1}{2} & \text{if } \theta_a \leq \frac{1}{2} \\ \frac{1}{2} & \text{if } \theta_s > \frac{1}{2} \end{cases} \quad (15)$$

Based on these payoff functions and the profit function in (5) it is now straightforward to determine the equilibria of the signalling game.

3.2 Equilibria of the signalling game

In a signalling game we have to consider three possible types of equilibria: Pooling equilibria where no information is revealed, separating equilibria where the signal gives perfect information about the type and semi-separating equilibria where

some types play mixed strategies. To obtain these equilibria we apply the concept of Perfect Bayesian Equilibrium that combines subgame perfection with Bayesian updating in order to obtain reasonable beliefs on the equilibrium path. Where appropriate, we add some additional refinements to rule out unreasonable out-of-equilibrium beliefs. In a first step we restrict attention to pure strategies and check whether any combination of signals $(i(h), i(l))$ by the low and high valuation type yields an equilibrium at permissible beliefs of the monopolist. However, as will be shown no separating equilibrium exists and the existing pooling equilibria are only supported by unreasonable out-of-equilibrium beliefs. Thus, in a second step, we consider mixed strategies and derive a semi-separating equilibrium that seems to be the most likely outcome of the game.

3.2.1 Pooling equilibria

In a pooling equilibrium both types of consumers choose the same strategy $s^h = s^l = s_c^p$ and therefore disclose the same amount of information i about their location. At the equilibrium path the monopolist updates its belief according to Bayes' rule, properly assuming that both types l and h play the equilibrium strategy with probability one. Thus in any pooling equilibrium the monopolist's ex-post belief will comply with the common prior distribution of the consumer types, i. e. he still assesses a probability of 1/2 for each type. In a Perfect Bayesian Equilibrium it is assumed that the monopolist is free to choose any out-of-equilibrium beliefs if he observes a deviation from the equilibrium path. Thus a pooling equilibrium has to satisfy the following conditions:

- The monopolist's beliefs are consistent with common priors that are updated according to Bayes rule whenever possible. On the equilibrium path of the pooling equilibrium beliefs therefore correspond to the common prior distribution of types h and l : $\theta_{i|i=s_c^p} = 1/2$.
- Given some out-of-equilibrium beliefs of the monopolist (i. e. for $i \neq s_c^p$), neither type has an incentive to deviate from its equilibrium strategy.

These requirements are sufficient to eliminate a pooling equilibrium where both consumer types do not disclose any information, i. e. choose strategy i_n . In accordance

with an equilibrium belief $\theta_n = 1/2$ such a strategy would result in a monopoly price $p_n = 3/8$. However, a type h consumer would have an incentive to deviate: Choosing strategy i_s would generate an expected utility between $1/4$ and $1/2$ (the exact value depends on the firm's belief θ_s) while the expected utility of strategy i_n just amounts to the lower value of $25/128$.

However, two other pooling equilibria may be constructed by assuming extreme beliefs at the deviation paths:

- An equilibrium (P1) where both types choose $s_c^p = i_s$, i. e. partially disclose their location information, is sustained by the customers fear that a deviation will be interpreted by the monopolist as a signal for a high willingness to pay. To be more exact, the beliefs $\theta_s = 1/2$ at the equilibrium path and $\theta_a > 1/2$ and $\theta_n = 1$ off the equilibrium path support this pooling equilibrium. The corresponding price strategy s_c^M of the monopolist is characterised by $p_s = 1/2$, $p_n = 1/2$ and $p_a = 1$.
- An pooling equilibrium (P2) with $s_c^p = i_a$, i. e. both types reveal all information, can be supported in a similar manner if the monopolist has a belief of $\theta_a = 1/2$ at the equilibrium path and expects the consumer to be of type h with probability $\theta_s > 1/2$ and $\theta_n = 1$, respectively, when observing a deviation. The corresponding prices are $p_a = 1/2$, $p_n = 1/2$ and $p_s = 1/2$.

Although both pooling equilibria are Perfect Bayesian Equilibria, forward induction arguments render them quite unreasonable: Note that in both pooling equilibria, P1 and P2, the high valuation consumer would be strictly worse off if he deviates while type l is just indifferent between all strategies. However, the out-of-equilibrium beliefs assert that the monopolist assumes that type l will deviate with a lower probability! Also note that equilibrium P1 is pareto dominated by P2, while P2 can be ruled out by the intuitive criterion (see *Cho/Kreps*, 1987): For any out-of-equilibrium belief of the monopolist the type h consumer cannot benefit by deviating from the equilibrium strategy. Thus, for this type of consumer, any action outside P2 is equilibrium dominated. This is not the case for type l : Deviation to i_n will be profitable if the monopolist does assign a sufficiently low probability for deviations

by type h (for whom, as just discussed, this action is equilibrium dominated and thus quite unlikely).

3.2.2 Pure strategy separating equilibria

Now consider separating equilibria: In a separating equilibrium each type of a player must choose a different pure strategy. Playing an equilibrium strategy $s^t = i(t)$ will thus inform the seller that the consumer is of type t and the monopolist will update his beliefs accordingly. Updating will thus result in a belief of $\theta_{i|i=s^h} = 1$ if the equilibrium strategy of type h has been observed while the monopolist will base its pricing decision on $\theta_{i|i=s^l} = 0$ when the equilibrium strategy assigned to type l has been played. When he observes a strategy that deviates from any equilibrium path, the monopolist is once again free to choose any belief about the customer's type.

Obviously, no type will reveal all information:

- Let us first consider type h : Choosing i_a would yield a price $p_a = 1$. This can not be an equilibrium strategy because the expected utility of the consumer equals zero and is therefore lower than for any other strategy (especially compared with a strategy that imitates the l -type consumer).
- A separating equilibrium where the low valuation consumer discloses all information is also not feasible. In this case the monopolist would set a price $p_a = 1/2$ which gives an incentive for type h to mimic type l .

Thus only two candidates for a separating equilibrium remain:

- First consider the strategy profile $\{(s^h = i_n, s^l = i_s), (p_n = 1/2, p_s = 1/4, p_a = 1)\}$ combined with the corresponding beliefs $\{\theta_n = 1, \theta_s = 0, \theta_a > 1/2\}$, that is, the monopolist expects the sender to be of type h when no information is revealed while i_s signals type l . Provided these beliefs, however, the high valuation customer could do better if he mimics the type l consumer: Sending a signal i_n would increase his expected utility from $1/8$ to $1/2$.
- The reverse case can be ruled out in a similar manner: Consider strategy profile $(\{s^l = i_n, s^h = i_s\}, \{p_n = 1/4, p_s = 1/2, p_a = 1\})$ and beliefs $\{\theta_n =$

$0, \theta_s = 1, \theta_a > 1/2\}$. Again type h has an incentive to mimic the low valuation consumer because that would raise his expected surplus from $1/4$ to $9/32$.

As a consequence no pure strategy separating equilibria exist in the signalling game.

3.2.3 Mixed strategies and Semi-separating equilibrium

We now proceed by examining the existence of mixed strategy equilibria in which at least one type randomizes over his pure strategies. Whenever the probability distributions applied for randomizing differ between the two types, the monopolist is able to update his prior beliefs when observing some signal i . If only one type plays some strategy with positive probability, the monopolist obtains perfect information when he observes this signal. We therefore will refer to the latter case as a semi-separating equilibrium.

First note that neither type will choose complete information disclosure with positive probability in any mixed strategy equilibrium: Type l will never play i_a because he obtains zero utility from this strategy (i_a is thus weakly dominated by i_s and i_n). Now suppose that the monopolist has observed a consumer playing strategy i_a : According to the reasoning above the only consistent belief is to assume that the sender is of type h with probability $\theta_a = 1$. Thus the seller will set a price $p_a = 1$ which yields zero utility to the consumer. As a consequence, neither type will completely reveal his information to the monopolist.

Therefore we can restrict attention to mixed strategy equilibria where both types play only the pure strategies i_n and i_s with positive probability. Now consider a mixed strategy equilibrium in which type t chooses strategy i with probability w_i^t . Note that for each type of customer the probabilities w_n^t and w_s^t must add up to one as i_a is supported with probability zero. Because $w_s^h = 1 - w_n^h$ and $w_s^l = 1 - w_n^l$, we can identify the randomization strategy of a player of type t by w_n^t only. Based on the respective probability of each type we are also able to derive the consistent ex-post beliefs of the monopolist on the equilibrium path:

$$\theta_n = \frac{w_n^h}{w_n^h + w_n^l} \quad (16)$$

$$\theta_s = \frac{1 - w_n^h}{2 - w_n^h - w_n^l} \quad (17)$$

Note that a player who is willing to randomize between pure strategies has to be indifferent between playing any of the pure strategies that he plays with positive probability in the mixed strategy. Applied to our case his expected utility has to be the same whether he chooses i_n or i_s . For type h we therefore can state that

$$\frac{2 - 3\theta_s}{4 - 4\theta_s} = \frac{1}{32} (9 - 6\theta_n + \theta_n^2) \quad (18)$$

and after inserting (16) and (17) into equation (18) and rearranging terms appropriately, we get the condition

$$\frac{1 + w_n^h - 2w_n^l}{1 - w_n^l} = \frac{(2w_n^h + 3w_n^l)^2}{8(w_n^l + w_n^h)^2}. \quad (19)$$

Similarly, a low valuation consumer has to be indifferent between i_n and i_s , when choosing to randomize his strategies, i. e.

$$\left(\frac{1 - 2\theta_s}{4 - 4\theta_h} \right)^2 = \frac{1}{32} (1 - 2\theta_n + \theta_n^2) \quad (20)$$

which yields

$$\left(\frac{w_n^h - w_n^l}{1 - w_n^l} \right)^2 = \frac{(w_n^l)^2}{2(w_n^h + w_n^l)^2}. \quad (21)$$

Now we must consider three possible scenarios:

- First let us assume that only type l randomizes. In this case w_n^h will either be zero or one while w_n^l has to lie between these values. Hence type h chooses a pure strategy.
- Similarly type h may choose a mixed strategy while type l decides in favour of strategy i_n or i_s .
- If both types choose mixed strategies, the solution to the game will correspond to the solution of a linear system that consists of equations (19) and (21).

An examination of all these cases shows that a unique semi-separating Perfect Bayesian Equilibrium in mixed strategies exists: Only consumers of type h randomize between i_n and i_s while the low valuation customers always conceal their location information. To be more precise, the equilibrium is characterised by a strategy profile $(s_f^*, s_c^*) \equiv \{(s^h = w_n^h = \sqrt{2}/2 - 1/2, s^l = w_n^l = 1), (p_n^* = 1 - \sqrt{2}/2, p_s^* =$

$1/2, p_a^* = 1\}$. From equations (16) and 17) we receive the corresponding equilibrium beliefs $\{(\theta_n^* = 3 - 2\sqrt{2}, \theta_s^* = 1, \theta_a^* > 1/2)\}$.

The expected utility for both consumer types, u^l and u^h , can then be calculated by using the equations (10), (13) and (14):

$$u^l(i_n, \theta_n^*(s_c^*)) = \frac{3}{8} - \frac{1}{4}\sqrt{2} \quad (22)$$

$$u^h(i_n, \theta_n^*(s_c^*)) = \frac{1}{4} \quad (23)$$

$$u^h(i_s, \theta_n^*(s_c^*)) = \frac{1}{4} \quad (24)$$

To compute the expected total profit, Π^M , we must first determine total demand Q . Note that we obtain the signal proportions w_i by adding up the respective conditional probabilities and recognising the common prior distribution of the types: $w_i = (w_i^h + w_i^l)/2$. To determine total expected profit, these shares have to be multiplied by $p_i^* \cdot x_i(p_i^*, \theta_i^*)$:

$$\Pi(s_f^*, s_c^*) = \sum_i w_i^* \cdot p_i^* \cdot x_i(p_i^*, \theta_i^*) \quad (25)$$

$$\Pi(s_f^*, s_c^*) = x_s \cdot p_s^* \cdot \frac{(1 - w_n^l) + (1 - w_n^h)}{2} + x_n \cdot p_n^* \cdot \frac{w_n^l + w_n^h}{2} \quad (26)$$

By inserting the equilibrium values of θ_i^* , p_i^* , w_i^l , and demand functions (2) to (4) into equation (26), we finally obtain total profit:

$$\Pi(s_f^*, s_c^*) = (1 - 2p_s^*(1 - \theta_s^*))p_s^* \cdot \frac{1 - w_n^h}{2} + \left(\frac{1}{2} + \frac{1}{2}\theta_n^* - p_n^*\right)p_n^* \cdot \frac{1 + w_n^h}{2} \quad (27)$$

$$= \frac{3}{8} - \frac{1}{8}\sqrt{2} + \left(1 - \frac{1}{2}\sqrt{2}\right) \left(1 - \frac{1}{2}\sqrt{2}\right) \left(\frac{1}{4} + \frac{1}{4}\sqrt{2}\right) \quad (28)$$

$$= \frac{1}{4} \quad (29)$$

Note that total profit of the monopolist and consumer surplus of the high valuation type are lower than in the (unreasonable) pooling equilibrium P2, while both high valuation customer and the monopolist receive exactly the same payoff values as in the less efficient pooling equilibrium P1. However, the low valuation consumer is better off because he will now obtain a positive expected surplus, while this surplus does not exceed zero when a pooling equilibrium P1 or P2 is played. Therefore, the inefficient pooling equilibrium P1 is also dominated by the semi-separating equilibrium.

To summarize the arguments mentioned above, both pooling equilibria P1 and P2 fail to resist to some forward induction arguments: (i) P1 does not appear to be a reasonable equilibrium because of two reasons: Firstly, it is pareto dominated by the semi-separating equilibrium. Secondly, it is based on implausible beliefs. (ii) P2 can be ruled out by the intuitive criterion. As a consequence, the semi-separating equilibrium — which is indeed not affected by any refinements — remains as the only reasonable outcome of the signalling game.

4 Price commitments and screening

Until now we have assumed that the consumer first chooses the amount of information disclosed and, observing this choice, the monopolist decides on his product location and the profit maximizing price offer. While the timing with respect to the location decision seems quite reasonable, the monopolist might have an incentive to set his price before the consumer moves. Such a commitment is likely to be feasible under the specific assumptions of our model, however, it might be more difficult to obtain in more realistic settings where the exact product location may affect production costs and a priori unknown extra costs for full personalization may exist.

Nevertheless we will now consider how our results change if we allow the monopolist to post a price offer in stage zero of the game, i. e. before the consumer chooses i . Note that such an offer could be made binding insofar as a higher price is no longer feasible by signing an enforceable contract stating that the product has to be provided at the prespecified price. However, because it might be in the interest of both the monopolist and the consumer to reduce this price if the consumer has disclosed less information than expected (i. e. deviated from the equilibrium path), the contract must be renegotiation proof in the following sense: Even after a pareto improving reduction of the price a deviating consumer must get a lower utility than he would get in the equilibrium.

The idea behind this restriction on feasible price offers may be best illustrated in the setting with only one type. Consider the case of a consumer with $v_{max} = 1$. As shown in section 2, the privately optimal amount of information revelation by

such a consumer is given by $i^* = 1/2$ and the monopolist will accordingly set a price $p^* = 1/2$. While this yields efficient consumption (the price is low enough to ensure that the product is sold for any location l_f chosen by the monopolist), total surplus is not maximized because the product is not perfectly personalized. This is due to the fear of the consumer that after choosing $i = 1$ the seller would set a price $p^M(i = 1) = 1$ which yields zero surplus for the buyer. By choosing the equilibrium value $i^* = 1/2$, however, the consumer can ensure an expected utility $E[u^h|i = 1/2] = 1/4$. What happens if the monopolist commits to a price $p^M = 1$? At first sight it seems as if an equilibrium ($i^e = 1, p^e = 1$) could be realized. However, suppose that the consumer deviates from this equilibrium by choosing $i < 1$: If the seller insists on $p^e = 1$ no trade occurs and both consumer and monopolist obtain zero surplus. The seller would therefore have an incentive to post a second offer that yields at least some probability of trade. Given these considerations, we must determine the optimal deviation of the consumer (the equilibrium amount of information disclosed in the game without commitment) and restrict the range of feasible price offers to these that guarantee at least the consumer surplus obtained under optimal deviation. In our example the expected utility is $1/4$ if the consumer chooses $i^* = 1/2$ instead of $i^e = 1$ and thus the price offer may not exceed $p = 3/4$ (which yields a consumer surplus of $1/4$ at $i = 1$). As easily can be shown the optimal price offer p^{PC} (with “PC” standing for “price commitment”) is generally given by $p^{PC} = v_{max} - (1 - i^*)/2$ and applying the formula for i^* in equation (1) from section 2 we obtain

$$p^{PC}(v_{max}) = 3/4v_{max} \quad \text{for } v_{max} \in [0, 2] \quad (30)$$

In the case with an homogenous willingness to pay we thus obtain the following result: The ability to set the monopoly price before the consumer decides about information revelation makes it possible to implement the surplus maximizing equilibrium ($i^{PC} = 1, p^{PC} = 3/4v_{max}$ (for $v_{max} \in [0, 2]$ — for higher v_{max} we get $p^{PC} = v_{max} - 1/2$). In this equilibrium the monopolist commits to not expropriate a consumer that reveals its information: This is done by setting a price that offers a consumer surplus equal to that obtained for optimal revelation in the situation without commitment.

How does this analysis extend to the case with two types? Pooling equilibria are not affected because higher prices than $p^* = 1/2$ would yield no trade with the low valua-

tion type (and thus no pooling) and committing to a lower price is not in the interest of the monopolist. When considering separating equilibria, however, commitment by the monopolist completely changes the situation: In section 3 there only existed a semi-separating equilibrium where the high valuation consumer partially disclosed information with positive probability. Now we are able to show that the monopolist may separate the two types by setting a pricing menu $(p_a = 3/4, p_n = 1 - 1/\sqrt{2})$.

The argument goes as follows: (i) At this menu type h has just no incentive to choose i_n to mimic type l because without information revelation his expected utility at $p = 1 - 1/\sqrt{2}$ is just equal to $1/4$, the utility derived for sure if he chooses i_a . (ii) A separating equilibrium where the low valuation type reveals more information than the consumer with the higher willingness to pay is neither feasible (type h would always have an incentive to mimic type l) nor in the interest of the monopolist (even if separation would work it would yield lower profits than the price menu considered above). (iii) There does not exist a combination of prices (p_a, p_s) that form a separating equilibrium: At $p_a = 3/4$ the price p_s that ensures that type h would not like to mimic type l must be at least $1/2$ and at this price there would be no trade with a type l consumer. Finally we have to check whether a marginal reduction of p_a that allows a price separating price $p_s < 1/2$ would be in the interest of the monopolist. This would be the case if the additional profit $p_s(1 - 2p_s)$ exceeds the profit loss due to the necessary reduction of p_a which is given by $1/2 - p_s$. The resulting condition $2p_s - 2p_s^2 - 1/2 > 0$, however, cannot be fulfilled in the relevant range $p_s \in [0, 1/2]$.

5 Conclusion

Would you like to be a prosumer and reveal detailed information about your preferences to a seller? We showed in a monopoly setting how the answer to this question might depend on your valuation of the product and on the commitment ability of the monopolistic seller you are facing:

- We first considered a situation where consumers differ with respect to the exact location in product space but have an identical valuation for the perfectly

personalized product. In this setting a prospective buyer will provide all information necessary for personalization if the monopolist is able to commit to a maximum price before he obtains this message. However, if commitment is not feasible, a consumer will only disclose some information. While this strategy reduces expected total surplus, it is chosen by the buyer to avoid total extraction of the surplus by the seller. In this setting we

obtained the result that the lower the probability of trade without disclosure (i. e. lower valuation) the larger the amount of information revealed.

- Matters get more complicated if consumers also differ with respect to their willingness to pay. In this case a signalling game results where low valuation consumers have an interest to be distinguishable from types with a high willingness to pay in order to obtain a lower price while high valuation consumers want to avoid expropriation of surplus and also have an incentive to mimic the type with the low willingness to pay. These effects render disclosure of information less attractive and thus without price commitment by the monopolist there only exist pooling equilibria with quite unreasonable beliefs and a mixed strategy separating equilibrium where only the high valuation consumers reveals part of his information with positive probability.

If, however, the monopolist is able to commit in advance to some maximum price, we have a screening game where the monopolist offers a menu of prices as a function of the amount of information disclosed. In this setting a pure strategy separating equilibrium exists where types with a high willingness to pay get a personalized product at a relatively high price while low valuation consumers provide no information and obtain a standardized but much cheaper product.

Note that we obtained these result in a model where personalization is costless and thus we are more likely to overstate the amount of information revealed. On the other hand, introducing competition might reduce the monopoly power of the seller and thus consumers may be less reluctant to provide the information. It should thus be interesting to combine the duopoly model from *Ulph/Vulkan* (2001) with the information revelation approach discussed in the present analysis. Another promising generalization is the consideration of a continuum of signals and/or types in the signalling game.

References

- Acquisti, A., Varian, H. R.* (2001), Conditioning Prices on Purchase History, University of California Berkeley: Discussion paper. (<http://www.sims.berkeley.edu/hal/people/hal/papers.html>)
- Bakos, J. Y.* (1997), Reducing Buyer Search Costs: Implications for Electronic Marketplaces, *Management Science* 43, 1676–1692.
- Cho, I., Kreps, D.* (1987), Signalling Games and Stable Equilibria, *Quarterly Journal of Economics* 103, 179–221.
- Salop, S. C.* (1979), Monopolistic Competition with Outside Goods, *Bell Journal of Economics* 10, 141-156.
- Toffler, A.* (1980), *The Third Wave*, New York : Morrow.
- Ulph, D., Vulkan, N.* (2000), Electronic Commerce and Competitive First-Degree Price Discrimination, University of Bristol: Economics Department Discussion Papers 00/484. (<http://else.econ.ucl.ac.uk/>)
- Ulph, D., Vulkan, N.* (2001), E-Commerce, Mass Customization and Price Discrimination, Working paper. (<http://else.econ.ucl.ac.uk/>)
- Varian, H. R.* (1997), Economic aspects of personal privacy, in: Irving, L. (ed.), *Privacy and Self-Regulation in the Information Age*. National Telecommunications and Information Administration. (http://www.ntia.doc.gov/reports/privacy/privacy_rpt.htm)
- Varian, H. R.* (2000), Versioning information goods, in: Kahin, B., Varian H. R. (eds.), *Internet Publishing and Beyond*. Cambridge MA: MIT Press.