"Final services' competition under an interconnection capacity regime"

Jose G. Aguilar
Universidad Autonoma de Baja California

and

Iñigo Herguera
Universidad Complutense de Madrid

Abstract: interconnection between fixed networks has been regulated across all countries on a per time regime basis. has been the model implemented in all fixed- voice telephony traffic untill recently. Some regulators face the prospect of regulating access or interconnection based on a capacity measure, i.e., Mgbites/ second of interconnection. The capacity regime is the one that actually applies to Internet and other types of traffic between networks.

In this paper we propose a simple oligopolistic model that allows to consider simple dynamic considerations to analyze the consequences of the introduction of the capacity interconnection regime in terms of final services competition outcomes, non- linear tariffs and the incentives that the new regime yield for the efficient use of the regulated network on the entrants.

The results show that gains in the allocation of traffic between different peak and off-peak hours that are allowed by the capacity regime could be very significant and may lead to a much more aggressive pricing in the final services. In any case, the simultaneous use of both interconnection models (i.e, the capacity and the per time based models) leads always to tougher competition in the final services markets and to higher efficiency (internal to the firm and allocative) gains. Nevertheless, the entrant needs a minimum scale before he can allocate his interconnection services in the capacity based regime. In addition, the convenience of the capacity model for the entrant will depend on the (price) reaction of the incumbent and the degree of exploitation of the economies of scale and scope that the new regime allows.

The new capacity based regime can accelerate the path of change occurred in all fixed operators in becoming simple carriers of a commodity and being less able to appropriate the rents of any service provision on the networks.

Key words: Interconnection per time, interconnection by capacity, dynamics of the competition, regulation, network dimension.

JEL classification: C72, L13, L51, L96.

1. Introduction and motivation

In practically all countries that liberalized the fixed telecommunications industry the interconnection or access services have been regulated based on two principles: the regulated price should recover all attributed costs of the local network and interconnection should be measured, and priced, mainly using the minute as the unit of account. It is well known the fact that local network costs are basically driven by very significant fixed investments in the network deployment and upgrading, where the variable costs attributed to an increase in traffic is almost negligible. Thus, a contradiction seemed to be imposed by regulation: the incumbent operators enjoy economies of scale in their local networks, partially caused by the interconnection traffic demanded by new entrants without own network. The entrants, on the other side, have to pay a regulated price based on the number of units-of-time (minutes) acquired on the wholesale service. This time- based interconnection regime (*IxT*) involves the imposition on the entrants of a cost structure for the local infrastructure that is very different to the one the incumbent faces and in fact, far from real. The incumbent operators do not face relatively high variable per-unit-of-time cost of usage in the local loop, while entrants have to pay this variable cost without being able to exploit any economy of scale or scope across time for the use of the infrastructure.

Additionally, the per minute price set by the regulators is a potential source of price squeeze strategies pursued by incumbent operators because while their average costs of network usage is diminishing with higher volumes of traffic, the final service price has to comply with a whole set of regulatory mechanisms that basically imposed (real) price reductions across time. From country to country final services prices are regulated in a different manner, but global price caps as well as direct (individual service) price regulation is most common among EU countries[1].

Under the *IxT* model then, the new operators cannot emulate the tariff strategies offered by historical operators to final consumers and have few options to maintain aggressive strategies unless they can build their own infrastructure. Furthermore, the entrant's incentives to increase the volume of traffic are very scarce since its average (input) costs are fixed on a per minute

[1] In fact in several contries of the EU, i.e., Spain, Italy and France, investigations have been opened on cases relating price sequeezing strategies by the incumbents in the pricing of local and national services.

basis. One additional basic characteristic of networks, the very different volumes switched in peak and off- peak hours, yields to the deployment of peak -adjusted capacity on the local networks that effectively is little used. One regulatory regime, one would desire, should try to give incentives for a better management of traffic between peak and off- peak hours. The *IxT* regime applied in voice fixed telephony while it defines one price for interconnection in peak hours greater than the regulated rate for off- peak hours, has not achieved in fact a better pattern of usage of the local networks across the day. [**Nota:** esto tienes razon hay que cambiarlo un poco y decir que aunque hay precios por minuto diferentes en peak y off-peak, el nuevoregimen de IxC de verdad trae consigo incentivos mucho mas claros para rellenar las off peak hours].

Interconnection price is based on the time of usage of the renting facility. However, tariff structures that regulators have in fact set do show different shapes. In the European Union for example, while Spain, France, Greece, Ireland, Italy, Germany, UK and Austria regulate the interconnection price only based on a per minute charge, Portugal and Holland decided to set a fixed per call interconnection fee plus a per minute charge. Norway, Sweden, Denmark and Belgium opted for the setting of three different charges for the wholesale local network services: a fixed charge (lump sum) fee, a fixed charge for each interconnection demanded and a per time price, always higher for peak hours than for off- peak times. In the next table we summarize the main interconnection regulated price structure of a selected group of countries in the EU and give the average wholesale price for a 3 minutes call in each country.

Table 1: Interconnection (i.e., termination rates) regulated tariff structures (peak periods) the EU (15 countries) in cents/ Euro for 2001.

	EU (15)	*EU (8)*	*Var. chrg. (8)*
Local	*0.87*	*0.91*	
Simple transit	*1.26*	*1.22*	*0.93*
Double transit	*1.85*	*1.77*	*1.28*

Sources: European Commission (2001) and Comisión del Mercado de las Telecomunicaciones (2002). – see file: int-1 where *EU(15)* represents the average interconnection charge in peak hour of a 3minutes call across 15 EU countries. Note that from the 15 countries considered, 8 price with two-part tariffs and 4 countries apply non-linear payments for interconnection.
EU(8) represents the average interconnection rate of a 3 minutes call applied in countries where termination is priced with a two-part tariff:
Var. chrg. (8) represents the marginal price per minute of interconnection charged in countries with a two-part tariff structure.

Since 1997, to choose a common date of departure, all countries in the EU have liberalized the data and voice fixed telecommunications industry. Entrants have to rent the local loop in almost all cases from the incumbent operators and while the former are free to set final prices the latter is usually restricted in the final service market by some form of price regulation ex ante. The strictness of the regulatory mechanism in the final service market depends on the years already passed since the opening of the market and the competitive conditions perceived by the regulator. After some years in the liberalization process in general average prices of voice telephony have been reduced very significantly but the volume of traffic has not augmented in the expected way. All operators assumed the volume of voice traffic in fixed networks was to increase by much more than the real figures show, more so after the drastic reduction in average (and marginal) prices that have occurred. Voice traffic seems to be a service that is not significantly sensitive to price changes. This has important implications for fixed telecommunications competition: entrants will achieve market share at the expense of historical operators. While the incumbents react to sequential price reductions offered by the entrants there is a limit to price competition that emerges: the level of (regulated) interconnection charges, usually set on a per time basis, below which no entrant can price its final service. Under the capacity model, by contrast, the lower bound of prices could be lower because long run wholesale costs shall be lower.

Several European regulators are considering the introduction of a new interconnection regime, one based on payments per units of *transmission capacity* rather than per minute regulated prices[2]. The switch of regime makes sense from a network perspective since the main bulk of total network costs are of fixed (or even sunk) nature. The new interconnection regime is based on the idea that entrants should receive a similar cost structure to the one really facing the incumbent so that they are able as well to exploit any economies of scale and of scope across services. It may well be that any entrant when buying a specific amount of capacity blocks can use them freely for voice as well as for IP traffic. Since the capacity block is defined on a per month basis, the entrants are able as well to optimally manage traffic among the different hours of the day (and month). Actually in Spain we can already observe the introduction of non linear tariffs for local and long distance (national) calls, significant bonus discounts by entrants, and

[2] The Spanish regulatory authority, Comision del Mercado de las Telecomunicaciones (CMT), has introduced such a capacity based interconnection regime (*IxC*) in August 2001 which is changing significantively the way operators charge for final services and manage the local network capacity they rent from the incumbent.

even in some cases a one “free- calls- day” to be chosen among the days of the week, given that the customer pre-selects the complete set of services offered by the entrant.

The capacity based regime of interconnection (*IxC*) is already known in other markets as in the traffic involving Internet backbones and some broadband wholesale markets[3]. There are important differences in each traffic type, though, because of the payments mechanisms at work in each case. We want to focus on the effects on competition, tariff structures and incentives for network dimensioning by the entrants due to the introduction of the capacity interconnection regime in fixed (one-way) interconnection telephony in the context of a simple oligopoly. This interconnection regime as applied to voice telephony in fixed circuit switched networks is rather novel and shall probably become more extended as fixed networks become more commodity carriers than value added service providers.

2. The new regulatory model

This model can be seen as the extreme of the different tariff schedules offered by regulators across the EU nowadays. As we can see from Table 1, some regulators offer non- linear input (wholesale) prices. In fact we model the situation where the regulator allows the entrants to make use of any of the two interconnection models: the one based on per minute payments for the wholesale service (*IxT*) and the new regime based on capacity contracting (*IxC*). With the new model the entrant’s cost structure changes importantly because where previous wholesale costs were mainly variable in nature, with the *IxC* model, by contrast, they become fixed (ex ante) costs. Now, the newcomers can exploit certain degree of economies of scale, not only along each block contracted but also, due to Erlangs formula for network dimensioning, across additional capacity blocks. Furthermore, they can switch traffic from peak to off –peak hours and achieve additional economies across the hours of the day. Additionally, as we shall show, the *IxC* regime allows the entrants to implement non-linear tariff structures and to bring about higher efficiencies at the retail level. The introduction of the capacity based regime (*IxC*) for interconnection has far reaching implications for consumers as well as for fixed operators.[**Nota:** no creo haga falta meter en el modelo las formulas de Erlang, creo complicaria las cosas- que bastante complicadas estan ya. Yo creo que esta explicacion de Erlang tables esta Ok porque da una idea de que

[3] In a number of EU countries flat rate Internet access call origination (FRIACO) at residenctial level already exists, as in France, Spain, Germany, Netherlands and UK.

estamos modelizandso algo tecnologico de un modo sensato. Quizas en este parrafo y en los que siguen, nos enrrollamos demasiado largo en explicar estas economias de escala, tu cortalo por donde quieras, aunque a mi me parece Ok. Es cierto que con el IxT tambien hay economias de alcance, y quizas deberiamos separar mejor ambas, pero lo que es seguro es que con el IxC las economias de escala, servicio a servicio, son importanes]

The paper is structured as follows: in section 2 the basic model is exposed, together with the specifities of the IxC model. We introduce also the possibility of the entrant to manage its network dimension capabilities as a "spark plug" of a more aggressive price competition. In section 3, a simple streamlined model is suggested to represent the *IxC* under different tariff schemes and the relative performance of each regime of interconnection is compared. Dynamic considerations are presented afterwards. Finally, section 4 contains the main conclusions.

3. The model

We depart from a situation where only one incumbent firm and one entrant compete for final consumers. The market is assumed to be covered in all periods, by one of the firm or by both, hence, operators compete for the whole market (e.g., a city). The incumbent operator owns the local infrastructure that is assumed in the short run to be a bottleneck facility, i.e., a natural monopoly. The regulator sets the interconnection price (either measured in units of time or units of capacity) ex ante. The fixed operators know in advance the characteristics of the regulated environment (regime of interconnection and relevant prices) and compete in the final market thereon. We further assume that the incumbent has enough local network capacity to satisfy the total demand for traffic. The capacity contracted can only be used in the present period, *t*. Both firms maximize profits in each period knowing the structure of the game. The dynamic considerations presented later are simple repetitions of the game played in each period that yield different parameter values for subsequent periods. Each firm, then, looks for a non cooperative equilibrium in each t.

Consumer demand

We build initially on a version of the model proposed by Laffont, Rey and Tirole (98a, b) and Bijl and Peitz (00). The market has a fixed size of *n* consumers all of them subscribing to one of the operators[4]. Each consumer chooses first to what operator he wants to subscribe and then chooses the amount of traffic, i.e., calls, he desires to make, given the relevant prices[5]. Demand for the service shall be inelastic given that the participation constraint is satisfied but demand for traffic, i.e., calls, is elastic with respect the price per minute that the operator charges. All consumers are homogeneous in that they bear the same utility function and demand parameters and operators are assumed not to segment the market. However, the consumers face a switching cost (*SC*) related to the disutility of changing operator that is assumed to be distributed uniformly across the measure of consumers assigned to the incumbent operator in each period. This introduces a way of differentiating the final service, i.e., a call, because even though the final service offered by any operator is a perfect substitute, conditional on belonging to a operator i, the total utility a consumer derives does not need to be the same irrespective of the operator he belongs to.

We incoporate the possibility of having non- linear prices. In particular, for operator *i*, let the (fixed) line rental fee be denoted by m_i and the retail per-minute-price[6] by p_i. The individual demand for traffic at p_i is $x(p_i)$ and the consumer derives direct utility[7], $u(x(p_i))$, which is expressed in monetary units and satisfies $u'(x) > 0$ and $u''(x) < 0 \quad \forall \in [0, \tilde{x}]$. Where $\tilde{x} = \arg\max_x \{u\}$. As usual, the individual demand arises from $u'(x) = p$.

It shall be useful to define the consumer indirect utility function as $v(p) \equiv \{u_0 + u(x) - xp - m\}$. Following Laffont, Rey, and Tirole (1998a, b), this implies that the consumers do not restrain to call when the prices increase but simply call less or make shorter calls. Income effects are rule out based on the assumption that total tariffs paid by end consumers do not bear a considerable amount of its monthly budget. As arguments of the indirect utility function we have added the subscription fee, *m*, and u_i^0. Let u_i^0 be an exogenous component of the utility corresponding with certain value intrinsically related to subscribing to an operator and not to the rival. It may represent loyalty to the operator, better quality, confidence, etc. Note that whereas the incumbent already has built this reputation or loyalty element, the newcomers, by contrast, have to build it

[4] Service demand is price inelastic whenever the participation restriction is not violated.
[5] The amount of traffic demanded is assumed to be of only one type of service, i.e., local or national long distance.
[6] We do not explicitly introduce other final price elements (e.g. an initial charge per call). We can think of this fixed per call charge as uniformly distributed in which case it would be irrelevant the relation of units of measurement with the call duration. If these elements were to be considered as separate the model would increase in complexity, i.e., in principle, it would be necessary to introduce a probability distribution for the duration of the calls.
[7] Continuous, quasiconcave and strictly monotone.

up through time with the market share they gain. We introduce this fixed utility component in order to avoid the possibility of entrants gaining market share "too rapidly".

3. Market share competition

In order to simplify the notation subscript 1 will be related to the incumbent and 2, to the entrant. Superscripts will be referred to time. At the beginning of the game, all the operators have a market share. Unless otherwise stated the game starts where $\varphi_1^0 = 1$, and $\varphi_2^0 = 0$.

Define σ as the inverse of the degree of substitutability among the services offered by the two operators. At the beginning of period *t* any consumer subscribed to the incumbent operator, 1, has a switching cost, *SC,* uniformly distributed in $\left[0, \sigma\varphi_i^{t-1}\right]$. This consumer is indifferent between staying with the incumbent or subscribing to the entrant, 2, if $v_2^t - \hat{SC} = v_1^t$,where $0 \leq \hat{SC} \leq \sigma\varphi_1^{t-1}$. This implies that all those consumers with $SC \in \left[0, v_2^t - v_1^t\right]$, leave the incumbent at the end of period *t-1*. Consequently, the percentage of consumers who stay with firm 1 is $1 + \left(\frac{v_1^t - v_2^t}{\sigma\varphi_1^{t-1}}\right)$. This in turn is multiplied by the market share already enjoyed in *t-1*, and yields the proportion of consumers that in *t* remain with the incumbent firm as

$$\varphi_1^t = \varphi_1^{t-1} + \frac{v_1^t - v_2^t}{\sigma}. \tag{1}$$

Equation (1) plays an important role in the dynamic considerations of the model because it is the link between periods (i.e., the state variable).

Similarly we have that all those consumers with a *sc ε[0, v_1-v_2]* switch to firm 2 at the end of period *t-1*, where the upper support of this uniform distribution is given by $\sigma\,\varphi^{-1}{}_1$. Hence, in *t* the entrant has a market share measured across consumers given by:

$$\varphi_1 = \varphi^{-1}{}_1 + (v_2 - v_1)/\sigma \qquad \text{(1b)}$$

2.3. The capacity interconnection regime

In order to understand the implications of the capacity based interconnection regime, *IxC*, a few technical clarifications are convenient for the aim of modelling. From the regulator's perspective, several measures are to be defined for the *IxC* regime to be implementable.

Assume every capacity block has a potential transmission of 2 Mbit/sec[8], each block containing a total of 30 circuits of 64 Kbit/sec transmission capacity each. The regulator has to set the minutes equivalent of this capacity before regulating the price of the block. To develop this equivalent the Erlang formula for the dimensioning of circuit- switched networks is used. First a quality level has to be defined, which in teletraffic engineering is defined as the probability of a call being blocked because the available channels are already in use. The regulator can set a grade of service, i.e., quality, which for example may be of *0.5%,* implying that one in 200 calls will be expected to be blocked[9]. Then the choice of a *0.5%* grade of service implies using Erlangs Tables that the maximum level of traffic in the average hour with the highest traffic intensity of the day is 63%[10]. This value is equivalent to having 19 circuits, out of the 30 available, each one conveying 60 minutes, which gives a total of 1.142 minutes[11]. Multiplying this amount by 25 working days per month and weighting the result by *11/12* to take care of lower traffic intensity during summer time, yields a total of *26,171* minutes of traffic in the (average) most congested hours of the day, across the month, which we call *r*. The regulator has next to decide what proportion of most congested daily hours has an average operator, which we call $\rho = r/K$, where *K* is the total average traffic each day (averaged across the month). By way of example, assume this parameter is set by the regulator at $\rho = 13\%$, which amounts to say that traffic in most congested hours represents *13%* of the total average daily traffic. Hence, the total (interconnection) traffic, in minutes, that an average operator receives for each 2 Mbite/sec block contracted is derived from $\rho^R = r/K$, where $r^R = 26.171$ min. and $\rho^R = 0.13$, which yields a total of *201,314* minutes/ month. We call this *K*, capacity per block[12]. Note that this figure crucially depends on the quality of service parameter and on the proportion of most congested hours' traffic assumed in a day for an average operator. The parameter values used in the calculation of *K* are picked from a real

[8] For example in Spain, the recently introduced capacity regime states that the standard block is of 2Mbit/sec but in reality the capacity can be contracted in smaller units, particularly in circuits of 64 Kbit/s each. Since this does not modify the essence of our model we maintain the 2 Mbit/s unit as the smallest contractable block of capacity.

[9] We are also implicitly assuming that all calls have the same duration. In the context of our model this is not crucial since x, traffic demanded, can be understood as total minutes of conversation performed by the individual consumer, or in our case, as well as total number of calls times the duration of each, assuming each call has the same duration.

[10] According to the Erlang formula, which relates the number of circuits to be offered for handling a specific traffic intensity with a predetermined grade of service parameter.

[11] To measure traffic intensity, which is accounted for in Erlangs, we need a predetermined time period, T. We use here the convention of T= 60 minutes.

[12] i.e., total minutes of interconnection capacity that the regulator decides an average operator can handle in a month per block contracted (2 Mbit/sec).

case[13] but are not relevant to the conclusions. More importantly, note how K depends (inversely) on the proportion of "most congested hours" which is an estimate by the regulator. Any entrant can manage less (more) traffic in most congested hours with the result of being able to convey more (less) minutes within the same block contracted.

We assume next that the regulator sets the per minute interconnection charge, α, and the per block price, P, in such a way as to make both models initially price equivalent, i.e., $\alpha = P/K$. In the next table we show the regulated prices, α and P, chosen by the Spanish regulator from where it is apparent if we look at the third and the sixth columns that both prices have been set at similar levels.

Table 2: Price comparison between the IxT and the IxC interconnection regimes for voice traffic in Spain.

	Interconnection per time			*Interconnection by capacity**		
Type of traffic	p/p/h	p/op/h	pond/p**	K	(2Mbit/s)/P	e/p/p
	euro-cents / min			minutes	euro-cents	euro-cents / min
Local	0.757	0.457	**0.661**	201,314	132,611	**0.659**
Simple	1.16	0.697	**1.012**	201,314	202,758	**1.007**
Double	2.218	1.334	**1.935**	201,314	387,380	**1.924**

***Source*:** Own elaboration with CMT data (2001).
p/p/h: price per minute in peak hours; **p/op/h**: price per minute in off-peak hours; **pond/p**: weighted price- where we have assumed that 68% of the traffic occurs in peak periods and 32% in off- peak hours. **(2Mbit/s)/P**: price per capacity block; **e/p/p**: equivalent price per minute;
* Combined capacity blocks (voice + Internet).

Network dimension *(d_i)*

Define d_i as the maximum volume of minutes that operator i is *effectively* able to manage in a month within each capacity block, which we call *network dimension*. Clearly, d_i is specific to the firm and may be related to the extent of own infrastructure deployment or on its ability to transfer traffic from peak to low usage periods. Note that r/K is the regulator's estimate of traffic proportion that is conveyed by the operator during the most congested hours with respect to total daily traffic (averaged across the month). We assume the regulator sets this ratio as an estimate

[13] We follow mostly the Spanish regulatory authority in the use of the different parameter values. See CMT (2001).

in advance, at $\rho^{-} = r/K = 13\%$, but in fact the ratio is endogenous to the firm[14]. In the new *IxC* regime network dimension plays an important role. Total interconnection capacity contracted by an operator does not need to equal the traffic indeed attended to him because the firm may develope own network skills that enable him to effectively manage more (less) traffic intensity that what the regulator's estimate implies. The entrant now has an added responsibility in dimensioning its total network resources, own and rented resources, to efficiently use the interconnection capacity contracted. It is all very well possible that $\rho_i > \rho^R = r/K$, where ρ_i is the ratio of most congested traffic hours as a total of overall traffic specific to firm i. Lets frame network dimension as a function of r^R and the parameter specific to the firm ρ_i, so that it can be expressed as

$$d_i(r^R/\rho_i)$$

where ρ_i is the proportion of traffic in most congested hours with respect total traffic (averaged for monthly figures) for operator i and r^R is the regulated target. We have additionally that $\partial d(.)/\partial \rho_i < 0$, i.e., an inverse relationship between network dimension and the proportion of most congested traffic for operator i. The ability of any firm to for example increase off-peak traffic will result in a higher ability to convey additional minutes of traffic for a given capacity block contracted. The operator has then an incentive to diminish his ρ_i[15] in order to increase his network dimension, d_i. Given that the block price is fixed *ex ante* at P, the way to achieve a reduction in ρ_i is by competing more aggressively in market shares, ignoring for the moment the strategic effect that this may induce. Then, $\rho_i < (>) .13 \rightarrow d_i > (<) \theta K$. Those operators that are not able to modify the pattern of traffic with respect the standards set by the regulator will not find the *IxC* model profitable.

Additionally, an important source of network dimensioning comes from Erlangs' Tables. As we mentioned before, the relationship existing in teletraffic engineering between the number of circuits needed to convey a specific amount of traffic intensity, given a quality parameter, is *concave*[16]. This implies that the number of minutes conveyed in one unit of 2 Mbit/sec block is *201.314* minutes. But, if two blocks are contracted, for the same quality level, the total amount of minutes conveyed is strictly higher than 2*(201314) minutes. As additional blocks of capacity

---

[14] The best (daily) traffic pattern for any operator would be a perfectly distributed one which would yield $\rho = \frac{1}{24} \approx 4.17\%$.

[15] Assume for example an increase in off-peak traffic.

are contracted a higher than proportional total of traffic (in minutes) will be transported over the local station where interconnection capacity has been contracted, hence achieving $d_i > \theta K$, where θ is the number of capacity blocks contracted. This is an important source of economies of scale because as the minutes-based traffic increases the per block price of capacity is set constant (per unit) by the regulator at price *P*. Hence, the entrant has incentives to increase traffic demanded to be able to exploit this source of decreasing average costs[17].

We capture these sources scale and scope economies in a simple way by making the effective maximum volume of traffic managed by firm i, d_i, dependent upon the market share it has in each period as,

$$d_2 = Z + Yn\phi_2, \qquad (2)$$

In equation (2) network dimension depends on the market share of the firm in the beginning of the period on proportion of the parameter *Y* (*>0*) and number of subscribers, *n*. The term Z is the management capacity when ϕ_2=0, e.g., at the beginning of the competition. This term captures specific characteristics of the operator that allow him to manage *Z* minutes and assumed to satisfy $r<Z<K$. The contribution of each consumer in the better block management is *Y*. It is important to note that when buying a capacity block the operator does not buy a fixed amount of minutes but a *transmission possibility K* set by the regulator which is simply an estimate based on a quality of service and a proportion of peak hours standards and calculated so as to make both interconnection regimes initially price equivalent. Since $\partial d_i/\partial \varphi_i > 0$, we have a source for scale economies that shall drive competition for subscribers over time in our context and will result on market share rivalry among the operators[18].

Note that d_i is affected by market share which in turn is determined by the operator's own pricing strategy. Consequently, network dimension is internalized. The entrant has an incentive to

---

[16] Variants of Erlangs formula are needed when there is traffic overflow in a particular route, or local station, but the concave relationship between number of circuits needed and traffic intensity remains.

[17] With the IxC model the regulator can achieve also a better traffic pattern of different services offered with the same unit of capacity, as for example the management of IP and voice traffic at the same time. The Spanish regulatory authority has introduced as well the possibility of a secondary market for interconnection capacity among operators that may have contracted too much capacity in the short term.

[18] An additional reason for specifying the relationship in equation (2) is that as any operator gains market share he is in a better position to extend his package of offered services to the final consumer (local telephony, long distance, Internet and data).

diminish $r/d_2 = 26{,}171/(Z + Yn\phi_2)$ where the regulator has fixed the numerator. Therefore, the entrant has clear incentives to increse d_2 as much as possible[19]. Given a set of prices, the entrant best strategy will be to maximize the use of his capacity. When referring to network dimension we mean the maximum possible level of d_i given a set of prices. It may be argued that the entrant has also to face the risk of contracting capacity that later is not effectively used[20]. In any case, the internalization of the network dimension is considered an added incentive to compete more aggressively for subscribers. This result may be highly considered by any regulator implementing the *IxC* regime.

3. Competition

We model competition that takes place between an incumbent (firm 1) and an entrant operator (firm 2). It is assumed that the regulator has defined the interconnection regime in the first instance, i.e., whether there is a time based interconnection (*IxT*) and its relevant price, α, or if there holds the capacity based regime (*IxC*) and the per block price, *P*, or even if the entrant can make use of any of the two regimes at the same time. Additionally we assume that the regulator sets both prices proportionally equivalent, i.e., $\alpha = \frac{P}{K}$, so as not to make the *IxC* regime more attractive from the start.

After the regulator has set the relevant prices, α and *P*, the entrant decides what regime to take advantage of and chooses the optimal combination of interconnection contracts. If he chooses the *IxC* regime then he shall demand as many blocks of capacity as the estimated demand traffic, which the entrant can forecast with precision. Afterwards, the firms compete at retail level in prices. We introduce the possibility of competing in two part tariffs: a fixed part , m_i, which represents the line rental and works as the instrument to attract new customers to any operator, and the per minute price of any call, p_i, which determines the volume of traffic demanded. Once all the choice variables are determined as a non cooperative equilibrium we calculate the values of the relevant variables (market shares, total demand). There are two subgames then: the first one captures the regime choice and the capacity demanded and the second represents the final market competition. The solution concept then is the subgame perfect equilibrium for every period t.

---

[19] Note that the operator could have interest in buying *IxC* even if the traffic does not reach *r* in any hour whenever $d_2>K$.

[20] We do not consider the risk sharing effect of the IxC regime in this formal model.

Note that the entrant may choose either regime ex ante but it is possible that in fact uses both regimes ex post. We allow here the regulator to impose that any traffic in excess of that previously contracted by the entrant has to be conducted based on the per time regulated price. Hence, if the entrant receives more traffic that initially forecast the excess traffic will be handled with the per unit of call input price, α. All operators, hence, will satisfy their effective demand in every period..

3. Analysis of the *IxC* under the application of different tariff schemes. Comparison with the *IxT*.

With interconnection prices regulated on a per minute basis, *IxT* regime, the profits functions are similar to that already analyzed in Bijl and Peitz (00). The entrant, it is assumed, has no local infrastructure, but only some long distance traffic conveyance facility of his own so that he needs (one-way) interconnection with the incumbent (firm 1)[21].

Define ω_i as the amount of total call minutes demanded from operator i , $\omega_i=n\varphi_i x_i$, and μ_j as its share of customers , $\mu_i= n\varphi_i$. Denote c_0, c and f_1 as the marginal costs of conveyance of an interconection minute, a complete call and the specific to each final customer cost component, respectively. The incumbent's profit function now can be expressed as,

$$\pi_1 = \omega_1 (p_1 - c_a) + \omega_2 (\alpha - c_b) + \mu_1 (m_1 - f_1).$$

Where the incumbent obtains profits from three sources: the sale of final services , the income generated by the subscription fee over its customers and the income generated by the sale of wholesale services to the entrant. We make a particular simplification by setting the on-net, c_a,, and the off-net, c_b, marginal costs equal to zero[22]

$$\pi_1 = \omega_1 p_1 + \omega_2 \alpha + \mu_1 (m_1 - f_1) - F_1 . \qquad (3)$$

[21] In Bijl and Peitz (00) the authors analyze the case of competition in two part tariffs only for the case of two symmetric operators, each having its own infrastructure (i.e., two way access).

[22] That $\alpha > c_b = 0$ occurs, could be imposed by the regulator to allow the recovery of some previous investment related to the network operation (e.g. access deficit related to the universal service provision). Anyway, the consequences of $\alpha = 0$ are reviewed later.

Where we have added a fixed (network) costs component, F_1, which is related to previous investments that will be recouped and considered constant throughout time. The entrant's profit function is composed of income generated by calls demanded, net of interconnection (variable) costs and the revenues due to subscribers' line rentals, net of the susbcriber's management costs,

$$\pi_2 = \omega_2 (p_2 - \alpha) + \mu_2 (m_2 - f_2), \tag{4}$$

3. The profit functions under the *IxC* model

Under the *IxC* model the incumbent sells θ blocks of capacity at a regulated price, P, each. We allow the possibility here that the entrants' final demand exceeds his contracted capacity and the resulting interconnection traffic has to be purchased at the per minute regulated price, α, thereafter. Hence, the profits for the incumbent now are, (where i=θ, number of blocks, in the following expressions)

$$\pi_1 = \omega_1 p_1 + iP + \mu_1 (m_1 - f_1) + (\omega_2 - id_2)\alpha - F_1, \tag{5}$$

The main difference of this profit function with respect the previous one lies in the fact that the income derived from the sale of call minutes of interconnection, $\omega_2\alpha$, was a direct function of the final price. In equation (5) we observe, by contrast, that wholesale services' income is derived from the selling of capacity blocks which itself is an indirect function of (present) prices[23]. Here arises an important difference among both interconnection regimes, in that in the *IxT* model the variable cost of interconnection to the entrant was relevant in the determination of final prices, and hence, competition is somehow relaxed. Under the *IxC* regime, by contrast, the costs of interconnection capacity θP, does not influence the final prices and hence, the *IxC* eliminates one important strategic component. The profits of the entrant are now for θ=1,2,....

$$\pi_2 = \begin{cases} \omega_2 p_2 - iP + \mu_2 (m_2 - f_2) & if \quad \omega_2 \le id_2 \\ id_2 p_2 + (\omega_2 - id_2)(p_2 - \alpha) - iP + \mu_2 (m_2 - f_2) & if \quad \omega_2 > id_2 \end{cases}, \tag{6}$$

where we have introduced the term related to network dimension by the entrant d_2, only active when $\omega_2 > \theta d_2$.

[23] though in the case of excess demand of traffic by the entrant, the incumbent also obtains $(\omega_2 - id_2)\alpha$ as revenues.

The entrant maximizes (6)choosing $\{\theta, p_2, m_2\}$. Note that the amount demanded and the amount contracted are two different variables. Note too, that the entrant does not confront any risk due to receiving excess of demand. If the amount demanded to the entrant is smaller that his specific network dimension, $\omega_2 \leq \theta\, d_2(\varphi_2)$ he can meet total demand with θ blocks. On the contrary, when $\theta d_2(\varphi_2) < \omega_2$, the entrant needs to contract additional interconnection services to satisfy his total demand[24].

3. Tariff schemes

Each operator maximizes profits in a non-cooperative way in every period. Next, we analyze the equilibrium results for three different tariff schemes: linear tariff, plain tariff, and two-part tariff (subscription fee and per-unit price). In the case of linear tariff we have, $m_1=m_2=0$ and in the case of a flat tariff, $p_1=p_2=0$[25]. In order to obtain the equilibrium prices we have used a quadratic functional form for the individual utility, as $u(x)= Ax - \frac{Bx^2}{2\gamma}$ that satisfies concavity for $0 \leq x \leq \frac{\gamma A}{B}$. From the utility function we derive individual demand for traffic as, $x = \frac{\gamma(A-p)}{B}$, where $A, B > 0$, are demand parameters and γ is an exogenous demand shock[26].

4. Linear tariff (per-minute price) only

We set $m_i=0$ for $i= 1,2$, and use the definitions of total (traffic) demand facing firm i as $\omega_i= n\, \varphi^t_i\, x_i$, and total number of subscribers as $\mu_i= n\, \varphi^t_i$, to express the profit function of each firm where φ^t_i was given in subsection 2.2. We take the FOC's from (3) with respect the variable price which yields[27],

$$\varphi^t_1\, x_1 + (\partial x_1/\partial p_1)\, \varphi^t_1\, p_1 + x_1\, p_1\, (\partial \varphi^t_1/\partial p_1) + \alpha\, x_2\, (\partial \varphi^t_2/\partial p_1) = 0$$

---

24 It is very well possible that even though across all hours of any day the traffic is smaller than r, the entrant may find profitable to contract capacity blocks.

25 A flat tariff is a unique payment per period independent of the amount demanded by the consumer. Note however that $p_2=0$, does not imply that π_2 is independent of d_2.

26 In the simulations some parameters have been arbitrarily fixed so that that an operator with $\varphi > .08$ (and a sufficiently high volume of traffic demanded) obtains marginal profits when contracting by capacity, i.e., $d_i\big|_{\varphi_i=.08} = K$.

27 At the cost of losing generality we set $f_1=f_2=0$ and $\gamma=1$.

Using the fact that, $\partial x_i/\partial p_i=-1/B$, that $\partial v_i/\partial p_i= - x_i$ by Roy's identity , $\partial \varphi^t_i/\partial p_i= (\partial v_i/\partial p_i) (1/\sigma)$ and $\partial \varphi^t_2/\partial p_1= x_1/\sigma$, from expressions (1) and (1b) and after simplifying we obtain the equilibrium prices under the *IxT* regime as [28]

$$p_1^* = \frac{Bx_1(\sigma\varphi_1 + \alpha x_2)}{\sigma\varphi_1 + Bx_1^2}, \tag{7a}$$

$$p_2^* = \alpha + \frac{Bx_2\sigma\varphi_2}{\sigma\varphi_2 + Bx_2^2}, \tag{7b}$$

where $x_1=x_1^*(p_1^*)$ and $x_2=x^*_2(p_2^*)$ and $\varphi_i= \varphi^{*t}_i$ for $\forall i$.

The equilibrium prices for the case when $\omega_2\leq \theta d_2$ and the firm contracts under the *IxC* model are,

$$p_1^* = \frac{Bx_1\sigma\varphi_1}{\sigma\varphi_1 + Bx_1^2}, \tag{8a}$$

$$p_2^* = \frac{Bx_2\sigma\varphi_2}{\sigma\varphi_2 + Bx_2^2}, \qquad \text{given} \quad \omega_2\leq\theta d_2 \tag{8b}$$

And

$$p^*_2= \alpha+ [Bx_2 (\sigma\, \varphi^t_2- \alpha\theta Y)]/[\sigma\varphi^t_2 + Bx^2_2], \quad \text{for } p_2> 0 \text{ and } \omega_2> \theta d_2$$

If the entrant's demand is higher than his network dimension, $w_2> \theta d_2$, the incumbent receives extra revenues either by the sale of an additional unit of capacity, in which case its final variable equilibrium price, $p_1^*(\theta/ \omega_2\leq \theta d_2)$ is not affected, orelse receives extra revenues due to the additional interconnection time contracted by the entrant on the excess demand traffic, $\alpha(\omega_2-\theta d_2)$. In this last case the incumbent best strategy is affected by the variable cost it imposes on the entrant with the per minute price of interconection, α. Even though the exceess demand traffic contracted is small in magnitude it results in higher equilibrium prices compared to the case when $\omega_2\leq \theta d_2$. The incument's variable traffic price becomes in this case,

[28] the the right hand side of the following expressions have been evaluated at equilibrium, though the use of * to denote this has been omitted.

$$p_1*(\theta/\ \omega_2 > \theta d_2) = p_1*(\theta/\ \omega_2 \leq \theta d_2) + [\alpha\ (x_2 - \theta Y)]/(\sigma\varphi^t_1 + B\ x_1)$$

which is $> p_1*(\theta/\ \omega_2 \leq \theta d_2)$. In fact comparing all three equilibrium (variable) prices for the incument we have that $p_1*(IxT) > p_1*(\theta/\ \omega_2 > \theta d_2) > p_1*(\theta/\ \omega_2 \leq \theta d_2)$ as long as $x_2' < x_2$, where x_2' is the excess demand interconnection contracted at price α.

The equilibrium profits for the entrant under the *IxT* regime are

$$\pi_2^* = \frac{n\sigma B\varphi_2^2 x_2^2}{\sigma\varphi_2 + Bx_2^2}, \tag{9}$$

The profit expression for the incumbent when $p_i>0$, y IxT, que son:

$$\Pi_1(IxT) = [B\ n\ x^2_1\ \varphi^t_1\ (\sigma\,\varphi^t_1 + \alpha x_2)]/(\sigma\,\varphi^t_1 + Bx^2_1) + \alpha(n\ \varphi^t_2\ x_2)\ .]$$

Using the price equivalence between IxT and IxT, i.e., $\alpha = P/K$, the equilibrium profits under the *IxC* model become [29]

$$\pi_2^* = \frac{n\sigma B\varphi_2^2 x_2^2}{\sigma\varphi_2 + Bx_2^2} - i\alpha K\ , \qquad \text{given} \qquad \omega_2 \leq \theta d_2 \tag{10a}$$

For the entrant, his profits when $\omega_2 > \theta d_2$, are

$\Pi_2*(\theta/\ \omega_2 > \theta d_2) = [n\sigma\ \varphi^{t2}_2\ (Bx^2_2 + \alpha\theta Y)]/(\sigma\ \varphi^t_2 + Bx^2_2) + \alpha\theta\ (Z - K)$, where $Z < K$,

Under the IxC regime the cuasi-reduced form profits expression when $p_i>0$ y $\omega_2 \leq \theta d_2$, are:

$\Pi_1(\theta/\ \omega_2 \leq \theta d_2) = (Bn\ \sigma\ \varphi^t_1\ x^2_1)/(\sigma\ \varphi^t_1 + B\ x^2_1) + \alpha\theta K$ when $\omega_2 \leq \theta d_2$

and when, $\omega_2 > \theta d_2$, we have that

$\Pi_1(\theta/\omega_2 > \theta d_2) = [B\ n\ \varphi^t_1\ x^2_1\ (\sigma\ \varphi^t_1 + \alpha\ (x_2 - \theta Y)]/(\sigma\ \varphi^t_1 + Bx^2_1) + \alpha\ n\ \varphi^t_2\ (x_2 - \theta Y) + \alpha\theta\ (K - Z)$

where $(x_2 - \theta Y)$ is the excess demand of interconnection traffic that the entrant contracts with the IxT regime.

The attractiveness of the *IxC* regime for the entrant lies in the possibility it offers him to exploit any economies of scale that market share enhances. Otherwise the entrant rather chooses the *IxT* regime, as does the incumbent.

Proposition 1: if the entrant cannot exploit economies of scale or scope ,i.e., $d_2=\theta K$, and both firms compete in variable prices (per minute) only ($p_i>0, m_i=0$), then both firms prefer the *IxT* regime rather than the *IxC* regime. Under *IxT* the equilibrium price is always higher and so are their profit levels.

It can be directly seen from the price and profit equilibrium expressions in each relevant case. *IxT* payoffs always dominate *IxC* payoffs except for the case when the entrant contracts θ capacity blocks and $\omega_2>\theta K$, in which case $\Pi_2*(IxT) =\Pi_2*(IxC/ \omega_2>\theta K)$.

From direct inspection of the equilibrium prices it is clear that, $p_i*(IxT) > p_i*(\theta/\omega_2> \theta d_2)> p_i*(\theta/ \omega_2\leq \theta d_2)$, for i= 1,2. In terms of profits, for the entrant between both interconnection regimes with linear prices we have that, $\Pi_2*(IxT)> \Pi_2*(\theta/\omega_2> \theta d_2) > \Pi_2*(\theta/ \omega_2\leq \theta d_2)$.

Under the time regime total traffic demanded is lower than under the capacity regime, prices are higher in the former and so are profits. The interconnection per minute price helps increasing the equilibrium variable price for both firms.
The incumbent also prefers always the *IxT* regime because with the regulated per minute interconnection price there is a strategic effect on the entrant best reply function, $(\partial\varphi^I_2/\partial v_1)\ (\partial v_1/\partial p_1) = (x_1/\sigma)> 0$ by which it gains always in higher profits. Given expressions (7a) and (8a) it is easy to check that $p_1*(IxT)> p_1*(IxC)$. Note that under the *IxC* and $\omega_2\leq \theta K$, the interconnection payment does not enter the best replies functions of any of the two rivals. Under *IxT* the entrant never chooses competition in flat rates, hence the pricing schedule of the entrant under *IxT* becomes restricted.

Proposition 3: if the entrant can exploit economies of scale across his market share, as long as he has enough size in the market, he prefers the IxC regime rather than the IxT model.

By comparing the profit expressions for the entrant in each relevant case it is clear that a big enough difference in market shares between both regimes, may yield higher profits to the entrant under the IxC.

3. Plain tariff (subscription fee) only

In the profit function we set now $p_i = 0$ and replace the subscription fee, m_i, by the difference between the gross and the net surpluses. The net surplus derived from being subscribed to operator i was defined as $v_i(p_i, m_i)$. Now denote the gross surplus as $k(p_i) = \{u^0 + u(x_i) - p_i x_i\}$ so that the line rental can be expressed as $\{k(p_i) - v_i(p_i, m_i)\}$. The firms when competing in flat tariffs are in fact competing for the surplus of the consumers. From the market share expressions note that $\partial\varphi^I_i/\partial v_i = \_\_\sigma$ and $\partial\varphi^I_k/\partial v_i = -1/\sigma$ for $i \neq k$. where σ is the (inverse) of the degree of substitution among the operators. After taking the FOC's with respect to v_i and simplifying the equilibrium line rentals in the *IxT* regime are obtained as,

$$m_1^* = \sigma\varphi_1 + \alpha x_2(0), \tag{11a}$$

$$m_2^* = \sigma\varphi_2 + \alpha x_2(0), \tag{11b}$$

Total traffic demands are evaluated at $x_i(0) = x_i^*(p_i = 0)$, which in our quadratic utilities case the maximum individual (traffic) demand become $x_i^* = A/B$.

The equilibrium flat tariffs depend directly on the market share attained by each firm.

In a similar way the equilibrium flat prices under the *IxC* regime are derived as,

$$m_1^* = \sigma\varphi_1 + \alpha x_2(0) - \frac{i\alpha Y}{n}, \tag{12a}$$

$$m_2^* = \sigma\varphi_2, \qquad \text{given} \quad \omega_2 \leq \theta d_2 \tag{12b}$$

$$m_2^* = \sigma\varphi_2 + \alpha x_2(0) - \frac{i\alpha Y}{n}, \quad \text{given} \quad \omega_2 > \theta d_2 \tag{12c}$$

Under the *IxC* regime the prices are inversely related to the network dimension parameter of the entrant, i.e., the hgher his ability to manage additional minutes in each block contracted, the lower the resulting equilibrium flat rates. In the beginning of the game, since the entrant shall enjoy smaller market shares than the entrant and both compete for the subcribers the entrant's rate is lower than the incumbent's one. If the variable price is zero, under the *IxC* model the demand of traffic by each consumer is maximum demand, $x^* = A/B$.

Comparing the previous equations it is observed that for the same market share under both models prices in *IxC* are always smaller than the prices in the *IxT* regime.

Proposition 3: if the entrant cannot exploit his network dimension, $d_2=\theta K$, whether competition takes place in linear prices or in flat rates, the equilibrium prices under the *IxC* regime are always smaller than under the *IxT* regime. This holds whatever the price schedules the firms compete over (linear or non- linear) and is robust also to the case when the entrant has excess demand of traffic, $w_2> \theta d_2$, and contracts the additional wholesale service either with additional capacity blocks or at price α.

IxT. The equilibrium profits are

$$\pi_2^* = n\sigma\varphi_2^2, \tag{13}$$

For the incumbent we have, $\Pi_1(\text{IxT})=n\ \sigma\varphi^{t2}{}_1 + \alpha n\ x_2(\varphi^t{}_1 + \varphi^t{}_2)]$

IxC. The equilibrium profits are

$$\pi_2^* = n\sigma\varphi_2^2 - i\alpha K, \qquad \text{given} \quad \omega_2 \leq \theta d_2 \tag{14a}$$

$$\pi_2^* = n\sigma\varphi_2^2 + i\alpha(Y-K), \qquad \text{given} \quad \omega_2 > \theta d_2 \tag{14b}$$

Proposition 4: if the entrant can manage network dimension, as long as the market share gain under IxC is big enough, compared to the market share it gained under IxT, he rather chooses the IxC regime.

3. Two part tariffs

Now we allow each operator to use the two parameters that define the (possibly) non linear outlay, p_i and m_i, for $i=1, 2$. Both firms compete simultaneously in two part tariffs under the two interconnection regimes.

IxT. The equilibrium prices are

$$p_1^* = 0, \tag{15a}$$

$$p_2^* = \alpha, \tag{15b}$$

$$m_1^* = \sigma\varphi_1 + \alpha x_2(\alpha), \qquad (15c)$$

$$m_2^* = \sigma\varphi_2, \qquad (15d)$$

IxC. The equilibrium prices are

$$p_1^* = 0, \qquad (16a)$$

$$p_2^* = 0, \quad \text{given} \quad \omega_2 \leq \theta d_2 \qquad (16b)$$

$$p_2^* = \alpha, \quad \text{given} \quad \omega_2 > \theta d_2 \qquad (16c)$$

$$m_1^* = \sigma\varphi_1 + \alpha x_2(\alpha) - \frac{i\alpha Y}{n}, \qquad (16d)$$

$$m_2^* = \sigma\varphi_2, \quad \text{given} \quad \omega_2 \leq \theta d_2 \qquad (16e)$$

$$m_2^* = \sigma\varphi_2 - \frac{i\alpha Y}{n}, \quad \text{given} \quad \omega_2 > \theta d_2 \qquad (16f)$$

As one would expect the variable price of the incumbent is set at zero (the assumed level of marginal costs of traffic) and both firms set $m_i > 0$ to extract surplus from their subscribers. Note that firms compete for subscribers basically in this case. The entrant has a disadvantage: his perceived marginal cost of traffic is positive, the interconnection rate a, and for the case when $\omega_2 > \theta d_2$ he cannot price traffic below that level.

Note also that in the IxC regime when the entrant needs additional interconnection services, $\omega_2 > \theta d_2$, then $\partial m_i/\partial(Y/n) < 0$, the flat rate tariff of both firms depend inversely on the network ability parameter of the entrant.

Comparing the previous equations it is observed that for the same market share under both models, the prices in *IxC* are always smaller to the prices under the *IxT* regime.

IxT. The equilibrium profits are

$$\pi_2^* = n\sigma\varphi_2^2, \qquad (17)$$

IxC. The equilibrium profits are

$$\pi_2^* = n\sigma\varphi_2^2 - i\alpha K, \quad \text{given} \quad \omega_2 \leq \theta d_2 \qquad (18a)$$

$$\pi_2^* = n\sigma\varphi_2^2 + i\alpha(Y - K), \quad \text{given} \quad \omega_2 \leq \theta d_2 \qquad (18b)$$

When both firms compete in the two part tariffs the entrant may find it more difficult to switch to the IxC regime. Remember that the entrant preferred the IxC regime only when the associated increase in market share under IxC model was high enough. When going from the IxT to the IxC regime under non linear prices, the jump in market share by the entrant is not as high as when linear prices apply. Hence, from the side of increased revenues the entrant shall find the IxC somehow less attractive.

With two-part tariffs, the entrant chooses p_2 equal to his (perceived) marginal cost whereas, the incumbent, due to his cost structure, chooses p_1 equal to zero (flat fees). It is evident that the latter prefers to move to a flat fee scenario. Thus, the incumbent may achieve losses on the entrant because the latter would not recover the fixed costs of capacity contracting. Additionally the incumbent would be able to increase his flat fee. If the market share was the same in both tariffs structures, the entrant is indifferent between the application of flat tariffs and the application of a two-part tariffs. Nonetheless, as said before, the incumbent is would choose a flat tariff. It is clear also that, if the market share was the same in both tariffs, both firms prefer non-linear tariffs.

4. Dynamic considerations

Evidently, the use of different tariff schemes and interconnection models has implications on the welfare. We take into account the following welfare measures: consumer surplus (*CS*), producer surplus (*PS*), and welfare or social surplus (*W*).

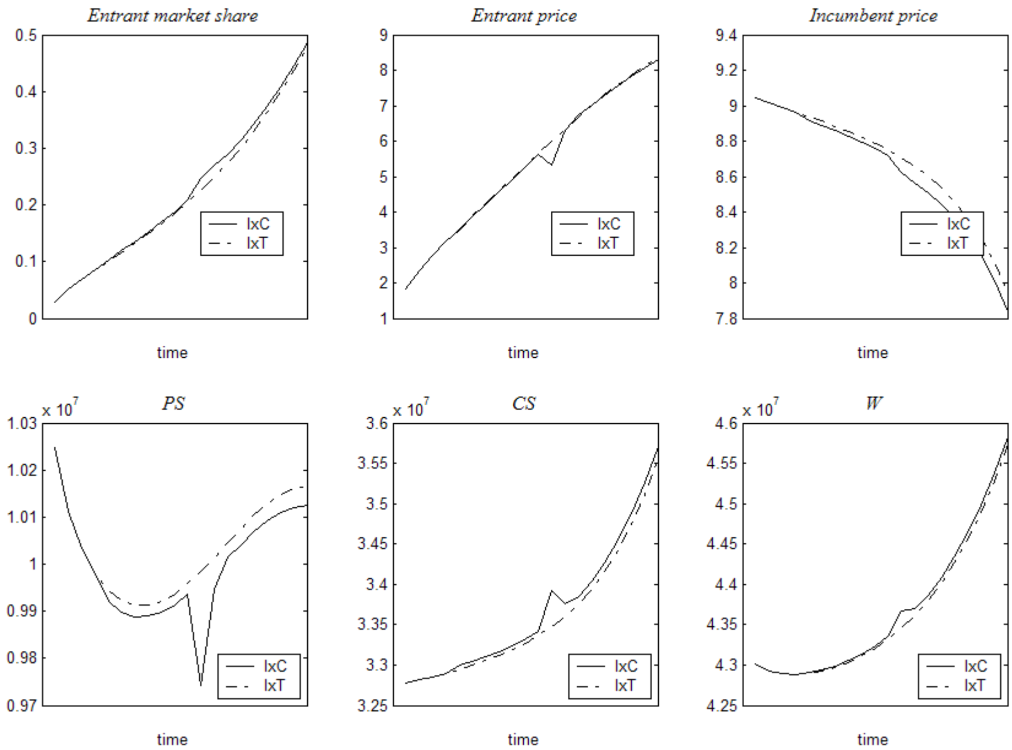
CS is the sum of the surplus of the clients of both firms (all the consumers) minus the added cost incurred by the consumers who changed their provider in the period, plus, in its case, the fine charged to the incumbent. Assume that this last amount is redistributed to the consumers by means of some lump-sum mechanism. The sum of the profits of all firms in the market is denoted by *PS*. Finally, *W* is the sum of *CS* and *PS*. All this is summarized in the following equations:

$$CS = n\left(v_1\phi_1 + v_2\phi_2 - \frac{(v_1 - v_2)^2}{2\sigma} \right) + \max\{0, \alpha\,(id_2 - \omega_2)\}, \tag{19a}$$

$$PS = \pi_1 + \pi_2\,, \tag{19b}$$

$$W = CS + PS\,. \tag{19c}$$

The efficiency measurement used to compare the models is the aggregate welfare throughout the periods, T=35[30]. Next, we show a set of graphs illustrating the main results.



Starting from the IxT model and going to the IxC regime we obtain (for linear tariffs) an aggregate decrease of .73% in *PS* but an increase of .53% *CS*. Finally an increase of .24% in *W* is obtained.

5. Conclusions

We have analyzed the new interconnection regime based on wholesale capacity payments in fixed telecommunications networks, a regime that applies to Internet traffic, some broadband traffic and now, some regukators face the prospect of applying it also to (narrowband) voice telephony. In Spain the regulator introduced this new interconnection regime on August 2001 and some effects can already be seen in the market. Some operators are offering close to flat-rates for long distance traffic.

We have modelled the capacity based interconnection regime, together with the possibility by the entrants to stick to the time based regime (interconnection wholesale payments based on a per minute basis). Additionally we allow for the possibility to use both regimes at the same tiem: the IxC and, if excess demand of traffic results at the end of each period, additional interconnection can be contracted on a regulated per minute price.

The capacity based model has one technical characteristics that makes it especially attractive. The relationship in teletraffic theory between traffic intensity (measured in Erlnags) and number of circuits needed in order to convey a specific amount of traffic, with a given quality parameter (probability of a call being blocked) is concave, i.e., there are economies of scale in the contracting of additional capacity blocks, measured in units of 2 Mgbites/sec, each. We allow for economies of scale (and possibly of scope) by the entrant via his *network dimension*, which measures his ability to manage more minutes than the ones originally included by the regulator in each capacity block.

Finally we allow for the final price to have a linear or non linear structure, i.e., we allow for competition to take place in linear prices, in flat (subscriber) rates or in two- part tariffs simultanmeously.

The results are derived from a repeated duopoly game and show that:

1.if the entrant cannot manage his network dimension, he prefers always the time based regime. It yields higher equilibrium prices and higher profits. The incumbent also prefers the time based regime.

2.if the entrangt can exploit economies of scale due to capacity contracting, the switch from the time to the capacity based regime if competition takes place in linear prices may be benefitial for the entrant only if he reaches a critical market share under the capacity based regime.

3.the new model transfers some risk to the entrant, and makes interconnection regulation less asymmetric.

3. if competition takes place in linear prices or in flat rates, the time based model dominates the capacity based regime only if the entrant cannot make use of economies of scale via its network dimension.

---

[30] With T=35 the changes in the equilibrium values from one period to the next are practically null.

5. the incumbent may prefer the capacity based model if he can induce losses on the entrant due to the higher fixed costs of capacity contracting and when the entran t does not reach a high enough market share as a result. This shall be easier under linear prices competition.

6. the time based interconnection allows both firms to increase final equilibrium prices at the cost of less traffic (but same number of clients)

7. With two- part tariffs, the entrant may find it difficult to choose the capacity based regime. The reason is that the gain in market share when going from the time based to the capacity based regime may not be large enough to ncompensate for the increment in fixed costs that he has to incurr.

The virtous circle, i.e., capacity contracting, higher volumes of traffic, more market share by entrants, lower prices, higher economies of scale, may not be achieved. It all will depend on the reaction of the incumbent to the capacity regime and structure of final prices implemented, and on the scope of economies of scale that the new interconnection capacity model implies.

Reference List

Little, I. and Wright,J. 2000 “Peering and settlement in the Internet: an economic analysis”, *Journal of Regulatory Economics*, 18:2, pp. 151- 173

Comision del Mercado de las Telecomunicaciones, 2001, *Resolución sobre la modificación de la oferta de interconexión de referencia de Telefónica de España, S.A.U. Resolución del 9 de agosto de 2001 en el expediente MTZ 2001/4036.* , Madrid, Spain.

Bijl, P. and Peitz, M. *Competition and Regulation in Telecommunications Markets.* CPB Netherlands Bureau for Economic Policy Analysis, The Hague, November 2000.

Aguilar , J. “Evaluación de los efectos dinámicos de la introducción del modelo de Interconexión por Capacidad”, *Economía Industrial*, 337. March 2002.

Laffont, J.J., P. Rey, and J. Tirole (1998a). “Network competition: I. Overview and nondiscriminatory pricing”, *RAND Journal of Economic*s, Vol. 29, No. 1, 1-37.

Laffont, J.J., P. Rey, and J. Tirole (1998b). “Network competition: II. Price discrimination”, *RAND Journal of Economic*s, Vol. 29, No. 1, 38-56.

Clark, M. P. 1991, *Networks and Telecommunications. Design and Operation*, John Willey and Sons Ltd., England.

Appendix

Appendix 1

Linear tariffs

In this appendix, we show (in separate graphs mode, for a particular parameters vector and a p_I) the strategic behavior intrinsic to the definition of the entrant benefit curves under *IxC*, for several different situations. The following cases are compared: a) when he only could manage K minutes (d_2=K), and, b) when he determines his network dimension according to $d_2=Y(1+\varphi_2)$.

We suppose that p_2◷ satisfy $\omega_2=\theta d_2$ when $d_2=K$, similarly, p_2◷◷ satisfy $\omega_2=\theta d_2$ when $d_2=Y(1+\varphi_2)$. Prices higher than p_2◷ and p_2◷◷, in each case, imply that the relevant profits function will be the one defined for ω_2✧id_2. Also, prices smaller than p_2◷ and p_2◷◷, imply that the relevant payoff function is the one defined for $\omega_2>id_2$. The price that finally maximizes the entrant's profits (and therefore the one that he chooses) will be p_2*.

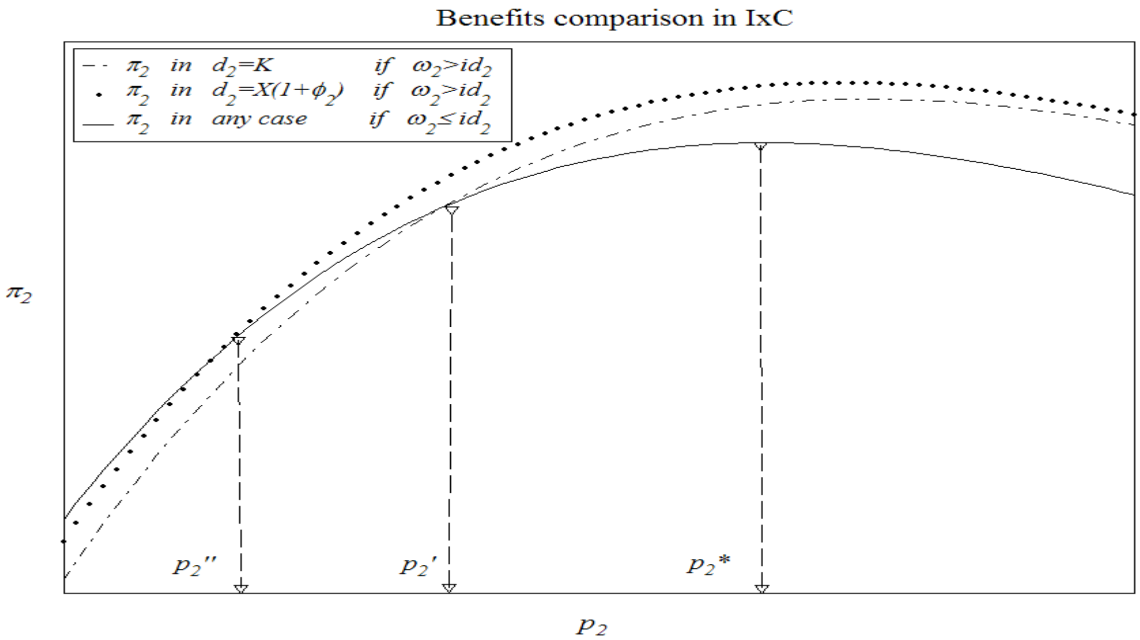


Figure 1: Passive condition (I). Direct and indirect effects.

As it is show in **Fehler! Verweisquelle konnte nicht gefunden werden.**, if the operator is sufficiently small itcould occur that the possibility of internalizing the network dimension does not have any effect in the equilibrium prices. The operator maximizes profits at p_2^*, that is, his total network dimension is higher than the received traffic demand[31], i.e., ω_2✧id_2. An aggressive behavior (in prices) does not bring any marginal profit. The entrant has excess capacity in equilibrium.

In **Fehler! Verweisquelle konnte nicht gefunden werden.**, the operator chooses p_2^* so that, at the end, his network dimension is higher than the traffic demanded ($\omega_2<\theta d_2$). However, if the operator had had a fixed (and equal to K) dimension would have schosen p_2◷ (where p_2◷ $>p_2^*$

[31] This happens because the entrant has not reached a critical mass. Then the cause is that ω_2 is small and not that d_2 is great.

and π_2◷ < π_2^*). Note that the operator does not take full advantage of the opportunity that the dimensioning provides him. In any case, the possibility to increase d_2 gives the entrant incentives to a more aggressive price behavior in the last stage and he ends up contracting excesss capacity.

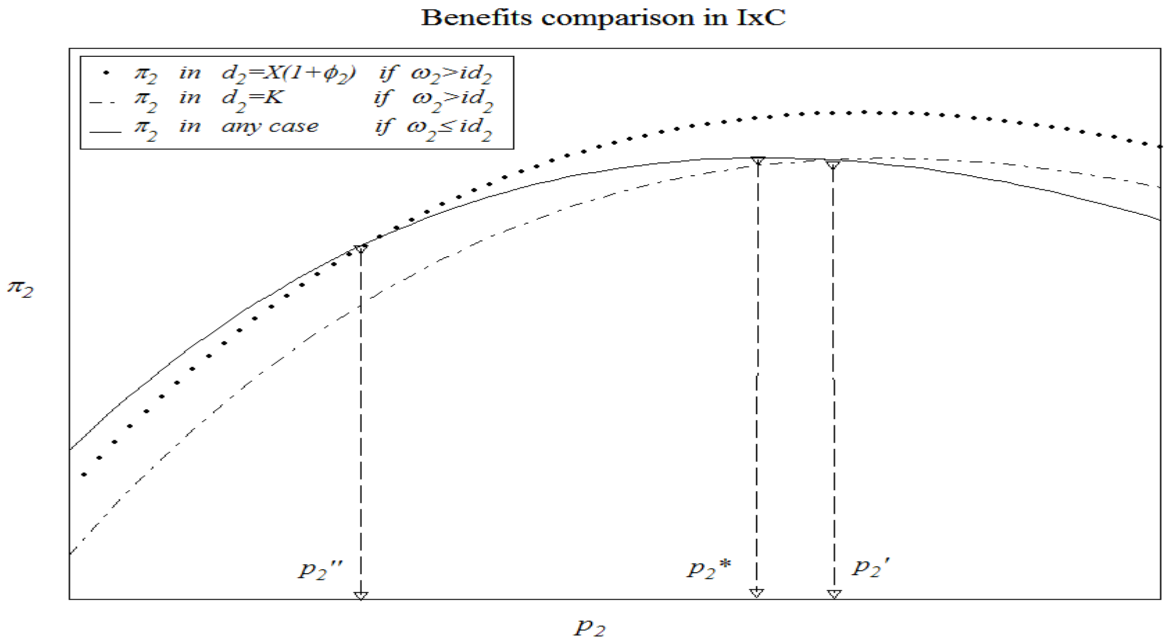


Figure 2: Active condition (partially). Direct effect.

As **Fehler! Verweisquelle konnte nicht gefunden werden.** shown, the operator increases his income using the maximum of his size network possibilities[32] (he has incentives to increase the variety of his services supply). Besides, the consumers enjoy the decrease in p_2^* that change from p_2◷ to p_2◷◷. It is interesting to observe that the threshold agrees with p_2^*. The entrant equals his capacity to his demand.

Fehler! Verweisquelle konnte nicht gefunden werden. represent the situation in which to use the dimension network at the maximum level is the entrant best choice. Nevertheless, it is not clear if this means an increase or a decrease in p_2 with respect to the situation where $d_2=K$, that will depend on the relation between the maximums (in this case implies a decrease). As it is observed, the thresholds are irrelevant in the determination of p_2^*. Anyway, in **Fehler! Verweisquelle konnte nicht gefunden werden.** the equilibrium price implies that the operator will require added interconnection.

[32] Is obvious that the new situation must be better than that that he leaves (π_2◷ < π_2◷◷). Otherwise, he did not do the change (since complete information exists). Then, the network dimension give to the entrant a maximum level but if he believe that is not recommendable exploit it, he could fix smaller values (being more passive as far as its supply of services for example). According to the values of the parameters, this is not the case.

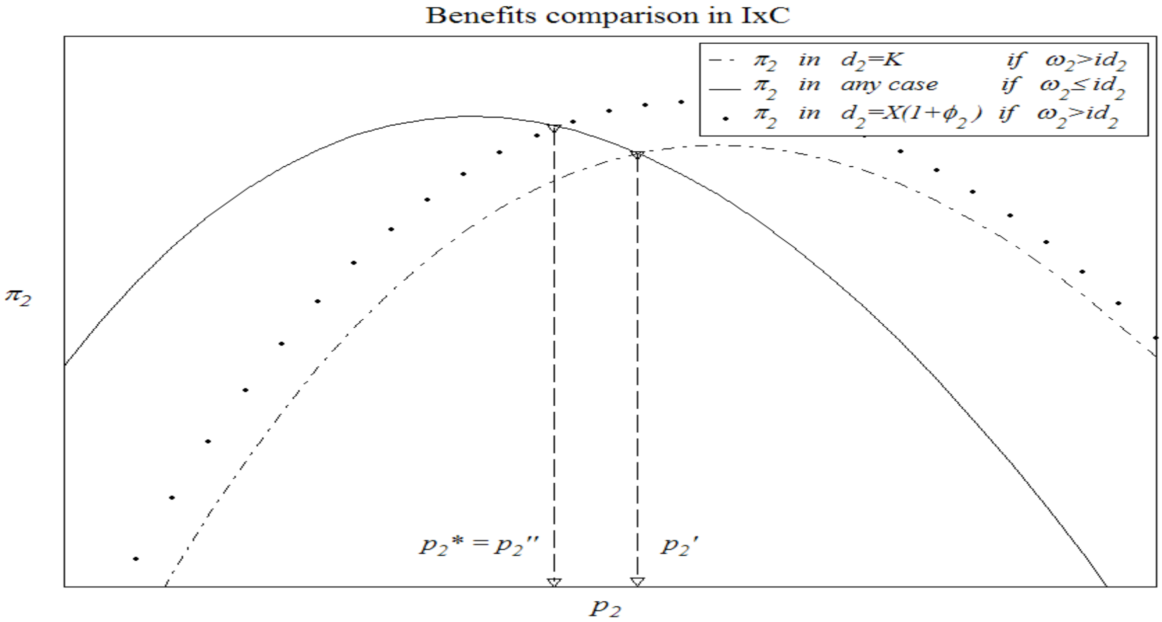


Figure 3: Active condition (totally). Direct effect.

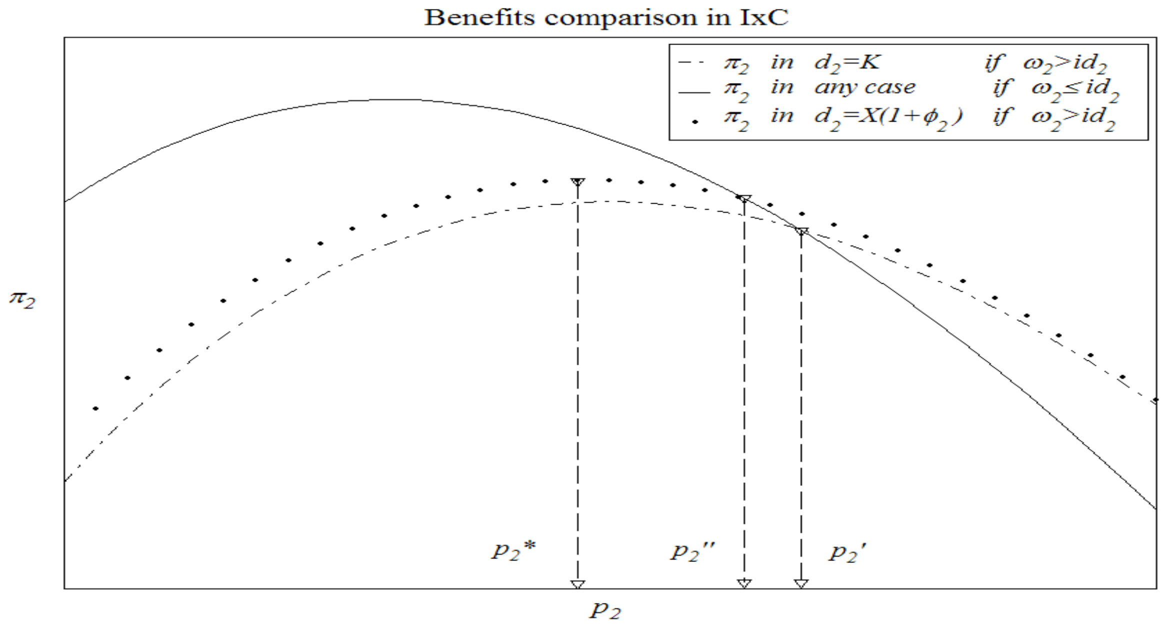


Figure 4: Passive condition (II). Direct effect.

The previous figures only include the direct effects generated by the possibility of network dimension[33]. To completely understand the competition process is necessary to take into account

[33] Although changing of a situation "a" to a situation "b" implies a transformation in the entrant benefit functions and therefore in his best respond function, can occur the case that the changes in the reaction occur to values sufficiently

the strategic effects triggered by the rival (indirect effects) as illustrated in **Fehler! Verweisquelle konnte nicht gefunden werden.** (that is the same situation that it is show in **Fehler! Verweisquelle konnte nicht gefunden werden.** but including all the effects) and **Fehler! Verweisquelle konnte nicht gefunden werden.**.

It is useful to suppose that we started in an equilibrium situation under $d_2=K$. But now the entrant is able to determine his network dimension in such a way that his benefit functions ends up moving. Evidently, also does his reaction function, changing from R_2🕔(p_1) to $R_2^o(p_1)$ (note that the reaction functions correspond to those of strategic substitutes operators in **Fehler! Verweisquelle konnte nicht gefunden werden.**). In this new situation the entrant fix p_2^o, where p_2🕔🕔< p_2^o<p_2🕔, with which he increase his benefits. At this price, the incumbent's best response is p_1^o that fulfills p_1^o<p_1🕔.

The increase in the aggressiveness will cause a general decrease in the entrant benefits function when $\omega_2>id_2$ (for an ample parameter rank). This is because he does not obtain the awaited market share but also because he has incurred *ex ante* in a fixed investment, reason why he is himself forced to make another marginal reduction in p_2 ("to recover part of the pie") to which the incumbent respond with a marginal decrease in p_1. In a final stage (the process finishes when the curves are totally adjusted, or what is the same, when no one operator wish to change his decisions given the decision of the rival) the entrant fix his price at p_2🕔🕔 and the incumbent responds with p_1🕔🕔.[34]

remote to the equilibrium (then, remaining constant). This is the case shown in **Fehler! Verweisquelle konnte nicht gefunden werden.**.

[34] In order to illustrate the competition process it has been tried to separate the effects in time and in its influence. Nevertheless, that happens simultaneously and affects the aggregate of functions.

Benefits comparison in IxC

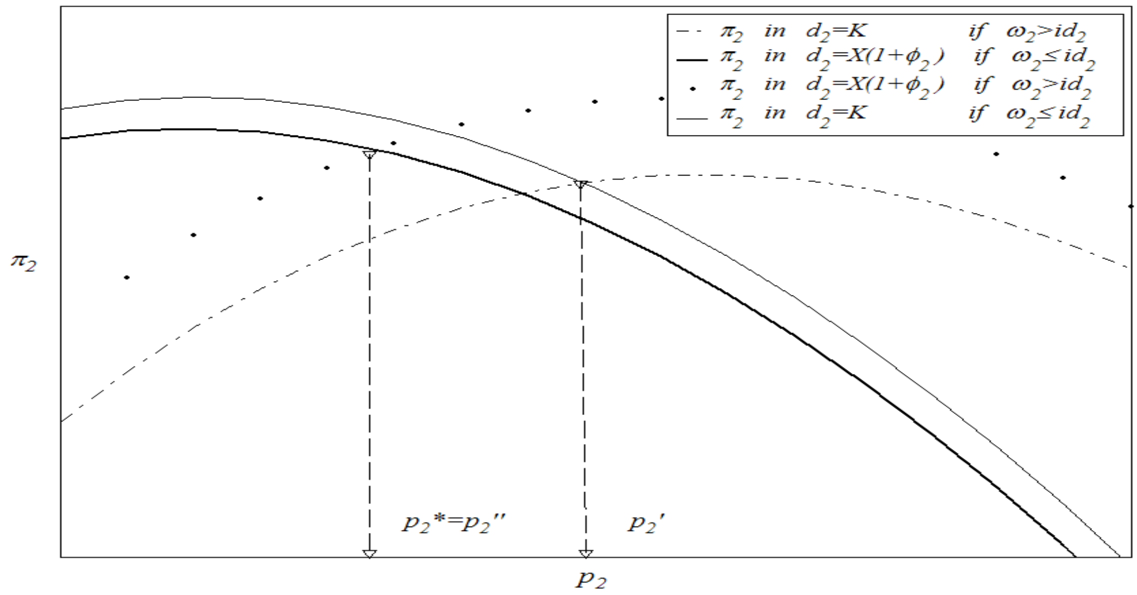


Figure 5: Active condition (totally). Direct and indirect effects.

Reaction functions in IxC

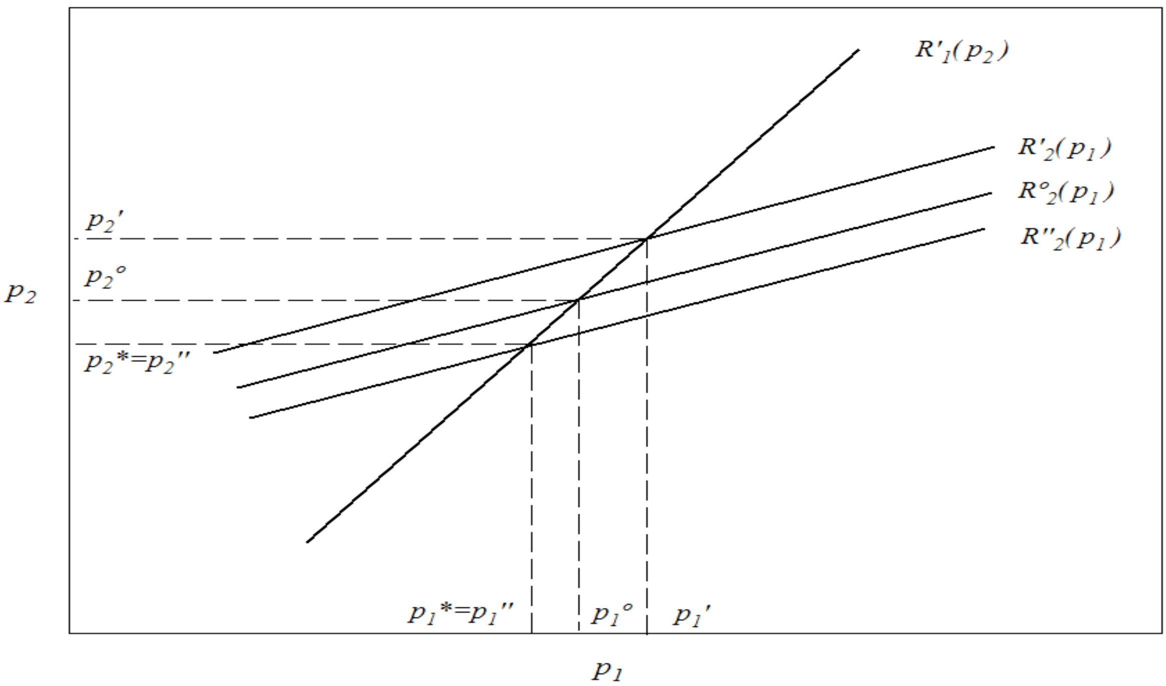


Figure 6: The competition process.

Appendix 2

Using**Fehler! Verweisquelle konnte nicht gefunden werden.**, and the fact that the entrant's market share fluctuates according to $0 \le \varphi_2 \le .6$, then the block dimension fluctuates according to $Z \le d_2 \le Z + .6nY$.

	Technical (maximum)	**Observed and suggested by the regulator (CMT)**	**Used in the paper**
d_2	[r, 24r]	[5r, 10r]	[7.12r, 10.68r]
r/d_2	[1, .042]	[.2, .1]	[.1404, .0936]

Block dimension limits.

The complete rank is very ample and is not observed in reality. In fact the CMT has indicated that firms who handle business voice or residential Internet have approximately r/d_2=.18 whereas more generalist operators can even reach r/d_2=.10. The incumbent can even be below this last value. The parameters Z and Y have been determined taking into account that $\phi_2 \in$[0, .6] and r/d_2=[.2, .1] as the relevant (feasible) ranges. Nevertheless as a result we obtain that K is very near to the superior limit of the rank, so it would be necessary a significant entrant's market share to reach it, so we arbitrarily have settled r/d_2= [.1404, .0936].

Next, we show that " choosing IxT dominates choosing IxC" by the entrant.

Independently of the tariff scheme used, taking the incumbent's prices as given, in the presence of both interconnection models, with interconnections prices proportionally equivalents, and without the possibility of modifying the productivity of the contracted capacity (d_2=K), "to choose IxT" dominates " to choose IxC".

Given the incumbent's prices we can rewrite equations

$$\pi_2^T(p_2, m_2) = \omega_2(p_2 - \alpha) + \mu_2(m_2 - f_2),$$

$$\pi_2^C(p_2, m_2 / \omega_2 > iK) = iKp_2 + (\omega_2 - id_2)(p_2 - \alpha) - iP + \mu_2(m_2 - f_2),$$

where exceptionally superscripts T and C have been used to distinguish between IxT and IxC respectively. Expanding the second of those expressions, grouping and remembering that $P \ge \alpha K$, is easy to arrive at

$$\pi_2^C(p_2, m_2 / \omega_2 > iK) = \omega_2(p_2 - \alpha) + \mu_2(m_2 - f_2),$$

thus,

$$\pi_2^T(p_2, m_2) \equiv \pi_2^C(p_2, m_2 / \omega_2 > iK).$$

Note that i=θ for the expressions that follow. That means that the entrant's profits when choosing *IxT* are the same ones he obtains when choosing *IxC* given $\omega_2 > id_2$. Then, if the option "choosing *IxC*" were attractive to him, that would have to happen when ω_2✧id_2, receiving

$$\pi_2^C(p_2, m_2 / \omega_2 \leq iK) = \omega_2 p_2 - iP + \mu_2(m_2 - f_2),$$

but using $\alpha = \frac{P}{K}$ the previous expression can be rewritten it as

$$\pi_2^C(p_2, m_2 / \omega_2 \leq iK) = \omega_2 p_2 - i\alpha K + \mu_2(m_2 - f_2),$$

that is, he has total costs of $i\alpha K$, nevertheless, when contracting *IxT* the (variable) costs are $\alpha\omega_2$ (assuming in both cases that f_2=0). Taking into account that this part of the analysis is valid when ω_2✧iK, there is no doubt that the expenses when contracting capacity are higher than those when contracting time, and only in the margin (when $\omega_2 = iK$) are equivalent. In other words, adding earlier results

$$\pi_2^T \equiv \pi_2^C(p_2, m_2 / \omega_2 > iK)$$

$$\pi_2^T \geq \pi_2^C(p_2, m_2 / \omega_2 \leq iK)$$

This result is independent of the way of modeling the incumbent's profit function.